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Different Digits Magic Squares and Number Patterns

Inder J. Taneja[1]

Abstract

*In this work, we shall brings **number patterns** based on magic square sums for the magic and **bimagic** squares constructed with different digits. In each case, as the digits increases in cells, the values of magic and **bimagic** squares sums also increases giving interesting **number patterns**. This is done for the magic squares of orders 5, 7, 8, 9 and 10.*

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[1]*Formerly, Professor of Mathematics, Universidade Federal de Santa Catarina, Florianópolis, SC, Brazil (1978-2012). Also worked at Delhi University, India (1976-1978).*
E-mail: *ijtaneja@gmail.com;*
Web-sites: *http://inderjtaneja.com;*
Twitter: *@IJTANEJA;*
Instagram: *@crazynumbers.*

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1 Introduction

Magic squares are generally constructed using sequential or concutive such as $1, 2, \ldots, n^2$. Here in this work, we shall write magic squares using non consecutive different digits numbers. This work we have done for the magic squares of orders 5, 7, 8, 9 and 10. As commonly, the entries are not in a sequential way, but each cell is with different digits. Also, all the entries in a magic square are with different numbers. The aim is to write in such a way that in each case as the number of digits increases, we get **number patterns with magic sums**, i.e, these sums are in symmetrical way. In case of orders 8 and 9 the magic squares considered are **pan diagonal**. In case of order 9, there are examples of **bimagic squares**, while in case of order 8 there are examples where there is **bimagic** only in rows sums. In both these cases there are interesting **number patterns** with **bimagic sums**. See below the **table of number patterns** for the different orders and digits that we worked in this paper:

Digits	Order 5	Order 7	Order 8	Order 9	Order 10
3	1665		3996	4995	
4	16665	31108	39996	49995	49995
5	166665	311008	399996	499995	499995
6	–	3110008	3999996	4999995	4999995
7	–	31100008	39999996	49999995	49999995
8	–	–	399999996	499999995	499999995
9	–	–	–	4999999995	4999999995
10	–	–	–	–	49999999995

Numbers Pattern Table: Numbers of Digits and Magic Sums

From the table we observe that still, we don't have magic squares with 3 digits in each cell for the magic square of orders 7 and 10. Magic squares of orders 4 and 6 are not dealt here, as in case of order 4, we have only two possibilities, i.e., for the digits 3 and 4, and for the magic squares of order 6, we have results for the digits 5 and 6, but for the case of 4 digits, it becomes **semi-magic square**. For details see author's previous work [13]. The **bimagic** sums pattern is for the , this we have obtained for the order 8 with **bimagic sums only in the rows**, and **bimagic sums** for the magic square order 9.

2 Pattern With Magic Sums of Order 5

This section deals with magic squares of order 5 written in different digits forming a number pattern with the magic squares sums for the digits 3, 4 and 5 different in each cell.

2.1 Different Digits Magic Squares of Order 5

Let's consider following composite magic square of order 5.

Example 2.1. *Let's consider a **pan magic square** of order 5 is given by*

		165	165	165	165	165
	11	22	33	44	55	165
165	43	54	15	21	32	165
165	25	31	42	53	14	165
165	52	13	24	35	41	165
165	34	45	51	12	23	165
	165	165	165	165	165	165

The above magic square is of 2-digits in each cell. It is not constructed with different digits, because in five places, we have 11, 22, 33, 44 and 55. The reason is that there are only 25 possibilities of using 1 to 5 in two digits forms, including repeated ones. When the number of digits increases, the possibilities also increases. Below are examples of magic square of order 5 with 3, 4 and 5 digits in each cell.

Example 2.2. *Let's consider a **3 digits cell** magic square of order 5.*

					1665
235	234	452	312	432	1665
431	531	143	345	215	1665
245	325	421	542	132	1665
541	254	214	124	532	1665
213	321	435	342	354	1665
1665	1665	1665	1665	1665	1665

Example 2.3. *Let's consider a **4 digits cell** magic square of order 5.*

					16665
2345	2354	4512	3142	4312	16665
4321	5321	1453	3425	2145	16665
2435	3215	4251	5412	1352	16665
5421	2534	2134	1234	5342	16665
2143	3241	4315	3452	3514	16665
16665	16665	16665	16665	16665	16665

Example 2.4. *Let's consider a **5 digits cell** magic square of order 5.*

					166665
12345	12354	34512	53142	54312	166665
54321	45321	21453	13425	32145	166665
12435	43215	34251	35412	41352	166665
35421	12534	52134	51234	15342	166665
52143	53241	24315	13452	23514	166665
166665	166665	166665	166665	166665	166665

The magic squares with digits 3 and 4 are obtained from the magic square of 7 digits. For details refer author's work [13].

Result 1. *The magic sums of magic squares of order 5 with 3, 4 and 5 digits in each cell lead us to a **number pattern** given by*

Number of Digits	*Magic Square Sums*
3	*1665*
4	*16665*
5	*166665*

Note 1. *We observe that there are total* $5! = 120$ *possibilities of writing 5 digits with different digits. Thus, there are much more possibilities of writing different digits magic squares with 5 different digits in each cell. We have written only one just to have an idea. Moreover, extending Example 2.1 to* ***palindromic numbers****, we observe that there are only 25* ***palindromes*** *written in terms of 3 digits. These digits forms a* ***pan diagonal*** *magic square of order 5 given by*

		1665	1665	1665	1665	1665
	111	222	333	444	555	1665
1665	434	545	151	212	323	1665
1665	252	313	424	535	141	1665
1665	525	131	242	353	414	1665
1665	343	454	515	121	232	1665
	1665	1665	1665	1665	1665	1665

Similar to Result 1, we can also write a ***number pattern*** *with different digits palindromic numbers. For more details on* ***palindromic-type*** *magic squares refer author's work [9, ?]*

3 Pattern With Magic Sums of Order 7

This section deals with magic squares of order 7 written in different digits forming a number pattern with the magic squares sums for the digits 4, 5, 6 and 7.

3.1 Different Digits Magic Squares of Order 7

Let's consider following composite pan diagonal magic square of order 7.

Example 3.1. *Let's consider a* ***pan diagonal*** *magic square of order 7 is given by*

		308	308	308	308	308	308	308
	11	22	33	44	55	66	77	308
308	65	76	17	21	32	43	54	308
308	42	53	64	75	16	27	31	308
308	26	37	41	52	63	74	15	308
308	73	14	25	36	47	51	62	308
308	57	61	72	13	24	35	46	308
308	34	45	56	67	71	12	23	308
	308	308	308	308	308	308	308	308

The above magic square is of 2-digits in each cell. It is not constructed with different digits, because in seven places, we have 11, 22, 33, 44, 55, 66 and 77. The reason is that there are only 49 possibilities of using 1 to 7 in two digits forms, including repeated ones. When the number of digits increases, the possibilities also increases. Below are examples of magic square of order 7 with different digits from 4 to 7 digits in each cell.

Example 3.2. *Let's consider a **4 digits cell** magic square of order 7.*

							31108
6271	4657	4526	2476	6152	2513	4513	31108
1567	3712	1275	7416	5214	5471	6453	31108
6317	7234	1752	5321	3562	2346	4576	31108
4612	3471	7264	5217	5246	3751	1547	31108
4213	3276	3651	4365	6725	5637	3241	31108
3561	4127	5126	4671	1742	4267	7614	31108
4567	4631	7514	1642	2467	7123	3164	31108
31108	31108	31108	31108	31108	31108	31108	31108

Example 3.3. *Let's consider a **5 digits cell** magic square of order 7.*

							311008
46271	34657	74526	12476	76152	42513	24513	311008
41567	53712	31275	27416	65214	65471	26453	311008
56317	17234	61752	45321	43562	52346	34576	311008
54612	53471	37264	65217	15246	23751	61547	311008
54213	43276	23651	74365	16725	45637	53241	311008
23561	54127	45126	54671	61742	14267	57614	311008
34567	54631	37514	31642	32467	67123	53164	311008
311008	311008	311008	311008	311008	311008	311008	311008

Example 3.4. *Let's consider a **6 digits cell** magic square of order 7.*

							3110008
546271	234657	374526	312476	376152	642513	624513	3110008
341567	653712	431275	327416	365214	265471	726453	3110008
256317	517234	461752	745321	143562	752346	234576	3110008
354612	653471	137264	465217	715246	423751	361547	3110008
654213	143276	723651	274365	416725	245637	653241	3110008
723561	654127	345126	254671	561742	314267	257614	3110008
234567	254631	637514	731642	532467	467123	253164	3110008
3110008	3110008	3110008	3110008	3110008	3110008	3110008	3110008

Example 3.5. *Let's consider a **7 digits cell** magic square of order 7.*

							31100008
3546271	1234657	1374526	5312476	4376152	7642513	7624513	31100008
2341567	4653712	6431275	5327416	7365214	3265471	1726453	31100008
4256317	6517234	3461752	6745321	7143562	1752346	1234576	31100008
7354612	2653471	5137264	3465217	3715246	6423751	2361547	31100008
7654213	5143276	4723651	1274365	3416725	1245637	7653241	31100008
4723561	3654127	7345126	3254671	3561742	5314267	3257614	31100008
1234567	7254631	2637514	5731642	1532467	5467123	7253164	31100008
31100008	31100008	31100008	31100008	31100008	31100008	31100008	31100008

The magic squares with digits 4, 5 and 6 are obtained from the magic square of 7 digits. For details refer author's work [13].

Result 2. *The magic sums of magic squares of order 7 with 4, 5, 6 and 7 digits in each cell lead us to a **number pattern** given by*

Number of Digits	Magic Square Sums
4	31108
5	311008
6	3110008
7	31100008

Note 2. *We observe that there are total* $7! = 5040$ *possibilities of writing 7 digits with different digits. Thus, there are much more possibilities of writing different digits magic squares with 7 digits in each cell. We have written only one just to have an idea. Moreover, there is a possibility of having a magic of order 7 with 3 digits in each cell. Still, we don't have this magic square. Any way below is a* ***pan diagonal*** *magic square with 3-digits* ***palindromic numbers*** *obtained from the Example 3.1 using the digits 1 to 7:*

		3108	3108	3108	3108	3108	3108	3108
	111	222	333	444	555	666	777	3108
3108	656	767	171	212	323	434	545	3108
3108	424	535	646	757	161	272	313	3108
3108	262	373	414	525	636	747	151	3108
3108	737	141	252	363	474	515	626	3108
3108	575	616	727	131	242	353	464	3108
3108	343	454	565	676	717	121	232	3108
	3108	3108	3108	3108	3108	3108	3108	3108

Similar to Result 2, we can also write a ***number pattern*** *with different digits* ***palindromic numbers****. For more details on* ***palindromic-type*** *magic squares refer author's work [9, ?].*

4 Pattern With Magic Sums of Order 8

This section deals with magic squares of order 8 written in different digits forming a number pattern with the magic sums and/or with semi-bimagic sums. The work if or the digits 3, 4, 5, 6, 7 and 8. This section is divided in two subsections. First on **semi-bimagic** square of order 8 and second on **pan diagonal** magic squares of order 8.

4.1 Different Digits Semi-Bimagic Squares of Order 8

In this section, we shall work with different digits semi-magic squares of order 8. The work is for the digits 4, 5, 6, 7 and 8. Lets consider following composite **pan diagonal** and **bimagic** square of order 8 with two digits cells.

Example 4.1. *Let's consider a* ***pan diagonal bimagic*** *square of order 8 given by*

		396	396	396	396	396	396	396	396
	28	61	54	15	43	86	77	32	396
396	42	87	76	33	25	64	51	18	396
396	11	58	65	24	36	73	82	47	396
396	37	72	83	46	14	55	68	21	396
396	56	13	22	67	71	38	45	84	396
396	74	35	48	81	57	12	23	66	396
396	63	26	17	52	88	41	34	75	396
396	85	44	31	78	62	27	16	53	396
	396	396	396	396	396	396	396	396	396

The above ***pan diagonal*** *magic square of 2-digits in each cell with magic and* ***bimagic*** *sums* $S_{8\times 8} := 396$ *and* $S_{8\times 8} := 23844$ *respectively. Also, each* 2×4 *blocks entries sum is same as of magic square, i.e., 396. Its construction is not with different digits, because there are are eight cells with repetition of digits, such as, 11, 22, 33, 44, 55, 66, 77 and 88. The reason is that there are only 64 possibilities of using the digits 1 to 8 in double digits form, including repeated ones. When the combinations are with more digits, the possibilities also increases. Below are examples of magic squares of order 8 with different digits. These examples are for the digits 4 to 8. The examples below are neither* ***pan diagonal*** *nor* ***bimagic****, but are with magic square. They are* ***bimagic*** *only in rows sums.*

Example 4.2. *Let's consider a* ***4 digits cell*** *magic square of order 8.*

								39996
6341	2785	1634	5278	8527	4163	3416	7852	39996
8563	4127	3452	7816	6385	2741	1678	5234	39996
5678	1234	2385	6741	7452	3816	4563	8127	39996
7416	3852	4527	8163	5634	1278	2341	6785	39996
1274	5638	6781	2345	3856	7412	8167	4523	39996
3812	7456	8123	4567	1238	5674	6745	2381	39996
2745	6381	5238	1674	4123	8567	7812	3456	39996
4167	8523	7856	3412	2781	6345	5274	1638	39996
39996	39996	39996	39996	39996	39996	39996	39996	39996

In this case, the magic and **bimagic (only in rows)** sums are $S_{8\times 8} := 39996$ and $S_{8\times 8} :=$ 241855004 respectively.

Example 4.3. *Let's consider a* ***5 digits cell*** *magic square of order 8.*

								399996
56341	12785	81634	45278	38527	74163	23416	67852	399996
78563	34127	63452	27816	16385	52741	41678	85234	399996
45678	81234	12385	56741	67452	23816	74563	38127	399996
27416	63852	34527	78163	85634	41278	52341	16785	399996
81274	45638	56781	12345	23856	67412	38167	74523	399996
63812	27456	78123	34567	41238	85674	16745	52381	399996
12745	56381	45238	81674	74123	38567	67812	23456	399996
34167	78523	27856	63412	52781	16345	85274	41638	399996
399996	399996	399996	399996	399996	399996	399996	399996	399996

In this case, the magic and **bimagic (only in rows)** sums are $S_{8\times 8} := 399996$ and $S_{8\times 8} := 24188655004$ respectively.

Example 4.4. *Let's consider a* ***6 digits cell*** *magic square of order 8.*

								3999996
856341	412785	781634	345278	638527	274163	523416	167852	3999996
278563	634127	163452	527816	416385	852741	341678	785234	3999996
345678	781234	412385	856741	167452	523816	274563	638127	3999996
527416	163852	634527	278163	785634	341278	852341	416785	3999996
381274	745638	456781	812345	123856	567412	238167	674523	3999996
563812	127456	678123	234567	741238	385674	816745	452381	3999996
812745	456381	745238	381674	674123	238567	567812	123456	3999996
234167	678523	127856	563412	452781	816345	385274	741638	3999996
3999996	3999996	3999996	3999996	3999996	3999996	3999996	3999996	3999996

In this case, the magic and **bimagic (only in rows)** sums are $S_{8\times 8} := 3999996$ and $S_{8\times 8} := 2418900655004$ respectively.

Example 4.5. *Let's consider a* ***7 digits cell*** *magic square of order 8.*

								39999996
7856341	3412785	2781634	6345278	1638527	5274163	8523416	4167852	39999996
1278563	5634127	8163452	4527816	7416385	3852741	2341678	6785234	39999996
2345678	6781234	7412385	3856741	8167452	4523816	1274563	5638127	39999996
8527416	4163852	1634527	5278163	2785634	6341278	7852341	3416785	39999996
6381274	2745638	3456781	7812345	4123856	8567412	5238167	1674523	39999996
4563812	8127456	5678123	1234567	6741238	2385674	3816745	7452381	39999996
3812745	7456381	6745238	2381674	5674123	1238567	4567812	8123456	39999996
5234167	1678523	4127856	8563412	3452781	7816345	6385274	2741638	39999996
39999996	39999996	39999996	39999996	39999996	39999996	39999996	39999996	39999996

In this case, the magic and **bimagic (only in rows)** sums are $S_{8\times 8} := 39999996$ and $S_{8\times 8} := 241890412655004$ respectively.

Example 4.6. *Let's consider a **8 digits cell** magic square of order 8.*

								399999996
27856341	63412785	52781634	16345278	41638527	85274163	78523416	34167852	399999996
41278563	85634127	78163452	34527816	27416385	63852741	52341678	16785234	399999996
12345678	56781234	67412385	23856741	38167452	74523816	81274563	45638127	399999996
38527416	74163852	81634527	45278163	12785634	56341278	67852341	23416785	399999996
56381274	12745638	23456781	67812345	74123856	38567412	45238167	81674523	399999996
74563812	38127456	45678123	81234567	56741238	12385674	23816745	67452381	399999996
63812745	27456381	16745238	52381674	85674123	41238567	34567812	78123456	399999996
85234167	41678523	34127856	78563412	63452781	27816345	16385274	52741638	399999996
399999996	399999996	399999996	399999996	399999996	399999996	399999996	399999996	399999996

In this case, the magic and **bimagic (only in rows)** sums are $S_{8\times 8} := 399999996$ and $S_{8\times 8} := 24189044812655004$ respectively.

The magic squares with digits 4, 5, 6 and 7 are obtained from the magic square of 8 digits. For details refer author's work [13].

Result 3. *The magic sums of magic squares of order 8 with 4, 5, 6, 7 and 8 digits in each cell lead us to a **number pattern** given by*

Number of Digits	*Magic Square Sums*	*Bimagic Sums of Rows*
4	*39996*	**2418 55004**
5	*399996*	**2418** 86 **55004**
6	*3999996*	**2418** 9006 **55004**
7	*39999996*	**2418** 904126 **55004**
8	*399999996*	**2418** 90448126 **55004**

Note 3. *We observe that there are total* 8! = 40320 *possibilities of writing different numbers with different digits of order 8. Thus, there are much more possibilities of writing different digits magic squares with 8 different digits in each cell. We have written only one just to have an idea. Moreover, there is a possibility of having a magic square of order 8 with 3 different digits in each cell. Still, we don't have this magic square. As an extension of Example 4.1 with 2 digits, below is a **pan diagonal** and **bimagic** square with 3-digits **palindromic numbers** using the digits 1 to 8:*

		3996	3996	3996	3996	3996	3996	3996	3996
	282	616	545	151	434	868	777	323	3996
3996	424	878	767	333	252	646	515	181	3996
3996	111	585	656	242	363	737	828	474	3996
3996	373	727	838	464	141	555	686	212	3996
3996	565	131	222	676	717	383	454	848	3996
3996	747	353	484	818	575	121	232	666	3996
3996	636	262	171	525	888	414	343	757	3996
3996	858	444	313	787	626	272	161	535	3996
	3996	3996	3996	3996	3996	3996	3996	3996	3996

*Similar to Result 3, we can also write a **number pattern** with different digits **palindromic numbers**. For details refer author's work [9, ?].*

4.2 Different Digits Pan Diagonal Magic Squares of Order 8

This subsection is a continuation of above subsection. In previous subsection, we worked with different digits magic square having 8 digits from 1 to 8, but the magic squares are not **pan diagonal**, and are **bimagic** only in rows. In this section we shall work with **pan diagonal** magic squares of 8 with different digits. Also, in the previous subsection, we were still unable to bring the magic square with 3 digits. Here we worked with magic squares of order 8 with 3 to 8 digits. Also, these magic squares have the extra property that each block of order with the equal sums entries.

Let's consider following composite **pan diagonal** square of order 8 with two digits cells.

Example 4.7. *Let's consider a **pan diagonal** square of order 8 given by*

		396	396	396	396	396	396	396	396
	11	85	44	58	21	75	34	68	396
396	48	54	15	81	38	64	25	71	396
396	55	41	88	14	65	31	78	24	396
396	84	18	51	45	74	28	61	35	396
396	12	86	43	57	22	76	33	67	396
396	47	53	16	82	37	63	26	72	396
396	56	42	87	13	66	32	77	23	396
396	83	17	52	46	73	27	62	36	396
	396	396	396	396	396	396	396	396	396

*The above magic square is **pan diagonal** of 2-digits in each cell with the property that each block of order 4 a magic square. Also, each 2 × 2 blocks are of equal sums entries. Its construction is not with different digits, because there are eight cells with repetition of digits, such as, 11, 22, 33, 44, 55, 66, 77 and 88. The reason is that there are only 64 possibilities of using the digits 1 to 8 in double digits form, including repeated ones. When the combinations are with more digits, the possibilities also increases. Below are examples of **pan diagonal** magic squares of **different digits** of order 8. All the blocks of order 4 chosen symmetrically are with equal sums entries. These examples are for the digits 3 to 8.*

Example 4.8. *Let's consider a **3 digits cell pan diagonal** magic square of order 8 given by*

		3996	3996	3996	3996	3996	3996	3996	3996
	487	156	715	648	823	532	371	264	3996
3996	265	378	537	826	641	714	153	482	3996
3996	632	723	164	471	256	387	548	815	3996
3996	814	541	382	253	478	165	726	637	3996
3996	176	467	628	735	512	843	284	351	3996
3996	358	285	846	517	734	621	462	173	3996
3996	743	612	451	184	367	276	835	528	3996
3996	521	834	273	362	185	458	617	746	3996
	3996	3996	3996	3996	3996	3996	3996	3996	3996

In this case, the **magic square** and **order 4 blocks entries** sums are $S_{8\times8} := 3996$ and $S_{16} := 7992$ respectively.

Example 4.9. *Let's consider a **4 digits cell pan diagonal** magic square of order 8 given by*

		39996	39996	39996	39996	39996	39996	39996	39996
	4876	1567	7153	6482	8234	5321	3715	2648	39996
39996	2658	3785	5371	8264	6412	7143	1537	4826	39996
39996	6321	7234	1648	4715	2567	3876	5482	8153	39996
39996	8143	5412	3826	2537	4785	1658	7264	6371	39996
39996	1765	4678	6284	7351	5123	8432	2846	3517	39996
39996	3587	2856	8462	5173	7341	6214	4628	1735	39996
39996	7432	6123	4517	1846	3678	2765	8351	5284	39996
39996	5214	8341	2735	3628	1856	4587	6173	7462	39996
	39996	39996	39996	39996	39996	39996	39996	39996	39996

In this case, the **magic square** and **order 4 blocks entries** sums are $S_{8\times8} := 39996$ and $S_{16} := 79992$ respectively.

Example 4.10. *Let's consider a **5 digits cell pan diagonal** magic square of order 8 given by*

		399996	399996	399996	399996	399996	399996	399996	399996
	48765	15678	71532	64823	82341	53214	37156	26487	399996
399996	26587	37856	53714	82641	64123	71432	15378	48265	399996
399996	63214	72341	16487	47156	25678	38765	54823	81532	399996
399996	81432	54123	38265	25378	47856	16587	72641	63714	399996
399996	17658	46785	62843	73512	51234	84321	28467	35176	399996
399996	35876	28567	84621	51734	73412	62143	46285	17358	399996
399996	74321	61234	45176	18467	36785	27658	83512	52843	399996
399996	52143	83412	27358	36285	18567	45876	61734	74621	399996
	399996	399996	399996	399996	399996	399996	399996	399996	399996

In this case, the **magic square** and **order 4 blocks entries** sums are $S_{8\times8} := 399996$ and $S_{16} := 799992$ respectively.

Example 4.11. *Let's consider a **6 digits cell pan diagonal** magic square of order 8 given by*

		3999996	3999996	3999996	3999996	3999996	3999996	3999996	3999996
	348765	215678	871532	564823	782341	653214	437156	126487	3999996
3999996	126587	437856	653714	782641	564123	871432	215378	348265	3999996
3999996	563214	872341	216487	347156	125678	438765	654823	781532	3999996
3999996	781432	654123	438265	125378	347856	216587	872641	563714	3999996
3999996	217658	346785	562843	873512	651234	784321	128467	435176	3999996
3999996	435876	128567	784621	651734	873412	562143	346285	217358	3999996
3999996	874321	561234	345176	218467	436785	127658	783512	652843	3999996
3999996	652143	783412	127358	436285	218567	345876	561734	874621	3999996
	3999996	3999996	3999996	3999996	3999996	3999996	3999996	3999996	3999996

In this case, the **magic square** and **order 4 blocks entries** sums are $S_{8\times 8} := 3999996$ and $S_{16} := 7999992$ respectively.

Example 4.12. *Let's consider a* ***7 digits cell pan diagonal*** *magic square of order 8 given by*

		39999996	39999996	39999996	39999996	39999996	39999996	39999996	39999996
	2348765	3215678	4871532	1564823	6782341	7653214	8437156	5126487	39999996
39999996	4126587	1437856	2653714	3782641	8564123	5871432	6215378	7348265	39999996
39999996	7563214	6872341	5216487	8347156	3125678	2438765	1654823	4781532	39999996
39999996	5781432	8654123	7438265	6125378	1347856	4216587	3872641	2563714	39999996
39999996	3217658	2346785	1562843	4873512	7651234	6784321	5128467	8435176	39999996
39999996	1435876	4128567	3784621	2651734	5873412	8562143	7346285	6217358	39999996
39999996	6874321	7561234	8345176	5218467	2436785	3127658	4783512	1652843	39999996
39999996	8652143	5783412	6127358	7436285	4218567	1345876	2561734	3874621	39999996
	39999996	39999996	39999996	39999996	39999996	39999996	39999996	39999996	39999996

In this case, the **magic square** and **order 4 blocks entries** sums are $S_{8\times 8} := 39999996$ and $S_{16} := 79999992$ respectively.

Example 4.13. *Let's consider a* ***8 digits cell pan diagonal*** *magic square of order 8 given by*

		399999996	399999996	399999996	399999996	399999996	399999996	399999996	399999996
	12348765	43215678	64871532	71564823	56782341	87653214	28437156	35126487	399999996
399999996	34126587	21437856	82653714	53782641	78564123	65871432	46215378	17348265	399999996
399999996	87563214	56872341	35216487	28347156	43125678	12438765	71654823	64781532	399999996
399999996	65781432	78654123	17438265	46125378	21347856	34216587	53872641	82563714	399999996
399999996	43217658	12346785	71562843	64873512	87651234	56784321	35128467	28435176	399999996
399999996	21435876	34128567	53784621	82651734	65873412	78562143	17346285	46217358	399999996
399999996	56874321	87561234	28345176	35218467	12436785	43127658	64783512	71652843	399999996
399999996	78652143	65783412	46127358	17436285	34218567	21345876	82561734	53874621	399999996
	399999996	399999996	399999996	399999996	399999996	399999996	399999996	399999996	399999996

In this case, the **magic square** and **order 4 blocks entries** sums are $S_{8\times8} := 399999996$ and $S_{16} := 799999992$ respectively.

The magic squares with digits 3, 4, 5, 6 and 7 are obtained from the magic square of 8 digits. For details refer author's work [13].

Result 4. *The magic sums and order 4 blocks entries sums of magic squares of order 8 with 4, 5, 6, 7 and 8 digits in each cell lead us to a **number pattern** given by*

Number of Digits	Magic Square Sums	Order 4 Blocks Entries Sum
3	3996	7992
4	39996	79992
5	399996	799992
6	3999996	7999992
7	39999996	79999992
8	399999996	799999992

Note 4. *We observe that there are total* $8! = 40320$ *possibilities of writing different numbers with different digits of order 8. Thus, there are much more possibilities of writing different digits magic squares with 8 different digits in each cell. We have written only one just to have an idea. As an extension of Example 4.7 with 2 digits, below is a **pan diagonal** magic square with 3-digits **palindromic numbers** using the digits 1 to 8:*

		3996	3996	3996	3996	3996	3996	3996	3996
	111	858	444	585	212	757	343	686	3996
3996	484	545	151	818	383	646	252	717	3996
3996	555	414	888	141	656	313	787	242	3996
3996	848	181	515	454	747	282	616	353	3996
3996	121	868	434	575	222	767	333	676	3996
3996	474	535	161	828	373	636	262	727	3996
3996	565	424	878	131	666	323	777	232	3996
3996	838	171	525	464	737	272	626	363	3996
	3996	3996	3996	3996	3996	3996	3996	3996	3996

*Each block of order 4 is a **pan diagonal** magic square of order 4. Similar to Result 4, we can also write a **number pattern** with different digits **palindromic numbers**. For more details on **palindromic-type** magic squares refer author's work [9, ?].*

5 Pattern With Magic Sums of Order 9

This section deals with magic squares of order 9 written in different digits forming a number pattern with the magic and with **bimagic** sums. The work is for the digits 3, 4, 5, 6, 7, 8 and 9. This section is divided in two subsections. First on **pan diagonal** magics squares of order 9 and second on **bimagic** squares of order 9.

5.1 Different Digits Pan Diagonal Magic Squares of Order 9

Let's consider following composite magic square of order 9.

Example 5.1. *Let's consider a **pan diagonal** magic square of order 9 is given by*

		495	495	495	495	495	495	495	495	495
	34	88	43	39	81	45	32	86	47	495
495	48	33	84	41	35	89	46	37	82	495
495	83	44	38	85	49	31	87	42	36	495
495	54	18	93	59	11	95	52	16	97	495
495	98	53	14	91	55	19	96	57	12	495
495	13	94	58	15	99	51	17	92	56	495
495	74	68	23	79	61	25	72	66	27	495
495	28	73	64	21	75	69	26	77	62	495
495	63	24	78	65	29	71	67	22	76	495
	495	495	495	495	495	495	495	495	495	495

Additionally it has property that each 3×3 *block is of same sum as of magic square, i.e.,* $S_9 = 495$*. Also each* 3×3 *block is a semi-magic square of order 3 (only in rows and columns).*

*The above **pan diagonal** magic square is with 2-digits in each cell. Its construction is not with different digits as in nine cells, there are numbers with repetition of digits, such as, 11, 22, 33, 44, 55, 66, 77, 88 and 99. The reason is that there are only 81 possibilities of using the digits 1 to 9 in double digits forms, including repeated ones. When the combinations are with more digits, the possibilities increases. Below are examples of **pan diagonal** magic squares of order 9 with different digits. These examples are for the digits 3 to 9 in each cell.*

Example 5.2. *Let's consider a **3 digits cell pan diagonal** magic square of order 9 given by*

http://doi.org/10.5281/zenodo.2555327

		4995	4995	4995	4995	4995	4995	4995	4995	4995
	186	642	837	783	348	534	489	945	231	4995
4995	531	789	345	237	486	942	834	183	648	4995
4995	948	234	483	645	831	189	342	537	786	4995
4995	264	423	978	861	129	675	567	726	372	4995
4995	672	867	126	378	564	723	975	261	429	4995
4995	729	375	561	426	972	267	123	678	864	4995
4995	315	591	759	912	297	456	618	894	153	4995
4995	453	918	294	159	615	891	756	312	597	4995
4995	897	156	612	594	753	318	291	459	915	4995
	4995	4995	4995	4995	4995	4995	4995	4995	4995	4995

Example 5.3. *Let's consider a* ***4 digits cell pan diagonal*** *magic square of order 9 given by*

		49995	49995	49995	49995	49995	49995	49995	49995	49995
	1864	6429	8372	7831	3486	5348	4897	9453	2315	49995
49995	5318	7891	3456	2375	4867	9423	8342	1834	6489	49995
49995	9483	2345	4837	6459	8312	1894	3426	5378	7861	49995
49995	2645	4237	9783	8612	1294	6759	5678	7261	3726	49995
49995	6729	8672	1264	3786	5648	7231	9753	2615	4297	49995
49995	7291	3756	5618	4267	9723	2675	1234	6789	8642	49995
49995	3156	5918	7591	9123	2975	4567	6189	8942	1534	49995
49995	4537	9183	2945	1594	6159	8912	7561	3126	5978	49995
49995	8972	1564	6129	5948	7531	3186	2915	4597	9153	49995
	49995	49995	49995	49995	49995	49995	49995	49995	49995	49995

Example 5.4. *Let's consider a* ***5 digits cell pan diagonal*** *magic square of order 9 given by*

		499995	499995	499995	499995	499995	499995	499995	499995	499995
	18642	64297	83726	78312	34867	53486	48972	94537	23156	499995
499995	53186	78912	34567	23756	48672	94237	83426	18342	64897	499995
499995	94837	23456	48372	64597	83126	18942	34267	53786	78612	499995
499995	26459	42375	97831	86129	12945	67591	56789	72615	37261	499995
499995	67291	86729	12645	37861	56489	72315	97531	26159	42975	499995
499995	72915	37561	56189	42675	97231	26759	12345	67891	86429	499995
499995	31564	59183	75918	91234	29753	45678	61894	89423	15348	499995
499995	45378	91834	29453	15948	61594	89123	75618	31264	59783	499995
499995	89723	15648	61294	59483	75318	31864	29153	45978	91534	499995
	499995	499995	499995	499995	499995	499995	499995	499995	499995	499995

Example 5.5. *Let's consider a* ***6 digits cell pan diagonal*** *magic square of order 9 given by*

		4999995	4999995	4999995	4999995	4999995	4999995	4999995	4999995	4999995
	186429	642975	837261	783126	348672	534867	489723	945378	231564	4999995
4999995	531864	789123	345678	237561	486729	942375	834267	183426	648972	4999995
4999995	948372	234567	483726	645978	831264	189423	342675	537861	786129	4999995
4999995	264597	423756	978312	861294	129453	675918	567891	726159	372615	4999995
4999995	672915	867291	126459	378612	564897	723156	975318	261594	429753	4999995
4999995	729153	375618	561894	426759	972315	267591	123456	678912	864297	4999995
4999995	315648	591834	759183	912345	297531	456789	618942	894237	153486	4999995
4999995	453786	918342	294537	159483	615948	891234	756189	312645	597831	4999995
4999995	897231	156489	612945	594837	753186	318642	291534	459783	915348	4999995
	4999995	4999995	4999995	4999995	4999995	4999995	4999995	4999995	4999995	4999995

Example 5.6. *Let's consider a* ***7 digits cell pan diagonal*** *magic square of order 9 given by*

		49999995	49999995	49999995	49999995	49999995	49999995	49999995	49999995	49999995
	1864297	6429753	8372615	7831264	3486729	5348672	4897231	9453786	2315648	49999995
49999995	5318642	7891234	3456789	2375618	4867291	9423756	8342675	1834267	6489723	49999995
49999995	9483726	2345678	4837261	6459783	8312645	1894237	3426759	5378612	7861294	49999995
49999995	2645978	4237561	9783126	8612945	1294537	6759183	5678912	7261594	3726159	49999995
49999995	6729153	8672915	1264597	3786129	5648972	7231564	9753186	2615948	4297531	49999995
49999995	7291534	3756189	5618942	4267591	9723156	2675918	1234567	6789123	8642975	49999995
49999995	3156489	5918342	7591834	9123456	2975318	4567891	6189423	8942375	1534867	49999995
49999995	4537861	9183426	2945378	1594837	6159483	8912345	7561894	3126459	5978312	49999995
49999995	8972315	1564897	6129453	5948372	7531864	3186429	2915348	4597831	9153486	49999995
	49999995	49999995	49999995	49999995	49999995	49999995	49999995	49999995	49999995	49999995

Example 5.7. *Let's consider a* ***8 digits cell pan diagonal*** *magic square of order 9 given by*

		499999995	499999995	499999995	499999995	499999995	499999995	499999995	499999995	499999995
	18642975	64297531	83726159	78312645	34867291	53486729	48972315	94537861	23156489	499999995
499999995	53186429	78912345	34567891	23756189	48672915	94237561	83426759	18342675	64897231	499999995
499999995	94837261	23456789	48372615	64597831	83126459	18942375	34267591	53786129	78612945	499999995
499999995	26459783	42375618	97831264	86129453	12945378	67591834	56789123	72615948	37261594	499999995
499999995	67291534	86729153	12645978	37861294	56489723	72315648	97531864	26159483	42975318	499999995
499999995	72915348	37561894	56189423	42675918	97231564	26759183	12345678	67891234	86429753	499999995
499999995	31564897	59183426	75918342	91234567	29753186	45678912	61894237	89423756	15348672	499999995
499999995	45378612	91834267	29453786	15948372	61594837	89123456	75618942	31264597	59783126	499999995
499999995	89723156	15648972	61294537	59483726	75318642	31864297	29153486	45978312	91534867	499999995
	499999995	499999995	499999995	499999995	499999995	499999995	499999995	499999995	499999995	499999995

Example 5.8. *Let's consider a* ***9 digits cell pan diagonal*** *magic square of order 9 given by*

		4999999995	4999999995	4999999995	4999999995	4999999995	4999999995	4999999995	4999999995	4999999995
	186429753	642975318	837261594	783126459	348672915	534867291	489723156	945378612	231564897	4999999995
4999999995	531864297	789123456	345678912	237561894	486729153	942375618	834267591	183426759	648972315	4999999995
4999999995	948372615	234567891	483726159	645978312	831264597	189423756	342675918	537861294	786129453	4999999995
4999999995	264597831	423756189	978312645	861294537	129453786	675918342	567891234	726159483	372615948	4999999995
4999999995	672915348	867291534	126459783	378612945	564897231	723156489	975318642	261594837	429753186	4999999995
4999999995	729153486	375618942	561894237	426759183	972315648	267591834	123456789	678912345	864297531	4999999995
4999999995	315648972	591834267	759183426	912345678	297531864	456789123	618942375	894237561	153486729	4999999995
4999999995	453786129	918342675	294537861	159483726	615948372	891234567	756189423	312645978	597831264	4999999995
4999999995	897231564	156489723	612945378	594837261	753186429	318642975	291534867	459783126	915348672	4999999995
	4999999995	4999999995	4999999995	4999999995	4999999995	4999999995	4999999995	4999999995	4999999995	4999999995

The magic squares with digits 3, 4, 5, 6 and 7 are obtained from the magic square of 8 digits. For details refer author's work [13].

Result 5. *The magic sums of magic squares of order 8 with 4, 5, 6, 7 and 8 digits in each cell lead us to a* ***number pattern*** *given by*

Number of Digits	Magic Square Sums
3	4995
4	49995
5	499995
6	4999995
7	49999995
8	499999995
9	4999999995

Note 5. *We observe that there are total* $9! = 362880$ *possibilities of writing different numbers with different digits of order 9. Thus, there are much more possibilities of writing different digits magic squares with 9 different digits in each cell. We have written only one just to have an idea. As an extension of Example 5.1 with 2 digits, below is a* ***pan diagonal*** *magic square with 3-digits* ***palindromic numbers*** *using the digits 1 to 9:*

		4995	4995	4995	4995	4995	4995	4995	4995	4995
	343	888	434	393	818	454	323	868	474	4995
4995	484	333	848	414	353	898	464	373	828	4995
4995	838	444	383	858	494	313	878	424	363	4995
4995	545	181	939	595	111	959	525	161	979	4995
4995	989	535	141	919	555	191	969	575	121	4995
4995	131	949	585	151	999	515	171	929	565	4995
4995	747	686	232	797	616	252	727	666	272	4995
4995	282	737	646	212	757	696	262	777	626	4995
4995	636	242	787	656	292	717	676	222	767	4995
	4995	4995	4995	4995	4995	4995	4995	4995	4995	4995

Additionally it has property that each 3×3 *block is of same sum as of magic square, i.e.,* $S_9 = 4995$*. Also each* 3×3 *block is a* ***semi-magic*** *square of order 3 (only in rows and columns).*

5.2 Different Digits Bimagic Squares of Order 9

Let's consider following composite magic square of order 9.

Example 5.9. *Let's consider a* ***bimagic*** *square of order 9 given by*

									495
11	29	35	48	54	63	76	82	97	495
46	52	67	71	89	95	18	24	33	495
78	84	93	16	22	37	41	59	65	495
39	15	21	64	43	58	92	77	86	495
62	47	56	99	75	81	34	13	28	495
94	73	88	32	17	26	69	45	51	495
25	31	19	53	68	44	87	96	72	495
57	66	42	85	91	79	23	38	14	495
83	98	74	27	36	12	55	61	49	495
495	495	495	495	495	495	495	495	495	495

The above magic square is ***bimagic****. The magic and* ***bimagic*** *sums are* $S_{9\times 9} = 495$ *and* $Sb_{9\times 9} = 33285$ *respectively. Each* 3×3 *block is of equal sum entries as of magic square, i.e.,* $S_9 = 495$*. Its construction is not with different digits as in nine cells, there are numbers with repetition of digits, such as, 11, 22, 33, 44, 55, 66, 77, 88 and 99. The reason is that there are* *only 81 possibilities of using the digits 1 to 9 in double digits forms, including repeated ones. When the combinations are with more digits, the possibilities increases. Below are examples of **bimagic** square of order 9 with different digits. These examples are for the digits 3 to 9.*

Example 5.10. *Let's consider a **3 digits cell bimagic** square of order 9 given by*

									4995
789	864	912	153	237	375	426	591	648	4995
126	291	348	489	564	612	753	837	975	4995
453	537	675	726	891	948	189	264	312	4995
945	723	897	318	186	261	672	459	534	4995
372	159	234	645	423	597	918	786	861	4995
618	486	561	972	759	834	345	123	297	4995
831	978	756	294	342	129	567	615	483	4995
267	315	183	531	678	456	894	942	729	4995
594	642	429	867	915	783	231	378	156	4995
4995	4995	4995	4995	4995	4995	4995	4995	4995	4995

The magic and **bimagic** sums are $S_{9\times 9} = 4995$ and $Sb_{9\times 9} = 3371625$ respectively.

Example 5.11. *Let's consider a **4 digits cell bimagic** square of order 9 given by*

									49995
6789	1864	8912	9153	4237	2375	3426	7591	5648	49995
3126	7291	5348	6489	1564	8612	9753	4837	2975	49995
9453	4537	2675	3726	7891	5948	6189	1264	8312	49995
2945	9723	4897	5318	3186	7261	8672	6459	1534	49995
8372	6159	1234	2645	9423	4597	5918	3786	7861	49995
5618	3486	7561	8972	6759	1834	2345	9123	4297	49995
7831	5978	3756	1294	8342	6129	4567	2615	9483	49995
4267	2315	9183	7531	5678	3456	1894	8942	6729	49995
1594	8642	6429	4867	2915	9783	7231	5378	3156	49995
49995	49995	49995	49995	49995	49995	49995	49995	49995	49995

The magic and **bimagic** sums are $S_{9\times 9} = 49995$ and $Sb_{9\times 9} = 337619625$ respectively.

Example 5.12. *Let's consider a **5 digits cell bimagic** square of order 9 given by*

									499995
56789	31864	78912	29153	94237	42375	83426	67591	15648	499995
83126	67291	15348	56489	31564	78612	29753	94837	42975	499995
29453	94537	42675	83726	67891	15948	56189	31264	78312	499995
12945	89723	64897	75318	53186	37261	48672	26459	91534	499995
48372	26159	91234	12645	89423	64597	75918	53786	37861	499995
75618	53486	37561	48972	26759	91834	12345	89123	64297	499995
97831	45978	23756	61294	18342	86129	34567	72615	59483	499995
34267	72315	59183	97531	45678	23456	61894	18942	86729	499995
61594	18642	86429	34867	72915	59783	97231	45378	23156	499995
499995	499995	499995	499995	499995	499995	499995	499995	499995	499995

The magic and **bimagic** sums are $S_{9\times 9} = 499995$ and $Sb_{9\times 9} = 33766859625$ respectively.

Example 5.13. *Let's consider a **6 digits cell bimagic** square of order 9 given by*

									4999995
456789	531864	678912	729153	894237	942375	183426	267591	315648	4999995
783126	867291	915348	156489	231564	378612	429753	594837	642975	4999995
129453	294537	342675	483726	567891	615948	756189	831264	978312	4999995
612945	489723	564897	975318	753186	837261	348672	126459	291534	4999995
948372	726159	891234	312645	189423	264597	675918	453786	537861	4999995
375618	153486	237561	648972	426759	591834	912345	789123	864297	4999995
597831	645978	423756	861294	918342	786129	234567	372615	159483	4999995
834267	972315	759183	297531	345678	123456	561894	618942	486729	4999995
261594	318642	186429	534867	672915	459783	897231	945378	723156	4999995
4999995	4999995	4999995	4999995	4999995	4999995	4999995	4999995	4999995	4999995

The magic and **bimagic** sums are $S_{9\times 9} = 4999995$ and $Sb_{9\times 9} = 3376735259625$ respectively.

Example 5.14. *Let's consider a **7 digits cell bimagic** square of order 9 given by*

									49999995
3456789	7531864	5678912	6729153	1894237	8942375	9183426	4267591	2315648	49999995
9783126	4867291	2915348	3156489	7231564	5378612	6429753	1594837	8642975	49999995
6129453	1294537	8342675	9483726	4567891	2615948	3756189	7831264	5978312	49999995
8612945	6489723	1564897	2975318	9753186	4837261	5348672	3126459	7291534	49999995
5948372	3726159	7891234	8312645	6189423	1264597	2675918	9453786	4537861	49999995
2375618	9153486	4237561	5648972	3426759	7591834	8912345	6789123	1864297	49999995
4597831	2645978	9423756	7861294	5918342	3786129	1234567	8372615	6159483	49999995
1834267	8972315	6759183	4297531	2345678	9123456	7561894	5618942	3486729	49999995
7261594	5318642	3186429	1534867	8672915	6459783	4897231	2945378	9723156	49999995
49999995	49999995	49999995	49999995	49999995	49999995	49999995	49999995	49999995	49999995

The magic and **bimagic** sums are $S_{9\times 9} = 49999995$ and $Sb_{9\times 9} = 337673983259625$ respectively.

Example 5.15. *Let's consider a **8 digits cell bimagic** square of order 9 given by*

									499999995
23456789	97531864	45678912	86729153	61894237	18942375	59183426	34267591	72315648	499999995
59783126	34867291	72915348	23156489	97231564	45378612	86429753	61594837	18642975	499999995
86129453	61294537	18342675	59483726	34567891	72615948	23756189	97831264	45978312	499999995
78612945	56489723	31564897	42975318	29753186	94837261	15348672	83126459	67291534	499999995
15948372	83726159	67891234	78312645	56189423	31264597	42675918	29453786	94537861	499999995
42375618	29153486	94237561	15648972	83426759	67591834	78912345	56789123	31864297	499999995
64597831	12645978	89423756	37861294	75918342	53786129	91234567	48372615	26159483	499999995
91834267	48972315	26759183	64297531	12345678	89123456	37561894	75618942	53486729	499999995
37261594	75318642	53186429	91534867	48672915	26459783	64897231	12945378	89723156	499999995
499999995	499999995	499999995	499999995	499999995	499999995	499999995	499999995	499999995	499999995

The magic and **bimagic** sums are $S_{9\times 9} = 499999995$ and $Sb_{9\times 9} = 33767403223259625$ respectively.

Example 5.16. *Let's consider a **9 digits cell bimagic** square of order 9 given by*

									4999999995
123456789	297531864	345678912	486729153	561894237	618942375	759183426	834267591	972315648	4999999995
459783126	534867291	672915348	723156489	897231564	945378612	186429753	261594837	318642975	4999999995
786129453	861294537	918342675	159483726	234567891	372615948	423756189	597831264	645978312	4999999995
378612945	156489723	231564897	642975318	429753186	594837261	915348672	783126459	867291534	4999999995
615948372	483726159	567891234	978312645	756189423	831264597	342675918	129453786	294537861	4999999995
942375618	729153486	894237561	315648972	183426759	267591834	678912345	456789123	531864297	4999999995
264597831	312645978	189423756	537861294	675918342	453786129	891234567	948372615	726159483	4999999995
591834267	648972315	426759183	864297531	912345678	789123456	237561894	375618942	153486729	4999999995
837261594	975318642	753186429	291534867	348672915	126459783	564897231	612945378	489723156	4999999995
4999999995	4999999995	4999999995	4999999995	4999999995	4999999995	4999999995	4999999995	4999999995	4999999995

The magic and **bimagic** sums are $S_{9\times 9} = 4999999995$ and $Sb_{9\times 9} = 3376740371623259625$ respectively.

The **bimagic** squares with digits 3, 4, 5, 6, 7 and 8 are obtained from the **bimagic** square of 9 digits. For details refer author's work [13].

Result 6. *The magic sums of magic squares of order 9 with 3, 4, 5, 6, 7, 8 and 9 digits in each cell lead us to a* ***number pattern*** *given by*

Number of Digits	Magic Square Sums	Bimagic Sums of Rows
3	4995	**337** 1 **625**
4	49995	**337** 619 **625**
5	499995	**337** 66859 **625**
6	4999995	**337** 6735259 **625**
7	49999995	**337** 673983259 **625**
8	499999995	**337** 67403223259 **625**
9	4999999995	**337** 6740371623259 **625**

Note 6. *From 4 digits onwards, we have better pattern with bimagic sums. See below:*

Number of Digits	Magic Square Sums	Bimagic Sums of Rows
4	49995	**3376** 1 **9625**
5	499995	**3376** 685 **9625**
6	4999995	**3376** 73525 **9625**
7	49999995	**3376** 7398325 **9625**
8	499999995	**3376** 740322325 **9625**
9	4999999995	**3376** 74037162325 **9625**

This pattern is still better from 6 digits onwards. See below:

Number of Digits	Magic Square Sums	Bimagic Sums of Rows
6	4999995	**33767** 35 **259625**
7	49999995	**33767** 3983 **259625**
8	499999995	**33767** 403223 **259625**
9	4999999995	**33767** 40371623 **259625**

Note 7. *We observe that there are total* $9! = 362880$ *possibilities of writing different numbers with different digits of order 9. Thus, there are much more possibilities of writing different digits magic squares with 9 different digits in each cell. We have written only one just to have an idea. As an extension of Example 5.9 with 2 digits, below is a* ***bimagic*** *square with 3-digits* ***palindromic numbers*** *using the digits 1 to 9:*

									4995
111	292	353	484	545	636	767	828	979	4995
464	525	676	717	898	959	181	242	333	4995
787	848	939	161	222	373	414	595	656	4995
393	151	212	646	434	585	929	777	868	4995
626	474	565	999	757	818	343	131	282	4995
949	737	888	323	171	262	696	454	515	4995
252	313	191	535	686	444	878	969	727	4995
575	666	424	858	919	797	232	383	141	4995
838	989	747	272	363	121	555	616	494	4995
4995	4995	4995	4995	4995	4995	4995	4995	4995	4995

Similar to Result 6, we can also write a ***number pattern*** *with different digits* ***palindromic numbers****. For more details on* ***palindromic-type*** *magic squares refer author's work [9, ?].*

6 Pattern With Magic Sums of Order 10

This section deals with magic squares of order 10 written in terms of different digits forming a number pattern with the magic squares sums for the digits 5, 6, 7, 8, 9 and 10.

6.1 Different Digits Magic Squares of Order 10

Let's consider following composite magic square of order 10.

Example 6.1. *Let's consider a* ***2 digits cell*** *magic square of order 10 is given by*

										495
00	98	45	61	17	73	54	86	29	32	495
75	11	97	42	59	38	80	03	64	26	495
41	67	22	89	35	16	78	50	93	04	495
69	06	74	33	20	82	47	91	15	58	495
53	30	68	76	44	21	95	19	02	87	495
84	43	10	28	96	55	09	62	37	71	495
27	52	39	05	81	94	66	48	70	13	495
36	24	83	90	08	49	12	77	51	65	495
92	79	56	14	63	07	31	25	88	40	495
18	85	01	57	72	60	23	34	46	99	495
495	495	495	495	495	495	495	495	495	495	495

The above bimagic square is of 2-digits in each cell. Its construction is not with different digits as in ten cells, there are numbers with repetition of digits, such as, 00, 11, 22, 33, 44, 55, 66, 77, 88 and 99. The reason is that there are only 100 possibiliites of using the digits 0 to 9 in double digits forms, including repeated ones. When the combinations are with more digits, the possibilities increases. Below are examples of magic square of order 10 with different digits. These examples are for the digits 4 to 10.

Example 6.2. *Let's consider a* ***4 digits cell*** *magic square of order 10 is given by*

										49995
4375	0539	8941	7892	6283	3714	2106	1057	9468	5620	49995
9460	1058	2807	6713	5674	4329	3285	7196	8942	0531	49995
8951	7192	3286	4625	9530	0468	5379	6714	2803	1047	49995
7802	6783	5374	0469	1948	8057	9531	4620	3215	2196	49995
3716	5624	9560	8051	2197	7803	1942	0438	4379	6285	49995
0148	8207	7613	3574	4069	5931	6420	2385	1796	9852	49995
5039	9841	1792	2386	3425	6570	7614	8203	0157	4968	49995
6524	4960	0158	1207	7316	2685	8793	9842	5031	3479	49995
2683	3475	4039	9148	8702	1296	0857	5961	6520	7314	49995
1297	2316	6425	5930	0851	9142	4068	3579	7684	8703	49995
49995	49995	49995	49995	49995	49995	49995	49995	49995	49995	49995

Example 6.3. *Let's consider a* ***5 digits cell*** *magic square of order 10 is given by*

										499995
43719	05371	89462	78953	62805	37196	21047	10538	94620	56284	499995
94628	10537	28053	67194	56780	43215	32806	71942	89461	05379	499995
89562	71946	32805	46280	95371	04629	53714	67193	28057	10438	499995
78053	67894	53719	04628	19462	80531	95370	46285	32106	21947	499995
37194	56280	95671	80537	21946	78052	19468	04329	43715	62803	499995
01437	82053	76194	35719	40628	59370	64285	23806	17942	98561	499995
50371	98462	17946	23805	34219	65784	76193	82057	01538	49620	499995
65280	49628	01537	12046	73194	26803	87952	98461	50379	34715	499995
26805	34719	40328	91462	87053	12947	08531	59670	65284	73196	499995
12946	23105	64280	59371	08537	91468	40629	35714	76893	87052	499995
499995	499995	499995	499995	499995	499995	499995	499995	499995	499995	499995

Example 6.4. *Let's consider a* ***6 digits cell*** *magic square of order 10 is given by*

										4999995
437195	053719	894621	789532	628053	371964	210476	105387	946208	562840	4999995
946280	105378	280537	671943	567804	432159	328065	719426	894612	053791	4999995
895621	719462	328056	462805	953710	046298	537149	671934	280573	104387	4999995
780532	678943	537194	046289	194628	805317	953701	462850	321065	219476	4999995
371946	562804	956710	805371	219467	780523	194682	043298	437159	628035	4999995
014378	820537	761943	357194	406289	593701	642850	238065	179426	985612	4999995
503719	984621	179462	238056	342195	657840	761934	820573	015387	496208	4999995
652804	496280	015378	120467	731946	268035	879523	984612	503791	347159	4999995
268053	347195	403289	914628	870532	129476	085317	596701	652840	731964	4999995
129467	231056	642805	593710	085371	914682	406298	357149	768934	870523	4999995
4999995	4999995	4999995	4999995	4999995	4999995	4999995	4999995	4999995	4999995	4999995

Example 6.5. *Let's consider a* ***7 digits cell*** *magic square of order 10 is given by*

										49999995
4371956	0537194	8946210	7895321	6280537	3719642	2104768	1053879	9462085	5628403	49999995
9462805	1053789	2805371	6719432	5678043	4321596	3280657	7194268	8946120	0537914	49999995
8956210	7194628	3280567	4628053	9537104	0462985	5371496	6719342	2805731	1043879	49999995
7805321	6789432	5371946	0462895	1946280	8053179	9537014	4628503	3210657	2194768	49999995
3719462	5628043	9567104	8053719	2194678	7805231	1946820	0432985	4371596	6280357	49999995
0143789	8205371	7619432	3571946	4062895	5937014	6428503	2380657	1794268	9856120	49999995
5037194	9846210	1794628	2380567	3421956	6578403	7619342	8205731	0153879	4962085	49999995
6528043	4962805	0153789	1204678	7319462	2680357	8795231	9846120	5037914	3471596	49999995
2680537	3471956	4032895	9146280	8705321	1294768	0853179	5967014	6528403	7319642	49999995
1294678	2310567	6428053	5937104	0853719	9146820	4062985	3571496	7689342	8705231	49999995
49999995	49999995	49999995	49999995	49999995	49999995	49999995	49999995	49999995	49999995	49999995

Example 6.6. *Let's consider a **8 digits cell** magic square of order 10 is given by*

										499999995
37195628	53719462	94621053	89532104	28053719	71964280	10476895	05387946	46208537	62840371	499999995
46280537	05378946	80537194	71943280	67804321	32159678	28065719	19426805	94612053	53791462	499999995
95621043	19462805	28056719	62805371	53710462	46298537	37149628	71934280	80573194	04387956	499999995
80532194	78943210	37194628	46289537	94628053	05317946	53701462	62850371	21065789	19476805	499999995
71946280	62804371	56710432	05371946	19467805	80523194	94682053	43298567	37159628	28035719	499999995
14378956	20537194	61943280	57194628	06289537	93701462	42850371	38065719	79426805	85612043	499999995
03719462	84621053	79462805	38056719	42195678	57840321	61934280	20573194	15387946	96208537	499999995
52804371	96280537	15378946	20467895	31946280	68035719	79523104	84612053	03791462	47159628	499999995
68053719	47195628	03289567	14628053	70532194	29476805	85317946	96701432	52840371	31964280	499999995
29467805	31056789	42805371	93710462	85371946	14682053	06298537	57149628	68934210	70523194	499999995
499999995	499999995	499999995	499999995	499999995	499999995	499999995	499999995	499999995	499999995	499999995

Example 6.7. *Let's consider a **9 digits cell** magic square of order 10 is given by*

										4999999995
437195628	053719462	894621053	789532104	628053719	371964280	210476895	105387946	946208537	562840371	4999999995
946280537	105378946	280537194	671943280	567804321	432159678	328065719	719426805	894612053	053791462	4999999995
895621043	719462805	328056719	462805371	953710462	046298537	537149628	671934280	280573194	104387956	4999999995
780532194	678943210	537194628	046289537	194628053	805317946	953701462	462850371	321065789	219476805	4999999995
371946280	562804371	956710432	805371946	219467805	780523194	194682053	043298567	437159628	628035719	4999999995
014378956	820537194	761943280	357194628	406289537	593701462	642850371	238065719	179426805	985612043	4999999995
503719462	984621053	179462805	238056719	342195678	657840321	761934280	820573194	015387946	496208537	4999999995
652804371	496280537	015378946	120467895	731946280	268035719	879523104	984612053	503791462	347159628	4999999995
268053719	347195628	403289567	914628053	870532194	129476805	085317946	596701432	652840371	731964280	4999999995
129467805	231056789	642805371	593710462	085371946	914682053	406298537	357149628	768934210	870523194	4999999995
4999999995	4999999995	4999999995	4999999995	4999999995	4999999995	4999999995	4999999995	4999999995	4999999995	4999999995

Example 6.8. *Let's consider a **10 digits cell** magic square of order 10 is given by*

										49999999995
0437195628	8053719462	7894621053	6789532104	4628053719	5371964280	3210476895	2105387946	1946208537	9562840371	49999999995
1946280537	2105378946	6280537194	5671943280	9567804321	0432159678	4328065719	3719426805	7894612053	8053791462	49999999995
7895621043	3719462805	4328056719	9462805371	8953710462	1046298537	0537149628	5671934280	6280573194	2104387956	49999999995
6780532194	5678943210	0537194628	1046289537	7194628053	2805317946	8953701462	9462850371	4321065789	3219476805	49999999995
5371946280	9562804371	8956710432	2805371946	3219467805	6780523194	7194682053	1043298567	0437159628	4628035719	49999999995
2014378956	6820537194	5761943280	0357194628	1406289537	8593701462	9642850371	4238065719	3179426805	7985612043	49999999995
8503719462	7984621053	3179462805	4238056719	0342195678	9657840321	5761934280	6820573194	2015387946	1496208537	49999999995
9652804371	1496280537	2015378946	3120467895	5731946280	4268035719	6879523104	7984612053	8503791462	0347159628	49999999995
4268053719	0347195628	1403289567	7914628053	6870532194	3129476805	2085317946	8596701432	9652840371	5731964280	49999999995
3129467805	4231056789	9642805371	8593710462	2085371946	7914682053	1406298537	0357149628	5768934210	6870523194	49999999995
49999999995	49999999995	49999999995	49999999995	49999999995	49999999995	49999999995	49999999995	49999999995	49999999995	49999999995

The magic squares with digits 3, 4, 5, 6 and 7 are obtained from the magic square of 8 digits. For details refer author's work [13].

Result 7. *The magic sums of magic squares of order 10 with 5, 6, 7, 8, 9 and 10 digits in each cell lead us to a* ***number pattern*** *given by*

Number of Digits	Magic Square Sums
4	49995
5	499995
6	4999995
7	49999995
8	499999995
9	4999999995
10	49999999995

Still, there are magic squares of order 10 with ten numbers 0 to 9, considering 3 digits in each cell. This shall be studied elsewhere.

Note 8. *We observe that there are total* $10! = 3628800$ *possibilities of writing different numbers with different digits of order 10. Thus, there are much more possibilities of writing different digits magic squares with 10 different digits in each cell. We have written only one just to have an idea. As an extension of Example 6.1 with 2 digits, below is a magic square with 3-digits* ***palindromic numbers*** *using the digits 0 to 9:*

										4995
000	989	454	616	171	737	545	868	292	323	4995
757	111	979	424	595	383	808	030	646	262	4995
414	676	222	898	353	161	787	505	939	040	4995
696	060	747	333	202	828	474	919	151	585	4995
535	303	686	767	444	212	959	191	020	878	4995
848	434	101	282	969	555	090	626	373	717	4995
272	525	393	050	818	949	666	484	707	131	4995
363	242	838	909	080	494	121	777	515	656	4995
929	797	565	141	636	070	313	252	888	404	4995
181	858	010	575	727	606	232	343	464	999	4995
4995	4995	4995	4995	4995	4995	4995	4995	4995	4995	4995

7 Final Comments

This work is continuation of author's previous work on different digits done in 2015. Most of the results appearing here are already given in [13], but we put them here with some improvements. Morever, interest here is to put the magic squares sums in **number patterns**. The sum increases as the number of digits increases. Main work is concentrated on bimagic squares of order 9. In this case we very interesting pattern. See again below:

Digits	Order 5	Order 7	Order 8	Order 9	Order 10
3	1665		3996	4995	
4	16665	31108	39996	49995	49995
5	166665	311008	399996	499995	499995
6	–	3110008	3999996	4999995	4999995
7	–	31100008	39999996	49999995	49999995
8	–	–	399999996	499999995	499999995
9	–	–	–	4999999995	4999999995
10	–	–	–	–	49999999995

Numbers Pattern Table: Numbers of Digits and Magic Sums

From the table we observe that still, we don't have magic squares with 3 digits in each cell for the magic square of orders 7 and 10. Magic squares of orders 4 and 6 is not dealt here, as in case of order 4, we have only two possibilities, i.e., for the digits 3 and 4, and for the magic squares of order 6, we have results for the digits 5 and 6, but for the digits 4 it becomes **semi-magic square**. For details see author's previous work [13]. In case of **bimagic** sums patterns, this we have obtained for the order 8 with **bimagic sums only in the rows**, and **bimagic squares sums** for the magic square order 9.

Also from the examples, we observed that the work with **palindromic types magic squares**, can be extended to get **number patterns** for the magic squares sums by increasing numbers of digits in each cell. For more details on **palindromic-type** magic squares refer author's work [9, **?**].

7.1 Author's Contributions to Magic Squares

The item-wise author's work on magic squares is as follows:

(i) **Digital Numbers** Magic Squares – [1, 2, 3, 4, 5, 6];

(ii) **Block-Wise Construction** of **Bimagic Squares** – [7];

(iii) Connections with **Genetic Tables** and **Shannon's entropy** – [8];

(iv) **Selfie** and **palindromic-type** Magic Squares – [9, 24];

(v) **Intervally Distributed** and **Block-Wise** Magic Squares – [10, 11, 12];

(vi) **Multi-digits** Magic Squares – [13];

(vii) **Perfect Square Sum** Magic Squares with **Uniformity**, **Minimum Sum** and **Pythagorean Triples** – [14, 15];

(viii) **Block-Wise** Constructions of Magic and Bimagic Squares – [16, 17, 18, 19, 23];

(ix) **Magic Crosses:** Repeated and Non Repeated Entries – [20];

(x) Representations of **Letters** and **Numbers** With Equal Sums Magic Squares of Orders 4 and 6 – [22, 23].

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