Overview of Kaon physics

Kei Yamamoto
(University of Zurich/Hiroshima University)

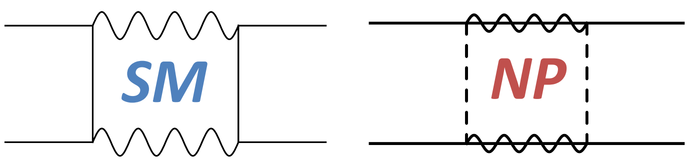
CKM2018 (Heidelberg)
17th Sep 2018

Why Kaon?

- Kaon observables are sensitive to NP at a very high scale, which is not accessible at the LHC

  - FCNC and CP violation in Kaon system are suppressed in the SM

c.f. meson mixing



$$\mathcal{L}_{eff} = \mathcal{L}^{SM} + \frac{1}{\Lambda_{NP}^2} \sum_i C_i \mathcal{O}_i^{\text{dim6}}$$

If $|C_{\text{NP}}| \sim 1$

$$\Lambda_{\text{NP}} \sim \begin{cases} \mathcal{O}(10^5\,\text{TeV}) & : K^0 \\ \mathcal{O}(10^4\,\text{TeV}) & : D^0 \\ \mathcal{O}(10^3\,\text{TeV}) & : B_{d,s} \end{cases}$$

- Several on-going experiments for Kaon observables (KOTO/NA62/LHCb + KLOE-2/TREK…)

- Using recent result of lattice calculation, there is discrepancy in ε’/ε between SM value and data (discuss in detail later)

Outline

- ϵ
- ϵ'/ϵ
- $K_L \to \pi^0 \nu\bar{\nu}$

  $K^+ \to \pi^+ \nu\bar{\nu}$
- $K_S \to \mu\mu$

ε and ε’

1964 $K_L \to 2\pi$ was observed *Discovery of CP violation*

CP odd CP even

$$|K_L\rangle = |K_2\rangle + \epsilon|K_1\rangle$$

Indirect CPV (mixing) ϵ

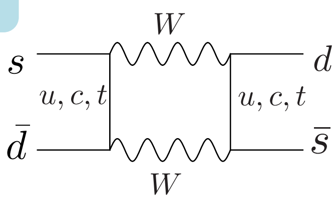
Direct CPV (decay) ϵ'

CP even $|2\pi\rangle$

$$\eta_{00} \equiv \frac{A(K_L \to \pi^0\pi^0)}{A(K_S \to \pi^0\pi^0)} \equiv \epsilon - 2\epsilon' \qquad \eta_{+-} \equiv \frac{A(K_L \to \pi^+\pi^-)}{A(K_S \to \pi^+\pi^-)} \equiv \epsilon + \epsilon'$$

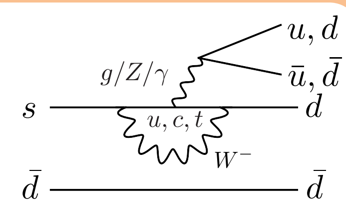
Indirect CPV (mixing) ϵ

$$|\epsilon| \simeq \frac{1}{3}\left(|\eta_{00}| + 2|\eta_{+-}|\right)$$



Direct CPV (decay) ϵ'

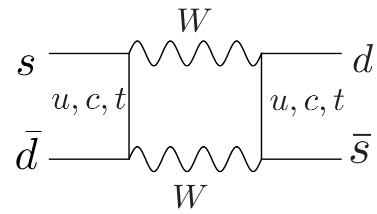
$$\left|\frac{\eta_{00}}{\eta_{+-}}\right|^2 \simeq 1 - 6\mathrm{Re}\left(\frac{\epsilon'}{\epsilon}\right)$$



$$\epsilon = \mathcal{O}(10^{-3}) \quad \mathrm{Re}\left(\frac{\epsilon'}{\epsilon}\right) = \mathcal{O}(10^{-3}) \quad \epsilon' = \mathcal{O}(10^{-6})$$

Highly suppressed and sensitive to NP

ε



Indirect CP violation ε gives severe constraint on NP

SM prediction of ε is sensitive to |Vcb|

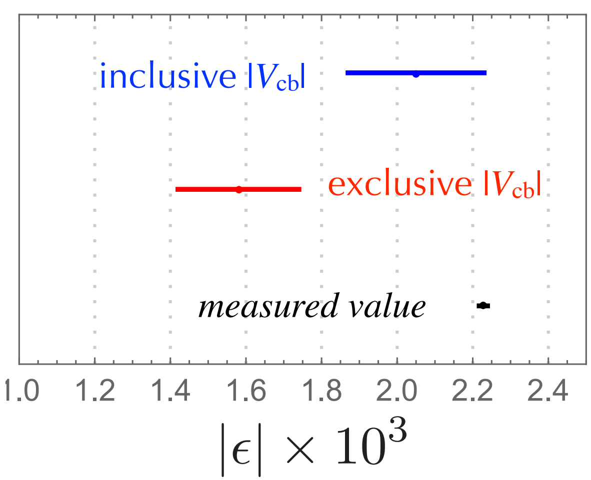
$\lambda_i = V_{is}^* V_{id}, \;\; S_0$: Inami-Lim function

η_i : QCD correction   NNLO : Brod and Gorbahn 1108.2036

$$\epsilon(SD) \propto \mathrm{Im}\lambda_t \Big[\mathrm{Re}\lambda_c \eta_{cc} S_0(x_c) - \mathrm{Re}\lambda_t \eta_{tt} S_0(x_t) - (\mathrm{Re}\lambda_c - \mathrm{Re}\lambda_t)\eta_{ct} S_0(x_c, x_t) \Big]$$
$$\simeq |V_{cb}|^2 \lambda^2 \bar{\eta} \Big[|V_{cb}|^2 (1-\bar{\rho}) \eta_{tt} S_0(x_t) + \eta_{ct} S_0(x_c, x_t) - \eta_{cc} S_0(x_c) \Big]$$

ε evaluated from inclusive |*V*cb| is consistent with the measured value
On the other hand there is 4σ tension with exclusive |*V*cb| (still tension in averaged)

See e.g. Bailey, Lee, Lee, Leem 1808.09657

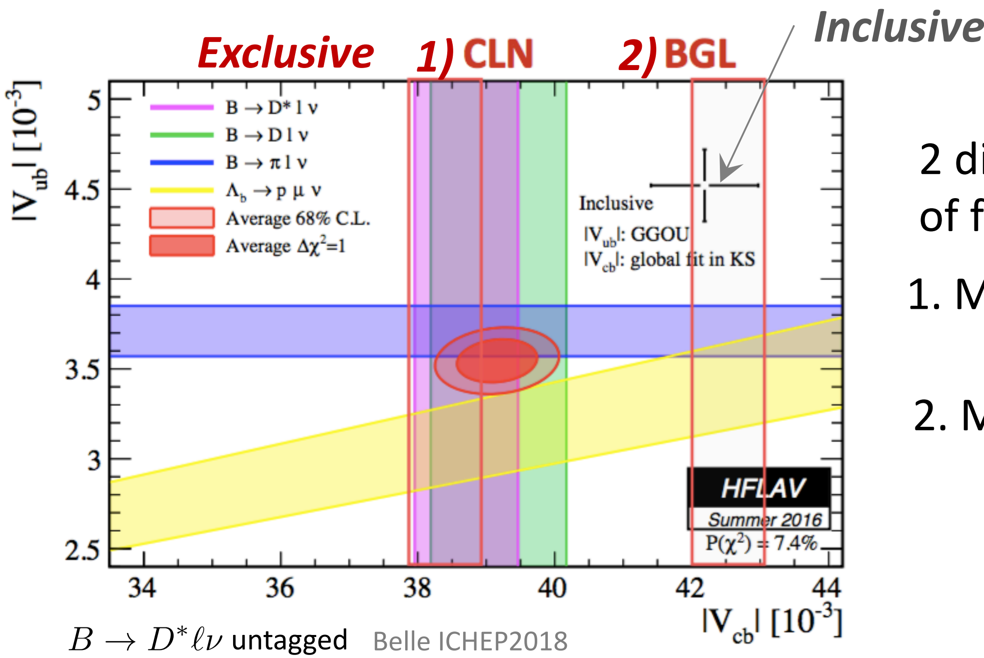


Vcb exclusive vs. inclusive problem

$$|\epsilon|_{\mathrm{exp}} = 2.228(11) \times 10^{-3}$$

$\mathcal{O}(10\%)$ **NP room on ε is still allowed**

ε: Vcb exclusive vs. inclusive



$B \to D^* \ell \nu$ untagged   Belle ICHEP2018

2 different methods for functional form of form factors:

1. Model-dependent method : ***CLN***
   Caprini, Lellouch, and Neubert (CLN) hep-ph/9712417
2. Model-independent method : ***BGL***
   Boyd, Grinstein, and Lebed (BGL) hep-ph/9705252

Recent discussions on exclusive Vcb :

The gap might be explained in part with ***BGL*** method   Bigi, Gambino and Schacht 1703.06124/1707.09509; Grinstein and Kobach 1703.08170

Large deviation from heavy quark symmetry ?   Bernlochner, Ligeti, Papucci and Robinson 1708.07134

The situation of exclusive Vcb is still unclear   *See talk WG2*

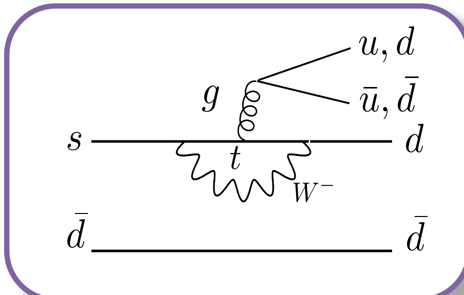
ε'/ε

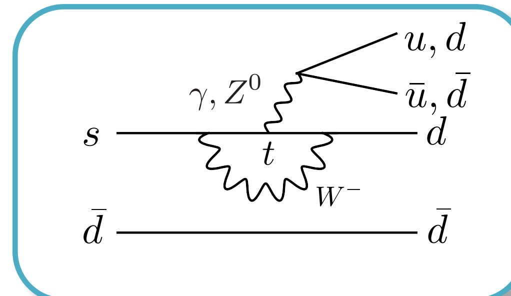
$$A(K^0 \to (\pi\pi)_{I=0,2}) = A_{0,2} e^{i\delta_{0,2}}$$

$$\frac{\epsilon'}{\epsilon} = -\frac{\omega}{\sqrt{2}\,|\epsilon|_{\rm exp}\,{\rm Re}A_0}\left({\rm Im}A_0 - \frac{1}{\omega}{\rm Im}A_2\right)$$

QCD penguin operator

EW penguin operator





ΔI=1/2 rule

$$\frac{{\rm Re}A_0}{{\rm Re}A_2} \equiv \frac{1}{\omega} = 22.46 \quad \text{(exp.)}$$

In the SM, there is accidental cancellation between $\mathrm{Im}A_0$ and $\mathrm{Im}A_2$ due to the enhancement factor $1/\omega$

EW penguin is comparable to QCD penguin due to the enhancement factor

ε'/ε discrepancy

Decay amplitude

Short distance / Matrix element

$$\langle(\pi\pi)_I|\mathcal{H}|K^0\rangle = \sum_n C_n \langle(\pi\pi)_I|\mathcal{O}_n|K^0\rangle$$

- Short distance
  - NLO result has been available since early 90's
  - NNLO QCD calculation is in progress *Cerda-Sevilla, Gorbahn, Jager, Kokulu 1611.08276*
- Long distance (Matrix elements)
  - First lattice result by **RBC-UKQCD** in 2015 *1502.00263 1505.07863*

From the lattice result, ε'/ε has been calculated in SM using data for Re$A_{0,2}$

SM with Lattice

$$\left(\frac{\epsilon'}{\epsilon}\right)_{\rm SM} = (1.06 \pm 5.07) \times 10^{-4}$$

Kitahara, Nierste and Tremper, 1607.06727

c.f. RBC-UKQCD / Buras, Gorbahn, Jager and Jamin 1507.06345

Exp

$$\left(\frac{\epsilon'}{\epsilon}\right)_{\rm exp} = (16.6 \pm 2.3) \times 10^{-4}$$

average of NA48 and KTeV

2.8σ difference $\mathcal{O}(1)$ **NP in ε'/ε ?**

ε'/ε discrepancy

SM with Lattice

$$\left(\frac{\epsilon'}{\epsilon}\right)_{\rm SM} = (1.06 \pm 5.07) \times 10^{-4}$$

2.8σ difference

Exp

$$\left(\frac{\epsilon'}{\epsilon}\right)_{\rm exp} = (16.6 \pm 2.3) \times 10^{-4}$$

- O_6 & O_8 have dominant effects on ε'/ε due to chiral enhancement

$$\langle(\pi\pi)_0|\mathcal{O}_6|K\rangle \propto B_6^{(1/2)}$$
$$\langle(\pi\pi)_2|\mathcal{O}_8|K\rangle \propto B_8^{(3/2)}$$

Non-perturbative parameter

QCD penguin $O_6 = (\bar{s}_\alpha d_\beta)_{V-A} \sum_q (\bar{q}_\beta q_\alpha)_{V+A}$

EW penguin $O_8 = \frac{3}{2}(\bar{s}_\alpha d_\beta)_{V-A} \sum_q e_q (\bar{q}_\beta q_\alpha)_{V+A}$

- Values extracted from the lattice result

Buras, Buttazzo, Girrbach-Noe and Knegjens 1503.02693

$$B_6^{(1/2)} = 0.57 \pm 0.19 \qquad B_8^{(3/2)} = 0.76 \pm 0.05$$

- Error for ε'/ε is dominated by $B_6^{(1/2)}$

- Two ways of analytic approaches

Large N_C **Dual QCD** approach

Buras and Gérard 1507.06326

$$B_6^{(1/2)} \leq B_8^{(3/2)} < 1 \qquad \left(\frac{\epsilon'}{\epsilon}\right)_{\rm SM} < (6.0 \pm 2.4) \times 10^{-4}$$

1805.11096

ChPT (FSI)

Gisbert and Pich 1712.06147
hep-ph/0007208

$$B_6^{(1/2)} \sim 1.5 \qquad B_8^{(3/2)} \sim 0.9 \qquad \left(\frac{\epsilon'}{\epsilon}\right)_{\rm SM} = (15 \pm 7) \times 10^{-4}$$

Result in DQCD approach gives a strong support to lattice result. On the other hand, result in ChPT is consistent with data

See J.Aebischer talk (WG3, Wed)

- Wait for improved lattice results *See C. Kelly talk (next)*

Interpretation of ε'/ε discrepancy

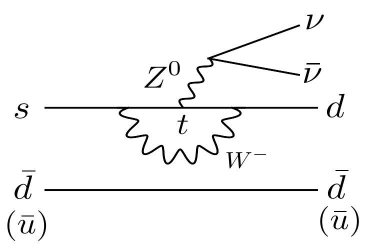
- Motivated by ε'/ε discrepancy, several new physics models have been studied

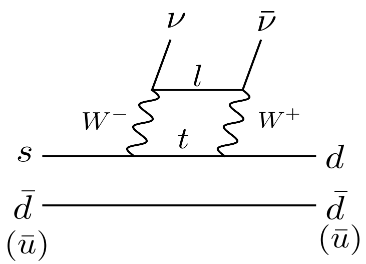
Little Higgs Model with T-parity	*Blanke, Buras and Recksiegel 1507.06316*
Modified Z scenario	*Buras, Buttazzo and Knegjens1507.08672/Buras, 1601.00005*
	Endo, Kitahara, Mishima and KY 1612.08839/Bobeth, Buras, Celis and Jung 1703.04753
Z' models	*Buras, Buttazzo, Knegjens 1507.08672 /Buras 1601.00005*
331 model	*Buras and De Fazio 1512.02869/1604.02344*
MSSM Chargino Z penguin	*Endo, Mishima, Ueda and KY 1608.01444*
Gluino Z penguin	*Tanimoto and KY 1603.07960*
	Endo, Goto, Kitahara, Mishima, Ueda and KY 1712.04959
Gluino Box	*Kitahara, Nierste and Tremper 1604.07400,1703.05786*
	Crivellin, D'Ambrosio, Kitahara, Nierste 1712.04959
	Chobanova, D'Ambrosio,Kitahara, Martínez, Santos, Fernández and KY 1711.11030
Vector-like quarks	*Bobeth, Buras, Celis and Jung 1609.04783*
Right handed current	*Cirigliano, Dekens, Vries and Mereghetti 1612.03914*
	Alioli, Cirigliano, Dekens, de Vries and Mereghetti 1703.04751
Leptoquark	*Bobeth and Buras 1712.01295*
LR symmetric model	*Haba,Umeeda and Yamada 1802.09903/1806.0342*
Type-III 2HDM	*Chen and Nomura 1805.07522/1808.04097*
Chiral-flavorful vectors	*Matsuzaki, Nishiwaki and KY 1806.02312*
Diquark model	*Chen and Nomura 1808.04097*

- Different implications (correlations & predictions) for other observables appear depending on models $\Rightarrow$ Possibility of model discriminations

Clean signal : **K→ πνν**

$K_L \to \pi^0 \nu\bar{\nu}$ and $K^+ \to \pi^+ \nu\bar{\nu}$





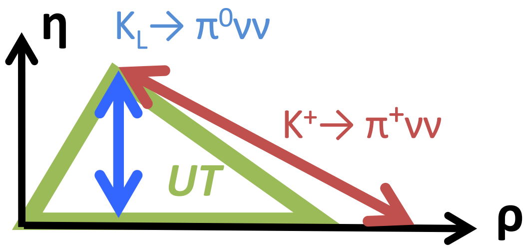
- Highly suppressed in the SM : BR_{SM}~10^{-11} *Buras, Buttazzo, Girrbach-Noe and Knegjens 1503.02693*

$$BR(K_L \to \pi^0 \nu\bar{\nu})_{\rm SM} = (3.00 \pm 0.30) \times 10^{-11}$$

$$BR(K^+ \to \pi^+ \nu\bar{\nu})_{\rm SM} = (9.11 \pm 0.72) \times 10^{-11}$$

- Theoretically clean (Hadronic matrix element can be estimated using isospin symmetry)

- Neutral decay $K_L \to \pi^0 \nu\nu$ is purely CP violating mode



$K_L \to \pi^0 \nu\bar{\nu}$ and $K^+ \to \pi^+ \nu\bar{\nu}$

$BR(K_L \to \pi^0 \nu\bar{\nu})_{\rm exp}$ *KOTO@J-PARC*

- KOTO at J-PARC reported new result from the 2015 data this summer

$$BR(K_L \to \pi^0 \nu\bar{\nu})_{\rm SM} = (3.00 \pm 0.30) \times 10^{-11}$$

$$BR(K_L \to \pi^0 \nu\bar{\nu})_{\rm exp} < 2.6 \times 10^{-8} \ (90\%{\rm C.L.})$$ *E391a*

$$< 3.0 \times 10^{-9} (90\%{\rm C.L.})$$ ***New 2018*** *KOTO@ICHEP2018*

- KOTO-phase2 aims to measure at 10% accuracy

$BR(K^+ \to \pi^+ \nu\bar{\nu})_{\rm exp}$ *NA62@CERN*

- NA62 at CERN observed one event in 2016 data

$$BR(K^+ \to \pi^+ \nu\bar{\nu})_{\rm SM} = (9.11 \pm 0.72) \times 10^{-11}$$

$$BR(K^+ \to \pi^+ \nu\bar{\nu})_{\rm exp} = (1.73^{+1.15}_{-1.05}) \times 10^{-10}$$ *BNL 949/E787*

$$< 14 \times 10^{-10} (95\%{\rm C.L.})$$ ***New 2018*** *NA62@FPCP2018*

- Expected about 20 SM events from the 2017-2018 data sample

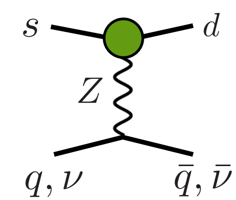
See M. Koval talk (WG3, Wed)

ε'/ε ⇔ K→πνν *- Examples -*

Z scenario *Buras, Buttazzo and Knegjens 1507.08672 / Buras 1601.00005 / Bobeth, Buras, Celis and Jung 1703.04753*

There are interesting correlations between Kaon observables depending on the chiral structure of coupling (LH and/or RH)

Δ_L and/or Δ_R



$$\mathrm{Re}\left(\frac{\epsilon'}{\epsilon}\right) \propto -\mathrm{Im}\,\Delta_L^{sd} - 3\,\mathrm{Im}\,\Delta_R^{sd} + \cdots$$

CPV ⇒ Strong correlation

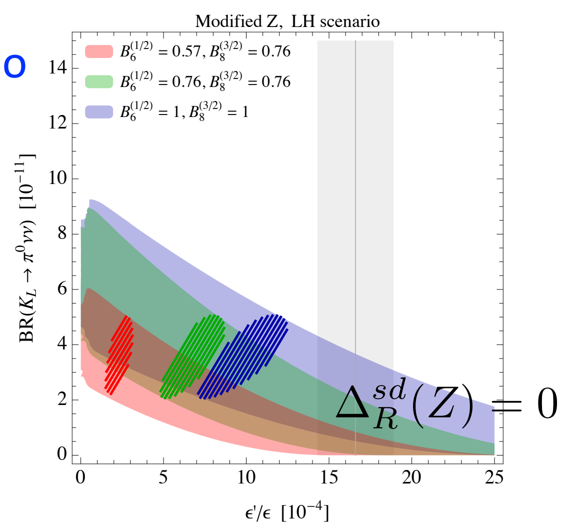
$$\mathrm{BR}(K^+ \to \pi^+\nu\bar{\nu}) \propto |X + \cdots|^2$$

$$\mathrm{BR}(K_L \to \pi^0\nu\bar{\nu}) \propto (\mathrm{Im}\,X)^2$$

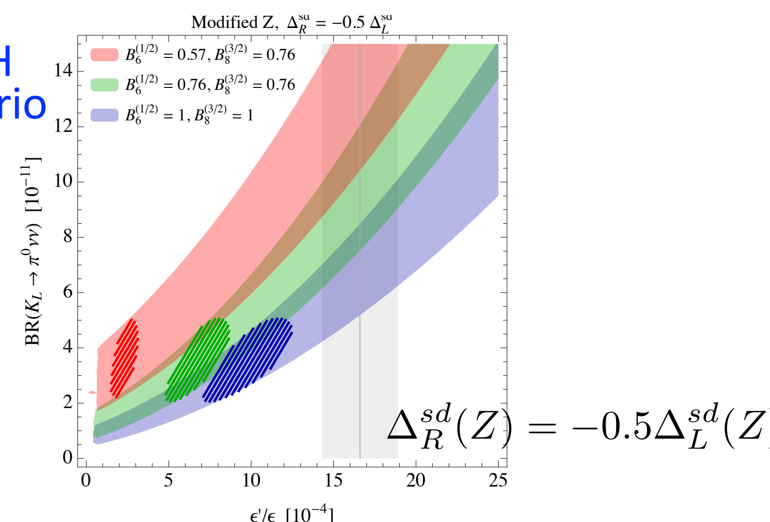
$$X = X(x_t)_{\mathrm{SM}} + \frac{\pi^2}{2M_W^2 G_F^2}\frac{\Delta_L^{\nu\nu}}{V_{ts}^* V_{td} M_Z^2}(\Delta_L^{sd} + \Delta_R^{sd})$$

$$|\epsilon_K| \propto \mathrm{Im}\Big[(\Delta_L^{sd})^2 + (\Delta_R^{sd})^2 - 240\Delta_L^{sd}\Delta_R^{sd}\Big]$$

LH scenario



LH+RH scenario

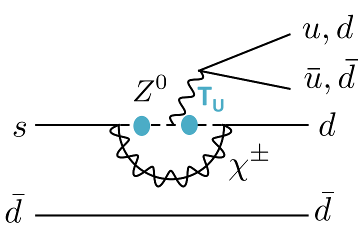


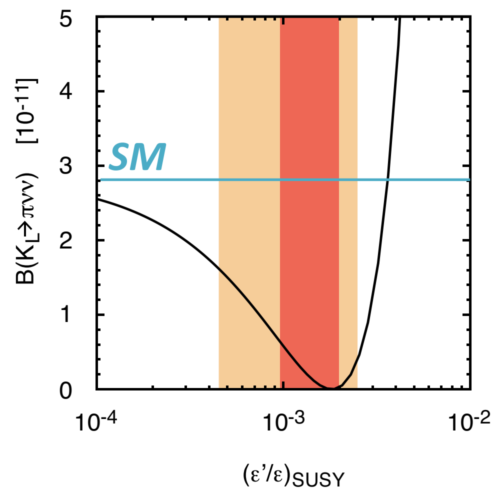
ε'/ε ⇔ K→πνν *- Examples -*

Chargino Z penguin *Endo, Mishima, Ueda and KY 1608.01444*

Large trilinear couplings bring enhancement of ε'/ε

LH Z scenario ⇒ negative correlation btwn ε'/ε and KL→π0νν

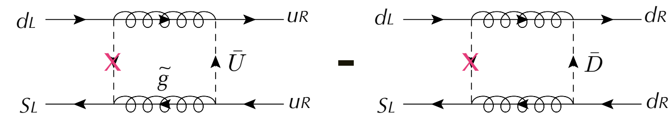


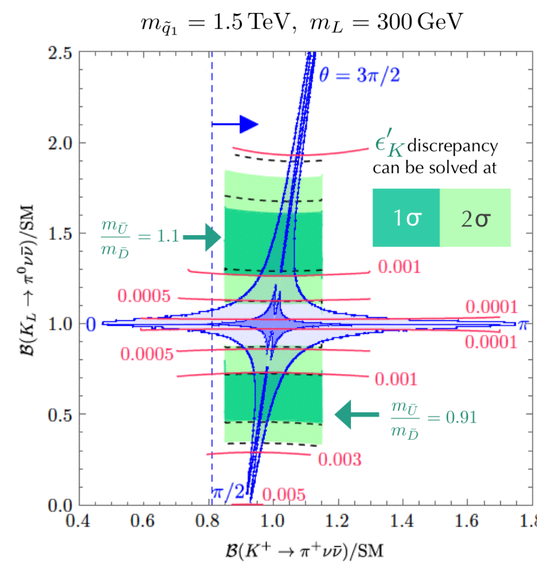


ε'/ε ⇔ SUSY scale < 4-6 TeV

Gluino box *Crivellin, D'Ambrosio, Kitahara and Nierste 1703.05786*

Large isospin breaking ($m_{\tilde{U}} \neq m_{\tilde{D}}$) gives effect on ImA2





$BR(K_L \to \pi^0\nu\bar{\nu})$	< 0.6 SM	< 1.4 SM	➡ ***KOTO***
$BR(K^+ \to \pi^+\nu\bar{\nu})$	O(10~100%) effect	< 1.2 SM	➡ ***NA62***

Different correlations between ε'/ε and K→πνν may allow to distinguish among models

Recent other progress for ε'/ε

See J.Aebischer talk (WG3, Wed)

- Chromomagnetic operator
  - ETM collaboration has reported the first Lattice result for the K→π matrix element of the chrome magnetic operator *ETM collaboration, 1712.09824*
  - DQCD gives a similar result *Buras and Gérard 1803.08052*

  CMO contribution is not significant in the SM, but it could be important in some NP models

- BSM operators *Aebischer, Buras and Gérard 1807.01709 / Aebischer, Bobeth, Buras, Gérard and Straub 1807.02520*

  Matrix elements of BSM operator are calculated in DQCD

  Master formula including BSM operators is derived with DQCD

  Scalar & tensor operators, which have chiral enhancement, are important

- SMEFT study : *(SM effective field theory) [SU(2) × U(1) inv.] ($\mu_{EW} < \mu < \mu_{NP}$)*

$$\mathcal{L} = \mathcal{L}_{SM} + \sum_i \mathcal{C}_i \mathcal{O}_i^{d\geq 5}$$

Due to the RG effect, observables have correlation

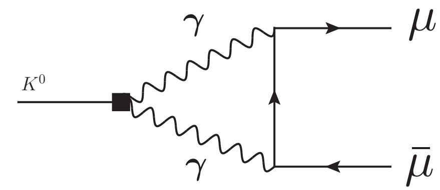
-Model independent approach *Aebischer, Bobeth, Buras and Straub 1808.00466*

The constraints from K^0 and D^0 mixing as well as EDM are potentially important

- Z penguin scenario *Bobeth, Buras, Celis and Jung 1703.04753*
Endo, Kitahara, Mishima and KY 1612.08839/ Endo, Goto, Kitahara, Mishima, Ueda and KY 1712.04959

ΔS=1 operators generate ΔS=2 contributions, through top-Yukawa enhanced RG evolution

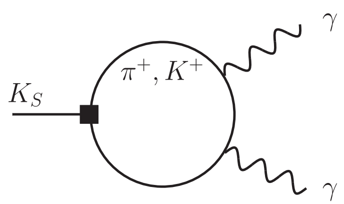
K→μμ



It could be used to probe NP contributions.
However, there are LD contributions (K→γγ) in SM:

$K_S \to \mu\mu$

K→γγ is evaluated in ChPT at O(p^4)



$$\mathcal{B}(K_S \to \mu^+\mu^-)_{\rm SM}$$
$$= (4.99\,(\rm LD) + 0.19\,(\rm SD)) \times 10^{-12}$$
$$= (5.18 \pm 1.50 \pm 0.02) \times 10^{-12}$$

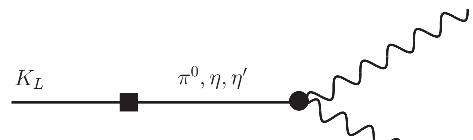
Ecker and Pich Nucl.Phys. B366 (1991) 189-20
Isidori and Unterdorfer hep-ph/0311084

$$\mathcal{B}(K_S \to \mu^+\mu^-)_{\rm exp} < 0.8 \times 10^{-9}$$ *LHCb Run1*

SM sensitivity at LHCb Run3 (2021-)

$K_L \to \mu\mu$

Leading O(p^4) contributions for K→γγ vanish



$\to A(K_L \to \mu\mu)_{\rm LD}$

Note: $\mathrm{CP}|\pi^0\rangle = -|\pi^0\rangle$

Determined from $B(K_L \to \gamma\gamma)_{\rm exp}$

➡ Sign ambiguity in $A(K_L \to \mu\mu)_{\rm LD}$

$$\mathcal{B}(K_L \to \mu^+\mu^-)_{\rm SM} = \begin{cases} (6.85 \pm 0.80 \pm 0.06) \times 10^{-9} & \text{destructive} \\ (8.11 \pm 1.49 \pm 0.13) \times 10^{-9} & \text{constructive} \end{cases}$$

Isidori and Unterdorfer hep-ph/0311084
Gorbahn and Haisch hep-ph/0605203

$$\mathcal{B}(K_L \to \mu^+\mu^-)_{\rm exp} = (6.84 \pm 0.11) \times 10^{-9}$$

PDG

$K_S \to \mu\mu$ - Interference contribution -

A state of K^0 (or $\bar{K}^0$) at t=0 evolves into mixture of K_S and K_L states

If # of K^0 & $\bar{K}^0$ in beam are different from each other, interference contribution btwn K_S and K_L exists

The decay intensity of neutral Kaon beam into *f* :

$N(K^0)$: # of K^0

$$I(t) = \frac{N(K^0)}{N(K^0)+N(\overline{K}^0)}\left|\langle f|\mathcal{H}_{eff}|K^0(t)\rangle\right|^2 + \frac{N(\overline{K}^0)}{N(K^0)+N(\overline{K}^0)}\left|\langle f|\mathcal{H}_{eff}|\overline{K}^0(t)\rangle\right|^2$$

$$= \frac{1}{2}|\mathcal{A}(K_S)|^2 e^{-\Gamma_S t} + \frac{1}{2}|\mathcal{A}(K_L)|^2 e^{-\Gamma_L t} + D\ \mathrm{Re}\left[e^{-i\Delta M_K t}\mathcal{A}(K_S)^*\mathcal{A}(K_L)\right]e^{-\frac{\Gamma_S+\Gamma_L}{2}t} + \mathcal{O}(\bar{\epsilon})$$

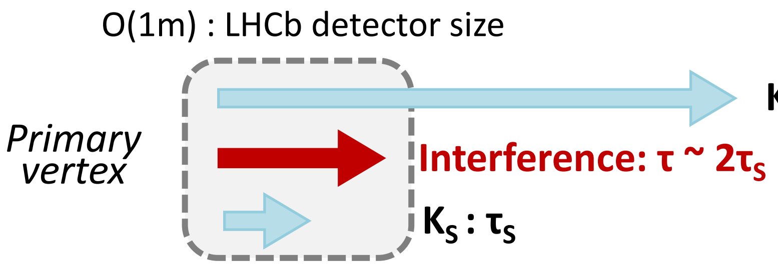
Interference contribution

Dilution factor $D \equiv \dfrac{N(K^0)-N(\bar{K}^0)}{N(K^0)+N(\bar{K}^0)}$

※Nonzero *D* can be achieved by an accompanying charged kaon tagging $pp \to K^0 K^- X$

Decay length of the interference contribution is around $2\tau_S$

➡ decay inside of LHCb detector



} $\mathcal{B}(K_S \to \mu^+\mu^-)_{\rm eff}$

$K_S \to \mu\mu$ - Interference contribution -

$$\mathcal{B}(K_S \to \mu^+\mu^-)_{\rm eff} = \mathcal{B}(K_S \to \mu^+\mu^-) + D\ \mathcal{B}(K \to \mu^+\mu^-)_{\rm int}$$

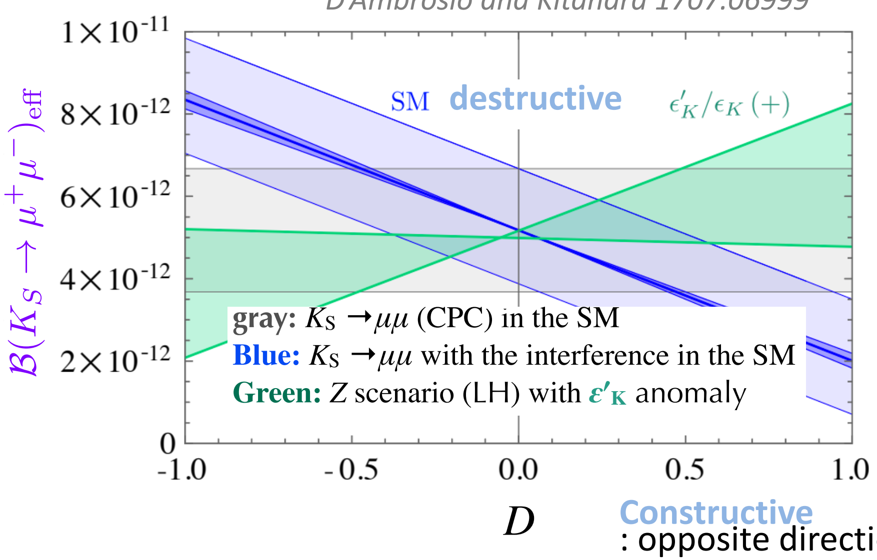
$$\mathcal{B}(K \to \mu^+\mu^-)_{\rm int} \propto \mathcal{A}(K_S \to \mu^+\mu^-)^* \mathcal{A}(K_L \to \mu^+\mu^-) \supset {\rm Im}A_{\rm SD} A(K_L \to \mu\mu)_{\rm LD}$$

$$\supset {\rm Im}A_{\rm SD} \qquad \propto A(K_L \to \mu\mu)_{\rm LD}$$

SD effect becomes comparable in size to LD, due to large $A(K_L \to \mu\mu)_{\rm LD}$ in interference effect

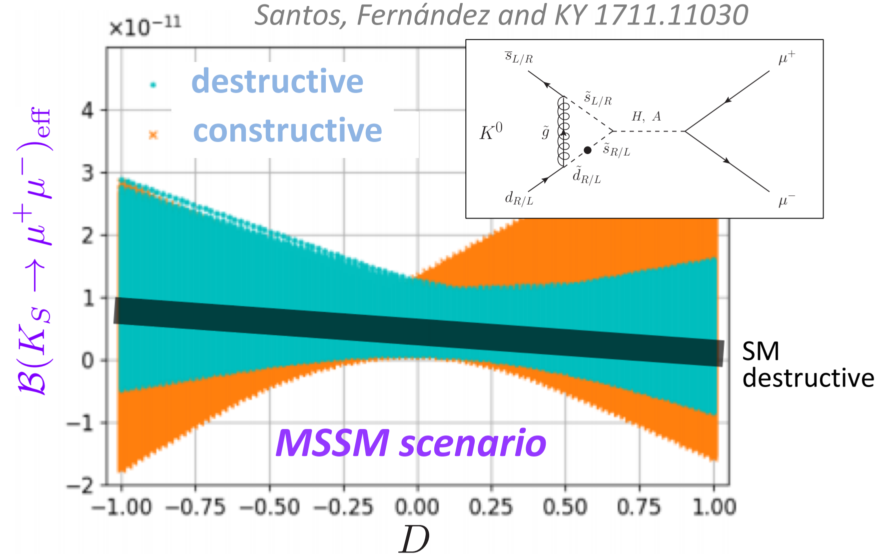
The sign of LD $A(K_L \to \mu\mu)_{\rm LD}$ can be determined by a measurement of the interference

D'Ambrosio and Kitahara 1707.06999



Constructive
: opposite direction

Chobanova, D'Ambrosio, Kitahara, Martínez, Santos, Fernández and KY 1711.11030



The interference changes Br. at O(60%) level in SM [more significant in MSSM] and the sign of the LD in $K_L \to \mu\mu$ can be determined if D=O(1) → ***LHCb***

Summary

Many recent progresses in Kaon physics

$\mathcal{O}(10\%)$ NP ϵ ← Vcb

Discrepancy ? ϵ'/ϵ ← Lattice

NP studies → check ↑

clean obs.
- $K_L \to \pi^0 \nu\bar{\nu}$ ← KOTO
- $K^+ \to \pi^+ \nu\bar{\nu}$ ← NA62

$K_S \to \mu\mu$ ← LHCb

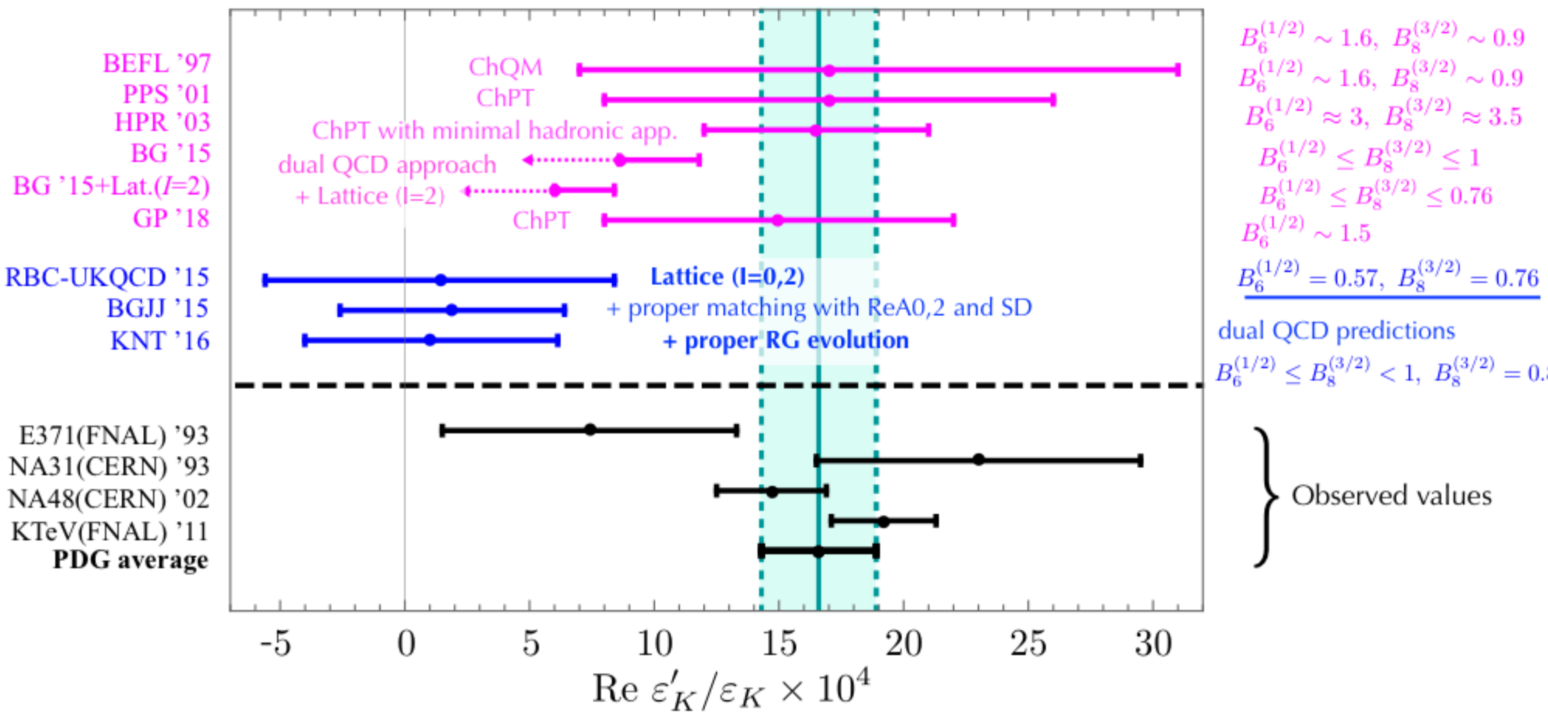
Many other interesting topics (not covered in this talk)

e.g.
- LFU test ($K_{e2}/K_{\mu 2}$)
- LFV ($K_L \to \mu e$)
- $K \to \pi\ell\ell$ *see A. Juettner talk (WG3 Wed)*
- correlation with B physics ($R_{D^{(*)}}, R_{K^{(*)}},,,$) *see M. Bordone talk (WG3 Wed)*

Kaon physics will continue to offer a powerful probe for NP!

Current situation of $\varepsilon'_K/\varepsilon_K$

$$\propto \underbrace{\mathrm{Im}A_0}_{\propto B_6^{(1/2)}} - \left(\frac{\mathrm{Re}A_0}{\mathrm{Re}A_2}\right) \underbrace{\mathrm{Im}A_2}_{\propto B_8^{(3/2)}}$$



$\Delta I = 1/2$ rule $\left(\frac{\mathrm{Re}A_0}{\mathrm{Re}A_2}\right)$	Exp.	ChPT	dual QCD	Lattice
	22.45 ± 0.05	~ 14	16.0 ± 1.5	31.0 ± 11.1

RBC-UKQCD lattice result

[1] Buras, et al., 1507.06345
[3] RBC-UKQCD, 1505.07863
[15] RBC-UKQCD, 1502.00263

Amplitude	Lattice QCD	Exp. data
$\mathrm{Re}A_0$ $[10^{-7}$ GeV$]$	$4.66 \pm 1.00 \pm 1.26$ [3]	3.322 ± 0.001 [1]
$\mathrm{Im}A_0$ $[10^{-11}$ GeV$]$	$-1.90 \pm 1.23 \pm 1.08$ [3]	—
$\mathrm{Re}A_2$ $[10^{-8}$ GeV$]$	$1.50 \pm 0.04 \pm 0.14$ [15]	1.479 ± 0.003 [1]
$\mathrm{Im}A_2$ $[10^{-13}$ GeV$]$	$-6.99 \pm 0.20 \pm 0.84$ [15]	—

- The real parts are consistent with those extracted from the data.
- Scattering phase-shifts are determined from the two-pion energy levels in a finite Euclidean volume on the lattice.

[Lellouch & Lüscher]

- Caveat: The calculated $I = 0$ $\pi\pi$ phase-shift is smaller than the data:

$$\delta_0 = 23.8(4.9)(1.2)^\circ \Longleftrightarrow (\delta_0)_{\mathrm{exp}} = 38.3(1.3)^\circ$$