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Block-Wise Construction of Magic and Bimagic Squares of Orders 39 to 45

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Abstract

*This paper brings **block-wise** construction of magic squares of order 39 to 45. In order to construct these magic squares we applied the previous known magic squares of orders 3 to 14, except order 12. In each case these are written again. Specially, in case of magic square of order 40, one of the possibility is **bimagic** square. The **block-wise** construction of magic squares of orders 8 to 36 can be seen in author's work [23, 24, 25, 26, 28, 31].*

Contents

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1 Introduction

In the previous works [23, 24, 25, 26, 28, 31], the author worked with **block-wise** constructions of magic squares. The work is from the orders 8 to 36. This paper brings **block-wise** construction of magic squares from the orders 39 to 42, i.e., for the orders 39, 40 and 42. In each case, all the possibilities are considered, for example, in case of order 42, we have the possibilities such as, 3×14, 6×7, 7×6 and 14×3. It depends on the magic sums division, where we shall have equal magic sums blocks or different magic sums blocks. Let's see how it works? The magic sums of order n of consecutive numbers from 1 to n^2 is given by

$$S_{n\times n} := \frac{n \times (1+n^2)}{2}, n \geq 3.$$

This formula is applied to know the possible blocks of magic squares from orders 8 to 45. Especially, in this paper for the magic squares of orders 39 to 45. That is, whenever is possible, we tried to bring equal sums blocks magic squares. In some cases, they are **semi-magic** or **pandiagonal**. The **semi-magic** happens for the blocks of order 3.

For example, in case of order 44, the magic sum is $S_{44\times 44} := 42614$. The possible blocks are 4×11, 11×4, and 22×2. This sums is divisible by 11 and 2, but not by 4. See below:

$$\begin{aligned}
&(i) && \frac{42614}{11} = 3874 && \Longrightarrow && \text{equal blocks of order 4;}\\
&(ii) && \frac{42614}{4} = 10653.5 && \Longrightarrow && \text{unequal blocks of order 11;}\\
&(iii) && \frac{42614}{2} = 21307 && \Longrightarrow && \text{equal blocks of order 22.}
\end{aligned}$$

This means that we can construct magic square of order 44 with equal magic sums of blocks 4 and 22. In case of blocks of magic squares of order 11, the magic sums are different. This philosophy is applied to all possible blocks of magic squares from orders 8 to 45. Especially, in this paper for the magic squares of orders 39 to 45. That is, whenever is possible, we tried to bring equal sums blocks magic squares. In some cases, they are semi-magic or pan diagonal. The semi-magic happens in case of blocks of order 3.

2 Magic Squares of Order 39

Block-wise construction of magic squares of order 39 depends on the product 3×13, i.e., either we can construct it by blocks of order 3 or by blocks of order 13. The magic square sum of order 39 is given by

$$S_{39\times 39} := \frac{39 \times (1+39^2)}{2} = 29679.$$

This sums is divisible by 3 and 13. See below:

$$\begin{aligned}
&(i) && \frac{29679}{13} = 2293 && \Longrightarrow && \text{equal blocks of order 3;}\\
&(ii) && \frac{29679}{3} = 9893 && \Longrightarrow && \text{equal blocks of order 13.}
\end{aligned}$$

This implies that we can made **block-wise** construction of magic square of order 39 where each block of order 3 is of same magic sum. Also each block of order 13 of is of same magic sum. In order to construct these two magic squares we need magic squares of orders 3 and 13. These are given below:

Example 2.1. *Let's consider a* ***magic square of 3*** *with Latin squares decomposition and* ***composite*** *magic square:*

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L			6
2	3	1	6
1	2	3	6
3	1	2	6
6	6	6	6

M			6
1	3	2	6
3	2	1	6
2	1	3	6
6	6	6	6

M_3			15
4	9	2	15
3	5	7	15
8	1	6	15
15	15	15	15

C_3			66
21	33	12	66
13	22	31	66
32	11	23	66
66	66	66	66

The magic squares M_3 and C_3 are obtained by using the operations

$$3 \times (A-1) + B := M_3 \quad \text{and} \quad 10 \times A + B := C_3,$$

respectively. The M_3 is magic square of order 3 of consecutive numbers from 1 to 9, and C_3 is the **composite** *magic square.*

Example 2.2. *Let's consider a* **pan diagonal** *magic square of 13 with Latin squares decomposition and* **composite** *magic square:*

L		91	91	91	91	91	91	91	91	91	91	91	91	91
	1	2	3	4	5	6	7	8	9	10	11	12	13	91
91	12	13	1	2	3	4	5	6	7	8	9	10	11	91
91	10	11	12	13	1	2	3	4	5	6	7	8	9	91
91	8	9	10	11	12	13	1	2	3	4	5	6	7	91
91	6	7	8	9	10	11	12	13	1	2	3	4	5	91
91	4	5	6	7	8	9	10	11	12	13	1	2	3	91
91	2	3	4	5	6	7	8	9	10	11	12	13	1	91
91	13	1	2	3	4	5	6	7	8	9	10	11	12	91
91	11	12	13	1	2	3	4	5	6	7	8	9	10	91
91	9	10	11	12	13	1	2	3	4	5	6	7	8	91
91	7	8	9	10	11	12	13	1	2	3	4	5	6	91
91	5	6	7	8	9	10	11	12	13	1	2	3	4	91
91	3	4	5	6	7	8	9	10	11	12	13	1	2	91
	91	91	91	91	91	91	91	91	91	91	91	91	91	91

M		91	91	91	91	91	91	91	91	91	91	91	91	91
	1	12	10	8	6	4	2	13	11	9	7	5	3	91
91	2	13	11	9	7	5	3	1	12	10	8	6	4	91
91	3	1	12	10	8	6	4	2	13	11	9	7	5	91
91	4	2	13	11	9	7	5	3	1	12	10	8	6	91
91	5	3	1	12	10	8	6	4	2	13	11	9	7	91
91	6	4	2	13	11	9	7	5	3	1	12	10	8	91
91	7	5	3	1	12	10	8	6	4	2	13	11	9	91
91	8	6	4	2	13	11	9	7	5	3	1	12	10	91
91	9	7	5	3	1	12	10	8	6	4	2	13	11	91
91	10	8	6	4	2	13	11	9	7	5	3	1	12	91
91	11	9	7	5	3	1	12	10	8	6	4	2	13	91
91	12	10	8	6	4	2	13	11	9	7	5	3	1	91
91	13	11	9	7	5	3	1	12	10	8	6	4	2	91
	91	91	91	91	91	91	91	91	91	91	91	91	91	91

M_{13}		1105	1105	1105	1105	1105	1105	1105	1105	1105	1105	1105	1105	1105
	1	25	36	47	58	69	80	104	115	126	137	148	159	1105
1105	145	169	11	22	33	44	55	66	90	101	112	123	134	1105
1105	120	131	155	166	8	19	30	41	65	76	87	98	109	1105
1105	95	106	130	141	152	163	5	16	27	51	62	73	84	1105
1105	70	81	92	116	127	138	149	160	2	26	37	48	59	1105
1105	45	56	67	91	102	113	124	135	146	157	12	23	34	1105
1105	20	31	42	53	77	88	99	110	121	132	156	167	9	1105
1105	164	6	17	28	52	63	74	85	96	107	118	142	153	1105
1105	139	150	161	3	14	38	49	60	71	82	93	117	128	1105
1105	114	125	136	147	158	13	24	35	46	57	68	79	103	1105
1105	89	100	111	122	133	144	168	10	21	32	43	54	78	1105
1105	64	75	86	97	108	119	143	154	165	7	18	29	40	1105
1105	39	50	61	72	83	94	105	129	140	151	162	4	15	1105
	1105	1105	1105	1105	1105	1105	1105	1105	1105	1105	1105	1105	1105	1105

11	2T	3R	48	56	64	72	8U	9S	R9	S7	T5	U3
T2	UU	1S	29	37	45	53	61	7T	8R	98	R6	S4
R3	S1	TT	UR	18	26	34	42	5U	6S	79	87	95
84	92	RU	SS	T9	U7	15	23	31	4T	5R	68	76
65	73	81	9T	RR	S8	T6	U4	12	2U	3S	49	57
46	54	62	7U	8S	99	R7	S5	T3	U1	1T	2R	38
27	35	43	51	6T	7R	88	96	R4	S2	TU	US	19
U8	16	24	32	4U	5S	69	77	85	93	R1	ST	TR
S9	T7	U5	13	21	3T	4R	58	66	74	82	9U	RS
9R	R8	S6	T4	U2	1U	2S	39	47	55	63	71	8T
7S	89	97	R5	S3	T1	UT	1R	28	36	44	52	6U
5T	6R	78	86	94	R2	SU	TS	U9	17	25	33	41
3U	4S	59	67	75	83	91	RT	SR	T8	U6	14	22
						C_{13}						

where $R := 10$, $S := 11$, $T := 12$, and $U := 13$. The magic squares M_{13} and C_{13} are obtained by using the operations

$$13 \times (A-1) + B := M_{13} \quad \text{and} \quad 10 \times A + B := C_{13},$$

respectively. The M_{13} is magic square of order 13 of consecutive numbers from 1 to 169, and C_{13} is the ***composite*** *magic square.*

2.1 Blocks Order 3

In order to construct magic square of order 39 as sub-blocks of order 3, we shall use the magic square of order 3 given in Example 2.1. Also we shall make use of magic rectangle of order 3 × 13 given in example below:

Example 2.3. *The magic rectangle of order 3 × 13 is given by*

	1	2	3	4	5	6	7	8	9	10	11	12	13	Total
1	34	2	21	26	8	22	10	31	16	33	37	5	15	260
2	1	23	36	27	28	29	20	11	12	13	4	17	39	260
3	25	35	3	7	24	9	30	18	32	14	19	38	6	260
Total	60	60	60	60	60	60	60	60	60	60	60	60	60	

Distribution 2.1. *Let's consider following composite distribution:*

1.1	2.1	3.1	4.1	5.1	6.1	7.1	8.1	9.1	10.1	11.1	12.1	13.1
1.2	2.2	3.2	4.2	5.2	6.2	7.2	8.2	9.2	10.2	11.2	12.2	13.2
1.3	2.3	3.3	4.3	5.3	6.3	7.3	8.3	9.3	10.3	11.3	12.3	13.3
1.4	2.4	3.4	4.4	5.4	6.4	7.4	8.4	9.4	10.4	11.4	12.4	13.4
1.5	2.5	3.5	4.5	5.5	6.5	7.5	8.5	9.5	10.5	11.5	12.5	13.5
1.6	2.6	3.6	4.6	5.6	6.6	7.6	8.6	9.6	10.6	11.6	12.6	13.6
1.7	2.7	3.7	4.7	5.7	6.7	7.7	8.7	9.7	10.7	11.7	12.7	13.7
1.8	2.8	3.8	4.8	5.8	6.8	7.8	8.8	9.8	10.8	11.8	12.8	13.8
1.9	2.9	3.9	4.9	5.9	6.9	7.9	8.9	9.9	10.9	11.9	12.9	13.9
1.10	2.10	3.10	4.10	5.10	6.10	7.10	8.10	9.10	10.10	11.10	12.10	13.10
1.11	2.11	3.11	4.11	5.11	6.11	7.11	8.11	9.11	10.11	11.11	12.11	13.11
1.12	2.12	3.12	4.12	5.12	6.12	7.12	8.12	9.12	10.12	11.12	12.12	13.12
1.13	2.13	3.13	4.13	5.13	6.13	7.13	8.13	9.13	10.13	11.13	12.13	13.13

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We shall construct 169 blocks of order 3, and put them according to Distribution 2.1. Below are few examples of **semi-magic** squares of order 3 constructed by applying the columns values given in Example 2.3 over the Example 2.1 by using the operation $M_3 := 39 \times (A-1) + B$. Below are few examples:

• Block 1.3

(1)			60
1	25	34	60
34	1	25	60
25	34	1	60
60	60	60	3

(3)			108
21	3	36	60
3	36	21	60
36	21	3	60
60	60	60	60

(1.3)			2331
21	939	1323	2283
1290	36	957	2283
972	1308	3	2283
2283	2283	2283	60

• Block 3.2

(3)			60
36	3	21	60
21	36	3	60
3	21	36	60
60	60	60	108

(2)			69
2	35	23	60
35	23	2	60
23	2	35	60
60	60	60	60

(3.2)			2292
1367	113	803	2283
815	1388	80	2283
101	782	1400	2283
2283	2283	2283	4155

• Block 5.4

(5)			60
28	24	8	60
8	28	24	60
24	8	28	60
60	60	60	84

(4)			81
26	7	27	60
7	27	26	60
27	26	7	60
60	60	60	60

(5.4)			2304
1079	904	300	2283
280	1080	923	2283
924	299	1060	2283
2283	2283	2283	3219

Based on similar procedure we construct all the 169 blocks of **semi-magic** squares (in rows and columns) and put them according to Distribution 2.1, we get the required **pan diagonal** magic square of order 39 given in example below.

Example 2.4. . *The **block-wise pan diagonal** magic square of order 39 with **semi-magic** blocks of order 3 of equal sums is given by*

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Pan diagonal magic square of order 39 with each block of order 3 a semi-magic square with equal magic sums, S(3x3):=2283 (rows and columns). Magic sum S(39x39):=29679.

		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	
		pan	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679
1	29679	34	961	1288	860	1361	62	1386	81	816	1040	241	1002	1061	921	301	1114	321	848	751	1161	371	421	681	1181	445	1241	597	501	521	1261	154	721	1408	629	1481	173	1497	201	585	29679
2	29679	1312	1	970	74	881	1328	783	1401	99	982	1041	260	297	1081	905	828	1121	334	381	761	1141	1188	401	694	617	441	1225	1262	481	540	1423	121	739	194	641	1448	552	1521	210	29679
3	29679	937	1321	25	1349	41	893	114	801	1368	261	1001	1021	925	281	1077	341	841	1101	1151	361	771	674	1201	408	1221	601	461	520	1281	482	706	1441	136	1460	161	662	234	561	1488	29679
4	29679	661	1462	160	1487	233	563	15	942	1326	892	1351	40	1367	113	803	1035	237	1011	1079	904	300	1100	336	847	763	1140	380	400	693	1190	460	1227	596	484	539	1260	150	716	1417	29679
5	29679	175	628	1480	584	1499	200	1293	39	951	64	859	1360	815	1388	80	978	1050	255	280	1080	923	843	1120	320	360	770	1153	1200	410	673	603	440	1240	1280	480	523	1418	130	735	29679
6	29679	1447	193	643	212	551	1520	975	1302	6	1327	73	883	101	782	1400	270	996	1017	924	299	1060	340	827	1116	1160	373	750	683	1180	420	1220	616	447	519	1264	500	715	1437	131	29679
7	29679	499	525	1259	133	734	1416	657	1457	169	1519	214	550	5	974	1304	873	1332	78	1399	103	781	1016	269	998	1074	900	309	1118	319	846	749	1155	379	412	672	1199	439	1239	605	29679
8	29679	1266	479	538	1436	129	718	170	637	1476	565	1486	232	1325	17	941	45	897	1341	805	1366	112	1010	1037	236	276	1089	918	826	1119	338	375	769	1139	1179	419	685	615	449	1219	29679
9	29679	518	1279	486	714	1420	149	1456	189	638	199	583	1501	953	1292	38	1365	54	864	79	814	1390	257	977	1049	933	294	1056	339	845	1099	1159	359	765	692	1192	399	1229	595	459	29679
10	29679	398	687	1198	451	1218	614	478	537	1268	148	720	1415	640	1475	168	1515	209	559	37	955	1291	863	1364	56	1380	84	819	1048	259	976	1055	932	296	1113	315	855	767	1138	378	29679
11	29679	1194	418	671	594	458	1231	1278	488	517	1422	128	733	188	636	1459	560	1495	228	1306	4	973	77	875	1331	786	1404	93	1000	1015	268	308	1076	899	822	1128	333	358	768	1157	29679
12	29679	691	1178	414	1238	607	438	527	1258	498	713	1435	135	1455	172	656	208	579	1496	940	1324	19	1343	44	896	117	795	1371	235	1009	1039	920	275	1088	348	840	1095	1158	377	748	29679
13	29679	1094	347	842	762	1134	387	416	670	1197	437	1233	613	490	516	1277	127	732	1424	655	1461	167	1498	227	558	33	950	1300	895	1345	43	1370	116	797	1029	240	1014	1087	922	274	29679
14	29679	854	1115	314	354	777	1152	1177	417	689	609	457	1217	1257	497	529	1434	137	712	174	635	1474	578	1494	211	1301	13	969	58	862	1363	818	1382	83	981	1053	249	298	1054	931	29679
15	29679	335	821	1127	1167	372	744	690	1196	397	1237	593	453	536	1270	477	722	1414	147	1454	187	642	207	562	1514	949	1320	14	1330	76	877	95	785	1403	273	990	1020	898	307	1078	29679
16	29679	1019	272	992	1068	903	312	1126	337	820	743	1166	374	411	666	1206	455	1216	612	476	531	1276	139	711	1433	634	1473	176	1513	213	557	16	968	1299	891	1340	52	1402	97	784	29679
17	29679	1013	1031	239	279	1092	912	844	1093	346	386	764	1133	1173	426	684	592	456	1235	1272	496	515	1413	146	724	186	644	1453	564	1493	226	1319	12	952	53	871	1359	799	1369	115	29679
18	29679	251	980	1052	936	288	1059	313	853	1117	1154	353	776	699	1191	393	1236	611	436	535	1256	492	731	1426	126	1463	166	654	206	577	1500	948	1303	32	1339	72	872	82	817	1384	29679
19	29679	874	1358	51	1398	92	793	1051	253	979	1058	935	290	1107	318	858	775	1156	352	392	698	1193	450	1212	621	494	514	1275	125	726	1432	646	1452	185	1492	225	566	31	954	1298	29679
20	29679	71	870	1342	794	1378	111	994	1018	271	311	1070	902	825	1131	327	376	742	1165	1205	413	665	588	465	1230	1255	495	533	1428	145	710	165	653	1465	576	1502	205	1305	11	967	29679
21	29679	1338	55	890	91	813	1379	238	1012	1033	914	278	1091	351	834	1098	1132	385	766	686	1172	425	1245	606	432	534	1274	475	730	1412	141	1472	178	633	215	556	1512	947	1318	18	29679
22	29679	1504	204	575	10	966	1307	889	1344	50	1381	110	792	1047	248	988	1090	916	277	1097	350	836	756	1137	390	424	688	1171	431	1244	608	489	510	1284	143	709	1431	632	1467	184	29679
23	29679	555	1511	217	1317	20	946	57	869	1357	812	1377	94	989	1027	267	292	1057	934	857	1109	317	357	780	1146	1195	391	697	620	452	1211	1251	504	528	1411	144	728	180	652	1451	29679
24	29679	224	568	1491	956	1297	30	1337	70	876	90	796	1397	247	1008	1028	901	310	1072	329	824	1130	1170	366	747	664	1204	415	1232	587	464	543	1269	471	729	1430	124	1471	164	648	29679
25	29679	138	705	1440	650	1450	183	1490	219	574	22	945	1316	868	1356	59	1396	96	791	1030	266	987	1086	911	286	1129	331	823	746	1169	368	405	669	1209	463	1234	586	470	542	1271	29679
26	29679	1407	153	723	163	651	1469	570	1510	203	1296	29	958	69	878	1336	798	1376	109	1007	1026	250	287	1066	930	838	1096	349	389	758	1136	1176	429	678	610	430	1243	1283	491	509	29679
27	29679	738	1425	120	1470	182	631	223	554	1506	965	1309	9	1346	49	888	89	811	1383	246	991	1046	910	306	1067	316	856	1111	1148	356	779	702	1185	396	1210	619	454	530	1250	503	29679
28	29679	444	1215	624	502	532	1249	119	737	1427	645	1446	192	1508	202	573	8	960	1315	880	1335	68	1375	108	800	1045	252	986	1069	929	285	1125	326	832	778	1150	355	395	701	1187	29679
29	29679	591	468	1224	1273	469	541	1439	140	704	159	660	1464	553	1509	221	1311	28	944	48	887	1348	810	1385	88	993	1025	265	305	1065	913	833	1105	345	370	745	1168	1208	407	668	29679
30	29679	1248	600	435	508	1282	493	725	1406	152	1479	177	627	222	572	1489	964	1295	24	1355	61	867	98	790	1395	245	1006	1032	909	289	1085	325	852	1106	1135	388	760	680	1175	428	29679
31	29679	774	1145	364	427	682	1174	434	1247	602	483	513	1287	151	727	1405	626	1478	179	1503	198	582	26	943	1314	866	1350	67	1387	87	809	1024	264	995	1084	915	284	1108	344	831	29679
32	29679	365	754	1164	1189	394	700	623	446	1214	1254	507	522	1429	118	736	191	647	1445	549	1518	216	1294	27	962	63	886	1334	789	1394	100	1005	1034	244	291	1064	928	851	1104	328	29679
33	29679	1144	384	755	667	1207	409	1226	590	467	546	1263	474	703	1438	142	1466	158	659	231	567	1485	963	1313	7	1354	47	882	107	802	1374	254	985	1044	908	304	1071	324	835	1124	29679
34	29679	1063	927	293	1123	330	830	757	1163	363	423	677	1183	466	1228	589	473	545	1265	132	708	1443	658	1468	157	1484	230	569	21	939	1323	884	1333	66	1373	102	808	1036	243	1004	29679
35	29679	303	1073	907	837	1103	343	383	753	1147	1184	403	696	604	433	1246	1286	485	512	1410	156	717	181	625	1477	581	1505	197	1290	36	957	46	885	1352	804	1393	86	984	1043	256	29679
36	29679	917	283	1083	323	850	1110	1143	367	773	676	1203	404	1213	622	448	524	1253	506	741	1419	123	1444	190	649	218	548	1517	972	1308	3	1353	65	865	106	788	1389	263	997	1023	29679
37	29679	1391	85	807	1022	258	1003	1075	906	302	1102	342	839	772	1149	362	406	695	1182	462	1223	598	505	526	1252	122	740	1421	639	1449	195	1516	220	547	2	971	1310	879	1329	75	29679
38	29679	787	1392	104	999	1042	242	282	1082	919	849	1112	322	369	752	1162	1202	402	679	599	442	1242	1267	472	544	1442	134	707	162	663	1458	571	1483	229	1322	23	938	42	894	1347	29679
39		105	806	1372	262	983	1038	926	295	1062	332	829	1122	1142	382	759	675	1186	422	1222	618	443	511	1285	487	719	1409	155	1482	171	630	196	580	1507	959	1289	35	1362	60	861	29679
		29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679

Thus we observe that we have a **pan diagonal** magic square of order 39 with 169 blocks of order 3 with equal **semi-magic** sums. The magic and semi-magic sums are $S_{39\times 39} := 29679$ and $S_{3\times 3} := 2283$ (only rows and columns) respectively.

2.2 Blocks of Order 13

In order to construct magic square of order 39 as sub-blocks of order 13, we shall use the magic square of order 13 given in Example 2.2. Also we shall again make use of magic rectangle given in Example 2.3. For simplicity, let's write it in vertical type, i.e., magic rectangle of order 13×3:

Example 2.5. *The magic rectangle of order* 13×3 *is given by*

	1	2	3	Total
1	34	1	25	60
2	2	23	35	60
3	21	36	3	60
4	26	27	7	60
5	8	28	24	60
6	22	29	9	60
7	10	20	30	60
8	31	11	18	60
9	16	12	32	60
10	33	13	14	60
11	37	4	19	60
12	5	17	38	60
13	15	39	6	60
Total	260	260	260	

Distribution 2.2. *Let's consider following distribution of order 3:*

11	21	31
12	22	32
13	23	33

We shall construct 9 block of order 13, and put them according to Distribution 2.2. Below are few examples of **pan diagonal** magic squares of order 13 constructed by applying the columns values given in magic rectangle 2.5 over the Example 2.2 by using the operation $M_{13} := 39 \times (A-1) + B$:

- **Block 23**

(2)		260	260	260	260	260	260	260	260	260	260	260	260	260
	25	35	3	7	24	9	30	18	32	14	19	38	6	260
260	38	6	25	35	3	7	24	9	30	18	32	14	19	260
260	14	19	38	6	25	35	3	7	24	9	30	18	32	260
260	18	32	14	19	38	6	25	35	3	7	24	9	30	260
260	9	30	18	32	14	19	38	6	25	35	3	7	24	260
260	7	24	9	30	18	32	14	19	38	6	25	35	3	260
260	35	3	7	24	9	30	18	32	14	19	38	6	25	260
260	6	25	35	3	7	24	9	30	18	32	14	19	38	260
260	19	38	6	25	35	3	7	24	9	30	18	32	14	260
260	32	14	19	38	6	25	35	3	7	24	9	30	18	260
260	30	18	32	14	19	38	6	25	35	3	7	24	9	260
260	24	9	30	18	32	14	19	38	6	25	35	3	7	260
260	3	7	24	9	30	18	32	14	19	38	6	25	35	260
	260	260	260	260	260	260	260	260	260	260	260	260	260	260

(3)		260	260	260	260	260	260	260	260	260	260	260	260	260
	1	17	13	11	29	27	23	39	4	12	20	28	36	260
260	23	39	4	12	20	28	36	1	17	13	11	29	27	260
260	36	1	17	13	11	29	27	23	39	4	12	20	28	260
260	27	23	39	4	12	20	28	36	1	17	13	11	29	260
260	28	36	1	17	13	11	29	27	23	39	4	12	20	260
260	29	27	23	39	4	12	20	28	36	1	17	13	11	260
260	20	28	36	1	17	13	11	29	27	23	39	4	12	260
260	11	29	27	23	39	4	12	20	28	36	1	17	13	260
260	12	20	28	36	1	17	13	11	29	27	23	39	4	260
260	13	11	29	27	23	39	4	12	20	28	36	1	17	260
260	4	12	20	28	36	1	17	13	11	29	27	23	39	260
260	17	13	11	29	27	23	39	4	12	20	28	36	1	260
260	39	4	12	20	28	36	1	17	13	11	29	27	23	260
	260	260	260	260	260	260	260	260	260	260	260	260	260	260

(23)		*9893*	*9893*	*9893*	*9893*	*9893*	*9893*	*9893*	*9893*	*9893*	*9893*	*9893*	*9893*	*9893*
	937	1343	91	245	926	339	1154	702	1213	519	722	1471	231	*9893*
9893	1466	234	940	1338	98	262	933	313	1148	676	1220	536	729	*9893*
9893	543	703	1460	208	947	1355	105	257	936	316	1143	683	1237	*9893*
9893	690	1232	546	706	1455	215	964	1362	79	251	910	323	1160	*9893*
9893	340	1167	664	1226	520	713	1472	222	959	1365	82	246	917	*9893*
9893	263	924	335	1170	667	1221	527	730	1479	196	953	1339	89	*9893*
9893	1346	106	270	898	329	1144	674	1238	534	725	1482	199	948	*9893*
9893	206	965	1353	101	273	901	324	1151	691	1245	508	719	1456	*9893*
9893	714	1463	223	972	1327	95	247	908	341	1158	686	1248	511	*9893*
9893	1222	518	731	1470	218	975	1330	90	254	925	348	1132	680	*9893*
9893	1135	675	1229	535	738	1444	212	949	1337	107	261	920	351	*9893*
9893	914	325	1142	692	1236	530	741	1447	207	956	1354	114	235	*9893*
9893	117	238	909	332	1159	699	1210	524	715	1454	224	963	1349	*9893*
	9893	*9893*	*9893*	*9893*	*9893*	*9893*	*9893*	*9893*	*9893*	*9893*	*9893*	*9893*	*9893*	*9893*

In the similar way, let's construct other 8 blocks of order 13 and put them according to Distributions 2.2, we get a **pan diagonal** magic square of order 39 given in example below:

Example 2.6. *The **block-wise pan diagonal** magic square of order 39 with pan diagonal magic square blocks of order 13 is given by*

https://doi.org/10.5281/zenodo.2555889

Pan diagonal magic square of order 39 with equal sums magic squares of order 13. Magic sums are S(39x39):=29679 and S(13x13):=9893.

		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	
	pan	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679
1	29679	41	190	304	373	562	595	785	856	1008	1196	1269	1302	1412	2	151	421	451	484	634	746	895	1047	1079	1113	1380	1490	80	229	265	334	523	673	707	934	969	1157	1230	1341	1451	29679
2	29679	1292	1441	72	182	294	366	554	587	814	850	997	1186	1258	1370	1519	33	143	411	444	476	626	775	889	1036	1069	1102	1331	1480	111	221	255	327	515	665	736	928	958	1147	1219	29679
3	29679	1178	1250	1321	1435	61	172	283	356	583	618	806	840	990	1061	1094	1399	1513	22	133	400	434	505	657	767	879	1029	1139	1211	1360	1474	100	211	244	317	544	696	728	918	951	29679
4	29679	829	980	1207	1281	1313	1425	54	164	275	385	577	607	796	868	1019	1090	1125	1391	1503	15	125	392	463	499	646	757	907	941	1168	1242	1352	1464	93	203	236	346	538	685	718	29679
5	29679	600	788	821	1009	1201	1270	1303	1414	44	193	306	377	567	639	749	860	1048	1084	1114	1381	1492	5	154	423	455	489	678	710	899	970	1162	1231	1342	1453	83	232	267	338	528	29679
6	29679	367	556	590	817	852	1001	1191	1263	1295	1406	73	187	295	445	478	629	778	891	1040	1074	1107	1373	1484	34	148	412	328	517	668	739	930	962	1152	1224	1334	1445	112	226	256	29679
7	29679	177	288	359	548	619	811	841	991	1180	1253	1324	1437	65	138	405	437	470	658	772	880	1030	1063	1097	1402	1515	26	216	249	320	509	697	733	919	952	1141	1214	1363	1476	104	29679
8	29679	1426	55	166	278	388	579	611	801	834	983	1172	1282	1318	1504	16	127	395	466	501	650	762	873	1022	1055	1126	1396	1465	94	205	239	349	540	689	723	912	944	1133	1243	1357	29679
9	29679	1274	1308	1419	47	158	307	382	568	601	790	824	1012	1203	1118	1386	1497	8	119	424	460	490	640	751	863	1051	1086	1235	1347	1458	86	197	268	343	529	679	712	902	973	1164	29679
10	29679	1006	1192	1264	1297	1409	76	189	299	372	561	593	782	853	1045	1075	1108	1375	1487	37	150	416	450	483	632	743	892	967	1153	1225	1336	1448	115	228	260	333	522	671	704	931	29679
11	29679	813	845	996	1185	1256	1289	1438	70	178	289	361	551	622	774	884	1035	1068	1100	1367	1516	31	139	406	439	473	661	735	923	957	1146	1217	1328	1477	109	217	250	322	512	700	29679
12	29679	580	616	802	835	985	1175	1285	1320	1430	60	171	281	353	502	655	763	874	1024	1058	1129	1398	1508	21	132	398	431	541	694	724	913	946	1136	1246	1359	1469	99	210	242	314	29679
13	29679	310	384	572	606	795	827	977	1204	1279	1309	1420	49	161	427	462	494	645	756	866	1016	1087	1123	1387	1498	10	122	271	345	533	684	717	905	938	1165	1240	1348	1459	88	200	29679
14	29679	40	192	301	374	563	597	784	858	1004	1197	1268	1300	1415	1	153	418	452	485	636	745	897	1043	1080	1112	1378	1493	79	231	262	335	524	675	706	936	965	1158	1229	1339	1454	29679
15	29679	1291	1443	68	183	293	364	557	586	816	847	998	1187	1260	1369	1521	29	144	410	442	479	625	777	886	1037	1070	1104	1330	1482	107	222	254	325	518	664	738	925	959	1148	1221	29679
16	29679	1181	1249	1323	1432	62	173	285	355	585	614	807	839	988	1064	1093	1401	1510	23	134	402	433	507	653	768	878	1027	1142	1210	1362	1471	101	212	246	316	546	692	729	917	949	29679
17	29679	831	979	1209	1277	1314	1424	52	167	274	387	574	608	797	870	1018	1092	1121	1392	1502	13	128	391	465	496	647	758	909	940	1170	1238	1353	1463	91	206	235	348	535	686	719	29679
18	29679	598	791	820	1011	1198	1271	1304	1416	43	195	302	378	566	637	752	859	1050	1081	1115	1382	1494	4	156	419	456	488	676	713	898	972	1159	1232	1343	1455	82	234	263	339	527	29679
19	29679	368	558	589	819	848	1002	1190	1261	1298	1405	75	184	296	446	480	628	780	887	1041	1073	1105	1376	1483	36	145	413	329	519	667	741	926	963	1151	1222	1337	1444	114	223	257	29679
20	29679	176	286	362	547	621	808	842	992	1182	1252	1326	1433	66	137	403	440	469	660	769	881	1031	1065	1096	1404	1511	27	215	247	323	508	699	730	920	953	1143	1213	1365	1472	105	29679
21	29679	1427	56	168	277	390	575	612	800	832	986	1171	1284	1315	1505	17	129	394	468	497	651	761	871	1025	1054	1128	1393	1466	95	207	238	351	536	690	722	910	947	1132	1245	1354	29679
22	29679	1275	1307	1417	50	157	309	379	569	602	792	823	1014	1199	1119	1385	1495	11	118	426	457	491	641	753	862	1053	1082	1236	1346	1456	89	196	270	340	530	680	714	901	975	1160	29679
23	29679	1003	1193	1265	1299	1408	78	185	300	371	559	596	781	855	1042	1076	1109	1377	1486	39	146	417	449	481	635	742	894	964	1154	1226	1338	1447	117	224	261	332	520	674	703	933	29679
24	29679	809	846	995	1183	1259	1288	1440	67	179	290	363	550	624	770	885	1034	1066	1103	1366	1518	28	140	407	441	472	663	731	924	956	1144	1220	1327	1479	106	218	251	324	511	702	29679
25	29679	582	613	803	836	987	1174	1287	1316	1431	59	169	284	352	504	652	764	875	1026	1057	1131	1394	1509	20	130	401	430	543	691	725	914	948	1135	1248	1355	1470	98	208	245	313	29679
26	29679	312	380	573	605	793	830	976	1206	1276	1310	1421	51	160	429	458	495	644	754	869	1015	1089	1120	1388	1499	12	121	273	341	534	683	715	908	937	1167	1237	1349	1460	90	199	29679
27	29679	42	191	303	375	564	594	786	857	1007	1195	1267	1301	1411	3	152	420	453	486	633	747	896	1046	1078	1111	1379	1489	81	230	264	336	525	672	708	935	968	1156	1228	1340	1450	29679
28	29679	1293	1442	71	181	292	365	553	588	815	849	999	1188	1257	1371	1520	32	142	409	443	475	627	776	888	1038	1071	1101	1332	1481	110	220	253	326	514	666	737	927	960	1149	1218	29679
29	29679	1177	1251	1322	1434	63	174	282	357	584	617	805	838	989	1060	1095	1400	1512	24	135	399	435	506	656	766	877	1028	1138	1212	1361	1473	102	213	243	318	545	695	727	916	950	29679
30	29679	828	981	1208	1280	1312	1423	53	163	276	386	576	609	798	867	1020	1091	1124	1390	1501	14	124	393	464	498	648	759	906	942	1169	1241	1351	1462	92	202	237	347	537	687	720	29679
31	29679	599	787	822	1010	1200	1272	1305	1413	45	194	305	376	565	638	748	861	1049	1083	1116	1383	1491	6	155	422	454	487	677	709	900	971	1161	1233	1344	1452	84	233	266	337	526	29679
32	29679	369	555	591	818	851	1000	1189	1262	1294	1407	74	186	297	447	477	630	779	890	1039	1072	1106	1372	1485	35	147	414	330	516	669	740	929	961	1150	1223	1333	1446	113	225	258	29679
33	29679	175	287	358	549	620	810	843	993	1179	1254	1325	1436	64	136	404	436	471	659	771	882	1032	1062	1098	1403	1514	25	214	248	319	510	698	732	921	954	1140	1215	1364	1475	103	29679
34	29679	1428	57	165	279	389	578	610	799	833	982	1173	1283	1317	1506	18	126	396	467	500	649	760	872	1021	1056	1127	1395	1467	96	204	240	350	539	688	721	911	943	1134	1244	1356	29679
35	29679	1273	1306	1418	46	159	308	381	570	603	789	825	1013	1202	1117	1384	1496	7	120	425	459	492	642	750	864	1052	1085	1234	1345	1457	85	198	269	342	531	681	711	903	974	1163	29679
36	29679	1005	1194	1266	1296	1410	77	188	298	370	560	592	783	854	1044	1077	1110	1374	1488	38	149	415	448	482	631	744	893	966	1155	1227	1335	1449	116	227	259	331	521	670	705	932	29679
37	29679	812	844	994	1184	1255	1290	1439	69	180	291	360	552	623	773	883	1033	1067	1099	1368	1517	30	141	408	438	474	662	734	922	955	1145	1216	1329	1478	108	219	252	321	513	701	29679
38	29679	581	615	804	837	984	1176	1286	1319	1429	58	170	280	354	503	654	765	876	1023	1059	1130	1397	1507	19	131	397	432	542	693	726	915	945	1137	1247	1358	1468	97	209	241	315	29679
39		311	383	571	604	794	826	978	1205	1278	1311	1422	48	162	428	461	493	643	755	865	1017	1088	1122	1389	1500	9	123	272	344	532	682	716	904	939	1166	1239	1350	1461	87	201	29679
		29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679	29679

Thus we observe that we have a magic square of order 39 with 9 blocks of order 13 with equal magic sums. The magic sums are $S_{39\times 39} := 29679$ and $S_{13\times 13} := 9893$.

3 Magic Squares of Order 40

Block-wise construction of magic squares of order 40 depends on the products 4×10, 5×8, 8×5 and 10×4, i.e., we can construct magic square of order 40 as blocks of order 4, 5, 8 and 10. The magic sum of order 40 is given by

$$S_{40\times 40} := \frac{40\times(1+40^2)}{2} = 32020.$$

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This sums is divisible by 10, 5 and 4, but not by 8. See below:

$$(i) \quad \frac{32020}{10} = 3202 \quad \Longrightarrow \quad \text{equal blocks of order 4;}$$
$$(ii) \quad \frac{32020}{8} = 4002.5 \quad \Longrightarrow \quad \text{unequal blocks of order 5;}$$
$$(iii) \quad \frac{32020}{5} = 6404 \quad \Longrightarrow \quad \text{equal blocks of order 8;}$$
$$(iv) \quad \frac{32020}{4} = 8005 \quad \Longrightarrow \quad \text{equal blocks of order 10.}$$

This implies that we can made **block-wise** construction of magic square of order 40 with equal magic sums blocks of orders 4, 8 and 10. In case of blocks of order 5, this construction shall be made by different magic sums. In case of blocks of order 8, the magic square constructed is **bimagic**, where each block of order 8 is either **bimagic** or **semi-bimagic**. In order to construct these magic squares we need magic squares of orders 4, 5, 8 and 10. These are given in examples below:

Example 3.1. *Let's consider Latin squares decomposition of magic square of order 4 given by*

L		10	10	10	10
	2	3	1	4	10
10	1	4	2	3	10
10	4	1	3	2	10
10	3	2	4	1	10
	10	10	10	10	10

M		10	10	10	10
	3	4	1	2	10
10	2	1	4	3	10
10	4	3	2	1	10
10	1	2	3	4	10
	10	10	10	10	10

M_4		34	34	34	34
	7	12	1	14	34
34	2	13	8	11	34
34	16	3	10	5	34
34	9	6	15	4	34
	34	34	34	34	34

C_4		110	110	110	110
	23	34	11	42	110
110	12	41	24	33	110
110	44	13	32	21	110
110	31	22	43	14	110
	110	110	110	110	110

The magic squares M_4 and C_4 are obtained by using the operations

$$4 \times (A-1) + B := M_4 \quad \textit{and} \quad 10 \times A + B := C_4,$$

respectively. The M_4 is magic square of order 4 of consecutive numbers from 1 to 16, and C_4 is the ***composite*** *magic square.*

Example 3.2. *Let's consider Latin squares decomposition of magic square of order 5 given by*

L		15	15	15	15	15
	1	2	3	4	5	15
15	4	5	1	2	3	15
15	2	3	4	5	1	15
15	5	1	2	3	4	15
15	3	4	5	1	2	15
	15	15	15	15	15	15

M		15	15	15	15	15
	1	2	3	4	5	15
15	3	4	5	1	2	15
15	5	1	2	3	4	15
15	2	3	4	5	1	15
15	4	5	1	2	3	15
	15	15	15	15	15	15

C_5		165	165	165	165	165
	11	22	33	44	55	165
165	43	54	15	21	32	165
165	25	31	42	53	14	165
165	52	13	24	35	41	165
165	34	45	51	12	23	165
	165	165	165	165	165	165

M_5		65	65	65	65	65
	1	9	12	20	23	65
65	17	25	3	6	14	65
65	8	11	19	22	5	65
65	24	2	10	13	16	65
65	15	18	21	4	7	65
	65	65	65	65	65	65

The magic squares M_5 and C_5 are obtained by using the operations

$$5 \times (A-1) + B := M_5 \quad \textit{and} \quad 10 \times A + B := C_5,$$

respectively. The M_5 is magic square of order 5 of consecutive numbers from 1 to 25, and C_5 is the ***composite*** *magic square.*

Example 3.3. *Let's consider Latin squares decomposition of magic square of order 8 given by*

L		36	36	36	36	36	36	36	36
	2	6	5	1	4	8	7	3	36
36	4	8	7	3	2	6	5	1	36
36	1	5	6	2	3	7	8	4	36
36	3	7	8	4	1	5	6	2	36
36	5	1	2	6	7	3	4	8	36
36	7	3	4	8	5	1	2	6	36
36	6	2	1	5	8	4	3	7	36
36	8	4	3	7	6	2	1	5	36
	36	36	36	36	36	36	36	36	36

M		36	36	36	36	36	36	36	36
	8	1	4	5	3	6	7	2	36
36	2	7	6	3	5	4	1	8	36
36	1	8	5	4	6	3	2	7	36
36	7	2	3	6	4	5	8	1	36
36	6	3	2	7	1	8	5	4	36
36	4	5	8	1	7	2	3	6	36
36	3	6	7	2	8	1	4	5	36
36	5	4	1	8	2	7	6	3	36
	36	36	36	36	36	36	36	36	36

										11180
M_8		260	260	260	260	260	260	260	260	
	16	41	36	5	27	62	55	18	260	11180
260	26	63	54	19	13	44	33	8	260	11180
260	1	40	45	12	22	51	58	31	260	11180
260	23	50	59	30	4	37	48	9	260	11180
260	38	3	10	47	49	24	29	60	260	11180
260	52	21	32	57	39	2	11	46	260	11180
260	43	14	7	34	64	25	20	53	260	11180
260	61	28	17	56	42	15	6	35	260	11180
	260	260	260	260	260	260	260	260	260	
	11180	11180	11180	11180	11180	11180	11180	11180		11180

										23844
C_8		396	396	396	396	396	396	396	396	
	28	61	54	15	43	86	77	32	396	23844
396	42	87	76	33	25	64	51	18	396	23844
396	11	58	65	24	36	73	82	47	396	23844
396	37	72	83	46	14	55	68	21	396	23844
396	56	13	22	67	71	38	45	84	396	23844
396	74	35	48	81	57	12	23	66	396	23844
396	63	26	17	52	88	41	34	75	396	23844
396	85	44	31	78	62	27	16	53	396	23844
	396	396	396	396	396	396	396	396	396	
	23844	23844	23844	23844	23844	23844	23844	23844		23844

The magic squares M_8 and C_8 are obtained by using the operations

$$8 \times (A-1) + B := M_8 \quad \text{and} \quad 10 \times A + B := C_8,$$

respectively. The M_8 is magic square of order 8 of consecutive numbers from 1 to 64, and C_8 is the ***composite*** *magic square. In this case both M_8 and C_8 are* ***bimagic***

Example 3.4. *Let's consider a pan diagonal magic square of order 8 given by*

		260	260	260	260	260	260	260	260
	29	40	1	60	21	48	9	52	260
260	4	57	32	37	12	49	24	45	260
260	64	5	36	25	56	13	44	17	260
260	33	28	61	8	41	20	53	16	260
260	30	39	2	59	22	47	10	51	260
260	3	58	31	38	11	50	23	46	260
260	63	6	35	26	55	14	43	18	260
260	34	27	62	7	42	19	54	15	260
	260	260	260	260	260	260	260	260	260

This **pan diagonal** magic square is not **bimagic** but it has properties similar to magic square of order 4 given in Example 3.1. Also all the 4 blocks along with middle block are **pan diagonal** magic squares with qual magic sums, i.e., $S_{4\times 4} := 130$.

Example 3.5. *Let's consider Latin squares decomposition of magic square of order 10 given by*

L										45
0	6	8	1	9	7	3	4	2	5	45
4	1	3	0	5	8	9	2	7	6	45
6	9	2	5	8	4	7	1	0	3	45
9	8	0	3	2	1	4	5	6	7	45
3	7	1	8	4	6	0	9	5	2	45
1	2	6	7	0	5	8	3	9	4	45
7	4	5	2	1	9	6	8	3	0	45
5	0	9	4	6	3	2	7	1	8	45
2	3	4	9	7	0	5	6	8	1	45
8	5	7	6	3	2	1	0	4	9	45
45	45	45	45	45	45	45	45	45	45	45

M										45
0	3	4	2	8	1	5	9	7	6	45
2	1	6	8	3	0	7	5	9	4	45
9	5	2	0	6	7	4	3	1	8	45
4	7	9	3	0	6	1	8	5	2	45
7	0	5	9	4	8	3	6	2	1	45
8	9	1	6	7	5	2	4	3	0	45
3	8	7	4	9	2	6	1	0	5	45
1	6	0	5	2	9	8	7	4	3	45
6	2	3	1	5	4	9	0	8	7	45
5	4	8	7	1	3	0	2	6	9	45
45	45	45	45	45	45	45	45	45	45	45

M_{10}										505
1	80	65	97	39	22	48	86	53	14	505
98	12	9	66	90	74	55	33	41	27	505
47	81	23	79	16	35	94	60	62	8	505
70	57	88	34	2	91	29	15	76	43	505
84	99	52	11	45	68	73	7	30	36	505
13	38	44	10	77	56	82	21	95	69	505
75	46	40	83	28	19	67	92	4	51	505
59	24	96	42	61	3	20	78	37	85	505
26	5	17	58	93	50	31	64	89	72	505
32	63	71	25	54	87	6	49	18	100	505
505	505	505	505	505	505	505	505	505	505	505

C_{10}										495
00	63	84	12	98	71	35	49	27	56	495
42	11	36	08	53	80	97	25	79	64	495
69	95	22	50	86	47	74	13	01	38	495
94	87	09	33	20	16	41	58	65	72	495
37	70	15	89	44	68	03	96	52	21	495
18	29	61	76	07	55	82	34	93	40	495
73	48	57	24	19	92	66	81	30	05	495
51	06	90	45	62	39	28	77	14	83	495
26	32	43	91	75	04	59	60	88	17	495
85	54	78	67	31	23	10	02	46	99	495
495	495	495	495	495	495	495	495	495	495	495

The magic squares M_{10} and C_{10} are obtained by using the operations

$$10 \times (A-1) + B := M_{10} \quad \text{and} \quad 10 \times A + B := C_{10},$$

respectively. The M_{10} is magic square of order 10 of consecutive numbers from 1 to 100, and C_{10} is the ***composite*** *magic square.*

3.1 Blocks of Order 4

In order to construct magic square of order 40 with sub-blocks of order 4 we shall distribute the total number of entries according to following distribution.

Distribution 3.1. *Let's distribute the numbers from 1 to 1600 in 100 blocks with 16 members each. The distribution is in such a way that all the blocks are of equal sums:*

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Total
A1	1	200	201	400	401	600	601	800	801	1000	1001	1200	1201	1400	1401	1600	12808
A2	2	199	202	399	402	599	602	799	802	999	1002	1199	1202	1399	1402	1599	12808
A3	3	198	203	398	403	598	603	798	803	998	1003	1198	1203	1398	1403	1598	12808
A4	4	197	204	397	404	597	604	797	804	997	1004	1197	1204	1397	1404	1597	12808
A5	5	196	205	396	405	596	605	796	805	996	1005	1196	1205	1396	1405	1596	12808
A6	6	195	206	395	406	595	606	795	806	995	1006	1195	1206	1395	1406	1595	12808
A7	7	194	207	394	407	594	607	794	807	994	1007	1194	1207	1394	1407	1594	12808
A8	8	193	208	393	408	593	608	793	808	993	1008	1193	1208	1393	1408	1593	12808
A9	9	192	209	392	409	592	609	792	809	992	1009	1192	1209	1392	1409	1592	12808
A10	10	191	210	391	410	591	610	791	810	991	1010	1191	1210	1391	1410	1591	12808
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
A52	52	149	252	349	452	549	652	749	852	949	1052	1149	1252	1349	1452	1549	12808
A53	53	148	253	348	453	548	653	748	853	948	1053	1148	1253	1348	1453	1548	12808
A54	54	147	254	347	454	547	654	747	854	947	1054	1147	1254	1347	1454	1547	12808
A55	55	146	255	346	455	546	655	746	855	946	1055	1146	1255	1346	1455	1546	12808
A56	56	145	256	345	456	545	656	745	856	945	1056	1145	1256	1345	1456	1545	12808
A57	57	144	257	344	457	544	657	744	857	944	1057	1144	1257	1344	1457	1544	12808
A58	58	143	258	343	458	543	658	743	858	943	1058	1143	1258	1343	1458	1543	12808
A59	59	142	259	342	459	542	659	742	859	942	1059	1142	1259	1342	1459	1542	12808
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
A91	91	110	291	310	491	510	691	710	891	910	1091	1110	1291	1310	1491	1510	12808
A92	92	109	292	309	492	509	692	709	892	909	1092	1109	1292	1309	1492	1509	12808
A93	93	108	293	308	493	508	693	708	893	908	1093	1108	1293	1308	1493	1508	12808
A94	94	107	294	307	494	507	694	707	894	907	1094	1107	1294	1307	1494	1507	12808
A95	95	106	295	306	495	506	695	706	895	906	1095	1106	1295	1306	1495	1506	12808
A96	96	105	296	305	496	505	696	705	896	905	1096	1105	1296	1305	1496	1505	12808
A97	97	104	297	304	497	504	697	704	897	904	1097	1104	1297	1304	1497	1504	12808
A98	98	103	298	303	498	503	698	703	898	903	1098	1103	1298	1303	1498	1503	12808
A99	99	102	299	302	499	502	699	702	899	902	1099	1102	1299	1302	1499	1502	12808
A100	100	101	300	301	500	501	700	701	900	901	1100	1101	1300	1301	1500	1501	12808

Distribution 3.2. *Let's organize the 100 blocks A1 to A100 according to following table:*

A1	A2	A3	A4	A5	A6	A7	A8	A9	A10
A11	A12	A13	A14	A15	A16	A17	A18	A19	A20
A21	A22	A23	A24	A25	A26	A27	A28	A29	A30
A31	A32	A33	A34	A35	A36	A37	A38	A39	A40
A41	A42	A43	A44	A45	A46	A47	A48	A49	A50
A51	A52	A53	A54	A55	A56	A57	A58	A59	A60
A61	A62	A63	A64	A65	A66	A67	A68	A69	A70
A71	A72	A73	A74	A75	A76	A77	A78	A79	A80
A81	A82	A83	A84	A85	A86	A87	A88	A89	A90
A91	A92	A93	A94	A95	A96	A97	A98	A99	A100

We shall construct 100 magic square of order 4 by applying the Example 3.1 of equal magic sums. See below some examples:

A5		3204	3204	3204	3204
	605	1196	5	1396	3204
3204	196	1205	796	1005	3204
3204	1596	205	996	405	3204
3204	805	596	1405	396	3204
	3204	3204	3204	3204	3204

A12		3204	3204	3204	3204
	612	1189	12	1389	3204
3204	189	1212	789	1012	3204
3204	1589	212	989	412	3204
3204	812	589	1412	389	3204
	3204	3204	3204	3204	3204

A36		3204	3204	3204	3204
	636	1165	36	1365	3204
3204	165	1236	765	1036	3204
3204	1565	236	965	436	3204
3204	836	565	1436	365	3204
	3204	3204	3204	3204	3204

A43		3204	3204	3204	3204
	643	1158	43	1358	3204
3204	158	1243	758	1043	3204
3204	1558	243	958	443	3204
3204	843	558	1443	358	3204
	3204	3204	3204	3204	3204

A69		3204	3204	3204	3204
	669	1132	69	1332	3204
3204	132	1269	732	1069	3204
3204	1532	269	932	469	3204
3204	869	532	1469	332	3204
	3204	3204	3204	3204	3204

A98		3204	3204	3204	3204
	698	1103	98	1303	3204
3204	103	1298	703	1098	3204
3204	1503	298	903	498	3204
3204	898	503	1498	303	3204
	3204	3204	3204	3204	3204

In the similar way we can complete 100 blocks of order 4 with equal magic sums. Let's put these 100 blocks according Distribution 5.2 we get a **pan diagonal** magic square of order 40 given by

Example 3.6. *The **pan diagonal** magic square of order 40 with equal magic sum blocks of square of order 4 is given by*

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Pan Diagonal Magic Square of Order 40 with equal magic square sums of order 4: Magic square sums: S(40x40):=32020 and S(4x4):=3202. Each block of order 2x2 with equal sum entries as of magic square of order 4

		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	
		pan	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020
1	32020	601	1200	1	1400	602	1199	2	1399	603	1198	3	1398	604	1197	4	1397	605	1196	5	1396	606	1195	6	1395	607	1194	7	1394	608	1193	8	1393	609	1192	9	1392	610	1191	10	1391	32020
2	32020	200	1201	800	1001	199	1202	799	1002	198	1203	798	1003	197	1204	797	1004	196	1205	796	1005	195	1206	795	1006	194	1207	794	1007	193	1208	793	1008	192	1209	792	1009	191	1210	791	1010	32020
3	32020	1600	201	1000	401	1599	202	999	402	1598	203	998	403	1597	204	997	404	1596	205	996	405	1595	206	995	406	1594	207	994	407	1593	208	993	408	1592	209	992	409	1591	210	991	410	32020
4	32020	801	600	1401	400	802	599	1402	399	803	598	1403	398	804	597	1404	397	805	596	1405	396	806	595	1406	395	807	594	1407	394	808	593	1408	393	809	592	1409	392	810	591	1410	391	32020
5	32020	611	1190	11	1390	612	1189	12	1389	613	1188	13	1388	614	1187	14	1387	615	1186	15	1386	616	1185	16	1385	617	1184	17	1384	618	1183	18	1383	619	1182	19	1382	620	1181	20	1381	32020
6	32020	190	1211	790	1011	189	1212	789	1012	188	1213	788	1013	187	1214	787	1014	186	1215	786	1015	185	1216	785	1016	184	1217	784	1017	183	1218	783	1018	182	1219	782	1019	181	1220	781	1020	32020
7	32020	1590	211	990	411	1589	212	989	412	1588	213	988	413	1587	214	987	414	1586	215	986	415	1585	216	985	416	1584	217	984	417	1583	218	983	418	1582	219	982	419	1581	220	981	420	32020
8	32020	811	590	1411	390	812	589	1412	389	813	588	1413	388	814	587	1414	387	815	586	1415	386	816	585	1416	385	817	584	1417	384	818	583	1418	383	819	582	1419	382	820	581	1420	381	32020
9	32020	621	1180	21	1380	622	1179	22	1379	623	1178	23	1378	624	1177	24	1377	625	1176	25	1376	626	1175	26	1375	627	1174	27	1374	628	1173	28	1373	629	1172	29	1372	630	1171	30	1371	32020
10	32020	180	1221	780	1021	179	1222	779	1022	178	1223	778	1023	177	1224	777	1024	176	1225	776	1025	175	1226	775	1026	174	1227	774	1027	173	1228	773	1028	172	1229	772	1029	171	1230	771	1030	32020
11	32020	1580	221	980	421	1579	222	979	422	1578	223	978	423	1577	224	977	424	1576	225	976	425	1575	226	975	426	1574	227	974	427	1573	228	973	428	1572	229	972	429	1571	230	971	430	32020
12	32020	821	580	1421	380	822	579	1422	379	823	578	1423	378	824	577	1424	377	825	576	1425	376	826	575	1426	375	827	574	1427	374	828	573	1428	373	829	572	1429	372	830	571	1430	371	32020
13	32020	631	1170	31	1370	632	1169	32	1369	633	1168	33	1368	634	1167	34	1367	635	1166	35	1366	636	1165	36	1365	637	1164	37	1364	638	1163	38	1363	639	1162	39	1362	640	1161	40	1361	32020
14	32020	170	1231	770	1031	169	1232	769	1032	168	1233	768	1033	167	1234	767	1034	166	1235	766	1035	165	1236	765	1036	164	1237	764	1037	163	1238	763	1038	162	1239	762	1039	161	1240	761	1040	32020
15	32020	1570	231	970	431	1569	232	969	432	1568	233	968	433	1567	234	967	434	1566	235	966	435	1565	236	965	436	1564	237	964	437	1563	238	963	438	1562	239	962	439	1561	240	961	440	32020
16	32020	831	570	1431	370	832	569	1432	369	833	568	1433	368	834	567	1434	367	835	566	1435	366	836	565	1436	365	837	564	1437	364	838	563	1438	363	839	562	1439	362	840	561	1440	361	32020
17	32020	641	1160	41	1360	642	1159	42	1359	643	1158	43	1358	644	1157	44	1357	645	1156	45	1356	646	1155	46	1355	647	1154	47	1354	648	1153	48	1353	649	1152	49	1352	650	1151	50	1351	32020
18	32020	160	1241	760	1041	159	1242	759	1042	158	1243	758	1043	157	1244	757	1044	156	1245	756	1045	155	1246	755	1046	154	1247	754	1047	153	1248	753	1048	152	1249	752	1049	151	1250	751	1050	32020
19	32020	1560	241	960	441	1559	242	959	442	1558	243	958	443	1557	244	957	444	1556	245	956	445	1555	246	955	446	1554	247	954	447	1553	248	953	448	1552	249	952	449	1551	250	951	450	32020
20	32020	841	560	1441	360	842	559	1442	359	843	558	1443	358	844	557	1444	357	845	556	1445	356	846	555	1446	355	847	554	1447	354	848	553	1448	353	849	552	1449	352	850	551	1450	351	32020
21	32020	651	1150	51	1350	652	1149	52	1349	653	1148	53	1348	654	1147	54	1347	655	1146	55	1346	656	1145	56	1345	657	1144	57	1344	658	1143	58	1343	659	1142	59	1342	660	1141	60	1341	32020
22	32020	150	1251	750	1051	149	1252	749	1052	148	1253	748	1053	147	1254	747	1054	146	1255	746	1055	145	1256	745	1056	144	1257	744	1057	143	1258	743	1058	142	1259	742	1059	141	1260	741	1060	32020
23	32020	1550	251	950	451	1549	252	949	452	1548	253	948	453	1547	254	947	454	1546	255	946	455	1545	256	945	456	1544	257	944	457	1543	258	943	458	1542	259	942	459	1541	260	941	460	32020
24	32020	851	550	1451	350	852	549	1452	349	853	548	1453	348	854	547	1454	347	855	546	1455	346	856	545	1456	345	857	544	1457	344	858	543	1458	343	859	542	1459	342	860	541	1460	341	32020
25	32020	661	1140	61	1340	662	1139	62	1339	663	1138	63	1338	664	1137	64	1337	665	1136	65	1336	666	1135	66	1335	667	1134	67	1334	668	1133	68	1333	669	1132	69	1332	670	1131	70	1331	32020
26	32020	140	1261	740	1061	139	1262	739	1062	138	1263	738	1063	137	1264	737	1064	136	1265	736	1065	135	1266	735	1066	134	1267	734	1067	133	1268	733	1068	132	1269	732	1069	131	1270	731	1070	32020
27	32020	1540	261	940	461	1539	262	939	462	1538	263	938	463	1537	264	937	464	1536	265	936	465	1535	266	935	466	1534	267	934	467	1533	268	933	468	1532	269	932	469	1531	270	931	470	32020
28	32020	861	540	1461	340	862	539	1462	339	863	538	1463	338	864	537	1464	337	865	536	1465	336	866	535	1466	335	867	534	1467	334	868	533	1468	333	869	532	1469	332	870	531	1470	331	32020
29	32020	671	1130	71	1330	672	1129	72	1329	673	1128	73	1328	674	1127	74	1327	675	1126	75	1326	676	1125	76	1325	677	1124	77	1324	678	1123	78	1323	679	1122	79	1322	680	1121	80	1321	32020
30	32020	130	1271	730	1071	129	1272	729	1072	128	1273	728	1073	127	1274	727	1074	126	1275	726	1075	125	1276	725	1076	124	1277	724	1077	123	1278	723	1078	122	1279	722	1079	121	1280	721	1080	32020
31	32020	1530	271	930	471	1529	272	929	472	1528	273	928	473	1527	274	927	474	1526	275	926	475	1525	276	925	476	1524	277	924	477	1523	278	923	478	1522	279	922	479	1521	280	921	480	32020
32	32020	871	530	1471	330	872	529	1472	329	873	528	1473	328	874	527	1474	327	875	526	1475	326	876	525	1476	325	877	524	1477	324	878	523	1478	323	879	522	1479	322	880	521	1480	321	32020
33	32020	681	1120	81	1320	682	1119	82	1319	683	1118	83	1318	684	1117	84	1317	685	1116	85	1316	686	1115	86	1315	687	1114	87	1314	688	1113	88	1313	689	1112	89	1312	690	1111	90	1311	32020
34	32020	120	1281	720	1081	119	1282	719	1082	118	1283	718	1083	117	1284	717	1084	116	1285	716	1085	115	1286	715	1086	114	1287	714	1087	113	1288	713	1088	112	1289	712	1089	111	1290	711	1090	32020
35	32020	1520	281	920	481	1519	282	919	482	1518	283	918	483	1517	284	917	484	1516	285	916	485	1515	286	915	486	1514	287	914	487	1513	288	913	488	1512	289	912	489	1511	290	911	490	32020
36	32020	881	520	1481	320	882	519	1482	319	883	518	1483	318	884	517	1484	317	885	516	1485	316	886	515	1486	315	887	514	1487	314	888	513	1488	313	889	512	1489	312	890	511	1490	311	32020
37	32020	691	1110	91	1310	692	1109	92	1309	693	1108	93	1308	694	1107	94	1307	695	1106	95	1306	696	1105	96	1305	697	1104	97	1304	698	1103	98	1303	699	1102	99	1302	700	1101	100	1301	32020
38	32020	110	1291	710	1091	109	1292	709	1092	108	1293	708	1093	107	1294	707	1094	106	1295	706	1095	105	1296	705	1096	104	1297	704	1097	103	1298	703	1098	102	1299	702	1099	101	1300	701	1100	32020
39	32020	1510	291	910	491	1509	292	909	492	1508	293	908	493	1507	294	907	494	1506	295	906	495	1505	296	905	496	1504	297	904	497	1503	298	903	498	1502	299	902	499	1501	300	901	500	32020
40		891	510	1491	310	892	509	1492	309	893	508	1493	308	894	507	1494	307	895	506	1495	306	896	505	1496	305	897	504	1497	304	898	503	1498	303	899	502	1499	302	900	501	1500	301	32020
		32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020

All the blocks of order 4 are **pan diagonal** magic squares.

3.2 Blocks of Order 5

As we have seen above, it is not possible to construct magic square of order 40 with blocks of order 5 of equal magic sums. In this case, we shall construct **pan diagonal** magic square of order 40 with different magic sums of order 5. In order to construct magic square of order 40, we shall make use of magic squares of order 5 and order 8 given in Examples 3.2 and 3.4 respectively.

Distribution 3.3. *Let's distribute the numbers from 1 to 1600 in 64 blocks of 25 each with different sums:*

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	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	Total
A1	1	65	129	193	257	321	385	449	513	577	641	705	769	833	897	961	1025	1089	1153	1217	1281	1345	1409	1473	1537	19225
A2	2	66	130	194	258	322	386	450	514	578	642	706	770	834	898	962	1026	1090	1154	1218	1282	1346	1410	1474	1538	19250
A3	3	67	131	195	259	323	387	451	515	579	643	707	771	835	899	963	1027	1091	1155	1219	1283	1347	1411	1475	1539	19275
A4	4	68	132	196	260	324	388	452	516	580	644	708	772	836	900	964	1028	1092	1156	1220	1284	1348	1412	1476	1540	19300
A5	5	69	133	197	261	325	389	453	517	581	645	709	773	837	901	965	1029	1093	1157	1221	1285	1349	1413	1477	1541	19325
A6	6	70	134	198	262	326	390	454	518	582	646	710	774	838	902	966	1030	1094	1158	1222	1286	1350	1414	1478	1542	19350
A7	7	71	135	199	263	327	391	455	519	583	647	711	775	839	903	967	1031	1095	1159	1223	1287	1351	1415	1479	1543	19375
A8	8	72	136	200	264	328	392	456	520	584	648	712	776	840	904	968	1032	1096	1160	1224	1288	1352	1416	1480	1544	19400
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
A29	29	93	157	221	285	349	413	477	541	605	669	733	797	861	925	989	1053	1117	1181	1245	1309	1373	1437	1501	1565	19925
A30	30	94	158	222	286	350	414	478	542	606	670	734	798	862	926	990	1054	1118	1182	1246	1310	1374	1438	1502	1566	19950
A31	31	95	159	223	287	351	415	479	543	607	671	735	799	863	927	991	1055	1119	1183	1247	1311	1375	1439	1503	1567	19975
A32	32	96	160	224	288	352	416	480	544	608	672	736	800	864	928	992	1056	1120	1184	1248	1312	1376	1440	1504	1568	20000
A33	33	97	161	225	289	353	417	481	545	609	673	737	801	865	929	993	1057	1121	1185	1249	1313	1377	1441	1505	1569	20025
A34	34	98	162	226	290	354	418	482	546	610	674	738	802	866	930	994	1058	1122	1186	1250	1314	1378	1442	1506	1570	20050
A35	35	99	163	227	291	355	419	483	547	611	675	739	803	867	931	995	1059	1123	1187	1251	1315	1379	1443	1507	1571	20075
A36	36	100	164	228	292	356	420	484	548	612	676	740	804	868	932	996	1060	1124	1188	1252	1316	1380	1444	1508	1572	20100
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
A57	57	121	185	249	313	377	441	505	569	633	697	761	825	889	953	1017	1081	1145	1209	1273	1337	1401	1465	1529	1593	20625
A58	58	122	186	250	314	378	442	506	570	634	698	762	826	890	954	1018	1082	1146	1210	1274	1338	1402	1466	1530	1594	20650
A59	59	123	187	251	315	379	443	507	571	635	699	763	827	891	955	1019	1083	1147	1211	1275	1339	1403	1467	1531	1595	20675
A60	60	124	188	252	316	380	444	508	572	636	700	764	828	892	956	1020	1084	1148	1212	1276	1340	1404	1468	1532	1596	20700
A61	61	125	189	253	317	381	445	509	573	637	701	765	829	893	957	1021	1085	1149	1213	1277	1341	1405	1469	1533	1597	20725
A62	62	126	190	254	318	382	446	510	574	638	702	766	830	894	958	1022	1086	1150	1214	1278	1342	1406	1470	1534	1598	20750
A63	63	127	191	255	319	383	447	511	575	639	703	767	831	895	959	1023	1087	1151	1215	1279	1343	1407	1471	1535	1599	20775
A64	64	128	192	256	320	384	448	512	576	640	704	768	832	896	960	1024	1088	1152	1216	1280	1344	1408	1472	1536	1600	20800

By the application of Example 3.2 let's construct some sample examples of magic square of order 5 based on the values given in Distributions 3.3:

A10		3890	3890	3890	3890	3890
	10	394	778	1162	1546	3890
3890	1098	1482	266	330	714	3890
3890	586	650	1034	1418	202	3890
3890	1354	138	522	906	970	3890
3890	842	1226	1290	74	458	3890
	3890	3890	3890	3890	3890	3890

A29		3984	3984	3984	3984	3984
	29	413	797	1181	1565	3984
3984	1117	1501	285	349	733	3984
3984	605	669	1053	1437	221	3984
3984	1373	157	541	925	989	3984
3984	861	1245	1309	93	477	3984
	3984	3984	3984	3984	3984	3984

A38		4030	4030	4030	4030	4030
	38	422	806	1190	1574	4030
4030	1126	1510	294	358	742	4030
4030	614	678	1062	1446	230	4030
4030	1382	166	550	934	998	4030
4030	870	1254	1318	102	486	4030
	4030	4030	4030	4030	4030	4030

A42		4050	4050	4050	4050	4050
	42	426	810	1194	1578	4050
4050	1130	1514	298	362	746	4050
4050	618	682	1066	1450	234	4050
4050	1386	170	554	938	1002	4050
4050	874	1258	1322	106	490	4050
	4050	4050	4050	4050	4050	4050

A57		4125	4125	4125	4125	4125
	57	441	825	1209	1593	4125
4125	1145	1529	313	377	761	4125
4125	633	697	1081	1465	249	4125
4125	1401	185	569	953	1017	4125
4125	889	1273	1337	121	505	4125
	4125	4125	4125	4125	4125	4125

A64		4160	4160	4160	4160	4160
	64	448	832	1216	1600	4160
4160	1152	1536	320	384	768	4160
4160	640	704	1088	1472	256	4160
4160	1408	192	576	960	1024	4160
4160	896	1280	1344	128	512	4160
	4160	4160	4160	4160	4160	4160

In the similar, we can construct other 58 blocks of order 5 using the entries given in Distributions 3.3. Now put these 64 blocks of order 5 according to **pan diagonal** magic square of order 8 given in Example 3.4 as below:

Distribution 3.4. *Let's distribute the 1600 numbers from 1 to 1600 in 64 blocks of 25 with different sums:*

A29	A40	A1	A60	A21	A48	A9	A52
A4	A57	A32	A37	A12	A49	A24	A45
A64	A5	A36	A25	A56	A13	A44	A17
A33	A28	A61	A8	A41	A20	A53	A16
A30	A39	A2	A59	A22	A47	A10	A51
A3	A58	A31	A38	A11	A50	A23	A46
A63	A6	A35	A26	A55	A14	A43	A18
A34	A27	A62	A7	A42	A19	A54	A15

Above distribution gives us a **pan diagonal** magic square of order 40 given in example below.

Example 3.7. *The **block-wise pan diagonal** magic square of order 40 with each block of order 5 having different magic sums is given by*

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Pan Diagonal Magic Square of Order 40 - All blocks of order 5 are magic squares with different magic sums. Blocks of order 10 are magic squares with equal sums entries - 80050

		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	
		pan	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020
1	32020	29	413	797	1181	1565	40	424	808	1192	1576	1	385	769	1153	1537	60	444	828	1212	1596	21	405	789	1173	1557	48	432	816	1200	1584	9	393	777	1161	1545	52	436	820	1204	1588	32020
2	32020	1117	1501	285	349	733	1128	1512	296	360	744	1089	1473	257	321	705	1148	1532	316	380	764	1109	1493	277	341	725	1136	1520	304	368	752	1097	1481	265	329	713	1140	1524	308	372	756	32020
3	32020	605	669	1053	1437	221	616	680	1064	1448	232	577	641	1025	1409	193	636	700	1084	1468	252	597	661	1045	1429	213	624	688	1072	1456	240	585	649	1033	1417	201	628	692	1076	1460	244	32020
4	32020	1373	157	541	925	989	1384	168	552	936	1000	1345	129	513	897	961	1404	188	572	956	1020	1365	149	533	917	981	1392	176	560	944	1008	1353	137	521	905	969	1396	180	564	948	1012	32020
5	32020	861	1245	1309	93	477	872	1256	1320	104	488	833	1217	1281	65	449	892	1276	1340	124	508	853	1237	1301	85	469	880	1264	1328	112	496	841	1225	1289	73	457	884	1268	1332	116	500	32020
6	32020	4	388	772	1156	1540	57	441	825	1209	1593	32	416	800	1184	1568	37	421	805	1189	1573	12	396	780	1164	1548	49	433	817	1201	1585	24	408	792	1176	1560	45	429	813	1197	1581	32020
7	32020	1092	1476	260	324	708	1145	1529	313	377	761	1120	1504	288	352	736	1125	1509	293	357	741	1100	1484	268	332	716	1137	1521	305	369	753	1112	1496	280	344	728	1133	1517	301	365	749	32020
8	32020	580	644	1028	1412	196	633	697	1081	1465	249	608	672	1056	1440	224	613	677	1061	1445	229	588	652	1036	1420	204	625	689	1073	1457	241	600	664	1048	1432	216	621	685	1069	1453	237	32020
9	32020	1348	132	516	900	964	1401	185	569	953	1017	1376	160	544	928	992	1381	165	549	933	997	1356	140	524	908	972	1393	177	561	945	1009	1368	152	536	920	984	1389	173	557	941	1005	32020
10	32020	836	1220	1284	68	452	889	1273	1337	121	505	864	1248	1312	96	480	869	1253	1317	101	485	844	1228	1292	76	460	881	1265	1329	113	497	856	1240	1304	88	472	877	1261	1325	109	493	32020
11	32020	64	448	832	1216	1600	5	389	773	1157	1541	36	420	804	1188	1572	25	409	793	1177	1561	56	440	824	1208	1592	13	397	781	1165	1549	44	428	812	1196	1580	17	401	785	1169	1553	32020
12	32020	1152	1536	320	384	768	1093	1477	261	325	709	1124	1508	292	356	740	1113	1497	281	345	729	1144	1528	312	376	760	1101	1485	269	333	717	1132	1516	300	364	748	1105	1489	273	337	721	32020
13	32020	640	704	1088	1472	256	581	645	1029	1413	197	612	676	1060	1444	228	601	665	1049	1433	217	632	696	1080	1464	248	589	653	1037	1421	205	620	684	1068	1452	236	593	657	1041	1425	209	32020
14	32020	1408	192	576	960	1024	1349	133	517	901	965	1380	164	548	932	996	1369	153	537	921	985	1400	184	568	952	1016	1357	141	525	909	973	1388	172	556	940	1004	1361	145	529	913	977	32020
15	32020	896	1280	1344	128	512	837	1221	1285	69	453	868	1252	1316	100	484	857	1241	1305	89	473	888	1272	1336	120	504	845	1229	1293	77	461	876	1260	1324	108	492	849	1233	1297	81	465	32020
16	32020	33	417	801	1185	1569	28	412	796	1180	1564	61	445	829	1213	1597	8	392	776	1160	1544	41	425	809	1193	1577	20	404	788	1172	1556	53	437	821	1205	1589	16	400	784	1168	1552	32020
17	32020	1121	1505	289	353	737	1116	1500	284	348	732	1149	1533	317	381	765	1096	1480	264	328	712	1129	1513	297	361	745	1108	1492	276	340	724	1141	1525	309	373	757	1104	1488	272	336	720	32020
18	32020	609	673	1057	1441	225	604	668	1052	1436	220	637	701	1085	1469	253	584	648	1032	1416	200	617	681	1065	1449	233	596	660	1044	1428	212	629	693	1077	1461	245	592	656	1040	1424	208	32020
19	32020	1377	161	545	929	993	1372	156	540	924	988	1405	189	573	957	1021	1352	136	520	904	968	1385	169	553	937	1001	1364	148	532	916	980	1397	181	565	949	1013	1360	144	528	912	976	32020
20	32020	865	1249	1313	97	481	860	1244	1308	92	476	893	1277	1341	125	509	840	1224	1288	72	456	873	1257	1321	105	489	852	1236	1300	84	468	885	1269	1333	117	501	848	1232	1296	80	464	32020
21	32020	30	414	798	1182	1566	39	423	807	1191	1575	2	386	770	1154	1538	59	443	827	1211	1595	22	406	790	1174	1558	47	431	815	1199	1583	10	394	778	1162	1546	51	435	819	1203	1587	32020
22	32020	1118	1502	286	350	734	1127	1511	295	359	743	1090	1474	258	322	706	1147	1531	315	379	763	1110	1494	278	342	726	1135	1519	303	367	751	1098	1482	266	330	714	1139	1523	307	371	755	32020
23	32020	606	670	1054	1438	222	615	679	1063	1447	231	578	642	1026	1410	194	635	699	1083	1467	251	598	662	1046	1430	214	623	687	1071	1455	239	586	650	1034	1418	202	627	691	1075	1459	243	32020
24	32020	1374	158	542	926	990	1383	167	551	935	999	1346	130	514	898	962	1403	187	571	955	1019	1366	150	534	918	982	1391	175	559	943	1007	1354	138	522	906	970	1395	179	563	947	1011	32020
25	32020	862	1246	1310	94	478	871	1255	1319	103	487	834	1218	1282	66	450	891	1275	1339	123	507	854	1238	1302	86	470	879	1263	1327	111	495	842	1226	1290	74	458	883	1267	1331	115	499	32020
26	32020	3	387	771	1155	1539	58	442	826	1210	1594	31	415	799	1183	1567	38	422	806	1190	1574	11	395	779	1163	1547	50	434	818	1202	1586	23	407	791	1175	1559	46	430	814	1198	1582	32020
27	32020	1091	1475	259	323	707	1146	1530	314	378	762	1119	1503	287	351	735	1126	1510	294	358	742	1099	1483	267	331	715	1138	1522	306	370	754	1111	1495	279	343	727	1134	1518	302	366	750	32020
28	32020	579	643	1027	1411	195	634	698	1082	1466	250	607	671	1055	1439	223	614	678	1062	1446	230	587	651	1035	1419	203	626	690	1074	1458	242	599	663	1047	1431	215	622	686	1070	1454	238	32020
29	32020	1347	131	515	899	963	1402	186	570	954	1018	1375	159	543	927	991	1382	166	550	934	998	1355	139	523	907	971	1394	178	562	946	1010	1367	151	535	919	983	1390	174	558	942	1006	32020
30	32020	835	1219	1283	67	451	890	1274	1338	122	506	863	1247	1311	95	479	870	1254	1318	102	486	843	1227	1291	75	459	882	1266	1330	114	498	855	1239	1303	87	471	878	1262	1326	110	494	32020
31	32020	63	447	831	1215	1599	6	390	774	1158	1542	35	419	803	1187	1571	26	410	794	1178	1562	55	439	823	1207	1591	14	398	782	1166	1550	43	427	811	1195	1579	18	402	786	1170	1554	32020
32	32020	1151	1535	319	383	767	1094	1478	262	326	710	1123	1507	291	355	739	1114	1498	282	346	730	1143	1527	311	375	759	1102	1486	270	334	718	1131	1515	299	363	747	1106	1490	274	338	722	32020
33	32020	639	703	1087	1471	255	582	646	1030	1414	198	611	675	1059	1443	227	602	666	1050	1434	218	631	695	1079	1463	247	590	654	1038	1422	206	619	683	1067	1451	235	594	658	1042	1426	210	32020
34	32020	1407	191	575	959	1023	1350	134	518	902	966	1379	163	547	931	995	1370	154	538	922	986	1399	183	567	951	1015	1358	142	526	910	974	1387	171	555	939	1003	1362	146	530	914	978	32020
35	32020	895	1279	1343	127	511	838	1222	1286	70	454	867	1251	1315	99	483	858	1242	1306	90	474	887	1271	1335	119	503	846	1230	1294	78	462	875	1259	1323	107	491	850	1234	1298	82	466	32020
36	32020	34	418	802	1186	1570	27	411	795	1179	1563	62	446	830	1214	1598	7	391	775	1159	1543	42	426	810	1194	1578	19	403	787	1171	1555	54	438	822	1206	1590	15	399	783	1167	1551	32020
37	32020	1122	1506	290	354	738	1115	1499	283	347	731	1150	1534	318	382	766	1095	1479	263	327	711	1130	1514	298	362	746	1107	1491	275	339	723	1142	1526	310	374	758	1103	1487	271	335	719	32020
38	32020	610	674	1058	1442	226	603	667	1051	1435	219	638	702	1086	1470	254	583	647	1031	1415	199	618	682	1066	1450	234	595	659	1043	1427	211	630	694	1078	1462	246	591	655	1039	1423	207	32020
39	32020	1378	162	546	930	994	1371	155	539	923	987	1406	190	574	958	1022	1351	135	519	903	967	1386	170	554	938	1002	1363	147	531	915	979	1398	182	566	950	1014	1359	143	527	911	975	32020
40		866	1250	1314	98	482	859	1243	1307	91	475	894	1278	1342	126	510	839	1223	1287	71	455	874	1258	1322	106	490	851	1235	1299	83	467	886	1270	1334	118	502	847	1231	1295	79	463	32020
		32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020

3.3 Blocks of Order 8: Bimagic Square

Let's consider the distribution of 40 numbers 1 to 40 as given below:

Distribution 3.5. *Let's consider distribution of 40 numbers from 1 to 40 as given by*

	1	2	3	4	5	6	7	8	Total
1	1	10	11	20	21	30	31	40	164
2	2	9	12	19	22	29	32	39	164
3	3	8	13	18	23	28	33	38	164
4	4	7	14	17	24	27	34	37	164
5	5	6	15	16	25	26	35	36	164

Distribution 3.6. *Let's consider following distribution of order 5:*

11	22	33	44	55
43	54	15	21	32
25	31	42	53	14
52	13	24	35	41
34	45	51	12	23

This is same as composite magic square of order 5 given in Example 3.2. Let's see some examples based on Distribution 3.6 applied over Latin square decomposition of magic square of order 8 given in Example 3.3. Here the formula applied is $M := 40 \times (A-1) + B$.

• Block 43

(4)		164	164	164	164	164	164	164	164
	7	27	24	4	17	37	34	14	164
164	17	37	34	14	7	27	24	4	164
164	4	24	27	7	14	34	37	17	164
164	14	34	37	17	4	24	27	7	164
164	24	4	7	27	34	14	17	37	164
164	34	14	17	37	24	4	7	27	164
164	27	7	4	24	37	17	14	34	164
164	37	17	14	34	27	7	4	24	164
	164	164	164	164	164	164	164	164	164

(3)		164	164	164	164	164	164	164	164
	38	3	18	23	13	28	33	8	164
164	8	33	28	13	23	18	3	38	164
164	3	38	23	18	28	13	8	33	164
164	33	8	13	28	18	23	38	3	164
164	28	13	8	33	3	38	23	18	164
164	18	23	38	3	33	8	13	28	164
164	13	28	33	8	38	3	18	23	164
164	23	18	3	38	8	33	28	13	164
	164	164	164	164	164	164	164	164	164

										6749852
(43)		6404	6404	6404	6404	6404	6404	6404	6404	
	278	1043	938	143	653	1468	1353	528	6404	6756252
6404	648	1473	1348	533	263	1058	923	158	6404	6756252
6404	123	958	1063	258	548	1333	1448	673	6404	6756252
6404	553	1328	1453	668	138	943	1078	243	6404	6756252
6404	948	133	248	1073	1323	558	663	1458	6404	6756252
6404	1338	543	678	1443	953	128	253	1068	6404	6756252
6404	1053	268	153	928	1478	643	538	1343	6404	6756252
6404	1463	658	523	1358	1048	273	148	933	6404	6756252
	6404	6404	6404	6404	6404	6404	6404	6404	6404	
	6756252	6756252	6756252	6756252	6756252	6756252	6756252	6756252		6749852

• Block 33

(3)		164	164	164	164	164	164	164	164
	8	28	23	3	18	38	33	13	164
164	18	38	33	13	8	28	23	3	164
164	3	23	28	8	13	33	38	18	164
164	13	33	38	18	3	23	28	8	164
164	23	3	8	28	33	13	18	38	164
164	33	13	18	38	23	3	8	28	164
164	28	8	3	23	38	18	13	33	164
164	38	18	13	33	28	8	3	23	164
	164	164	164	164	164	164	164	164	164

(3)		164	164	164	164	164	164	164	164
	38	3	18	23	13	28	33	8	164
164	8	33	28	13	23	18	3	38	164
164	3	38	23	18	28	13	8	33	164
164	33	8	13	28	18	23	38	3	164
164	28	13	8	33	3	38	23	18	164
164	18	23	38	3	33	8	13	28	164
164	13	28	33	8	38	3	18	23	164
164	23	18	3	38	8	33	28	13	164
	164	164	164	164	164	164	164	164	164

										6807452
(33)		6404	6404	6404	6404	6404	6404	6404	6404	
	318	1083	898	103	693	1508	1313	488	6404	6807452
6404	688	1513	1308	493	303	1098	883	118	6404	6807452
6404	83	918	1103	298	508	1293	1488	713	6404	6807452
6404	513	1288	1493	708	98	903	1118	283	6404	6807452
6404	908	93	288	1113	1283	518	703	1498	6404	6807452
6404	1298	503	718	1483	913	88	293	1108	6404	6807452
6404	1093	308	113	888	1518	683	498	1303	6404	6807452
6404	1503	698	483	1318	1088	313	108	893	6404	6807452
	6404	6404	6404	6404	6404	6404	6404	6404	6404	
	6807452	6807452	6807452	6807452	6807452	6807452	6807452	6807452		6807452

We observe that the **Block 43** is **semi-bimagic** and the **Block 33** is **bimagic**. In this way, among other 23 blocks, they are either **bimagic** or **semi-bimagic**.

Example 3.8. *Combining all the 25 blocks of **bimagic** or **semi-bimagic** squares of order 8 and putting them according to Distribution 3.6, we get a **bimagic** square of order 40 given by*

https://doi.org/10.5281/zenodo.2555889

Pan Diagonal Bimagic Square of Order 40 - All blocks of order 8 are of equal magic sums with different bimagic or semi-bimagic sums.

			1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39		34165340
		pan	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	
1	32020	400	1161	820	21	771	1590	1231	410	359	1122	859	62	732	1549	1272	449	318	1083	898	103	693	1508	1313	488	277	1044	937	144	654	1467	1354	527	236	1005	976	185	615	1426	1395	566	32020	34165340
2	32020	770	1591	1230	411	381	1180	801	40	729	1552	1269	452	342	1139	842	79	688	1513	1308	493	303	1098	883	118	647	1474	1347	534	264	1057	924	157	606	1435	1386	575	225	1016	965	196	32020	34165340
3	32020	1	840	1181	380	430	1211	1570	791	42	879	1142	339	469	1252	1529	752	83	918	1103	298	508	1293	1488	713	124	957	1064	257	547	1334	1447	674	165	996	1025	216	586	1375	1406	635	32020	34165340
4	32020	431	1210	1571	790	20	821	1200	361	472	1249	1532	749	59	862	1159	322	513	1288	1493	708	98	903	1118	283	554	1327	1454	667	137	944	1077	244	595	1366	1415	626	176	985	1036	205	32020	34165340
5	32020	830	11	370	1191	1201	440	781	1580	869	52	329	1152	1242	479	742	1539	908	93	288	1113	1283	518	703	1498	947	134	247	1074	1324	557	664	1457	986	175	206	1035	1365	596	625	1416	32020	34165340
6	32020	1220	421	800	1561	831	10	371	1190	1259	462	759	1522	872	49	332	1149	1298	503	718	1483	913	88	293	1108	1337	544	677	1444	954	127	254	1067	1376	585	636	1405	995	166	215	1026	32020	34165340
7	32020	1171	390	31	810	1600	761	420	1221	1132	349	72	849	1559	722	459	1262	1093	308	113	888	1518	683	498	1303	1054	267	154	927	1477	644	537	1344	1015	226	195	966	1436	605	576	1385	32020	34165340
8	32020	1581	780	401	1240	1170	391	30	811	1542	739	442	1279	1129	352	69	852	1503	698	483	1318	1088	313	108	893	1464	657	524	1357	1047	274	147	934	1425	616	565	1396	1006	235	186	975	32020	34165340
9	32020	278	1043	938	143	653	1468	1353	528	237	1004	977	184	614	1427	1394	567	396	1165	816	25	775	1586	1235	406	360	1121	860	61	731	1550	1271	450	319	1082	899	102	692	1509	1312	489	32020	34165340
10	32020	648	1473	1348	533	263	1058	923	158	607	1434	1387	574	224	1017	964	197	766	1595	1226	415	385	1176	805	36	730	1551	1270	451	341	1140	841	80	689	1512	1309	492	302	1099	882	119	32020	34165340
11	32020	123	958	1063	258	548	1333	1448	673	164	997	1024	217	587	1374	1407	634	5	836	1185	376	426	1215	1566	795	41	880	1141	340	470	1251	1530	751	82	919	1102	299	509	1292	1489	712	32020	34165340
12	32020	553	1328	1453	668	138	943	1078	243	594	1367	1414	627	177	984	1037	204	435	1206	1575	786	16	825	1196	365	471	1250	1531	750	60	861	1160	321	512	1289	1492	709	99	902	1119	282	32020	34165340
13	32020	948	133	248	1073	1323	558	663	1458	987	174	207	1034	1364	597	624	1417	826	15	366	1195	1205	436	785	1576	870	51	330	1151	1241	480	741	1540	909	92	289	1112	1282	519	702	1499	32020	34165340
14	32020	1338	543	678	1443	953	128	253	1068	1377	584	637	1404	994	167	214	1027	1216	425	796	1565	835	6	375	1186	1260	461	760	1521	871	50	331	1150	1299	502	719	1482	912	89	292	1109	32020	34165340
15	32020	1053	268	153	928	1478	643	538	1343	1014	227	194	967	1437	604	577	1384	1175	386	35	806	1596	765	416	1225	1131	350	71	850	1560	721	460	1261	1092	309	112	889	1519	682	499	1302	32020	34165340
16	32020	1463	658	523	1358	1048	273	148	933	1424	617	564	1397	1007	234	187	974	1585	776	405	1236	1166	395	26	815	1541	740	441	1280	1130	351	70	851	1502	699	482	1319	1089	312	109	892	32020	34165340
17	32020	356	1125	856	65	735	1546	1275	446	320	1081	900	101	691	1510	1311	490	279	1042	939	142	652	1469	1352	529	238	1003	978	183	613	1428	1393	568	397	1164	817	24	774	1587	1234	407	32020	34165340
18	32020	726	1555	1266	455	345	1136	845	76	690	1511	1310	491	301	1100	881	120	649	1472	1349	532	262	1059	922	159	608	1433	1388	573	223	1018	963	198	767	1594	1227	414	384	1177	804	37	32020	34165340
19	32020	45	876	1145	336	466	1255	1526	755	81	920	1101	300	510	1291	1490	711	122	959	1062	259	549	1332	1449	672	163	998	1023	218	588	1373	1408	633	4	837	1184	377	427	1214	1567	794	32020	34165340
20	32020	475	1246	1535	746	56	865	1156	325	511	1290	1491	710	100	901	1120	281	552	1329	1452	669	139	942	1079	242	593	1368	1413	628	178	983	1038	203	434	1207	1574	787	17	824	1197	364	32020	34165340
21	32020	866	55	326	1155	1245	476	745	1536	910	91	290	1111	1281	520	701	1500	949	132	249	1072	1322	559	662	1459	988	173	208	1033	1363	598	623	1418	827	14	367	1194	1204	437	784	1577	32020	34165340
22	32020	1256	465	756	1525	875	46	335	1146	1300	501	720	1481	911	90	291	1110	1339	542	679	1442	952	129	252	1069	1378	583	638	1403	993	168	213	1028	1217	424	797	1564	834	7	374	1187	32020	34165340
23	32020	1135	346	75	846	1556	725	456	1265	1091	310	111	890	1520	681	500	1301	1052	269	152	929	1479	642	539	1342	1013	228	193	968	1438	603	578	1383	1174	387	34	807	1597	764	417	1224	32020	34165340
24	32020	1545	736	445	1276	1126	355	66	855	1501	700	481	1320	1090	311	110	891	1462	659	522	1359	1049	272	149	932	1423	618	563	1398	1008	233	188	973	1584	777	404	1237	1167	394	27	814	32020	34165340
25	32020	239	1002	979	182	612	1429	1392	569	398	1163	818	23	773	1588	1233	408	357	1124	857	64	734	1547	1274	447	316	1085	896	105	695	1506	1315	486	280	1041	940	141	651	1470	1351	530	32020	34165340
26	32020	609	1432	1389	572	222	1019	962	199	768	1593	1228	413	383	1178	803	38	727	1554	1267	454	344	1137	844	77	686	1515	1306	495	305	1096	885	116	650	1471	1350	531	261	1060	921	160	32020	34165340
27	32020	162	999	1022	219	589	1372	1409	632	3	838	1183	378	428	1213	1568	793	44	877	1144	337	467	1254	1527	754	85	916	1105	296	506	1295	1486	715	121	960	1061	260	550	1331	1450	671	32020	34165340
28	32020	592	1369	1412	629	179	982	1039	202	433	1208	1573	788	18	823	1198	363	474	1247	1534	747	57	864	1157	324	515	1286	1495	706	96	905	1116	285	551	1330	1451	670	140	941	1080	241	32020	34165340
29	32020	989	172	209	1032	1362	599	622	1419	828	13	368	1193	1203	438	783	1578	867	54	327	1154	1244	477	744	1537	906	95	286	1115	1285	516	705	1496	950	131	250	1071	1321	560	661	1460	32020	34165340
30	32020	1379	582	639	1402	992	169	212	1029	1218	423	798	1563	833	8	373	1188	1257	464	757	1524	874	47	334	1147	1296	505	716	1485	915	86	295	1106	1340	541	680	1441	951	130	251	1070	32020	34165340
31	32020	1012	229	192	969	1439	602	579	1382	1173	388	33	808	1598	763	418	1223	1134	347	74	847	1557	724	457	1264	1095	306	115	886	1516	685	496	1305	1051	270	151	930	1480	641	540	1341	32020	34165340
32	32020	1422	619	562	1399	1009	232	189	972	1583	778	403	1238	1168	393	28	813	1544	737	444	1277	1127	354	67	854	1505	696	485	1316	1086	315	106	895	1461	660	521	1360	1050	271	150	931	32020	34165340
33	32020	317	1084	897	104	694	1507	1314	487	276	1045	936	145	655	1466	1355	526	240	1001	980	181	611	1430	1391	570	399	1162	819	22	772	1589	1232	409	358	1123	858	63	733	1548	1273	448	32020	34165340
34	32020	687	1514	1307	494	304	1097	884	117	646	1475	1346	535	265	1056	925	156	610	1431	1390	571	221	1020	961	200	769	1592	1229	412	382	1179	802	39	728	1553	1268	453	343	1138	843	78	32020	34165340
35	32020	84	917	1104	297	507	1294	1487	714	125	956	1065	256	546	1335	1446	675	161	1000	1021	220	590	1371	1410	631	2	839	1182	379	429	1212	1569	792	43	878	1143	338	468	1253	1528	753	32020	34165340
36	32020	514	1287	1494	707	97	904	1117	284	555	1326	1455	666	136	945	1076	245	591	1370	1411	630	180	981	1040	201	432	1209	1572	789	19	822	1199	362	473	1248	1533	748	58	863	1158	323	32020	34165340
37	32020	907	94	287	1114	1284	517	704	1497	946	135	246	1075	1325	556	665	1456	990	171	210	1031	1361	600	621	1420	829	12	369	1192	1202	439	782	1579	868	53	328	1153	1243	478	743	1538	32020	34165340
38	32020	1297	504	717	1484	914	87	294	1107	1336	545	676	1445	955	126	255	1066	1380	581	640	1401	991	170	211	1030	1219	422	799	1562	832	9	372	1189	1258	463	758	1523	873	48	333	1148	32020	34165340
39	32020	1094	307	114	887	1517	684	497	1304	1055	266	155	926	1476	645	536	1345	1011	230	191	970	1440	601	580	1381	1172	389	32	809	1599	762	419	1222	1133	348	73	848	1558	723	458	1263	32020	34165340
40		1504	697	484	1317	1087	314	107	894	1465	656	525	1356	1046	275	146	935	1421	620	561	1400	1010	231	190	971	1582	779	402	1239	1169	392	29	812	1543	738	443	1278	1128	353	68	853	32020	34165340
		32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	
		34165340	34165340	34165340	34165340	34165340	34165340	34165340	34165340	34165340	34165340	34165340	34165340	34165340	34165340	34165340	34165340	34165340	34165340	34165340	34165340	34165340	34165340	34165340	34165340	34165340	34165340	34165340	34165340	34165340	34165340	34165340	34165340	34165340	34165340	34165340	34165340	34165340	34165340	34165340	34165340		34165340

Note 3.1. *Let's put 25 blocks of order 8 according to following distribution:*

11	21	31	41	51
12	22	32	42	52
13	23	33	43	53
14	24	34	44	54
15	25	35	45	55

*In this case, we don't get **bimagic** square of order 40. We get only magic square.*

3.4 Blocks of Order 10

Let's consider the distribution of 40 numbers from 1 to 40 as given below:

Distribution 3.7. *Let's consider distribution of 40 numbers from 1 to 40 as given by*

	1	2	3	4	5	6	7	8	9	10	Total
1	1	8	9	16	17	24	25	32	33	40	205
2	2	7	10	15	18	23	26	31	34	39	205
3	3	6	11	14	19	22	27	30	35	38	205
4	4	5	12	13	20	21	28	29	36	37	205

Distribution 3.8. *Let's consider following distribution of order 4:*

11	12	13	14
21	22	23	24
31	32	33	34
41	42	43	44

Let's see some examples based on Distribution 3.7 applied over Latin square decomposition of magic square of order 10 given in Example 3.5. Here the formula applied is $M := 40 \times (A-1) + B$

• Block 12

①										205
1	40	17	25	8	32	24	33	9	16	205
32	8	40	17	24	16	33	1	25	9	205
17	25	9	33	16	8	32	24	40	1	205
25	1	32	16	9	33	17	40	8	24	205
24	16	25	32	17	9	40	8	1	33	205
2	17	8	9	40	24	1	25	16	32	205
9	24	16	1	33	40	25	17	32	8	205
16	9	33	40	1	17	8	32	24	25	205
40	32	24	8	25	1	16	9	33	17	205
8	33	1	24	32	25	9	16	17	40	205
205	205	205	205	205	205	205	205	205	205	205

②										205
2	34	23	7	31	15	18	26	39	10	205
23	7	31	10	39	34	2	15	18	26	205
7	31	10	39	23	26	34	2	15	18	205
39	26	18	15	2	10	31	7	23	34	205
15	2	34	26	18	7	23	39	10	31	205
18	15	2	34	26	23	39	10	31	7	205
31	10	39	23	7	18	26	34	2	15	205
26	18	15	2	34	39	10	31	7	23	205
10	39	26	18	15	31	7	23	34	2	205
34	23	7	31	10	2	15	18	26	39	205
205	205	205	205	205	205	205	205	205	205	205

12										8005
2	1594	663	967	311	1255	938	1306	359	610	8005
1263	287	1591	650	959	634	1282	15	978	346	8005
647	991	330	1319	623	306	1274	922	1575	18	8005
999	26	1258	615	322	1290	671	1567	303	954	8005
935	602	994	1266	658	327	1583	319	10	1311	8005
58	655	282	354	1586	943	39	970	631	1247	8005
351	930	639	23	1287	1578	986	674	1242	295	8005
626	338	1295	1562	34	679	290	1271	927	983	8005
1570	1279	946	298	975	31	607	343	1314	642	8005
314	1303	7	951	1250	962	335	618	666	1599	8005
8005	8005	8005	8005	8005	8005	8005	8005	8005	8005	8005

- **Block 44**

4										205
4	37	20	28	5	29	21	36	12	13	205
29	5	37	20	21	13	36	4	28	12	205
20	28	12	36	13	5	29	21	37	4	205
28	4	29	13	12	36	20	37	5	21	205
21	13	28	29	20	12	37	5	4	36	205
36	20	5	12	37	21	4	28	13	29	205
12	21	13	4	36	37	28	20	29	5	205
13	12	36	37	4	20	5	29	21	28	205
37	29	21	5	28	4	13	12	36	20	205
5	36	4	21	29	28	12	13	20	37	205
205	205	205	205	205	205	205	205	205	205	205

4										205
4	36	21	5	29	13	20	28	37	12	205
21	5	29	12	37	36	4	13	20	28	205
5	29	12	37	21	28	36	4	13	20	205
37	28	20	13	4	12	29	5	21	36	205
13	4	36	28	20	5	21	37	12	29	205
20	13	4	36	28	21	37	12	29	5	205
29	12	37	21	5	20	28	36	4	13	205
28	20	13	4	36	37	12	29	5	21	205
12	37	28	20	13	29	5	21	36	4	205
36	21	5	29	12	4	13	20	28	37	205
205	205	205	205	205	205	205	205	205	205	205

44										8005
124	1476	781	1085	189	1133	820	1428	477	492	8005
1141	165	1469	772	837	516	1404	133	1100	468	8005
765	1109	452	1437	501	188	1156	804	1453	140	8005
1117	148	1140	493	444	1412	789	1445	181	836	8005
813	484	1116	1148	780	445	1461	197	132	1429	8005
1420	773	164	476	1468	821	157	1092	509	1125	8005
469	812	517	141	1405	1460	1108	796	1124	173	8005
508	460	1413	1444	156	797	172	1149	805	1101	8005
1452	1157	828	180	1093	149	485	461	1436	764	8005
196	1421	125	829	1132	1084	453	500	788	1477	8005
8005	8005	8005	8005	8005	8005	8005	8005	8005	8005	8005

In this way we can construct other 14 blocks of order 10. Putting all these 16 blocks of order 10 according to Distribution 5.3, we get a magic square of order 40 given example below.

Example 3.9. *The* ***block-wise*** *magic square of order 40 with magic square blocks of order 10 is given by*

Magic square of order 40 with equal magic sums of order 10: Magic square sums: S(40x40):=32020 and S(10x10):=8005.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	32020
1	1	1593	664	968	312	1256	937	1305	360	609	2	1594	663	967	311	1255	938	1306	359	610	3	1595	662	966	310	1254	939	1307	358	611	4	1596	661	965	309	1253	940	1308	357	612	32020
2	1264	288	1592	649	960	633	1281	16	977	345	1263	287	1591	650	959	634	1282	15	978	346	1262	286	1590	651	958	635	1283	14	979	347	1261	285	1589	652	957	636	1284	13	980	348	32020
3	648	992	329	1320	624	305	1273	921	1576	17	647	991	330	1319	623	306	1274	922	1575	18	646	990	331	1318	622	307	1275	923	1574	19	645	989	332	1317	621	308	1276	924	1573	20	32020
4	1000	25	1257	616	321	1289	672	1568	304	953	999	26	1258	615	322	1290	671	1567	303	954	998	27	1259	614	323	1291	670	1566	302	955	997	28	1260	613	324	1292	669	1565	301	956	32020
5	936	601	993	1265	657	328	1584	320	9	1312	935	602	994	1266	658	327	1583	319	10	1311	934	603	995	1267	659	326	1582	318	11	1310	933	604	996	1268	660	325	1581	317	12	1309	32020
6	1297	656	281	353	1585	944	40	969	632	1248	1298	655	282	354	1586	943	39	970	631	1247	1299	654	283	355	1587	942	38	971	630	1246	1300	653	284	356	1588	941	37	972	629	1245	32020
7	352	929	640	24	1288	1577	985	673	1241	296	351	930	639	23	1287	1578	986	674	1242	295	350	931	638	22	1286	1579	987	675	1243	294	349	932	637	21	1285	1580	988	676	1244	293	32020
8	625	337	1296	1561	33	680	289	1272	928	984	626	338	1295	1562	34	679	290	1271	927	983	627	339	1294	1563	35	678	291	1270	926	982	628	340	1293	1564	36	677	292	1269	925	981	32020
9	1569	1280	945	297	976	32	608	344	1313	641	1570	1279	946	298	975	31	607	343	1314	642	1571	1278	947	299	974	30	606	342	1315	643	1572	1277	948	300	973	29	605	341	1316	644	32020
10	313	1304	8	952	1249	961	336	617	665	1600	314	1303	7	951	1250	962	335	618	666	1599	315	1302	6	950	1251	963	334	619	667	1598	316	1301	5	949	1252	964	333	620	668	1597	32020
11	41	1553	704	1008	272	1216	897	1345	400	569	42	1554	703	1007	271	1215	898	1346	399	570	43	1555	702	1006	270	1214	899	1347	398	571	44	1556	701	1005	269	1213	900	1348	397	572	32020
12	1224	248	1552	689	920	593	1321	56	1017	385	1223	247	1551	690	919	594	1322	55	1018	386	1222	246	1550	691	918	595	1323	54	1019	387	1221	245	1549	692	917	596	1324	53	1020	388	32020
13	688	1032	369	1360	584	265	1233	881	1536	57	687	1031	370	1359	583	266	1234	882	1535	58	686	1030	371	1358	582	267	1235	883	1534	59	685	1029	372	1357	581	268	1236	884	1533	60	32020
14	1040	65	1217	576	361	1329	712	1528	264	913	1039	66	1218	575	362	1330	711	1527	263	914	1038	67	1219	574	363	1331	710	1526	262	915	1037	68	1220	573	364	1332	709	1525	261	916	32020
15	896	561	1033	1225	697	368	1544	280	49	1352	895	562	1034	1226	698	367	1543	279	50	1351	894	563	1035	1227	699	366	1542	278	51	1350	893	564	1036	1228	700	365	1541	277	52	1349	32020
16	1337	696	241	393	1545	904	80	1009	592	1208	1338	695	242	394	1546	903	79	1010	591	1207	1339	694	243	395	1547	902	78	1011	590	1206	1340	693	244	396	1548	901	77	1012	589	1205	32020
17	392	889	600	64	1328	1537	1025	713	1201	256	391	890	599	63	1327	1538	1026	714	1202	255	390	891	598	62	1326	1539	1027	715	1203	254	389	892	597	61	1325	1540	1028	716	1204	253	32020
18	585	377	1336	1521	73	720	249	1232	888	1024	586	378	1335	1522	74	719	250	1231	887	1023	587	379	1334	1523	75	718	251	1230	886	1022	588	380	1333	1524	76	717	252	1229	885	1021	32020
19	1529	1240	905	257	1016	72	568	384	1353	681	1530	1239	906	258	1015	71	567	383	1354	682	1531	1238	907	259	1014	70	566	382	1355	683	1532	1237	908	260	1013	69	565	381	1356	684	32020
20	273	1344	48	912	1209	1001	376	577	705	1560	274	1343	47	911	1210	1002	375	578	706	1559	275	1342	46	910	1211	1003	374	579	707	1558	276	1341	45	909	1212	1004	373	580	708	1557	32020
21	81	1513	744	1048	232	1176	857	1385	440	529	82	1514	743	1047	231	1175	858	1386	439	530	83	1515	742	1046	230	1174	859	1387	438	531	84	1516	741	1045	229	1173	860	1388	437	532	32020
22	1184	208	1512	729	880	553	1361	96	1057	425	1183	207	1511	730	879	554	1362	95	1058	426	1182	206	1510	731	878	555	1363	94	1059	427	1181	205	1509	732	877	556	1364	93	1060	428	32020
23	728	1072	409	1400	544	225	1193	841	1496	97	727	1071	410	1399	543	226	1194	842	1495	98	726	1070	411	1398	542	227	1195	843	1494	99	725	1069	412	1397	541	228	1196	844	1493	100	32020
24	1080	105	1177	536	401	1369	752	1488	224	873	1079	106	1178	535	402	1370	751	1487	223	874	1078	107	1179	534	403	1371	750	1486	222	875	1077	108	1180	533	404	1372	749	1485	221	876	32020
25	856	521	1073	1185	737	408	1504	240	89	1392	855	522	1074	1186	738	407	1503	239	90	1391	854	523	1075	1187	739	406	1502	238	91	1390	853	524	1076	1188	740	405	1501	237	92	1389	32020
26	1377	736	201	433	1505	864	120	1049	552	1168	1378	735	202	434	1506	863	119	1050	551	1167	1379	734	203	435	1507	862	118	1051	550	1166	1380	733	204	436	1508	861	117	1052	549	1165	32020
27	432	849	560	104	1368	1497	1065	753	1161	216	431	850	559	103	1367	1498	1066	754	1162	215	430	851	558	102	1366	1499	1067	755	1163	214	429	852	557	101	1365	1500	1068	756	1164	213	32020
28	545	417	1376	1481	113	760	209	1192	848	1064	546	418	1375	1482	114	759	210	1191	847	1063	547	419	1374	1483	115	758	211	1190	846	1062	548	420	1373	1484	116	757	212	1189	845	1061	32020
29	1489	1200	865	217	1056	112	528	424	1393	721	1490	1199	866	218	1055	111	527	423	1394	722	1491	1198	867	219	1054	110	526	422	1395	723	1492	1197	868	220	1053	109	525	421	1396	724	32020
30	233	1384	88	872	1169	1041	416	537	745	1520	234	1383	87	871	1170	1042	415	538	746	1519	235	1382	86	870	1171	1043	414	539	747	1518	236	1381	85	869	1172	1044	413	540	748	1517	32020
31	121	1473	784	1088	192	1136	817	1425	480	489	122	1474	783	1087	191	1135	818	1426	479	490	123	1475	782	1086	190	1134	819	1427	478	491	124	1476	781	1085	189	1133	820	1428	477	492	32020
32	1144	168	1472	769	840	513	1401	136	1097	465	1143	167	1471	770	839	514	1402	135	1098	466	1142	166	1470	771	838	515	1403	134	1099	467	1141	165	1469	772	837	516	1404	133	1100	468	32020
33	768	1112	449	1440	504	185	1153	801	1456	137	767	1111	450	1439	503	186	1154	802	1455	138	766	1110	451	1438	502	187	1155	803	1454	139	765	1109	452	1437	501	188	1156	804	1453	140	32020
34	1120	145	1137	496	441	1409	792	1448	184	833	1119	146	1138	495	442	1410	791	1447	183	834	1118	147	1139	494	443	1411	790	1446	182	835	1117	148	1140	493	444	1412	789	1445	181	836	32020
35	816	481	1113	1145	777	448	1464	200	129	1432	815	482	1114	1146	778	447	1463	199	130	1431	814	483	1115	1147	779	446	1462	198	131	1430	813	484	1116	1148	780	445	1461	197	132	1429	32020
36	1417	776	161	473	1465	824	160	1089	512	1128	1418	775	162	474	1466	823	159	1090	511	1127	1419	774	163	475	1467	822	158	1091	510	1126	1420	773	164	476	1468	821	157	1092	509	1125	32020
37	472	809	520	144	1408	1457	1105	793	1121	176	471	810	519	143	1407	1458	1106	794	1122	175	470	811	518	142	1406	1459	1107	795	1123	174	469	812	517	141	1405	1460	1108	796	1124	173	32020
38	505	457	1416	1441	153	800	169	1152	808	1104	506	458	1415	1442	154	799	170	1151	807	1103	507	459	1414	1443	155	798	171	1150	806	1102	508	460	1413	1444	156	797	172	1149	805	1101	32020
39	1449	1160	825	177	1096	152	488	464	1433	761	1450	1159	826	178	1095	151	487	463	1434	762	1451	1158	827	179	1094	150	486	462	1435	763	1452	1157	828	180	1093	149	485	461	1436	764	32020
40	193	1424	128	832	1129	1081	456	497	785	1480	194	1423	127	831	1130	1082	455	498	786	1479	195	1422	126	830	1131	1083	454	499	787	1478	196	1421	125	829	1132	1084	453	500	788	1477	32020
	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020	32020

Thus we observe that we have a magic square of order 40 with 16 blocks of order 10 with equal magic sums. The magic sums are $S_{40\times 40} := 32020$ and $S_{10\times 10} := 3202$.

4 Magic Squares of Order 42

The **Block-wise** construction of magic squares of order 42 depends on the product 3×14, 6×7, 6×6 and 14×3, i.e., we can construct magic square of order 42 as blocks of orders 3, 6, 7 and 14. The magic sum of order 42 is given by

$$S_{42\times 42} := \frac{42 \times (1+42^2)}{2} = 37065.$$

This sum is divisible by 7 and 14, but not by 3 and 6. See below:

$$\begin{aligned} &(i) && \frac{37065}{14} = 2647.5 && \Longrightarrow && \text{unequal blocks of order 3;} \\ &(ii) && \frac{37065}{7} = 5295 && \Longrightarrow && \text{equal blocks of order 6;} \\ &(iii) && \frac{37065}{6} = 6176.5 && \Longrightarrow && \text{unequal blocks of order 7;} \\ &(iv) && \frac{37065}{3} = 12355 && \Longrightarrow && \text{equal blocks of order 14.} \end{aligned}$$

This implies that we can made **block-wise** construction of magic square of order 42 with equal magic sums blocks of order 6 and 14. In case of blocks of orders 3 and 7, the construction of magic squares of order 42 is with different magic sums. In order to construct these magic squares we shall need magic squares of orders 3, 6, 7 and 14. The magic square of order 3 is already given in Example 2.1. The magic squares of orders 6, 7 and 14 are given below.

- **Magic Square of Order 6**

Example 4.1. *Let's consider a magic square of order 6.*

(L)						21
1	6	6	6	1	1	21
5	2	5	2	2	5	21
4	4	3	3	4	3	21
3	3	4	4	3	4	21
2	5	2	5	5	2	21
6	1	1	1	6	6	21
21	21	21	21	21	21	21

(M)						21
1	5	4	3	2	6	21
6	2	4	3	5	1	21
6	5	3	4	2	1	21
6	2	3	4	5	1	21
1	2	4	3	5	6	21
1	5	3	4	2	6	21
21	21	21	21	21	21	21

M_6						111
1	35	34	33	2	6	111
30	8	28	9	11	25	111
24	23	15	16	20	13	111
18	14	21	22	17	19	111
7	26	10	27	29	12	111
31	5	3	4	32	36	111
111	111	111	111	111	111	111

C_6						231
11	65	64	63	12	16	231
56	22	54	23	25	51	231
46	45	33	34	42	31	231
36	32	43	44	35	41	231
21	52	24	53	55	26	231
61	15	13	14	62	66	231
231	231	231	231	231	231	231

The magic squares M_6 and C_6 are obtained by using the operations

$$6 \times (A-1) + B := M_6 \quad \text{and} \quad 10 \times A + B := C_6,$$

respectively. The M_6 is magic square of order 6 of consecutive numbers from 1 to 36, and C_6 is the ***composite*** *magic square.*

- **Pan Diagonal Magic Square of Order 7**

Example 4.2. *Let's consider Latin squares decomposition of magic square of order 7 given by*

L		28	28	28	28	28	28	28
	1	2	3	4	5	6	7	28
28	6	7	1	2	3	4	5	28
28	4	5	6	7	1	2	3	28
28	2	3	4	5	6	7	1	28
28	7	1	2	3	4	5	6	28
28	5	6	7	1	2	3	4	28
28	3	4	5	6	7	1	2	28
	28	28	28	28	28	28	28	28

M		28	28	28	28	28	28	28
	1	2	3	4	5	6	7	28
28	5	6	7	1	2	3	4	28
28	2	3	4	5	6	7	1	28
28	6	7	1	2	3	4	5	28
28	3	4	5	6	7	1	2	28
28	7	1	2	3	4	5	6	28
28	4	5	6	7	1	2	3	28
	28	28	28	28	28	28	28	28

M_7		175	175	175	175	175	175	175
	1	9	17	25	33	41	49	175
175	40	48	7	8	16	24	32	175
175	23	31	39	47	6	14	15	175
175	13	21	22	30	38	46	5	175
175	45	4	12	20	28	29	37	175
175	35	36	44	3	11	19	27	175
175	18	26	34	42	43	2	10	175
	175	175	175	175	175	175	175	175

C_7		308	308	308	308	308	308	308
	11	22	33	44	55	66	77	308
308	65	76	17	21	32	43	54	308
308	42	53	64	75	16	27	31	308
308	26	37	41	52	63	74	15	308
308	73	14	25	36	47	51	62	308
308	57	61	72	13	24	35	46	308
308	34	45	56	67	71	12	23	308
	308	308	308	308	308	308	308	308

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The magic squares M_7 and C_7 are obtained by using the operations

$$7 \times (A-1) + B := M_7 \quad \text{and} \quad 10 \times A + B := C_7,$$

respectively. The M_7 is magic square of order 7 of consecutive numbers from 1 to 49, and C_7 is the ***composite*** *magic square.*

- **Magic Square of Order 14**

Example 4.3. *Let's consider a magic square of 14 with Latin square decomposition and* ***composite*** *magic square:*

(L)														105
1	4	11	5	8	2	9	3	7	14	12	13	6	10	105
14	2	5	11	6	3	10	4	8	12	13	7	1	9	105
12	14	3	6	11	4	1	5	9	13	8	2	10	7	105
13	12	14	4	7	5	2	6	10	9	3	1	8	11	105
10	13	12	14	5	6	3	7	1	4	2	9	11	8	105
7	8	9	10	1	**11**	**13**	**14**	**12**	6	5	4	3	2	105
9	10	1	2	3	**14**	**12**	**11**	**13**	8	7	6	5	4	105
6	7	8	9	10	**12**	**14**	**13**	**11**	5	4	3	2	1	105
8	9	10	1	2	**13**	**11**	**12**	**14**	7	6	5	4	3	105
3	11	4	7	9	1	8	2	6	10	14	12	13	5	105
11	3	6	8	4	10	7	1	5	2	9	14	12	13	105
2	5	7	3	13	9	6	10	4	11	1	8	14	12	105
4	6	2	13	12	8	5	9	3	1	11	10	7	14	105
5	1	13	12	14	7	4	8	2	3	10	11	9	6	105
105	105	105	105	105	105	105	105	105	105	105	105	105	105	105

(M)														105
1	13	10	14	11	6	7	9	8	3	4	2	12	5	105
4	2	13	1	14	7	8	10	9	5	3	12	6	11	105
6	5	3	13	2	8	9	1	10	4	12	7	11	14	105
5	7	6	4	13	9	10	2	1	12	8	11	14	3	105
12	6	8	7	5	10	1	3	2	9	11	14	4	13	105
9	10	1	2	3	**11**	**14**	**12**	**13**	8	7	6	5	4	105
2	3	4	5	6	**13**	**12**	**14**	**11**	1	10	9	8	7	105
3	4	5	6	7	**14**	**11**	**13**	**12**	2	1	10	9	8	105
7	8	9	10	1	**12**	**13**	**11**	**14**	6	5	4	3	2	105
13	9	14	11	4	5	6	8	7	10	2	3	1	12	105
8	14	11	3	12	4	5	7	6	13	9	1	2	10	105
14	11	2	12	9	3	4	6	5	7	13	8	10	1	105
11	1	12	8	10	2	3	5	4	14	6	13	7	9	105
10	12	7	9	8	1	2	4	3	11	14	5	13	6	105
105	105	105	105	105	105	105	105	105	105	105	105	105	105	105

M_{14}														1379
1	95	191	112	138	78	58	32	17	159	146	178	125	49	1379
147	16	110	176	188	127	87	117	4	53	70	83	38	163	1379
126	162	31	137	172	63	80	6	47	26	97	99	149	184	1379
190	111	150	46	155	115	5	77	34	170	23	96	140	67	1379
164	84	55	12	121	86	29	101	133	62	173	25	186	148	1379
104	10	89	37	21	**166**	**179**	**154**	**195**	74	44	134	59	113	1379
19	35	132	57	3	**153**	**196**	**165**	**180**	92	122	51	72	102	1379
45	65	71	90	36	**193**	**152**	**181**	**168**	119	130	2	103	24	1379
30	43	60	73	94	**182**	**167**	**194**	**151**	107	7	118	22	131	1379
68	144	156	27	75	48	105	93	120	136	183	42	11	171	1379
81	187	175	114	69	18	135	52	85	14	106	143	160	40	1379
139	174	8	161	142	108	116	15	79	185	39	61	54	98	1379
177	129	123	192	56	33	20	64	100	141	82	158	91	13	1379
88	124	28	145	109	9	50	128	66	41	157	189	169	76	1379
1379	1379	1379	1379	1379	1379	1379	1379	1379	1379	1379	1379	1379	1379	1379

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11	4U	SR	5V	8S	26	97	39	78	V3	T4	U2	6T	R5
V4	22	5U	S1	6V	37	R8	4R	89	T5	U3	7T	16	9S
T6	V5	33	6U	S2	48	19	51	9R	U4	8T	27	RS	7V
U5	T7	V6	44	7U	59	2R	62	R1	9T	38	1S	8V	S3
RT	U6	T8	V7	55	6R	31	73	12	49	2S	9V	S4	8U
79	8R	91	R2	13	**SS**	**UV**	**VT**	**TU**	68	57	46	35	24
92	R3	14	25	36	**VU**	**TT**	**SV**	**US**	81	7R	69	58	47
63	74	85	96	R7	**TV**	**VS**	**UU**	**ST**	52	41	3R	29	18
87	98	R9	1R	21	**UT**	**SU**	**TS**	**VV**	76	65	54	43	32
3U	S9	4V	7S	94	15	86	28	67	RR	V2	T3	U1	5T
S8	3V	6S	83	4T	R4	75	17	56	2U	99	V1	T2	UR
2V	5S	72	3T	U9	93	64	R6	45	S7	1U	88	VR	T1
4S	61	2T	U8	TR	82	53	95	34	1V	S6	RU	77	V9
5R	1T	U7	T9	V8	71	42	84	23	3S	RV	S5	9U	66
							C_{14}						

where $R := 10$, $S := 11$, $T := 12$, $U := 13$ and $V := 14$. The magic squares M_{14} and C_{14} are obtained by using the operations

$$14 \times (A - 1) + B := M_{14} \quad \text{and} \quad 10 \times A + B := C_{14},$$

respectively. The M_{14} is magic square of order 14 of consecutive numbers from 1 to 196, and C_{14} is the **composite** *magic square.*

In case of magic square M_{14}, the inner 4×4 block is a **pan diagonal magic square** of order 4 with magic sum $S_{4\times 4} := 694$.

4.1 Blocks of Order 3

As we have seen above, it is not possible to construct magic square of order 42 with blocks of equal sums magic squares of order 3. In this case, we shall construct magic square of order 42 with different magic sums of order 3. In order to make this magic square we shall make use of magic squares of order 3 and order 14 given in Examples 2.1 and 4.3 respectively.

Distribution 4.1. *Let's consider the following distribution of 1 to 42 numbers divided in 3 groups of 14 each:*

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	Total
1	1	4	7	10	13	16	19	22	25	28	31	34	37	40	287
2	2	5	8	11	14	17	20	23	26	29	32	35	38	41	301
3	3	6	9	12	15	18	21	24	27	30	33	36	39	42	315

Distribution 4.2. *Let's rewrite the composite C_{14} magic square of order 14 given in Example 4.3 as follows:*

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1.1	4.13	11.10	5.14	8.11	2.6	9.7	3.9	7.8	14.3	12.4	13.2	6.12	10.5
14.4	2.2	5.13	11.1	614	3.7	10.8	4.10	8.9	12.5	13.3	7.12	1.6	9.11
12.6	14.5	3.3	6.13	11.2	4.8	1.9	5.1	9.10	13.4	8.12	2.7	10.11	7.14
13.5	12.7	14.6	4.4	7.13	5.9	2.10	6.2	10.1	9.12	3.8	1.11	8.14	11.3
10.12	13.6	12.8	14.7	5.5	6.10	3.1	7.3	1.2	4.9	2.11	9.14	11.4	8.13
7.9	8.10	9.1	10.2	1.3	11.11	13.14	14.12	12.13	6.8	5.7	4.6	3.5	2.4
9.2	10.3	1.4	2.5	3.6	14.13	12.12	11.14	13.11	8.1	7.10	6.9	5.8	4.7
6.3	7.4	8.5	9.6	10.7	12.14	14.11	13.13	11.12	5.2	4.1	3.10	2.9	1.8
8.7	9.8	10.9	1.10	2.1	13.12	11.13	12.11	14.14	7.6	6.5	5.4	4.3	3.2
3.13	11.9	4.14	7.11	9.4	1.5	8.6	2.8	6.7	10.10	14.2	12.3	13.1	5.12
11.8	3.14	6.11	8.3	4.12	10.4	7.5	1.7	5.6	2.13	9.9	14.1	12.2	13.10
2.14	5.11	7.2	3.12	13.9	9.3	6.4	10.6	4.5	11.7	1.13	8.8	14.10	12.1
4.11	6.1	2.12	13.8	12.10	8.2	5.3	9.5	3.4	1.14	11.6	10.13	7.7	14.9
5.10	1.12	13.7	12.9	14.8	7.1	4.2	8.4	2.3	3.11	10.14	11.5	9.13	6.6

We shall construct 194 block of order 3 and put them according to Distribution 4.1. Below are few examples of magic squares of order 3 constructed by applying the columns values given in Distribution 4.1 over the Example 2.1 by using the operation $M_3 := 42 \times (A-1) + B$:

- **Block 8.10**

8			69
23	24	22	69
22	23	24	69
24	22	23	69
69	69	69	69

10			89
28	30	29	89
30	29	28	89
29	28	30	89
89	89	89	89

8.10			2859
952	996	911	2859
912	953	994	2859
995	910	954	2859
2859	2859	2859	2859

• **Block 9.3**

(9)			*78*
26	27	25	*78*
25	26	27	*78*
27	25	26	*78*
78	*78*	*78*	*78*

(3)			*24*
7	9	8	*24*
9	8	7	*24*
8	7	9	*24*
24	*24*	*24*	*24*

(9.3)			*3174*
1057	1101	1016	*3174*
1017	1058	1099	*3174*
1100	1015	1059	*3174*
3174	*3174*	*3174*	*3174*

• **Block 14.12**

(14)			*123*
41	42	40	*123*
40	41	42	*123*
42	40	41	*123*
123	*123*	*123*	*123*

(12)			*105*
34	36	35	*105*
36	35	34	*105*
35	34	36	*105*
105	*105*	*105*	*105*

(14.12)			*5145*
1714	1758	1673	*5145*
1674	1715	1756	*5145*
1757	1672	1716	*5145*
5145	*5145*	*5145*	*5145*

Based on similar procedure we construct all the 196 blocks magic squares of order 3 and put them according to Distributions 4.2, we get the required magic square of order 42 given in example below.

Example 4.4. *The **block-wise** magic square of order 42 with blocks of order 3 is given by*

Magic square of order 42. All blocks of order 3 are magic squares with different magic sums forming a magic square of order 14.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	37065
1	43	87	2	457	501	416	1330	1374	1289	586	630	545	955	999	914	184	228	143	1069	1113	1028	319	363	278	820	864	779	1687	1731	1646	1438	1482	1397	1558	1602	1517	706	750	665	1189	1233	1148	37065
2	3	44	85	417	458	499	1290	1331	1372	546	587	628	915	956	997	144	185	226	1029	1070	1111	279	320	361	780	821	862	1647	1688	1729	1398	1439	1480	1518	1559	1600	666	707	748	1149	1190	1231	37065
3	86	1	45	500	415	459	1373	1288	1332	629	544	588	998	913	957	227	142	186	1112	1027	1071	362	277	321	863	778	822	1730	1645	1689	1481	1396	1440	1601	1516	1560	749	664	708	1232	1147	1191	37065
4	1690	1734	1649	172	216	131	583	627	542	1303	1347	1262	712	756	671	313	357	272	1198	1242	1157	448	492	407	949	993	908	1441	1485	1400	1561	1605	1520	832	876	791	58	102	17	1081	1125	1040	37065
5	1650	1691	1732	132	173	214	543	584	625	1263	1304	1345	672	713	754	273	314	355	1158	1199	1240	408	449	490	909	950	991	1401	1442	1483	1521	1562	1603	792	833	874	18	59	100	1041	1082	1123	37065
6	1733	1648	1692	215	130	174	626	541	585	1346	1261	1305	755	670	714	356	271	315	1241	1156	1200	491	406	450	992	907	951	1484	1399	1443	1604	1519	1563	875	790	834	101	16	60	1124	1039	1083	37065
7	1444	1488	1403	1693	1737	1652	301	345	260	709	753	668	1306	1350	1265	442	486	401	67	111	26	547	591	506	1078	1122	1037	1564	1608	1523	958	1002	917	187	231	146	1207	1251	1166	838	882	797	37065
8	1404	1445	1486	1653	1694	1735	261	302	343	669	710	751	1266	1307	1348	402	443	484	27	68	109	507	548	589	1038	1079	1120	1524	1565	1606	918	959	1000	147	188	229	1167	1208	1249	798	839	880	37065
9	1487	1402	1446	1736	1651	1695	344	259	303	752	667	711	1349	1264	1308	485	400	444	110	25	69	590	505	549	1121	1036	1080	1607	1522	1566	1001	916	960	230	145	189	1250	1165	1209	881	796	840	37065
10	1567	1611	1526	1447	1491	1406	1696	1740	1655	430	474	389	835	879	794	571	615	530	196	240	155	676	720	635	1177	1221	1136	1084	1128	1043	316	360	275	73	117	32	964	1008	923	1309	1353	1268	37065
11	1527	1568	1609	1407	1448	1489	1656	1697	1738	390	431	472	795	836	877	531	572	613	156	197	238	636	677	718	1137	1178	1219	1044	1085	1126	276	317	358	33	74	115	924	965	1006	1269	1310	1351	37065
12	1610	1525	1569	1490	1405	1449	1739	1654	1698	473	388	432	878	793	837	614	529	573	239	154	198	719	634	678	1220	1135	1179	1127	1042	1086	359	274	318	116	31	75	1007	922	966	1352	1267	1311	37065
13	1210	1254	1169	1570	1614	1529	1450	1494	1409	1699	1743	1658	559	603	518	700	744	659	295	339	254	805	849	764	46	90	5	445	489	404	199	243	158	1090	1134	1049	1312	1356	1271	961	1005	920	37065
14	1170	1211	1252	1530	1571	1612	1410	1451	1492	1659	1700	1741	519	560	601	660	701	742	255	296	337	765	806	847	6	47	88	405	446	487	159	200	241	1050	1091	1132	1272	1313	1354	921	962	1003	37065
15	1253	1168	1212	1613	1528	1572	1493	1408	1452	1742	1657	1701	602	517	561	743	658	702	338	253	297	848	763	807	89	4	48	488	403	447	242	157	201	1133	1048	1092	1355	1270	1314	1004	919	963	37065
16	823	867	782	952	996	911	1051	1095	1010	1180	1224	1139	49	93	8	1333	1377	1292	1594	1638	1553	1714	1758	1673	1465	1509	1424	694	738	653	565	609	524	436	480	395	307	351	266	178	222	137	37065
17	783	824	865	912	953	994	1011	1052	1093	1140	1181	1222	9	50	91	1293	1334	1375	1554	1595	1636	1674	1715	1756	1425	1466	1507	654	695	736	525	566	607	396	437	478	267	308	349	138	179	220	37065
18	866	781	825	995	910	954	1094	1009	1053	1223	1138	1182	92	7	51	1376	1291	1335	1637	1552	1596	1757	1672	1716	1508	1423	1467	737	652	696	608	523	567	479	394	438	350	265	309	221	136	180	37065
19	1054	1098	1013	1183	1227	1142	52	96	11	181	225	140	310	354	269	1717	1761	1676	1462	1506	1421	1342	1386	1301	1585	1629	1544	925	969	884	826	870	785	697	741	656	568	612	527	439	483	398	37065
20	1014	1055	1096	1143	1184	1225	12	53	94	141	182	223	270	311	352	1677	1718	1759	1422	1463	1504	1302	1343	1384	1545	1586	1627	885	926	967	786	827	868	657	698	739	528	569	610	399	440	481	37065
21	1097	1012	1056	1226	1141	1185	95	10	54	224	139	183	353	268	312	1760	1675	1719	1505	1420	1464	1385	1300	1344	1628	1543	1587	968	883	927	869	784	828	740	655	699	611	526	570	482	397	441	37065
22	679	723	638	808	852	767	937	981	896	1066	1110	1025	1195	1239	1154	1468	1512	1427	1711	1755	1670	1591	1635	1550	1336	1380	1295	550	594	509	421	465	380	322	366	281	193	237	152	64	108	23	37065
23	639	680	721	768	809	850	897	938	979	1026	1067	1108	1155	1196	1237	1428	1469	1510	1671	1712	1753	1551	1592	1633	1296	1337	1378	510	551	592	381	422	463	282	323	364	153	194	235	24	65	106	37065
24	722	637	681	851	766	810	980	895	939	1109	1024	1068	1238	1153	1197	1511	1426	1470	1754	1669	1713	1634	1549	1593	1379	1294	1338	593	508	552	464	379	423	365	280	324	236	151	195	107	22	66	37065
25	943	987	902	1072	1116	1031	1201	1245	1160	70	114	29	169	213	128	1588	1632	1547	1339	1383	1298	1459	1503	1418	1720	1764	1679	814	858	773	685	729	644	556	600	515	427	471	386	298	342	257	37065
26	903	944	985	1032	1073	1114	1161	1202	1243	30	71	112	129	170	211	1548	1589	1630	1299	1340	1381	1419	1460	1501	1680	1721	1762	774	815	856	645	686	727	516	557	598	387	428	469	258	299	340	37065
27	986	901	945	1115	1030	1074	1244	1159	1203	113	28	72	212	127	171	1631	1546	1590	1382	1297	1341	1502	1417	1461	1763	1678	1722	857	772	816	728	643	687	599	514	558	470	385	429	341	256	300	37065
28	331	375	290	1327	1371	1286	460	504	419	829	873	788	1060	1104	1019	55	99	14	940	984	899	190	234	149	691	735	650	1204	1248	1163	1684	1728	1643	1435	1479	1394	1555	1599	1514	580	624	539	37065
29	291	332	373	1287	1328	1369	420	461	502	789	830	871	1020	1061	1102	15	56	97	900	941	982	150	191	232	651	692	733	1164	1205	1246	1644	1685	1726	1395	1436	1477	1515	1556	1597	540	581	622	37065
30	374	289	333	1370	1285	1329	503	418	462	872	787	831	1103	1018	1062	98	13	57	983	898	942	233	148	192	734	649	693	1247	1162	1206	1727	1642	1686	1478	1393	1437	1598	1513	1557	623	538	582	37065
31	1324	1368	1283	334	378	293	703	747	662	931	975	890	454	498	413	1186	1230	1145	811	855	770	61	105	20	562	606	521	205	249	164	1075	1119	1034	1681	1725	1640	1432	1476	1391	1582	1626	1541	37065
32	1284	1325	1366	294	335	376	663	704	745	891	932	973	414	455	496	1146	1187	1228	771	812	853	21	62	103	522	563	604	165	206	247	1035	1076	1117	1641	1682	1723	1392	1433	1474	1542	1583	1624	37065
33	1367	1282	1326	377	292	336	746	661	705	974	889	933	497	412	456	1229	1144	1188	854	769	813	104	19	63	605	520	564	248	163	207	1118	1033	1077	1724	1639	1683	1475	1390	1434	1625	1540	1584	37065
34	208	252	167	577	621	536	802	846	761	328	372	287	1579	1623	1538	1057	1101	1016	682	726	641	1192	1236	1151	433	477	392	1321	1365	1280	79	123	38	946	990	905	1708	1752	1667	1429	1473	1388	37065
35	168	209	250	537	578	619	762	803	844	288	329	370	1539	1580	1621	1017	1058	1099	642	683	724	1152	1193	1234	393	434	475	1281	1322	1363	39	80	121	906	947	988	1668	1709	1750	1389	1430	1471	37065
36	251	166	210	620	535	579	845	760	804	371	286	330	1622	1537	1581	1100	1015	1059	725	640	684	1235	1150	1194	476	391	435	1364	1279	1323	122	37	81	989	904	948	1751	1666	1710	1472	1387	1431	37065
37	451	495	410	673	717	632	202	246	161	1576	1620	1535	1456	1500	1415	928	972	887	553	597	512	1063	1107	1022	304	348	263	82	126	41	1318	1362	1277	1213	1257	1172	817	861	776	1705	1749	1664	37065
38	411	452	493	633	674	715	162	203	244	1536	1577	1618	1416	1457	1498	888	929	970	513	554	595	1023	1064	1105	264	305	346	42	83	124	1278	1319	1360	1173	1214	1255	777	818	859	1665	1706	1747	37065
39	494	409	453	716	631	675	245	160	204	1619	1534	1578	1499	1414	1458	971	886	930	596	511	555	1106	1021	1065	347	262	306	125	40	84	1361	1276	1320	1256	1171	1215	860	775	819	1748	1663	1707	37065
40	574	618	533	76	120	35	1573	1617	1532	1453	1497	1412	1702	1746	1661	799	843	758	424	468	383	934	978	893	175	219	134	325	369	284	1216	1260	1175	1315	1359	1274	1087	1131	1046	688	732	647	37065
41	534	575	616	36	77	118	1533	1574	1615	1413	1454	1495	1662	1703	1744	759	800	841	384	425	466	894	935	976	135	176	217	285	326	367	1176	1217	1258	1275	1316	1357	1047	1088	1129	648	689	730	37065
42	617	532	576	119	34	78	1616	1531	1575	1496	1411	1455	1745	1660	1704	842	757	801	467	382	426	977	892	936	218	133	177	368	283	327	1259	1174	1218	1358	1273	1317	1130	1045	1089	731	646	690	37065
	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065

4.2 Blocks of Order 6

In this subsection, we shall give construction of a magic square of order 42 in 49 blocks of magic squares of order 6 with equal magic sums. In order to construct it let's divide 1764 numbers in 49 equal parts according to following distribution.

Distribution 4.3. *Let's consider the following distribution of 1 to 1764 numbers in 49 blocks of equal sums:*

	1	2	3	4	5	6	...	...	16	17	18	19	20	21	...	...	31	32	33	34	35	36	Total
A1	1	98	99	196	197	294	...	...	784	785	882	883	980	981	...	...	1471	1568	1569	1666	1667	1764	31770
A2	2	97	100	195	198	293	...	...	783	786	881	884	979	982	...	...	1472	1567	1570	1665	1668	1763	31770
A3	3	96	101	194	199	292	...	...	782	787	880	885	978	983	...	...	1473	1566	1571	1664	1669	1762	31770
A4	4	95	102	193	200	291	...	...	781	788	879	886	977	984	...	...	1474	1565	1572	1663	1670	1761	31770
A5	5	94	103	192	201	290	...	...	780	789	878	887	976	985	...	...	1475	1564	1573	1662	1671	1760	31770
A6	6	93	104	191	202	289	...	...	779	790	877	888	975	986	...	...	1476	1563	1574	1661	1672	1759	31770
A7	7	92	105	190	203	288	...	...	778	791	876	889	974	987	...	...	1477	1562	1575	1660	1673	1758	31770
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
A22	22	77	120	175	218	273	...	...	763	806	861	904	959	1002	...	...	1492	1547	1590	1645	1688	1743	31770
A23	23	76	121	174	219	272	...	...	762	807	860	905	958	1003	...	...	1493	1546	1591	1644	1689	1742	31770
A24	24	75	122	173	220	271	...	...	761	808	859	906	957	1004	...	...	1494	1545	1592	1643	1690	1741	31770
A25	25	74	123	172	221	270	...	...	760	809	858	907	956	1005	...	...	1495	1544	1593	1642	1691	1740	31770
A26	26	73	124	171	222	269	...	...	759	810	857	908	955	1006	...	...	1496	1543	1594	1641	1692	1739	31770
A27	27	72	125	170	223	268	...	...	758	811	856	909	954	1007	...	...	1497	1542	1595	1640	1693	1738	31770
A28	28	71	126	169	224	267	...	...	757	812	855	910	953	1008	...	...	1498	1541	1596	1639	1694	1737	31770
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
A42	42	57	140	155	238	253	...	...	743	826	841	924	939	1022	...	...	1512	1527	1610	1625	1708	1723	31770
A43	43	56	141	154	239	252	...	...	742	827	840	925	938	1023	...	...	1513	1526	1611	1624	1709	1722	31770
A44	44	55	142	153	240	251	...	...	741	828	839	926	937	1024	...	...	1514	1525	1612	1623	1710	1721	31770
A45	45	54	143	152	241	250	...	...	740	829	838	927	936	1025	...	...	1515	1524	1613	1622	1711	1720	31770
A46	46	53	144	151	242	249	...	...	739	830	837	928	935	1026	...	...	1516	1523	1614	1621	1712	1719	31770
A47	47	52	145	150	243	248	...	...	738	831	836	929	934	1027	...	...	1517	1522	1615	1620	1713	1718	31770
A48	48	51	146	149	244	247	...	...	737	832	835	930	933	1028	...	...	1518	1521	1616	1619	1714	1717	31770
A49	49	50	147	148	245	246	...	...	736	833	834	931	932	1029	...	...	1519	1520	1617	1618	1715	1716	31770

Let's construct 36 magic squares of order 6 according to Example 4.1, and put them according to following structure.

Structure 4.1. *Let's consider 36 blocks of magic squares of order 6 as below:*

A1	A2	A3	A4	A5	A6	A7
A8	A9	A10	A11	A12	A13	A14
A15	A16	A17	A18	A19	A20	A21
A22	A23	A24	A25	A26	A27	A28
A29	A30	A31	A32	A33	A34	A35
A36	A37	A38	A39	A40	A41	A42
A43	A44	A45	A46	A47	A48	A49

We shall construct 49 magic square of order 6 by applying the Example 4.1 of equal magic sums. See below some examples:

https://doi.org/10.5281/zenodo.2555889

A16						5295
16	1682	1651	1584	83	279	5295
1455	377	1357	408	506	1192	5295
1161	1094	702	769	965	604	5295
867	671	996	1063	800	898	5295
310	1259	475	1290	1388	573	5295
1486	212	114	181	1553	1749	5295
5295	5295	5295	5295	5295	5295	5295

A27						5295
27	1693	1640	1595	72	268	5295
1444	366	1346	419	517	1203	5295
1150	1105	713	758	954	615	5295
856	660	1007	1052	811	909	5295
321	1248	464	1301	1399	562	5295
1497	223	125	170	1542	1738	5295
5295	5295	5295	5295	5295	5295	5295

A31						5295
31	1697	1636	1599	68	264	5295
1440	362	1342	423	521	1207	5295
1146	1109	717	754	950	619	5295
852	656	1011	1048	815	913	5295
325	1244	460	1305	1403	558	5295
1501	227	129	166	1538	1734	5295
5295	5295	5295	5295	5295	5295	5295

A46						5295
46	1712	1621	1614	53	249	5295
1425	347	1327	438	536	1222	5295
1131	1124	732	739	935	634	5295
837	641	1026	1033	830	928	5295
340	1229	445	1320	1418	543	5295
1516	242	144	151	1523	1719	5295
5295	5295	5295	5295	5295	5295	5295

In the similar way we can construct other 45 magic squares of order 6. Putting all these 49 blocks of equal sums magic squares of order 6 according to Distribution 4.1 we get a magic square of order 42 given in example below.

Example 4.5. *The magic square of order 42 with equal sum blocks of square of order 6 is given by*

https://doi.org/10.5281/zenodo.2555889

Magic square of order 42 with equal sums magic squares of order 6: Magic square sums: S(42x42):=37065 and S(6x6):=5295.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	37065
1	1	1667	1666	1569	98	294	2	1668	1665	1570	97	293	3	1669	1664	1571	96	292	4	1670	1663	1572	95	291	5	1671	1662	1573	94	290	6	1672	1661	1574	93	289	7	1673	1660	1575	92	288	37065
2	1470	392	1372	393	491	1177	1469	391	1371	394	492	1178	1468	390	1370	395	493	1179	1467	389	1369	396	494	1180	1466	388	1368	397	495	1181	1465	387	1367	398	496	1182	1464	386	1366	399	497	1183	37065
3	1176	1079	687	784	980	589	1175	1080	688	783	979	590	1174	1081	689	782	978	591	1173	1082	690	781	977	592	1172	1083	691	780	976	593	1171	1084	692	779	975	594	1170	1085	693	778	974	595	37065
4	882	686	981	1078	785	883	881	685	982	1077	786	884	880	684	983	1076	787	885	879	683	984	1075	788	886	878	682	985	1074	789	887	877	681	986	1073	790	888	876	680	987	1072	791	889	37065
5	295	1274	490	1275	1373	588	296	1273	489	1276	1374	587	297	1272	488	1277	1375	586	298	1271	487	1278	1376	585	299	1270	486	1279	1377	584	300	1269	485	1280	1378	583	301	1268	484	1281	1379	582	37065
6	1471	197	99	196	1568	1764	1472	198	100	195	1567	1763	1473	199	101	194	1566	1762	1474	200	102	193	1565	1761	1475	201	103	192	1564	1760	1476	202	104	191	1563	1759	1477	203	105	190	1562	1758	37065
7	8	1674	1659	1576	91	287	9	1675	1658	1577	90	286	10	1676	1657	1578	89	285	11	1677	1656	1579	88	284	12	1678	1655	1580	87	283	13	1679	1654	1581	86	282	14	1680	1653	1582	85	281	37065
8	1463	385	1365	400	498	1184	1462	384	1364	401	499	1185	1461	383	1363	402	500	1186	1460	382	1362	403	501	1187	1459	381	1361	404	502	1188	1458	380	1360	405	503	1189	1457	379	1359	406	504	1190	37065
9	1169	1086	694	777	973	596	1168	1087	695	776	972	597	1167	1088	696	775	971	598	1166	1089	697	774	970	599	1165	1090	698	773	969	600	1164	1091	699	772	968	601	1163	1092	700	771	967	602	37065
10	875	679	988	1071	792	890	874	678	989	1070	793	891	873	677	990	1069	794	892	872	676	991	1068	795	893	871	675	992	1067	796	894	870	674	993	1066	797	895	869	673	994	1065	798	896	37065
11	302	1267	483	1282	1380	581	303	1266	482	1283	1381	580	304	1265	481	1284	1382	579	305	1264	480	1285	1383	578	306	1263	479	1286	1384	577	307	1262	478	1287	1385	576	308	1261	477	1288	1386	575	37065
12	1478	204	106	189	1561	1757	1479	205	107	188	1560	1756	1480	206	108	187	1559	1755	1481	207	109	186	1558	1754	1482	208	110	185	1557	1753	1483	209	111	184	1556	1752	1484	210	112	183	1555	1751	37065
13	15	1681	1652	1583	84	280	16	1682	1651	1584	83	279	17	1683	1650	1585	82	278	18	1684	1649	1586	81	277	19	1685	1648	1587	80	276	20	1686	1647	1588	79	275	21	1687	1646	1589	78	274	37065
14	1456	378	1358	407	505	1191	1455	377	1357	408	506	1192	1454	376	1356	409	507	1193	1453	375	1355	410	508	1194	1452	374	1354	411	509	1195	1451	373	1353	412	510	1196	1450	372	1352	413	511	1197	37065
15	1162	1093	701	770	966	603	1161	1094	702	769	965	604	1160	1095	703	768	964	605	1159	1096	704	767	963	606	1158	1097	705	766	962	607	1157	1098	706	765	961	608	1156	1099	707	764	960	609	37065
16	868	672	995	1064	799	897	867	671	996	1063	800	898	866	670	997	1062	801	899	865	669	998	1061	802	900	864	668	999	1060	803	901	863	667	1000	1059	804	902	862	666	1001	1058	805	903	37065
17	309	1260	476	1289	1387	574	310	1259	475	1290	1388	573	311	1258	474	1291	1389	572	312	1257	473	1292	1390	571	313	1256	472	1293	1391	570	314	1255	471	1294	1392	569	315	1254	470	1295	1393	568	37065
18	1485	211	113	182	1554	1750	1486	212	114	181	1553	1749	1487	213	115	180	1552	1748	1488	214	116	179	1551	1747	1489	215	117	178	1550	1746	1490	216	118	177	1549	1745	1491	217	119	176	1548	1744	37065
19	22	1688	1645	1590	77	273	23	1689	1644	1591	76	272	24	1690	1643	1592	75	271	25	1691	1642	1593	74	270	26	1692	1641	1594	73	269	27	1693	1640	1595	72	268	28	1694	1639	1596	71	267	37065
20	1449	371	1351	414	512	1198	1448	370	1350	415	513	1199	1447	369	1349	416	514	1200	1446	368	1348	417	515	1201	1445	367	1347	418	516	1202	1444	366	1346	419	517	1203	1443	365	1345	420	518	1204	37065
21	1155	1100	708	763	959	610	1154	1101	709	762	958	611	1153	1102	710	761	957	612	1152	1103	711	760	956	613	1151	1104	712	759	955	614	1150	1105	713	758	954	615	1149	1106	714	757	953	616	37065
22	861	665	1002	1057	806	904	860	664	1003	1056	807	905	859	663	1004	1055	808	906	858	662	1005	1054	809	907	857	661	1006	1053	810	908	856	660	1007	1052	811	909	855	659	1008	1051	812	910	37065
23	316	1253	469	1296	1394	567	317	1252	468	1297	1395	566	318	1251	467	1298	1396	565	319	1250	466	1299	1397	564	320	1249	465	1300	1398	563	321	1248	464	1301	1399	562	322	1247	463	1302	1400	561	37065
24	1492	218	120	175	1547	1743	1493	219	121	174	1546	1742	1494	220	122	173	1545	1741	1495	221	123	172	1544	1740	1496	222	124	171	1543	1739	1497	223	125	170	1542	1738	1498	224	126	169	1541	1737	37065
25	29	1695	1638	1597	70	266	30	1696	1637	1598	69	265	31	1697	1636	1599	68	264	32	1698	1635	1600	67	263	33	1699	1634	1601	66	262	34	1700	1633	1602	65	261	35	1701	1632	1603	64	260	37065
26	1442	364	1344	421	519	1205	1441	363	1343	422	520	1206	1440	362	1342	423	521	1207	1439	361	1341	424	522	1208	1438	360	1340	425	523	1209	1437	359	1339	426	524	1210	1436	358	1338	427	525	1211	37065
27	1148	1107	715	756	952	617	1147	1108	716	755	951	618	1146	1109	717	754	950	619	1145	1110	718	753	949	620	1144	1111	719	752	948	621	1143	1112	720	751	947	622	1142	1113	721	750	946	623	37065
28	854	658	1009	1050	813	911	853	657	1010	1049	814	912	852	656	1011	1048	815	913	851	655	1012	1047	816	914	850	654	1013	1046	817	915	849	653	1014	1045	818	916	848	652	1015	1044	819	917	37065
29	323	1246	462	1303	1401	560	324	1245	461	1304	1402	559	325	1244	460	1305	1403	558	326	1243	459	1306	1404	557	327	1242	458	1307	1405	556	328	1241	457	1308	1406	555	329	1240	456	1309	1407	554	37065
30	1499	225	127	168	1540	1736	1500	226	128	167	1539	1735	1501	227	129	166	1538	1734	1502	228	130	165	1537	1733	1503	229	131	164	1536	1732	1504	230	132	163	1535	1731	1505	231	133	162	1534	1730	37065
31	36	1702	1631	1604	63	259	37	1703	1630	1605	62	258	38	1704	1629	1606	61	257	39	1705	1628	1607	60	256	40	1706	1627	1608	59	255	41	1707	1626	1609	58	254	42	1708	1625	1610	57	253	37065
32	1435	357	1337	428	526	1212	1434	356	1336	429	527	1213	1433	355	1335	430	528	1214	1432	354	1334	431	529	1215	1431	353	1333	432	530	1216	1430	352	1332	433	531	1217	1429	351	1331	434	532	1218	37065
33	1141	1114	722	749	945	624	1140	1115	723	748	944	625	1139	1116	724	747	943	626	1138	1117	725	746	942	627	1137	1118	726	745	941	628	1136	1119	727	744	940	629	1135	1120	728	743	939	630	37065
34	847	651	1016	1043	820	918	846	650	1017	1042	821	919	845	649	1018	1041	822	920	844	648	1019	1040	823	921	843	647	1020	1039	824	922	842	646	1021	1038	825	923	841	645	1022	1037	826	924	37065
35	330	1239	455	1310	1408	553	331	1238	454	1311	1409	552	332	1237	453	1312	1410	551	333	1236	452	1313	1411	550	334	1235	451	1314	1412	549	335	1234	450	1315	1413	548	336	1233	449	1316	1414	547	37065
36	1506	232	134	161	1533	1729	1507	233	135	160	1532	1728	1508	234	136	159	1531	1727	1509	235	137	158	1530	1726	1510	236	138	157	1529	1725	1511	237	139	156	1528	1724	1512	238	140	155	1527	1723	37065
37	43	1709	1624	1611	56	252	44	1710	1623	1612	55	251	45	1711	1622	1613	54	250	46	1712	1621	1614	53	249	47	1713	1620	1615	52	248	48	1714	1619	1616	51	247	49	1715	1618	1617	50	246	37065
38	1428	350	1330	435	533	1219	1427	349	1329	436	534	1220	1426	348	1328	437	535	1221	1425	347	1327	438	536	1222	1424	346	1326	439	537	1223	1423	345	1325	440	538	1224	1422	344	1324	441	539	1225	37065
39	1134	1121	729	742	938	631	1133	1122	730	741	937	632	1132	1123	731	740	936	633	1131	1124	732	739	935	634	1130	1125	733	738	934	635	1129	1126	734	737	933	636	1128	1127	735	736	932	637	37065
40	840	644	1023	1036	827	925	839	643	1024	1035	828	926	838	642	1025	1034	829	927	837	641	1026	1033	830	928	836	640	1027	1032	831	929	835	639	1028	1031	832	930	834	638	1029	1030	833	931	37065
41	337	1232	448	1317	1415	546	338	1231	447	1318	1416	545	339	1230	446	1319	1417	544	340	1229	445	1320	1418	543	341	1228	444	1321	1419	542	342	1227	443	1322	1420	541	343	1226	442	1323	1421	540	37065
42	1513	239	141	154	1526	1722	1514	240	142	153	1525	1721	1515	241	143	152	1524	1720	1516	242	144	151	1523	1719	1517	243	145	150	1522	1718	1518	244	146	149	1521	1717	1519	245	147	148	1520	1716	37065
	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065

In this case the magic square sums are $S_{42\times 42} := 37065$ and $S_{6\times 6} := 5295$.

4.3 Blocks of Order 7

As we have seen above, it is not possible to construct magic square of order 42 with blocks of equal sums magic squares of order 7. In this case, we shall construct magic square of order 42 with different magic sums of order 7. In order to make this magic square we shall make use of magic squares of order 7 and order 6 given in Examples 4.2 and 4.1 respectively.

Distribution 4.4. *Let's consider the following distribution of 1 to 42 numbers divided in 3 groups of 14 each:*

	1	2	3	4	5	6	7	Total
1	1	12	13	24	25	36	37	148
2	2	11	14	23	26	35	38	149
3	3	10	15	22	27	34	39	150
4	4	9	16	21	28	33	40	151
5	5	8	17	20	29	32	41	152
6	6	7	18	19	30	31	42	153

Let's rewrite the composite magic square of order 6 given in Example 4.1.

Distribution 4.5. *Let's rewrite the composite C_6 magic square of order 6 given in Example 4.1 as follows:*

11	65	64	63	12	16
56	22	54	23	25	51
46	45	33	34	42	31
36	32	43	44	35	41
21	52	24	53	55	26
61	15	13	14	62	66

We shall construct 36 blocks of order 7 and put them according to Distribution 4.5. Below are few examples of magic squares of order 7 constructed by applying the columns values given in Distribution 4.4 over the Example 4.2 by using the operation $M_7 := 42 \times (A-1) + B$. See below some examples.

- **Block 22**

②		149	149	149	149	149	149	149
	2	11	14	23	26	35	38	149
149	35	38	2	11	14	23	26	149
149	23	26	35	38	2	11	14	149
149	11	14	23	26	35	38	2	149
149	38	2	11	14	23	26	35	149
149	26	35	38	2	11	14	23	149
149	14	23	26	35	38	2	11	149
	149	149	149	149	149	149	149	149

②		149	149	149	149	149	149	149
	2	11	14	23	26	35	38	149
149	26	35	38	2	11	14	23	149
149	11	14	23	26	35	38	2	149
149	35	38	2	11	14	23	26	149
149	14	23	26	35	38	2	11	149
149	38	2	11	14	23	26	35	149
149	23	26	35	38	2	11	14	149
	149	149	149	149	149	149	149	149

(22)		6113	6113	6113	6113	6113	6113	6113
	44	431	560	947	1076	1463	1592	6113
6113	1454	1589	80	422	557	938	1073	6113
6113	935	1064	1451	1580	77	458	548	6113
6113	455	584	926	1061	1442	1577	68	6113
6113	1568	65	446	581	962	1052	1439	6113
6113	1088	1430	1565	56	443	572	959	6113
6113	569	950	1085	1466	1556	53	434	6113
	6113	6113	6113	6113	6113	6113	6113	6113

- **Block 34**

(3)		150	150	150	150	150	150	150
	3	10	15	22	27	34	39	150
150	34	39	3	10	15	22	27	150
150	22	27	34	39	3	10	15	150
150	10	15	22	27	34	39	3	150
150	39	3	10	15	22	27	34	150
150	27	34	39	3	10	15	22	150
150	15	22	27	34	39	3	10	150
	150	150	150	150	150	150	150	150

(4)		151	151	151	151	151	151	151
	4	9	16	21	28	33	40	151
151	28	33	40	4	9	16	21	151
151	9	16	21	28	33	40	4	151
151	33	40	4	9	16	21	28	151
151	16	21	28	33	40	4	9	151
151	40	4	9	16	21	28	33	151
151	21	28	33	40	4	9	16	151
	151	151	151	151	151	151	151	151

(34)		6157	6157	6157	6157	6157	6157	6157
	88	387	604	903	1120	1419	1636	6157
6157	1414	1629	124	382	597	898	1113	6157
6157	891	1108	1407	1624	117	418	592	6157
6157	411	628	886	1101	1402	1617	112	6157
6157	1612	105	406	621	922	1096	1395	6157
6157	1132	1390	1605	100	399	616	915	6157
6157	609	910	1125	1426	1600	93	394	6157
	6157	6157	6157	6157	6157	6157	6157	6157

Based on similar procedure we construct all the 196 blocks magic squares of order 3 and put them according to Distributions 4.2, we get the required magic square of order 42 given in example below.

Example 4.6.. *The **block-wise** magic square of order 42 with blocks of order 7 is given by*

Magic square of order 42 with 36 blocks of pan diagonal magic squares of order 7 with different magics sums.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	37065
1	1	474	517	990	1033	1506	1549	215	260	731	776	1247	1292	1763	214	261	730	777	1246	1293	1762	213	262	729	778	1245	1294	1761	2	473	518	989	1034	1505	1550	6	469	522	985	1038	1501	1554	37065
2	1495	1548	37	463	516	979	1032	1289	1754	251	257	722	773	1238	1288	1755	250	256	723	772	1239	1287	1756	249	255	724	771	1240	1496	1547	38	464	515	980	1031	1500	1543	42	468	511	984	1027	37065
3	978	1021	1494	1537	36	499	505	764	1235	1280	1751	242	293	719	765	1234	1281	1750	243	292	718	766	1233	1282	1749	244	291	717	977	1022	1493	1538	35	500	506	973	1026	1489	1542	31	504	510	37065
4	498	541	967	1020	1483	1536	25	284	755	761	1226	1277	1742	239	285	754	760	1227	1276	1743	238	286	753	759	1228	1275	1744	237	497	542	968	1019	1484	1535	26	493	546	972	1015	1488	1531	30	37065
5	1525	24	487	540	1003	1009	1482	1739	230	281	746	797	1223	1268	1738	231	280	747	796	1222	1269	1737	232	279	748	795	1221	1270	1526	23	488	539	1004	1010	1481	1530	19	492	535	1008	1014	1477	37065
6	1045	1471	1524	13	486	529	1002	1259	1265	1730	227	272	743	788	1258	1264	1731	226	273	742	789	1257	1263	1732	225	274	741	790	1046	1472	1523	14	485	530	1001	1050	1476	1519	18	481	534	997	37065
7	528	991	1044	1507	1513	12	475	734	785	1250	1301	1727	218	269	735	784	1251	1300	1726	219	268	736	783	1252	1299	1725	220	267	527	992	1043	1508	1514	11	476	523	996	1039	1512	1518	7	480	37065
8	174	301	690	817	1206	1333	1722	44	431	560	947	1076	1463	1592	172	303	688	819	1204	1335	1720	45	430	561	946	1077	1462	1593	47	428	563	944	1079	1460	1595	169	306	685	822	1201	1338	1717	37065
9	1332	1711	210	300	679	816	1195	1454	1589	80	422	557	938	1073	1330	1713	208	298	681	814	1197	1455	1588	81	423	556	939	1072	1457	1586	83	425	554	941	1070	1327	1716	205	295	684	811	1200	37065
10	805	1194	1321	1710	199	336	678	935	1064	1451	1580	77	458	548	807	1192	1323	1708	201	334	676	934	1065	1450	1581	76	459	549	932	1067	1448	1583	74	461	551	810	1189	1326	1705	204	331	673	37065
11	325	714	804	1183	1320	1699	198	455	584	926	1061	1442	1577	68	327	712	802	1185	1318	1701	196	454	585	927	1060	1443	1576	69	452	587	929	1058	1445	1574	71	330	709	799	1188	1315	1704	193	37065
12	1698	187	324	703	840	1182	1309	1568	65	446	581	962	1052	1439	1696	189	322	705	838	1180	1311	1569	64	447	580	963	1053	1438	1571	62	449	578	965	1055	1436	1693	192	319	708	835	1177	1314	37065
13	1218	1308	1687	186	313	702	829	1088	1430	1565	56	443	572	959	1216	1306	1689	184	315	700	831	1089	1431	1564	57	442	573	958	1091	1433	1562	59	440	575	956	1213	1303	1692	181	318	697	834	37065
14	691	828	1207	1344	1686	175	312	569	950	1085	1466	1556	53	434	693	826	1209	1342	1684	177	310	568	951	1084	1467	1557	52	435	566	953	1082	1469	1559	50	437	696	823	1212	1339	1681	180	307	37065
15	132	343	648	859	1164	1375	1680	131	344	647	860	1163	1376	1679	87	388	603	904	1119	1420	1635	88	387	604	903	1120	1419	1636	128	347	644	863	1160	1379	1676	85	390	601	906	1117	1422	1633	37065
16	1374	1669	168	342	637	858	1153	1373	1670	167	341	638	857	1154	1413	1630	123	381	598	897	1114	1414	1629	124	382	597	898	1113	1370	1673	164	338	641	854	1157	1411	1632	121	379	600	895	1116	37065
17	847	1152	1363	1668	157	378	636	848	1151	1364	1667	158	377	635	892	1107	1408	1623	118	417	591	891	1108	1407	1624	117	418	592	851	1148	1367	1664	161	374	632	894	1105	1410	1621	120	415	589	37065
18	367	672	846	1141	1362	1657	156	368	671	845	1142	1361	1658	155	412	627	885	1102	1401	1618	111	411	628	886	1101	1402	1617	112	371	668	842	1145	1358	1661	152	414	625	883	1104	1399	1620	109	37065
19	1656	145	366	661	882	1140	1351	1655	146	365	662	881	1139	1352	1611	106	405	622	921	1095	1396	1612	105	406	621	922	1096	1395	1652	149	362	665	878	1136	1355	1609	108	403	624	919	1093	1398	37065
20	1176	1350	1645	144	355	660	871	1175	1349	1646	143	356	659	872	1131	1389	1606	99	400	615	916	1132	1390	1605	100	399	616	915	1172	1346	1649	140	359	656	875	1129	1387	1608	97	402	613	918	37065
21	649	870	1165	1386	1644	133	354	650	869	1166	1385	1643	134	353	610	909	1126	1425	1599	94	393	609	910	1125	1426	1600	93	394	653	866	1169	1382	1640	137	350	612	907	1128	1423	1597	96	391	37065
22	90	385	606	901	1122	1417	1638	86	389	602	905	1118	1421	1634	129	346	645	862	1161	1378	1677	130	345	646	861	1162	1377	1678	89	386	605	902	1121	1418	1637	127	348	643	864	1159	1380	1675	37065
23	1416	1627	126	384	595	900	1111	1412	1631	122	380	599	896	1115	1371	1672	165	339	640	855	1156	1372	1671	166	340	639	856	1155	1415	1628	125	383	596	899	1112	1369	1674	163	337	642	853	1158	37065
24	889	1110	1405	1626	115	420	594	893	1106	1409	1622	119	416	590	850	1149	1366	1665	160	375	633	849	1150	1365	1666	159	376	634	890	1109	1406	1625	116	419	593	852	1147	1368	1663	162	373	631	37065
25	409	630	888	1099	1404	1615	114	413	626	884	1103	1400	1619	110	370	669	843	1144	1359	1660	153	369	670	844	1143	1360	1659	154	410	629	887	1100	1403	1616	113	372	667	841	1146	1357	1662	151	37065
26	1614	103	408	619	924	1098	1393	1610	107	404	623	920	1094	1397	1653	148	363	664	879	1137	1354	1654	147	364	663	880	1138	1353	1613	104	407	620	923	1097	1394	1651	150	361	666	877	1135	1356	37065
27	1134	1392	1603	102	397	618	913	1130	1388	1607	98	401	614	917	1173	1347	1648	141	358	657	874	1174	1348	1647	142	357	658	873	1133	1391	1604	101	398	617	914	1171	1345	1650	139	360	655	876	37065
28	607	912	1123	1428	1602	91	396	611	908	1127	1424	1598	95	392	652	867	1168	1383	1641	136	351	651	868	1167	1384	1642	135	352	608	911	1124	1427	1601	92	395	654	865	1170	1381	1639	138	349	37065
29	43	432	559	948	1075	1464	1591	170	305	686	821	1202	1337	1718	46	429	562	945	1078	1461	1594	171	304	687	820	1203	1336	1719	173	302	689	818	1205	1334	1721	48	427	564	943	1080	1459	1596	37065
30	1453	1590	79	421	558	937	1074	1328	1715	206	296	683	812	1199	1456	1587	82	424	555	940	1071	1329	1714	207	297	682	813	1198	1331	1712	209	299	680	815	1196	1458	1585	84	426	553	942	1069	37065
31	936	1063	1452	1579	78	457	547	809	1190	1325	1706	203	332	674	933	1066	1449	1582	75	460	550	808	1191	1324	1707	202	333	675	806	1193	1322	1709	200	335	677	931	1068	1447	1584	73	462	552	37065
32	456	583	925	1062	1441	1578	67	329	710	800	1187	1316	1703	194	453	586	928	1059	1444	1575	70	328	711	801	1186	1317	1702	195	326	713	803	1184	1319	1700	197	451	588	930	1057	1446	1573	72	37065
33	1567	66	445	582	961	1051	1440	1694	191	320	707	836	1178	1313	1570	63	448	579	964	1054	1437	1695	190	321	706	837	1179	1312	1697	188	323	704	839	1181	1310	1572	61	450	577	966	1056	1435	37065
34	1087	1429	1566	55	444	571	960	1214	1304	1691	182	317	698	833	1090	1432	1563	58	441	574	957	1215	1305	1690	183	316	699	832	1217	1307	1688	185	314	701	830	1092	1434	1561	60	439	576	955	37065
35	570	949	1086	1465	1555	54	433	695	824	1211	1340	1682	179	308	567	952	1083	1468	1558	51	436	694	825	1210	1341	1683	178	309	692	827	1208	1343	1685	176	311	565	954	1081	1470	1560	49	438	37065
36	211	264	727	780	1243	1296	1759	5	470	521	986	1037	1502	1553	3	472	519	988	1035	1504	1551	4	471	520	987	1036	1503	1552	212	263	728	779	1244	1295	1760	216	259	732	775	1248	1291	1764	37065
37	1285	1758	247	253	726	769	1242	1499	1544	41	467	512	983	1028	1497	1546	39	465	514	981	1030	1498	1545	40	466	513	982	1029	1286	1757	248	254	725	770	1241	1290	1753	252	258	721	774	1237	37065
38	768	1231	1284	1747	246	289	715	974	1025	1490	1541	32	503	509	976	1023	1492	1539	34	501	507	975	1024	1491	1540	33	502	508	767	1232	1283	1748	245	290	716	763	1236	1279	1752	241	294	720	37065
39	288	751	757	1230	1273	1746	235	494	545	971	1016	1487	1532	29	496	543	969	1018	1485	1534	27	495	544	970	1017	1486	1533	28	287	752	758	1229	1274	1745	236	283	756	762	1225	1278	1741	240	37065
40	1735	234	277	750	793	1219	1272	1529	20	491	536	1007	1013	1478	1527	22	489	538	1005	1011	1480	1528	21	490	537	1006	1012	1479	1736	233	278	749	794	1220	1271	1740	229	282	745	798	1224	1267	37065
41	1255	1261	1734	223	276	739	792	1049	1475	1520	17	482	533	998	1047	1473	1522	15	484	531	1000	1048	1474	1521	16	483	532	999	1256	1262	1733	224	275	740	791	1260	1266	1729	228	271	744	787	37065
42	738	781	1254	1297	1723	222	265	524	995	1040	1511	1517	8	479	526	993	1042	1509	1515	10	477	525	994	1041	1510	1516	9	478	737	782	1253	1298	1724	221	266	733	786	1249	1302	1728	217	270	37065
	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065

4.4 Blocks of Order 14

In this subsection, we shall give construction of a magic square of order 42 in 9 blocks of magic squares of order 14 with equal magic sums. In order to construct it let's divide 1764 numbers in 9 equal parts according to following distribution.

Distribution 4.6. *Let's consider the following distribution of 1 to 1764 numbers in 49 blocks of equal sums:*

https://doi.org/10.5281/zenodo.2555889

	1	2	3	4	5	6	7	8	9	10	...	...	187	188	189	190	191	192	193	194	195	196	Total
A1	1	18	19	36	37	54	55	72	73	90	...	...	1675	1692	1693	1710	1711	1728	1729	1746	1747	1764	172970
A2	2	17	20	35	38	53	56	71	74	89	...	...	1676	1691	1694	1709	1712	1727	1730	1745	1748	1763	172970
A3	3	16	21	34	39	52	57	70	75	88	...	...	1677	1690	1695	1708	1713	1726	1731	1744	1749	1762	172970
A4	4	15	22	33	40	51	58	69	76	87	...	...	1678	1689	1696	1707	1714	1725	1732	1743	1750	1761	172970
A5	5	14	23	32	41	50	59	68	77	86	...	...	1679	1688	1697	1706	1715	1724	1733	1742	1751	1760	172970
A6	6	13	24	31	42	49	60	67	78	85	...	...	1680	1687	1698	1705	1716	1723	1734	1741	1752	1759	172970
A7	7	12	25	30	43	48	61	66	79	84	...	...	1681	1686	1699	1704	1717	1722	1735	1740	1753	1758	172970
A8	8	11	26	29	44	47	62	65	80	83	...	...	1682	1685	1700	1703	1718	1721	1736	1739	1754	1757	172970
A9	9	10	27	28	45	46	63	64	81	82	...	...	1683	1684	1701	1702	1719	1720	1737	1738	1755	1756	172970

Distribution 4.7. *Let's put 9 blocks of order 14 according to following table:*

A1	A2	A3
A4	A5	A6
A7	A8	A9

We shall construct 9 magic square of order 14 by applying the Example 4.3 of equal magic sums. Before, let's see some examples:

- **Block A4**

A4														12355
4	850	1714	1005	1239	699	519	285	148	1426	1311	1599	1120	436	12355
1318	141	987	1581	1689	1138	778	1048	33	472	627	742	339	1462	12355
1131	1455	274	1228	1545	562	717	51	418	231	868	886	1336	1653	12355
1707	994	1347	411	1390	1030	40	688	303	1527	202	861	1257	598	12355
1473	753	490	105	1084	771	256	904	1192	555	1552	220	1671	1329	12355
933	87	796	328	184	1491	1606	1383	1750	663	393	1203	526	1012	12355
166	310	1185	508	22	1372	1761	1480	1617	825	1095	454	645	915	12355
400	580	634	807	321	1732	1365	1624	1509	1066	1167	15	922	213	12355
267	382	537	652	843	1635	1498	1743	1354	958	58	1059	195	1174	12355
609	1293	1401	238	670	429	940	832	1077	1221	1642	375	94	1534	12355
724	1678	1570	1023	616	159	1210	465	760	123	951	1282	1437	357	12355
1246	1563	69	1444	1275	969	1041	130	706	1660	346	544	483	879	12355
1588	1156	1102	1725	501	292	177	573	897	1264	735	1419	814	112	12355
789	1113	249	1300	976	76	447	1149	591	364	1408	1696	1516	681	12355
12355	12355	12355	12355	12355	12355	12355	12355	12355	12355	12355	12355	12355	12355	12355

- **Block A7**

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A7														*12355*
7	853	1717	1002	1236	696	516	282	151	1429	1308	1596	1123	439	*12355*
1321	138	984	1578	1686	1141	781	1051	30	475	624	745	336	1465	*12355*
1128	1452	277	1231	1542	565	714	48	421	228	871	889	1339	1650	*12355*
1704	997	1344	408	1393	1033	43	691	300	1524	205	858	1254	601	*12355*
1470	750	493	102	1087	768	259	907	1195	552	1555	223	1668	1326	*12355*
930	84	799	331	187	1488	1609	1380	1753	660	390	1200	529	1015	*12355*
169	313	1182	511	25	1375	1758	1483	1614	822	1092	457	642	912	*12355*
403	583	637	804	318	1735	1362	1627	1506	1069	1164	12	925	210	*12355*
264	385	534	655	840	1632	1501	1740	1357	961	61	1056	192	1177	*12355*
606	1290	1398	241	673	426	943	835	1074	1218	1645	372	97	1537	*12355*
727	1681	1573	1020	619	156	1213	462	763	120	948	1285	1434	354	*12355*
1249	1560	66	1447	1272	966	1038	133	709	1663	349	547	480	876	*12355*
1591	1159	1105	1722	498	295	174	570	894	1267	732	1416	817	115	*12355*
786	1110	246	1303	979	79	444	1146	588	367	1411	1699	1519	678	*12355*
12355	*12355*	*12355*	*12355*	*12355*	*12355*	*12355*	*12355*	*12355*	*12355*	*12355*	*12355*	*12355*	*12355*	*12355*

In the similar way we can construct other 7 blocks of order 14. Let's put these 9 blocks of equal magic sums order 14 according to Distribution 4.7. This gives a magic square of order 42 written in example below.

Example 4.7.. *The* ***block-wise*** *magic square of order 42 with equal magic sums blocks of order 14 is given by*

https://doi.org/10.5281/zenodo.2555889

Magic square of order 42 with 9 blocks of magic squares of order 14 with equal magic sums. Magic sums are S(42x42):=37042 and S(14x14):=12355.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	37065
1	1	847	1711	1008	1242	702	522	288	145	1423	1314	1602	1117	433	2	848	1712	1007	1241	701	521	287	146	1424	1313	1601	1118	434	3	849	1713	1006	1240	700	520	286	147	1425	1312	1600	1119	435	37065
2	1315	144	990	1584	1692	1135	775	1045	36	469	630	739	342	1459	1316	143	989	1583	1691	1136	776	1046	35	470	629	740	341	1460	1317	142	988	1582	1690	1137	777	1047	34	471	628	741	340	1461	37065
3	1134	1458	271	1225	1548	559	720	54	415	234	865	883	1333	1656	1133	1457	272	1226	1547	560	719	53	416	233	866	884	1334	1655	1132	1456	273	1227	1546	561	718	52	417	232	867	885	1335	1654	37065
4	1710	991	1350	414	1387	1027	37	685	306	1530	199	864	1260	595	1709	992	1349	413	1388	1028	38	686	305	1529	200	863	1259	596	1708	993	1348	412	1389	1029	39	687	304	1528	201	862	1258	597	37065
5	1476	756	487	108	1081	774	253	901	1189	558	1549	217	1674	1332	1475	755	488	107	1082	773	254	902	1190	557	1550	218	1673	1331	1474	754	489	106	1083	772	255	903	1191	556	1551	219	1672	1330	37065
6	936	90	793	325	181	1494	1603	1386	1747	666	396	1206	523	1009	935	89	794	326	182	1493	1604	1385	1748	665	395	1205	524	1010	934	88	795	327	183	1492	1605	1384	1749	664	394	1204	525	1011	37065
7	163	307	1188	505	19	1369	1764	1477	1620	828	1098	451	648	918	164	308	1187	506	20	1370	1763	1478	1619	827	1097	452	647	917	165	309	1186	507	21	1371	1762	1479	1618	826	1096	453	646	916	37065
8	397	577	631	810	324	1729	1368	1621	1512	1063	1170	18	919	216	398	578	632	809	323	1730	1367	1622	1511	1064	1169	17	920	215	399	579	633	808	322	1731	1366	1623	1510	1065	1168	16	921	214	37065
9	270	379	540	649	846	1638	1495	1746	1351	955	55	1062	198	1171	269	380	539	650	845	1637	1496	1745	1352	956	56	1061	197	1172	268	381	538	651	844	1636	1497	1744	1353	957	57	1060	196	1173	37065
10	612	1296	1404	235	667	432	937	829	1080	1224	1639	378	91	1531	611	1295	1403	236	668	431	938	830	1079	1223	1640	377	92	1532	610	1294	1402	237	669	430	939	831	1078	1222	1641	376	93	1533	37065
11	721	1675	1567	1026	613	162	1207	468	757	126	954	1279	1440	360	722	1676	1568	1025	614	161	1208	467	758	125	953	1280	1439	359	723	1677	1569	1024	615	160	1209	466	759	124	952	1281	1438	358	37065
12	1243	1566	72	1441	1278	972	1044	127	703	1657	343	541	486	882	1244	1565	71	1442	1277	971	1043	128	704	1658	344	542	485	881	1245	1564	70	1443	1276	970	1042	129	705	1659	345	543	484	880	37065
13	1585	1153	1099	1728	504	289	180	576	900	1261	738	1422	811	109	1586	1154	1100	1727	503	290	179	575	899	1262	737	1421	812	110	1587	1155	1101	1726	502	291	178	574	898	1263	736	1420	813	111	37065
14	792	1116	252	1297	973	73	450	1152	594	361	1405	1693	1513	684	791	1115	251	1298	974	74	449	1151	593	362	1406	1694	1514	683	790	1114	250	1299	975	75	448	1150	592	363	1407	1695	1515	682	37065
15	4	850	1714	1005	1239	699	519	285	148	1426	1311	1599	1120	436	5	851	1715	1004	1238	698	518	284	149	1427	1310	1598	1121	437	6	852	1716	1003	1237	697	517	283	150	1428	1309	1597	1122	438	37065
16	1318	141	987	1581	1689	1138	778	1048	33	472	627	742	339	1462	1319	140	986	1580	1688	1139	779	1049	32	473	626	743	338	1463	1320	139	985	1579	1687	1140	780	1050	31	474	625	744	337	1464	37065
17	1131	1455	274	1228	1545	562	717	51	418	231	868	886	1336	1653	1130	1454	275	1229	1544	563	716	50	419	230	869	887	1337	1652	1129	1453	276	1230	1543	564	715	49	420	229	870	888	1338	1651	37065
18	1707	994	1347	411	1390	1030	40	688	303	1527	202	861	1257	598	1706	995	1346	410	1391	1031	41	689	302	1526	203	860	1256	599	1705	996	1345	409	1392	1032	42	690	301	1525	204	859	1255	600	37065
19	1473	753	490	105	1084	771	256	904	1192	555	1552	220	1671	1329	1472	752	491	104	1085	770	257	905	1193	554	1553	221	1670	1328	1471	751	492	103	1086	769	258	906	1194	553	1554	222	1669	1327	37065
20	933	87	796	328	184	1491	1606	1383	1750	663	393	1203	526	1012	932	86	797	329	185	1490	1607	1382	1751	662	392	1202	527	1013	931	85	798	330	186	1489	1608	1381	1752	661	391	1201	528	1014	37065
21	166	310	1185	508	22	1372	1761	1480	1617	825	1095	454	645	915	167	311	1184	509	23	1373	1760	1481	1616	824	1094	455	644	914	168	312	1183	510	24	1374	1759	1482	1615	823	1093	456	643	913	37065
22	400	580	634	807	321	1732	1365	1624	1509	1066	1167	15	922	213	401	581	635	806	320	1733	1364	1625	1508	1067	1166	14	923	212	402	582	636	805	319	1734	1363	1626	1507	1068	1165	13	924	211	37065
23	267	382	537	652	843	1635	1498	1743	1354	958	58	1059	195	1174	266	383	536	653	842	1634	1499	1742	1355	959	59	1058	194	1175	265	384	535	654	841	1633	1500	1741	1356	960	60	1057	193	1176	37065
24	609	1293	1401	238	670	429	940	832	1077	1221	1642	375	94	1534	608	1292	1400	239	671	428	941	833	1076	1220	1643	374	95	1535	607	1291	1399	240	672	427	942	834	1075	1219	1644	373	96	1536	37065
25	724	1678	1570	1023	616	159	1210	465	760	123	951	1282	1437	357	725	1679	1571	1022	617	158	1211	464	761	122	950	1283	1436	356	726	1680	1572	1021	618	157	1212	463	762	121	949	1284	1435	355	37065
26	1246	1563	69	1444	1275	969	1041	130	706	1660	346	544	483	879	1247	1562	68	1445	1274	968	1040	131	707	1661	347	545	482	878	1248	1561	67	1446	1273	967	1039	132	708	1662	348	546	481	877	37065
27	1588	1156	1102	1725	501	292	177	573	897	1264	735	1419	814	112	1589	1157	1103	1724	500	293	176	572	896	1265	734	1418	815	113	1590	1158	1104	1723	499	294	175	571	895	1266	733	1417	816	114	37065
28	789	1113	249	1300	976	76	447	1149	591	364	1408	1696	1516	681	788	1112	248	1301	977	77	446	1148	590	365	1409	1697	1517	680	787	1111	247	1302	978	78	445	1147	589	366	1410	1698	1518	679	37065
29	7	853	1717	1002	1236	696	516	282	151	1429	1308	1596	1123	439	8	854	1718	1001	1235	695	515	281	152	1430	1307	1595	1124	440	9	855	1719	1000	1234	694	514	280	153	1431	1306	1594	1125	441	37065
30	1321	138	984	1578	1686	1141	781	1051	30	475	624	745	336	1465	1322	137	983	1577	1685	1142	782	1052	29	476	623	746	335	1466	1323	136	982	1576	1684	1143	783	1053	28	477	622	747	334	1467	37065
31	1128	1452	277	1231	1542	565	714	48	421	228	871	889	1339	1650	1127	1451	278	1232	1541	566	713	47	422	227	872	890	1340	1649	1126	1450	279	1233	1540	567	712	46	423	226	873	891	1341	1648	37065
32	1704	997	1344	408	1393	1033	43	691	300	1524	205	858	1254	601	1703	998	1343	407	1394	1034	44	692	299	1523	206	857	1253	602	1702	999	1342	406	1395	1035	45	693	298	1522	207	856	1252	603	37065
33	1470	750	493	102	1087	768	259	907	1195	552	1555	223	1668	1326	1469	749	494	101	1088	767	260	908	1196	551	1556	224	1667	1325	1468	748	495	100	1089	766	261	909	1197	550	1557	225	1666	1324	37065
34	930	84	799	331	187	1488	1609	1380	1753	660	390	1200	529	1015	929	83	800	332	188	1487	1610	1379	1754	659	389	1199	530	1016	928	82	801	333	189	1486	1611	1378	1755	658	388	1198	531	1017	37065
35	169	313	1182	511	25	1375	1758	1483	1614	822	1092	457	642	912	170	314	1181	512	26	1376	1757	1484	1613	821	1091	458	641	911	171	315	1180	513	27	1377	1756	1485	1612	820	1090	459	640	910	37065
36	403	583	637	804	318	1735	1362	1627	1506	1069	1164	12	925	210	404	584	638	803	317	1736	1361	1628	1505	1070	1163	11	926	209	405	585	639	802	316	1737	1360	1629	1504	1071	1162	10	927	208	37065
37	264	385	534	655	840	1632	1501	1740	1357	961	61	1056	192	1177	263	386	533	656	839	1631	1502	1739	1358	962	62	1055	191	1178	262	387	532	657	838	1630	1503	1738	1359	963	63	1054	190	1179	37065
38	606	1290	1398	241	673	426	943	835	1074	1218	1645	372	97	1537	605	1289	1397	242	674	425	944	836	1073	1217	1646	371	98	1538	604	1288	1396	243	675	424	945	837	1072	1216	1647	370	99	1539	37065
39	727	1681	1573	1020	619	156	1213	462	763	120	948	1285	1434	354	728	1682	1574	1019	620	155	1214	461	764	119	947	1286	1433	353	729	1683	1575	1018	621	154	1215	460	765	118	946	1287	1432	352	37065
40	1249	1560	66	1447	1272	966	1038	133	709	1663	349	547	480	876	1250	1559	65	1448	1271	965	1037	134	710	1664	350	548	479	875	1251	1558	64	1449	1270	964	1036	135	711	1665	351	549	478	874	37065
41	1591	1159	1105	1722	498	295	174	570	894	1267	732	1416	817	115	1592	1160	1106	1721	497	296	173	569	893	1268	731	1415	818	116	1593	1161	1107	1720	496	297	172	568	892	1269	730	1414	819	117	37065
42	786	1110	246	1303	979	79	444	1146	588	367	1411	1699	1519	678	785	1109	245	1304	980	80	443	1145	587	368	1412	1700	1520	677	784	1108	244	1305	981	81	442	1144	586	369	1413	1701	1521	676	37065
	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065	37065

In this case the magic sums are $S_{42\times42} := 37065$ and $S_{14\times14} := 12355$. All the middle blocks of order 4 are pan diagonal magic square with equal magic sums, $S_{4\times4} := 6230$.

5 Magic Squares of Order 44

The **Block-wise** construction of magic squares of order 44 depends on the product 4×11, 11×4 and 22×2, i.e., either we can construct it by blocks of order 4 or by blocks of order 11 or blocks of order 22. The magic square sum of order

44 is given by

$$S_{44\times 44} := \frac{44\times(1+44^2)}{2} = 42614.$$

This sums is divisible by 11 and 2, but not by 4. See below:

$$\begin{aligned}
(i) \quad & \frac{42614}{11} = 3874 && \Longrightarrow && \text{equal blocks of order 4;}\\
(ii) \quad & \frac{42614}{4} = 10653.5 && \Longrightarrow && \text{unequal blocks of order 11;}\\
(iii) \quad & \frac{42614}{2} = 21307 && \Longrightarrow && \text{equal blocks of order 22.}
\end{aligned}$$

This implies that we can make **block-wise** construction of magic square of order 44, where the blocks of orders 4 and 22 are of same magic sums. In case of blocks of order 11, we shall have magic square of order 44 with different magic sums of order 11. In order to construct magic squares of order 44, we need the magic squares of order 4, 11 and 22. The magic square of order 4 is given in Example 3.1. The magic squares of orders 11 and 22 are given below:

- **Magic Square of Order 11**

Example 5.1. *Let's consider Latin squares decomposition of magic square of order 11 given by*

(L)		66	66	66	66	66	66	66	66	66	66	66
	1	2	3	4	5	6	7	8	9	10	11	66
66	10	11	1	2	3	4	5	6	7	8	9	66
66	8	9	10	11	1	2	3	4	5	6	7	66
66	6	7	8	9	10	11	1	2	3	4	5	66
66	4	5	6	7	8	9	10	11	1	2	3	66
66	2	3	4	5	6	7	8	9	10	11	1	66
66	11	1	2	3	4	5	6	7	8	9	10	66
66	9	10	11	1	2	3	4	5	6	7	8	66
66	7	8	9	10	11	1	2	3	4	5	6	66
66	5	6	7	8	9	10	11	1	2	3	4	66
66	3	4	5	6	7	8	9	10	11	1	2	66
	66	66	66	66	66	66	66	66	66	66	66	66

(M)		66	66	66	66	66	66	66	66	66	66	66
	1	2	3	4	5	6	7	8	9	10	11	66
66	9	10	11	1	2	3	4	5	6	7	8	66
66	6	7	8	9	10	11	1	2	3	4	5	66
66	3	4	5	6	7	8	9	10	11	1	2	66
66	11	1	2	3	4	5	6	7	8	9	10	66
66	8	9	10	11	1	2	3	4	5	6	7	66
66	5	6	7	8	9	10	11	1	2	3	4	66
66	2	3	4	5	6	7	8	9	10	11	1	66
66	10	11	1	2	3	4	5	6	7	8	9	66
66	7	8	9	10	11	1	2	3	4	5	6	66
66	4	5	6	7	8	9	10	11	1	2	3	66
	66	66	66	66	66	66	66	66	66	66	66	66

M_{11}		671	671	671	671	671	671	671	671	671	671	671
	1	13	25	37	49	61	73	85	97	109	121	671
671	108	120	11	12	24	36	48	60	72	84	96	671
671	83	95	107	119	10	22	23	35	47	59	71	671
671	58	70	82	94	106	118	9	21	33	34	46	671
671	44	45	57	69	81	93	105	117	8	20	32	671
671	19	31	43	55	56	68	80	92	104	116	7	671
671	115	6	18	30	42	54	66	67	79	91	103	671
671	90	102	114	5	17	29	41	53	65	77	78	671
671	76	88	89	101	113	4	16	28	40	52	64	671
671	51	63	75	87	99	100	112	3	15	27	39	671
671	26	38	50	62	74	86	98	110	111	2	14	671
	671	671	671	671	671	671	671	671	671	671	671	671

11	22	33	44	55	66	77	88	99	RR	SS
R9	SR	1S	21	32	43	54	65	76	87	98
86	97	R8	S9	1R	2S	31	42	53	64	75
63	74	85	96	R7	S8	19	2R	3S	41	52
4S	51	62	73	84	95	R6	S7	18	29	3R
28	39	4R	5S	61	72	83	94	R5	S6	17
S5	16	27	38	49	5R	6S	71	82	93	R4
92	R3	S4	15	26	37	48	59	6R	7S	81
7R	8S	91	R2	S3	14	25	36	47	58	69
57	68	79	8R	9S	R1	S2	13	24	35	46
34	45	56	67	78	89	9R	RS	S1	12	23
					C_{11}					

where $R := 10$ and $S := 11$. The magic squares M_{11} and C_{11} are obtained by using the operations

$$11 \times (A-1) + B := M_{11} \quad \text{and} \quad 10 \times A + B := C_{11},$$

respectively. The M_{11} is magic square of order 11 of consecutive numbers from 1 to 121, and C_{11} is the ***composite*** *magic square.*

• Magic Square of Order 22

This magic square is of similar nature as of magic square of orders 10 and 14. These are double of prime numbers i.e., 2×5, 2×7 and 2×11. There is no systematic way to construct these magic squares. There construction can be done either by **a pair of mutually orthogonal diagonal Latin squares** or by **self orthogonal diagonal Latin squares**. In case of order 10 and 14, we use the old constructions due to [5, 6, 7] done as **pair of mutually orthogonal diagonal Latin**

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squares. In case of order 22, we shall use the idea **self orthogonal diagonal Latin squares** using the software due to W. Harry [3]. See below magic square of order 22:

Example 5.2.. *The magic square of order 22 based on* ***self orthogonal diagonal Latin squares*** *is given by*

Magic Square of Order 22: 12 pan diagonal magic squares of order 5 and 1 of order 7

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	5335
1	1	94	76	200	114	387	477	457	432	412	25	49	238	262	198	173	153	275	360	342	315	295	5335
2	115	24	6	98	201	435	410	390	475	455	48	67	176	151	241	260	196	320	293	273	363	338	5335
3	202	111	47	28	10	478	453	433	413	388	71	90	263	194	174	154	239	361	341	316	298	271	5335
4	32	203	112	70	50	411	391	476	456	431	89	3	152	242	261	197	172	294	276	359	339	319	5335
5	72	54	199	113	93	454	434	409	389	479	2	26	195	175	150	240	264	337	317	297	272	364	5335
6	282	372	352	327	307	116	250	230	210	183	163	143	375	465	445	420	400	13	103	83	58	38	5335
7	330	305	285	370	350	166	139	119	253	226	206	186	423	398	378	463	443	61	36	16	101	81	5335
8	373	348	328	308	283	209	182	162	142	122	249	229	466	441	421	401	376	104	79	59	39	14	5335
9	306	286	371	351	326	252	232	205	185	165	138	118	399	379	464	444	419	37	17	102	82	57	5335
10	349	329	304	284	374	141	121	248	228	208	188	161	442	422	397	377	467	80	60	35	15	105	5335
11	46	69	92	5	23	184	164	144	117	251	231	204	322	345	368	281	299	437	460	483	396	414	5335
12	91	4	27	45	68	227	207	187	160	140	120	254	367	280	303	321	344	482	395	418	436	459	5335
13	385	470	452	425	405	18	108	88	63	43	301	325	277	358	340	212	126	221	245	181	156	136	5335
14	430	403	383	473	448	66	41	21	106	86	324	343	127	300	270	362	213	159	134	224	243	179	5335
15	471	451	426	408	381	109	84	64	44	19	347	366	214	123	323	292	274	246	177	157	137	222	5335
16	404	386	469	449	429	42	22	107	87	62	365	279	296	215	124	346	314	135	225	244	180	155	5335
17	447	427	407	382	474	85	65	40	20	110	278	302	336	318	211	125	369	178	158	133	223	247	5335
18	233	257	193	168	148	265	355	335	310	290	416	440	11	96	78	51	31	392	468	450	217	131	5335
19	171	146	236	255	191	313	288	268	353	333	439	458	56	29	9	99	74	132	415	380	472	218	5335
20	258	189	169	149	234	356	331	311	291	266	462	481	97	77	52	34	7	219	128	438	402	384	5335
21	147	237	256	192	167	289	269	354	334	309	480	394	30	12	95	75	55	406	220	129	461	424	5335
22	190	170	145	235	259	332	312	287	267	357	393	417	73	53	33	8	100	446	428	216	130	484	5335
	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335

The above magic square is with 12 blocks of **pan diagonal** magic squares of order 4 (indicated by yellow color) and one block of **pan diagonal** magic square of order 7 (indicated by blue color).

5.1 Blocks of Order 4

In order to construct magic square of order 44 with sub-blocks of order 4 we shall distribute the total number of entries according to following distribution.

Distribution 5.1. *Let's distribute the 1936 numbers from 1 to 1936 in 121 blocks of 16 each in such a way that all the blocks are of equal sums:*

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Total
A1	1	242	243	484	485	726	727	968	969	1210	1211	1452	1453	1694	1695	1936	15496
A2	2	241	244	483	486	725	728	967	970	1209	1212	1451	1454	1693	1696	1935	15496
A3	3	240	245	482	487	724	729	966	971	1208	1213	1450	1455	1692	1697	1934	15496
A4	4	239	246	481	488	723	730	965	972	1207	1214	1449	1456	1691	1698	1933	15496
A5	5	238	247	480	489	722	731	964	973	1206	1215	1448	1457	1690	1699	1932	15496
A6	6	237	248	479	490	721	732	963	974	1205	1216	1447	1458	1689	1700	1931	15496
A7	7	236	249	478	491	720	733	962	975	1204	1217	1446	1459	1688	1701	1930	15496
A8	8	235	250	477	492	719	734	961	976	1203	1218	1445	1460	1687	1702	1929	15496
A9	9	234	251	476	493	718	735	960	977	1202	1219	1444	1461	1686	1703	1928	15496
A10	10	233	252	475	494	717	736	959	978	1201	1220	1443	1462	1685	1704	1927	15496
A11	11	232	253	474	495	716	737	958	979	1200	1221	1442	1463	1684	1705	1926	15496
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
A112	112	131	354	373	596	615	838	857	1080	1099	1322	1341	1564	1583	1806	1825	15496
A113	113	130	355	372	597	614	839	856	1081	1098	1323	1340	1565	1582	1807	1824	15496
A114	114	129	356	371	598	613	840	855	1082	1097	1324	1339	1566	1581	1808	1823	15496
A115	115	128	357	370	599	612	841	854	1083	1096	1325	1338	1567	1580	1809	1822	15496
A116	116	127	358	369	600	611	842	853	1084	1095	1326	1337	1568	1579	1810	1821	15496
A117	117	126	359	368	601	610	843	852	1085	1094	1327	1336	1569	1578	1811	1820	15496
A118	118	125	360	367	602	609	844	851	1086	1093	1328	1335	1570	1577	1812	1819	15496
A119	119	124	361	366	603	608	845	850	1087	1092	1329	1334	1571	1576	1813	1818	15496
A120	120	123	362	365	604	607	846	849	1088	1091	1330	1333	1572	1575	1814	1817	15496
A121	121	122	363	364	605	606	847	848	1089	1090	1331	1332	1573	1574	1815	1816	15496

Distribution 5.2. *Let's distribute the 121 blocks, i.e., from A1 to A121 according to following table:*

A1	A2	A3	A4	A5	A6	A7	A8	A9	A10	A11
A12	A13	A14	A15	A16	A17	A18	A19	A20	A21	A22
A23	A24	A25	A26	A27	A28	A29	A30	A31	A32	A33
A34	A35	A36	A37	A38	A39	A40	A41	A42	A43	A44
A45	A46	A47	A48	A49	A50	A51	A52	A53	A54	A55
A56	A57	A58	A59	A60	A61	A62	A63	A64	A65	A66
A67	A68	A69	A70	A71	A72	A73	A74	A75	A76	A77
A78	A79	A80	A81	A82	A83	A84	A85	A86	A87	A88
A89	A90	A91	A92	A93	A94	A95	A96	A97	A98	A99
A100	A101	A102	A103	A104	A105	A106	A107	A108	A109	A110
A111	A112	A113	A114	A115	A116	A117	A118	A119	A120	A121

We shall construct 121 magic square of order 4 by applying the Example 3.1. See below some examples:

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A5		*3874*	*3874*	*3874*	*3874*
	731	1448	5	1690	*3874*
3874	238	1457	964	1215	*3874*
3874	1932	247	1206	489	*3874*
3874	973	722	1699	480	*3874*
	3874	*3874*	*3874*	*3874*	*3874*

A13		*3874*	*3874*	*3874*	*3874*
	739	1440	13	1682	*3874*
3874	230	1465	956	1223	*3874*
3874	1924	255	1198	497	*3874*
3874	981	714	1707	472	*3874*
	3874	*3874*	*3874*	*3874*	*3874*

A28		*3874*	*3874*	*3874*	*3874*
	754	1425	28	1667	*3874*
3874	215	1480	941	1238	*3874*
3874	1909	270	1183	512	*3874*
3874	996	699	1722	457	*3874*
	3874	*3874*	*3874*	*3874*	*3874*

A54		*3874*	*3874*	*3874*	*3874*
	780	1399	54	1641	*3874*
3874	189	1506	915	1264	*3874*
3874	1883	296	1157	538	*3874*
3874	1022	673	1748	431	*3874*
	3874	*3874*	*3874*	*3874*	*3874*

A70		*3874*	*3874*	*3874*	*3874*
	796	1383	70	1625	*3874*
3874	173	1522	899	1280	*3874*
3874	1867	312	1141	554	*3874*
3874	1038	657	1764	415	*3874*
	3874	*3874*	*3874*	*3874*	*3874*

A105		*3874*	*3874*	*3874*	*3874*
	831	1348	105	1590	*3874*
3874	138	1557	864	1315	*3874*
3874	1832	347	1106	589	*3874*
3874	1073	622	1799	380	*3874*
	3874	*3874*	*3874*	*3874*	*3874*

Similar to above 6 examples, let's construct 121 blocks of order 4 of equal magic sums, and put them according Distribution 5.1 we get a **pan diagonal** magic square of order 44 given in example below.

Example 5.3. *The **pan diagonal** magic square of order 44 with equal magic sums blocks of **pan diagonal** magic square of order 4 is given by*

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Pan diagonal magic square of order 44 with equal magic sums of order 4. Magic sums are S(44x44):=42614 and S(4x4):=3874.

		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	
		pan	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614
1	42614	727	1452	1	1694	728	1451	2	1693	729	1450	3	1692	730	1449	4	1691	731	1448	5	1690	732	1447	6	1689	733	1446	7	1688	734	1445	8	1687	735	1444	9	1686	736	1443	10	1685	737	1442	11	1684	42614
2	42614	242	1453	968	1211	241	1454	967	1212	240	1455	966	1213	239	1456	965	1214	238	1457	964	1215	237	1458	963	1216	236	1459	962	1217	235	1460	961	1218	234	1461	960	1219	233	1462	959	1220	232	1463	958	1221	42614
3	42614	1936	243	1210	485	1935	244	1209	486	1934	245	1208	487	1933	246	1207	488	1932	247	1206	489	1931	248	1205	490	1930	249	1204	491	1929	250	1203	492	1928	251	1202	493	1927	252	1201	494	1926	253	1200	495	42614
4	42614	969	726	1695	484	970	725	1696	483	971	724	1697	482	972	723	1698	481	973	722	1699	480	974	721	1700	479	975	720	1701	478	976	719	1702	477	977	718	1703	476	978	717	1704	475	979	716	1705	474	42614
5	42614	738	1441	12	1683	739	1440	13	1682	740	1439	14	1681	741	1438	15	1680	742	1437	16	1679	743	1436	17	1678	744	1435	18	1677	745	1434	19	1676	746	1433	20	1675	747	1432	21	1674	748	1431	22	1673	42614
6	42614	231	1464	957	1222	230	1465	956	1223	229	1466	955	1224	228	1467	954	1225	227	1468	953	1226	226	1469	952	1227	225	1470	951	1228	224	1471	950	1229	223	1472	949	1230	222	1473	948	1231	221	1474	947	1232	42614
7	42614	1925	254	1199	496	1924	255	1198	497	1923	256	1197	498	1922	257	1196	499	1921	258	1195	500	1920	259	1194	501	1919	260	1193	502	1918	261	1192	503	1917	262	1191	504	1916	263	1190	505	1915	264	1189	506	42614
8	42614	980	715	1706	473	981	714	1707	472	982	713	1708	471	983	712	1709	470	984	711	1710	469	985	710	1711	468	986	709	1712	467	987	708	1713	466	988	707	1714	465	989	706	1715	464	990	705	1716	463	42614
9	42614	749	1430	23	1672	750	1429	24	1671	751	1428	25	1670	752	1427	26	1669	753	1426	27	1668	754	1425	28	1667	755	1424	29	1666	756	1423	30	1665	757	1422	31	1664	758	1421	32	1663	759	1420	33	1662	42614
10	42614	220	1475	946	1233	219	1476	945	1234	218	1477	944	1235	217	1478	943	1236	216	1479	942	1237	215	1480	941	1238	214	1481	940	1239	213	1482	939	1240	212	1483	938	1241	211	1484	937	1242	210	1485	936	1243	42614
11	42614	1914	265	1188	507	1913	266	1187	508	1912	267	1186	509	1911	268	1185	510	1910	269	1184	511	1909	270	1183	512	1908	271	1182	513	1907	272	1181	514	1906	273	1180	515	1905	274	1179	516	1904	275	1178	517	42614
12	42614	991	704	1717	462	992	703	1718	461	993	702	1719	460	994	701	1720	459	995	700	1721	458	996	699	1722	457	997	698	1723	456	998	697	1724	455	999	696	1725	454	1000	695	1726	453	1001	694	1727	452	42614
13	42614	760	1419	34	1661	761	1418	35	1660	762	1417	36	1659	763	1416	37	1658	764	1415	38	1657	765	1414	39	1656	766	1413	40	1655	767	1412	41	1654	768	1411	42	1653	769	1410	43	1652	770	1409	44	1651	42614
14	42614	209	1486	935	1244	208	1487	934	1245	207	1488	933	1246	206	1489	932	1247	205	1490	931	1248	204	1491	930	1249	203	1492	929	1250	202	1493	928	1251	201	1494	927	1252	200	1495	926	1253	199	1496	925	1254	42614
15	42614	1903	276	1177	518	1902	277	1176	519	1901	278	1175	520	1900	279	1174	521	1899	280	1173	522	1898	281	1172	523	1897	282	1171	524	1896	283	1170	525	1895	284	1169	526	1894	285	1168	527	1893	286	1167	528	42614
16	42614	1002	693	1728	451	1003	692	1729	450	1004	691	1730	449	1005	690	1731	448	1006	689	1732	447	1007	688	1733	446	1008	687	1734	445	1009	686	1735	444	1010	685	1736	443	1011	684	1737	442	1012	683	1738	441	42614
17	42614	771	1408	45	1650	772	1407	46	1649	773	1406	47	1648	774	1405	48	1647	775	1404	49	1646	776	1403	50	1645	777	1402	51	1644	778	1401	52	1643	779	1400	53	1642	780	1399	54	1641	781	1398	55	1640	42614
18	42614	198	1497	924	1255	197	1498	923	1256	196	1499	922	1257	195	1500	921	1258	194	1501	920	1259	193	1502	919	1260	192	1503	918	1261	191	1504	917	1262	190	1505	916	1263	189	1506	915	1264	188	1507	914	1265	42614
19	42614	1892	287	1166	529	1891	288	1165	530	1890	289	1164	531	1889	290	1163	532	1888	291	1162	533	1887	292	1161	534	1886	293	1160	535	1885	294	1159	536	1884	295	1158	537	1883	296	1157	538	1882	297	1156	539	42614
20	42614	1013	682	1739	440	1014	681	1740	439	1015	680	1741	438	1016	679	1742	437	1017	678	1743	436	1018	677	1744	435	1019	676	1745	434	1020	675	1746	433	1021	674	1747	432	1022	673	1748	431	1023	672	1749	430	42614
21	42614	782	1397	56	1639	783	1396	57	1638	784	1395	58	1637	785	1394	59	1636	786	1393	60	1635	787	1392	61	1634	788	1391	62	1633	789	1390	63	1632	790	1389	64	1631	791	1388	65	1630	792	1387	66	1629	42614
22	42614	187	1508	913	1266	186	1509	912	1267	185	1510	911	1268	184	1511	910	1269	183	1512	909	1270	182	1513	908	1271	181	1514	907	1272	180	1515	906	1273	179	1516	905	1274	178	1517	904	1275	177	1518	903	1276	42614
23	42614	1881	298	1155	540	1880	299	1154	541	1879	300	1153	542	1878	301	1152	543	1877	302	1151	544	1876	303	1150	545	1875	304	1149	546	1874	305	1148	547	1873	306	1147	548	1872	307	1146	549	1871	308	1145	550	42614
24	42614	1024	671	1750	429	1025	670	1751	428	1026	669	1752	427	1027	668	1753	426	1028	667	1754	425	1029	666	1755	424	1030	665	1756	423	1031	664	1757	422	1032	663	1758	421	1033	662	1759	420	1034	661	1760	419	42614
25	42614	793	1386	67	1628	794	1385	68	1627	795	1384	69	1626	796	1383	70	1625	797	1382	71	1624	798	1381	72	1623	799	1380	73	1622	800	1379	74	1621	801	1378	75	1620	802	1377	76	1619	803	1376	77	1618	42614
26	42614	176	1519	902	1277	175	1520	901	1278	174	1521	900	1279	173	1522	899	1280	172	1523	898	1281	171	1524	897	1282	170	1525	896	1283	169	1526	895	1284	168	1527	894	1285	167	1528	893	1286	166	1529	892	1287	42614
27	42614	1870	309	1144	551	1869	310	1143	552	1868	311	1142	553	1867	312	1141	554	1866	313	1140	555	1865	314	1139	556	1864	315	1138	557	1863	316	1137	558	1862	317	1136	559	1861	318	1135	560	1860	319	1134	561	42614
28	42614	1035	660	1761	418	1036	659	1762	417	1037	658	1763	416	1038	657	1764	415	1039	656	1765	414	1040	655	1766	413	1041	654	1767	412	1042	653	1768	411	1043	652	1769	410	1044	651	1770	409	1045	650	1771	408	42614
29	42614	804	1375	78	1617	805	1374	79	1616	806	1373	80	1615	807	1372	81	1614	808	1371	82	1613	809	1370	83	1612	810	1369	84	1611	811	1368	85	1610	812	1367	86	1609	813	1366	87	1608	814	1365	88	1607	42614
30	42614	165	1530	891	1288	164	1531	890	1289	163	1532	889	1290	162	1533	888	1291	161	1534	887	1292	160	1535	886	1293	159	1536	885	1294	158	1537	884	1295	157	1538	883	1296	156	1539	882	1297	155	1540	881	1298	42614
31	42614	1859	320	1133	562	1858	321	1132	563	1857	322	1131	564	1856	323	1130	565	1855	324	1129	566	1854	325	1128	567	1853	326	1127	568	1852	327	1126	569	1851	328	1125	570	1850	329	1124	571	1849	330	1123	572	42614
32	42614	1046	649	1772	407	1047	648	1773	406	1048	647	1774	405	1049	646	1775	404	1050	645	1776	403	1051	644	1777	402	1052	643	1778	401	1053	642	1779	400	1054	641	1780	399	1055	640	1781	398	1056	639	1782	397	42614
33	42614	815	1364	89	1606	816	1363	90	1605	817	1362	91	1604	818	1361	92	1603	819	1360	93	1602	820	1359	94	1601	821	1358	95	1600	822	1357	96	1599	823	1356	97	1598	824	1355	98	1597	825	1354	99	1596	42614
34	42614	154	1541	880	1299	153	1542	879	1300	152	1543	878	1301	151	1544	877	1302	150	1545	876	1303	149	1546	875	1304	148	1547	874	1305	147	1548	873	1306	146	1549	872	1307	145	1550	871	1308	144	1551	870	1309	42614
35	42614	1848	331	1122	573	1847	332	1121	574	1846	333	1120	575	1845	334	1119	576	1844	335	1118	577	1843	336	1117	578	1842	337	1116	579	1841	338	1115	580	1840	339	1114	581	1839	340	1113	582	1838	341	1112	583	42614
36	42614	1057	638	1783	396	1058	637	1784	395	1059	636	1785	394	1060	635	1786	393	1061	634	1787	392	1062	633	1788	391	1063	632	1789	390	1064	631	1790	389	1065	630	1791	388	1066	629	1792	387	1067	628	1793	386	42614
37	42614	826	1353	100	1595	827	1352	101	1594	828	1351	102	1593	829	1350	103	1592	830	1349	104	1591	831	1348	105	1590	832	1347	106	1589	833	1346	107	1588	834	1345	108	1587	835	1344	109	1586	836	1343	110	1585	42614
38	42614	143	1552	869	1310	142	1553	868	1311	141	1554	867	1312	140	1555	866	1313	139	1556	865	1314	138	1557	864	1315	137	1558	863	1316	136	1559	862	1317	135	1560	861	1318	134	1561	860	1319	133	1562	859	1320	42614
39	42614	1837	342	1111	584	1836	343	1110	585	1835	344	1109	586	1834	345	1108	587	1833	346	1107	588	1832	347	1106	589	1831	348	1105	590	1830	349	1104	591	1829	350	1103	592	1828	351	1102	593	1827	352	1101	594	42614
40	42614	1068	627	1794	385	1069	626	1795	384	1070	625	1796	383	1071	624	1797	382	1072	623	1798	381	1073	622	1799	380	1074	621	1800	379	1075	620	1801	378	1076	619	1802	377	1077	618	1803	376	1078	617	1804	375	42614
41	42614	837	1342	111	1584	838	1341	112	1583	839	1340	113	1582	840	1339	114	1581	841	1338	115	1580	842	1337	116	1579	843	1336	117	1578	844	1335	118	1577	845	1334	119	1576	846	1333	120	1575	847	1332	121	1574	42614
42	42614	132	1563	858	1321	131	1564	857	1322	130	1565	856	1323	129	1566	855	1324	128	1567	854	1325	127	1568	853	1326	126	1569	852	1327	125	1570	851	1328	124	1571	850	1329	123	1572	849	1330	122	1573	848	1331	42614
43	42614	1826	353	1100	595	1825	354	1099	596	1824	355	1098	597	1823	356	1097	598	1822	357	1096	599	1821	358	1095	600	1820	359	1094	601	1819	360	1093	602	1818	361	1092	603	1817	362	1091	604	1816	363	1090	605	42614
44		1079	616	1805	374	1080	615	1806	373	1081	614	1807	372	1082	613	1808	371	1083	612	1809	370	1084	611	1810	369	1085	610	1811	368	1086	609	1812	367	1087	608	1813	366	1088	607	1814	365	1089	606	1815	364	42614
		42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614

5.2 Blocks of Order 11

In order to construct magic square of order 44 as sub-blocks of order 11, we shall use the magic square of order 11 given in 5.1. Let's consider following distribution of 44 numbers as 11 × 4.

Example 5.4. *Let's consider the following distribution of 44 numbers from 1 to 44 divided in 4 parts with different sums.*

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	1	2	3	4	5	6	7	8	9	10	11	Total
1	1	8	9	16	17	24	25	32	33	40	41	246
2	2	7	10	15	18	23	26	31	34	39	42	247
3	3	6	11	14	19	22	27	30	35	38	43	248
4	4	5	12	13	20	21	28	29	36	37	44	249

Let's rewrite the composite magic square of order 4 given in Example 3.1 as below.

Distribution 5.3. *Let's rewrite the composite magic square of order 4 given in Example 3.1 as follows:*

23	34	11	42
12	41	24	33
44	13	32	21
31	22	43	14

We shall construct 16 block of order 11 and put them according to Distribution 5.3. Below are few examples of **pan diagonal** magic squares of order 11 constructed by considering row values given in table 5.4 over the Example 5.1 by using the operation $M_{44} := 44 \times (A-1) + B$:

- **Block 14**

①		246	246	246	246	246	246	246	246	246	246	246
	1	8	9	16	17	24	25	32	33	40	41	246
246	40	41	1	8	9	16	17	24	25	32	33	246
246	32	33	40	41	1	8	9	16	17	24	25	246
246	24	25	32	33	40	41	1	8	9	16	17	246
246	16	17	24	25	32	33	40	41	1	8	9	246
246	8	9	16	17	24	25	32	33	40	41	1	246
246	41	1	8	9	16	17	24	25	32	33	40	246
246	33	40	41	1	8	9	16	17	24	25	32	246
246	25	32	33	40	41	1	8	9	16	17	24	246
246	17	24	25	32	33	40	41	1	8	9	16	246
246	9	16	17	24	25	32	33	40	41	1	8	246
	246	246	246	246	246	246	246	246	246	246	246	246

④		249	249	249	249	249	249	249	249	249	249	249
	4	5	12	13	20	21	28	29	36	37	44	249
249	36	37	44	4	5	12	13	20	21	28	29	249
249	21	28	29	36	37	44	4	5	12	13	20	249
249	12	13	20	21	28	29	36	37	44	4	5	249
249	44	4	5	12	13	20	21	28	29	36	37	249
249	29	36	37	44	4	5	12	13	20	21	28	249
249	20	21	28	29	36	37	44	4	5	12	13	249
249	5	12	13	20	21	28	29	36	37	44	4	249
249	37	44	4	5	12	13	20	21	28	29	36	249
249	28	29	36	37	44	4	5	12	13	20	21	249
249	13	20	21	28	29	36	37	44	4	5	12	249
	249	249	249	249	249	249	249	249	249	249	249	249

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14		10589	10589	10589	10589	10589	10589	10589	10589	10589	10589	10589
	4	313	364	673	724	1033	1084	1393	1444	1753	1804	10589
10589	1752	1797	44	312	357	672	717	1032	1077	1392	1437	10589
10589	1385	1436	1745	1796	37	352	356	665	716	1025	1076	10589
10589	1024	1069	1384	1429	1744	1789	36	345	396	664	709	10589
10589	704	708	1017	1068	1377	1428	1737	1788	29	344	389	10589
10589	337	388	697	748	1016	1061	1376	1421	1736	1781	28	10589
10589	1780	21	336	381	696	741	1056	1060	1369	1420	1729	10589
10589	1413	1728	1773	20	329	380	689	740	1049	1100	1368	10589
10589	1093	1408	1412	1721	1772	13	328	373	688	733	1048	10589
10589	732	1041	1092	1401	1452	1720	1765	12	321	372	681	10589
10589	365	680	725	1040	1085	1400	1445	1760	1764	5	320	10589
	10589	10589	10589	10589	10589	10589	10589	10589	10589	10589	10589	10589

- **Block 32**

3		248	248	248	248	248	248	248	248	248	248	248
	3	6	11	14	19	22	27	30	35	38	43	248
248	38	43	3	6	11	14	19	22	27	30	35	248
248	30	35	38	43	3	6	11	14	19	22	27	248
248	22	27	30	35	38	43	3	6	11	14	19	248
248	14	19	22	27	30	35	38	43	3	6	11	248
248	6	11	14	19	22	27	30	35	38	43	3	248
248	43	3	6	11	14	19	22	27	30	35	38	248
248	35	38	43	3	6	11	14	19	22	27	30	248
248	27	30	35	38	43	3	6	11	14	19	22	248
248	19	22	27	30	35	38	43	3	6	11	14	248
248	11	14	19	22	27	30	35	38	43	3	6	248
	248	248	248	248	248	248	248	248	248	248	248	248

2		247	247	247	247	247	247	247	247	247	247	247
	2	7	10	15	18	23	26	31	34	39	42	247
247	34	39	42	2	7	10	15	18	23	26	31	247
247	23	26	31	34	39	42	2	7	10	15	18	247
247	10	15	18	23	26	31	34	39	42	2	7	247
247	42	2	7	10	15	18	23	26	31	34	39	247
247	31	34	39	42	2	7	10	15	18	23	26	247
247	18	23	26	31	34	39	42	2	7	10	15	247
247	7	10	15	18	23	26	31	34	39	42	2	247
247	39	42	2	7	10	15	18	23	26	31	34	247
247	26	31	34	39	42	2	7	10	15	18	23	247
247	15	18	23	26	31	34	39	42	2	7	10	247
	247	247	247	247	247	247	247	247	247	247	247	247

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(32)		10675	10675	10675	10675	10675	10675	10675	10675	10675	10675	10675
	90	227	450	587	810	947	1170	1307	1530	1667	1890	10675
10675	1662	1887	130	222	447	582	807	942	1167	1302	1527	10675
10675	1299	1522	1659	1882	127	262	442	579	802	939	1162	10675
10675	934	1159	1294	1519	1654	1879	122	259	482	574	799	10675
10675	614	794	931	1154	1291	1514	1651	1874	119	254	479	10675
10675	251	474	611	834	926	1151	1286	1511	1646	1871	114	10675
10675	1866	111	246	471	606	831	966	1146	1283	1506	1643	10675
10675	1503	1638	1863	106	243	466	603	826	963	1186	1278	10675
10675	1183	1318	1498	1635	1858	103	238	463	598	823	958	10675
10675	818	955	1178	1315	1538	1630	1855	98	235	458	595	10675
10675	455	590	815	950	1175	1310	1535	1670	1850	95	230	10675
	10675	10675	10675	10675	10675	10675	10675	10675	10675	10675	10675	10675

In the similar way, let's construct other 14 blocks of order 11, and put them according to Distributions 5.3, we get a **pan diagonal** magic square of order 44 given in example below.

Example 5.5. *The **block-wise pan diagonal** magic square of order 44 with blocks of **pan diagonal** magic squares of order 11 is given by*

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Pan diagonal magic square of order 44 with16 blocks of pan diagonal magic squares of order 11 with different magic sums.

		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	
		pan	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614
1	42614	47	270	407	630	767	990	1127	1350	1487	1710	1847	92	225	452	585	812	945	1172	1305	1532	1665	1892	1	316	361	676	721	1036	1081	1396	1441	1756	1801	134	183	494	543	854	903	1214	1263	1574	1623	1934	42614
2	42614	1707	1842	87	267	402	627	762	987	1122	1347	1482	1664	1885	132	224	445	584	805	944	1165	1304	1525	1749	1800	41	309	360	669	720	1029	1080	1389	1440	1618	1931	174	178	491	538	851	898	1211	1258	1571	42614
3	42614	1342	1479	1702	1839	82	307	399	622	759	982	1119	1297	1524	1657	1884	125	264	444	577	804	937	1164	1388	1433	1748	1793	40	349	353	668	713	1028	1073	1255	1566	1615	1926	171	218	486	535	846	895	1206	42614
4	42614	979	1114	1339	1474	1699	1834	79	302	439	619	754	936	1157	1296	1517	1656	1877	124	257	484	576	797	1021	1072	1381	1432	1741	1792	33	348	393	661	712	890	1203	1250	1563	1610	1923	166	215	526	530	843	42614
5	42614	659	751	974	1111	1334	1471	1694	1831	74	299	434	616	796	929	1156	1289	1516	1649	1876	117	256	477	701	705	1020	1065	1380	1425	1740	1785	32	341	392	570	838	887	1198	1247	1558	1607	1918	163	210	523	42614
6	42614	294	431	654	791	971	1106	1331	1466	1691	1826	71	249	476	609	836	928	1149	1288	1509	1648	1869	116	340	385	700	745	1013	1064	1373	1424	1733	1784	25	207	518	567	878	882	1195	1242	1555	1602	1915	158	42614
7	42614	1823	66	291	426	651	786	1011	1103	1326	1463	1686	1868	109	248	469	608	829	968	1148	1281	1508	1641	1777	24	333	384	693	744	1053	1057	1372	1417	1732	1910	155	202	515	562	875	922	1190	1239	1550	1599	42614
8	42614	1458	1683	1818	63	286	423	646	783	1006	1143	1323	1501	1640	1861	108	241	468	601	828	961	1188	1280	1416	1725	1776	17	332	377	692	737	1052	1097	1365	1547	1594	1907	150	199	510	559	870	919	1230	1234	42614
9	42614	1138	1363	1455	1678	1815	58	283	418	643	778	1003	1181	1320	1500	1633	1860	101	240	461	600	821	960	1096	1405	1409	1724	1769	16	325	376	685	736	1045	1227	1274	1542	1591	1902	147	194	507	554	867	914	42614
10	42614	775	998	1135	1358	1495	1675	1810	55	278	415	638	820	953	1180	1313	1540	1632	1853	100	233	460	593	729	1044	1089	1404	1449	1717	1768	9	324	369	684	862	911	1222	1271	1582	1586	1899	142	191	502	551	42614
11	42614	410	635	770	995	1130	1355	1490	1715	1807	50	275	453	592	813	952	1173	1312	1533	1672	1852	93	232	368	677	728	1037	1088	1397	1448	1757	1761	8	317	499	546	859	906	1219	1266	1579	1626	1894	139	186	42614
12	42614	2	315	362	675	722	1035	1082	1395	1442	1755	1802	133	184	493	544	853	904	1213	1264	1573	1624	1933	48	269	408	629	768	989	1128	1349	1488	1709	1848	91	226	451	586	811	946	1171	1306	1531	1666	1891	42614
13	42614	1750	1799	42	310	359	670	719	1030	1079	1390	1439	1617	1932	173	177	492	537	852	897	1212	1257	1572	1708	1841	88	268	401	628	761	988	1121	1348	1481	1663	1886	131	223	446	583	806	943	1166	1303	1526	42614
14	42614	1387	1434	1747	1794	39	350	354	667	714	1027	1074	1256	1565	1616	1925	172	217	485	536	845	896	1205	1341	1480	1701	1840	81	308	400	621	760	981	1120	1298	1523	1658	1883	126	263	443	578	803	938	1163	42614
15	42614	1022	1071	1382	1431	1742	1791	34	347	394	662	711	889	1204	1249	1564	1609	1924	165	216	525	529	844	980	1113	1340	1473	1700	1833	80	301	440	620	753	935	1158	1295	1518	1655	1878	123	258	483	575	798	42614
16	42614	702	706	1019	1066	1379	1426	1739	1786	31	342	391	569	837	888	1197	1248	1557	1608	1917	164	209	524	660	752	973	1112	1333	1472	1693	1832	73	300	433	615	795	930	1155	1290	1515	1650	1875	118	255	478	42614
17	42614	339	386	699	746	1014	1063	1374	1423	1734	1783	26	208	517	568	877	881	1196	1241	1556	1601	1916	157	293	432	653	792	972	1105	1332	1465	1692	1825	72	250	475	610	835	927	1150	1287	1510	1647	1870	115	42614
18	42614	1778	23	334	383	694	743	1054	1058	1371	1418	1731	1909	156	201	516	561	876	921	1189	1240	1549	1600	1824	65	292	425	652	785	1012	1104	1325	1464	1685	1867	110	247	470	607	830	967	1147	1282	1507	1642	42614
19	42614	1415	1726	1775	18	331	378	691	738	1051	1098	1366	1548	1593	1908	149	200	509	560	869	920	1229	1233	1457	1684	1817	64	285	424	645	784	1005	1144	1324	1502	1639	1862	107	242	467	602	827	962	1187	1279	42614
20	42614	1095	1406	1410	1723	1770	15	326	375	686	735	1046	1228	1273	1541	1592	1901	148	193	508	553	868	913	1137	1364	1456	1677	1816	57	284	417	644	777	1004	1182	1319	1499	1634	1859	102	239	462	599	822	959	42614
21	42614	730	1043	1090	1403	1450	1718	1767	10	323	370	683	861	912	1221	1272	1581	1585	1900	141	192	501	552	776	997	1136	1357	1496	1676	1809	56	277	416	637	819	954	1179	1314	1539	1631	1854	99	234	459	594	42614
22	42614	367	678	727	1038	1087	1398	1447	1758	1762	7	318	500	545	860	905	1220	1265	1580	1625	1893	140	185	409	636	769	996	1129	1356	1489	1716	1808	49	276	454	591	814	951	1174	1311	1534	1671	1851	94	231	42614
23	42614	136	181	496	541	856	901	1216	1261	1576	1621	1936	3	314	363	674	723	1034	1083	1394	1443	1754	1803	90	227	450	587	810	947	1170	1307	1530	1667	1890	45	272	405	632	765	992	1125	1352	1485	1712	1845	42614
24	42614	1620	1929	176	180	489	540	849	900	1209	1260	1569	1751	1798	43	311	358	671	718	1031	1078	1391	1438	1662	1887	130	222	447	582	807	942	1167	1302	1527	1705	1844	85	265	404	625	764	985	1124	1345	1484	42614
25	42614	1253	1568	1613	1928	169	220	488	533	848	893	1208	1386	1435	1746	1795	38	351	355	666	715	1026	1075	1299	1522	1659	1882	127	262	442	579	802	939	1162	1344	1477	1704	1837	84	305	397	624	757	984	1117	42614
26	42614	892	1201	1252	1561	1612	1921	168	213	528	532	841	1023	1070	1383	1430	1743	1790	35	346	395	663	710	934	1159	1294	1519	1654	1879	122	259	482	574	799	977	1116	1337	1476	1697	1836	77	304	437	617	756	42614
27	42614	572	840	885	1200	1245	1560	1605	1920	161	212	521	703	707	1018	1067	1378	1427	1738	1787	30	343	390	614	794	931	1154	1291	1514	1651	1874	119	254	479	657	749	976	1109	1336	1469	1696	1829	76	297	436	42614
28	42614	205	520	565	880	884	1193	1244	1553	1604	1913	160	338	387	698	747	1015	1062	1375	1422	1735	1782	27	251	474	611	834	926	1151	1286	1511	1646	1871	114	296	429	656	789	969	1108	1329	1468	1689	1828	69	42614
29	42614	1912	153	204	513	564	873	924	1192	1237	1552	1597	1779	22	335	382	695	742	1055	1059	1370	1419	1730	1866	111	246	471	606	831	966	1146	1283	1506	1643	1821	68	289	428	649	788	1009	1101	1328	1461	1688	42614
30	42614	1545	1596	1905	152	197	512	557	872	917	1232	1236	1414	1727	1774	19	330	379	690	739	1050	1099	1367	1503	1638	1863	106	243	466	603	826	963	1186	1278	1460	1681	1820	61	288	421	648	781	1008	1141	1321	42614
31	42614	1225	1276	1544	1589	1904	145	196	505	556	865	916	1094	1407	1411	1722	1771	14	327	374	687	734	1047	1183	1318	1498	1635	1858	103	238	463	598	823	958	1140	1361	1453	1680	1813	60	281	420	641	780	1001	42614
32	42614	864	909	1224	1269	1584	1588	1897	144	189	504	549	731	1042	1091	1402	1451	1719	1766	11	322	371	682	818	955	1178	1315	1538	1630	1855	98	235	458	595	773	1000	1133	1360	1493	1673	1812	53	280	413	640	42614
33	42614	497	548	857	908	1217	1268	1577	1628	1896	137	188	366	679	726	1039	1086	1399	1446	1759	1763	6	319	455	590	815	950	1175	1310	1535	1670	1850	95	230	412	633	772	993	1132	1353	1492	1713	1805	52	273	42614
34	42614	89	228	449	588	809	948	1169	1308	1529	1668	1889	46	271	406	631	766	991	1126	1351	1486	1711	1846	135	182	495	542	855	902	1215	1262	1575	1622	1935	4	313	364	673	724	1033	1084	1393	1444	1753	1804	42614
35	42614	1661	1888	129	221	448	581	808	941	1168	1301	1528	1706	1843	86	266	403	626	763	986	1123	1346	1483	1619	1930	175	179	490	539	850	899	1210	1259	1570	1752	1797	44	312	357	672	717	1032	1077	1392	1437	42614
36	42614	1300	1521	1660	1881	128	261	441	580	801	940	1161	1343	1478	1703	1838	83	306	398	623	758	983	1118	1254	1567	1614	1927	170	219	487	534	847	894	1207	1385	1436	1745	1796	37	352	356	665	716	1025	1076	42614
37	42614	933	1160	1293	1520	1653	1880	121	260	481	573	800	978	1115	1338	1475	1698	1835	78	303	438	618	755	891	1202	1251	1562	1611	1922	167	214	527	531	842	1024	1069	1384	1429	1744	1789	36	345	396	664	709	42614
38	42614	613	793	932	1153	1292	1513	1652	1873	120	253	480	658	750	975	1110	1335	1470	1695	1830	75	298	435	571	839	886	1199	1246	1559	1606	1919	162	211	522	704	708	1017	1068	1377	1428	1737	1788	29	344	389	42614
39	42614	252	473	612	833	925	1152	1285	1512	1645	1872	113	295	430	655	790	970	1107	1330	1467	1690	1827	70	206	519	566	879	883	1194	1243	1554	1603	1914	159	337	388	697	748	1016	1061	1376	1421	1736	1781	28	42614
40	42614	1865	112	245	472	605	832	965	1145	1284	1505	1644	1822	67	290	427	650	787	1010	1102	1327	1462	1687	1911	154	203	514	563	874	923	1191	1238	1551	1598	1780	21	336	381	696	741	1056	1060	1369	1420	1729	42614
41	42614	1504	1637	1864	105	244	465	604	825	964	1185	1277	1459	1682	1819	62	287	422	647	782	1007	1142	1322	1546	1595	1906	151	198	511	558	871	918	1231	1235	1413	1728	1773	20	329	380	689	740	1049	1100	1368	42614
42	42614	1184	1317	1497	1636	1857	104	237	464	597	824	957	1139	1362	1454	1679	1814	59	282	419	642	779	1002	1226	1275	1543	1590	1903	146	195	506	555	866	915	1093	1408	1412	1721	1772	13	328	373	688	733	1048	42614
43	42614	817	956	1177	1316	1537	1629	1856	97	236	457	596	774	999	1134	1359	1494	1674	1811	54	279	414	639	863	910	1223	1270	1583	1587	1898	143	190	503	550	732	1041	1092	1401	1452	1720	1765	12	321	372	681	42614
44		456	589	816	949	1176	1309	1536	1669	1849	96	229	411	634	771	994	1131	1354	1491	1714	1806	51	274	498	547	858	907	1218	1267	1578	1627	1895	138	187	365	680	725	1040	1085	1400	1445	1760	1764	5	320	42614
		42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614

Thus we observe that we have a **pan diagonal** magic square of order 44 with 16 blocks of order 11 with different magic sums.

5.3 Blocks of Order 22

As we have seen in previous subsection, the magic square of order 44 in terms of different magic sums blocks of order 11. Here we shall give magic square of order 44 as sub-blocks of order 22 with equal magic sums. In order to construct this magic square we need to divide the numbers 1 to 1936 in four equal parts. See below this division.

Distribution 5.4. *Let's divide the numbers 1 to 1936 in four equal parts as follows:*

	1	2	3	4	5	6	7	8	9	10	...	...	239	240	241	242	243	244	245	...	...	475	476	477	478	479	480	481	482	483	484	Total
A1	1	8	9	16	17	24	25	32	33	40	...	...	953	960	961	968	969	976	977	...	...	1897	1904	1905	1912	1913	1920	1921	1928	1929	1936	468754
A2	2	7	10	15	18	23	26	31	34	39	...	...	954	959	962	967	970	975	978	...	...	1898	1903	1906	1911	1914	1919	1922	1927	1930	1935	468754
A4	3	6	11	14	19	22	27	30	35	38	...	...	955	958	963	966	971	974	979	...	...	1899	1902	1907	1910	1915	1918	1923	1926	1931	1934	468754
A3	4	5	12	13	20	21	28	29	36	37	...	...	956	957	964	965	972	973	980	...	...	1900	1901	1908	1909	1916	1917	1924	1925	1932	1933	468754

Distribution 5.5. *Let's consider following distribution of* 2×2*:*

A1	A2
A3	A4

Constructing 4 magic squares of order 22 by using the entries given in 5.4 and applying over Example 5.2 we get a magic square of order 44 given in example below.

Example 5.6. *The* ***block-wise*** *magic square of order 44 with sub-blocks of order 22 with equal magic sums is given by*

https://doi.org/10.5281/zenodo.2555889

Magic Square of Order 44 - 4 blocks of equal magic sums of order 22. Magic Sums are S(44x44):=42614 and S(22x22):=21307.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	42614
1	1	376	304	800	456	1545	1905	1825	1728	1648	97	193	952	1048	792	689	609	1097	1440	1368	1257	1177	2	375	303	799	455	1546	1906	1826	1727	1647	98	194	951	1047	791	690	610	1098	1439	1367	1258	1178	42614
2	457	96	24	392	801	1737	1640	1560	1897	1817	192	265	704	601	961	1040	784	1280	1169	1089	1449	1352	458	95	23	391	802	1738	1639	1559	1898	1818	191	266	703	602	962	1039	783	1279	1170	1090	1450	1351	42614
3	808	441	185	112	40	1912	1809	1729	1649	1552	281	360	1049	776	696	616	953	1441	1361	1264	1192	1081	807	442	186	111	39	1911	1810	1730	1650	1551	282	359	1050	775	695	615	954	1442	1362	1263	1191	1082	42614
4	128	809	448	280	200	1641	1561	1904	1824	1721	353	9	608	968	1041	785	688	1176	1104	1433	1353	1273	127	810	447	279	199	1642	1562	1903	1823	1722	354	10	607	967	1042	786	687	1175	1103	1434	1354	1274	42614
5	288	216	793	449	369	1816	1736	1633	1553	1913	8	104	777	697	600	960	1056	1345	1265	1185	1088	1456	287	215	794	450	370	1815	1735	1634	1554	1914	7	103	778	698	599	959	1055	1346	1266	1186	1087	1455	42614
6	1128	1488	1408	1305	1225	464	1000	920	840	729	649	569	1497	1857	1777	1680	1600	49	409	329	232	152	1127	1487	1407	1306	1226	463	999	919	839	730	650	570	1498	1858	1778	1679	1599	50	410	330	231	151	42614
7	1320	1217	1137	1480	1400	664	553	473	1009	904	824	744	1689	1592	1512	1849	1769	241	144	64	401	321	1319	1218	1138	1479	1399	663	554	474	1010	903	823	743	1690	1591	1511	1850	1770	242	143	63	402	322	42614
8	1489	1392	1312	1232	1129	833	728	648	568	488	993	913	1864	1761	1681	1601	1504	416	313	233	153	56	1490	1391	1311	1231	1130	834	727	647	567	487	994	914	1863	1762	1682	1602	1503	415	314	234	154	55	42614
9	1224	1144	1481	1401	1304	1008	928	817	737	657	552	472	1593	1513	1856	1776	1673	145	65	408	328	225	1223	1143	1482	1402	1303	1007	927	818	738	658	551	471	1594	1514	1855	1775	1674	146	66	407	327	226	42614
10	1393	1313	1216	1136	1496	561	481	992	912	832	752	641	1768	1688	1585	1505	1865	320	240	137	57	417	1394	1314	1215	1135	1495	562	482	991	911	831	751	642	1767	1687	1586	1506	1866	319	239	138	58	418	42614
11	184	273	368	17	89	736	656	576	465	1001	921	816	1288	1377	1472	1121	1193	1745	1840	1929	1584	1656	183	274	367	18	90	735	655	575	466	1002	922	815	1287	1378	1471	1122	1194	1746	1839	1930	1583	1655	42614
12	361	16	105	177	272	905	825	745	640	560	480	1016	1465	1120	1209	1281	1376	1928	1577	1672	1744	1833	362	15	106	178	271	906	826	746	639	559	479	1015	1466	1119	1210	1282	1375	1927	1578	1671	1743	1834	42614
13	1537	1880	1808	1697	1617	72	432	352	249	169	1201	1297	1105	1432	1360	848	504	881	977	721	624	544	1538	1879	1807	1698	1618	71	431	351	250	170	1202	1298	1106	1431	1359	847	503	882	978	722	623	543	42614
14	1720	1609	1529	1889	1792	264	161	81	424	344	1296	1369	505	1200	1080	1448	849	633	536	896	969	713	1719	1610	1530	1890	1791	263	162	82	423	343	1295	1370	506	1199	1079	1447	850	634	535	895	970	714	42614
15	1881	1801	1704	1632	1521	433	336	256	176	73	1385	1464	856	489	1289	1168	1096	984	705	625	545	888	1882	1802	1703	1631	1522	434	335	255	175	74	1386	1463	855	490	1290	1167	1095	983	706	626	546	887	42614
16	1616	1544	1873	1793	1713	168	88	425	345	248	1457	1113	1184	857	496	1384	1256	537	897	976	720	617	1615	1543	1874	1794	1714	167	87	426	346	247	1458	1114	1183	858	495	1383	1255	538	898	975	719	618	42614
17	1785	1705	1625	1528	1896	337	257	160	80	440	1112	1208	1344	1272	841	497	1473	712	632	529	889	985	1786	1706	1626	1527	1895	338	258	159	79	439	1111	1207	1343	1271	842	498	1474	711	631	530	890	986	42614
18	929	1025	769	672	592	1057	1417	1337	1240	1160	1664	1760	41	384	312	201	121	1568	1872	1800	865	521	930	1026	770	671	591	1058	1418	1338	1239	1159	1663	1759	42	383	311	202	122	1567	1871	1799	866	522	42614
19	681	584	944	1017	761	1249	1152	1072	1409	1329	1753	1832	224	113	33	393	296	528	1657	1520	1888	872	682	583	943	1018	762	1250	1151	1071	1410	1330	1754	1831	223	114	34	394	295	527	1658	1519	1887	871	42614
20	1032	753	673	593	936	1424	1321	1241	1161	1064	1848	1921	385	305	208	136	25	873	512	1752	1608	1536	1031	754	674	594	935	1423	1322	1242	1162	1063	1847	1922	386	306	207	135	26	874	511	1751	1607	1535	42614
21	585	945	1024	768	665	1153	1073	1416	1336	1233	1920	1576	120	48	377	297	217	1624	880	513	1841	1696	586	946	1023	767	666	1154	1074	1415	1335	1234	1919	1575	119	47	378	298	218	1623	879	514	1842	1695	42614
22	760	680	577	937	1033	1328	1248	1145	1065	1425	1569	1665	289	209	129	32	400	1784	1712	864	520	1936	759	679	578	938	1034	1327	1247	1146	1066	1426	1570	1666	290	210	130	31	399	1783	1711	863	519	1935	42614
23	3	374	302	798	454	1547	1907	1827	1726	1646	99	195	950	1046	790	691	611	1099	1438	1366	1259	1179	4	373	301	797	453	1548	1908	1828	1725	1645	100	196	949	1045	789	692	612	1100	1437	1365	1260	1180	42614
24	459	94	22	390	803	1739	1638	1558	1899	1819	190	267	702	603	963	1038	782	1278	1171	1091	1451	1350	460	93	21	389	804	1740	1637	1557	1900	1820	189	268	701	604	964	1037	781	1277	1172	1092	1452	1349	42614
25	806	443	187	110	38	1910	1811	1731	1651	1550	283	358	1051	774	694	614	955	1443	1363	1262	1190	1083	805	444	188	109	37	1909	1812	1732	1652	1549	284	357	1052	773	693	613	956	1444	1364	1261	1189	1084	42614
26	126	811	446	278	198	1643	1563	1902	1822	1723	355	11	606	966	1043	787	686	1174	1102	1435	1355	1275	125	812	445	277	197	1644	1564	1901	1821	1724	356	12	605	965	1044	788	685	1173	1101	1436	1356	1276	42614
27	286	214	795	451	371	1814	1734	1635	1555	1915	6	102	779	699	598	958	1054	1347	1267	1187	1086	1454	285	213	796	452	372	1813	1733	1636	1556	1916	5	101	780	700	597	957	1053	1348	1268	1188	1085	1453	42614
28	1126	1486	1406	1307	1227	462	998	918	838	731	651	571	1499	1859	1779	1678	1598	51	411	331	230	150	1125	1485	1405	1308	1228	461	997	917	837	732	652	572	1500	1860	1780	1677	1597	52	412	332	229	149	42614
29	1318	1219	1139	1478	1398	662	555	475	1011	902	822	742	1691	1590	1510	1851	1771	243	142	62	403	323	1317	1220	1140	1477	1397	661	556	476	1012	901	821	741	1692	1589	1509	1852	1772	244	141	61	404	324	42614
30	1491	1390	1310	1230	1131	835	726	646	566	486	995	915	1862	1763	1683	1603	1502	414	315	235	155	54	1492	1389	1309	1229	1132	836	725	645	565	485	996	916	1861	1764	1684	1604	1501	413	316	236	156	53	42614
31	1222	1142	1483	1403	1302	1006	926	819	739	659	550	470	1595	1515	1854	1774	1675	147	67	406	326	227	1221	1141	1484	1404	1301	1005	925	820	740	660	549	469	1596	1516	1853	1773	1676	148	68	405	325	228	42614
32	1395	1315	1214	1134	1494	563	483	990	910	830	750	643	1766	1686	1587	1507	1867	318	238	139	59	419	1396	1316	1213	1133	1493	564	484	989	909	829	749	644	1765	1685	1588	1508	1868	317	237	140	60	420	42614
33	182	275	366	19	91	734	654	574	467	1003	923	814	1286	1379	1470	1123	1195	1747	1838	1931	1582	1654	181	276	365	20	92	733	653	573	468	1004	924	813	1285	1380	1469	1124	1196	1748	1837	1932	1581	1653	42614
34	363	14	107	179	270	907	827	747	638	558	478	1014	1467	1118	1211	1283	1374	1926	1579	1670	1742	1835	364	13	108	180	269	908	828	748	637	557	477	1013	1468	1117	1212	1284	1373	1925	1580	1669	1741	1836	42614
35	1539	1878	1806	1699	1619	70	430	350	251	171	1203	1299	1107	1430	1358	846	502	883	979	723	622	542	1540	1877	1805	1700	1620	69	429	349	252	172	1204	1300	1108	1429	1357	845	501	884	980	724	621	541	42614
36	1718	1611	1531	1891	1790	262	163	83	422	342	1294	1371	507	1198	1078	1446	851	635	534	894	971	715	1717	1612	1532	1892	1789	261	164	84	421	341	1293	1372	508	1197	1077	1445	852	636	533	893	972	716	42614
37	1883	1803	1702	1630	1523	435	334	254	174	75	1387	1462	854	491	1291	1166	1094	982	707	627	547	886	1884	1804	1701	1629	1524	436	333	253	173	76	1388	1461	853	492	1292	1165	1093	981	708	628	548	885	42614
38	1614	1542	1875	1795	1715	166	86	427	347	246	1459	1115	1182	859	494	1382	1254	539	899	974	718	619	1613	1541	1876	1796	1716	165	85	428	348	245	1460	1116	1181	860	493	1381	1253	540	900	973	717	620	42614
39	1787	1707	1627	1526	1894	339	259	158	78	438	1110	1206	1342	1270	843	499	1475	710	630	531	891	987	1788	1708	1628	1525	1893	340	260	157	77	437	1109	1205	1341	1269	844	500	1476	709	629	532	892	988	42614
40	931	1027	771	670	590	1059	1419	1339	1238	1158	1662	1758	43	382	310	203	123	1566	1870	1798	867	523	932	1028	772	669	589	1060	1420	1340	1237	1157	1661	1757	44	381	309	204	124	1565	1869	1797	868	524	42614
41	683	582	942	1019	763	1251	1150	1070	1411	1331	1755	1830	222	115	35	395	294	526	1659	1518	1886	870	684	581	941	1020	764	1252	1149	1069	1412	1332	1756	1829	221	116	36	396	293	525	1660	1517	1885	869	42614
42	1030	755	675	595	934	1422	1323	1243	1163	1062	1846	1923	387	307	206	134	27	875	510	1750	1606	1534	1029	756	676	596	933	1421	1324	1244	1164	1061	1845	1924	388	308	205	133	28	876	509	1749	1605	1533	42614
43	587	947	1022	766	667	1155	1075	1414	1334	1235	1918	1574	118	46	379	299	219	1622	878	515	1843	1694	588	948	1021	765	668	1156	1076	1413	1333	1236	1917	1573	117	45	380	300	220	1621	877	516	1844	1693	42614
44	758	678	579	939	1035	1326	1246	1147	1067	1427	1571	1667	291	211	131	30	398	1782	1710	862	518	1934	757	677	580	940	1036	1325	1245	1148	1068	1428	1572	1668	292	212	132	29	397	1781	1709	861	517	1933	42614
	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614	42614

According to Example 5.2 we have 48 (4×12) **pan diagonal** magic squares of order 5 and 4 **pan diagonal** magic square of order 7.

6 Magic Squares of Order 45

The **Block-wise** construction of magic squares of order 45 depends on the products 3×15, 5×9, 9×5 and 15×3, i.e., we can construct magic square of order 45 as blocks of orders 3, 5, 9 and 15. The magic sum of order 45 is given by

$$S_{45\times 45} := \frac{45 \times (1+45^2)}{2} = 45585.$$

Let's see the divisions of 45585 by 15, 9, 5 and 3. See below:

$$\begin{array}{llll} (i) & \frac{45585}{15} = 3039 & \Longrightarrow & \text{equal blocks of order 3;} \\ (ii) & \frac{45585}{9} = 5065 & \Longrightarrow & \text{equal blocks of order 5;} \\ (iii) & \frac{45585}{5} = 9117 & \Longrightarrow & \text{equal blocks of order 9;} \\ (iv) & \frac{45585}{3} = 15195 & \Longrightarrow & \text{equal blocks of order 15.} \end{array}$$

This implies that we can made **block-wise** constructions of magic squares of order 45 with equal magic sums blocks of orders 3, 5, 9 and 15. In order to produce these magic squares of order 45, we need magic squares of order 3, 5, 9 and 15. The magic squares of orders 3 and 5 are already given in Examples 2.1 and 3.2 respectively. The blocks of order 15 are not done as it is included in the case of the order 5 giving equal sums magic squares of order 45. This covers the possibility of blocks of order 15. In case of blocks of order 3, we will get equal sums of **semi-magic** squares. Since order 3 gives semi-magic sums, we shall work with two types of order 9. One of equal magic sums and another with different magic sums. Below is a **pan diagonal** magic square of order 9.

- **Pan Diagonal Squares of Order 9**

Example 6.1. *Let's consider Latin squares decomposition of magic square of order 9 resulting in **pan diagonal** magic square:*

Ⓛ		45	45	45	45	45	45	45	45	45
	1	6	8	7	3	5	4	9	2	45
45	5	7	3	2	4	9	8	1	6	45
45	9	2	4	6	8	1	3	5	7	45
45	2	4	9	8	1	6	5	7	3	45
45	6	8	1	3	5	7	9	2	4	45
45	7	3	5	4	9	2	1	6	8	45
45	3	5	7	9	2	4	6	8	1	45
45	4	9	2	1	6	8	7	3	5	45
45	8	1	6	5	7	3	2	4	9	45
	45	45	45	45	45	45	45	45	45	45

Ⓜ		45	45	45	45	45	45	45	45	45
	8	4	3	7	6	2	9	5	1	45
45	1	9	5	3	8	4	2	7	6	45
45	6	2	7	5	1	9	4	3	8	45
45	5	1	9	4	3	8	6	2	7	45
45	7	6	2	9	5	1	8	4	3	45
45	3	8	4	2	7	6	1	9	5	45
45	2	7	6	1	9	5	3	8	4	45
45	4	3	8	6	2	7	5	1	9	45
45	9	5	1	8	4	3	7	6	2	45
	45	45	45	45	45	45	45	45	45	45

M_9		369	369	369	369	369	369	369	369	369
	8	49	66	61	24	38	36	77	10	369
369	37	63	23	12	35	76	65	7	51	369
369	78	11	34	50	64	9	22	39	62	369
369	14	28	81	67	3	53	42	56	25	369
369	52	69	2	27	41	55	80	13	30	369
369	57	26	40	29	79	15	1	54	68	369
369	20	43	60	73	18	32	48	71	4	369
369	31	75	17	6	47	70	59	19	45	369
369	72	5	46	44	58	21	16	33	74	369
	369	369	369	369	369	369	369	369	369	369

C_9		495	495	495	495	495	495	495	495	495
	18	64	83	77	36	52	49	95	21	495
495	51	79	35	23	48	94	82	17	66	495
495	96	22	47	65	81	19	34	53	78	495
495	25	41	99	84	13	68	56	72	37	495
495	67	86	12	39	55	71	98	24	43	495
495	73	38	54	42	97	26	11	69	85	495
495	32	57	76	91	29	45	63	88	14	495
495	44	93	28	16	62	87	75	31	59	495
495	89	15	61	58	74	33	27	46	92	495
	495	495	495	495	495	495	495	495	495	495

The magic squares M_9 and C_9 are obtained by using the operations

$$9 \times (A-1) + B := M_9 \quad \textit{and} \quad 10 \times A + B := C_9,$$

respectively. The M_9 is a magic square of order 9 of consecutive numbers from 1 to 81, and C_9 is the **composite** *magic square.*

Additionally it has the property that each 3×3 block is of same sum as of magic square, i.e., $S_9 = 369$. Also each 3×3 block is a **semi-magic** square of order 3 (only in rows and columns).

6.1 Blocks of Order 3

In order to construct magic square of order 45 as sub-blocks of order 3, we shall use the magic square of order 3 given in 2.1. Also we shall make use of magic rectangle of order 3×15 given in example below:

Example 6.2. *The magic rectangle of order* 3×15 *is given by*

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	Total
1	1	28	26	4	9	27	25	12	35	36	17	22	43	44	16	345
2	38	39	40	41	31	32	33	23	13	14	15	5	6	7	8	345
3	30	2	3	24	29	10	11	34	21	19	37	42	20	18	45	345
Total	69	69	69	69	69	69	69	69	69	69	69	69	69	69	69	

Let's consider following composition distribution. It will help to put blocks of order 3 to bring magic square of order 45.

Distribution 6.1. *Let's consider following distribution* 15×15 *in composite form:*

1.1	2.1	3.1	4.1	5.1	6.1	7.1	8.1	9.1	10.1	11.1	12.1	13.1	14.1	15.1
1.2	2.2	3.2	4.2	5.2	6.2	7.2	8.2	9.2	10.2	11.2	12.2	13.2	14.2	15.2
1.3	2.3	3.3	4.3	5.3	6.3	7.3	8.3	9.3	10.3	11.3	12.3	13.3	14.3	15.3
1.4	2.4	3.4	4.4	5.4	6.4	7.4	8.4	9.4	10.4	11.4	12.4	13.4	14.4	15.4
1.5	2.5	3.5	4.5	5.5	6.5	7.5	8.5	9.5	10.5	11.5	12.5	13.5	14.5	15.5
1.6	2.6	3.6	4.6	5.6	6.6	7.6	8.6	9.6	10.6	11.6	12.6	13.6	14.6	15.6
1.7	2.7	3.7	4.7	5.7	6.7	7.7	8.7	9.7	10.7	11.7	12.7	13.7	14.7	15.7
1.8	2.8	3.8	4.8	5.8	6.8	7.8	8.8	9.8	10.8	11.8	12.8	13.8	14.8	15.8
1.9	2.9	3.9	4.9	5.9	6.9	7.9	8.9	9.9	10.9	11.9	12.9	13.9	14.9	15.9
1.10	2.10	3.10	4.10	5.10	6.10	7.10	8.10	9.10	10.10	11.10	12.10	13.10	14.10	15.10
1.11	2.11	3.11	4.11	5.11	6.11	7.11	8.11	9.11	10.11	11.11	12.11	13.11	14.11	15.11
1.12	2.12	3.12	4.12	5.12	6.12	7.12	8.12	9.12	10.12	11.12	12.12	13.12	14.12	15.12
1.13	2.13	3.13	4.13	5.13	6.13	7.13	8.13	9.13	10.13	11.13	12.13	13.13	14.13	15.13
1.14	2.14	3.14	4.14	5.14	6.14	7.14	8.14	9.14	10.14	11.14	12.14	13.14	14.14	15.14
1.15	2.15	3.15	4.15	5.15	6.15	7.15	8.15	9.15	10.15	11.15	12.15	13.15	14.15	15.15

We shall construct 225 blocks of order 3 and put them according to Distribution 6.1. Below are few examples of **semi-magic** squares of order 3 constructed by applying the columns values given of magic rectangle given in Example 6.2 over the Example 2.1 by using the operation $M_3 := 45 \times (A-1) + B$:

- **Block 2.7**

2			69
39	2	28	69
28	39	2	69
2	28	39	69
69	69	69	117

7			99
25	11	33	69
11	33	25	69
33	25	11	69
69	69	69	69

2.7			3039
1735	56	1248	3039
1226	1743	70	3039
78	1240	1721	3039
3039	3039	3039	5199

- **Block 3.13**

3			69
40	3	26	69
26	40	3	69
3	26	40	69
69	69	69	120

13			18
43	20	6	69
20	6	43	69
6	43	20	69
69	69	69	69

3.13			2988
1798	110	1131	3039
1145	1761	133	3039
96	1168	1775	3039
3039	3039	3039	5334

• **Block 7.9**

7			69
33	11	25	69
25	33	11	69
11	25	33	69
69	69	69	99

9			45
35	21	13	69
21	13	35	69
13	35	21	69
69	69	69	69

7.9			3009
1475	471	1093	3039
1101	1453	485	3039
463	1115	1461	3039
3039	3039	3039	4389

Based on similar procedure we construct all the 225 blocks of **semi-magic** squares (in rows and columns) and put them according to Distributions 6.1, we get the required **pan diagonal** magic square of order 45 given in example below.

Example 6.3.. *The **block-wise pan diagonal** magic square of order 45 with **semi-magic** blocks of order 3 is given by*

https://doi.org/10.5281/zenodo.2555889

Pan diagonal magic square of order 45. All block of order 3 are of equal semi-magic sums. Magic sums are S(45x45):=45585 and S(3x3):=3039 (only rows and columns).

		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	
	pan	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585
1	45585	1666	1335	38	1711	75	1253	1756	120	1163	1801	1065	173	1351	1290	398	1396	435	1208	1441	480	1118	991	1515	533	541	930	1568	586	840	1613	631	1650	758	181	1875	983	226	885	1928	271	795	1973	316	2010	713	45585
2	45585	30	1703	1306	1245	1748	46	1155	1793	91	165	1838	1036	390	1388	1261	1200	1433	406	1110	1478	451	525	1028	1486	1560	578	901	1605	623	811	750	668	1621	975	218	1846	1920	263	856	1965	308	766	705	353	1981	45585
3	45585	1343	1	1695	83	1216	1740	128	1126	1785	1073	136	1830	1298	361	1380	443	1171	1425	488	1081	1470	1523	496	1020	938	1531	570	848	1576	615	1658	721	660	1883	946	210	893	1891	255	803	1936	300	2018	676	345	45585
4	45585	1693	1307	39	1738	47	1254	1783	92	1164	1828	1037	174	1378	1262	399	1423	407	1209	1468	452	1119	1018	1487	534	568	902	1569	613	812	1614	658	1622	759	208	1847	984	253	857	1929	298	767	1974	343	1982	714	45585
5	45585	2	1704	1333	1217	1749	73	1127	1794	118	137	1839	1063	362	1389	1288	1172	1434	433	1082	1479	478	497	1029	1513	1532	579	928	1577	624	838	722	669	1648	947	219	1873	1892	264	883	1937	309	793	677	354	2008	45585
6	45585	1344	28	1667	84	1243	1712	129	1153	1757	1074	163	1802	1299	388	1352	444	1198	1397	489	1108	1442	1524	523	992	939	1558	542	849	1603	587	1659	748	632	1884	973	182	894	1918	227	804	1963	272	2019	703	317	45585
7	45585	1691	1308	40	1736	48	1255	1781	93	1165	1826	1038	175	1376	1263	400	1421	408	1210	1466	453	1120	1016	1488	535	566	903	1570	611	813	1615	656	1623	760	206	1848	985	251	858	1930	296	768	1975	341	1983	715	45585
8	45585	3	1705	1331	1218	1750	71	1128	1795	116	138	1840	1061	363	1390	1286	1173	1435	431	1083	1480	476	498	1030	1511	1533	580	926	1578	625	836	723	670	1646	948	220	1871	1893	265	881	1938	310	791	678	355	2006	45585
9	45585	1345	26	1668	85	1241	1713	130	1151	1758	1075	161	1803	1300	386	1353	445	1196	1398	490	1106	1443	1525	521	993	940	1556	543	850	1601	588	1660	746	633	1885	971	183	895	1916	228	805	1961	273	2020	701	318	45585
10	45585	1669	1329	41	1714	69	1256	1759	114	1166	1804	1059	176	1354	1284	401	1399	429	1211	1444	474	1121	994	1509	536	544	924	1571	589	834	1616	634	1644	761	184	1869	986	229	879	1931	274	789	1976	319	2004	716	45585
11	45585	24	1706	1309	1239	1751	49	1149	1796	94	159	1841	1039	384	1391	1264	1194	1436	409	1104	1481	454	519	1031	1489	1554	581	904	1599	626	814	744	671	1624	969	221	1849	1914	266	859	1959	311	769	699	356	1984	45585
12	45585	1346	4	1689	86	1219	1734	131	1129	1779	1076	139	1824	1301	364	1374	446	1174	1419	491	1084	1464	1526	499	1014	941	1534	564	851	1579	609	1661	724	654	1886	949	204	896	1894	249	806	1939	294	2021	679	339	45585
13	45585	1674	1334	31	1719	74	1246	1764	119	1156	1809	1064	166	1359	1289	391	1404	434	1201	1449	479	1111	999	1514	526	549	929	1561	594	839	1606	639	1649	751	189	1874	976	234	884	1921	279	794	1966	324	2009	706	45585
14	45585	29	1696	1314	1244	1741	54	1154	1786	99	164	1831	1044	389	1381	1269	1199	1426	414	1109	1471	459	524	1021	1494	1559	571	909	1604	616	819	749	661	1629	974	211	1854	1919	256	864	1964	301	774	704	346	1989	45585
15	45585	1336	9	1694	76	1224	1739	121	1134	1784	1066	144	1829	1291	369	1379	436	1179	1424	481	1089	1469	1516	504	1019	931	1539	569	841	1584	614	1651	729	659	1876	954	209	886	1899	254	796	1944	299	2011	684	344	45585
16	45585	1692	1315	32	1737	55	1247	1782	100	1157	1827	1045	167	1377	1270	392	1422	415	1202	1467	460	1112	1017	1495	527	567	910	1562	612	820	1607	657	1630	752	207	1855	977	252	865	1922	297	775	1967	342	1990	707	45585
17	45585	10	1697	1332	1225	1742	72	1135	1787	117	145	1832	1062	370	1382	1287	1180	1427	432	1090	1472	477	505	1022	1512	1540	572	927	1585	617	837	730	662	1647	955	212	1872	1900	257	882	1945	302	792	685	347	2007	45585
18	45585	1337	27	1675	77	1242	1720	122	1152	1765	1067	162	1810	1292	387	1360	437	1197	1405	482	1107	1450	1517	522	1000	932	1557	550	842	1602	595	1652	747	640	1877	972	190	887	1917	235	797	1962	280	2012	702	325	45585
19	45585	1690	1316	33	1735	56	1248	1780	101	1158	1825	1046	168	1375	1271	393	1420	416	1203	1465	461	1113	1015	1496	528	565	911	1563	610	821	1608	655	1631	753	205	1856	978	250	866	1923	295	776	1968	340	1991	708	45585
20	45585	11	1698	1330	1226	1743	70	1136	1788	115	146	1833	1060	371	1383	1285	1181	1428	430	1091	1473	475	506	1023	1510	1541	573	925	1586	618	835	731	663	1645	956	213	1870	1901	258	880	1946	303	790	686	348	2005	45585
21	45585	1338	25	1676	78	1240	1721	123	1150	1766	1068	160	1811	1293	385	1361	438	1195	1406	483	1105	1451	1518	520	1001	933	1555	551	843	1600	596	1653	745	641	1878	970	191	888	1915	236	798	1960	281	2013	700	326	45585
22	45585	1677	1339	23	1722	79	1238	1767	124	1148	1812	1069	158	1362	1294	383	1407	439	1193	1452	484	1103	1002	1519	518	552	934	1553	597	844	1598	642	1654	743	192	1879	968	237	889	1913	282	799	1958	327	2014	698	45585
23	45585	34	1688	1317	1249	1733	57	1159	1778	102	169	1823	1047	394	1373	1272	1204	1418	417	1114	1463	462	529	1013	1497	1564	563	912	1609	608	822	754	653	1632	979	203	1857	1924	248	867	1969	293	777	709	338	1992	45585
24	45585	1328	12	1699	68	1227	1744	113	1137	1789	1058	147	1834	1283	372	1384	428	1182	1429	473	1092	1474	1508	507	1024	923	1542	574	833	1587	619	1643	732	664	1868	957	214	878	1902	259	788	1947	304	2003	687	349	45585
25	45585	1700	1326	13	1745	66	1228	1790	111	1138	1835	1056	148	1385	1281	373	1430	426	1183	1475	471	1093	1025	1506	508	575	921	1543	620	831	1588	665	1641	733	215	1866	958	260	876	1903	305	786	1948	350	2001	688	45585
26	45585	21	1678	1340	1236	1723	80	1146	1768	125	156	1813	1070	381	1363	1295	1191	1408	440	1101	1453	485	516	1003	1520	1551	553	935	1596	598	845	741	643	1655	966	193	1880	1911	238	890	1956	283	800	696	328	2015	45585
27	45585	1318	35	1686	58	1250	1731	103	1160	1776	1048	170	1821	1273	395	1371	418	1205	1416	463	1115	1461	1498	530	1011	913	1565	561	823	1610	606	1633	755	651	1858	980	201	868	1925	246	778	1970	291	1993	710	336	45585
28	45585	1701	1324	14	1746	64	1229	1791	109	1139	1836	1054	149	1386	1279	374	1431	424	1184	1476	469	1094	1026	1504	509	576	919	1544	621	829	1589	666	1639	734	216	1864	959	261	874	1904	306	784	1949	351	1999	689	45585
29	45585	19	1679	1341	1234	1724	81	1144	1769	126	154	1814	1071	379	1364	1296	1189	1409	441	1099	1454	486	514	1004	1521	1549	554	936	1594	599	846	739	644	1656	964	194	1881	1909	239	891	1954	284	801	694	329	2016	45585
30	45585	1319	36	1684	59	1251	1729	104	1161	1774	1049	171	1819	1274	396	1369	419	1206	1414	464	1116	1459	1499	531	1009	914	1566	559	824	1611	604	1634	756	649	1859	981	199	869	1926	244	779	1971	289	1994	711	334	45585
31	45585	1682	1342	15	1727	82	1230	1772	127	1140	1817	1072	150	1367	1297	375	1412	442	1185	1457	487	1095	1007	1522	510	557	937	1545	602	847	1590	647	1657	735	197	1882	960	242	892	1905	287	802	1950	332	2017	690	45585
32	45585	37	1680	1322	1252	1725	62	1162	1770	107	172	1815	1052	397	1365	1277	1207	1410	422	1117	1455	467	532	1005	1502	1567	555	917	1612	600	827	757	645	1637	982	195	1862	1927	240	872	1972	285	782	712	330	1997	45585
33	45585	1320	17	1702	60	1232	1747	105	1142	1792	1050	152	1837	1275	377	1387	420	1187	1432	465	1097	1477	1500	512	1027	915	1547	577	825	1592	622	1635	737	667	1860	962	217	870	1907	262	780	1952	307	1995	692	352	45585
34	45585	1687	1347	5	1732	87	1220	1777	132	1130	1822	1077	140	1372	1302	365	1417	447	1175	1462	492	1085	1012	1527	500	562	942	1535	607	852	1580	652	1662	725	202	1887	950	247	897	1895	292	807	1940	337	2022	680	45585
35	45585	42	1670	1327	1257	1715	67	1167	1760	112	177	1805	1057	402	1355	1282	1212	1400	427	1122	1445	472	537	995	1507	1572	545	922	1617	590	832	762	635	1642	987	185	1867	1932	230	877	1977	275	787	717	320	2002	45585
36	45585	1310	22	1707	50	1237	1752	95	1147	1797	1040	157	1842	1265	382	1392	410	1192	1437	455	1102	1482	1490	517	1032	905	1552	582	815	1597	627	1625	742	672	1850	967	222	860	1912	267	770	1957	312	1985	697	357	45585
37	45585	1708	1325	6	1753	65	1221	1798	110	1131	1843	1055	141	1393	1280	366	1438	425	1176	1483	470	1086	1033	1505	501	583	920	1536	628	830	1581	673	1640	726	223	1865	951	268	875	1896	313	785	1941	358	2000	681	45585
38	45585	20	1671	1348	1235	1716	88	1145	1761	133	155	1806	1078	380	1356	1303	1190	1401	448	1100	1446	493	515	996	1528	1550	546	943	1595	591	853	740	636	1663	965	186	1888	1910	231	898	1955	276	808	695	321	2023	45585
39	45585	1311	43	1685	51	1258	1730	96	1168	1775	1041	178	1820	1266	403	1370	411	1213	1415	456	1123	1460	1491	538	1010	906	1573	560	816	1618	605	1626	763	650	1851	988	200	861	1933	245	771	1978	290	1986	718	335	45585
40	45585	1709	1323	7	1754	63	1222	1799	108	1132	1844	1053	142	1394	1278	367	1439	423	1177	1484	468	1087	1034	1503	502	584	918	1537	629	828	1582	674	1638	727	224	1863	952	269	873	1897	314	783	1942	359	1998	682	45585
41	45585	18	1672	1349	1233	1717	89	1143	1762	134	153	1807	1079	378	1357	1304	1188	1402	449	1098	1447	494	513	997	1529	1548	547	944	1593	592	854	738	637	1664	963	187	1889	1908	232	899	1953	277	809	693	322	2024	45585
42	45585	1312	44	1683	52	1259	1728	97	1169	1773	1042	179	1818	1267	404	1368	412	1214	1413	457	1124	1458	1492	539	1008	907	1574	558	817	1619	603	1627	764	648	1852	989	198	862	1934	243	772	1979	288	1987	719	333	45585
43	45585	1681	1350	8	1726	90	1223	1771	135	1133	1816	1080	143	1366	1305	368	1411	450	1178	1456	495	1088	1006	1530	503	556	945	1538	601	855	1583	646	1665	728	196	1890	953	241	900	1898	286	810	1943	331	2025	683	45585
44	45585	45	1673	1321	1260	1718	61	1170	1763	106	180	1808	1051	405	1358	1276	1215	1403	421	1125	1448	466	540	998	1501	1575	548	916	1620	593	826	765	638	1636	990	188	1861	1935	233	871	1980	278	781	720	323	1996	45585
45		1313	16	1710	53	1231	1755	98	1141	1800	1043	151	1845	1268	376	1395	413	1186	1440	458	1096	1485	1493	511	1035	908	1546	585	818	1591	630	1628	736	675	1853	961	225	863	1906	270	773	1951	315	1988	691	360	45585
		45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585

Thus we observe that we have a **pan diagonal** magic square of order 45 with 2259 blocks of order 3 with equal **semi-magic** sums. The magic and **semi-magic** sums are $S_{45\times 45} := 45585$ and $S_{3\times 3} := 3039$ (only rows and columns) respectively.

6.2 Blocks of Order 5

In order to construct magic square of order 45 as sub-blocks of order 5, we shall use the magic square of order 5 given in 3.2. Also we shall make use of magic rectangle of order 5×9 given in example below:

Example 6.4. *The magic rectangle of order* 5×9 *is given by*

	1	2	3	4	5	6	7	8	9	Total
1	20	43	19	21	7	12	9	31	45	207
2	17	22	18	38	14	40	10	44	4	207
3	35	33	5	16	23	30	41	13	11	207
4	42	2	36	6	32	8	28	24	29	207
5	1	15	37	34	39	25	27	3	26	207
Total	115	115	115	115	115	115	115	115	115	

Let's consider following composition distribution. It will help to put blocks of order 5 to bring magic square of order 45.

Distribution 6.2. *Let's consider following distribution of* 15×15 *in composite form:*

11	21	31	41	51	61	71	81	91
12	22	32	42	52	62	72	82	92
13	23	33	43	53	63	73	83	93
15	25	35	45	55	65	75	85	95
12	22	32	42	52	62	72	82	92
13	23	33	43	53	63	73	83	93
14	24	34	44	54	64	74	84	94
15	25	35	45	55	65	75	85	95
16	26	36	46	56	66	76	86	96
17	27	37	47	57	67	77	87	97
18	28	38	48	58	68	78	88	98
19	29	39	49	59	69	79	89	99

We shall construct 81 blocks of order 5 and put them according to Distribution 6.2. Below are few examples of **pan diagonal** magic squares of order 5 constructed by applying the columns values of magic rectangle given in Example 6.4 over the Example 3.2 by using the operation $M_5 := 45 \times (A-1) + B$:

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- **Block 24**

(2)		*115*	*115*	*115*	*115*	*115*
	43	22	33	2	15	*115*
115	2	15	43	22	33	*115*
115	22	33	2	15	43	*115*
115	15	43	22	33	2	*115*
115	33	2	15	43	22	*115*
	115	*115*	*115*	*115*	*115*	*115*

(4)		*115*	*115*	*115*	*115*	*115*
	21	6	38	34	16	*115*
115	38	34	16	21	6	*115*
115	16	21	6	38	34	*115*
115	6	38	34	16	21	*115*
115	34	16	21	6	38	*115*
	115	*115*	*115*	*115*	*115*	*115*

(24)		*5065*	*5065*	*5065*	*5065*	*5065*
	1911	951	1478	79	646	*5065*
5065	83	664	1906	966	1446	*5065*
5065	961	1461	51	668	1924	*5065*
5065	636	1928	979	1456	66	*5065*
5065	1474	61	651	1896	983	*5065*
	5065	*5065*	*5065*	*5065*	*5065*	*5065*

- **Block 56**

(5)		*115*	*115*	*115*	*115*	*115*
	7	14	23	32	39	*115*
115	32	39	7	14	23	*115*
115	14	23	32	39	7	*115*
115	39	7	14	23	32	*115*
115	23	32	39	7	14	*115*
	115	*115*	*115*	*115*	*115*	*115*

(6)		*115*	*115*	*115*	*115*	*115*
	12	8	40	25	30	*115*
115	40	25	30	12	8	*115*
115	30	12	8	40	25	*115*
115	8	40	25	30	12	*115*
115	25	30	12	8	40	*115*
	115	*115*	*115*	*115*	*115*	*115*

(56)		*5065*	*5065*	*5065*	*5065*	*5065*
	282	593	1030	1420	1740	*5065*
5065	1435	1735	300	597	998	*5065*
5065	615	1002	1403	1750	295	*5065*
5065	1718	310	610	1020	1407	*5065*
5065	1015	1425	1722	278	625	*5065*
	5065	*5065*	*5065*	*5065*	*5065*	*5065*

- **Block 97**

(9)		*115*	*115*	*115*	*115*	*115*
	45	4	11	29	26	*115*
115	29	26	45	4	11	*115*
115	4	11	29	26	45	*115*
115	26	45	4	11	29	*115*
115	11	29	26	45	4	*115*
	115	*115*	*115*	*115*	*115*	*115*

(7)		*115*	*115*	*115*	*115*	*115*
	9	28	10	27	41	*115*
115	10	27	41	9	28	*115*
115	41	9	28	10	27	*115*
115	28	10	27	41	9	*115*
115	27	41	9	28	10	*115*
	115	*115*	*115*	*115*	*115*	*115*

(97)		*5065*	*5065*	*5065*	*5065*	*5065*
	1989	163	460	1287	1166	*5065*
5065	1270	1152	2021	144	478	*5065*
5065	176	459	1288	1135	2007	*5065*
5065	1153	1990	162	491	1269	*5065*
5065	477	1301	1134	2008	145	*5065*
	5065	*5065*	*5065*	*5065*	*5065*	*5065*

Based on similar procedure we construct all the 225 blocks of **pan diagonal** magic squares of order 5, and put them according to Distributions 6.2, we get the required **pan diagonal** magic square of order 45 given in example below.

Example 6.5. . *The **block-wise pan diagonal** magic square of order 45 with **pan diagonal** blocks of order 5 is given by*

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Pan diagonal magic square of order 45. All block of order 5 are pan diagonal magic squares with equal magic sums. Magic sums are S(45x45):=45585 and S(5x5):=5065.

		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	
		pan	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585
1	45585	875	762	1547	1846	35	1910	987	1457	46	665	830	807	197	1576	1655	920	1707	692	226	1520	290	627	1007	1396	1745	515	1797	1322	316	1115	380	447	1817	1216	1205	1370	1977	557	1036	125	2000	177	467	1261	1160	45585
2	45585	1862	1	890	740	1572	62	631	1925	965	1482	1592	1621	845	785	222	242	1486	935	1685	717	1412	1711	305	605	1032	332	1081	530	1775	1347	1232	1171	395	425	1842	1052	91	1385	1955	582	1277	1126	2015	155	492	45585
3	45585	755	1550	1887	17	856	980	1460	87	647	1891	800	200	1617	1637	811	1700	695	267	1502	901	620	1010	1437	1727	271	1790	1325	357	1097	496	440	1820	1257	1187	361	1970	560	1077	107	1351	170	470	1302	1142	1981	45585
4	45585	42	872	721	1565	1865	672	1907	946	1475	65	1662	827	766	215	1595	1527	917	1666	710	245	1752	287	586	1025	1415	1122	512	1756	1340	335	1212	377	406	1835	1235	132	1367	1936	575	1055	1167	1997	136	485	1280	45585
5	45585	1531	1880	20	897	737	1441	80	650	1932	962	181	1610	1640	852	782	676	260	1505	942	1682	991	1430	1730	312	602	1306	350	1100	537	1772	1801	1250	1190	402	422	541	1070	110	1392	1952	451	1295	1145	2022	152	45585
6	45585	898	722	1552	1860	33	1933	947	1462	60	663	853	767	202	1590	1653	943	1667	697	240	1518	313	587	1012	1410	1743	538	1757	1327	330	1113	403	407	1822	1230	1203	1393	1937	562	1050	123	2023	137	472	1275	1158	45585
7	45585	1867	15	888	763	1532	67	645	1923	988	1442	1597	1635	843	808	182	247	1500	933	1708	677	1417	1725	303	628	992	337	1095	528	1798	1307	1237	1185	393	448	1802	1057	105	1383	1978	542	1282	1140	2013	178	452	45585
8	45585	753	1573	1847	22	870	978	1483	47	652	1905	798	223	1577	1642	825	1698	718	227	1507	915	618	1033	1397	1732	285	1788	1348	317	1102	510	438	1843	1217	1192	375	1968	583	1037	112	1365	168	493	1262	1147	1995	45585
9	45585	2	877	735	1563	1888	632	1912	960	1473	88	1622	832	780	213	1618	1487	922	1680	708	268	1712	292	600	1023	1438	1082	517	1770	1338	358	1172	382	420	1833	1258	92	1372	1950	573	1078	1127	2002	150	483	1303	45585
10	45585	1545	1878	43	857	742	1455	78	673	1892	967	195	1608	1663	812	787	690	258	1528	902	1687	1005	1428	1753	272	607	1320	348	1123	497	1777	1815	1248	1213	362	427	555	1068	133	1352	1957	465	1293	1168	1982	157	45585
11	45585	874	756	1548	1882	5	1909	981	1458	82	635	829	801	198	1612	1625	919	1701	693	262	1490	289	621	1008	1432	1715	514	1791	1323	352	1085	379	441	1818	1252	1175	1369	1971	558	1072	95	1999	171	468	1297	1130	45585
12	45585	1863	37	860	739	1566	63	667	1895	964	1476	1593	1657	815	784	216	243	1522	905	1684	711	1413	1747	275	604	1026	333	1117	500	1774	1341	1233	1207	365	424	1836	1053	127	1355	1954	576	1278	1162	1985	154	486	45585
13	45585	725	1549	1881	18	892	950	1459	81	648	1927	770	199	1611	1638	847	1670	694	261	1503	937	590	1009	1431	1728	307	1760	1324	351	1098	532	410	1819	1251	1188	397	1940	559	1071	108	1387	140	469	1296	1143	2017	45585
14	45585	36	873	757	1535	1864	666	1908	982	1445	64	1656	828	802	185	1594	1521	918	1702	680	244	1746	288	622	995	1414	1116	513	1792	1310	334	1206	378	442	1805	1234	126	1368	1972	545	1054	1161	1998	172	455	1279	45585
15	45585	1567	1850	19	891	738	1477	50	649	1926	963	217	1580	1639	846	783	712	230	1504	936	1683	1027	1400	1729	306	603	1342	320	1099	531	1773	1837	1220	1189	396	423	577	1040	109	1386	1953	487	1265	1144	2016	153	45585
16	45585	876	726	1568	1879	16	1911	951	1478	79	646	831	771	218	1609	1636	921	1671	713	259	1501	291	591	1028	1429	1726	516	1761	1343	349	1096	381	411	1838	1249	1186	1371	1941	578	1069	106	2001	141	488	1294	1141	45585
17	45585	1883	34	871	741	1536	83	664	1906	966	1446	1613	1654	826	786	186	263	1519	916	1686	681	1433	1744	286	606	996	353	1114	511	1776	1311	1253	1204	376	426	1806	1073	124	1366	1956	546	1298	1159	1996	156	456	45585
18	45585	736	1551	1851	38	889	961	1461	51	668	1924	781	201	1581	1658	844	1681	696	231	1523	934	601	1011	1401	1748	304	1771	1326	321	1118	529	421	1821	1221	1208	394	1951	561	1041	128	1384	151	471	1266	1163	2014	45585
19	45585	6	893	754	1546	1866	636	1928	979	1456	66	1626	848	799	196	1596	1491	938	1699	691	246	1716	308	619	1006	1416	1086	533	1789	1321	336	1176	398	439	1816	1236	96	1388	1969	556	1056	1131	2018	169	466	1281	45585
20	45585	1564	1861	21	861	758	1474	61	651	1896	983	214	1591	1641	816	803	709	241	1506	906	1703	1024	1411	1731	276	623	1339	331	1101	501	1793	1834	1231	1191	366	443	574	1051	111	1356	1973	484	1276	1146	1986	173	45585
21	45585	862	752	1544	1884	23	1897	977	1454	84	653	817	797	194	1614	1643	907	1697	689	264	1508	277	617	1004	1434	1733	502	1787	1319	354	1103	367	437	1814	1254	1193	1357	1967	554	1074	113	1987	167	464	1299	1148	45585
22	45585	1859	39	878	727	1562	59	669	1913	952	1472	1589	1659	833	772	212	239	1524	923	1672	707	1409	1749	293	592	1022	329	1119	518	1762	1337	1229	1209	383	412	1832	1049	129	1373	1942	572	1274	1164	2003	142	482	45585
23	45585	743	1537	1877	14	894	968	1447	77	644	1929	788	187	1607	1634	849	1688	682	257	1499	939	608	997	1427	1724	309	1778	1312	347	1094	534	428	1807	1247	1184	399	1958	547	1067	104	1389	158	457	1292	1139	2019	45585
24	45585	32	869	759	1553	1852	662	1904	984	1463	52	1652	824	804	203	1582	1517	914	1704	698	232	1742	284	624	1013	1402	1112	509	1794	1328	322	1202	374	444	1823	1222	122	1364	1974	563	1042	1157	1994	174	473	1267	45585
25	45585	1569	1868	7	887	734	1479	68	637	1922	959	219	1598	1627	842	779	714	248	1492	932	1679	1029	1418	1717	302	599	1344	338	1087	527	1769	1839	1238	1177	392	419	579	1058	97	1382	1949	489	1283	1132	2012	149	45585
26	45585	867	728	1570	1870	30	1902	953	1480	70	660	822	773	220	1600	1650	912	1673	715	250	1515	282	593	1030	1420	1740	507	1763	1345	340	1110	372	413	1840	1240	1200	1362	1943	580	1060	120	1992	143	490	1285	1155	45585
27	45585	1885	25	885	732	1538	85	655	1920	957	1448	1615	1645	840	777	188	265	1510	930	1677	683	1435	1735	300	597	998	355	1105	525	1767	1313	1255	1195	390	417	1808	1075	115	1380	1947	548	1300	1150	2010	147	458	45585
28	45585	750	1542	1853	40	880	975	1452	53	670	1915	795	192	1583	1660	835	1695	687	233	1525	925	615	1002	1403	1750	295	1785	1317	323	1120	520	435	1812	1223	1210	385	1965	552	1043	130	1375	165	462	1268	1165	2005	45585
29	45585	8	895	745	1560	1857	638	1930	970	1470	57	1628	850	790	210	1587	1493	940	1690	705	237	1718	310	610	1020	1407	1088	535	1780	1335	327	1178	400	430	1830	1227	98	1390	1960	570	1047	1133	2020	160	480	1272	45585
30	45585	1555	1875	12	863	760	1465	75	642	1898	985	205	1605	1632	818	805	700	255	1497	908	1705	1015	1425	1722	278	625	1330	345	1092	503	1795	1825	1245	1182	368	445	565	1065	102	1358	1975	475	1290	1137	1988	175	45585
31	45585	864	748	1540	1872	41	1899	973	1450	72	671	819	793	190	1602	1661	909	1693	685	252	1526	279	613	1000	1422	1751	504	1783	1315	342	1121	369	433	1810	1242	1211	1359	1963	550	1062	131	1989	163	460	1287	1166	45585
32	45585	1855	27	896	729	1558	55	657	1931	954	1468	1585	1647	851	774	208	235	1512	941	1674	703	1405	1737	311	594	1018	325	1107	536	1764	1333	1225	1197	401	414	1828	1045	117	1391	1944	568	1270	1152	2021	144	478	45585
33	45585	761	1539	1873	10	882	986	1449	73	640	1917	806	189	1603	1630	837	1706	684	253	1495	927	626	999	1423	1720	297	1796	1314	343	1090	522	446	1809	1243	1180	387	1976	549	1063	100	1377	176	459	1288	1135	2007	45585
34	45585	28	865	747	1571	1854	658	1900	972	1481	54	1648	820	792	221	1584	1513	910	1692	716	234	1738	280	612	1031	1404	1108	505	1782	1346	324	1198	370	432	1841	1224	118	1360	1962	581	1044	1153	1990	162	491	1269	45585
35	45585	1557	1886	9	883	730	1467	86	639	1918	955	207	1616	1629	838	775	702	266	1494	928	1675	1017	1436	1719	298	595	1332	356	1089	523	1765	1827	1256	1179	388	415	567	1076	99	1378	1945	477	1301	1134	2008	145	45585
36	45585	886	744	1574	1848	13	1921	969	1484	48	643	841	789	224	1578	1633	931	1689	719	228	1498	301	609	1034	1398	1723	526	1779	1349	318	1093	391	429	1844	1218	1183	1381	1959	584	1038	103	2011	159	494	1263	1138	45585
37	45585	1889	3	868	751	1554	89	633	1903	976	1464	1619	1623	823	796	204	269	1488	913	1696	699	1439	1713	283	616	1014	359	1083	508	1786	1329	1259	1173	373	436	1824	1079	93	1363	1966	564	1304	1128	1993	166	474	45585
38	45585	733	1561	1869	44	858	958	1471	69	674	1893	778	211	1599	1664	813	1678	706	249	1529	903	598	1021	1419	1754	273	1768	1336	339	1124	498	418	1831	1239	1214	363	1948	571	1059	134	1353	148	481	1284	1169	1983	45585
39	45585	24	899	723	1543	1876	654	1934	948	1453	76	1644	854	768	193	1606	1509	944	1668	688	256	1734	314	588	1003	1426	1104	539	1758	1318	346	1194	404	408	1813	1246	114	1394	1938	553	1066	1149	2024	138	463	1291	45585
40	45585	1533	1858	31	879	764	1443	58	661	1914	989	183	1588	1651	834	809	678	238	1516	924	1709	993	1408	1741	294	629	1308	328	1111	519	1799	1803	1228	1201	384	449	543	1048	121	1374	1979	453	1273	1156	2004	179	45585
41	45585	900	749	1534	1871	11	1935	974	1444	71	641	855	794	184	1601	1631	945	1694	679	251	1496	315	614	994	1421	1721	540	1784	1309	341	1091	405	434	1804	1241	1181	1395	1964	544	1061	101	2025	164	454	1286	1136	45585
42	45585	1849	26	866	765	1559	49	656	1901	990	1469	1579	1646	821	810	209	229	1511	911	1710	704	1399	1736	281	630	1019	319	1106	506	1800	1334	1219	1196	371	450	1829	1039	116	1361	1980	569	1264	1151	1991	180	479	45585
43	45585	731	1575	1874	4	881	956	1485	74	634	1916	776	225	1604	1624	836	1676	720	254	1489	926	596	1035	1424	1714	296	1766	1350	344	1084	521	416	1845	1244	1174	386	1946	585	1064	94	1376	146	495	1289	1129	2006	45585
44	45585	29	859	746	1541	1890	659	1894	971	1451	90	1649	814	791	191	1620	1514	904	1691	686	270	1739	274	611	1001	1440	1109	499	1781	1316	360	1199	364	431	1811	1260	119	1354	1961	551	1080	1154	1984	161	461	1305	45585
45		1556	1856	45	884	724	1466	56	675	1919	949	206	1586	1665	839	769	701	236	1530	929	1669	1016	1406	1755	299	589	1331	326	1125	524	1759	1826	1226	1215	389	409	566	1046	135	1379	1939	476	1271	1170	2009	139	45585
		45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585

Thus we observe that we have a **pan diagonal** magic square of order 45 with equal magic sums blocks of **pan diagonal** magic squares of order 5. The magic sums are $S_{45\times 45} := 45585$ and $S_{5\times 5} := 5065$. The blocks of orders 10, 15, 20, 25, 30, 35 and 40 are also **pan diagonal** magic squares. Obviously, entries in these cases are not consecutive. The entries are consecutive only in case of order 45.

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6.3 Blocks of Order 9

In this subsection, we shall present **block-wise** construction of **pan diagonal** magic square of order 45. The blocks are **pan diagonal** magic square of order 9 with different magic sums. Each block of order 9 is with **semi-magic** equal sums magic squares of order 3. In order to construct magic square of order 45 we shall use **pan diagonal** magic squares of orders 5 and 9 given in Examples 3.2 and 6.1 respectively. This is divided in two subsections again. One is with different magic sums and another with equal magic sums.

6.3.1 Different Magic Sums Blocks of Order 9

In this subsection, we shall construct **pan diagonal** magic square of order 45 by sub-blocks of **pan diagonal** magic squares of order 9 with different magic sums. Let's divide the total numbers 2025 from 1 to 2035 in 25 blocks of with 81 in each block. It is given in distribution below.

Distribution 6.3. *Let's consider following distribution of 2025 numbers from 1 to 2025 in 25 blocks with 81 in each block:*

	1	2	3	4	5	6	7	8	9			38	39	40	41	42	43			73	74	75	76	77	78	79	80	81	Total
A1	1	26	51	76	101	126	151	176	201	...	...	926	951	976	1001	1026	1051	...	...	1801	1826	1851	1876	1901	1926	1951	1976	2001	81081
A2	2	27	52	77	102	127	152	177	202	...	...	927	952	977	1002	1027	1052	...	...	1802	1827	1852	1877	1902	1927	1952	1977	2002	81162
A3	3	28	53	78	103	128	153	178	203	...	...	928	953	978	1003	1028	1053	...	...	1803	1828	1853	1878	1903	1928	1953	1978	2003	81243
A4	4	29	54	79	104	129	154	179	204	...	...	929	954	979	1004	1029	1054	...	...	1804	1829	1854	1879	1904	1929	1954	1979	2004	81324
A5	5	30	55	80	105	130	155	180	205	...	...	930	955	980	1005	1030	1055	...	...	1805	1830	1855	1880	1905	1930	1955	1980	2005	81405
A6	6	31	56	81	106	131	156	181	206	...	...	931	956	981	1006	1031	1056	...	...	1806	1831	1856	1881	1906	1931	1956	1981	2006	81486
A7	7	32	57	82	107	132	157	182	207	...	...	932	957	982	1007	1032	1057	...	...	1807	1832	1857	1882	1907	1932	1957	1982	2007	81567
A8	8	33	58	83	108	133	158	183	208	...	...	933	958	983	1008	1033	1058	...	...	1808	1833	1858	1883	1908	1933	1958	1983	2008	81648
A9	9	34	59	84	109	134	159	184	209	...	...	934	959	984	1009	1034	1059	...	...	1809	1834	1859	1884	1909	1934	1959	1984	2009	81729
A10	10	35	60	85	110	135	160	185	210	...	...	935	960	985	1010	1035	1060	...	...	1810	1835	1860	1885	1910	1935	1960	1985	2010	81810
A11	11	36	61	86	111	136	161	186	211	...	...	936	961	986	1011	1036	1061	...	...	1811	1836	1861	1886	1911	1936	1961	1986	2011	81891
A12	12	37	62	87	112	137	162	187	212	...	...	937	962	987	1012	1037	1062	...	...	1812	1837	1862	1887	1912	1937	1962	1987	2012	81972
A13	13	38	63	88	113	138	163	188	213	...	...	938	963	988	1013	1038	1063	...	...	1813	1838	1863	1888	1913	1938	1963	1988	2013	82053
A14	14	39	64	89	114	139	164	189	214	...	...	939	964	989	1014	1039	1064	...	...	1814	1839	1864	1889	1914	1939	1964	1989	2014	82134
A15	15	40	65	90	115	140	165	190	215	...	...	940	965	990	1015	1040	1065	...	...	1815	1840	1865	1890	1915	1940	1965	1990	2015	82215
A16	16	41	66	91	116	141	166	191	216	...	...	941	966	991	1016	1041	1066	...	...	1816	1841	1866	1891	1916	1941	1966	1991	2016	82296
A17	17	42	67	92	117	142	167	192	217	...	...	942	967	992	1017	1042	1067	...	...	1817	1842	1867	1892	1917	1942	1967	1992	2017	82377
A18	18	43	68	93	118	143	168	193	218	...	...	943	968	993	1018	1043	1068	...	...	1818	1843	1868	1893	1918	1943	1968	1993	2018	82458
A19	19	44	69	94	119	144	169	194	219	...	...	944	969	994	1019	1044	1069	...	...	1819	1844	1869	1894	1919	1944	1969	1994	2019	82539
A20	20	45	70	95	120	145	170	195	220	...	...	945	970	995	1020	1045	1070	...	...	1820	1845	1870	1895	1920	1945	1970	1995	2020	82620
A21	21	46	71	96	121	146	171	196	221	...	...	946	971	996	1021	1046	1071	...	...	1821	1846	1871	1896	1921	1946	1971	1996	2021	82701
A22	22	47	72	97	122	147	172	197	222	...	...	947	972	997	1022	1047	1072	...	...	1822	1847	1872	1897	1922	1947	1972	1997	2022	82782
A23	23	48	73	98	123	148	173	198	223	...	...	948	973	998	1023	1048	1073	...	...	1823	1848	1873	1898	1923	1948	1973	1998	2023	82863
A24	24	49	74	99	124	149	174	199	224	...	...	949	974	999	1024	1049	1074	...	...	1824	1849	1874	1899	1924	1949	1974	1999	2024	82944
A25	25	50	75	100	125	150	175	200	225	...	...	950	975	1000	1025	1050	1075	...	...	1825	1850	1875	1900	1925	1950	1975	2000	2025	83025

Let's rewrite the magic square of order 5 given in 3.2 and put them in terms of blocks given above.

Distribution 6.4. *Let's consider following distribution of order 5:*

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A1	A7	A13	A19	A25
A18	A24	A5	A6	A12
A10	A11	A17	A23	A4
A22	A3	A9	A15	A16
A14	A20	A21	A2	A8

We shall construct 25 magic squares of order 9 using the values given in 6.3 and put them according to 6.4. The **pan diagonal** magic squares are constructed based the Example 6.1. See below some examples:

- **Block A3**

A3		9027	9027	9027	9027	9027	9027	9027	9027	9027
	178	1203	1628	1503	578	928	878	1903	228	9027
9027	903	1553	553	278	853	1878	1603	153	1253	9027
9027	1928	253	828	1228	1578	203	528	953	1528	9027
9027	328	678	2003	1653	53	1303	1028	1378	603	9027
9027	1278	1703	28	653	1003	1353	1978	303	728	9027
9027	1403	628	978	703	1953	353	3	1328	1678	9027
9027	478	1053	1478	1803	428	778	1178	1753	78	9027
9027	753	1853	403	128	1153	1728	1453	453	1103	9027
9027	1778	103	1128	1078	1428	503	378	803	1828	9027
	9027	9027	9027	9027	9027	9027	9027	9027	9027	9027

- **Block A11**

A11		9099	9099	9099	9099	9099	9099	9099	9099	9099
	186	1211	1636	1511	586	936	886	1911	236	9099
9099	911	1561	561	286	861	1886	1611	161	1261	9099
9099	1936	261	836	1236	1586	211	536	961	1536	9099
9099	336	686	2011	1661	61	1311	1036	1386	611	9099
9099	1286	1711	36	661	1011	1361	1986	311	736	9099
9099	1411	636	986	711	1961	361	11	1336	1686	9099
9099	486	1061	1486	1811	436	786	1186	1761	86	9099
9099	761	1861	411	136	1161	1736	1461	461	1111	9099
9099	1786	111	1136	1086	1436	511	386	811	1836	9099
	9099	9099	9099	9099	9099	9099	9099	9099	9099	9099

- **Block A23**

A23		*9207*	*9207*	*9207*	*9207*	*9207*	*9207*	*9207*	*9207*	*9207*
	198	1223	1648	1523	598	948	898	1923	248	*9207*
9207	923	1573	573	298	873	1898	1623	173	1273	*9207*
9207	1948	273	848	1248	1598	223	548	973	1548	*9207*
9207	348	698	2023	1673	73	1323	1048	1398	623	*9207*
9207	1298	1723	48	673	1023	1373	1998	323	748	*9207*
9207	1423	648	998	723	1973	373	23	1348	1698	*9207*
9207	498	1073	1498	1823	448	798	1198	1773	98	*9207*
9207	773	1873	423	148	1173	1748	1473	473	1123	*9207*
9207	1798	123	1148	1098	1448	523	398	823	1848	*9207*
	9207	*9207*	*9207*	*9207*	*9207*	*9207*	*9207*	*9207*	*9207*	*9207*

In the similar way we can construct other 22 **pan diagonal** magic squares of order 9. Arranging all these 25 blocks of magic squares of order 9 in according to Distribution 6.4 we get a pan diagonal magic square of order 45 given in example below.

Example 6.6. . *The* ***block-wise pan diagonal*** *magic square of order 45 with pan diagonal blocks of order 9 is given by*

https://doi.org/10.5281/zenodo.2555889

Pan diagonal magic square of order 45. All block of order 9 are pan diagonal magic squares with different magic sums. Block of order 3 are semi-magic square of order 3.

		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	
		pan	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585
1	45585	176	1201	1626	1501	576	926	876	1901	226	182	1207	1632	1507	582	932	882	1907	232	188	1213	1638	1513	588	938	888	1913	238	194	1219	1644	1519	594	944	894	1919	244	200	1225	1650	1525	600	950	900	1925	250	45585
2	45585	901	1551	551	276	851	1876	1601	151	1251	907	1557	557	282	857	1882	1607	157	1257	913	1563	563	288	863	1888	1613	163	1263	919	1569	569	294	869	1894	1619	169	1269	925	1575	575	300	875	1900	1625	175	1275	45585
3	45585	1926	251	826	1226	1576	201	526	951	1526	1932	257	832	1232	1582	207	532	957	1532	1938	263	838	1238	1588	213	538	963	1538	1944	269	844	1244	1594	219	544	969	1544	1950	275	850	1250	1600	225	550	975	1550	45585
4	45585	326	676	2001	1651	51	1301	1026	1376	601	332	682	2007	1657	57	1307	1032	1382	607	338	688	2013	1663	63	1313	1038	1388	613	344	694	2019	1669	69	1319	1044	1394	619	350	700	2025	1675	75	1325	1050	1400	625	45585
5	45585	1276	1701	26	651	1001	1351	1976	301	726	1282	1707	32	657	1007	1357	1982	307	732	1288	1713	38	663	1013	1363	1988	313	738	1294	1719	44	669	1019	1369	1994	319	744	1300	1725	50	675	1025	1375	2000	325	750	45585
6	45585	1401	626	976	701	1951	351	1	1326	1676	1407	632	982	707	1957	357	7	1332	1682	1413	638	988	713	1963	363	13	1338	1688	1419	644	994	719	1969	369	19	1344	1694	1425	650	1000	725	1975	375	25	1350	1700	45585
7	45585	476	1051	1476	1801	426	776	1176	1751	76	482	1057	1482	1807	432	782	1182	1757	82	488	1063	1488	1813	438	788	1188	1763	88	494	1069	1494	1819	444	794	1194	1769	94	500	1075	1500	1825	450	800	1200	1775	100	45585
8	45585	751	1851	401	126	1151	1726	1451	451	1101	757	1857	407	132	1157	1732	1457	457	1107	763	1863	413	138	1163	1738	1463	463	1113	769	1869	419	144	1169	1744	1469	469	1119	775	1875	425	150	1175	1750	1475	475	1125	45585
9	45585	1776	101	1126	1076	1426	501	376	801	1826	1782	107	1132	1082	1432	507	382	807	1832	1788	113	1138	1088	1438	513	388	813	1838	1794	119	1144	1094	1444	519	394	819	1844	1800	125	1150	1100	1450	525	400	825	1850	45585
10	45585	193	1218	1643	1518	593	943	893	1918	243	199	1224	1649	1524	599	949	899	1924	249	180	1205	1630	1505	580	930	880	1905	230	181	1206	1631	1506	581	931	881	1906	231	187	1212	1637	1512	587	937	887	1912	237	45585
11	45585	918	1568	568	293	868	1893	1618	168	1268	924	1574	574	299	874	1899	1624	174	1274	905	1555	555	280	855	1880	1605	155	1255	906	1556	556	281	856	1881	1606	156	1256	912	1562	562	287	862	1887	1612	162	1262	45585
12	45585	1943	268	843	1243	1593	218	543	968	1543	1949	274	849	1249	1599	224	549	974	1549	1930	255	830	1230	1580	205	530	955	1530	1931	256	831	1231	1581	206	531	956	1531	1937	262	837	1237	1587	212	537	962	1537	45585
13	45585	343	693	2018	1668	68	1318	1043	1393	618	349	699	2024	1674	74	1324	1049	1399	624	330	680	2005	1655	55	1305	1030	1380	605	331	681	2006	1656	56	1306	1031	1381	606	337	687	2012	1662	62	1312	1037	1387	612	45585
14	45585	1293	1718	43	668	1018	1368	1993	318	743	1299	1724	49	674	1024	1374	1999	324	749	1280	1705	30	655	1005	1355	1980	305	730	1281	1706	31	656	1006	1356	1981	306	731	1287	1712	37	662	1012	1362	1987	312	737	45585
15	45585	1418	643	993	718	1968	368	18	1343	1693	1424	649	999	724	1974	374	24	1349	1699	1405	630	980	705	1955	355	5	1330	1680	1406	631	981	706	1956	356	6	1331	1681	1412	637	987	712	1962	362	12	1337	1687	45585
16	45585	493	1068	1493	1818	443	793	1193	1768	93	499	1074	1499	1824	449	799	1199	1774	99	480	1055	1480	1805	430	780	1180	1755	80	481	1056	1481	1806	431	781	1181	1756	81	487	1062	1487	1812	437	787	1187	1762	87	45585
17	45585	768	1868	418	143	1168	1743	1468	468	1118	774	1874	424	149	1174	1749	1474	474	1124	755	1855	405	130	1155	1730	1455	455	1105	756	1856	406	131	1156	1731	1456	456	1106	762	1862	412	137	1162	1737	1462	462	1112	45585
18	45585	1793	118	1143	1093	1443	518	393	818	1843	1799	124	1149	1099	1449	524	399	824	1849	1780	105	1130	1080	1430	505	380	805	1830	1781	106	1131	1081	1431	506	381	806	1831	1787	112	1137	1087	1437	512	387	812	1837	45585
19	45585	185	1210	1635	1510	585	935	885	1910	235	186	1211	1636	1511	586	936	886	1911	236	192	1217	1642	1517	592	942	892	1917	242	198	1223	1648	1523	598	948	898	1923	248	179	1204	1629	1504	579	929	879	1904	229	45585
20	45585	910	1560	560	285	860	1885	1610	160	1260	911	1561	561	286	861	1886	1611	161	1261	917	1567	567	292	867	1892	1617	167	1267	923	1573	573	298	873	1898	1623	173	1273	904	1554	554	279	854	1879	1604	154	1254	45585
21	45585	1935	260	835	1235	1585	210	535	960	1535	1936	261	836	1236	1586	211	536	961	1536	1942	267	842	1242	1592	217	542	967	1542	1948	273	848	1248	1598	223	548	973	1548	1929	254	829	1229	1579	204	529	954	1529	45585
22	45585	335	685	2010	1660	60	1310	1035	1385	610	336	686	2011	1661	61	1311	1036	1386	611	342	692	2017	1667	67	1317	1042	1392	617	348	698	2023	1673	73	1323	1048	1398	623	329	679	2004	1654	54	1304	1029	1379	604	45585
23	45585	1285	1710	35	660	1010	1360	1985	310	735	1286	1711	36	661	1011	1361	1986	311	736	1292	1717	42	667	1017	1367	1992	317	742	1298	1723	48	673	1023	1373	1998	323	748	1279	1704	29	654	1004	1354	1979	304	729	45585
24	45585	1410	635	985	710	1960	360	10	1335	1685	1411	636	986	711	1961	361	11	1336	1686	1417	642	992	717	1967	367	17	1342	1692	1423	648	998	723	1973	373	23	1348	1698	1404	629	979	704	1954	354	4	1329	1679	45585
25	45585	485	1060	1485	1810	435	785	1185	1760	85	486	1061	1486	1811	436	786	1186	1761	86	492	1067	1492	1817	442	792	1192	1767	92	498	1073	1498	1823	448	798	1198	1773	98	479	1054	1479	1804	429	779	1179	1754	79	45585
26	45585	760	1860	410	135	1160	1735	1460	460	1110	761	1861	411	136	1161	1736	1461	461	1111	767	1867	417	142	1167	1742	1467	467	1117	773	1873	423	148	1173	1748	1473	473	1123	754	1854	404	129	1154	1729	1454	454	1104	45585
27	45585	1785	110	1135	1085	1435	510	385	810	1835	1786	111	1136	1086	1436	511	386	811	1836	1792	117	1142	1092	1442	517	392	817	1842	1798	123	1148	1098	1448	523	398	823	1848	1779	104	1129	1079	1429	504	379	804	1829	45585
28	45585	197	1222	1647	1522	597	947	897	1922	247	178	1203	1628	1503	578	928	878	1903	228	184	1209	1634	1509	584	934	884	1909	234	190	1215	1640	1515	590	940	890	1915	240	191	1216	1641	1516	591	941	891	1916	241	45585
29	45585	922	1572	572	297	872	1897	1622	172	1272	903	1553	553	278	853	1878	1603	153	1253	909	1559	559	284	859	1884	1609	159	1259	915	1565	565	290	865	1890	1615	165	1265	916	1566	566	291	866	1891	1616	166	1266	45585
30	45585	1947	272	847	1247	1597	222	547	972	1547	1928	253	828	1228	1578	203	528	953	1528	1934	259	834	1234	1584	209	534	959	1534	1940	265	840	1240	1590	215	540	965	1540	1941	266	841	1241	1591	216	541	966	1541	45585
31	45585	347	697	2022	1672	72	1322	1047	1397	622	328	678	2003	1653	53	1303	1028	1378	603	334	684	2009	1659	59	1309	1034	1384	609	340	690	2015	1665	65	1315	1040	1390	615	341	691	2016	1666	66	1316	1041	1391	616	45585
32	45585	1297	1722	47	672	1022	1372	1997	322	747	1278	1703	28	653	1003	1353	1978	303	728	1284	1709	34	659	1009	1359	1984	309	734	1290	1715	40	665	1015	1365	1990	315	740	1291	1716	41	666	1016	1366	1991	316	741	45585
33	45585	1422	647	997	722	1972	372	22	1347	1697	1403	628	978	703	1953	353	3	1328	1678	1409	634	984	709	1959	359	9	1334	1684	1415	640	990	715	1965	365	15	1340	1690	1416	641	991	716	1966	366	16	1341	1691	45585
34	45585	497	1072	1497	1822	447	797	1197	1772	97	478	1053	1478	1803	428	778	1178	1753	78	484	1059	1484	1809	434	784	1184	1759	84	490	1065	1490	1815	440	790	1190	1765	90	491	1066	1491	1816	441	791	1191	1766	91	45585
35	45585	772	1872	422	147	1172	1747	1472	472	1122	753	1853	403	128	1153	1728	1453	453	1103	759	1859	409	134	1159	1734	1459	459	1109	765	1865	415	140	1165	1740	1465	465	1115	766	1866	416	141	1166	1741	1466	466	1116	45585
36	45585	1797	122	1147	1097	1447	522	397	822	1847	1778	103	1128	1078	1428	503	378	803	1828	1784	109	1134	1084	1434	509	384	809	1834	1790	115	1140	1090	1440	515	390	815	1840	1791	116	1141	1091	1441	516	391	816	1841	45585
37	45585	189	1214	1639	1514	589	939	889	1914	239	195	1220	1645	1520	595	945	895	1920	245	196	1221	1646	1521	596	946	896	1921	246	177	1202	1627	1502	577	927	877	1902	227	183	1208	1633	1508	583	933	883	1908	233	45585
38	45585	914	1564	564	289	864	1889	1614	164	1264	920	1570	570	295	870	1895	1620	170	1270	921	1571	571	296	871	1896	1621	171	1271	902	1552	552	277	852	1877	1602	152	1252	908	1558	558	283	858	1883	1608	158	1258	45585
39	45585	1939	264	839	1239	1589	214	539	964	1539	1945	270	845	1245	1595	220	545	970	1545	1946	271	846	1246	1596	221	546	971	1546	1927	252	827	1227	1577	202	527	952	1527	1933	258	833	1233	1583	208	533	958	1533	45585
40	45585	339	689	2014	1664	64	1314	1039	1389	614	345	695	2020	1670	70	1320	1045	1395	620	346	696	2021	1671	71	1321	1046	1396	621	327	677	2002	1652	52	1302	1027	1377	602	333	683	2008	1658	58	1308	1033	1383	608	45585
41	45585	1289	1714	39	664	1014	1364	1989	314	739	1295	1720	45	670	1020	1370	1995	320	745	1296	1721	46	671	1021	1371	1996	321	746	1277	1702	27	652	1002	1352	1977	302	727	1283	1708	33	658	1008	1358	1983	308	733	45585
42	45585	1414	639	989	714	1964	364	14	1339	1689	1420	645	995	720	1970	370	20	1345	1695	1421	646	996	721	1971	371	21	1346	1696	1402	627	977	702	1952	352	2	1327	1677	1408	633	983	708	1958	358	8	1333	1683	45585
43	45585	489	1064	1489	1814	439	789	1189	1764	89	495	1070	1495	1820	445	795	1195	1770	95	496	1071	1496	1821	446	796	1196	1771	96	477	1052	1477	1802	427	777	1177	1752	77	483	1058	1483	1808	433	783	1183	1758	83	45585
44	45585	764	1864	414	139	1164	1739	1464	464	1114	770	1870	420	145	1170	1745	1470	470	1120	771	1871	421	146	1171	1746	1471	471	1121	752	1852	402	127	1152	1727	1452	452	1102	758	1858	408	133	1158	1733	1458	458	1108	45585
45		1789	114	1139	1089	1439	514	389	814	1839	1795	120	1145	1095	1445	520	395	820	1845	1796	121	1146	1096	1446	521	396	821	1846	1777	102	1127	1077	1427	502	377	802	1827	1783	108	1133	1083	1433	508	383	808	1833	45585
		45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585

6.3.2 Equal Magic Sums Blocks of Order 9

In the previous subsection, we constructed **pan diagonal** magic square of order 45 by sub-blocks of **pan diagonal** magic squares of order 9 with different magic sums. This subsection brings construction of **pan diagonal** magic square of order 45 by sub-blocks of magic squares of order 9 with equal magic sums. The construction is based on a magic rectangle of order 5×9 given in Example 6.4. For simplicity, lets rewrite it as 9×5

Example 6.7. *The magic rectangle of order* 9×5 *is given by*

	1	2	3	4	5	Total
1	20	17	35	42	1	115
2	43	22	33	2	15	115
3	19	18	5	36	37	115
4	21	38	16	6	34	115
5	7	14	23	32	39	115
6	12	40	30	8	25	115
7	9	10	41	28	27	115
8	31	44	13	24	3	115
9	45	4	11	29	26	115
Total	207	207	207	207	207	

Distribution 6.5. *Let's consider following distribution of order* 5×5 *in composite form:*

11	21	31	41	51
12	22	32	42	52
13	23	33	43	53
14	24	34	44	54
15	25	35	45	55

Using the columns of magic rectangle 6.7 over the Example 6.1, we shall construct 25 blocks and them put them according to Distribution 6.5 to get a **pan diagonal** magic square of order 45. We observe that even though the Example 6.1 is pan diagonal, but new construction of magic squares of order 9 is not pan diagonal. The operation applied is $M_9 := 45 \times (A-1) + B$, where A and B are as given in Example 6.1. See below some examples,

- **Block 21**

(2)									207
17	40	44	10	18	14	38	4	22	207
14	10	18	22	38	4	44	17	40	207
4	22	38	40	44	17	18	14	10	207
22	38	4	44	17	40	14	10	18	207
40	44	17	18	14	10	4	22	38	207
10	18	14	38	4	22	17	40	44	207
18	14	10	4	22	38	40	44	17	207
38	4	22	17	40	44	10	18	14	207
44	17	40	14	10	18	22	38	4	207
207	207	207	207	207	207	207	207	207	207

(1)									207
31	21	19	9	12	43	45	7	20	207
20	45	7	19	31	21	43	9	12	207
12	43	9	7	20	45	21	19	31	207
7	20	45	21	19	31	12	43	9	207
9	12	43	45	7	20	31	21	19	207
19	31	21	43	9	12	20	45	7	207
43	9	12	20	45	7	19	31	21	207
21	19	31	12	43	9	7	20	45	207
45	7	20	31	21	19	9	12	43	207
207	207	207	207	207	207	207	207	207	207

(21)									9177
751	1776	1954	414	777	628	1710	142	965	9177
605	450	772	964	1696	156	1978	729	1767	9177
147	988	1674	1762	1955	765	786	604	436	9177
952	1685	180	1956	739	1786	597	448	774	9177
1764	1947	763	810	592	425	166	966	1684	9177
424	796	606	1708	144	957	740	1800	1942	9177
808	594	417	155	990	1672	1774	1966	741	9177
1686	154	976	732	1798	1944	412	785	630	9177
1980	727	1775	616	426	784	954	1677	178	9177
9177	9177	9177	9177	9177	9177	9177	9177	9177	9177

- **Block 43**

(4)									207
42	8	24	28	36	32	6	29	2	207
32	28	36	2	6	29	24	42	8	207
29	2	6	8	24	42	36	32	28	207
2	6	29	24	42	8	32	28	36	207
8	24	42	36	32	28	29	2	6	207
28	36	32	6	29	2	42	8	24	207
36	32	28	29	2	6	8	24	42	207
6	29	2	42	8	24	28	36	32	207
24	42	8	32	28	36	2	6	29	207
207	207	207	207	207	207	207	207	207	207

(3)									207
13	16	5	41	30	33	11	23	35	207
35	11	23	5	13	16	33	41	30	207
30	33	41	23	35	11	16	5	13	207
23	35	11	16	5	13	30	33	41	207
41	30	33	11	23	35	13	16	5	207
5	13	16	33	41	30	35	11	23	207
33	41	30	35	11	23	5	13	16	207
16	5	13	30	33	41	23	35	11	207
11	23	35	13	16	5	41	30	33	207
207	207	207	207	207	207	207	207	207	207

43									9177
1858	331	1040	1256	1605	1428	236	1283	80	9177
1430	1226	1598	50	238	1276	1068	1886	345	9177
1290	78	266	338	1070	1856	1591	1400	1228	9177
68	260	1271	1051	1850	328	1425	1248	1616	9177
356	1065	1878	1586	1418	1250	1273	61	230	9177
1220	1588	1411	258	1301	75	1880	326	1058	9177
1608	1436	1245	1295	56	248	320	1048	1861	9177
241	1265	58	1875	348	1076	1238	1610	1406	9177
1046	1868	350	1408	1231	1580	86	255	1293	9177
9177	9177	9177	9177	9177	9177	9177	9177	9177	9177

- **Block 52**

5									207
1	25	3	27	37	39	34	26	15	207
39	27	37	15	34	26	3	1	25	207
26	15	34	25	3	1	37	39	27	207
15	34	26	3	1	25	39	27	37	207
25	3	1	37	39	27	26	15	34	207
27	37	39	34	26	15	1	25	3	207
37	39	27	26	15	34	25	3	1	207
34	26	15	1	25	3	27	37	39	207
3	1	25	39	27	37	15	34	26	207
207	207	207	207	207	207	207	207	207	207

2									207
44	38	18	10	40	22	4	14	17	207
17	4	14	18	44	38	22	10	40	207
40	22	10	14	17	4	38	18	44	207
14	17	4	38	18	44	40	22	10	207
10	40	22	4	14	17	44	38	18	207
18	44	38	22	10	40	17	4	14	207
22	10	40	17	4	14	18	44	38	207
38	18	44	40	22	10	14	17	4	207
4	14	17	44	38	18	10	40	22	207
207	207	207	207	207	207	207	207	207	207

52									9177
44	1118	108	1180	1660	1732	1489	1139	647	9177
1727	1174	1634	648	1529	1163	112	10	1120	9177
1165	652	1495	1094	107	4	1658	1728	1214	9177
644	1502	1129	128	18	1124	1750	1192	1630	9177
1090	130	22	1624	1724	1187	1169	668	1503	9177
1188	1664	1748	1507	1135	670	17	1084	104	9177
1642	1720	1210	1142	634	1499	1098	134	38	9177
1523	1143	674	40	1102	100	1184	1637	1714	9177
94	14	1097	1754	1208	1638	640	1525	1147	9177
9177	9177	9177	9177	9177	9177	9177	9177	9177	9177

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In the similar way, we construct other 22 blocks of order 9. Putting all the 25 blocks of order 9 in the Distribution 6.5 we get a **pan diagonal** magic square of order 45 given in exemple below.

Example 6.8.. *The **block-wise pan diagonal** magic square of order 45 with equal magic sums block of order 9 is given by*

Pan diagonal magic square of order 45. All block of order 9 are magic squares with equal magic sums. Magic square sums are S(45x45):=45585 and S(9x9):=9117.

		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	
	pan	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585
1	45585	886	516	1369	369	822	313	945	1987	1910	751	1776	1954	414	777	628	1710	142	965	1561	1326	559	1809	192	1033	720	457	1460	1876	336	1054	1224	1587	1438	270	1267	65	31	1101	109	1179	1632	1753	1530	1132	650	45585
2	45585	290	405	817	1909	931	2001	1393	864	507	605	450	772	964	1696	156	1978	729	1767	1010	1845	187	1459	706	471	583	1539	1317	1415	1260	1582	64	256	1281	1078	1854	327	1730	1215	1627	649	1516	1146	133	9	1092	45585
3	45585	1992	1933	909	502	1370	900	831	289	391	147	988	1674	1762	1955	765	786	604	436	462	1483	684	1312	560	1575	201	1009	1831	1272	88	234	322	1055	1890	1596	1414	1246	1137	673	1494	1087	110	45	1641	1729	1201	45585
4	45585	1897	920	2025	1371	874	526	282	403	819	952	1685	180	1956	739	1786	597	448	774	1447	695	495	561	1549	1336	1002	1843	189	52	245	1305	1056	1864	346	1407	1258	1584	637	1505	1170	111	19	1111	1722	1213	1629	45585
5	45585	504	1362	898	855	277	380	2011	1911	919	1764	1947	763	810	592	425	166	966	1684	1314	552	1573	225	997	1820	481	1461	694	324	1047	1888	1620	1402	1235	1291	66	244	1089	102	43	1665	1717	1190	1156	651	1504	45585
6	45585	379	841	291	943	1989	1902	875	540	1357	424	796	606	1708	144	957	740	1800	1942	1819	211	1011	718	459	1452	1550	1350	547	1234	1606	1416	268	1269	57	1865	360	1042	1189	1651	1731	1528	1134	642	20	1125	97	45585
7	45585	853	279	372	2000	1935	907	514	1381	876	808	594	417	155	990	1672	1774	1966	741	223	999	1812	470	1485	682	1324	571	1551	1618	1404	1227	1280	90	232	334	1066	1866	1663	1719	1182	1145	675	1492	1099	121	21	45585
8	45585	921	1999	1921	867	538	1359	367	830	315	1686	154	976	732	1798	1944	412	785	630	696	469	1471	1542	1348	549	1807	200	1035	246	1279	76	1857	358	1044	1222	1595	1440	1506	1144	661	12	1123	99	1177	1640	1755	45585
9	45585	1395	862	515	301	381	829	1899	912	2023	1980	727	1775	616	426	784	954	1677	178	585	1537	1325	1021	1821	199	1449	687	493	1080	1852	335	1426	1236	1594	54	237	1303	135	7	1100	1741	1191	1639	639	1497	1168	45585
10	45585	899	533	1368	370	850	292	904	1994	1907	764	1793	1953	415	805	607	1669	149	962	1574	1343	558	1810	220	1012	679	464	1457	1889	353	1053	1225	1615	1417	229	1274	62	44	1118	108	1180	1660	1732	1489	1139	647	45585
11	45585	287	364	824	1908	944	2018	1372	865	535	602	409	779	963	1709	173	1957	730	1795	1007	1804	194	1458	719	488	562	1540	1345	1412	1219	1589	63	269	1298	1057	1855	355	1727	1174	1634	648	1529	1163	112	10	1120	45585
12	45585	2020	1912	910	509	1367	859	848	288	404	175	967	1675	1769	1952	724	803	603	449	490	1462	685	1319	557	1534	218	1008	1844	1300	67	235	329	1052	1849	1613	1413	1259	1165	652	1495	1094	107	4	1658	1728	1214	45585
13	45585	1904	917	1984	1388	873	539	310	382	820	959	1682	139	1973	738	1799	625	427	775	1454	692	454	578	1548	1349	1030	1822	190	59	242	1264	1073	1863	359	1435	1237	1585	644	1502	1129	128	18	1124	1750	1192	1630	45585
14	45585	505	1390	877	814	284	377	2024	1928	918	1765	1975	742	769	599	422	179	983	1683	1315	580	1552	184	1004	1817	494	1478	693	325	1075	1867	1579	1409	1232	1304	83	243	1090	130	22	1624	1724	1187	1169	668	1503	45585
15	45585	378	854	308	922	1990	1930	872	499	1364	423	809	623	1687	145	985	737	1759	1949	1818	224	1028	697	460	1480	1547	1309	554	1233	1619	1433	247	1270	85	1862	319	1049	1188	1664	1748	1507	1135	670	17	1084	104	45585
16	45585	832	280	400	1997	1894	914	513	1394	893	787	595	445	152	949	1679	1773	1979	758	202	1000	1840	467	1444	689	1323	584	1568	1597	1405	1255	1277	49	239	333	1079	1883	1642	1720	1210	1142	634	1499	1098	134	38	45585
17	45585	938	1998	1934	895	517	1360	374	827	274	1703	153	989	760	1777	1945	419	782	589	713	468	1484	1570	1327	550	1814	197	994	263	1278	89	1885	337	1045	1229	1592	1399	1523	1143	674	40	1102	100	1184	1637	1714	45585
18	45585	1354	869	512	314	398	828	1900	940	2002	1939	734	1772	629	443	783	955	1705	157	544	1544	1322	1034	1838	198	1450	715	472	1039	1859	332	1439	1253	1593	55	265	1282	94	14	1097	1754	1208	1638	640	1525	1147	45585
19	45585	868	511	1355	401	840	303	911	2003	1925	733	1771	1940	446	795	618	1676	158	980	1543	1321	545	1841	210	1023	686	473	1475	1858	331	1040	1256	1605	1428	236	1283	80	13	1096	95	1211	1650	1743	1496	1148	665	45585
20	45585	305	371	833	1895	913	1996	1383	896	525	620	416	788	950	1678	151	1968	761	1785	1025	1811	203	1445	688	466	573	1571	1335	1430	1226	1598	50	238	1276	1068	1886	345	1745	1181	1643	635	1498	1141	123	41	1110	45585
21	45585	2010	1923	941	518	1385	866	826	275	373	165	978	1706	1778	1970	731	781	590	418	480	1473	716	1328	575	1541	196	995	1813	1290	78	266	338	1070	1856	1591	1400	1228	1155	663	1526	1103	125	11	1636	1715	1183	45585
22	45585	1913	935	1991	1366	860	508	300	393	851	968	1700	146	1951	725	1768	615	438	806	1463	710	461	556	1535	1318	1020	1833	221	68	260	1271	1051	1850	328	1425	1248	1616	653	1520	1136	106	5	1093	1740	1203	1661	45585
23	45585	536	1380	888	821	293	395	1993	1906	905	1796	1965	753	776	608	440	148	961	1670	1346	570	1563	191	1013	1835	463	1456	680	356	1065	1878	1586	1418	1250	1273	61	230	1121	120	33	1631	1733	1205	1138	646	1490	45585
24	45585	365	823	286	933	2021	1920	890	506	1373	410	778	601	1698	176	975	755	1766	1958	1805	193	1006	708	491	1470	1565	1316	563	1220	1588	1411	258	1301	75	1880	326	1058	1175	1633	1726	1518	1166	660	35	1091	113	45585
25	45585	843	311	390	2015	1901	923	500	1363	871	798	626	435	170	956	1688	1760	1948	736	213	1031	1830	485	1451	698	1310	553	1546	1608	1436	1245	1295	56	248	320	1048	1861	1653	1751	1200	1160	641	1508	1085	103	16	45585
26	45585	916	1985	1903	885	528	1391	383	845	281	1681	140	958	750	1788	1976	428	800	596	691	455	1453	1560	1338	581	1823	215	1001	241	1265	58	1875	348	1076	1238	1610	1406	1501	1130	643	30	1113	131	1193	1655	1721	45585
27	45585	1361	878	530	283	376	815	1931	930	2013	1946	743	1790	598	421	770	986	1695	168	551	1553	1340	1003	1816	185	1481	705	483	1046	1868	350	1408	1231	1580	86	255	1293	101	23	1115	1723	1186	1625	671	1515	1158	45585
28	45585	879	501	1386	388	818	272	929	2012	1932	744	1761	1971	433	773	587	1694	167	987	1554	1311	576	1828	188	992	704	482	1482	1869	321	1071	1243	1583	1397	254	1292	87	24	1086	126	1198	1628	1712	1514	1157	672	45585
29	45585	312	389	842	1926	924	1986	1352	883	503	627	434	797	981	1689	141	1937	748	1763	1032	1829	212	1476	699	456	542	1558	1313	1437	1244	1607	81	249	1266	1037	1873	323	1752	1199	1652	666	1509	1131	92	28	1088	45585
30	45585	1988	1892	928	527	1392	884	816	306	384	143	947	1693	1787	1977	749	771	621	429	458	1442	703	1337	582	1559	186	1026	1824	1268	47	253	347	1077	1874	1581	1431	1239	1133	632	1513	1112	132	29	1626	1746	1194	45585
31	45585	1922	942	2009	1356	891	519	278	362	838	977	1707	164	1941	756	1779	593	407	793	1472	717	479	546	1566	1329	998	1802	208	77	267	1289	1041	1881	339	1403	1217	1603	662	1527	1154	96	36	1104	1718	1172	1648	45585
32	45585	523	1358	857	839	302	402	2004	1896	936	1783	1943	722	794	617	447	159	951	1701	1333	548	1532	209	1022	1842	474	1446	711	343	1043	1847	1604	1427	1257	1284	51	261	1108	98	2	1649	1742	1212	1149	636	1521	45585
33	45585	396	834	276	902	2008	1898	897	524	1382	441	789	591	1667	163	953	762	1784	1967	1836	204	996	677	478	1448	1572	1334	572	1251	1599	1401	227	1288	53	1887	344	1067	1206	1644	1716	1487	1153	638	42	1109	122	45585
34	45585	812	298	368	2022	1919	932	531	1374	861	767	613	413	177	974	1697	1791	1959	726	182	1018	1808	492	1469	707	1341	564	1536	1577	1423	1223	1302	74	257	351	1059	1851	1622	1738	1178	1167	659	1517	1116	114	6	45585
35	45585	906	2016	1914	863	497	1378	392	852	299	1671	171	969	728	1757	1963	437	807	614	681	486	1464	1538	1307	568	1832	222	1019	231	1296	69	1853	317	1063	1247	1617	1424	1491	1161	654	8	1082	118	1202	1662	1739	45585
36	45585	1379	887	537	294	366	846	1918	908	1982	1964	752	1797	609	411	801	973	1673	137	569	1562	1347	1014	1806	216	1468	683	452	1064	1877	357	1419	1221	1611	73	233	1262	119	32	1122	1734	1176	1656	658	1493	1127	45585
37	45585	858	529	1387	387	835	285	926	2019	1891	723	1789	1972	432	790	600	1691	174	946	1533	1339	577	1827	205	1005	701	489	1441	1848	349	1072	1242	1600	1410	251	1299	46	3	1114	127	1197	1645	1725	1511	1164	631	45585
38	45585	271	386	849	1927	903	2014	1365	882	520	586	431	804	982	1668	169	1950	747	1780	991	1826	219	1477	678	484	555	1557	1330	1396	1241	1614	82	228	1294	1050	1872	340	1711	1196	1659	667	1488	1159	105	27	1105	45585
39	45585	2005	1905	927	534	1351	881	844	307	363	160	960	1692	1794	1936	746	799	622	408	475	1455	702	1344	541	1556	214	1027	1803	1285	60	252	354	1036	1871	1609	1432	1218	1150	645	1512	1119	91	26	1654	1747	1173	45585
40	45585	1929	901	2006	1384	892	498	295	375	837	984	1666	161	1969	757	1758	610	420	792	1479	676	476	574	1567	1308	1015	1815	207	84	226	1286	1069	1882	318	1420	1230	1602	669	1486	1151	124	37	1083	1735	1185	1647	45585
41	45585	522	1375	870	836	309	361	1983	1924	937	1782	1960	735	791	624	406	138	979	1702	1332	565	1545	206	1029	1801	453	1474	712	342	1060	1860	1601	1434	1216	1263	79	262	1107	115	15	1646	1749	1171	1128	664	1522	45585
42	45585	397	813	304	915	2007	1915	856	521	1389	442	768	619	1680	162	970	721	1781	1974	1837	183	1024	690	477	1465	1531	1331	579	1252	1578	1429	240	1287	70	1846	341	1074	1207	1623	1744	1500	1152	655	1	1106	129	45585
43	45585	825	297	385	1981	1916	939	532	1353	889	780	612	430	136	971	1704	1792	1938	754	195	1017	1825	451	1466	714	1342	543	1564	1590	1422	1240	1261	71	264	352	1038	1879	1635	1737	1195	1126	656	1524	1117	93	34	45585
44	45585	934	2017	1893	880	510	1377	399	811	296	1699	172	948	745	1770	1962	444	766	611	709	487	1443	1555	1320	567	1839	181	1016	259	1297	48	1870	330	1062	1254	1576	1421	1519	1162	633	25	1095	117	1209	1621	1736	45585
45		1376	894	496	273	394	847	1917	925	1995	1961	759	1756	588	439	802	972	1690	150	566	1569	1306	993	1834	217	1467	700	465	1061	1884	316	1398	1249	1612	72	250	1275	116	39	1081	1713	1204	1657	657	1510	1140	45585
		45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585	45585

7 Final Comments

In the previous works [23, 24, 25, 26, 28, 31], the author worked with **block-wise** constructions of magic squares of orders 8 to 36. In this work we brought **block-wise** constructions of magic squares for the orders 39 to 42. In each case, all the possibilities are considered. It depends on the magic sums division, where we shall have equal magic sums blocks or different magic sums blocks. Let's see how it works? The magic sum of order n of consecutive numbers from 1 to n^2 is given by

$$S_{n\times n} := \frac{n\times(1+n^2)}{2}, n\geq 3.$$

Let's analyse each case separately.

- **Magic Square of order 39**

The magic sum of order 39 is given by

$$S_{39\times 39} := \frac{39\times(1+39^2)}{2} = 29679.$$

This sums is divisible by 3 and 13. See below:

$$\begin{array}{llll} (i) & \frac{29679}{13} = 2293 & \Longrightarrow & \text{equal blocks of order 3;} \\ (ii) & \frac{29679}{3} = 9893 & \Longrightarrow & \text{equal blocks of order 13.} \end{array}$$

- **Magic Square of order 40**

The magic sum of order 40 is given by

$$S_{40\times 40} := \frac{40\times(1+40^2)}{2} = 32020.$$

This sums is divisible by 10, 5 and 4, but not by 8. See below:

$$\begin{array}{llll} (i) & \frac{32020}{10} = 3202 & \Longrightarrow & \text{equal blocks of order 4;} \\ (ii) & \frac{32020}{8} = 4002.5 & \Longrightarrow & \text{unequal blocks of order 5;} \\ (iii) & \frac{32020}{5} = 6404 & \Longrightarrow & \text{equal blocks of order 8;} \\ (iv) & \frac{32020}{4} = 8005 & \Longrightarrow & \text{equal blocks of order 10.} \end{array}$$

- **Magic Square of order 42**

The magic sum of order 42 is given by

$$S_{42\times 42} := \frac{42\times(1+42^2)}{2} = 37065.$$

This sum is divisible by 7 and 14, but not by 3 and 6. See below:

$$\begin{array}{llll} (i) & \frac{37065}{14} = 2647.5 & \Longrightarrow & \text{unequal blocks of order 3;} \\ (ii) & \frac{37065}{7} = 5295 & \Longrightarrow & \text{equal blocks of order 6;} \\ (iii) & \frac{37065}{6} = 6176.5 & \Longrightarrow & \text{unequal blocks of order 7;} \\ (iv) & \frac{37065}{3} = 12355 & \Longrightarrow & \text{equal blocks of order 14.} \end{array}$$

- **Magic Square of order 44**

The magic sum of order 44 is given by

$$S_{44\times 44} := \frac{44 \times (1+44^2)}{2} = 42614.$$

This sums is divisible by 11 and 2, but not by 4. See below:

$$\begin{array}{llll} (i) & \frac{42614}{11} = 3874 & \Longrightarrow & \text{equal blocks of order 4;} \\ (ii) & \frac{42614}{4} = 10653.5 & \Longrightarrow & \text{unequal blocks of order 11;} \\ (iii) & \frac{42614}{2} = 21307 & \Longrightarrow & \text{equal blocks of order 22.} \end{array}$$

- **Magic Square of order 45**

The magic sum of order 45 is given by

$$S_{45\times 45} := \frac{45 \times (1+45^2)}{2} = 45585.$$

Let's see the divisions of 45585 by 15, 9, 5 and 3. See below:

$$\begin{array}{llll} (i) & \frac{45585}{15} = 3039 & \Longrightarrow & \text{equal blocks of order 3;} \\ (ii) & \frac{45585}{9} = 5065 & \Longrightarrow & \text{equal blocks of order 5;} \\ (iii) & \frac{45585}{5} = 9117 & \Longrightarrow & \text{equal blocks of order 9;} \\ (iv) & \frac{45585}{3} = 15195 & \Longrightarrow & \text{equal blocks of order 15.} \end{array}$$

This philosophy is applied to all possible blocks of magic squares from orders 8 to 45. Especially, in this paper for the magic squares of orders 39 to 45. That is, whenever possible, we tried to bring blocks of equal sums magic squares. In some cases, they are **semi-magic** or **pan diagonal**. The **semi-magic** happens in case of blocks of order 3. In case of **bimagic** square of order 40, some sub-blocks are **semi-bimagics**. The **pan diagonal** doesn't happens for for sub-blocks of orders 6, 10, 14 and 22. During last few years author worked on magic squares in different sitations. Below are the details:

7.1 Author's Contributions to Magic Squares

The item-wise author's work on magic squares is as follows:

(i) **Digital Numbers** Magic Squares - [8, 9, 10, 11, 12, 13];
(ii) **Block-Wise Construction** of **Bimagic Squares** - [14];
(iii) Connections with **Genetic Tables** and **Shannon's entropy** - [15];
(iv) **Selfie** and **Palindromic-type** Magic Squares - [16, 31];
(v) **Intervally Distributed** and **Block-Wise** Magic Squares - [17, 18, 19, 31];
(vi) **Multi-digits** and **Number Patterns** Magic Squares - [20, 30];
(vii) **Perfect Square Sum** Magic Squares with **Uniformity**, **Minimum Sum** and **Pythagorean Triples** - [21, 22];
(viii) **Block-Wise** Constructions of Magic and Bimagic Squares - [23, 24, 25, 26, 29];
(ix) **Magic Crosses:** Repeated and Non Repeated Entries - [27];
(x) Representations of **Letters** and **Numbers** With Equal Sums Magic Squares of Orders 4 and 6 - [28];

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References

[1] **Aale de Winkel**, The Magic Encyclopedia, *http://magichypercubes.com/Encyclopedia/*

[2] **C.A. Pickover**, The Zen of Magic Squares, Circles, and Stars, Princeton University Press, 2002.

[3] **H. White**, Magic Squares - *http://budshaw.ca/SODLS.html*

[4] **G. Pfeffermann**, Les Tablettes du Chercheur, Journal des Jeux d'Esprit et de Combinaisons, (fortnightly magazine) issues of 1891 Paris.

[5] **W.D. Wallis and L. Zhu**, Four Orthogonal Diagonal Latin Squares of Order 12, Utilitas Math., 21 C (1982), 205-207.

[6] **L. Zhu**, Orthogonal Diagonal Latin Squares of Order Fourteen, J. Austral. Math. Soc. (Series A), 36(1984), 1-3.

[7] **W.D. Wallis**, Completion of the Spectrum of Orthogonal Diagonal Latin Squares, In Graphs Matrices, and Designs, pp 43-49, Marcel Dekker, New York, 1992 – Edited by J.W. Brown, F. Cherry, L. Most, M. Most, E.T. Parker and W.D. Wallis.

[8] **I.J. TANEJA**, Digital Era: Magic Squares and 8th May 2010 (08.05.2010), May, 2010, pp. 1-4, *https://arxiv.org/abs/1005.1384* - *http://bit.ly/2Q1jRgR*.

[9] **I.J. TANEJA**, Universal Bimagic Squares and the day 10th October 2010 (10.10.10), Oct, 2010, pp. 1-5, *https://arxiv.org/abs/1010.2083* - textithttp://bit.ly/2DnPZK3.

[10] **I.J. TANEJA**, DIGITAL ERA: Universal Bimagic Squares, Oct, 2010, pp. 1-8, *https://arxiv.org/abs/1010.2541*; *http://bit.ly/2OGRiF0*.

[11] **I.J. TANEJA**, Upside Down Numerical Equation, Bimagic Squares, and the day September 11, Oct. 2010, pp. 1-7, *https://arxiv.org/abs/1010.4186*; *http://bit.ly/2O2V*.

[12] **I.J. TANEJA**, Equivalent Versions of "Khajuraho" and "Lo-Shu" Magic Squares and the day 1st October 2010 (01.10.2010), Nov. 2010, pp. 1-7, *https://arxiv.org/abs/1011.0451*; *http://bit.ly/2O6x9Ls*.

[13] **I.J. TANEJA**, Upside Down Magic, Bimagic, Palindromic Squares and Pythagoras Theorem on a Palindromic Day - 11.02.2011, Feb. 2011, pp.1-9, *https://arxiv.org/abs/1102.2394*; *http://bit.ly/2PXVtws*

[14] **I.J. TANEJA**, Bimagic Squares of Bimagic Squares and an Open Problem, Feb. 2011, pp. 1-14, *https://arxiv.org/abs/1102.3052*;*http://bit.ly/2xz0gxv*.

[15] **I.J. TANEJA**, Representations of Genetic Tables, Bimagic Squares, Hamming Distances and Shannon Entropy, Jun. 2012, pp. 1-19, *https://arxiv.org/abs/1206.2220*; *http://bit.ly/2zpgxXk*.

[16] **I.J. TANEJA**, Selfie Palindromic Magic Squares, RGMIA Research Report Collection, **18**(2015), Art. 98, pp. 1-15. *http://rgmia.org/papers/v18/v18a98.pdf*; *http://bit.ly/2QMCAOo*.

[17] **I.J. TANEJA**, Intervally Distributed, Palindromic, Selfie Magic Squares, and Double Colored Patterns, RGMIA Research Report Collection, **18**(2015), Art. 127, pp. 1-45. *http://rgmia.org/papers/v18/v18a127.pdf*; *http://bit.ly/2MVmNhV*.

[18] **I.J. TANEJA**, Intervally Distributed, Palindromic and Selfie Magic Squares: Genetic Table and Colored Pattern – Orders 11 to 20, RGMIA Research Report Collection, **18**(2015), Art. 140, pp. 1-43. *http://rgmia.org/papers/v18/v18a140.pdf -http://bit.ly/2OCadRn*.

[19] **I.J. TANEJA**, Intervally Distributed, Palindromic and Selfie Magic Squares – Orders 21 to 25 , **18**(2015), Art. 151, pp. 1-33. *http://rgmia.org/papers/v18/v18a151.pdf*; *http://bit.ly/2NnYyZM*.

[20] **I.J. TANEJA**, Multi-Digits Magic Squares, RGMIA Research Report Collection, **18**(2015), Art. 159, pp. 1-22. *http://rgmia.org/papers/v18/v18a159.pdf*; *http://bit.ly/2O31yKy*.

[21] **I.J. TANEJA**, Magic Squares with Perfect Square Number Sums, Research Report Collection, **20**(2017), Article 11, pp. 1-24, *http://rgmia.org/papers/v20/v20a11.pdf*; *http://bit.ly/2vJTjJi*.

[22] **I.J. TANEJA**, Pythagorean Triples and Perfect Square Sum Magic Squares, RGMIA Research Report Collection, **20**(2017), Art. 128, pp. 1-22, *http://rgmia.org/papers/v20/v20a128.pdf* - *http://bit.ly/2BeRitx*.

[23] **I.J. TANEJA**, Block-Wise Equal Sums Pandiagonal Magic Squares of Order $4k$, **Zenodo**, Open Access, January 31, 2019, pp. 1-17, http://doi.org/10.5281/zenodo.2554288 - *http://bit.ly/2RrKtaY*.

[24] **I.J. TANEJA**, Block-Wise Equal Sums Magic Squares of Orders $3k$ and $6k$, , **Zenodo**, Open Access, Febuary 01, 2019, pp. 1-55 http://doi.org/10.5281/zenodo.2554895; http://bit.ly/2MIm14o.

[25] **I.J. TANEJA**, Block-Wise Unequal Sums Magic Squares, **Zenodo**, Open Access, Febuary 01, 2019, pp. 1-55 http://doi.org/10.5281/zenodo.2555260; http://bit.ly/2GiQWmD.

[26] **I.J. TANEJA**, Magic Rectangles in Construction of Block-Wise Pandiagonal Magic Squares, **Zenodo**, Open Access, January 31, 2019, pp. 1-49 , http://doi.org/10.5281/zenodo.2554520; http://bit.ly/2GfSDB9.

[27] **I.J. TANEJA**, Magic Crosses: Repeated and Non Repeated Entries, **Zenodo**, Open Access, Open Access, Febuary 01, 2019, pp. 1-37, http://doi.org/10.5281/zenodo.2554623; http://bit.ly/2DLIlaD.

[28] **I.J. TANEJA**, Representations of Letters and Numbers With Equal Sums Magic Squares of Orders 4 and 6, **Zenodo**, Open Access, Febuary 01, 2019, pp. 1-82, http://doi.org/10.5281/zenodo.2555287; http://bit.ly/2CWJ72A.

[29] **I.J. TANEJA**, Block-Wise Magic and Bimagic Squares of Orders 12 to 36, **Zenodo**, Open Access, Febuary 01, 2019, pp. 1-53, http://doi.org/10.5281/zenodo.2555343; http://bit.ly/2WA6N6c.

[30] **I.J. TANEJA**, Different Digits Magic Squares and Number Patterns, **Zenodo**, Open Access, Febuary 01, 2019, pp. 1-34, http://doi.org/10.5281/zenodo.2555327; http://bit.ly/2Tr3ufr, pp. 1-34.

[31] **I.J. TANEJA**, Palindromic, Patterned Magic Sums, Composite, and Colored Patterns in Magic Squares, **Zenodo**, Open Access, Febuary 02, 2019, pp. 1-99, http://doi.org/10.5281/zenodo.2555741; http://bit.ly/2UybT0P.