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Single Letter Patterned Representations and Fibonacci Sequence Values[1]

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Abstract

This work brings representations of ***palindromic*** *and* ***number patterns*** *in terms of single letter "a". Some examples of prime patterns are also considered. Different classifications of palindromic patterns are considered, such as palindromic decomposition, double symmetric patterns, number patterns decompositions, etc. Number patterns with powers are also studied. Some extensions to* ***Pythagorean triples*** *are also given. Study towards* ***Fibonacci sequence*** *and its extensions are also made. This work is revised and enlarged version of author's previous work done in 2015.*

Contents

1 Single Letter Representations

Let us consider

$$f^n(10) = 10^n + 10^{n-1} + ... + 10^2 + 10 + 10^0,$$

For $a \in \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, we can write

$$a\, f^n(10) = \underbrace{aaa...a}_{(n+1)-times},$$

[1]It is revised and enlarged version of authors previous work [11, 12] appeared in 2015

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In particular,

$$
\begin{aligned}
aa &= f^1(10) = a10 + a && \Longrightarrow 11 \quad := \frac{aa}{a} \\
aaa &= f^2(10) = a10^2 + a10 + a && \Longrightarrow 111 \quad := \frac{aaa}{a} \\
aaaa &= f^3(10) = a10^3 + a10^2 + a10 + a && \Longrightarrow 1111 \quad := \frac{aaaa}{a} \\
aaaaa &= f^4(10) = a10^4 + a10^3 + a10^2 + a10 + a && \Longrightarrow 11111 := \frac{aaaaa}{a} \\
&\ldots\ldots
\end{aligned}
$$

In [12, 14] author wrote natural numbers in terms of single letter "a". See some examples below:

$$
\begin{aligned}
5 &:= \frac{aa - a}{a + a} \\
56 &:= \frac{aaa + a}{a + a} \\
582 &:= \frac{aaaaa + aaaa}{aa + aa - a} \\
1233 &:= \frac{aaaa + aaa + aa}{a} \\
4950 &:= \frac{aaaaa - aaaa - aaa + aa}{a + a}.
\end{aligned}
$$

for any $\in \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$.

The work on representations of natural numbers using **single letter** is first of its kind [20, 21]. Author also worked [16, 17, 18, 19], with representation of numbers using **single digit** for each value from 1 to 9 separately. Representation of numbers using all the digits from 1 to 9 in increasing and decreasing orders is done by author [7, 8, 9]. Comments to this work can be seen at author's site [10]. Different studies on numbers, such as, **selfie numbers, running expressions**, etc. refer to author's work web-site: *https://inderjtaneja.com*.

Our aim is to write **number patterns** using **single letter** "a". For studies on patterns refer to [1, 2, 4]. This work revised and enlarged version of authors previous work [11, 12] done in 2015.

2 Representations of Palindromic and Number Patterns

Below are examples of *palindromic* and *number patterns* in terms of single letter "a". These examples are divided in subsections. All the examples are followed by their respective decompositions. The following patterns are considered:

2.1 Number Patterns with Palindromic Decompositions

Here below are examples of palindromic patterns with palindromic decompositions.

Example 1. *The following example is represented in two different forms. One with product decomposition and another with potentiation.*

(i) ***Product***

$$
\begin{aligned}
\mathbf{121 = 11 \times 11} &:= \frac{aa \times aa}{a \times a} \\
\mathbf{12321 = 111 \times 111} &:= \frac{aaa \times aaa}{a \times a} \\
\mathbf{1234321 = 1111 \times 1111} &:= \frac{aaaa \times aaaa}{a \times a} \\
\mathbf{123454321 = 11111 \times 11111} &:= \frac{aaaaa \times aaaaa}{a \times a} \\
\mathbf{12345654321 = 111111 \times 111111} &:= \frac{aaaaaa \times aaaaaa}{a \times a} \\
\mathbf{1234567654321 = 1111111 \times 1111111} &:= \frac{aaaaaaa \times aaaaaaa}{a \times a} \\
\mathbf{123456787654321 = 11111111 \times 11111111} &:= \frac{aaaaaaaa \times aaaaaaaa}{a \times a} \\
\mathbf{12345678987654321 = 111111111 \times 111111111} &:= \frac{aaaaaaaaa \times aaaaaaaaa}{a \times a}
\end{aligned}
$$

Since $11 \times 11 = 11^2$, $111 \times 111 = 111^2$, *etc. Below is an alternative way of writing the same pattern:*

(ii) ***Potentiation***

$$
\begin{aligned}
\mathbf{121 = 11^2} &:= \left(\frac{aa}{a}\right)^{\frac{a+a}{a}} \\
\mathbf{12321 = 111^2} &:= \left(\frac{aaa}{a}\right)^{\frac{a+a}{a}} \\
\mathbf{1234321 = 1111^2} &:= \left(\frac{aaaa}{a}\right)^{\frac{a+a}{a}} \\
\mathbf{123454321 = 11111^2} &:= \left(\frac{aaaaa}{a}\right)^{\frac{a+a}{a}} \\
\mathbf{12345654321 = 111111^2} &:= \left(\frac{aaaaaa}{a}\right)^{\frac{a+a}{a}} \\
\mathbf{1234567654321 = 1111111^2} &:= \left(\frac{aaaaaa}{a}\right)^{\frac{a+a}{a}} \\
\mathbf{123456787654321 = 11111111^2} &:= \left(\frac{aaaaaaaa}{a}\right)^{\frac{a+a}{a}} \\
\mathbf{12345678987654321 = 111111111^2} &:= \left(\frac{aaaaaaaaa}{a}\right)^{\frac{a+a}{a}}.
\end{aligned}
$$

Example 2.

$$
\begin{aligned}
11 &= 1 \times 11 &&:= \frac{a \times aa}{a \times a}\\
1221 &= 11 \times 111 &&:= \frac{aa \times aaa}{a \times a}\\
123321 &= 111 \times 1111 &&:= \frac{aaa \times aaaa}{a \times a}\\
12344321 &= 1111 \times 11111 &&:= \frac{aaaa \times aaaaa}{a \times a}\\
1234554321 &= 11111 \times 111111 &&:= \frac{aaaaa \times aaaaaa}{a \times a}\\
123456654321 &= 111111 \times 1111111 &&:= \frac{aaaaaa \times aaaaaaa}{a \times a}\\
12345677654321 &= 1111111 \times 11111111 &&:= \frac{aaaaaaa \times aaaaaaaa}{a \times a}\\
1234567887654321 &= 11111111 \times 111111111 &&:= \frac{aaaaaaaa \times aaaaaaaaa}{a \times a}\\
123456789987654321 &= 111111111 \times 1111111111 &&:= \frac{aaaaaaaaa \times aaaaaaaaaa}{a \times a}
\end{aligned}
$$

• Palindromic-Type Pythagorean Triples

Similar to Example 1, there are lot of **palindromic-type patterns** with **Pythagorean triples**. See below the two examples,

Example 3.

$$
\begin{aligned}
099^2 + 20^2 &= 1\,01^2\\
12\,099^2 + 220^2 &= 121\,01^2\\
1232\,099^2 + 2220^2 &= 12321\,01^2\\
123432\,099^2 + 22220^2 &= 1234321\,01^2\\
12345432\,099^2 + 222220^2 &= 123454321\,01^2\\
1234565432\,099^2 + 2222220^2 &= 12345654321\,01^2\\
123456765432\,099^2 + 22222220^2 &= 1234567654321\,01^2\\
12345678765432\,099^2 + 222222220^2 &= 123456787654321\,01^2\\
1234567898765432\,099^2 + 2222222220^2 &= 12345678987654321\,01^2
\end{aligned}
$$

It is based on the **pythagorean triple** $(99, 20, 101)$.

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Example 4.

$$\begin{aligned}
019^2 + 180^2 &= 1\,81^2\\
12\,019^2 + 1980^2 &= 121\,81^2\\
1232\,019^2 + 19980^2 &= 12321\,81^2\\
123432\,019^2 + 199980^2 &= 1234321\,81^2\\
12345432\,019^2 + 1999980^2 &= 123454321\,81^2\\
1234565432\,019^2 + 19999980^2 &= 12345654321\,81^2\\
123456765432\,019^2 + 199999980^2 &= 1234567654321\,81^2\\
12345678765432\,019^2 + 1999999980^2 &= 123456787654321\,81^2\\
1234567898765432\,019^2 + 19999999980^2 &= 12345678987654321\,81^2
\end{aligned}$$

It is based on the **pythagorean triple** $(19, 180, 181)$. More study on the Examples 3 and 4 can be seen in author's [22, 23].

Example 5. *The following example is represented in two different forms. One with product decomposition and another with potentiation.*

(i) ***Product***

$$\begin{aligned}
1089 = 11 \times 99 &:= \frac{aa \times (aaa - aa - a)}{a \times a}\\
110889 = 111 \times 999 &:= \frac{aaa \times (aaaa - aaa - a)}{a \times a}\\
11108889 = 1111 \times 9999 &:= \frac{aaaa \times (aaaaa - aaaa - a)}{a \times a}\\
1111088889 = 11111 \times 99999 &:= \frac{aaaaa \times (aaaaaa - aaaaa - a)}{a \times a}\\
111110888889 = 111111 \times 999999 &:= \frac{aaaaaa \times (aaaaaaa - aaaaaa - a)}{a \times a}.
\end{aligned}$$

The above decomposition can also we written as $11 \times 99 = 33^2$, $111 \times 999 = 333^2$, *etc. We can rewrite the above representation using as potentiation:*

(ii) ***Potentiation***

$$\begin{aligned}
1089 = 11 \times 99 = 33^2 &:= \left(\frac{aa + aa + aa}{a}\right)^{\left(\frac{a+a}{a}\right)}\\
110889 = 111 \times 999 = 333^2 &:= \left(\frac{aaa + aaa + aaa}{a}\right)^{\left(\frac{a+a}{a}\right)}\\
11108889 = 1111 \times 9999 = 3333^2 &:= \left(\frac{aaaa + aaaa + aaaa}{a}\right)^{\left(\frac{a+a}{a}\right)}\\
1111088889 = 11111 \times 99999 = 33333^2 &:= \left(\frac{aaaaa + aaaaa + aaaaa}{a}\right)^{\left(\frac{a+a}{a}\right)}\\
111110888889 = 111111 \times 999999 = 333333^2 &:= \left(\frac{aaaaaa + aaaaaa + aaaaaa}{a}\right)^{\left(\frac{a+a}{a}\right)}
\end{aligned}$$

The reverse of 1089 is 9801 has following interesting pattern:

Example 6. *The following example is represented in two different forms. One with product decomposition and another with potentiation.*

(i) ***Potentiation***

$$9801 = 99^2 := \left(\frac{aaa - aa - a}{a \times a}\right)^{\frac{a+a}{a}}$$
$$998001 = 999^2 := \left(\frac{aaaa - aa - a}{a \times a}\right)^{\frac{a+a}{a}}$$
$$99980001 = 9999^2 := \left(\frac{aaaaa - aa - a}{a \times a}\right)^{\frac{a+a}{a}}$$
$$9999800001 = 99999^2 := \left(\frac{aaaaaa - aa - a}{a \times a}\right)^{\frac{a+a}{a}}$$
$$999998000001 = 999999^2 := \left(\frac{aaaaaaa - aa - a}{a \times a}\right)^{\frac{a+a}{a}}$$

We can rewrite the above representation in a direct way without potentiation:

(ii) ***Without Potentiation***

$$9801 = 99^2 := \frac{(aaaa - aa - aa) \times (aa - a - a)}{a \times a}$$
$$998001 = 999^2 := \frac{(aaaaaa - aaa - aaa) \times (aa - a - a)}{a \times a}$$
$$99980001 = 9999^2 := \frac{(aaaaaaaa - aaaa - aaaa) \times (aa - a - a)}{a \times a}$$
$$9999800001 = 99999^2 := \frac{(aaaaaaaaaa - aaaaa - aaaaa) \times (aa - a - a)}{a \times a}$$
$$999998000001 = 999999^2 := \frac{(aaaaaaaaaaaa - aaaaaa - aaaaaa) \times (aa - a - a)}{a \times a}$$

More examples on potentiation are given in Section 2.4. See examples 32, 33, 34, 35 and 36.

Example 7.

$$7623 = 11 \times 9 \times 77 := \frac{aa \times (aaa - aa - a) \times (aa - a - a - a - a)}{a \times a \times a}$$
$$776223 = 111 \times 9 \times 777 := \frac{aaa \times (aaaa - aaa - a) \times (aa - a - a - a - a)}{a \times a \times a}$$
$$77762223 = 1111 \times 9 \times 7777 := \frac{aaaa \times (aaaaa - aaaa - a) \times (aa - a - a - a - a)}{a \times a \times a}$$
$$7777622223 = 11111 \times 9 \times 77777 := \frac{aaaaa \times (aaaaaa - aaaaa - a) \times (aa - a - a - a - a)}{a \times a \times a}$$
$$777776222223 = 111111 \times 9 \times 777777 := \frac{aaaaaa \times (aaaaaaa - aaaaaa - a) \times (aa - a - a - a - a)}{a \times a \times a}$$

2.2 Palindromic Patterns with Number Pattern Decompositions

Here below are examples palindromic patterns decomposed in number patterns.

Example 8.

$$\begin{aligned}
1 &= 0 \times 9 + 1 &&:= \frac{a}{a} \\
11 &= 1 \times 9 + 2 &&:= \frac{aa}{a} \\
111 &= 12 \times 9 + 3 &&:= \frac{aaa}{a} \\
1111 &= 123 \times 9 + 4 &&:= \frac{aaaa}{a} \\
11111 &= 1234 \times 9 + 5 &&:= \frac{aaaaa}{a} \\
111111 &= 12345 \times 9 + 6 &&:= \frac{aaaaaa}{a} \\
1111111 &= 123456 \times 9 + 7 &&:= \frac{aaaaaaa}{a} \\
11111111 &= 1234567 \times 9 + 8 &&:= \frac{aaaaaaaa}{a} \\
111111111 &= 12345678 \times 9 + 9 &&:= \frac{aaaaaaaaa}{a} \\
1111111111 &= 123456789 \times 9 + 10 &&:= \frac{aaaaaaaaaa}{a}
\end{aligned}$$

Example 9.

$$\begin{aligned}
77 &= 76 + 1 &&:= \frac{aa}{a} \times \frac{aa-a-a-a-a}{a} \\
777 &= 765 + 12 &&:= \frac{aaa}{a} \times \frac{aa-a-a-a-a}{a} \\
7777 &= 7654 + 123 &&:= \frac{aaaa}{a} \times \frac{aa-a-a-a-a}{a} \\
777777 &= 76543 + 1234 &&:= \frac{aaaaa}{a} \times \frac{aa-a-a-a-a}{a} \\
7777777 &= 765432 + 12345 &&:= \frac{aaaaaa}{a} \times \frac{aa-a-a-a-a}{a} \\
77777777 &= 7654321 + 123456 &&:= \frac{aaaaaaa}{a} \times \frac{aa-a-a-a-a}{a} \\
777777777 &= 76543210 + 1234567 &&:= \frac{aaaaaaaa}{a} \times \frac{aa-a-a-a-a}{a}
\end{aligned}$$

Example 10.

$$
\begin{aligned}
88 &= 9\times 9+7 &&:=\frac{aa-a-a-a}{a}\times\frac{aa}{a}\\
888 &= 98\times 9+6 &&:=\frac{aa-a-a-a}{a}\times\frac{aaa}{a}\\
8888 &= 987\times 9+5 &&:=\frac{aa-a-a-a}{a}\times\frac{aaaa}{a}\\
88888 &= 9876\times 9+4 &&:=\frac{aa-a-a-a}{a}\times\frac{aaaaa}{a}\\
888888 &= 98765\times 9+3 &&:=\frac{aa-a-a-a}{a}\times\frac{aaaaaa}{a}\\
8888888 &= 987654\times 9+2 &&:=\frac{aa-a-a-a}{a}\times\frac{aaaaaaa}{a}\\
88888888 &= 9876543\times 9+1 &&:=\frac{aa-a-a-a}{a}\times\frac{aaaaaaaa}{a}\\
888888888 &= 98765432\times 9+0 &&:=\frac{aa-a-a-a}{a}\times\frac{aaaaaaaaa}{a}\\
8888888888 &= 987654321\times 9-1 &&:=\frac{aa-a-a-a}{a}\times\frac{aaaaaaaaaa}{a}\\
88888888888 &= 9876543210\times 9-2 &&:=\frac{aa-a-a-a}{a}\times\frac{aaaaaaaaaaa}{a}
\end{aligned}
$$

Example 11.

$$
\begin{aligned}
99 &= 98+1 &&:=\frac{aaa-aa-a}{a\times a}\\
999 &= 987+12 &&:=\frac{aaaa-aaa-a}{a\times a}\\
9999 &= 9876+123 &&:=\frac{aaaaa-aaaa-a}{a\times a}\\
99999 &= 98765+1234 &&:=\frac{aaaaaa-aaaaa-a}{a\times a}\\
999999 &= 987654+12345 &&:=\frac{aaaaaaa-aaaaaa-a}{a\times a}\\
9999999 &= 9876543+123456 &&:=\frac{aaaaaaaa-aaaaaaa-a}{a\times a}\\
99999999 &= 98765432+1234567 &&:=\frac{aaaaaaaaa-aaaaaaaa-a}{a\times a}\\
999999999 &= 987654321+12345678 &&:=\frac{aaaaaaaaaa-aaaaaaaaa-a}{a\times a}\\
9999999999 &= 9876543210+123456789 &&:=\frac{aaaaaaaaaaa-aaaaaaaaaa-a}{a\times a}
\end{aligned}
$$

Example 12.

$$
\begin{aligned}
\mathbf{33 = 12 + 21} &:= \frac{aa}{a} \times \frac{a+a+a}{a} \\
\mathbf{444 = 123 + 321} &:= \frac{aaa}{a} \times \frac{a+a+a+a}{a} \\
\mathbf{5555 = 1234 + 4321} &:= \frac{aaaa}{a} \times \frac{a+a+a+a+a}{a} \\
\mathbf{66666 = 12345 + 54321} &:= \frac{aaaaa}{a} \times \frac{a+a+a+a+a+a}{a} \\
\mathbf{777777 = 123456 + 654321} &:= \frac{aaaaaa}{a} \times \frac{aa-a-a-a-a}{a} \\
\mathbf{8888888 = 1234567 + 7654321} &:= \frac{aaaaaaa}{a} \times \frac{aa-a-a-a}{a} \\
\mathbf{99999999 = 12345678 + 87654321} &:= \frac{aaaaaaaa}{a} \times \frac{aa-a-a}{a}
\end{aligned}
$$

Example 13.

$$
\begin{aligned}
\mathbf{2772 = 4 \times 693} &:= \frac{aaaaa-aa-aa-a}{a+a+a+a} \\
\mathbf{27772 = 4 \times 6943} &:= \frac{aaaaaa-aa-aa-a}{a+a+a+a} \\
\mathbf{277772 = 4 \times 69443} &:= \frac{aaaaaaa-aa-aa-a}{a+a+a+a} \\
\mathbf{2777772 = 4 \times 694443} &:= \frac{aaaaaaaa-aa-aa-a}{a+a+a+a} \\
\mathbf{27777772 = 4 \times 6944443} &:= \frac{aaaaaaaaa-aa-aa-a}{a+a+a+a}
\end{aligned}
$$

In this example the previous number $272 := (aaaa-aa-aa-a)/(a+a+a+a)$ is also a palindrome, but its decomposition $272 = 4 \times 68$ is not symmetrical to other values of the patterns.

Example 14. *This example is little irregular in terms of number patterns. But, later making proper choices, we can* *bring two different regular patterns.*

$$\begin{aligned}
101 &= 101 &&:= \frac{aaa - aa + a}{a}\\
1001 &= 11 \times 91 &&:= \frac{aaaa - aaa + a}{a}\\
10001 &= 73 \times 137 &&:= \frac{aaaaa - aaaa + a}{a}\\
100001 &= 11 \times 9091 &&:= \frac{aaaaaa - aaaaa + a}{a}\\
1000001 &= 101 \times 9901 &&:= \frac{aaaaaaa - aaaaaa + a}{a}\\
10000001 &= 11 \times 909091 &&:= \frac{aaaaaaaa - aaaaaaa + a}{a}\\
100000001 &= 17 \times 5882353 &&:= \frac{aaaaaaaaa - aaaaaaaa + a}{a}\\
1000000001 &= 11 \times 90909091 &&:= \frac{aaaaaaaaaa - aaaaaaaaa + a}{a}\\
10000000001 &= 101 \times 99009901 &&:= \frac{aaaaaaaaaaa - aaaaaaaaaa + a}{a}\\
100000000001 &= 11 \times 9090909091 &&:= \frac{aaaaaaaaaaaa - aaaaaaaaaaa + a}{a}
\end{aligned}$$

This example shows that it not necessary that every palindromic pattern can be decomposed to number pattern. By considering only even number terms, i.e., 2nd, 4th, 6th, ..., we get a number pattern decomposition. Also considering 1st, 5th, 9th, ...terms, we get another regular number pattern decomposition. See below:

$$\begin{aligned}
1001 &= 11 \times 91 &&:= \frac{aaaa - aaa + a}{a}\\
100001 &= 11 \times 9091 &&:= \frac{aaaaaa - aaaaa + a}{a}\\
10000001 &= 11 \times 909091 &&:= \frac{aaaaaaaa - aaaaaaa + a}{a}\\
1000000001 &= 11 \times 90909091 &&:= \frac{aaaaaaaaaa - aaaaaaaaa + a}{a}\\
100000000001 &= 11 \times 9090909091 &&:= \frac{aaaaaaaaaaaa - aaaaaaaaaaa + a}{a}
\end{aligned}$$

$$\begin{aligned}
101 &= 101 &&:= \frac{aaa - aa + a}{a}\\
1000001 &= 101 \times 9901 &&:= \frac{aaaaaaa - aaaaaa + a}{a}\\
10000000001 &= 101 \times 99009901 &&:= \frac{aaaaaaaaaaa - aaaaaaaaaa + a}{a}\\
100000000000001 &= 101 \times 990099009901 &&:= \frac{aaaaaaaaaaaaaaa - aaaaaaaaaaaaaa + a}{a}
\end{aligned}$$

Another interesting pattern for above palindromic pattern is the following example.

Example 15. *This example brings repeated patterns*

$$\begin{aligned}
\mathbf{11} &= \mathbf{1 \times 7 + 3} &&:= \frac{aa - a + a}{a} \\
\mathbf{101} &= \mathbf{14 \times 7 + 3} &&:= \frac{aaa - aa + a}{a} \\
\mathbf{1001} &= \mathbf{142 \times 7 + 7} &&:= \frac{aaaa - aaa + a}{a} \\
\mathbf{10001} &= \mathbf{1428 \times 7 + 5} &&:= \frac{aaaaa - aaaa + a}{a} \\
\mathbf{100001} &= \mathbf{14285 \times 7 + 6} &&:= \frac{aaaaaa - aaaaa + a}{a} \\
\mathbf{1000001} &= \mathbf{142857 \times 7 + 2} &&:= \frac{aaaaaaa - aaaaaa + a}{a} \\
\mathbf{10000001} &= \mathbf{142857\,1 \times 7 + 4} &&:= \frac{aaaaaaaa - aaaaaaa + a}{a} \\
\mathbf{100000001} &= \mathbf{142857\,14 \times 7 + 3} &&:= \frac{aaaaaaaaa - aaaaaaaa + a}{a} \\
\mathbf{1000000001} &= \mathbf{142857\,142 \times 7 + 7} &&:= \frac{aaaaaaaaaa - aaaaaaaaa + a}{a} \\
\mathbf{10000000001} &= \mathbf{142857\,1428 \times 7 + 5} &&:= \frac{aaaaaaaaaaa - aaaaaaaaaa + a}{a} \\
\mathbf{100000000001} &= \mathbf{142857\,14285 \times 7 + 6} &&:= \frac{aaaaaaaaaaaa - aaaaaaaaaaa + a}{a} \\
\mathbf{1000000000001} &= \mathbf{142857\,142857 \times 7 + 2} &&:= \frac{aaaaaaaaaaaaa - aaaaaaaaaaaa + a}{a}
\end{aligned}$$

It is interesting to observe that the number 142857 make an interesting triangle in first six lines, and last members in each line make a sequence of numbers 1, 4, 2, 8, 5 and 7. The same repeats again in 7th to 12th line. More properties of the this number can be seen in examples 38 given in section 3.5. Another interesting property of the above palindromic pattern is given by

$$\begin{aligned}
101 \times ab &= ab\,ab \\
1001 \times abc &= abc\,abc \\
10001 \times abcd &= abcd\,abcd \\
100001 \times abcde &= abcde\,abcde \\
1000001 \times abcdef &= abcdef\,abcdef \\
10000001 \times abcdefg &= abcdefg\,abcdefg \\
&....
\end{aligned}$$

where, $abc = a \times 10^2 + b \times 10 + c$, etc. For example, $101 \times 23 = 23\,23$, $1001 \times 347 = 347\,347$, etc.

Example 16.

$$\begin{aligned}
\mathbf{111111111 = 12345679 \times 9 \times 1} &:= \frac{aaaaaaaaa}{a} \times \frac{a}{a}\\
\mathbf{222222222 = 12345679 \times 9 \times 2} &:= \frac{aaaaaaaaa}{a} \times \frac{a+a}{a}\\
\mathbf{333333333 = 12345679 \times 9 \times 3} &:= \frac{aaaaaaaaa}{a} \times \frac{a+a+a}{a}\\
\mathbf{444444444 = 12345679 \times 9 \times 4} &:= \frac{aaaaaaaaa}{a} \times \frac{a+a+a+a}{a}\\
\mathbf{555555555 = 12345679 \times 9 \times 5} &:= \frac{aaaaaaaaa}{a} \times \frac{a+a+a+a+a}{a}\\
\mathbf{666666666 = 12345679 \times 9 \times 6} &:= \frac{aaaaaaaaa}{a} \times \frac{a+a+a+a+a+a}{a}\\
\mathbf{777777777 = 12345679 \times 9 \times 7} &:= \frac{aaaaaaaaa}{a} \times \frac{aa-a-a-a-a}{a}\\
\mathbf{888888888 = 12345679 \times 9 \times 8} &:= \frac{aaaaaaaaa}{a} \times \frac{aa-a-a-a}{a}\\
\mathbf{999999999 = 12345679 \times 9 \times 9} &:= \frac{aaaaaaaaa}{a} \times \frac{aa-a-a}{a}
\end{aligned}$$

The number 12345679 appearing above can be written as a division of 1/81, i.e.

$$\frac{1}{81} = 0.012345679\ 012345679\ 012345679\ 012345679\ .... = 0.\overline{012345679}$$

More situations of similar kind with less number of repetitions are given in examples 26, **??** and **??**.

Example 17. *Multiplying by 3 the number 12345679, appearing in previous example, we get*

$$12345679 \times 3 = 37037037.$$

The number 37037037 has very interesting properties. Multiplying it from 1 to 27 and reorganizing the values, we get very interesting patterns:

$$\begin{array}{lll}
37037037 \times 03 = 111111111 & 37037037 \times 1 = 037\ 037\ 037 & 37037037 \times 5 = 185\ 185\ 185\\
37037037 \times 06 = 222222222 & 37037037 \times 10 = 370\ 370\ 370 & 37037037 \times 14 = 518\ 518\ 518\\
37037037 \times 09 = 333333333 & 37037037 \times 19 = 703\ 703\ 703 & 37037037 \times 23 = 851\ 851\ 851\\
37037037 \times 12 = 444444444 & & \\
37037037 \times 15 = 555555555 & 37037037 \times 2 = 074\ 074\ 074 & 37037037 \times 7 = 259\ 259\ 259\\
37037037 \times 18 = 666666666 & 37037037 \times 11 = 407\ 407\ 407 & 37037037 \times 16 = 592\ 592\ 592\\
37037037 \times 21 = 777777777 & 37037037 \times 20 = 740\ 740\ 740 & 37037037 \times 25 = 925\ 925\ 925\\
37037037 \times 24 = 888888888 & & \\
37037037 \times 27 = 999999999 & 37037037 \times 4 = 148\ 148\ 148 & 37037037 \times 8 = 296\ 296\ 296\\
 & 37037037 \times 13 = 481\ 481\ 481 & 37037037 \times 17 = 629\ 629\ 629\\
 & 37037037 \times 22 = 814\ 814\ 814 & 37037037 \times 26 = 962\ 962\ 962.
\end{array}$$

Example 18. *Dividing 37037037 by 37 we get palindromic number 1001001. Any other number with the similar kind of pattern divided by last two digits always give the same palindromic number. See below*

$$17017017/17{=}1001001$$
$$19019019/19{=}1001001$$
$$23023023/23{=}1001001$$
$$45045045/45{=}1001001.$$

Let us make similar kind of multiplications as in previous example with number 17017017, we get symmetrical values, but not as beautiful as in previous example. See below:

$17017017 \times 01 := 017\ 017\ 017$
$17017017 \times 02 := 034\ 034\ 034$
$17017017 \times 03 := 051\ 051\ 051$
$17017017 \times 04 := 068\ 068\ 068$
$17017017 \times 05 := 085\ 085\ 085$
$17017017 \times 06 := 102\ 102\ 102$
$17017017 \times 07 := 119\ 119\ 119$
$17017017 \times 08 := 136\ 136\ 136$
$17017017 \times 09 := 153\ 153\ 153$
$17017017 \times 10 := 170\ 170\ 170$
$17017017 \times 11 := 187\ 187\ 187$
$17017017 \times 12 := 204\ 204\ 204$
$17017017 \times 13 := 221\ 221\ 221$
$17017017 \times 14 := 238\ 238\ 238$
$17017017 \times 15 := 255\ 255\ 255$
$17017017 \times 16 := 272\ 272\ 272$
$17017017 \times 17 := 289\ 289\ 289$
$17017017 \times 18 := 306\ 306\ 306$
$17017017 \times 19 := 323\ 323\ 323$
$17017017 \times 20 := 340\ 340\ 340$
$17017017 \times 21 := 357\ 357\ 357$
$17017017 \times 22 := 374\ 374\ 374$
$17017017 \times 23 := 391\ 391\ 391$
$17017017 \times 24 := 408\ 408\ 408$
$17017017 \times 25 := 425\ 425\ 425$
$17017017 \times 26 := 442\ 442\ 442$
$17017017 \times 27 := 459\ 459\ 459$
$17017017 \times 28 := 476\ 476\ 476$
$17017017 \times 29 := 493\ 493\ 493$
$17017017 \times 30 := 510\ 510\ 510$
$17017017 \times 31 := 527\ 527\ 527$
$17017017 \times 32 := 544\ 544\ 544$
$17017017 \times 33 := 561\ 561\ 561$
$17017017 \times 34 := 578\ 578\ 578$
$17017017 \times 35 := 595\ 595\ 595$
$17017017 \times 36 := 612\ 612\ 612$
$17017017 \times 37 := 629\ 629\ 629$
$17017017 \times 38 := 646\ 646\ 646$
$17017017 \times 39 := 663\ 663\ 663$
$17017017 \times 40 := 680\ 680\ 680$
$17017017 \times 41 := 697\ 697\ 697$
$17017017 \times 42 := 714\ 714\ 714$
$17017017 \times 43 := 731\ 731\ 731$
$17017017 \times 44 := 748\ 748\ 748$
$17017017 \times 45 := 765\ 765\ 765$
$17017017 \times 46 := 782\ 782\ 782$
$17017017 \times 47 := 799\ 799\ 799$
$17017017 \times 48 := 816\ 816\ 816$
$17017017 \times 49 := 833\ 833\ 833$
$17017017 \times 50 := 850\ 850\ 850$
$17017017 \times 51 := 867\ 867\ 867$
$17017017 \times 52 := 884\ 884\ 884$
$17017017 \times 53 := 901\ 901\ 901$
$17017017 \times 54 := 918\ 918\ 918$
$17017017 \times 55 := 935\ 935\ 935$
$17017017 \times 56 := 952\ 952\ 952$
$17017017 \times 57 := 969\ 969\ 969$
$17017017 \times 58 := 986\ 986\ 986$

Above multiplications are done only up to 58, but it can't go more. For example if we consider multiplication by 59, we get nonsymmetric result:

$$17017017 \times 59 := 100\ 400\ 4003.$$

Example 19. *The number 717 can be decomposed in two different ways, i.e.,* $717 = 88 \times 8 + 13$ *and* $717 = 56 \times 13 - 1$. *The first decomposition is* ***palindromic-type****, while the second is just* ***number pattern****. Below are both* *decompositions:*

$$717 = 88 \times 8 + 13 = 56 \times 13 - 11 := \frac{aaa + a}{a + a} \times \frac{aa + a + a}{a} - \frac{aa}{a}$$
$$7117 = 888 \times 8 + 13 = 556 \times 13 - 111 := \frac{aaaa + a}{a + a} \times \frac{aa + a + a}{a} - \frac{aaa}{a}$$
$$71117 = 8888 \times 8 + 13 = 5556 \times 13 - 1111 := \frac{aaaaa + a}{a + a} \times \frac{aa + a + a}{a} - \frac{aaaa}{a}$$
$$711117 = 88888 \times 8 + 13 = 55556 \times 13 - 11111 := \frac{aaaaaa + a}{a + a} \times \frac{aa + a + a}{a} - \frac{aaaaa}{a}$$
$$7111117 = 888888 \times 8 + 13 = 555556 \times 13 - 111111 := \frac{aaaaaaa + a}{a + a} \times \frac{aa + a + a}{a} - \frac{aaaaaa}{a}$$

Or,

$$717 = 88 \times 8 + 13 = 56 \times 13 - 11 := \frac{(aaa - aa - aa - a) \times (aa - a - a - a)}{a \times a} + \frac{aa + a + a}{a}$$
$$7117 = 888 \times 8 + 13 = 556 \times 13 - 111 := \frac{(aaaa - aaa - aaa - a) \times (aa - a - a - a)}{a \times a} + \frac{aa + a + a}{a}$$
$$71117 = 8888 \times 8 + 13 = 5556 \times 13 - 1111 := \frac{(aaaaa - aaaa - aaaa - a) \times (aa - a - a - a)}{a \times a} + \frac{aa + a + a}{a}$$
$$711117 = 88888 \times 8 + 13 = 55556 \times 13 - 11111 := \frac{(aaaaaa - aaaaa - aaaaa - a) \times (aa - a - a - a)}{a \times a} + \frac{aa + a + a}{a}$$
$$7111117 = 888888 \times 8 + 13 = 555556 \times 13 - 111111 := \frac{(aaaaaaa - aaaaaa - aaaaaa - a) \times (aa - a - a - a)}{a \times a} + \frac{aa + a + a}{a}$$

In the similar way, we can have many palindromic patterns. See subsection below.

2.3 Palindromic Patterns with Palindromic Decompositions

In this subsections the palindromic patterns are decomposed in another palindromic patterns.

Example 20.

$$121 = 11 \times 11 := \frac{aa \times aa}{a \times a \times a}$$
$$1221 = 11 \times 111 := \frac{aa \times aaa}{a \times a \times a}$$
$$12221 = 11 \times 1111 := \frac{aa \times aaaa}{a \times a \times a}$$
$$122221 = 11 \times 11111 := \frac{aa \times aaaaa}{a \times a \times a}$$
$$1222221 = 11 \times 111111 := \frac{aa \times aaaaaa}{a \times a \times a}$$

Example 21.

$$1331 = 11 \times 11 \times 11 := \frac{aa \times aa \times aa}{a \times a \times a}$$
$$13431 = 11 \times 11 \times 111 := \frac{aa \times aa \times aaa}{a \times a \times a}$$
$$134431 = 11 \times 11 \times 1111 := \frac{aa \times aa \times aaaa}{a \times a \times a}$$
$$1344431 = 11 \times 11 \times 11111 := \frac{aa \times aa \times aaaaa}{a \times a \times a}$$
$$13444431 = 11 \times 11 \times 111111 := \frac{aa \times aa \times aaaaaa}{a \times a \times a}$$

Single Letter Patterned Representations and Fibonacci Sequence Values,
Zenodo, Feb. 06, 2019, pp. 1-40, https://doi.org/10.5281/zenodo.2558522

Example 22.

$$\begin{aligned}
\mathbf{99 = 9 \times 11} &:= \frac{aaa - aa - a}{a} \\
\mathbf{999 = 9 \times 111} &:= \frac{aaaa - aaa - a}{a} \\
\mathbf{9999 = 9 \times 1111} &:= \frac{aaaaa - aaaa - a}{a} \\
\mathbf{99999 = 9 \times 11111} &:= \frac{aaaaaa - aaaaa - a}{a} \\
\mathbf{999999 = 9 \times 111111} &:= \frac{aaaaaaa - aaaaaa - a}{a}
\end{aligned}$$

Example 23.

$$\begin{aligned}
\mathbf{1001 = 13 \times 77} &:= \frac{aa \times (aaaa - aaa + a)}{aa \times a} \\
\mathbf{10101 = 13 \times 777} &:= \frac{aaa \times (aaaa - aaa + a)}{aa \times a} \\
\mathbf{101101 = 13 \times 7777} &:= \frac{aaaa \times (aaaa - aaa + a)}{aa \times a} \\
\mathbf{1011101 = 13 \times 77777} &:= \frac{aaaaa \times (aaaa - aaa + a)}{aa \times a} \\
\mathbf{10111101 = 13 \times 777777} &:= \frac{aaaaaa \times (aaaa - aaa + a)}{aa \times a}
\end{aligned}$$

Below are **even order decompositions** resulting again in palindromic patterns.

Example 24.

$$\begin{aligned}
\mathbf{1111 = 11 \times 101} &:= \frac{aaaa}{a} \times \frac{a}{a} \\
\mathbf{2222 = 11 \times 202} &:= \frac{aaaa}{a} \times \frac{a+a}{a} \\
\mathbf{3333 = 11 \times 303} &:= \frac{aaaa}{a} \times \frac{a+a+a}{a} \\
\mathbf{4444 = 11 \times 404} &:= \frac{aaaa}{a} \times \frac{a+a+a+a}{a} \\
\mathbf{5555 = 11 \times 505} &:= \frac{aaaa}{a} \times \frac{a+a+a+a+a}{a} \\
\mathbf{6666 = 11 \times 606} &:= \frac{aaaa}{a} \times \frac{a+a+a+a+a+a}{a} \\
\mathbf{7777 = 11 \times 707} &:= \frac{aaaa}{a} \times \frac{aa-a-a-a-a}{a} \\
\mathbf{8888 = 11 \times 808} &:= \frac{aaaa}{a} \times \frac{aa-a-a-a}{a} \\
\mathbf{9999 = 11 \times 909} &:= \frac{aaaa}{a} \times \frac{aa-a-a}{a}
\end{aligned}$$

Example 25.

$$\begin{aligned}
\mathbf{111111 = 11 \times 10101} &:= \frac{aaaaaa}{a} \times \frac{a}{a}\\
\mathbf{222222 = 11 \times 20202} &:= \frac{aaaaaa}{a} \times \frac{a+a}{a}\\
\mathbf{333333 = 11 \times 30303} &:= \frac{aaaaaa}{a} \times \frac{a+a+a}{a}\\
\mathbf{444444 = 11 \times 40404} &:= \frac{aaaaaa}{a} \times \frac{a+a+a+a}{a}\\
\mathbf{555555 = 11 \times 50505} &:= \frac{aaaaaa}{a} \times \frac{a+a+a+a+a}{a}\\
\mathbf{666666 = 11 \times 60606} &:= \frac{aaaaaa}{a} \times \frac{a+a+a+a+a+a}{a}\\
\mathbf{777777 = 11 \times 70707} &:= \frac{aaaaaa}{a} \times \frac{aa-a-a-a-a}{a}\\
\mathbf{888888 = 11 \times 80808} &:= \frac{aaaaaa}{a} \times \frac{aa-a-a-a}{a}\\
\mathbf{999999 = 11 \times 90909} &:= \frac{aaaaaa}{a} \times \frac{aa-a-a}{a}
\end{aligned}$$

Example 26.

$$\begin{aligned}
\mathbf{11111111 = 11 \times 1010101} &:= \frac{aaaaaaaa}{a} \times \frac{a}{a}\\
\mathbf{22222222 = 11 \times 2020202} &:= \frac{aaaaaaaa}{a} \times \frac{a+a}{a}\\
\mathbf{33333333 = 11 \times 3030303} &:= \frac{aaaaaaaa}{a} \times \frac{a+a+a}{a}\\
\mathbf{44444444 = 11 \times 4040404} &:= \frac{aaaaaaaa}{a} \times \frac{a+a+a+a}{a}\\
\mathbf{55555555 = 11 \times 5050505} &:= \frac{aaaaaaaa}{a} \times \frac{a+a+a+a+a}{a}\\
\mathbf{66666666 = 11 \times 6060606} &:= \frac{aaaaaaaa}{a} \times \frac{a+a+a+a+a+a}{a}\\
\mathbf{77777777 = 11 \times 7070707} &:= \frac{aaaaaaaa}{a} \times \frac{aa-a-a-a-a}{a}\\
\mathbf{88888888 = 11 \times 8080808} &:= \frac{aaaaaaaa}{a} \times \frac{aa-a-a-a}{a}\\
\mathbf{99999999 = 11 \times 9090909} &:= \frac{aaaaaaaa}{a} \times \frac{aa-a-a}{a}
\end{aligned}$$

Above example 26 brings palindromes on both sides of the expressions, while example **??** is written in terms of number pattern. Here the repetition of same digit is eight times, i.e., 11111111, while in example **??** same digits repeats nine times, i.e., 111111111 . When we work with repetition of even number of digits, i.e, 2, 4, 6, etc., the decomposition can be palindromic or number pattern. In case of odd numbers the situation is different. The following two examples brings patterns working with 3,4, 5, 6 and 7 times repetitions of same digit.Below are three **odd order decompositions**, resulting in prime factors.

Example 27.

$$111 = 37 \times 3 \times 1 \; := \frac{aaa \times a}{a \times a}$$
$$222 = 37 \times 3 \times 2 \; := \frac{aaa \times (a+a)}{a \times a}$$
$$333 = 37 \times 3 \times 3 \; := \frac{aaa \times (a+a+a)}{a \times a}$$
$$444 = 37 \times 3 \times 4 \; := \frac{aaa \times (a+a+a+a)}{a \times a}$$
$$555 = 37 \times 3 \times 5 \; := \frac{aaa \times (a+a+a+a+a)}{a \times a}$$
$$666 = 37 \times 3 \times 6 \; := \frac{aaa \times (a+a+a+a+a+a)}{a \times a}$$
$$777 = 37 \times 3 \times 7 \; := \frac{aaa \times (aa-a-a-a-a)}{a \times a}$$
$$888 = 37 \times 3 \times 8 \; := \frac{aaa \times (aa-a-a-a)}{a \times a}$$
$$999 = 37 \times 3 \times 9 \; := \frac{aaa \times (aa-a-a)}{a \times a}$$

Example 28.

$$11111 = 41 \times 271 \times 1 \; := \frac{aaaaa \times a}{a \times a}$$
$$22222 = 41 \times 271 \times 2 \; := \frac{aaaaa \times (a+a)}{a \times a}$$
$$33333 = 41 \times 271 \times 3 \; := \frac{aaaaa \times (a+a+a)}{a \times a}$$
$$44444 = 41 \times 271 \times 4 \; := \frac{aaaaa \times (a+a+a+a)}{a \times a}$$
$$55555 = 41 \times 271 \times 5 \; := \frac{aaaaa \times (a+a+a+a+a)}{a \times a}$$
$$66666 = 41 \times 271 \times 6 \; := \frac{aaaaa \times (a+a+a+a+a+a)}{a \times a}$$
$$77777 = 41 \times 271 \times 7 \; := \frac{aaaaa \times (aa-a-a-a-a)}{a \times a}$$
$$88888 = 41 \times 271 \times 8 \; := \frac{aaaaa \times (aa-a-a-a)}{a \times a}$$
$$99999 = 41 \times 271 \times 9 \; := \frac{aaaaa \times (aa-a-a)}{a \times a}$$

Example 29.

$$\begin{aligned}
\mathbf{1111111} = \mathbf{239} \times \mathbf{4649} \times \mathbf{1} &:= \frac{aaaaaaa \times a}{a \times a} \\
\mathbf{2222222} = \mathbf{239} \times \mathbf{4649} \times \mathbf{2} &:= \frac{aaaaaaa \times (a+a)}{a \times a} \\
\mathbf{3333333} = \mathbf{239} \times \mathbf{4649} \times \mathbf{3} &:= \frac{aaaaaaa \times (a+a+a)}{a \times a} \\
\mathbf{4444444} = \mathbf{239} \times \mathbf{4649} \times \mathbf{4} &:= \frac{aaaaaaa \times (a+a+a+a)}{a \times a} \\
\mathbf{5555555} = \mathbf{239} \times \mathbf{4649} \times \mathbf{5} &:= \frac{aaaaaaa \times (a+a+a+a+a)}{a \times a} \\
\mathbf{6666666} = \mathbf{239} \times \mathbf{4649} \times \mathbf{6} &:= \frac{aaaaaaa \times (a+a+a+a+a+a)}{a \times a} \\
\mathbf{7777777} = \mathbf{239} \times \mathbf{4649} \times \mathbf{7} &:= \frac{aaaaaaa \times (aa-a-a-a-a)}{a \times a} \\
\mathbf{8888888} = \mathbf{239} \times \mathbf{4649} \times \mathbf{8} &:= \frac{aaaaaaa \times (aa-a-a-a)}{a \times a} \\
\mathbf{9999999} = \mathbf{239} \times \mathbf{4649} \times \mathbf{9} &:= \frac{aaaaaaa \times (aa-a-a)}{a \times a}
\end{aligned}$$

See the example below for 6 digits repetition written in two different patterns.

Example 30.

$$\begin{aligned}
\mathbf{111111} &= 15873 \times 7 \times 1 = 37037 \times 3 \times 1 \\
\mathbf{222222} &= 15873 \times 7 \times 2 = 37037 \times 3 \times 2 \\
\mathbf{333333} &= 15873 \times 7 \times 3 = 37037 \times 3 \times 3 \\
\mathbf{444444} &= 15873 \times 7 \times 4 = 37037 \times 3 \times 4 \\
\mathbf{555555} &= 15873 \times 7 \times 5 = 37037 \times 3 \times 5 \\
\mathbf{666666} &= 15873 \times 7 \times 6 = 37037 \times 3 \times 6 \\
\mathbf{777777} &= 15873 \times 7 \times 7 = 37037 \times 3 \times 7 \\
\mathbf{888888} &= 15873 \times 7 \times 8 = 37037 \times 3 \times 8 \\
\mathbf{999999} &= 15873 \times 7 \times 9 = 37037 \times 3 \times 9.
\end{aligned}$$

2.4 Squared Number Patterns

Examples 1, 6 and 7 are with squares of 11, 33 and 99 respectively. Here we shall bring more examples considering squared number patterns.

Example 31.

$$16^2 = 256 := \left(\frac{aa+aa+aa-a}{a+a}\right)^{\frac{a+a}{a}}$$

$$166^2 = 27556 := \left(\frac{aaa+aaa+aaa-a}{a+a)}\right)^{\frac{a+a}{a}}$$

$$1666^2 = 2775556 := \left(\frac{aaaa+aaaa+aaaa-a}{a+a}\right)^{\frac{a+a}{a}}$$

$$16666^2 = 277755556 := \left(\frac{aaaaa+aaaaa+aaaaa-a}{a+a}\right)^{\frac{a+a}{a}}$$

$$166666^2 = 27777555556 := \left(\frac{aaaaaa+aaaaaa+aaaaaa-a}{a+a}\right)^{\frac{a+a}{a}}$$

Example 32.

$$19^2 = 361 := \left(\frac{aa+aa-a-a-a}{a}\right)^{\frac{a+a}{a}}$$

$$199^2 = 39601 := \left(\frac{aaa+aaa-aa-aa-a}{a}\right)^{\frac{a+a}{a}}$$

$$1999^2 = 3996001 := \left(\frac{aaaa+aaaa-aaa-aaa-a}{a}\right)^{\frac{a+a}{a}}$$

$$19999^2 = 399960001 := \left(\frac{aaaaa+aaaaa-aaaa-aaaa-a}{a}\right)^{\frac{a+a}{a}}$$

$$199999^2 = 39999600001 := \left(\frac{aaaaaa+aaaaaa-aaaaa-aaaaa-a}{a}\right)^{\frac{a+a}{a}}$$

Example 33.

$$34^2 = 1156 := \left(\frac{aa+aa+aa+a}{a}\right)^{\frac{a+a}{a}}$$

$$334^2 = 111556 := \left(\frac{aaa+aaa+aaa+a}{a}\right)^{\frac{a+a}{a}}$$

$$3334^2 = 11115556 := \left(\frac{aaaa+aaaa+aaaa+a}{a}\right)^{\frac{a+a}{a}}$$

$$33334^2 = 1111155556 := \left(\frac{aaaaa+aaaaa+aaaaa+a}{a}\right)^{\frac{a+a}{a}}$$

$$333334^2 = 111111555556 := \left(\frac{aaaaaa+aaaaaa+aaaaaa+a}{a}\right)^{\frac{a+a}{a}}$$

Example 34.

$$43^2 = 1849 := \left(\frac{aa+aa+aa+aa-a}{a}\right)^{\frac{a+a}{a}}$$

$$433^2 = 187489 := \left(\frac{aaa+aaa+aaa+aaa-aa}{a}\right)^{\frac{a+a}{a}}$$

$$4333^2 = 18774889 := \left(\frac{aaaa+aaaa+aaaa+aaaa-aaa}{a}\right)^{\frac{a+a}{a}}$$

$$43333^2 = 1877748889 := \left(\frac{aaaaa+aaaaa+aaaaa+aaaaa-aaaa}{a}\right)^{\frac{a+a}{a}}$$

$$433333^2 = 187777488889 := \left(\frac{aaaaaa+aaaaaa+aaaaaa+aaaaaa-aaaaa}{a}\right)^{\frac{a+a}{a}}$$

Example 35.

$$67^2 = 4489 := \left(\frac{aaa+aa+aa+a}{a+a}\right)^{\frac{a+a}{a}}$$

$$667^2 = 444889 := \left(\frac{aaaa+aaa+aaa+a}{a+a}\right)^{\frac{a+a}{a}}$$

$$6667^2 = 44448889 := \left(\frac{aaaaa+aaaa+aaaa+a}{a+a}\right)^{\frac{a+a}{a}}$$

$$66667^2 = 4444488889 := \left(\frac{aaaaaa+aaaaa+aaaaa+a}{a+a}\right)^{\frac{a+a}{a}}$$

$$666667^2 = 444444888889 := \left(\frac{aaaaaaa+aaaaaa+aaaaaa+a}{a+a}\right)^{\frac{a+a}{a}}$$

Example 36.

$$91^2 = 828 := \left(\frac{aaa-aa-aa+a+a}{a}\right)^{\frac{a+a}{a}}$$

$$991^2 = 98208 := \left(\frac{aaaa-aaa-aa+a+a}{a}\right)^{\frac{a+a}{a}}$$

$$9991^2 = 9982008 := \left(\frac{aaaaa-aaaa-aa+a+a}{a}\right)^{\frac{a+a}{a}}$$

$$99991^2 = 999820008 := \left(\frac{aaaaaa-aaaaa-aa+a+a}{a}\right)^{\frac{a+a}{a}}$$

$$999991^2 = 99998200008 := \left(\frac{xaaaaaaa-aaaaaa-aa+a+a}{a}\right)^{\frac{a+a}{a}}$$

Example 37. *When we work with equal digits, the numbers 11, 33, 66 and 99 give following beautiful patterns:*

$$\begin{aligned}
\mathbf{11}^2 &= 121 & \mathbf{66}^2 &= 4356\\
\mathbf{111}^2 &= 12321 & \mathbf{666}^2 &= 443556\\
\mathbf{1111}^2 &= 1234321 & \mathbf{6666}^2 &= 44435556\\
\mathbf{11111}^2 &= 123454321 & \mathbf{66666}^2 &= 4444355556\\
\mathbf{111111}^2 &= 12345654321 & \mathbf{666666}^2 &= 444443555556
\end{aligned}$$

$$\begin{aligned}
\mathbf{33}^2 &= 1089 & \mathbf{99}^2 &= 9801\\
\mathbf{333}^2 &= 110889 & \mathbf{999}^2 &= 998001\\
\mathbf{3333}^2 &= 11108889 & \mathbf{9999}^2 &= 99980001\\
\mathbf{33333}^2 &= 1111088889 & \mathbf{99999}^2 &= 9999800001\\
\mathbf{333333}^2 &= 111110888889 & \mathbf{999999}^2 &= 999998000001
\end{aligned}$$

Only in the first case the result is palindromic, others are just number patterns. They can also be brought to palindromic by making proper divisions in each case, i.e., dividing by 3^2, 6^2 *and* 9^2 *respectively. See below*

$$\begin{aligned}
\mathbf{11}^2 &= \frac{33^2}{3^2} = \frac{66^2}{6^2} = \frac{99^2}{9^2} = 121\\
\mathbf{111}^2 &= \frac{333^2}{3^2} = \frac{666^2}{6^2} = \frac{999^2}{9^2} = 12321\\
\mathbf{1111}^2 &= \frac{3333^2}{3^2} = \frac{6666^2}{6^2} = \frac{9999^2}{9^2} = 1234321\\
\mathbf{11111}^2 &= \frac{33333^2}{3^2} = \frac{66666^2}{6^2} = \frac{99999^2}{9^2} = 123454321\\
\mathbf{111111}^2 &= \frac{333333^2}{3^2} = \frac{666666^2}{6^2} = \frac{999999^2}{9^2} = 12345654321
\end{aligned}$$

Above process works also for numbers $\frac{22^2}{2^2}$, $\frac{44^2}{4^2}$, $\frac{55^2}{5^2}$, $\frac{77^2}{7^2}$ *and* $\frac{88^2}{8^2}$, *and still the result remains the same.*

2.5 Cyclic-Type Patterns

In this subsection, we shall present situations, where the patterns are formed by repetition of digits. In each case, the repetitions are in different forms. We can write

$$999999 = 3 \times 7 \times 11 \times 13 \times 37.$$

Division by 3, 11 and 37 always bring palindromes, i.e.,

$$\begin{aligned}
999999/3 &= 333333\\
999999/11 &= 90909\\
999999/37 &= 9009 \times 3.
\end{aligned}$$

The division by other two numbers, i.e., by 7 and 13 gives two different numbers, i.e.,

$$\begin{aligned}
999999/7 &= 142857\\
999999/13 &= 76923.
\end{aligned}$$

The following three examples are based on the numbers 142857 and 76923 with repetition of digits. The number 142857 also appeared in example 15 with interesting pattern.

Example 38. *This example is based on the number 142857. Multiplying by certain numbers, we get numbers with repetition of same digits. See below*

$$\begin{aligned}
\mathbf{142857 \times 1 = 142857} &:= \frac{aaaaaa \times (aa-a-a)}{(aa-a-a-a-a) \times a} \times \frac{a}{a} \\
\mathbf{142857 \times 3 = 428571} &:= \frac{aaaaaa \times (aa-a-a)}{(aa-a-a-a-a) \times a} \times \frac{a+a+a}{a} \\
\mathbf{142857 \times 2 = 285714} &:= \frac{aaaaaa \times (aa-a-a)}{(aa-a-a-a-a) \times a} \times \frac{a+a}{a} \\
\mathbf{142857 \times 6 = 857142} &:= \frac{aaaaaa \times (aa-a-a)}{(aa-a-a-a-a) \times a} \times \frac{a+a+a+a+a+a}{a} \\
\mathbf{142857 \times 4 = 571428} &:= \frac{aaaaaa \times (aa-a-a)}{(aa-a-a-a-a) \times a} \times \frac{a+a+a+a}{a} \\
\mathbf{142857 \times 5 = 714285} &:= \frac{aaaaaa \times (aa-a-a)}{(aa-a-a-a-a) \times a} \times \frac{a+a+a+a+a}{a}
\end{aligned}$$

Multiplying by 7 and dividing by 9, above numbers, we get interesting pattern:

$$\begin{aligned}
142857 \times 7/9 &= 111111 \\
428571 \times 7/9 &= 333333 \\
285714 \times 7/9 &= 222222 \\
857142 \times 7/9 &= 666666 \\
571428 \times 7/9 &= 444444 \\
714285 \times 7/9 &= 555555
\end{aligned}$$

Also, we can write

$$\frac{1}{7} = 0.142857\ 142857\ 142857\ 142857\ .. = 0.\overline{142857}.$$

Example 39. *This example is based on the number 76923. After multiplication by different numbers, we get a numbers with repetitions of same digits.*

$$\begin{aligned}
\mathbf{76923 \times 1 \ = 076923} &:= \frac{aaaaaa \times (aa-a-a)}{(aa+a+a) \times a} \times \frac{a}{a} \\
\mathbf{76923 \times 10 = 769230} &:= \frac{aaaaaa \times (aa-a-a)}{(aa+a+a) \times a} \times \frac{aa-a}{a} \\
\mathbf{76923 \times 9 \ = 692307} &:= \frac{aaaaaa \times (aa-a-a)}{(aa+a+a) \times a} \times \frac{aa-a-a}{a} \\
\mathbf{76923 \times 12 = 923076} &:= \frac{aaaaaa \times (aa-a-a)}{(aa+a+a) \times a} \times \frac{aa+a}{a} \\
\mathbf{76923 \times 3 \ = 230769} &:= \frac{aaaaaa \times (aa-a-a)}{(aa+a+a) \times a} \times \frac{a+a+a}{a} \\
\mathbf{76923 \times 4 \ = 307692} &:= \frac{aaaaaa \times (aa-a-a)}{(aa+a+a) \times a} \times \frac{a+a+a+a}{a}
\end{aligned}$$

Here also we have similar property as of previous example. Multiplying by 13 and dividing by 9, above numbers, we get interesting pattern:

$$\begin{aligned} 076923 \times 13/9 &= 111111 \\ 230769 \times 13/9 &= 333333 \\ 307692 \times 13/9 &= 444444 \\ 692307 \times 13/9 &= 999999. \end{aligned}$$

Here we don't have symmetrical values for 769230 and 923076. Also, we have

$$\frac{1}{13} := 0.076923\ 076923\ 076923\ 076923\ ... = 0.\overline{076923}$$

Example 40. *This example also deals with the number 76923. After multiplication by different numbers, we get a numbers with repetitions of same digits.*

$$\begin{aligned}
\mathbf{76923 \times 2} &= \mathbf{153846} := \frac{aaaaaa \times (aa-a-a)}{(aa+a+a) \times a} \times \frac{a+a}{a} \\
\mathbf{76923 \times 7} &= \mathbf{538461} := \frac{aaaaaa \times (aa-a-a)}{(aa+a+a) \times a} \times \frac{aa-a-a-a-a}{a} \\
\mathbf{76923 \times 5} &= \mathbf{384615} := \frac{aaaaaa \times (aa-a-a)}{(aa+a+a) \times a} \times \frac{a+a+a+a+a}{a} \\
\mathbf{76923 \times 11} &= \mathbf{846153} := \frac{aaaaaa \times (aa-a-a)}{(aa+a+a) \times a} \times \frac{aa}{a} \\
\mathbf{76923 \times 6} &= \mathbf{461538} := \frac{aaaaaa \times (aa-a-a)}{(aa+a+a) \times a} \times \frac{a+a+a+a+a+a}{a} \\
\mathbf{76923 \times 8} &= \mathbf{615384} := \frac{aaaaaa \times (aa-a-a)}{(aa+a+a) \times a} \times \frac{aa-a-a-a}{a}
\end{aligned}$$

Example 41. *The division 1/7 and 1/13 also works, we consider multiplications of 7 and 13. Here below are two situations for multiplication of 13:*

$$\frac{1}{26} = 0.038461\ 538461\ 538461\ 538461\ 538461\ 538461... = 0.038461\ \overline{538461}$$

and

$$\frac{1}{39} = 0.025641\ 025641\ 025641\ 025641\ 025641 = 0.\overline{025641}$$

In the second case we have following repetition of digits:

$$\mathbf{025641 \times 1 \quad = 025641} := \frac{aaaaaa \times (a+a+a)}{(aa+a+a) \times a} \times \frac{a}{a}$$
$$\mathbf{025641 \times 10 = 256410} := \frac{aaaaaa \times (a+a+a)}{(aa+a+a) \times a} \times \frac{aa-a}{a}$$
$$\mathbf{025641 \times 22 = 564102} := \frac{aaaaaa \times (a+a+a)}{(aa+a+a) \times a} \times \frac{aa+aa}{a}$$
$$\mathbf{025641 \times 25 = 641025} := \frac{aaaaaa \times (a+a+a)}{(aa+a+a) \times a} \times \frac{aa+aa+a+a+a}{a}$$
$$\mathbf{025641 \times 16 = 410256} := \frac{aaaaaa \times (a+a+a)}{(aa+a+a) \times a} \times \frac{aa+a+a+a+a+a}{a}$$
$$\mathbf{025641 \times 4 \quad = 102564} := \frac{aaaaaa \times (a+a+a)}{(aa+a+a) \times a} \times \frac{a+a+a+a}{a}$$

The numbers 142857, 076923, 153846 and 025641 respectively appearing in examples 38, 39, 40 and 41 satisfies common properties given by

$14+28+57=99$	$07+69+23=99$	$15+38+46=99$	$02+56+41=99$
$42+85+71=99\times 2$	$76+92+30=99\times 2$	$53+84+61=99\times 2$	$25+64+10=99$
$28+57+14=99$	$69+23+07=99$	$38+46+15=99$	$56+41+02=99$
$85+71+42=99\times 2$	$92+30+76=99\times 2$	$84+61+53=99\times 2$	$64+10+25=99$
$57+14+28=99$	$23+07+69=99$	$46+15+38=99$	$41+02+56=99$
$71+42+85=99$	$30+76+92=99\times 2$	$61+53+84=99\times 2.$	$10+25+64=99.$

and

$142+857=999$	$076+923=999$	$153+846=999$	$025+641=666$
$428+571=999$	$769+230=999$	$538+461=999$	$256+410=666$
$285+714=999$	$692+307=999$	$384+615=999$	$564+102=666$
$857+142=999$	$923+076=999$	$846+153=999$	$641+025=666$
$571+428=999$	$230+769=999$	$461+538=999$	$410+256=666$
$714+285=999$	$307+692=999$	$615+384=999$	$102+564=666.$

Example 42. *Multiplication of 1089 with* 1, 2, 3, 4, 5, 6, 7, 8 *and* 9 *brings very interesting pattern. See below:*

$$\begin{aligned}
\mathbf{1089 \times 1 = 1089} &:= \left(\frac{aaaa - aa - aa}{a}\right) \times \frac{a}{a}\\
\mathbf{1089 \times 2 = 2178} &:= \left(\frac{aaaa - aa - aa}{a}\right) \times \frac{a+a}{a}\\
\mathbf{1089 \times 3 = 3267} &:= \left(\frac{aaaa - aa - aa}{a}\right) \times \frac{a+a+a}{a}\\
\mathbf{1089 \times 4 = 4356} &:= \left(\frac{aaaa - aa - aa}{a}\right) \times \frac{a+a+a+a}{a}\\
\mathbf{1089 \times 5 = 5445} &:= \left(\frac{aaaa - aa - aa}{a}\right) \times \frac{a+a+a+a+a}{a}\\
\mathbf{1089 \times 6 = 6534} &:= \left(\frac{aaaa - aa - aa}{a}\right) \times \frac{a+a+a+a+a+a}{a}\\
\mathbf{1089 \times 7 = 7623} &:= \left(\frac{aaaa - aa - aa}{a}\right) \times \frac{aa-a-a-a-a}{a}\\
\mathbf{1089 \times 8 = 8712} &:= \left(\frac{aaaa - aa - aa}{a}\right) \times \frac{aa-a-a-a}{a}\\
\mathbf{1089 \times 9 = 9801} &:= \left(\frac{aaaa - aa - aa}{a}\right) \times \frac{aa-a-a}{a}
\end{aligned}$$

In each column, the digits are in consecutive way (increasing or decreasing). In pairs, they are reverse of each other, i.e., (1089, 9801), (2178, 8712), (3267, 7623) and (4356, 6534). Another interesting property of these nine numbers is with magic squares, i.e., considering members of 2, 3 or 4 columns, they always forms magic squares of order 3x3 [1].

Example 43. *Another number having similar kind of properties of previous example is 9109. Multiplying it by 1, 2, 3, 4, 5, 6, 7, 8 and 9, we get*

$$\begin{aligned}
\mathbf{9109 \times 1 = 09109} &:= \left(\frac{aaaaa - aaaaaa \times (a+a)}{aaa}\right) \times \frac{a}{a \times a}\\
\mathbf{9109 \times 2 = 18218} &:= \left(\frac{aaaaa - aaaaaa \times (a+a)}{aaa}\right) \times \frac{a+a}{a \times a}\\
\mathbf{9109 \times 3 = 27327} &:= \left(\frac{aaaaa - aaaaaa \times (a+a)}{aaa}\right) \times \frac{a+a+a}{a \times a}\\
\mathbf{9109 \times 4 = 36436} &:= \left(\frac{aaaaa - aaaaaa \times (a+a)}{aaa}\right) \times \frac{a+a+a+a}{a \times a}\\
\mathbf{9109 \times 5 = 45545} &:= \left(\frac{aaaaa - aaaaaa \times (a+a)}{aaa}\right) \times \frac{a+a+a+a+a}{a \times a}\\
\mathbf{9109 \times 6 = 54654} &:= \left(\frac{aaaaa - aaaaaa \times (a+a)}{aaa}\right) \times \frac{a+a+a+a+a+a}{a \times a}\\
\mathbf{9109 \times 7 = 63763} &:= \left(\frac{aaaaa - aaaaaa \times (a+a)}{aaa}\right) \times \frac{aa-a-a-a-a}{a \times a}\\
\mathbf{9109 \times 8 = 72872} &:= \left(\frac{aaaaa - aaaaaa \times (a+a)}{aaa}\right) \times \frac{aa-a-a-a}{a \times a}\\
\mathbf{9109 \times 9 = 81981} &:= \left(\frac{aaaaa - aaaaaa \times (a+a)}{aaa}\right) \times \frac{aa-a-a}{a \times a}
\end{aligned}$$

Column members are in increasing and decreasing orders of 0 to 8 or 9. Also first and last two digit of each number are same, and are multiple of 9. Moreover, the number 9109 is a prime number. Here also, all the new nine numbers make a magic square of order 3×3. Another interesting property of above pattern is

$$\begin{aligned} 100 &= \frac{091+09}{1} = \frac{182+18}{2} = \frac{273+27}{3} \\ &= \frac{364+36}{4} = \frac{455+45}{5} = \frac{546+54}{6} \\ &= \frac{637+63}{7} = \frac{728+72}{8} = \frac{819+81}{9} \end{aligned}$$

Example 44. *This example is based on the property,* $8712 = 2178 \times 4 = 1089 \times 2 \times 4$, *i.e., after multiplication by 4 the number 2178 becomes its reverse, i.e., 8712. Also it is a multiple of 8 with 1089.*

$$\begin{aligned} \mathbf{8712 = 2178 \times 4} &:= \frac{(aaaa - aa - aa) \times (a+a)}{a \times a} \times \frac{a+a+a+a}{a} \\ \mathbf{87912 = 21978 \times 4} &:= \frac{(aaaaa - aaa - aa) \times (a+a)}{a \times a} \times \frac{a+a+a+a}{a} \\ \mathbf{879912 = 219978 \times 4} &:= \frac{(aaaaaa - aaaa - aa) \times (a+a)}{a \times a} \times \frac{a+a+a+a}{a} \\ \mathbf{8799912 = 2199978 \times 4} &:= \frac{(aaaaaaa - aaaaa - aa) \times (a+a)}{a \times a} \times \frac{a+a+a+a}{a} \\ \mathbf{87999912 = 21999978 \times 4} &:= \frac{(aaaaaaaa - aaaaaa - aa) \times (a+a)}{a \times a} \times \frac{a+a+a+a}{a} \end{aligned}$$

Example 45. *If we multiply 1089 by 9, we get 9801, i.e., reverse of 1089. The same happens with next members of the patterns.*

$$\begin{aligned} \mathbf{9801 =} \quad & \mathbf{1089 \times 9 = 99 \times 99} &&:= \frac{aaa - aa - a}{a} \times \frac{aaa - aa - a}{a} \\ \mathbf{98901 =} \quad & \mathbf{10989 \times 9 = 99 \times 999} &&:= \frac{aaa - aa - a}{a} \times \frac{aaaa - aaa - a}{a} \\ \mathbf{989901 =} \quad & \mathbf{109989 \times 9 = 99 \times 9999} &&:= \frac{aaa - aa - a}{a} \times \frac{aaaaa - aaaa - a}{a} \\ \mathbf{9899901 =} \quad & \mathbf{1099989 \times 9 = 99 \times 99999} &&:= \frac{aaa - aa - a}{a} \times \frac{aaaaaa - aaaaa - a}{a} \\ \mathbf{98999901 =} \quad & \mathbf{10999989 \times 9 = 99 \times 999999} &&:= \frac{aaa - aa - a}{a} \times \frac{aaaaaaa - aaaaaa - a}{a} \end{aligned}$$

Tricks for Making Pattern. [2] Let us consider numbers of 3, 4, 5 and 6 digits, for example, 183, 3568, 19757 and 876456. Changing first digit with last and vice-versa, in each case, we get, 381, 8563, 79751 and 676458 respectively. Let us consider the difference among the respective values (higher minus lesser), i.e.,

$$\begin{aligned} 381 - 183 &= 198 \\ 8563 - 3568 &= 4995 \\ 79751 - 19757 &= 59994 \\ 876456 - 676458 &= 199998 \end{aligned}$$

Changing again last digit with first and vice-versa, and adding we get the required pattern, i.e.,

$$\begin{aligned} 198 + 891 &= 1089 \\ 4995 + 5994 &= 10989 \\ 59994 + 49995 &= 109989 \\ 199998 + 899991 &= 1099989 \end{aligned}$$

Proceeding further with higher digits, we get further values of the pattern. Here the condition is that, the difference in each case should be bigger than 1, for example, $3453 - 3453 = 0$ is not valid number. Second condition is that if this differences come to 99, 999, etc, i.e., $201 - 102 = 99$, $4433 - 3434 = 999$, etc. In this situation, we have to sum twice, i.e, $99 + 99 = 198$ and $999 + 999 = 1998$, and then $198 + 891 = 1089$ and $1998 + 8991 = 10989$, etc. Alternatively, we can write it as **palindromic-type**. See below

- **Palindromic-Type Pattern**

$$\begin{aligned} 198 + 891 &= 1089 \\ 1998 + 8991 &= 10989 \\ 19998 + 89991 &= 109989 \\ 199998 + 899991 &= 1099989 \end{aligned}$$

For more examples of this kind refer author's another work [13, 14, 15].

2.6 Doubly Symmetric Patterns

Below are three examples of *doubly symmetric patterns*, i.e., we can write, $99 \times 5 = 9 \times 55$, $99 \times 7 = 9 \times 77$ and $99 \times 8 = 9 \times 88$, etc.

Example 46.

$$\begin{aligned} \mathbf{18} = \quad & \mathbf{9 \times 2 = 9 \times 2} &:= \frac{a \times (aa - a - a) \times (a + a)}{a \times a \times a} \\ \mathbf{198} = \quad & \mathbf{99 \times 2 = 9 \times 22} &:= \frac{aa \times (aa - a - a) \times (a + a)}{a \times a \times a} \\ \mathbf{1998} = \quad & \mathbf{999 \times 2 = 9 \times 222} &:= \frac{aaa \times (aa - a - a) \times (a + a)}{a \times a \times a} \\ \mathbf{19998} = \quad & \mathbf{9999 \times 2 = 9 \times 2222} &:= \frac{aaaa \times (aa - a - a) \times (a + a)}{a \times a \times a} \\ \mathbf{199998} = \quad & \mathbf{99999 \times 2 = 9 \times 22222} &:= \frac{aaaaa \times (aa - a - a) \times (a + a)}{a \times a \times a} \end{aligned}$$

Example 47.

$$27 = 9 \times 3 = 9 \times 3 := \frac{a \times (aa - a - a) \times (a + a + a)}{a \times a \times a}$$

$$297 = 99 \times 3 = 9 \times 33 := \frac{aa \times (aa - a - a) \times (a + a + a)}{a \times a \times a}$$

$$2997 = 999 \times 3 = 9 \times 333 := \frac{aaa \times (aa - a - a) \times (a + a + a)}{a \times a \times a}$$

$$29997 = 9999 \times 3 = 9 \times 3333 := \frac{aaaa \times (aa - a - a) \times (a + a + a)}{a \times a \times a}$$

$$299997 = 99999 \times 3 = 9 \times 33333 := \frac{aaaaa \times (aa - a - a) \times (a + a + a)}{a \times a \times a}$$

Example 48.

$$36 = 9 \times 4 = 9 \times 4 := \frac{a \times (aa + a) \times (a + a + a)}{a \times a \times a}$$

$$396 = 99 \times 4 = 9 \times 44 := \frac{aa \times (aa + a) \times (a + a + a)}{a \times a \times a}$$

$$3996 = 999 \times 4 = 9 \times 444 := \frac{aaa \times (aa + a) \times (a + a + a)}{a \times a \times a}$$

$$39996 = 9999 \times 4 = 9 \times 4444 := \frac{aaaa \times (aa + a) \times (a + a + a)}{a \times a \times a}$$

$$399996 = 99999 \times 4 = 9 \times 44444 := \frac{aaaaa \times (aa + a) \times (a + a + a)}{a \times a \times a}$$

Example 49.

$$45 = 9 \times 5 = 9 \times 5 := \frac{aaa - aa - aa + a}{a + a}$$

$$495 = 99 \times 5 = 9 \times 55 := \frac{aaaa - aaa - aa + a}{a + a}$$

$$4995 = 999 \times 5 = 9 \times 555 := \frac{aaaaa - aaaa - aa + a}{a + a}$$

$$49995 = 9999 \times 5 = 9 \times 5555 := \frac{aaaaaa - aaaaa - aa + a}{a + a}$$

$$499995 = 99999 \times 5 = 9 \times 55555 := \frac{aaaaaaa - aaaaaa - aa + a}{a + a}$$

Example 50.

$$54 = 9 \times 6 = 9 \times 6 := \frac{a \times (aaa - a - a - a)}{(a + a) \times a}$$

$$594 = 99 \times 6 = 9 \times 66 := \frac{aa \times (aaa - a - a - a)}{(a + a) \times a}$$

$$5994 = 999 \times 6 = 9 \times 666 := \frac{aaa \times (aaa - a - a - a)}{(a + a) \times a}$$

$$59994 = 9999 \times 6 = 9 \times 6666 := \frac{aaaa \times (aaa - a - a - a)}{(a + a) \times a}$$

$$599994 = 99999 \times 6 = 9 \times 66666 := \frac{aaaaa \times (aaa - a - a - a)}{(a + a) \times a}$$

Example 51.

$$\begin{aligned}
63 &= 9\times 7 = 9\times 7 &&:= \frac{(a+a+a)\times(aa+aa-a)}{a\times a}\\
693 &= 99\times 7 = 9\times 77 &&:= \frac{(aa+aa+aa)\times(aa+aa-a)}{a\times a}\\
6993 &= 999\times 7 = 9\times 777 &&:= \frac{(aaa+aaa+aaa)\times(aa+aa-a)}{a\times a}\\
69993 &= 9999\times 7 = 9\times 7777 &&:= \frac{(aaaa+aaaa+aaaa)\times(aa+aa-a)}{a\times a}\\
699993 &= 99999\times 7 = 9\times 77777 &&:= \frac{(aaaaa+aaaaa+aaaaa)\times(aa+aa-a)}{a\times a}
\end{aligned}$$

Example 52.

$$\begin{aligned}
72 &= 9\times 8 = 9\times 8 &&:= \frac{(aa+a)\times(aa+a)\times a}{(a+a)\times a\times a}\\
792 &= 99\times 8 = 9\times 88 &&:= \frac{(aa+a)\times(aa+a)\times aa}{(a+a)\times a\times a}\\
7992 &= 999\times 8 = 9\times 888 &&:= \frac{(aa+a)\times(aa+a)\times aaa}{(a+a)\times a\times a}\\
79992 &= 9999\times 8 = 9\times 8888 &&:= \frac{(aa+a)\times(aa+a)\times aaaa}{(a+a)\times a\times a}\\
799992 &= 99999\times 8 = 9\times 88888 &&:= \frac{(aa+a)\times(aa+a)\times aaaaa}{(a+a)\times a\times a}
\end{aligned}$$

Example 53. *(**General Way**). Examples 50, 51 and 52 can be written in a general way. Let us consider a and b, such that*

$$a,b\in\{1,2,3,4,5,6,7,8,9\},\ a+b=9,\ a>b,$$

then, examples 51 and 52 can be summarized as

$$\begin{aligned}
a9b &= 99\times A &&= 9\times AA\\
a99b &= 999\times A &&= 9\times AAA\\
a999b &= 9999\times A &&= 9\times AAAA\\
a9999b &= 99999\times A &&= 9\times AAAAA\\
a99999b &= 999999\times A &&= 9\times AAAAAA.
\end{aligned}$$

where $$A=a+1, \text{and } AAA = A\times 10^2 + A\times 10^1 + A\times 10^0, \text{etc.}$$

If $b>a$ *as in case of example 50, writing* $B=b+1$*, the above pattern changes as*

$$\begin{aligned}
a9b &= 99\times B &&= 9\times BB\\
a99b &= 999\times B &&= 9\times BBB\\
a999b &= 9999\times B &&= 9\times BBBB\\
a9999b &= 99999\times B &&= 9\times BBBBB\\
a99999b &= 999999\times B &&= 9\times BBBBBB
\end{aligned}$$

We can apply this general way to get the pattern appearing in example 45. Let us change a with b and b with a in the above pattern, then sum both, we get

$$
\begin{aligned}
a9b + b9a &= 1089\\
a99b + b99a &= 10989\\
a999b + b999a &= 109989\\
a9999b + b9999a &= 1099989\\
a99999b + b99999a &= 10999989
\end{aligned}
$$

This is the same pattern appearing in example 45. Multiplying above pattern with 8 and 9, we get respectively the patterns appearing in examples 44 and 45. Multiplying by 5 we get palindromic numbers. Here below are these three multiplications:

$$
\begin{aligned}
1089 \times 5 &= 5445\\
10989 \times 5 &= 54445\\
109989 \times 5 &= 544445\\
1099989 \times 5 &= 5444445\\
10999989 \times 5 &= 54444445
\end{aligned}
\qquad
\begin{aligned}
1089 \times 8 &= 4 \times 2178 = 8712\\
10989 \times 8 &= 4 \times 21978 = 87912\\
109989 \times 8 &= 4 \times 219978 = 879912\\
1099989 \times 8 &= 4 \times 2199978 = 8799912\\
10999989 \times 8 &= 4 \times 21999978 = 87999912
\end{aligned}
\qquad
\begin{aligned}
1089 \times 9 &= 9801\\
10989 \times 9 &= 98901\\
109989 \times 9 &= 989901\\
1099989 \times 9 &= 9899901\\
10999989 \times 9 &= 98999901.
\end{aligned}
$$

In the first case results are palindromic patterns, while second and third brings reverse numbers. Here below is another interesting relations with 1, 6 and 9:

$$
\begin{aligned}
1089 &= (916 - 619) + (961 - 169)\\
10989 &= (9116 - 6119) + (9661 - 1669)\\
109989 &= (91116 - 61119) + (96661 - 16669)\\
1099989 &= (911116 - 611119) + (966661 - 166669).
\end{aligned}
$$

Also we can write

$$
\begin{aligned}
1089 &= (996 - 699) + (991 - 199)\\
1089 &= (966 - 669) + (911 - 119)\\
1089 &= (916 - 619) + (961 - 169)\\
1089 &= (1 + 1) \times (611 - 116) + 99\\
1089 &= (1 + 1) \times (661 - 166) + 99\\
1089 &= (1 + 1) \times (691 - 196) + 99\\
1089 &= (6 \times 6 + 6 \times 6 + 6 \times 6) \times (9 + 1) + 9\\
1089 &= (1 \times 6 \times 9 + 1 \times 6 \times 9) \times (9 + 1) + 9\\
1089 &= 11 \times 11 \times 9.
\end{aligned}
$$

2.7 Number Patterns with Number Pattern Decompositions

Example 54.

$$\begin{aligned}
11111111101 = 123456789 \times 9 \times 1 &:= \frac{(aaaaaaaaaaa - aa + a)}{a} \times \frac{a}{a}\\
22222222202 = 123456789 \times 9 \times 2 &:= \frac{(aaaaaaaaaaa - aa + a)}{a} \times \frac{a+a}{a}\\
33333333303 = 123456789 \times 9 \times 3 &:= \frac{(aaaaaaaaaaa - aa + a)}{a} \times \frac{a+a+a}{a}\\
44444444404 = 123456789 \times 9 \times 4 &:= \frac{(aaaaaaaaaaa - aa + a)}{a} \times \frac{a+a+a+a}{a}\\
55555555505 = 123456789 \times 9 \times 5 &:= \frac{(aaaaaaaaaaa - aa + a)}{a} \times \frac{a+a+a+a+a}{a}\\
66666666606 = 123456789 \times 9 \times 6 &:= \frac{(aaaaaaaaaaa - aa + a)}{a} \times \frac{a+a+a+a+a+a}{a}\\
77777777707 = 123456789 \times 9 \times 7 &:= \frac{(aaaaaaaaaaa - aa + a)}{a} \times \frac{aa-a-a-a-a}{a}\\
88888888808 = 123456789 \times 9 \times 8 &:= \frac{(aaaaaaaaaaa - aa + a)}{a} \times \frac{aa-a-a-a}{a}\\
99999999909 = 123456789 \times 9 \times 9 &:= \frac{(aaaaaaaaaaa - aa + a)}{a} \times \frac{aa-a-a}{a}
\end{aligned}$$

This example is an extension of example **??**. Instead considering 12345679, we have considered all the digits, i.e., 123456789. We get number pattern on both sides, while in example **??**, we have palindromic pattern on one side of the expression.

Example 55.

$$\begin{aligned}
9 = 1 \times 8 + 1 &:= \frac{aa - a - a}{a}\\
98 = 12 \times 8 + 2 &:= \frac{aaa - aa - a - a}{a}\\
987 = 123 \times 8 + 3 &:= \frac{aaaa - aaa - aa - a - a}{a}\\
9876 = 1234 \times 8 + 4 &:= \frac{aaaaa - aaaa - aaa - aa - a - a}{a}\\
98765 = 12345 \times 8 + 5 &:= \frac{aaaaaa - aaaaa - aaaa - aaa - aa - a - a}{a}\\
987654 = 123456 \times 8 + 6 &:= \frac{aaaaaaa - aaaaaa - aaaaa - aaaa - aaa - aa - a - a}{a}\\
9876543 = 1234567 \times 8 + 7 &:= \frac{aaaaaaaa - aaaaaaa - aaaaaa - aaaaa - aaaa - aaa - aa - a - a}{a}\\
98765432 = 12345678 \times 8 + 8 &:= \frac{aaaaaaaaa - aaaaaaaa - aaaaaaa - aaaaaa - aaaaa - aaaa - aaa - aa - a - a}{a}\\
987654321 = 123456789 \times 8 + 9 &:= \frac{aaaaaaaaaa - aaaaaaaaa - aaaaaaaa - aaaaaaa - aaaaaa - aaaaa - aaaa - aaa - aa - a - a}{a}
\end{aligned}$$

Example 56.

$$\begin{aligned}
9 &= 9\times 1 = 11-2 &&:= \frac{aa-a-a}{a}\\
108 &= 9\times 12 = 111-3 &&:= \frac{aaa-a-a-a}{a}\\
1107 &= 9\times 123 = 1111-4 &&:= \frac{aaaa-a-a-a-a}{a}\\
11106 &= 9\times 1234 = 11111-5 &&:= \frac{aaaaa-a-a-a-a-a}{a}\\
111105 &= 9\times 12345 = 111111-6 &&:= \frac{aaaaaa-a-a-a-a-a-a}{a}\\
1111104 &= 9\times 123456 = 1111111-9 &&:= \frac{aaaaaaa-aa+a+a+a+a}{a}\\
11111103 &= 9\times 1234567 = 11111111-8 &&:= \frac{aaaaaaaa-aa+a+a+a}{a}\\
111111102 &= 9\times 12345678 = 111111111-9 &&:= \frac{aaaaaaaaa-aa+a+a}{a}\\
1111111101 &= 9\times 123456789 = 1111111111-10 &&:= \frac{aaaaaaaaaa-aa+a}{a}
\end{aligned}$$

Example 57.

$$\begin{aligned}
9 &= 9\times 1 = 1\times 10-1 &&:= \frac{a\times(aa-a)}{a\times a}-\frac{a}{a}\\
189 &= 9\times 21 = 2\times 100-11 &&:= \frac{(a+a)\times(aaa-aa)}{a\times a}-\frac{aa}{a}\\
2889 &= 9\times 321 = 3\times 1000-111 &&:= \frac{(a+a+a)\times(aaaa-aaa)}{a\times a}-\frac{aaa}{a}\\
38889 &= 9\times 4321 = 4\times 10000-1111 &&:= \frac{(a+a+a+a)\times(aaaaa-aaaa)}{a\times a}-\frac{aaaa}{a}\\
488889 &= 9\times 54321 = 5\times 100000-11111 &&:= \frac{(a+a+a+a+a)\times(aaaaaa-aaaaa)}{a\times a}-\frac{aaaaa}{a}\\
5888889 &= 9\times 654321 = 6\times 1000000-111111 &&:= \frac{(a+a+a+a+a+a)\times(aaaaaaa-aaaaaa)}{a\times a}-\frac{aaaaaa}{a}\\
68888889 &= 9\times 7654321 = 7\times 10000000-1111111 &&:= \frac{(aa-a-a-a-a)\times(aaaaaaaa-aaaaaaa)}{a\times a}-\frac{aaaaaaa}{a}\\
788888889 &= 9\times 87654321 = 8\times 100000000-11111111 &&:= \frac{(aa-a-a-a)\times(aaaaaaaaa-aaaaaaaa)}{a\times a}-\frac{aaaaaaaa}{a}\\
8888888889 &= 9\times 987654321 = 9\times 1000000000-111111111 &&:= \frac{(aa-a-a)\times(aaaaaaaaaa-aaaaaaaaa)}{a\times a}-\frac{aaaaaaaaa}{a}
\end{aligned}$$

Example 58.

$$81 = 9 \times 9 = 80 + 1 := \frac{(aa - a - a - a) \times (aa - a)}{a \times a} + \frac{a}{a}$$
$$882 = 9 \times 98 = 880 + 2 := \frac{(aa - a - a - a) \times (aaa - a)}{a \times a} + \frac{a + a}{a}$$
$$8883 = 9 \times 987 = 8880 + 3 := \frac{(aa - a - a - a) \times (aaaa - a)}{a \times a} + \frac{a + a + a}{a}$$
$$88884 = 9 \times 9876 = 88880 + 4 := \frac{(aa - a - a - a) \times (aaaaa - a)}{a \times a} + \frac{a + a + a + a}{a}$$
$$888885 = 9 \times 98765 = 888880 + 5 := \frac{(aa - a - a - a) \times (aaaaaa - a)}{a \times a} + \frac{a + a + a + a + a}{a}$$
$$8888886 = 9 \times 987654 = 8888880 + 6 := \frac{(aa - a - a - a) \times (aaaaaaa - a)}{a \times a} + \frac{a + a + a + a + a + a}{a}$$
$$88888887 = 9 \times 9876543 = 88888880 + 7 := \frac{(aa - a - a - a) \times (aaaaaaaa - a)}{a \times a} + \frac{aa - a - a - a - a}{a}$$
$$888888888 = 9 \times 98765432 = 888888880 + 8 := \frac{(aa - a - a - a) \times (aaaaaaaaa - a)}{a \times a} + \frac{aa - a - a - a}{a}$$
$$8888888889 = 9 \times 987654321 = 8888888880 + 9 := \frac{(aa - a - a - a) \times (aaaaaaaaaa - a)}{a \times a} + \frac{aa - a - a}{a}$$

Example 59.

$$81 = 9 \times 9 = 91 - 10 := \frac{(aa - a - a) \times (aa - a)}{a \times a} + \frac{a - aa + a)}{a}$$
$$801 = 9 \times 89 = 811 - 10 := \frac{(aa - a - a - a) \times (aaa - aa)}{a \times a} + \frac{aa - aa + a}{a}$$
$$7101 = 9 \times 789 = 7111 - 10 := \frac{(aa - a - a - a - a) \times (aaaa - aaa)}{a \times a} + \frac{aaa - aa + a}{a}$$
$$61101 = 9 \times 6789 = 61111 - 10 := \frac{(a + a + a + a + a + a) \times (aaaaa - aaaa)}{a \times a} + \frac{aaaa - aa + a}{a}$$
$$511101 = 9 \times 56789 = 511111 - 10 := \frac{(a + a + a + a + a) \times (aaaaaa - aaaaa)}{a \times a} + \frac{aaaaa - aa + a}{a}$$
$$4111101 = 9 \times 456789 = 4111111 - 10 := \frac{(a + a + a + a) \times (aaaaaaa - aaaaaa)}{a \times a} + \frac{aaaaaa - aa + a}{a}$$
$$31111101 = 9 \times 3456789 = 31111111 - 10 := \frac{(a + a + a) \times (aaaaaaaa - aaaaaaa)}{a \times a} + \frac{aaaaaaa - aa + a}{a}$$
$$211111101 = 9 \times 23456789 = 211111111 - 10 := \frac{(a + a) \times (aaaaaaaaa - aaaaaaaa)}{a \times a} + \frac{aaaaaaaa - aa + a}{a}$$
$$1111111101 = 9 \times 123456789 = 1111111111 - 10 := \frac{a \times (aaaaaaaaaa - aaaaaaaaa)}{a \times a} + \frac{aaaaaaaaa - aa + a}{a}$$

Example 60.

$$91 = 10^2 - 10^1 + 1 := \frac{aaa - aa - aa + a + a}{a}$$
$$9901 = 10^4 - 10^2 + 1 := \frac{aaaaa - aaaa - aaa + aa + a}{a}$$
$$999001 = 10^6 - 10^3 + 1 : = \frac{aaaaaaa - aaaaaa - aaaa + aaa + a}{a}$$
$$99990001 = 10^8 - 10^4 + 1 := \frac{aaaaaaaaa - aaaaaaaa - aaaaa + aaaa + a}{a}$$
$$9999900001 = 10^{10} - 10^5 + 1 := \frac{aaaaaaaaaaa - aaaaaaaaaa - aaaaaa + aaaaa + a}{a}$$

2.8 Semi-Prime Patterns

Below are some examples of **prime number patterns**. It is not necessary that the further number each example be a prime number.

Example 61.

$$
\begin{aligned}
\mathbf{31} &:= \frac{aa+aa+aa-a-a}{a}\\
\mathbf{331} &:= \frac{aaa+aaa+aaa-a-a}{a}\\
\mathbf{3331} &:= \frac{aaaa+aaaa+aaaa-a-a}{a}\\
\mathbf{33331} &:= \frac{aaaaa+aaaaa+aaaaa-a-a}{a}\\
\mathbf{333331} &:= \frac{aaaaaa+aaaaaa+aaaaaa-a-a}{a}\\
\mathbf{3333331} &:= \frac{aaaaaaa+aaaaaaa+aaaaaaa-a-a}{a}\\
\mathbf{33333331} &:= \frac{aaaaaaaa+aaaaaaaa+aaaaaaaa-a-a}{a}
\end{aligned}
$$

The next number in this case is not a prime number, i.e., we can write $333333331 = 17 \times 19607843$.

Example 62.

$$
\begin{aligned}
\mathbf{59} &:= \frac{aa\times aa-a\times a}{(a+a)\times a}-\frac{a}{a}\\
\mathbf{599} &:= \frac{aaa\times aa-a\times a}{(a+a)\times a}-\frac{aa}{a}\\
\mathbf{5999} &:= \frac{aaaa\times aa-a\times a}{(a+a)\times a}-\frac{aaa}{a}\\
\mathbf{59999} &:= \frac{aaaaa\times aa-a\times a}{(a+a)\times a}-\frac{aaaa}{a}\\
\mathbf{599999} &:= \frac{aaaaaa\times aa-a\times a}{(a+a)\times a}-\frac{aaaaa}{a}\\
\mathbf{5999999} &:= \frac{aaaaaaa\times aa-a\times a}{(a+a)\times a}-\frac{aaaaaa}{a}
\end{aligned}
$$

In this example not all the numbers are primes, for example, $5999 = 7 \times 857$, $5999999 = 1013 \times 5923$, etc.

Example 63.

$$\begin{aligned}
\mathbf{23} &:= \frac{aa+aa+a}{a}\\
\mathbf{233} &:= \frac{aaa+aaa+aa}{a}\\
\mathbf{2333} &:= \frac{aaaa+aaaa+aaa}{a}\\
\mathbf{23333} &:= \frac{aaaaa+aaaaa+aaaa}{a}\\
\mathbf{233333} &:= \frac{aaaaaa+aaaaaa+aaaaa}{a}\\
\mathbf{2333333} &:= \frac{aaaaaaa+aaaaaaa+aaaaaa}{a}
\end{aligned}$$

Here also the next number is not prime, i.e., $233333 = 353 \times 661$. After this, the next prime number is 23333333333.

Example 64.

$$\begin{aligned}
\mathbf{19} &:= \frac{aa+aa-a-a-a}{a}\\
\mathbf{199} &:= \frac{aaa+aaa-aa-aa-a}{a}\\
\mathbf{1999} &:= \frac{aaaa+aaaa-aaa-aaa-a}{a}\\
\mathbf{199999} &:= \frac{aaaaaa+aaaaaa-aaaaa-aaaaa-a}{a}\\
\mathbf{19999999} &:= \frac{aaaaaaaa+aaaaaaaa-aaaaaaa-aaaaaaa-a}{a}
\end{aligned}$$

In this example the numbers $19999 = 7 \times 2857$ and $1999999 = 17 \times 71 \times 1657$ are not prime numbers. The next number is also not prime, i.e., $199999999 = 89 \times 1447 \times 1553$.

2.9 Fibonacci Sequences and Extensions

This subsection deals with the patterns related to *Fibonacci sequences* [3, 5, 6] and its extensions or variations to *Lucas sequences*, *Tribonacci sequence*, *Tetranacci sequence*, etc.

Example 65. *(**Fibonacci sequence** The recurring formula for Fibonacci sequence is given by*

$$F_0 = 1, \quad F_1 = 1, \quad F_{n+1} = F_n + F_{n-1}, \quad n \geq 1.$$

Its values are as follows:

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 10946, 17711, 28657, 46368, 75025, 121393, 196418, 317811, 514229, 832040, ...

The above values can be obtained by subsequent divisions given below:

$$\frac{1}{89} = 0.01\ 1\ 2\ 3\ 5\ 9550561...$$

$$\frac{1}{9899} = 0.0001\ 01\ 02\ 03\ 05\ 08\ 13\ 21\ 34\ 55\ 9046368...$$

$$\frac{1}{998999} = 0.000001\ 001\ 002\ 003\ 005\ 008\ 013\ 021\ 034\ 055\ 089\ 144\ 233\ 377\ 610\ 9885995...$$

$$\frac{1}{99989999} = 1.0001\ 0002\ 0003\ 0005\ 0008\ 0013\ 0021\ 0034\ 0055\ 0089\ 0144\ 0233\ 0377\ 0610\ 0987\ 1597\ 2584\ 4181\ 6766094777 \times 10^{-8}...$$

$$\frac{1}{9999899999} = 1.00001\ 00002\ 00003\ 00005\ 00008\ 00013\ 00021\ 00034\ 00055\ 00089\ 00144\ 00233\ 00377\ 00610\ 00987\ 01597\ 02584\ 04181\ 06765\ 10946\ 17711\ 28657\ 46368\ 750262139 \times 10^{-10}$$

$$\frac{1}{999998999999} = 1.000001\ 000002\ 000003\ 000005\ 000008\ 000013\ 000021\ 000034\ 000055\ 000089\ 000144\ 000233\ 000377\ 000610\ 000987\ 001597\ 002584\ 004181\ 006765\ 010946\ 017711\ 028657\ 046368\ 075025\ 121393\ 196418\ 317811\ 514229\ 83204134 \times 10^{-12}...$$

Here we observe that as 9 increases on both sides of the division $1/89$, the resulting decimal fractions approximates more near to Fibonacci sequence values. Denominator values lead us to following pattern:

$$\mathbf{89 = 10^2 - 11} := \frac{aaa - aa - aa}{a}$$

$$\mathbf{9899 = 10^4 - 101} := \frac{aaaaa - aaaa - aaa + aa - a}{a}$$

$$\mathbf{998999 = 10^6 - 1001} := \frac{aaaaaaa - aaaaaa - aaaa + aaa - a}{a}$$

$$\mathbf{99989999 = 10^8 - 10001} := \frac{aaaaaaaaa - aaaaaaaa - aaaaa + aaaa - a}{a}$$

$$\mathbf{9999899999 = 10^{10} - 100001} := \frac{aaaaaaaaaaa - aaaaaaaaaa - aaaaaa + aaaaa - a}{a}$$

We can also write in terms of **geometric series**:

$$\frac{1}{89} = \frac{1}{10^2} + \frac{11}{10^4} + \frac{11^2}{10^6} + ...$$

$$\frac{1}{9899} = \frac{1}{10^4} + \frac{101}{10^8} + \frac{101^2}{10^{12}} + ...$$

$$\frac{1}{998999} = \frac{1}{10^6} + \frac{1001}{10^{12}} + \frac{1001^2}{10^{18}} + ...$$

$$\frac{1}{99989999} = \frac{1}{10^8} + \frac{1001}{10^{16}} + \frac{1001^2}{10^{24}} + ...$$

Example 66. *As an extension or variation of **Fibonacci sequence** is **Lucas sequence**. It is given by the following recurring formula:*

$$L_0 = 2, \quad L_1 = 1, \quad L_n = L_{n-1} + F_{n-2}, \quad n \geq 2$$

Its values are as follows

$$2, 1, 3, 4, 7, 11, 18, 29, 47, 76, 123, 199, 322, 521, 843, 1364, 2207, 3571, 5778, 9349, 15127, 24476, 39603, 64079, 103682, 167761, 271443, 439204, 710647, 1149851, ...$$

The above values are obtained by subsequent divisions formed by following pattern:

$$\frac{19}{89} = 0.2\ 1\ 3\ 4\ 83146067...$$

$$\frac{199}{9899} = 0.02\ 01\ 03\ 04\ 07\ 11\ 18\ 29\ 47\ 77250227...$$

$$\frac{1999}{998999} = 0.002\ 001\ 003\ 004\ 007\ 011\ 018\ 029\ 047\ 076\ 123\ 199\ 322\ 521\ 844210576...$$

$$\frac{19999}{99989999} = 0.0002\ 0001\ 0003\ 0004\ 000\ 70011\ 0018\ 0029\ 0047\ 0076\ 0123\ 0199\ 0322\ 0521\ 0843\ 1364\ 2207\ 3571\ 5778\ 9350512944...$$

$$\frac{199999}{9999899999} = 0.00002\ 00001\ 00003\ 00004\ 00007\ 00011\ 00018\ 00029\ 00047\ 00076\ 00123\ 00199\ 00322\ 00521\ 00843\ 01364\ 02207\ 03571\ 05778\ 09349\ 15127\ 24476\ 39603\ 640800368...$$

$$\frac{1999999}{999998999999} = 0.000002\ 000001\ 000003\ 000004\ 000007\ 000011\ 000018\ 000029\ 000047\ 000076\ 000123\ 000199\ 000322\ 000521\ 000843\ 001364\ 002207\ 003571\ 005778\ 009349\ 015127\ 024476\ 039603\ 064079\ 103682\ 167761\ 271443\ 439204\ 71064814985...$$

We observe that above expressions are formed by two patterns, i.e., one in numerator and another in denominator. Single letter representations are given by

$$\frac{\mathbf{19}}{\mathbf{89}} := \frac{aa + aa - a - a - a}{aaa - aa - aa}$$

$$\frac{\mathbf{199}}{\mathbf{9899}} := \frac{aaa + aaa - aa - aa - a}{aaaaa - aaaa - aaa + aa - a}$$

$$\frac{\mathbf{1999}}{\mathbf{998999}} := \frac{aaaa + aaaa - aaa - aaa - a}{aaaaaaa - aaaaaa - aaaa + aaa - a}$$

$$\frac{\mathbf{19999}}{\mathbf{99989999}} := \frac{aaaaa + aaaaa - aaaa - aaaa - a}{aaaaaaaaa - aaaaaaaa - aaaaa + aaaa - a}$$

$$\frac{\mathbf{199999}}{\mathbf{9999899999}} := \frac{aaaaaa + aaaaaa - aaaaa - aaaaa - a}{aaaaaaaaaaa - aaaaaaaaaa - aaaaaa + aaaaa - a}.$$

Example 67. *(**Tribonacci Sequence**) The **Tribonacci sequence** is a generalization of the **Fibonacci sequence** defined by the recurrence formula:*

$$T_1 = 1, \quad T_2 = 1, \quad T_3 = 2, \quad T_n = T_{n-1} + T_{n-2} + T_{n-3}, \quad n \geq 4$$

Its values are given by

$$1, 1, 2, 4, 7, 13, 24, 44, 81, 149, 274, 504, 927, 1705, 3136, 5768, 10609, 19513, 35890, 66012, 121415, 223317, 410744, 755476, 1389537, 2555757, 4700770, 8646064, ...$$

The above sequence values are obtained in little different form of example 65. See below the respective division pattern:

$$\frac{1}{889} = 0.001\ 1\ 2\ 4\ 8593925....$$
$$\frac{1}{989899} = 0.000001\ 01\ 02\ 04\ 07\ 13\ 24\ 44\ 82517913...$$
$$\frac{1}{998998999} = 1.001\ 002\ 004\ 007\ 013\ 024\ 044\ 081\ 149\ 274\ 504\ 92870814 \times 10^{9}...$$
$$\frac{1}{999899989999} = 1.0001\ 0002\ 0004\ 0007\ 0013\ 0024\ 0044\ 0081\ 0149\ 0274\ 0504\ 0927\ 1705\ 3136\ 576906109 \times 10^{-12}....$$
$$\frac{1}{999989999899999} = 1.00001\ 00002\ 00004\ 00007\ 00013\ 00024\ 00044\ 00081\ 00149\ 00274\ 00504\ 00927\ 01705$$
$$03136\ 05768\ 10609\ 19513\ 35890\ 6601321417 \times 10 \times -15...$$

Single digit representation of denominator in division pattern is as follows:

$$889 = 10^3 - 111 := \frac{aaaa - aaa - aaa}{a}$$
$$989899 = 10^6 - 10101 := \frac{aaaaaaa - aaaaaa - aaaaa + aaaa - aaa + aa - a}{a}$$
$$998998999 = 10^9 - 1001001 := \frac{aaaaaaaaaa - aaaaaaaaa - aaaaaaa + aaaaaa - aaaa + aaa - a}{a}$$
$$999899989999 = 10^{12} - 100010001 := \frac{aaaaaaaaaaaaa - aaaaaaaaaaaa - aaaaaaaaa + aaaaaaaa - aaaaa + aaaa - a}{a}$$

Example 68. *(**Tetranacci Sequence**) The **Tetranacci sequence** is a generalization of the **Fibonacci sequence** defined by the recurrence formula:*

$$T_0 = 0, \quad T_1 = 1, \quad T_2 = 1, \quad T_3 = 2, \quad T_n = T_{n-1} + T_{n-2} + T_{n-3} + T_{n-4}, \quad n \geq 4$$

Its values are given by

$$0, 1, 1, 2, 4, 8, 15, 29, 56, 108, 208, 401, 773, 1490, 2872, 5536, 10671, 20569, 39648, 76424, ...$$

The above sequence values are obtained in little different form of example 65. See below the respective division pattern:

$$\frac{1}{8889} = 0.0001\ 1\ 2\ 4\ 98593767....$$
$$\frac{1}{98989899} = 1.01\ 02\ 04\ 08\ 15\ 29\ 5710120888...$$
$$\frac{1}{998998998999} = 1.001\ 002\ 004\ 008\ 015\ 029\ 056\ 108\ 208\ 401\ 7744928775...$$
$$\frac{1}{9998999899989999} = 1.0001\ 0002\ 0004\ 0008\ 0015\ 0029\ 0056\ 0108\ 0208\ 0401\ 0773\ 1490\ 2872\ 55370673....$$

We can write

$$8889 = 10^4 - \frac{1111}{1} := \left(\frac{aa-a}{a}\right)^{\left(\frac{a+a+a+a}{a}\right)} - \frac{aaaa}{a}$$
$$98989899 = 10^8 - \frac{11111111}{11} := \left(\frac{aa-a}{a}\right)^{\left(\frac{aa-a-a-a}{a}\right)} - \frac{aaaaaaaa}{aa}$$
$$998998998999 = 10^{12} - \frac{111111111111}{111} := \left(\frac{aa-a}{a}\right)^{\left(\frac{aa+a}{a}\right)} - \frac{aaaaaaaaaaaa}{aaa}$$
$$9998999899989999 = 10^{16} - \frac{1111111111111111}{1111} := \left(\frac{aa-a}{a}\right)^{\left(\frac{aa+a+a+a+a+a}{a}\right)} - \frac{aaaaaaaaaaaaaaaa}{aaaa}$$

Examples 65 and 66 have the same denominator. The difference is in numerator values; one is 1 and another is pattern 19, 199,.. Thus we have three different **denominators** in above four examples 65, 66, 67, and 68. Here below are these three different patterns:

- **Fibonacci sequence**

$$\mathbf{89} = 10^2 - 11$$
$$\mathbf{9899} = 10^4 - 101$$
$$\mathbf{998999} = 10^6 - 1001$$
$$\mathbf{99989999} = 10^8 - 10001$$

- **Tribonacci sequence**

$$\mathbf{889} = 10^3 - 111$$
$$\mathbf{989899} = 10^6 - 10101$$
$$\mathbf{998998999} = 10^9 - 1001001$$
$$\mathbf{999899989999} = 10^{12} - 100010001$$

- **Tetranacci sequence**

$$\mathbf{8889} = 10^4 - 1111$$
$$\mathbf{98989899} = 10^8 - 1010101$$
$$\mathbf{998998998999} = 10^{12} - 1001001001$$
$$\mathbf{9998999899989999} = 10^{16} - 1000100010001$$

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