Delivering an online course using STACK

George Kinnear[1]

Abstract: This paper reports on recent work to develop an online course in introductory university mathematics. The course is delivered entirely as a series of quizzes, which interleave textbook-style written content with STACK questions and other interactive elements. The paper describes how ideas from education research have informed the design of the course, with a particular focus on how individual tasks have been designed using STACK.

Keywords: Online learning; course design; task design

1 Introduction

Each year, the University of Edinburgh has around 600 students studying courses from year 1 of the mathematics program; this includes a course in linear algebra in the first semester [Sa18] and a course in calculus in the second. The students come from a diverse range of backgrounds, and have a wide range of prior mathematics study and attainment. To better address the needs of this diverse group, an additional course was introduced to the first semester in the 2018/19 academic year: *Fundamentals of Algebra and Calculus* (FAC). The aim of this course is to cover similar topics to advanced high school mathematics syllabuses, such as A-Level Further Mathematics or SQA Advanced Higher Mathematics [Sc16]. Some students in the cohort have not had the opportunity to take these advanced qualifications at school, and others would simply benefit from further practice. Students are advised to take FAC based on their entry grades and their performance in a short diagnostic test [Ki18]. In 2018/19 there were 113 students signed up to take the course for credit.

A novel feature of the course is that it is delivered almost entirely online, interleaving textbook-style exposition with videos of worked examples, interactive applets, and practice questions implemented in STACK. In the next section, we describe the course and its overall design in more detail. In section 3 we give details of three ways in which ideas from education research are put into practice in the course, making use of the features of STACK.

2 About the course

The topics in the course are based on the content of typical high school syllabuses, with a focus on calculus methods and supporting algebraic work. The differentiation content

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[1] University of Edinburgh, School of Mathematics G.Kinnear@ed.ac.uk

includes chain, product and quotient rules, as well as implicit, parametric and logarithmic differentiation. The integration content includes a basic appreciation of the definition in terms of Riemann sums, but mainly focuses on techniques such as substitution, integration by parts and integrating rational functions using partial fractions.

2.1   Course design

The course is 20 credits out of 120 in the year; this is equivalent to 10 ECTS credits, and notionally 200 hours of work for the student. Since the semester consists of 11 weeks, we have created 10 units which are studied in the first 10 weeks, with a final assessment taking place in week 11.

Each unit has a consistent structure, with

- a "Getting Started" section, which motivates the week's topic and reviews pre-requisite content (e.g. differentiation facts when starting integration),

- four sections of content, each of which is designed to take around 2 hours to complete (roughly equivalent to one lecture plus associated practice),

- a 90-minute "Practice Quiz" with a mix of questions on the week; this can be taken an unlimited number of times, with full feedback provided on each attempt, and

- a 90-minute "Final Test" which is similar in style to the Practice Quiz, but only allows a single attempt.

A student's grade on the course is determined by combining the results of the 10 weekly Final Tests (together worth 80% of the grade) with a final 2-hour test covering topics from the whole course (worth the remaining 20%).

3   Task design

In this section we give three examples of how education research has influenced the design of tasks within the course. In each case, the features of STACK are used to put the ideas into practice.

3.1   Faded worked examples

The *worked example effect* is a finding from cognitive science which suggests that learners benefit more from studying worked examples than from unguided problem solving [KSC06, p80]. In traditional teaching, it is quite common to give "example-problem pairs" where

Find the Maclaurin series of $f(x) = \sin(x)$.

We compute the derivatives and evaluate them at $x = 0$:

$$\begin{aligned}
f(x) &= \sin(x) & f(0) &= 0 \\
f'(x) &= \cos(x) & f'(0) &= 1 \\
f''(x) &= -\sin(x) & f''(0) &= 0 \\
f^{(3)}(x) &= -\cos(x) & f^{(3)}(0) &= -1 \\
f^{(4)}(x) &= \sin(x) & f^{(4)}(0) &= 0
\end{aligned}$$

and from here we see that the cycle of values $0, 1, 0, -1$ will repeat.

So the Maclaurin series begins:

[_____] (enter the first four nonzero terms)

The general term is

○ **(a)** $\dfrac{(-1)^n \, x^{2n+1}}{(2n+1)!}$

○ **(b)** $\dfrac{(-1)^{2n+1} \, x^{2n+1}}{(2n+1)!}$

○ **(c)** $\dfrac{(-1)^n \, x^n}{n!}$

○ **(d)** $\dfrac{(-1)^n \, x^{2n}}{(2n)!}$

Find the Maclaurin series of $f(x) = \cos(x)$.

We compute the derivatives and evaluate them at $x = 0$:

$$\begin{aligned}
f(x) &= \cos(x) & f(0) &= 1 \\
f'(x) &= -\sin(x) & f'(0) &= 0 \\
f''(x) &= -\cos(x) & f''(0) &= -1 \\
f^{(3)}(x) &= \sin(x) & f^{(3)}(0) &= 0 \\
f^{(4)}(x) &= \cos(x) & f^{(4)}(0) &= 1
\end{aligned}$$

and from here we see that the cycle of values $1, 0, -1, 0$ will repeat.

So the Maclaurin series begins:

[_____] (enter the first four nonzero terms)

The general term is [_____]

Note: for the last question, the general term would be typed as `(-1)^n*x^(2*n+1)/(2*n+1)!`

(a) Worked example with the final step as a question. (b) Next, the multiple-choice final answer is replaced with a constructed response input.

Find the Maclaurin series of $f(x) = \ln(x + 1)$.

We compute the derivatives and evaluate them at $x = 0$:

$f(x) = \ln(x + 1)$ $f(0) = 0$

$f'(x) =$ [_____] $f'(0) =$ [_____]

$f''(x) =$ [_____] $f''(0) =$ [_____]

$f'''(x) =$ [_____] $f'''(0) =$ [_____]

So the Maclaurin series begins:

[_____] (enter the first four nonzero terms)

The general term is [_____]

Find the Maclaurin series of $f(x) = e^{-4x}$.

(a) The first five nonzero terms are:

[_____]

(b) The general term is:

[_____]

(c) The fully scaffolded version of the question. (d) The final question in the sequence, with all scaffolding removed.

Fig. 1: A sequence of faded worked examples of calculating Maclaurin series.

the teacher presents a worked example then the learner solves a similar problem. Building on this, the idea of a *faded worked example* is to present a sequence of problems with varying amounts of the solution already worked out. For instance, the sequence may start with a worked example, followed by an almost-complete worked example with the last step missing, then conclude with an unguided problem; this pattern of fading the worked-out steps from the end has been found to be most favourable for learning [Re02].

Faded worked examples are used in numerous places in FAC. For instance, when introducing the procedure for computing terms of Maclaurin series, we start with a worked example

then follow up with a sequence of STACK questions which are faded worked examples, as shown in Figure 1.

In practice, such sequences do take a bit more effort to produce than simply copying the same question a few times to give an equivalent amount of practice. However, the extra effort is not so great since it mostly amounts to moving content between the question text and the worked solution. For instance, one good way to author these questions is to follow the same sequence as the students:

1. Write a worked example.

2. Starting with the worked example, construct a question with the final step replaced by a STACK input, and move that line of working to the general feedback.

3. Starting with a clone of that question, repeat the process of replacing a step in the working with a STACK input. This step should be repeated until you produce a question which is fully scaffolded.

4. Produce a "pure problem" version of the question, with no scaffolding. This could be done starting from scratch, or by starting with a clone of the most scaffolded version of the question and removing most of the inputs.

3.2 Retrieval practice

The *testing effect* is a consistent finding from cognitive science, showing that "retrieval of information from memory produces better retention than restudying the same information for an equivalent amount of time" [RB11, p20]. The testing effect has been seen in numerous lab studies [RK06], and also in classroom settings. This has given rise to the notion of "test-enhanced learning", where tests are used as a tool to help students to learn during a course [BB15]. For example, one study with a precalculus course for engineering students compared the effect of distributing questions on a topic across multiple quizzes rather than having them all in the quiz associated to the topic [Ho16]. Spacing out the questions in this way required students to engage in more effortful retrieval of the learned solution procedure from memory, and the study found that it resulted in students performing better on the exam and in a subsequent course.

The course design of FAC builds in numerous opportunities for retrieval practice. In a similar way to the precalculus course in [Ho16], assessments later in the course draw on skills from previous units. This often arises in a natural way; for instance, finding areas between curves using integration naturally builds in recall practice of the procedure for finding intersections of curves.

Another simple example of retrieval practice in the course is at the start of the unit on integration, where there is an opportunity to recall facts from the previous unit on

differentiation. While this could be done by presenting students with a table of the standard derivatives, we instead give a STACK question which asks the students to recall this table (see Figure 2).

Complete the following table of standard derivatives. Try to do this without checking your notes -- which of the most important standard derivatives can you remember?

Function	Derivative
x^n	

Check

Fig. 2: A STACK question which encourages retrieval practice of differentiation facts.

This question makes use of STACK's ability to assess the properties of the student's answer. In particular, we check that each row of the table is correct (with the right-hand entry being the derivative of the left-hand entry) without the student needing to complete the rows in any particular order. We also check that overall the set of functions given in the left-hand column is the same set that we were expecting.

3.3 Learner-generated examples

A "concept image" [TV81] is a cognitive structure associated with a mathematical concept, which goes beyond just the formal definition. Part of this concept image is the "example space" [GM08], and developing a rich example space is widely believed to be an important part of understanding a concept. Previous studies have concluded that "it may be beneficial to introduce students to new concepts by requiring them to generate their own examples" [DH96, p297]. Despite this, asking students to generate examples has been categorised among a larger set of "higher level mathematical skills" which are relatively under-represented in existing assessment practices [Sa03]. Fortunately, there are cases where STACK is ideally placed to assess learner-generated examples.

One example comes from the section where arithmetic sequences are introduced. Students have also recently worked with the definitions of increasing/decreasing sequences, and

In each case below, give an example of an arithmetic sequence with the stated property, by entering an expression for the general term. If it is not possible, enter `none` .

(a) Increasing

$u_n =$ [   ]

(b) Decreasing

$u_n =$ [   ]

(c) Bounded above

$u_n =$ [   ]

(d) Decreasing and bounded below

$u_n =$ [   ]

[Check]

Fig. 3: A question assessing learner-generated examples of arithmetic sequences with given properties.

sequences which are bounded above or below. The question in Figure 3 brings these ideas together, and asks for examples of arithmetic sequences with given properties. To assess the student responses, we extract from each input the coefficient of n (which will be the common difference d) and the constant term, then check the appropriate properties (e.g. that $d > 0$ for part (a)).

A further example, shown in Figure 4, asks students to give examples of quadratics with given numbers of intersections with $y = q$, where in this case $q = x^2$. Alongside this question, students can use a GeoGebra applet to explore the effect of changing the coefficients of $ax^2 + bx + c$, so they can experiment and test out their examples.

To assess the student's response (sa) in each of the first three entries, we check that:

1. sa is not `none`,

2. sa is a quadratic (i.e. has degree 2 and is equal to its degree 2 Taylor polynomial),

3. sa is not q, and

4. the quadratic equation $q = sa$ has the correct number of roots (since Maxima's `solve` command returns complex roots, we need to do this by analysing the discriminant of $q - sa$).

For each case below, type in a quadratic, e.g. `2x^2+3x+1`, whose graph has exactly the given number of intersections with $y = x^2$. If it is not possible, then enter `none`.

No intersection: $y =$ [   ]

1 intersection: $y =$ [   ]

2 intersections: $y =$ [   ]

3 intersections: $y =$ [   ]

[Check]

Fig. 4: A question assessing learner-generated examples of quadratics.

4 Conclusions

Developing the course was a significant effort, requiring around 100 STACK questions to be written for each week in addition to the textbook-type content. However, early indications are that the course has been successful, with good student evaluations and a strong overall performance on the final assessment (the mean score is around 80% and only two students failed to reach the 40% threshold required to pass). Further evaluation work will be conducted once exam results are available from the other year 1 mathematics courses. Now that the course is in place, it will also provide a platform to test different designs and investigate the effectiveness of the various approaches described above.

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