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Block-Wise Equal Sums Magic Squares of Orders $3k$ and $6k$

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Abstract

*This paper brings **block-wise** magic squares multiple of 3 and 6. The multiples of 6 means even order multiples of 3, such as, orders 12, 18, 24, 30, 36, etc. From magic squares of order 9 to 36, all the cases are of **pandiagonal** magic squares except the orders 18 and 30. The constructions are in such a way that in each case the **sub-blocks** are of same magic sums or sum of all entries are same. The work on **pandiagonal** squares of orders* $4k$ *is done by author [22]. In these cases, all the sub-blocks are of perfect **pandiagonal** magic squares of order 4 with same magic sums. In case of order 30, three different ways are given. The **pandiagonal** magic square of order 36 is done two different ways, one with 81 sub-blocks of **pandiagonal** magic squares of order 4, and second with 16 blocks of order 9 **pandiagonal** magic squares with different magic sums. This work is a combined version of author's previous two papers [23, 24] done in 2017*

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1 Introduction

Author [22], worked with **pandiagonal** magic squares of orders multiple of $4k$ in such a way that all the **sub-blocks** of order 4 in each case are of equal magic sums, and also are **pandiagonal** magic squares of order 4. The work done is for the magic squares of orders 8, 12, 16, 20, 24, 28 and 32. It is observed that for writing these magic squares just the knowledge of magic square of order 4 is sufficient. The magic square of order 4 used is Khajuraho's perfect **pandiagonal** magic square of order 4.

In this paper, we worked on magic squares of orders multiple of $3k$, such as, of orders 9, 12, 15, etc. There are two types of orders $3k$. One is odd orders, such as, orders 9, 15, 21, etc. The second is of even orders, such as, orders 12, 18, 24, etc. The second case is same as multiple of 6. The work is divided in two separate sections. One section is on odd orders magic squares, and another on even orders, i.e., multiple of 6. This work combines author's previous two works [23, 24] done in 2017.

2 Odd Order Magic Squares of Order $3k$

This section brings **block-wise pan magic squares** of order $3k$, $k \geq 3$. The constructions are in such a way that **sub-blocks** are of same magic sums. In some cases, the magic squares are pan magic squares. The example of odd order are from orders 9 to 33. The work is based on triples resulting in equal sums. For simiplicyt, let's consider a magic square of order 3:

Example 1. *A magic square of order 3 is given by*

			15
8	1	6	15
3	5	7	15
4	9	2	15
15	15	15	15

Let's organize the numbers of above Example 1 in increasing order and consider the following distribution:

Distribution 1. *Let's reorganize the members of above magic square in an increasing way:*

(1)	1	6	8	15
(2)	3	5	7	15
(3)	2	4	9	15

Structure 1. *Let's put the rows (1), (2) and (3) according to following table:*

(11)	(12)	(13)
(21)	(22)	(23)
(31)	(32)	(33)

Above distribution is useful to construct **pan magic square** of order 9. See the example below:

2.1 Pan Magic Square of Order 9

Example 2. *According Distribution 1 and Structure 1 the* ***pan magic square*** *of order 9 is given by*

		369	369	369	369	369	369	369	369	369
	8	49	66	61	24	38	36	77	10	369
369	37	63	23	12	35	76	65	7	51	369
369	78	11	34	50	64	9	22	39	62	369
369	14	28	81	67	3	53	42	56	25	369
369	52	69	2	27	41	55	80	13	30	369
369	57	26	40	29	79	15	1	54	68	369
369	20	43	60	73	18	32	48	71	4	369
369	31	75	17	6	47	70	59	19	45	369
369	72	5	46	44	58	21	16	33	74	369
	369	369	369	369	369	369	369	369	369	369

In this case, the magic sum is $S_{9\times 9} = 369$. The sum of all the entries of each 3×3 blocks are the same sums as of magic square, i.e., $S_9 = 369$. The middle block of orde 3is a magic square of order 3, and other 8 subgroups are semi-magic squares of order 3.

The magic square given in Example 2 is a combination of two **mutually orthogonal diagonal Latin squares**:

Example 3. *Two mutually orthogonal Latin squares composing magic square of order 9 given in Example 2 are given by*

(A)		45	45	45	45	45	45	45	45	45
	1	6	8	7	3	5	4	9	2	45
45	5	7	3	2	4	9	8	1	6	45
45	9	2	4	6	8	1	3	5	7	45
45	2	4	9	8	1	6	5	7	3	45
45	6	8	1	3	5	7	9	2	4	45
45	7	3	5	4	9	2	1	6	8	45
45	3	5	7	9	2	4	6	8	1	45
45	4	9	2	1	6	8	7	3	5	45
45	8	1	6	5	7	3	2	4	9	45
	45	45	45	45	45	45	45	45	45	45

(B)		45	45	45	45	45	45	45	45	45
	8	4	3	7	6	2	9	5	1	45
45	1	9	5	3	8	4	2	7	6	45
45	6	2	7	5	1	9	4	3	8	45
45	5	1	9	4	3	8	6	2	7	45
45	7	6	2	9	5	1	8	4	3	45
45	3	8	4	2	7	6	1	9	5	45
45	2	7	6	1	9	5	3	8	4	45
45	4	3	8	6	2	7	5	1	9	45
45	9	5	1	8	4	3	7	6	2	45
	45	45	45	45	45	45	45	45	45	45

Making the operation $9\times(A-1)+B$ *we get the magic square given in Example 2*

2.2 Pan Magic Square of Order 15

Distribution 2. *Let's consider the following distribution to construct* ***pan magic square*** *of order 15:*

(1)	1	6	8	12	13	40
(2)	3	5	7	11	14	40
(3)	2	4	9	10	15	40

The construction of **pan magic square** of order 15 with each sub-block of same sum is based on the following pan diagonal magic square of order 5 constructed using a pair of mutually orthogonal diagonal Latin squares:

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Example 4. *The **pan magic square** of order 5 is given by*

A		15	15	15	15	15
	1	2	3	4	5	15
15	4	5	1	2	3	15
15	2	3	4	5	1	15
15	5	1	2	3	4	15
15	3	4	5	1	2	15
	15	15	15	15	15	15

B		15	15	15	15	15
	1	4	2	5	3	15
15	2	5	3	1	4	15
15	3	1	4	2	5	15
15	4	2	5	3	1	15
15	5	3	1	4	2	15
	15	15	15	15	15	15

AB		65	65	65	65	65
	1	9	12	20	23	65
65	17	25	3	6	14	65
65	8	11	19	22	5	65
65	24	2	10	13	16	65
65	15	18	21	4	7	65
	65	65	65	65	65	65

The **pan magic square** appearing in ***AB*** is constructed according to the following formula:

$$AB := 5 \times (A-1) + B$$

where A and B are the **mutually orthogonal diagonal Latin squares** of order 5.

We shall construct 9 magic squares of order 5 using the Distribution 2 and Example 4, and put them according to Structure 1. In this case, the composition considered is as follows:

$$AB := 15 \times (A-1) + B$$

i.e., instead of 5, the multiplications is with 15.

Construction 1. *Below are 9 **pan magic squares** of order 5 constructed according to Example 4 using the Distribution 2:*

- **Block 11**

1		40	40	40	40	40
	1	6	8	12	13	40
40	12	13	1	6	8	40
40	6	8	12	13	1	40
40	13	1	6	8	12	40
40	8	12	13	1	6	40
	40	40	40	40	40	40

1		40	40	40	40	40
	1	12	6	13	8	40
40	6	13	8	1	12	40
40	8	1	12	6	13	40
40	12	6	13	8	1	40
40	13	8	1	12	6	40
	40	40	40	40	40	40

11		565	565	565	565	565
	1	87	111	178	188	565
565	171	193	8	76	117	565
565	83	106	177	186	13	565
565	192	6	88	113	166	565
565	118	173	181	12	81	565
	565	565	565	565	565	565

- **Block 12**

1		40	40	40	40	40
	1	6	8	12	13	40
40	12	13	1	6	8	40
40	6	8	12	13	1	40
40	13	1	6	8	12	40
40	8	12	13	1	6	40
	40	40	40	40	40	40

2		40	40	40	40	40
	3	11	5	14	7	40
40	5	14	7	3	11	40
40	7	3	11	5	14	40
40	11	5	14	7	3	40
40	14	7	3	11	5	40
	40	40	40	40	40	40

12		565	565	565	565	565
	3	86	110	179	187	565
565	170	194	7	78	116	565
565	82	108	176	185	14	565
565	191	5	89	112	168	565
565	119	172	183	11	80	565
	565	565	565	565	565	565

- **Block 13**

1		40	40	40	40	40
	1	6	8	12	13	40
40	12	13	1	6	8	40
40	6	8	12	13	1	40
40	13	1	6	8	12	40
40	8	12	13	1	6	40
	40	40	40	40	40	40

3		40	40	40	40	40
	2	10	4	15	9	40
40	4	15	9	2	10	40
40	9	2	10	4	15	40
40	10	4	15	9	2	40
40	15	9	2	10	4	40
	40	40	40	40	40	40

13		565	565	565	565	565
	2	85	109	180	189	565
565	169	195	9	77	115	565
565	84	107	175	184	15	565
565	190	4	90	114	167	565
565	120	174	182	10	79	565
	565	565	565	565	565	565

- **Block 21**

2		40	40	40	40	40
	3	5	7	11	14	40
40	11	14	3	5	7	40
40	5	7	11	14	3	40
40	14	3	5	7	11	40
40	7	11	14	3	5	40
	40	40	40	40	40	40

1		40	40	40	40	40
	1	12	6	13	8	40
40	6	13	8	1	12	40
40	8	1	12	6	13	40
40	12	6	13	8	1	40
40	13	8	1	12	6	40
	40	40	40	40	40	40

21		565	565	565	565	565
	31	72	96	163	203	565
565	156	208	38	61	102	565
565	68	91	162	201	43	565
565	207	36	73	98	151	565
565	103	158	196	42	66	565
	565	565	565	565	565	565

- **Block 22**

2		40	40	40	40	40
	3	5	7	11	14	40
40	11	14	3	5	7	40
40	5	7	11	14	3	40
40	14	3	5	7	11	40
40	7	11	14	3	5	40
	40	40	40	40	40	40

2		40	40	40	40	40
	3	11	5	14	7	40
40	5	14	7	3	11	40
40	7	3	11	5	14	40
40	11	5	14	7	3	40
40	14	7	3	11	5	40
	40	40	40	40	40	40

22		565	565	565	565	565
	33	71	95	164	202	565
565	155	209	37	63	101	565
565	67	93	161	200	44	565
565	206	35	74	97	153	565
565	104	157	198	41	65	565
	565	565	565	565	565	565

- **Block 23**

2		40	40	40	40	40
	3	5	7	11	14	40
40	11	14	3	5	7	40
40	5	7	11	14	3	40
40	14	3	5	7	11	40
40	7	11	14	3	5	40
	40	40	40	40	40	40

3		40	40	40	40	40
	2	10	4	15	9	40
40	4	15	9	2	10	40
40	9	2	10	4	15	40
40	10	4	15	9	2	40
40	15	9	2	10	4	40
	40	40	40	40	40	40

23		565	565	565	565	565
	32	70	94	165	204	565
565	154	210	39	62	100	565
565	69	92	160	199	45	565
565	205	34	75	99	152	565
565	105	159	197	40	64	565
	565	565	565	565	565	565

- **Block 31**

3		40	40	40	40	40
	2	4	9	10	15	40
40	10	15	2	4	9	40
40	4	9	10	15	2	40
40	15	2	4	9	10	40
40	9	10	15	2	4	40
	40	40	40	40	40	40

1		40	40	40	40	40
	1	12	6	13	8	40
40	6	13	8	1	12	40
40	8	1	12	6	13	40
40	12	6	13	8	1	40
40	13	8	1	12	6	40
	40	40	40	40	40	40

31		565	565	565	565	565
	16	57	126	148	218	565
565	141	223	23	46	132	565
565	53	121	147	216	28	565
565	222	21	58	128	136	565
565	133	143	211	27	51	565
	565	565	565	565	565	565

- **Block 32**

3		40	40	40	40	40
	2	4	9	10	15	40
40	10	15	2	4	9	40
40	4	9	10	15	2	40
40	15	2	4	9	10	40
40	9	10	15	2	4	40
	40	40	40	40	40	40

2		40	40	40	40	40
	3	11	5	14	7	40
40	5	14	7	3	11	40
40	7	3	11	5	14	40
40	11	5	14	7	3	40
40	14	7	3	11	5	40
	40	40	40	40	40	40

32		565	565	565	565	565
	18	56	125	149	217	565
565	140	224	22	48	131	565
565	52	123	146	215	29	565
565	221	20	59	127	138	565
565	134	142	213	26	50	565
	565	565	565	565	565	565

- **Block 33**

3		40	40	40	40	40
	2	4	9	10	15	40
40	10	15	2	4	9	40
40	4	9	10	15	2	40
40	15	2	4	9	10	40
40	9	10	15	2	4	40
	40	40	40	40	40	40

3		40	40	40	40	40
	2	10	4	15	9	40
40	4	15	9	2	10	40
40	9	2	10	4	15	40
40	10	4	15	9	2	40
40	15	9	2	10	4	40
	40	40	40	40	40	40

33		565	565	565	565	565
	17	55	124	150	219	565
565	139	225	24	47	130	565
565	54	122	145	214	30	565
565	220	19	60	129	137	565
565	135	144	212	25	49	565
	565	565	565	565	565	565

In all the 9 blocks, the composition is applied by use of following formula:

$$AB := 15 \times (A - 1) + B$$

Combining the above 9 **pan magic squares** of order 5 according to structure 1 we get a **pan magic square** of order 15 given in the example below.

Example 5. *According Distribution 2, Structure 1 and 9* ***pan magic squares*** *given above , we have a* ***pan magic square*** *of order 15 given by*

		1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695
	1	87	111	178	188	3	86	110	179	187	2	85	109	180	189	1695
1695	171	193	8	76	117	170	194	7	78	116	169	195	9	77	115	1695
1695	83	106	177	186	13	82	108	176	185	14	84	107	175	184	15	1695
1695	192	6	88	113	166	191	5	89	112	168	190	4	90	114	167	1695
1695	118	173	181	12	81	119	172	183	11	80	120	174	182	10	79	1695
1695	31	72	96	163	203	33	71	95	164	202	32	70	94	165	204	1695
1695	156	208	38	61	102	155	209	37	63	101	154	210	39	62	100	1695
1695	68	91	162	201	43	67	93	161	200	44	69	92	160	199	45	1695
1695	207	36	73	98	151	206	35	74	97	153	205	34	75	99	152	1695
1695	103	158	196	42	66	104	157	198	41	65	105	159	197	40	64	1695
1695	16	57	126	148	218	18	56	125	149	217	17	55	124	150	219	1695
1695	141	223	23	46	132	140	224	22	48	131	139	225	24	47	130	1695
1695	53	121	147	216	28	52	123	146	215	29	54	122	145	214	30	1695
1695	222	21	58	128	136	221	20	59	127	138	220	19	60	129	137	1695
1695	133	143	211	27	51	134	142	213	26	50	135	144	212	25	49	1695
	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695

In this case, the magic sum is $S_{15\times 15} = 1695$. Each 5×5 block is a **pan magic square** of order 5 with the same magic sum $S_{5\times 5} = 565$.

2.3 Pan Magic Square of Order 21

Distribution 3. *Let's consider the following distribution to construct **pan magic square** of order 21:*

1	1	6	8	12	13	182	19	77
2	3	5	7	11	14	17	20	77
3	2	4	9	10	15	16	21	77

The construction of **pan magic square** of order 21 with each sub-block of same sum is based on the following pan diagonal magic square of order 7 constructed using a pair of mutually orthogonal diagonal Latin squares:

Example 6. *The **pan diagonal magic square of order 7** is given by*

A		28	28	28	28	28	28	28
	1	2	3	4	5	6	7	28
28	6	7	1	2	3	4	5	28
28	4	5	6	7	1	2	3	28
28	2	3	4	5	6	7	1	28
28	7	1	2	3	4	5	6	28
28	5	6	7	1	2	3	4	28
28	3	4	5	6	7	1	2	28
	28	28	28	28	28	28	28	28

B		28	28	28	28	28	28	28
	1	2	3	4	5	6	7	28
28	5	6	7	1	2	3	4	28
28	2	3	4	5	6	7	1	28
28	6	7	1	2	3	4	5	28
28	3	4	5	6	7	1	2	28
28	7	1	2	3	4	5	6	28
28	4	5	6	7	1	2	3	28
	28	28	28	28	28	28	28	28

AB		175	175	175	175	175	175	175
	1	9	17	25	33	41	49	175
175	40	48	7	8	16	24	32	175
175	23	31	39	47	6	14	15	175
175	13	21	22	30	38	46	5	175
175	45	4	12	20	28	29	37	175
175	35	36	44	3	11	19	27	175
175	18	26	34	42	43	2	10	175
	175	175	175	175	175	175	175	175

The **pan magic square** appearing in ***AB*** is constructed according to the following formula:

$$AB := 7 \times (A - 1) + B$$

where A and B are the **mutually orthogonal diagonal Latin squares** of order 7.

We shall construct 9 magic squares of order 7 using the Distribution 3 and Example 6, and put them according to Structure 1. In this case, the composition considered is as follows:

$$AB := 21 \times (A - 1) + B$$

i.e., instead of 7, the multiplications is with 21.

Construction 2. *Below are 9* ***pan magic squares*** *of order 7 constructed according to Example 6 using the Distribution 3:*

- **Block 11**

1		77	77	77	77	77	77	77
	1	6	8	12	13	18	19	77
77	18	19	1	6	8	12	13	77
77	12	13	18	19	1	6	8	77
77	6	8	12	13	18	19	1	77
77	19	1	6	8	12	13	18	77
77	13	18	19	1	6	8	12	77
77	8	12	13	18	19	1	6	77
	77	77	77	77	77	77	77	77

1		77	77	77	77	77	77	77
	1	6	8	12	13	18	19	77
77	13	18	19	1	6	8	12	77
77	6	8	12	13	18	19	1	77
77	18	19	1	6	8	12	13	77
77	8	12	13	18	19	1	6	77
77	19	1	6	8	12	13	18	77
77	12	13	18	19	1	6	8	77
	77	77	77	77	77	77	77	77

11		1547	1547	1547	1547	1547	1547	1547
	1	111	155	243	265	375	397	1547
1547	370	396	19	106	153	239	264	1547
1547	237	260	369	391	18	124	148	1547
1547	123	166	232	258	365	390	13	1547
1547	386	12	118	165	250	253	363	1547
1547	271	358	384	8	117	160	249	1547
1547	159	244	270	376	379	6	113	1547
	1547	1547	1547	1547	1547	1547	1547	1547

- **Block 12**

(1)		77	77	77	77	77	77	77
	1	6	8	12	13	18	19	77
77	18	19	1	6	8	12	13	77
77	12	13	18	19	1	6	8	77
77	6	8	12	13	18	19	1	77
77	19	1	6	8	12	13	18	77
77	13	18	19	1	6	8	12	77
77	8	12	13	18	19	1	6	77
	77	77	77	77	77	77	77	77

(2)		77	77	77	77	77	77	77
	3	5	7	11	14	17	20	77
77	14	17	20	3	5	7	11	77
77	5	7	11	14	17	20	3	77
77	17	20	3	5	7	11	14	77
77	7	11	14	17	20	3	5	77
77	20	3	5	7	11	14	17	77
77	11	14	17	20	3	5	7	77
	77	77	77	77	77	77	77	77

(12)		1547	1547	1547	1547	1547	1547	1547
	3	110	154	242	266	374	398	1547
1547	371	395	20	108	152	238	263	1547
1547	236	259	368	392	17	125	150	1547
1547	122	167	234	257	364	389	14	1547
1547	385	11	119	164	251	255	362	1547
1547	272	360	383	7	116	161	248	1547
1547	158	245	269	377	381	5	112	1547
	1547	1547	1547	1547	1547	1547	1547	1547

- **Block 13**

(1)		77	77	77	77	77	77	77
	1	6	8	12	13	18	19	77
77	18	19	1	6	8	12	13	77
77	12	13	18	19	1	6	8	77
77	6	8	12	13	18	19	1	77
77	19	1	6	8	12	13	18	77
77	13	18	19	1	6	8	12	77
77	8	12	13	18	19	1	6	77
	77	77	77	77	77	77	77	77

(3)		77	77	77	77	77	77	77
	2	4	9	10	15	16	21	77
77	15	16	21	2	4	9	10	77
77	4	9	10	15	16	21	2	77
77	16	21	2	4	9	10	15	77
77	9	10	15	16	21	2	4	77
77	21	2	4	9	10	15	16	77
77	10	15	16	21	2	4	9	77
	77	77	77	77	77	77	77	77

(13)		1547	1547	1547	1547	1547	1547	1547
	2	109	156	241	267	373	399	1547
1547	372	394	21	107	151	240	262	1547
1547	235	261	367	393	16	126	149	1547
1547	121	168	233	256	366	388	15	1547
1547	387	10	120	163	252	254	361	1547
1547	273	359	382	9	115	162	247	1547
1547	157	246	268	378	380	4	114	1547
	1547	1547	1547	1547	1547	1547	1547	1547

- **Block 21**

(2)		77	77	77	77	77	77	77
	3	5	7	11	14	17	20	77
77	17	20	3	5	7	11	14	77
77	11	14	17	20	3	5	7	77
77	5	7	11	14	17	20	3	77
77	20	3	5	7	11	14	17	77
77	14	17	20	3	5	7	11	77
77	7	11	14	17	20	3	5	77
	77	77	77	77	77	77	77	77

(1)		77	77	77	77	77	77	77
	1	6	8	12	13	18	19	77
77	13	18	19	1	6	8	12	77
77	6	8	12	13	18	19	1	77
77	18	19	1	6	8	12	13	77
77	8	12	13	18	19	1	6	77
77	19	1	6	8	12	13	18	77
77	12	13	18	19	1	6	8	77
	77	77	77	77	77	77	77	77

(21)		1547	1547	1547	1547	1547	1547	1547
	43	90	134	222	286	354	418	1547
1547	349	417	61	85	132	218	285	1547
1547	216	281	348	412	60	103	127	1547
1547	102	145	211	279	344	411	55	1547
1547	407	54	97	144	229	274	342	1547
1547	292	337	405	50	96	139	228	1547
1547	138	223	291	355	400	48	92	1547
	1547	1547	1547	1547	1547	1547	1547	1547

- **Block 22**

(2)		77	77	77	77	77	77	77
	3	5	7	11	14	17	20	77
77	17	20	3	5	7	11	14	77
77	11	14	17	20	3	5	7	77
77	5	7	11	14	17	20	3	77
77	20	3	5	7	11	14	17	77
77	14	17	20	3	5	7	11	77
77	7	11	14	17	20	3	5	77
	77	77	77	77	77	77	77	77

(2)		77	77	77	77	77	77	77
	3	5	7	11	14	17	20	77
77	14	17	20	3	5	7	11	77
77	5	7	11	14	17	20	3	77
77	17	20	3	5	7	11	14	77
77	7	11	14	17	20	3	5	77
77	20	3	5	7	11	14	17	77
77	11	14	17	20	3	5	7	77
	77	77	77	77	77	77	77	77

(22)		1547	1547	1547	1547	1547	1547	1547
	45	89	133	221	287	353	419	1547
1547	350	416	62	87	131	217	284	1547
1547	215	280	347	413	59	104	129	1547
1547	101	146	213	278	343	410	56	1547
1547	406	53	98	143	230	276	341	1547
1547	293	339	404	49	95	140	227	1547
1547	137	224	290	356	402	47	91	1547
	1547	1547	1547	1547	1547	1547	1547	1547

- **Block 23**

(2)		77	77	77	77	77	77	77
	3	5	7	11	14	17	20	77
77	17	20	3	5	7	11	14	77
77	11	14	17	20	3	5	7	77
77	5	7	11	14	17	20	3	77
77	20	3	5	7	11	14	17	77
77	14	17	20	3	5	7	11	77
77	7	11	14	17	20	3	5	77
	77	77	77	77	77	77	77	77

(3)		77	77	77	77	77	77	77
	2	4	9	10	15	16	21	77
77	15	16	21	2	4	9	10	77
77	4	9	10	15	16	21	2	77
77	16	21	2	4	9	10	15	77
77	9	10	15	16	21	2	4	77
77	21	2	4	9	10	15	16	77
77	10	15	16	21	2	4	9	77
	77	77	77	77	77	77	77	77

(23)		1547	1547	1547	1547	1547	1547	1547
	44	88	135	220	288	352	420	1547
1547	351	415	63	86	130	219	283	1547
1547	214	282	346	414	58	105	128	1547
1547	100	147	212	277	345	409	57	1547
1547	408	52	99	142	231	275	340	1547
1547	294	338	403	51	94	141	226	1547
1547	136	225	289	357	401	46	93	1547
	1547	1547	1547	1547	1547	1547	1547	1547

- **Block 31**

(3)		77	77	77	77	77	77	77
	2	4	9	10	15	16	21	77
77	16	21	2	4	9	10	15	77
77	10	15	16	21	2	4	9	77
77	4	9	10	15	16	21	2	77
77	21	2	4	9	10	15	16	77
77	15	16	21	2	4	9	10	77
77	9	10	15	16	21	2	4	77
	77	77	77	77	77	77	77	77

(1)		77	77	77	77	77	77	77
	1	6	8	12	13	18	19	77
77	13	18	19	1	6	8	12	77
77	6	8	12	13	18	19	1	77
77	18	19	1	6	8	12	13	77
77	8	12	13	18	19	1	6	77
77	19	1	6	8	12	13	18	77
77	12	13	18	19	1	6	8	77
	77	77	77	77	77	77	77	77

(31)		1547	1547	1547	1547	1547	1547	1547
	22	69	176	201	307	333	439	1547
1547	328	438	40	64	174	197	306	1547
1547	195	302	327	433	39	82	169	1547
1547	81	187	190	300	323	432	34	1547
1547	428	33	76	186	208	295	321	1547
1547	313	316	426	29	75	181	207	1547
1547	180	202	312	334	421	27	71	1547
	1547	1547	1547	1547	1547	1547	1547	1547

- **Block 32**

3		77	77	77	77	77	77	77
	2	4	9	10	15	16	21	77
77	16	21	2	4	9	10	15	77
77	10	15	16	21	2	4	9	77
77	4	9	10	15	16	21	2	77
77	21	2	4	9	10	15	16	77
77	15	16	21	2	4	9	10	77
77	9	10	15	16	21	2	4	77
	77	77	77	77	77	77	77	77

2		77	77	77	77	77	77	77
	3	5	7	11	14	17	20	77
77	14	17	20	3	5	7	11	77
77	5	7	11	14	17	20	3	77
77	17	20	3	5	7	11	14	77
77	7	11	14	17	20	3	5	77
77	20	3	5	7	11	14	17	77
77	11	14	17	20	3	5	7	77
	77	77	77	77	77	77	77	77

32		1547	1547	1547	1547	1547	1547	1547
	24	68	175	200	308	332	440	1547
1547	329	437	41	66	173	196	305	1547
1547	194	301	326	434	38	83	171	1547
1547	80	188	192	299	322	431	35	1547
1547	427	32	77	185	209	297	320	1547
1547	314	318	425	28	74	182	206	1547
1547	179	203	311	335	423	26	70	1547
	1547	1547	1547	1547	1547	1547	1547	1547

- **Block 33**

3		77	77	77	77	77	77	77
	2	4	9	10	15	16	21	77
77	16	21	2	4	9	10	15	77
77	10	15	16	21	2	4	9	77
77	4	9	10	15	16	21	2	77
77	21	2	4	9	10	15	16	77
77	15	16	21	2	4	9	10	77
77	9	10	15	16	21	2	4	77
	77	77	77	77	77	77	77	77

3		77	77	77	77	77	77	77
	2	4	9	10	15	16	21	77
77	15	16	21	2	4	9	10	77
77	4	9	10	15	16	21	2	77
77	16	21	2	4	9	10	15	77
77	9	10	15	16	21	2	4	77
77	21	2	4	9	10	15	16	77
77	10	15	16	21	2	4	9	77
	77	77	77	77	77	77	77	77

33		1547	1547	1547	1547	1547	1547	1547
	23	67	177	199	309	331	441	1547
1547	330	436	42	65	172	198	304	1547
1547	193	303	325	435	37	84	170	1547
1547	79	189	191	298	324	430	36	1547
1547	429	31	78	184	210	296	319	1547
1547	315	317	424	30	73	183	205	1547
1547	178	204	310	336	422	25	72	1547
	1547	1547	1547	1547	1547	1547	1547	1547

In all the 9 blocks, the composition is applied by use of following formula:

$$AB := 15 \times (A-1) + B$$

Combining the above 9 **pan magic squares** of order 7 according to structure 1 we get a **pan magic square** of order 21 given in the example below.

Example 7. *According Distribution 3, Structure 1 and 9* ***pan magic squares*** *given above, we have a* ***pan magic square*** *of order 21 given by*

		4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641
	1	111	155	243	265	375	397	3	110	154	242	266	374	398	2	109	156	241	267	373	399	4641
4641	370	396	19	106	153	239	264	371	395	20	108	152	238	263	372	394	21	107	151	240	262	4641
4641	237	260	369	391	18	124	148	236	259	368	392	17	125	150	235	261	367	393	16	126	149	4641
4641	123	166	232	258	365	390	13	122	167	234	257	364	389	14	121	168	233	256	366	388	15	4641
4641	386	12	118	165	250	253	363	385	11	119	164	251	255	362	387	10	120	163	252	254	361	4641
4641	271	358	384	8	117	160	249	272	360	383	7	116	161	248	273	359	382	9	115	162	247	4641
4641	159	244	270	376	379	6	113	158	245	269	377	381	5	112	157	246	268	378	380	4	114	4641
4641	43	90	134	222	286	354	418	45	89	133	221	287	353	419	44	88	135	220	288	352	420	4641
4641	349	417	61	85	132	218	285	350	416	62	87	131	217	284	351	415	63	86	130	219	283	4641
4641	216	281	348	412	60	103	127	215	280	347	413	59	104	129	214	282	346	414	58	105	128	4641
4641	102	145	211	279	344	411	55	101	146	213	278	343	410	56	100	147	212	277	345	409	57	4641
4641	407	54	97	144	229	274	342	406	53	98	143	230	276	341	408	52	99	142	231	275	340	4641
4641	292	337	405	50	96	139	228	293	339	404	49	95	140	227	294	338	403	51	94	141	226	4641
4641	138	223	291	355	400	48	92	137	224	290	356	402	47	91	136	225	289	357	401	46	93	4641
4641	22	69	176	201	307	333	439	24	68	175	200	308	332	440	23	67	177	199	309	331	441	4641
4641	328	438	40	64	174	197	306	329	437	41	66	173	196	305	330	436	42	65	172	198	304	4641
4641	195	302	327	433	39	82	169	194	301	326	434	38	83	171	193	303	325	435	37	84	170	4641
4641	81	187	190	300	323	432	34	80	188	192	299	322	431	35	79	189	191	298	324	430	36	4641
4641	428	33	76	186	208	295	321	427	32	77	185	209	297	320	429	31	78	184	210	296	319	4641
4641	313	316	426	29	75	181	207	314	318	425	28	74	182	206	315	317	424	30	73	183	205	4641
4641	180	202	312	334	421	27	71	179	203	311	335	423	26	70	178	204	310	336	422	25	72	4641
	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641

2.4 Pan Magic Square of Order 27

Distribution 4. *Let's consider the following distribution to construct* ***pan magic square*** *of order 27:*

1	1	6	8	12	13	18	19	24	25	126
2	3	5	7	11	14	17	20	23	26	126
3	2	4	9	10	15	16	21	22	27	126

The construction of **pan magic square** of order 21 with each sub-block of same sum is based on the following pan diagonal magic square of order 7 constructed using a pair of mutually orthogonal diagonal Latin squares:

Example 8. *The* ***pan diagonal magic square of order 9*** *is given by*

(A)		45	45	45	45	45	45	45	45	45
	1	6	8	7	3	5	4	9	2	45
45	5	7	3	2	4	9	8	1	6	45
45	9	2	4	6	8	1	3	5	7	45
45	2	4	9	8	1	6	5	7	3	45
45	6	8	1	3	5	7	9	2	4	45
45	7	3	5	4	9	2	1	6	8	45
45	3	5	7	9	2	4	6	8	1	45
45	4	9	2	1	6	8	7	3	5	45
45	8	1	6	5	7	3	2	4	9	45
	45	45	45	45	45	45	45	45	45	45

(B)		45	45	45	45	45	45	45	45	45
	8	4	3	7	6	2	9	5	1	45
45	1	9	5	3	8	4	2	7	6	45
45	6	2	7	5	1	9	4	3	8	45
45	5	1	9	4	3	8	6	2	7	45
45	7	6	2	9	5	1	8	4	3	45
45	3	8	4	2	7	6	1	9	5	45
45	2	7	6	1	9	5	3	8	4	45
45	4	3	8	6	2	7	5	1	9	45
45	9	5	1	8	4	3	7	6	2	45
	45	45	45	45	45	45	45	45	45	45

(AB)		369	369	369	369	369	369	369	369	369
	8	49	66	61	24	38	36	77	10	369
369	37	63	23	12	35	76	65	7	51	369
369	78	11	34	50	64	9	22	39	62	369
369	14	28	81	67	3	53	42	56	25	369
369	52	69	2	27	41	55	80	13	30	369
369	57	26	40	29	79	15	1	54	68	369
369	20	43	60	73	18	32	48	71	4	369
369	31	75	17	6	47	70	59	19	45	369
369	72	5	46	44	58	21	16	33	74	369
	369	369	369	369	369	369	369	369	369	369

The **pan magic square** appearing in ***AB*** is constructed according to the following formula:

$$AB := 9 \times (A - 1) + B$$

where A and B are the **mutually orthogonal diagonal Latin squares** of order 9. This is the same Example 2 as given above. We have written it again to bring pan magic of order 27.

We shall construct 9 magic squares of order 9 using the distribution 4 and Example 8, and put them according to Structure 1. In this case the composition considered is as follows:

$$AB := 27 \times (A - 1) + B$$

i.e., instead of 9, the multiplications is with 27.

Construction 3. *Below are 9 magic squares of order 9 constructed according to Example 8 using the Distribution 4:*

- **Block 11**

1									126
1	18	22	19	8	13	12	27	6	126
13	19	8	6	12	27	22	1	18	126
27	6	12	18	22	1	8	13	19	126
6	12	27	22	1	18	13	19	8	126
18	22	1	8	13	19	27	6	12	126
19	8	13	12	27	6	1	18	22	126
8	13	19	27	6	12	18	22	1	126
12	27	6	1	18	22	19	8	13	126
22	1	18	13	19	8	6	12	27	126
126	126	126	126	126	126	126	126	126	126

1									126
22	12	8	19	18	6	27	13	1	126
1	27	13	8	22	12	6	19	18	126
18	6	19	13	1	27	12	8	22	126
13	1	27	12	8	22	18	6	19	126
19	18	6	27	13	1	22	12	8	126
8	22	12	6	19	18	1	27	13	126
6	19	18	1	27	13	8	22	12	126
12	8	22	18	6	19	13	1	27	126
27	13	1	22	12	8	19	18	6	126
126	126	126	126	126	126	126	126	126	126

11									3285
22	471	575	505	207	330	324	715	136	3285
325	513	202	143	319	714	573	19	477	3285
720	141	316	472	568	27	201	332	508	3285
148	298	729	579	8	481	342	492	208	3285
478	585	6	216	337	487	724	147	305	3285
494	211	336	303	721	153	1	486	580	3285
195	343	504	703	162	310	467	589	12	3285
309	710	157	18	465	586	499	190	351	3285
594	13	460	346	498	197	154	315	708	3285
3285	3285	3285	3285	3285	3285	3285	3285	3285	3285

- **Block 12**

1									126
1	18	22	19	8	13	12	27	6	126
13	19	8	6	12	27	22	1	18	126
27	6	12	18	22	1	8	13	19	126
6	12	27	22	1	18	13	19	8	126
18	22	1	8	13	19	27	6	12	126
19	8	13	12	27	6	1	18	22	126
8	13	19	27	6	12	18	22	1	126
12	27	6	1	18	22	19	8	13	126
22	1	18	13	19	8	6	12	27	126
126	126	126	126	126	126	126	126	126	126

2									126
23	11	7	20	17	5	26	14	3	126
3	26	14	7	23	11	5	20	17	126
17	5	20	14	3	26	11	7	23	126
14	3	26	11	7	23	17	5	20	126
20	17	5	26	14	3	23	11	7	126
7	23	11	5	20	17	3	26	14	126
5	20	17	3	26	14	7	23	11	126
11	7	23	17	5	20	14	3	26	126
26	14	3	23	11	7	20	17	5	126
126	126	126	126	126	126	126	126	126	126

12									3285
23	470	574	506	206	329	323	716	138	3285
327	512	203	142	320	713	572	20	476	3285
719	140	317	473	570	26	200	331	509	3285
149	300	728	578	7	482	341	491	209	3285
479	584	5	215	338	489	725	146	304	3285
493	212	335	302	722	152	3	485	581	3285
194	344	503	705	161	311	466	590	11	3285
308	709	158	17	464	587	500	192	350	3285
593	14	462	347	497	196	155	314	707	3285
3285	3285	3285	3285	3285	3285	3285	3285	3285	3285

- **Block 13**

1									126
1	18	22	19	8	13	12	27	6	126
13	19	8	6	12	27	22	1	18	126
27	6	12	18	22	1	8	13	19	126
6	12	27	22	1	18	13	19	8	126
18	22	1	8	13	19	27	6	12	126
19	8	13	12	27	6	1	18	22	126
8	13	19	27	6	12	18	22	1	126
12	27	6	1	18	22	19	8	13	126
22	1	18	13	19	8	6	12	27	126
126	126	126	126	126	126	126	126	126	126

3									126
24	10	9	21	16	4	25	15	2	126
2	25	15	9	24	10	4	21	16	126
16	4	21	15	2	25	10	9	24	126
15	2	25	10	9	24	16	4	21	126
21	16	4	25	15	2	24	10	9	126
9	24	10	4	21	16	2	25	15	126
4	21	16	2	25	15	9	24	10	126
10	9	24	16	4	21	15	2	25	126
25	15	2	24	10	9	21	16	4	126
126	126	126	126	126	126	126	126	126	126

13									3285
24	469	576	507	205	328	322	717	137	3285
326	511	204	144	321	712	571	21	475	3285
718	139	318	474	569	25	199	333	510	3285
150	299	727	577	9	483	340	490	210	3285
480	583	4	214	339	488	726	145	306	3285
495	213	334	301	723	151	2	484	582	3285
193	345	502	704	160	312	468	591	10	3285
307	711	159	16	463	588	501	191	349	3285
592	15	461	348	496	198	156	313	706	3285
3285	3285	3285	3285	3285	3285	3285	3285	3285	3285

- **Block 21**

2									126
3	17	23	20	7	14	11	26	5	126
14	20	7	5	11	26	23	3	17	126
26	5	11	17	23	3	7	14	20	126
5	11	26	23	3	17	14	20	7	126
17	23	3	7	14	20	26	5	11	126
20	7	14	11	26	5	3	17	23	126
7	14	20	26	5	11	17	23	3	126
11	26	5	3	17	23	20	7	14	126
23	3	17	14	20	7	5	11	26	126
126	126	126	126	126	126	126	126	126	126

1									126
22	12	8	19	18	6	27	13	1	126
1	27	13	8	22	12	6	19	18	126
18	6	19	13	1	27	12	8	22	126
13	1	27	12	8	22	18	6	19	126
19	18	6	27	13	1	22	12	8	126
8	22	12	6	19	18	1	27	13	126
6	19	18	1	27	13	8	22	12	126
12	8	22	18	6	19	13	1	27	126
27	13	1	22	12	8	19	18	6	126
126	126	126	126	126	126	126	126	126	126

21									3285
76	444	602	532	180	357	297	688	109	3285
352	540	175	116	292	687	600	73	450	3285
693	114	289	445	595	81	174	359	535	3285
121	271	702	606	62	454	369	519	181	3285
451	612	60	189	364	514	697	120	278	3285
521	184	363	276	694	126	55	459	607	3285
168	370	531	676	135	283	440	616	66	3285
282	683	130	72	438	613	526	163	378	3285
621	67	433	373	525	170	127	288	681	3285
3285	3285	3285	3285	3285	3285	3285	3285	3285	3285

- **Block 22**

2									126
3	17	23	20	7	14	11	26	5	126
14	20	7	5	11	26	23	3	17	126
26	5	11	17	23	3	7	14	20	126
5	11	26	23	3	17	14	20	7	126
17	23	3	7	14	20	26	5	11	126
20	7	14	11	26	5	3	17	23	126
7	14	20	26	5	11	17	23	3	126
11	26	5	3	17	23	20	7	14	126
23	3	17	14	20	7	5	11	26	126
126	126	126	126	126	126	126	126	126	126

2									126
23	11	7	20	17	5	26	14	3	126
3	26	14	7	23	11	5	20	17	126
17	5	20	14	3	26	11	7	23	126
14	3	26	11	7	23	17	5	20	126
20	17	5	26	14	3	23	11	7	126
7	23	11	5	20	17	3	26	14	126
5	20	17	3	26	14	7	23	11	126
11	7	23	17	5	20	14	3	26	126
26	14	3	23	11	7	20	17	5	126
126	126	126	126	126	126	126	126	126	126

22									3285
77	443	601	533	179	356	296	689	111	3285
354	539	176	115	293	686	599	74	449	3285
692	113	290	446	597	80	173	358	536	3285
122	273	701	605	61	455	368	518	182	3285
452	611	59	188	365	516	698	119	277	3285
520	185	362	275	695	125	57	458	608	3285
167	371	530	678	134	284	439	617	65	3285
281	682	131	71	437	614	527	165	377	3285
620	68	435	374	524	169	128	287	680	3285
3285	3285	3285	3285	3285	3285	3285	3285	3285	3285

- **Block 23**

2									126
3	17	23	20	7	14	11	26	5	126
14	20	7	5	11	26	23	3	17	126
26	5	11	17	23	3	7	14	20	126
5	11	26	23	3	17	14	20	7	126
17	23	3	7	14	20	26	5	11	126
20	7	14	11	26	5	3	17	23	126
7	14	20	26	5	11	17	23	3	126
11	26	5	3	17	23	20	7	14	126
23	3	17	14	20	7	5	11	26	126
126	126	126	126	126	126	126	126	126	126

2									126
24	10	9	21	16	4	25	15	2	126
2	25	15	9	24	10	4	21	16	126
16	4	21	15	2	25	10	9	24	126
15	2	25	10	9	24	16	4	21	126
21	16	4	25	15	2	24	10	9	126
9	24	10	4	21	16	2	25	15	126
4	21	16	2	25	15	9	24	10	126
10	9	24	16	4	21	15	2	25	126
25	15	2	24	10	9	21	16	4	126
126	126	126	126	126	126	126	126	126	126

23									3285
78	442	603	534	178	355	295	690	110	3285
353	538	177	117	294	685	598	75	448	3285
691	112	291	447	596	79	172	360	537	3285
123	272	700	604	63	456	367	517	183	3285
453	610	58	187	366	515	699	118	279	3285
522	186	361	274	696	124	56	457	609	3285
166	372	529	677	133	285	441	618	64	3285
280	684	132	70	436	615	528	164	376	3285
619	69	434	375	523	171	129	286	679	3285
3285	3285	3285	3285	3285	3285	3285	3285	3285	3285

- **Block 31**

3									126
2	16	24	21	9	15	10	25	4	126
15	21	9	4	10	25	24	2	16	126
25	4	10	16	24	2	9	15	21	126
4	10	25	24	2	16	15	21	9	126
16	24	2	9	15	21	25	4	10	126
21	9	15	10	25	4	2	16	24	126
9	15	21	25	4	10	16	24	2	126
10	25	4	2	16	24	21	9	15	126
24	2	16	15	21	9	4	10	25	126
126	126	126	126	126	126	126	126	126	126

1									126
22	12	8	19	18	6	27	13	1	126
1	27	13	8	22	12	6	19	18	126
18	6	19	13	1	27	12	8	22	126
13	1	27	12	8	22	18	6	19	126
19	18	6	27	13	1	22	12	8	126
8	22	12	6	19	18	1	27	13	126
6	19	18	1	27	13	8	22	12	126
12	8	22	18	6	19	13	1	27	126
27	13	1	22	12	8	19	18	6	126
126	126	126	126	126	126	126	126	126	126

31									3285
49	417	629	559	234	384	270	661	82	3285
379	567	229	89	265	660	627	46	423	3285
666	87	262	418	622	54	228	386	562	3285
94	244	675	633	35	427	396	546	235	3285
424	639	33	243	391	541	670	93	251	3285
548	238	390	249	667	99	28	432	634	3285
222	397	558	649	108	256	413	643	39	3285
255	656	103	45	411	640	553	217	405	3285
648	40	406	400	552	224	100	261	654	3285
3285	3285	3285	3285	3285	3285	3285	3285	3285	3285

- **Block 32**

3									126
2	16	24	21	9	15	10	25	4	126
15	21	9	4	10	25	24	2	16	126
25	4	10	16	24	2	9	15	21	126
4	10	25	24	2	16	15	21	9	126
16	24	2	9	15	21	25	4	10	126
21	9	15	10	25	4	2	16	24	126
9	15	21	25	4	10	16	24	2	126
10	25	4	2	16	24	21	9	15	126
24	2	16	15	21	9	4	10	25	126
126	126	126	126	126	126	126	126	126	126

2									126
23	11	7	20	17	5	26	14	3	126
3	26	14	7	23	11	5	20	17	126
17	5	20	14	3	26	11	7	23	126
14	3	26	11	7	23	17	5	20	126
20	17	5	26	14	3	23	11	7	126
7	23	11	5	20	17	3	26	14	126
5	20	17	3	26	14	7	23	11	126
11	7	23	17	5	20	14	3	26	126
26	14	3	23	11	7	20	17	5	126
126	126	126	126	126	126	126	126	126	126

(32)									3285
50	416	628	560	233	383	269	662	84	3285
381	566	230	88	266	659	626	47	422	3285
665	86	263	419	624	53	227	385	563	3285
95	246	674	632	34	428	395	545	236	3285
425	638	32	242	392	543	671	92	250	3285
547	239	389	248	668	98	30	431	635	3285
221	398	557	651	107	257	412	644	38	3285
254	655	104	44	410	641	554	219	404	3285
647	41	408	401	551	223	101	260	653	3285
3285	3285	3285	3285	3285	3285	3285	3285	3285	3285

- **Block 33**

(3)									126
2	16	24	21	9	15	10	25	4	126
15	21	9	4	10	25	24	2	16	126
25	4	10	16	24	2	9	15	21	126
4	10	25	24	2	16	15	21	9	126
16	24	2	9	15	21	25	4	10	126
21	9	15	10	25	4	2	16	24	126
9	15	21	25	4	10	16	24	2	126
10	25	4	2	16	24	21	9	15	126
24	2	16	15	21	9	4	10	25	126
126	126	126	126	126	126	126	126	126	126

(3)									126
24	10	9	21	16	4	25	15	2	126
2	25	15	9	24	10	4	21	16	126
16	4	21	15	2	25	10	9	24	126
15	2	25	10	9	24	16	4	21	126
21	16	4	25	15	2	24	10	9	126
9	24	10	4	21	16	2	25	15	126
4	21	16	2	25	15	9	24	10	126
10	9	24	16	4	21	15	2	25	126
25	15	2	24	10	9	21	16	4	126
126	126	126	126	126	126	126	126	126	126

(33)									3285
51	415	630	561	232	382	268	663	83	3285
380	565	231	90	267	658	625	48	421	3285
664	85	264	420	623	52	226	387	564	3285
96	245	673	631	36	429	394	544	237	3285
426	637	31	241	393	542	672	91	252	3285
549	240	388	247	669	97	29	430	636	3285
220	399	556	650	106	258	414	645	37	3285
253	657	105	43	409	642	555	218	403	3285
646	42	407	402	550	225	102	259	652	3285
3285	3285	3285	3285	3285	3285	3285	3285	3285	3285

In all the 9 blocks, the composition is applied by use of following formula:

$$AB := 27 \times (A - 1) + B$$

Combining the above 9 magic squares of order 9 according to structure 1 we get a **pan magic square** of order 27 given in the example below.

Example 9. *According Distribution 4, Structure 1 and 9* ***magic squares*** *given above , we have a* ***pan magic square*** *of order 27 given by*

I	1	2	3	4	5	6	7	8	9	10	11	12	13
		9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855
	22	471	575	505	207	330	324	715	136	23	470	574	506
9855	325	513	202	143	319	714	573	19	477	327	512	203	142
9855	720	141	316	472	568	27	201	332	508	719	140	317	473
9855	148	298	729	579	8	481	342	492	208	149	300	728	578
9855	478	585	6	216	337	487	724	147	305	479	584	5	215
9855	494	211	336	303	721	153	1	486	580	493	212	335	302
9855	195	343	504	703	162	310	467	589	12	194	344	503	705
9855	309	710	157	18	465	586	499	190	351	308	709	158	17
9855	594	13	460	346	498	197	154	315	708	593	14	462	347
9855	76	444	602	532	180	357	297	688	109	77	443	601	533
9855	352	540	175	116	292	687	600	73	450	354	539	176	115
9855	693	114	289	445	595	81	174	359	535	692	113	290	446
9855	121	271	702	606	62	454	369	519	181	122	273	701	605
9855	451	612	60	189	364	514	697	120	278	452	611	59	188
9855	521	184	363	276	694	126	55	459	607	520	185	362	275
9855	168	370	531	676	135	283	440	616	66	167	371	530	678
9855	282	683	130	72	438	613	526	163	378	281	682	131	71
9855	621	67	433	373	525	170	127	288	681	620	68	435	374
9855	49	417	629	559	234	384	270	661	82	50	416	628	560
9855	379	567	229	89	265	660	627	46	423	381	566	230	88
9855	666	87	262	418	622	54	228	386	562	665	86	263	419
9855	94	244	675	633	35	427	396	546	235	95	246	674	632
9855	424	639	33	243	391	541	670	93	251	425	638	32	242
9855	548	238	390	249	667	99	28	432	634	547	239	389	248
9855	222	397	558	649	108	256	413	643	39	221	398	557	651
9855	255	656	103	45	411	640	553	217	405	254	655	104	44
9855	648	40	406	400	552	224	100	261	654	647	41	408	401
	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855

14	15	16	17	18	19	20	21	22	23	24	25	26	27	(II)
9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855
206	329	323	716	138	24	469	576	507	205	328	322	717	137	9855
320	713	572	20	476	326	511	204	144	321	712	571	21	475	9855
570	26	200	331	509	718	139	318	474	569	25	199	333	510	9855
7	482	341	491	209	150	299	727	577	9	483	340	490	210	9855
338	489	725	146	304	480	583	4	214	339	488	726	145	306	9855
722	152	3	485	581	495	213	334	301	723	151	2	484	582	9855
161	311	466	590	11	193	345	502	704	160	312	468	591	10	9855
464	587	500	192	350	307	711	159	16	463	588	501	191	349	9855
497	196	155	314	707	592	15	461	348	496	198	156	313	706	9855
179	356	296	689	111	78	442	603	534	178	355	295	690	110	9855
293	686	599	74	449	353	538	177	117	294	685	598	75	448	9855
597	80	173	358	536	691	112	291	447	596	79	172	360	537	9855
61	455	368	518	182	123	272	700	604	63	456	367	517	183	9855
365	516	698	119	277	453	610	58	187	366	515	699	118	279	9855
695	125	57	458	608	522	186	361	274	696	124	56	457	609	9855
134	284	439	617	65	166	372	529	677	133	285	441	618	64	9855
437	614	527	165	377	280	684	132	70	436	615	528	164	376	9855
524	169	128	287	680	619	69	434	375	523	171	129	286	679	9855
233	383	269	662	84	51	415	630	561	232	382	268	663	83	9855
266	659	626	47	422	380	565	231	90	267	658	625	48	421	9855
624	53	227	385	563	664	85	264	420	623	52	226	387	564	9855
34	428	395	545	236	96	245	673	631	36	429	394	544	237	9855
392	543	671	92	250	426	637	31	241	393	542	672	91	252	9855
668	98	30	431	635	549	240	388	247	669	97	29	430	636	9855
107	257	412	644	38	220	399	556	650	106	258	414	645	37	9855
410	641	554	219	404	253	657	105	43	409	642	555	218	403	9855
551	223	101	260	653	646	42	407	402	550	225	102	259	652	9855
9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855

Combining parts (I) and (II) we get a pan magic square of order 27 with each block of order 9 a magic squares of equal sums. In this case, the magic sum is $S_{27\times 27} = 9855$. Each 9×9 block is a **magic square** of order 9 with equal magic sum $S_{9\times 9} = 3285$.

We observed that in the above example we get a pan magic square of order 27. But each block of order 9 is a magic square, but not pan. Below is example due to Dwane [2], where the each block of order 9 is also a pan diagonal.

Example 10. ***Dwane's Construction [2]:*** *The following example of **pan magic squares** of order 27 with each sub-block of order 9 also a **pan magic square**:*

(I)	1	2	3	4	5	6	7	8	9	10	11	12	13
		9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855
	22	471	575	505	207	330	324	715	136	23	470	574	506
9855	325	513	202	143	319	714	573	19	477	327	512	203	142
9855	720	141	316	472	568	27	201	332	508	719	140	317	473
9855	148	298	729	579	8	481	342	492	208	149	300	728	578
9855	478	585	6	216	337	487	724	147	305	479	584	5	215
9855	494	211	336	303	721	153	1	486	580	493	212	335	302
9855	195	343	504	703	162	310	467	589	12	194	344	503	705
9855	309	710	157	18	465	586	499	190	351	308	709	158	17
9855	594	13	460	346	498	197	154	315	708	593	14	462	347
9855	76	444	602	532	180	357	297	688	109	77	443	601	533
9855	352	540	175	116	292	687	600	73	450	354	539	176	115
9855	693	114	289	445	595	81	174	359	535	692	113	290	446
9855	121	271	702	606	62	454	369	519	181	122	273	701	605
9855	451	612	60	189	364	514	697	120	278	452	611	59	188
9855	521	184	363	276	694	126	55	459	607	520	185	362	275
9855	168	370	531	676	135	283	440	616	66	167	371	530	678
9855	282	683	130	72	438	613	526	163	378	281	682	131	71
9855	621	67	433	373	525	170	127	288	681	620	68	435	374
9855	49	417	629	559	234	384	270	661	82	50	416	628	560
9855	379	567	229	89	265	660	627	46	423	381	566	230	88
9855	666	87	262	418	622	54	228	386	562	665	86	263	419
9855	94	244	675	633	35	427	396	546	235	95	246	674	632
9855	424	639	33	243	391	541	670	93	251	425	638	32	242
9855	548	238	390	249	667	99	28	432	634	547	239	389	248
9855	222	397	558	649	108	256	413	643	39	221	398	557	651
9855	255	656	103	45	411	640	553	217	405	254	655	104	44
9855	648	40	406	400	552	224	100	261	654	647	41	408	401
	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855

14	15	16	17	18	19	20	21	22	23	24	25	26	27	(II)
9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855
206	329	323	716	138	24	469	576	507	205	328	322	717	137	9855
320	713	572	20	476	326	511	204	144	321	712	571	21	475	9855
570	26	200	331	509	718	139	318	474	569	25	199	333	510	9855
7	482	341	491	209	150	299	727	577	9	483	340	490	210	9855
338	489	725	146	304	480	583	4	214	339	488	726	145	306	9855
722	152	3	485	581	495	213	334	301	723	151	2	484	582	9855
161	311	466	590	11	193	345	502	704	160	312	468	591	10	9855
464	587	500	192	350	307	711	159	16	463	588	501	191	349	9855
497	196	155	314	707	592	15	461	348	496	198	156	313	706	9855
179	356	296	689	111	78	442	603	534	178	355	295	690	110	9855
293	686	599	74	449	353	538	177	117	294	685	598	75	448	9855
597	80	173	358	536	691	112	291	447	596	79	172	360	537	9855
61	455	368	518	182	123	272	700	604	63	456	367	517	183	9855
365	516	698	119	277	453	610	58	187	366	515	699	118	279	9855
695	125	57	458	608	522	186	361	274	696	124	56	457	609	9855
134	284	439	617	65	166	372	529	677	133	285	441	618	64	9855
437	614	527	165	377	280	684	132	70	436	615	528	164	376	9855
524	169	128	287	680	619	69	434	375	523	171	129	286	679	9855
233	383	269	662	84	51	415	630	561	232	382	268	663	83	9855
266	659	626	47	422	380	565	231	90	267	658	625	48	421	9855
624	53	227	385	563	664	85	264	420	623	52	226	387	564	9855
34	428	395	545	236	96	245	673	631	36	429	394	544	237	9855
392	543	671	92	250	426	637	31	241	393	542	672	91	252	9855
668	98	30	431	635	549	240	388	247	669	97	29	430	636	9855
107	257	412	644	38	220	399	556	650	106	258	414	645	37	9855
410	641	554	219	404	253	657	105	43	409	642	555	218	403	9855
551	223	101	260	653	646	42	407	402	550	225	102	259	652	9855
9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855

Combining parts (I) and (II) we get a pan magic square of order 27 with each block of order 9 a pan magic squares of equal sums. In this case, the magic sum is $S_{27\times27} = 9855$. Each 9×9 block is also **pan magic square** of order 9 with equal magic sum $S_{9\times9} = 3285$. 3×3 block also sums to same values, i.e., $S_9 = 3285$. This example is Dwane's construction. More details can be seen in his site [2].

2.5 Pan Magic Square of Order 33

Distribution 5. *Let's consider the following distribution to construct* ***pan magic square*** *of order 33:*

(1)	1	6	8	12	13	18	19	24	25	30	31	187
(2)	3	5	7	11	14	17	20	23	26	29	32	187
(3)	2	4	9	10	15	16	21	22	27	28	33	187

The construction of **pan magic square** of order 33 with each sub-block of same sum is based on the following pan diagonal magic square of order 11 constructed using a pair of mutually orthogonal diagonal Latin squares:

Example 11. *Let's consider the following* ***pan diagonal magic square*** *of order 11:*

AB		671	671	671	671	671	671	671	671	671	671	671
	1	13	25	37	49	61	73	85	97	109	121	671
671	108	120	11	12	24	36	48	60	72	84	96	671
671	83	95	107	119	10	22	23	35	47	59	71	671
671	58	70	82	94	106	118	9	21	33	34	46	671
671	44	45	57	69	81	93	105	117	8	20	32	671
671	19	31	43	55	56	68	80	92	104	116	7	671
671	115	6	18	30	42	54	66	67	79	91	103	671
671	90	102	114	5	17	29	41	53	65	77	78	671
671	76	88	89	101	113	4	16	28	40	52	64	671
671	51	63	75	87	99	100	112	3	15	27	39	671
671	26	38	50	62	74	86	98	110	111	2	14	671
	671	671	671	671	671	671	671	671	671	671	671	671

Example 12. *The above* ***pan magic square*** *of order 11 is constructed based on pair of* ***mutually diagonal Latin squares*** *A and B given by*

A		66	66	66	66	66	66	66	66	66	66	66
	1	2	3	4	5	6	7	8	9	10	11	66
66	10	11	1	2	3	4	5	6	7	8	9	66
66	8	9	10	11	1	2	3	4	5	6	7	66
66	6	7	8	9	10	11	1	2	3	4	5	66
66	4	5	6	7	8	9	10	11	1	2	3	66
66	2	3	4	5	6	7	8	9	10	11	1	66
66	11	1	2	3	4	5	6	7	8	9	10	66
66	9	10	11	1	2	3	4	5	6	7	8	66
66	7	8	9	10	11	1	2	3	4	5	6	66
66	5	6	7	8	9	10	11	1	2	3	4	66
66	3	4	5	6	7	8	9	10	11	1	2	66
	66	66	66	66	66	66	66	66	66	66	66	66

B		66	66	66	66	66	66	66	66	66	66	66
	1	2	3	4	5	6	7	8	9	10	11	66
66	9	10	11	1	2	3	4	5	6	7	8	66
66	6	7	8	9	10	11	1	2	3	4	5	66
66	3	4	5	6	7	8	9	10	11	1	2	66
66	11	1	2	3	4	5	6	7	8	9	10	66
66	8	9	10	11	1	2	3	4	5	6	7	66
66	5	6	7	8	9	10	11	1	2	3	4	66
66	2	3	4	5	6	7	8	9	10	11	1	66
66	10	11	1	2	3	4	5	6	7	8	9	66
66	7	8	9	10	11	1	2	3	4	5	6	66
66	4	5	6	7	8	9	10	11	1	2	3	66
	66	66	66	66	66	66	66	66	66	66	66	66

The **pan magic square** appearing in ***AB*** is constructed according to the following formula:

$$AB := 11 \times (A-1) + B$$

where A and B are the **mutually orthogonal diagonal Latin squares** of order 11 as given above.

We shall construct 9 magic squares of order 11 using the distribution 5 and Example 11, and put them according to Structure 1. In this case, the composition considered is as follows:

$$AB := 33 \times (A-1) + B$$

i.e., instead of 11, the multiplications is with 33.

In the Examples of orders 15, 21 and 27 respectively given in 5, 7 and 9, all the 9 blocks are constructed based on a pair of **mutual orthogonal diagonal Latin squares**. The same process is applied for the **pan magic square** of order 33. In this case, we won't write the Latin squares, such as A and B. Only composite magic squares of type AB are written to construct pan magic square of order 33.

Construction 4. *Below are 9 **pan magic squares** of order 11 constructed according to Example 11 using the Distribution 5:*

- **Block 11**

(11)		5995	5995	5995	5995	5995	5995	5995	5995	5995	5995	5995
	1	171	239	375	409	579	613	783	817	987	1021	5995
5995	982	1020	31	166	237	371	408	574	612	778	816	5995
5995	777	811	981	1015	30	196	232	369	404	573	607	5995
5995	569	606	772	810	976	1014	25	195	262	364	402	5995
5995	394	397	567	602	771	805	975	1009	24	190	261	5995
5995	189	256	393	427	562	600	767	804	970	1008	19	5995
5995	1003	18	184	255	388	426	592	595	765	800	969	5995
5995	798	965	1002	13	183	250	387	421	591	625	760	5995
5995	624	790	793	963	998	12	178	249	382	420	586	5995
5995	415	585	619	789	823	958	996	8	177	244	381	5995
5995	243	376	414	580	618	784	822	988	991	6	173	5995
	5995	5995	5995	5995	5995	5995	5995	5995	5995	5995	5995	5995

- **Block 12**

12		5995	5995	5995	5995	5995	5995	5995	5995	5995	5995	5995
	3	170	238	374	410	578	614	782	818	986	1022	5995
5995	983	1019	32	168	236	370	407	575	611	779	815	5995
5995	776	812	980	1016	29	197	234	368	403	572	608	5995
5995	568	605	773	809	977	1013	26	194	263	366	401	5995
5995	395	399	566	601	770	806	974	1010	23	191	260	5995
5995	188	257	392	428	564	599	766	803	971	1007	20	5995
5995	1004	17	185	254	389	425	593	597	764	799	968	5995
5995	797	964	1001	14	182	251	386	422	590	626	762	5995
5995	623	791	795	962	997	11	179	248	383	419	587	5995
5995	416	584	620	788	824	960	995	7	176	245	380	5995
5995	242	377	413	581	617	785	821	989	993	5	172	5995
	5995	5995	5995	5995	5995	5995	5995	5995	5995	5995	5995	5995

- **Block 13**

13		5995	5995	5995	5995	5995	5995	5995	5995	5995	5995	5995
	2	169	240	373	411	577	615	781	819	985	1023	5995
5995	984	1018	33	167	235	372	406	576	610	780	814	5995
5995	775	813	979	1017	28	198	233	367	405	571	609	5995
5995	570	604	774	808	978	1012	27	193	264	365	400	5995
5995	396	398	565	603	769	807	973	1011	22	192	259	5995
5995	187	258	391	429	563	598	768	802	972	1006	21	5995
5995	1005	16	186	253	390	424	594	596	763	801	967	5995
5995	796	966	1000	15	181	252	385	423	589	627	761	5995
5995	622	792	794	961	999	10	180	247	384	418	588	5995
5995	417	583	621	787	825	959	994	9	175	246	379	5995
5995	241	378	412	582	616	786	820	990	992	4	174	5995
	5995	5995	5995	5995	5995	5995	5995	5995	5995	5995	5995	5995

- **Block 21**

21		5995	5995	5995	5995	5995	5995	5995	5995	5995	5995	5995
	67	138	206	342	442	546	646	750	850	954	1054	5995
5995	949	1053	97	133	204	338	441	541	645	745	849	5995
5995	744	844	948	1048	96	163	199	336	437	540	640	5995
5995	536	639	739	843	943	1047	91	162	229	331	435	5995
5995	361	430	534	635	738	838	942	1042	90	157	228	5995
5995	156	223	360	460	529	633	734	837	937	1041	85	5995
5995	1036	84	151	222	355	459	559	628	732	833	936	5995
5995	831	932	1035	79	150	217	354	454	558	658	727	5995
5995	657	757	826	930	1031	78	145	216	349	453	553	5995
5995	448	552	652	756	856	925	1029	74	144	211	348	5995
5995	210	343	447	547	651	751	855	955	1024	72	140	5995
	5995	5995	5995	5995	5995	5995	5995	5995	5995	5995	5995	5995

- **Block 22**

22		5995	5995	5995	5995	5995	5995	5995	5995	5995	5995	5995
	69	137	205	341	443	545	647	749	851	953	1055	5995
5995	950	1052	98	135	203	337	440	542	644	746	848	5995
5995	743	845	947	1049	95	164	201	335	436	539	641	5995
5995	535	638	740	842	944	1046	92	161	230	333	434	5995
5995	362	432	533	634	737	839	941	1043	89	158	227	5995
5995	155	224	359	461	531	632	733	836	938	1040	86	5995
5995	1037	83	152	221	356	458	560	630	731	832	935	5995
5995	830	931	1034	80	149	218	353	455	557	659	729	5995
5995	656	758	828	929	1030	77	146	215	350	452	554	5995
5995	449	551	653	755	857	927	1028	73	143	212	347	5995
5995	209	344	446	548	650	752	854	956	1026	71	139	5995
	5995	5995	5995	5995	5995	5995	5995	5995	5995	5995	5995	5995

- **Block 23**

23		5995	5995	5995	5995	5995	5995	5995	5995	5995	5995	5995
	68	136	207	340	444	544	648	748	852	952	1056	5995
5995	951	1051	99	134	202	339	439	543	643	747	847	5995
5995	742	846	946	1050	94	165	200	334	438	538	642	5995
5995	537	637	741	841	945	1045	93	160	231	332	433	5995
5995	363	431	532	636	736	840	940	1044	88	159	226	5995
5995	154	225	358	462	530	631	735	835	939	1039	87	5995
5995	1038	82	153	220	357	457	561	629	730	834	934	5995
5995	829	933	1033	81	148	219	352	456	556	660	728	5995
5995	655	759	827	928	1032	76	147	214	351	451	555	5995
5995	450	550	654	754	858	926	1027	75	142	213	346	5995
5995	208	345	445	549	649	753	853	957	1025	70	141	5995
	5995	5995	5995	5995	5995	5995	5995	5995	5995	5995	5995	5995

- **Block 31**

31		5995	5995	5995	5995	5995	5995	5995	5995	5995	5995	5995
	34	105	272	309	475	513	679	717	883	921	1087	5995
5995	916	1086	64	100	270	305	474	508	678	712	882	5995
5995	711	877	915	1081	63	130	265	303	470	507	673	5995
5995	503	672	706	876	910	1080	58	129	295	298	468	5995
5995	328	463	501	668	705	871	909	1075	57	124	294	5995
5995	123	289	327	493	496	666	701	870	904	1074	52	5995
5995	1069	51	118	288	322	492	526	661	699	866	903	5995
5995	864	899	1068	46	117	283	321	487	525	691	694	5995
5995	690	724	859	897	1064	45	112	282	316	486	520	5995
5995	481	519	685	723	889	892	1062	41	111	277	315	5995
5995	276	310	480	514	684	718	888	922	1057	39	107	5995
	5995	5995	5995	5995	5995	5995	5995	5995	5995	5995	5995	5995

- **Block 32**

32		5995	5995	5995	5995	5995	5995	5995	5995	5995	5995	5995
	36	104	271	308	476	512	680	716	884	920	1088	5995
5995	917	1085	65	102	269	304	473	509	677	713	881	5995
5995	710	878	914	1082	62	131	267	302	469	506	674	5995
5995	502	671	707	875	911	1079	59	128	296	300	467	5995
5995	329	465	500	667	704	872	908	1076	56	125	293	5995
5995	122	290	326	494	498	665	700	869	905	1073	53	5995
5995	1070	50	119	287	323	491	527	663	698	865	902	5995
5995	863	898	1067	47	116	284	320	488	524	692	696	5995
5995	689	725	861	896	1063	44	113	281	317	485	521	5995
5995	482	518	686	722	890	894	1061	40	110	278	314	5995
5995	275	311	479	515	683	719	887	923	1059	38	106	5995
	5995	5995	5995	5995	5995	5995	5995	5995	5995	5995	5995	5995

- **Block 33**

33		5995	5995	5995	5995	5995	5995	5995	5995	5995	5995	5995
	35	103	273	307	477	511	681	715	885	919	1089	5995
5995	918	1084	66	101	268	306	472	510	676	714	880	5995
5995	709	879	913	1083	61	132	266	301	471	505	675	5995
5995	504	670	708	874	912	1078	60	127	297	299	466	5995
5995	330	464	499	669	703	873	907	1077	55	126	292	5995
5995	121	291	325	495	497	664	702	868	906	1072	54	5995
5995	1071	49	120	286	324	490	528	662	697	867	901	5995
5995	862	900	1066	48	115	285	319	489	523	693	695	5995
5995	688	726	860	895	1065	43	114	280	318	484	522	5995
5995	483	517	687	721	891	893	1060	42	109	279	313	5995
5995	274	312	478	516	682	720	886	924	1058	37	108	5995
	5995	5995	5995	5995	5995	5995	5995	5995	5995	5995	5995	5995

Combining the above 9 magic squares of order 11 according to structure 1 we get a **pan magic square** of order 33 given in the example below.

Example 13. *According Distribution 5, Structure 1 and 9* ***magic squares*** *given above, a* ***pan magic square*** *of order 33 is given in two parts* (I) *and* (I) *is as follows:*

(I)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
		17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985
17985	1	171	239	375	409	579	613	783	817	987	1021	3	170	238	374	410
17985	982	1020	31	166	237	371	408	574	612	778	816	983	1019	32	168	236
17985	777	811	981	1015	30	196	232	369	404	573	607	776	812	980	1016	29
17985	569	606	772	810	976	1014	25	195	262	364	402	568	605	773	809	977
17985	394	397	567	602	771	805	975	1009	24	190	261	395	399	566	601	770
17985	189	256	393	427	562	600	767	804	970	1008	19	188	257	392	428	564
17985	1003	18	184	255	388	426	592	595	765	800	969	1004	17	185	254	389
17985	798	965	1002	13	183	250	387	421	591	625	760	797	964	1001	14	182
17985	624	790	793	963	998	12	178	249	382	420	586	623	791	795	962	997
17985	415	585	619	789	823	958	996	8	177	244	381	416	584	620	788	824
17985	243	376	414	580	618	784	822	988	991	6	173	242	377	413	581	617
17985	67	138	206	342	442	546	646	750	850	954	1054	69	137	205	341	443
17985	949	1053	97	133	204	338	441	541	645	745	849	950	1052	98	135	203
17985	744	844	948	1048	96	163	199	336	437	540	640	743	845	947	1049	95
17985	536	639	739	843	943	1047	91	162	229	331	435	535	638	740	842	944
17985	361	430	534	635	738	838	942	1042	90	157	228	362	432	533	634	737
17985	156	223	360	460	529	633	734	837	937	1041	85	155	224	359	461	531
17985	1036	84	151	222	355	459	559	628	732	833	936	1037	83	152	221	356
17985	831	932	1035	79	150	217	354	454	558	658	727	830	931	1034	80	149
17985	657	757	826	930	1031	78	145	216	349	453	553	656	758	828	929	1030
17985	448	552	652	756	856	925	1029	74	144	211	348	449	551	653	755	857
17985	210	343	447	547	651	751	855	955	1024	72	140	209	344	446	548	650
17985	34	105	272	309	475	513	679	717	883	921	1087	36	104	271	308	476
17985	916	1086	64	100	270	305	474	508	678	712	882	917	1085	65	102	269
17985	711	877	915	1081	63	130	265	303	470	507	673	710	878	914	1082	62
17985	503	672	706	876	910	1080	58	129	295	298	468	502	671	707	875	911
17985	328	463	501	668	705	871	909	1075	57	124	294	329	465	500	667	704
17985	123	289	327	493	496	666	701	870	904	1074	52	122	290	326	494	498
17985	1069	51	118	288	322	492	526	661	699	866	903	1070	50	119	287	323
17985	864	899	1068	46	117	283	321	487	525	691	694	863	898	1067	47	116
17985	690	724	859	897	1064	45	112	282	316	486	520	689	725	861	896	1063
17985	481	519	685	723	889	892	1062	41	111	277	315	482	518	686	722	890
	276	310	480	514	684	718	888	922	1057	39	107	275	311	479	515	683
	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985

17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	(II)
17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985
578	614	782	818	986	1022	2	169	240	373	411	577	615	781	819	985	1023	17985
370	407	575	611	779	815	984	1018	33	167	235	372	406	576	610	780	814	17985
197	234	368	403	572	608	775	813	979	1017	28	198	233	367	405	571	609	17985
1013	26	194	263	366	401	570	604	774	808	978	1012	27	193	264	365	400	17985
806	974	1010	23	191	260	396	398	565	603	769	807	973	1011	22	192	259	17985
599	766	803	971	1007	20	187	258	391	429	563	598	768	802	972	1006	21	17985
425	593	597	764	799	968	1005	16	186	253	390	424	594	596	763	801	967	17985
251	386	422	590	626	762	796	966	1000	15	181	252	385	423	589	627	761	17985
11	179	248	383	419	587	622	792	794	961	999	10	180	247	384	418	588	17985
960	995	7	176	245	380	417	583	621	787	825	959	994	9	175	246	379	17985
785	821	989	993	5	172	241	378	412	582	616	786	820	990	992	4	174	17985
545	647	749	851	953	1055	68	136	207	340	444	544	648	748	852	952	1056	17985
337	440	542	644	746	848	951	1051	99	134	202	339	439	543	643	747	847	17985
164	201	335	436	539	641	742	846	946	1050	94	165	200	334	438	538	642	17985
1046	92	161	230	333	434	537	637	741	841	945	1045	93	160	231	332	433	17985
839	941	1043	89	158	227	363	431	532	636	736	840	940	1044	88	159	226	17985
632	733	836	938	1040	86	154	225	358	462	530	631	735	835	939	1039	87	17985
458	560	630	731	832	935	1038	82	153	220	357	457	561	629	730	834	934	17985
218	353	455	557	659	729	829	933	1033	81	148	219	352	456	556	660	728	17985
77	146	215	350	452	554	655	759	827	928	1032	76	147	214	351	451	555	17985
927	1028	73	143	212	347	450	550	654	754	858	926	1027	75	142	213	346	17985
752	854	956	1026	71	139	208	345	445	549	649	753	853	957	1025	70	141	17985
512	680	716	884	920	1088	35	103	273	307	477	511	681	715	885	919	1089	17985
304	473	509	677	713	881	918	1084	66	101	268	306	472	510	676	714	880	17985
131	267	302	469	506	674	709	879	913	1083	61	132	266	301	471	505	675	17985
1079	59	128	296	300	467	504	670	708	874	912	1078	60	127	297	299	466	17985
872	908	1076	56	125	293	330	464	499	669	703	873	907	1077	55	126	292	17985
665	700	869	905	1073	53	121	291	325	495	497	664	702	868	906	1072	54	17985
491	527	663	698	865	902	1071	49	120	286	324	490	528	662	697	867	901	17985
284	320	488	524	692	696	862	900	1066	48	115	285	319	489	523	693	695	17985
44	113	281	317	485	521	688	726	860	895	1065	43	114	280	318	484	522	17985
894	1061	40	110	278	314	483	517	687	721	891	893	1060	42	109	279	313	17985
719	887	923	1059	38	106	274	312	478	516	682	720	886	924	1058	37	108	17985
17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985

Combining parts (I) and (II) we get required result. In this case, the magic sum is $S_{33\times 33} = 17985$. Each 11×11 block is a **pan magic square** of order 11 with equal magic sums $S_{11\times 11} = 5995$.

3 Even Order Magic Squares of Order $3k$ or Orders $6k$

This section brings even order magic squares of type $3k$. In case of even order the distribution is much simple than what we did in the previous section on odd order. This distribution is given in each case separately. In this case we need magic square of order 6. This magic square is due to [3].

3.1 Magic Square of Order 6

Example 14. *Let's consider a magic square of order 6.*

						111
1	35	34	33	2	6	111
30	8	28	9	11	25	111
24	23	15	16	20	13	111
18	14	21	22	17	19	111
7	26	10	27	29	12	111
31	5	3	4	32	36	111
111	111	111	111	111	111	111

3.2 Magic Square of Order 12

In this subsection, we shall give construction of a magic square of order 12 in four blocks of magic squares of order 6 with equal magic sums. In order to construct, let's divide 144 numbers in four equal parts according to following distribution.

Distribution 6. *Let's consider the following distribution of 144 numbers in 4 blocks of 36 each giving equal sum:*

	1	2	3	4	5	6	...	...	31	32	33	34	35	36	Total
A1	1	8	9	16	17	24	...	...	121	128	129	136	137	144	2610
A2	2	7	10	15	18	23	...	...	122	127	130	135	138	143	2610
A3	3	6	11	14	19	22	...	...	123	126	131	134	139	142	2610
A4	4	5	12	13	20	21	...	...	124	125	132	133	140	141	2610

Let's construct 4 blocks of magic squares of order 6 according to Example 14, and put them according to following structure.

Structure 2. *Let's consider 4 blocks of order 6 as below:*

A1	A2
A3	A4

Example 15. *4 blocks of magic squares constructed according to Example 14 by using data given in Distribution 6, and putting them according to Structure 2, we get a magic square of order 12 given by*

												870
1	137	136	129	8	24	2	138	135	130	7	23	870
120	32	112	33	41	97	119	31	111	34	42	98	870
96	89	57	64	80	49	95	90	58	63	79	50	870
72	56	81	88	65	73	71	55	82	87	66	74	870
25	104	40	105	113	48	26	103	39	106	114	47	870
121	17	9	16	128	144	122	18	10	15	127	143	870
3	139	134	131	6	22	4	140	133	132	5	21	870
118	30	110	35	43	99	117	29	109	36	44	100	870
94	91	59	62	78	51	93	92	60	61	77	52	870
70	54	83	86	67	75	69	53	84	85	68	76	870
27	102	38	107	115	46	28	101	37	108	116	45	870
123	19	11	14	126	142	124	20	12	13	125	141	870
870	870	870	870	870	870	870	870	870	870	870	870	870

In this case, the magic sum is $S_{12\times 12} := 870$, and all four blocks of order 6 are magic squares with equal magic sums $S_{6\times 6} := 435$.

3.3 Magic Square of Order 18

In this subsection, we shall give construction of a magic square of order 18 in 9 blocks of magic squares of order 6 with equal magic sums. In order to construct it let's divide 324 numbers in 9 equal parts according to following distribution.

Distribution 7. *Let's consider the following distribution of 1 to 324 numbers in 9 blocks to construct **magic square** of order 18:*

	1	2	3	4	5	6	...	...	31	32	33	34	35	36	Total
A1	1	18	19	36	37	54	...	...	271	288	289	306	307	324	5850
A2	2	17	20	35	38	53	...	...	272	287	290	305	308	323	5850
A3	3	16	21	34	39	52	...	...	273	286	291	304	309	322	5850
A4	4	15	22	33	40	51	...	...	274	285	292	303	310	321	5850
A5	5	14	23	32	41	50	...	...	275	284	293	302	311	320	5850
A6	6	13	24	31	42	49	...	...	276	283	294	301	312	319	5850
A7	7	12	25	30	43	48	...	...	277	282	295	300	313	318	5850
A8	8	11	26	29	44	47	...	...	278	281	296	299	314	317	5850
A9	9	10	27	28	45	46	...	...	279	280	297	298	315	316	5850

Let's construct 9 blocks of magic squares of order 6 according to Example 14, and put them according to following structure.

Structure 3. *Let's consider 9 blocks of order 6 as below:*

A1	A2	A3
A4	A5	A6
A7	A8	A9

Example 16. *4 blocks of magic squares constructed according to Example 14 by using data given in Distribution 7, and putting them according to Structure 3, we get a magic square of order 18 given by*

																		2925
1	307	306	289	18	54	2	308	305	290	17	53	3	309	304	291	16	52	2925
270	72	252	73	91	217	269	71	251	74	92	218	268	70	250	75	93	219	2925
216	199	127	144	180	109	215	200	128	143	179	110	214	201	129	142	178	111	2925
162	126	181	198	145	163	161	125	182	197	146	164	160	124	183	196	147	165	2925
55	234	90	235	253	108	56	233	89	236	254	107	57	232	88	237	255	106	2925
271	37	19	36	288	324	272	38	20	35	287	323	273	39	21	34	286	322	2925
4	310	303	292	15	51	5	311	302	293	14	50	6	312	301	294	13	49	2925
267	69	249	76	94	220	266	68	248	77	95	221	265	67	247	78	96	222	2925
213	202	130	141	177	112	212	203	131	140	176	113	211	204	132	139	175	114	2925
159	123	184	195	148	166	158	122	185	194	149	167	157	121	186	193	150	168	2925
58	231	87	238	256	105	59	230	86	239	257	104	60	229	85	240	258	103	2925
274	40	22	33	285	321	275	41	23	32	284	320	276	42	24	31	283	319	2925
7	313	300	295	12	48	8	314	299	296	11	47	9	315	298	297	10	46	2925
264	66	246	79	97	223	263	65	245	80	98	224	262	64	244	81	99	225	2925
210	205	133	138	174	115	209	206	134	137	173	116	208	207	135	136	172	117	2925
156	120	187	192	151	169	155	119	188	191	152	170	154	118	189	190	153	171	2925
61	228	84	241	259	102	62	227	83	242	260	101	63	226	82	243	261	100	2925
277	43	25	30	282	318	278	44	26	29	281	317	279	45	27	28	280	316	2925
2925	2925	2925	2925	2925	2925	2925	2925	2925	2925	2925	2925	2925	2925	2925	2925	2925	2925	2925

The above magic square is with magic sum $S_{18\times 18} = 2925$, and all the nine blocks of order 6 are magic squares with equal magic sums $S_{6\times 6} := 975$.

3.4 Magic Square of Order 24

In this subsection, we shall give construction of a magic square of order 24 in 16 blocks of magic squares of order 6 with equal magic sums. In order to construct it let's divide 576 numbers in 16 equal parts according to following distribution.

Distribution 8. *Let's consider the following distribution of 1 to 576 numbers in 16 blocks of equal sums:*

	1	2	3	4	5	6	...	...	31	32	33	34	35	36	Total
A1	1	32	33	64	65	96	...	...	481	512	513	544	545	576	10386
A2	2	31	34	63	66	95	...	...	482	511	514	543	546	575	10386
A3	3	30	35	62	67	94	...	...	483	510	515	542	547	574	10386
A4	4	29	36	61	68	93	...	...	484	509	516	541	548	573	10386
A5	5	28	37	60	69	92	...	...	485	508	517	540	549	572	10386
A6	6	27	38	59	70	91	...	...	486	507	518	539	550	571	10386
A7	7	26	39	58	71	90	...	...	487	506	519	538	551	570	10386
A8	8	25	40	57	72	89	...	...	488	505	520	537	552	569	10386
A9	9	24	41	56	73	88	...	...	489	504	521	536	553	568	10386
A10	10	23	42	55	74	87	...	...	490	503	522	535	554	567	10386
A11	11	22	43	54	75	86	...	...	491	502	523	534	555	566	10386
A12	12	21	44	53	76	85	...	...	492	501	524	533	556	565	10386
A13	13	20	45	52	77	84	...	...	493	500	525	532	557	564	10386
A14	14	19	46	51	78	83	...	...	494	499	526	531	558	563	10386
A15	15	18	47	50	79	82	...	...	495	498	527	530	559	562	10386
A16	16	17	48	49	80	81	...	...	496	497	528	529	560	561	10386

Let's construct 16 blocks of magic squares of order 6 according to Example 14, and put them according to following structure.

Structure 4. *Let's consider 16 blocks of order 6 as below:*

A1	A2	A3	A4
A5	A6	A7	A8
A9	A10	A11	A12
A13	A14	A15	A16

Example 17. *16 blocks of magic squares constructed according to Example 14 by using data given in Distribution 8, and putting them according to Structure 4, we get a magic square of order 24 given by*

1	2	3	4	5	6	7	8	9	10	11	12
(I)											
1	545	544	513	32	96	2	546	543	514	31	95
480	128	448	129	161	385	479	127	447	130	162	386
384	353	225	256	320	193	383	354	226	255	319	194
288	224	321	352	257	289	287	223	322	351	258	290
97	416	160	417	449	192	98	415	159	418	450	191
481	65	33	64	512	576	482	66	34	63	511	575
5	549	540	517	28	92	6	550	539	518	27	91
476	124	444	133	165	389	475	123	443	134	166	390
380	357	229	252	316	197	379	358	230	251	315	198
284	220	325	348	261	293	283	219	326	347	262	294
101	412	156	421	453	188	102	411	155	422	454	187
485	69	37	60	508	572	486	70	38	59	507	571
9	553	536	521	24	88	10	554	535	522	23	87
472	120	440	137	169	393	471	119	439	138	170	394
376	361	233	248	312	201	375	362	234	247	311	202
280	216	329	344	265	297	279	215	330	343	266	298
105	408	152	425	457	184	106	407	151	426	458	183
489	73	41	56	504	568	490	74	42	55	503	567
13	557	532	525	20	84	14	558	531	526	19	83
468	116	436	141	173	397	467	115	435	142	174	398
372	365	237	244	308	205	371	366	238	243	307	206
276	212	333	340	269	301	275	211	334	339	270	302
109	404	148	429	461	180	110	403	147	430	462	179
493	77	45	52	500	564	494	78	46	51	499	563
6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924

13	14	15	16	17	18	19	20	21	22	23	24	
(II)												6924
3	547	542	515	30	94	4	548	541	516	29	93	6924
478	126	446	131	163	387	477	125	445	132	164	388	6924
382	355	227	254	318	195	381	356	228	253	317	196	6924
286	222	323	350	259	291	285	221	324	349	260	292	6924
99	414	158	419	451	190	100	413	157	420	452	189	6924
483	67	35	62	510	574	484	68	36	61	509	573	6924
7	551	538	519	26	90	8	552	537	520	25	89	6924
474	122	442	135	167	391	473	121	441	136	168	392	6924
378	359	231	250	314	199	377	360	232	249	313	200	6924
282	218	327	346	263	295	281	217	328	345	264	296	6924
103	410	154	423	455	186	104	409	153	424	456	185	6924
487	71	39	58	506	570	488	72	40	57	505	569	6924
11	555	534	523	22	86	12	556	533	524	21	85	6924
470	118	438	139	171	395	469	117	437	140	172	396	6924
374	363	235	246	310	203	373	364	236	245	309	204	6924
278	214	331	342	267	299	277	213	332	341	268	300	6924
107	406	150	427	459	182	108	405	149	428	460	181	6924
491	75	43	54	502	566	492	76	44	53	501	565	6924
15	559	530	527	18	82	16	560	529	528	17	81	6924
466	114	434	143	175	399	465	113	433	144	176	400	6924
370	367	239	242	306	207	369	368	240	241	305	208	6924
274	210	335	338	271	303	273	209	336	337	272	304	6924
111	402	146	431	463	178	112	401	145	432	464	177	6924
495	79	47	50	498	562	496	80	48	49	497	561	6924
6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924

Combining partes (I) and (II) we get the required result. In this case the magic sum is $S_{24\times 24} = 6924$, and all the 16 blocks of order 6 are magic squares with equal magic sums $S_{6\times 6} := 1731$.

3.5 Magic Square of Order 30

In this subsection, we shall give construction of a magic square of order 30 in 25 blocks of magic squares of order 6 with equal magic sums. In order to construct it let's divide 900 numbers in 25 equal parts according to following distribution.

Distribution 9. *Let's consider the following distribution of 1 to 900 numbers in 25 blocks of equal sums:*

	1	2	3	4	5	6	...	...	31	32	33	34	35	36	Total
A1	1	50	51	100	101	150	...	...	751	800	801	850	851	900	16218
A2	2	49	52	99	102	149	...	...	752	799	802	849	852	899	16218
A3	3	48	53	98	103	148	...	...	753	798	803	848	853	898	16218
A4	4	47	54	97	104	147	...	...	754	797	804	847	854	897	16218
A5	5	46	55	96	105	146	...	...	755	796	805	846	855	896	16218
A6	6	45	56	95	106	145	...	...	756	795	806	845	856	895	16218
A7	7	44	57	94	107	144	...	...	757	794	807	844	857	894	16218
A8	8	43	58	93	108	143	...	...	758	793	808	843	858	893	16218
A9	9	42	59	92	109	142	...	...	759	792	809	842	859	892	16218
A10	10	41	60	91	110	141	...	...	760	791	810	841	860	891	16218
A11	11	40	61	90	111	140	...	...	761	790	811	840	861	890	16218
A12	12	39	62	89	112	139	...	...	762	789	812	839	862	889	16218
A13	13	38	63	88	113	138	...	...	763	788	813	838	863	888	16218
A14	14	37	64	87	114	137	...	...	764	787	814	837	864	887	16218
A15	15	36	65	86	115	136	...	...	765	786	815	836	865	886	16218
A16	16	35	66	85	116	135	...	...	766	785	816	835	866	885	16218
A17	17	34	67	84	117	134	...	...	767	784	817	834	867	884	16218
A18	18	33	68	83	118	133	...	...	768	783	818	833	868	883	16218
A19	19	32	69	82	119	132	...	...	769	782	819	832	869	882	16218
A20	20	31	70	81	120	131	...	...	770	781	820	831	870	881	16218
A21	21	30	71	80	121	130	...	...	771	780	821	830	871	880	16218
A22	22	29	72	79	122	129	...	...	772	779	822	829	872	879	16218
A23	23	28	73	78	123	128	...	...	773	778	823	828	873	878	16218
A24	24	27	74	77	124	127	...	...	774	777	824	827	874	877	16218
A25	25	26	75	76	125	126	...	...	775	776	825	826	875	876	16218

Let's construct 25 magic squares of order 6 according to Example 14, and put them according to following structure.

Structure 5. *Let's consider 25 blocks of magic squares of order 6 as below:*

A1	A2	A3	A4	A5
A6	A7	A8	A9	A10
A11	A12	A13	A14	A15
A16	A17	A18	A19	A20
A21	A22	A23	A24	A25

Example 18. *25 blocks of magic squares constructed according to Example 14 by using data given in Distribution 9, and putting them according to Structure 5, we get a magic square of order 30 given by*

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
(I)																	
1	851	850	801	50	150	2	852	849	802	49	149	3	853	848	803	48	148
750	200	700	201	251	601	749	199	699	202	252	602	748	198	698	203	253	603
600	551	351	400	500	301	599	552	352	399	499	302	598	553	353	398	498	303
450	350	501	550	401	451	449	349	502	549	402	452	448	348	503	548	403	453
151	650	250	651	701	300	152	649	249	652	702	299	153	648	248	653	703	298
751	101	51	100	800	900	752	102	52	99	799	899	753	103	53	98	798	898
6	856	845	806	45	145	7	857	844	807	44	144	8	858	843	808	43	143
745	195	695	206	256	606	744	194	694	207	257	607	743	193	693	208	258	608
595	556	356	395	495	306	594	557	357	394	494	307	593	558	358	393	493	308
445	345	506	545	406	456	444	344	507	544	407	457	443	343	508	543	408	458
156	645	245	656	706	295	157	644	244	657	707	294	158	643	243	658	708	293
756	106	56	95	795	895	757	107	57	94	794	894	758	108	58	93	793	893
11	861	840	811	40	140	12	862	839	812	39	139	13	863	838	813	38	138
740	190	690	211	261	611	739	189	689	212	262	612	738	188	688	213	263	613
590	561	361	390	490	311	589	562	362	389	489	312	588	563	363	388	488	313
440	340	511	540	411	461	439	339	512	539	412	462	438	338	513	538	413	463
161	640	240	661	711	290	162	639	239	662	712	289	163	638	238	663	713	288
761	111	61	90	790	890	762	112	62	89	789	889	763	113	63	88	788	888
16	866	835	816	35	135	17	867	834	817	34	134	18	868	833	818	33	133
735	185	685	216	266	616	734	184	684	217	267	617	733	183	683	218	268	618
585	566	366	385	485	316	584	567	367	384	484	317	583	568	368	383	483	318
435	335	516	535	416	466	434	334	517	534	417	467	433	333	518	533	418	468
166	635	235	666	716	285	167	634	234	667	717	284	168	633	233	668	718	283
766	116	66	85	785	885	767	117	67	84	784	884	768	118	68	83	783	883
21	871	830	821	30	130	22	872	829	822	29	129	23	873	828	823	28	128
730	180	680	221	271	621	729	179	679	222	272	622	728	178	678	223	273	623
580	571	371	380	480	321	579	572	372	379	479	322	578	573	373	378	478	323
430	330	521	530	421	471	429	329	522	529	422	472	428	328	523	528	423	473
171	630	230	671	721	280	172	629	229	672	722	279	173	628	228	673	723	278
771	121	71	80	780	880	772	122	72	79	779	879	773	123	73	78	778	878
13515	13515	13515	13515	13515	13515	13515	13515	13515	13515	13515	13515	13515	13515	13515	13515	13515	13515

19	20	21	22	23	24	25	26	27	28	29	30	
(II)												13515
4	854	847	804	47	147	5	855	846	805	46	146	13515
747	197	697	204	254	604	746	196	696	205	255	605	13515
597	554	354	397	497	304	596	555	355	396	496	305	13515
447	347	504	547	404	454	446	346	505	546	405	455	13515
154	647	247	654	704	297	155	646	246	655	705	296	13515
754	104	54	97	797	897	755	105	55	96	796	896	13515
9	859	842	809	42	142	10	860	841	810	41	141	13515
742	192	692	209	259	609	741	191	691	210	260	610	13515
592	559	359	392	492	309	591	560	360	391	491	310	13515
442	342	509	542	409	459	441	341	510	541	410	460	13515
159	642	242	659	709	292	160	641	241	660	710	291	13515
759	109	59	92	792	892	760	110	60	91	791	891	13515
14	864	837	814	37	137	15	865	836	815	36	136	13515
737	187	687	214	264	614	736	186	686	215	265	615	13515
587	564	364	387	487	314	586	565	365	386	486	315	13515
437	337	514	537	414	464	436	336	515	536	415	465	13515
164	637	237	664	714	287	165	636	236	665	715	286	13515
764	114	64	87	787	887	765	115	65	86	786	886	13515
19	869	832	819	32	132	20	870	831	820	31	131	13515
732	182	682	219	269	619	731	181	681	220	270	620	13515
582	569	369	382	482	319	581	570	370	381	481	320	13515
432	332	519	532	419	469	431	331	520	531	420	470	13515
169	632	232	669	719	282	170	631	231	670	720	281	13515
769	119	69	82	782	882	770	120	70	81	781	881	13515
24	874	827	824	27	127	25	875	826	825	26	126	13515
727	177	677	224	274	624	726	176	676	225	275	625	13515
577	574	374	377	477	324	576	575	375	376	476	325	13515
427	327	524	527	424	474	426	326	525	526	425	475	13515
174	627	227	674	724	277	175	626	226	675	725	276	13515
774	124	74	77	777	877	775	125	75	76	776	876	13515
13515	13515	13515	13515	13515	13515	13515	13515	13515	13515	13515	13515	13515

Combining parts (I) and (II) we get the required result. In this case, the magic square sum is $S_{30\times 30} = 13515$, and all the 25 blocks of order 6 are magic squares with equal magic sums $S_{6\times 6} := 2703$.

3.6 Magic Square of Order 36

In this subsection, we shall give construction of a magic square of order 36 in 36 blocks of magic squares of order 6 with equal magic sums. In order to construct it let's divide 1296 numbers in 36 equal parts according to following distribution.

Distribution 10. *Let's consider the following distribution of 1 to 1296 numbers in 36 blocks of equal sums:*

https://doi.org/10.5281/zenodo.2554895

	1	2	3	4	5	6	...	...	31	32	33	34	35	36	Total
A1	1	72	73	144	145	216	...	...	1081	1152	1153	1224	1225	1296	23346
A2	2	71	74	143	146	215	...	...	1082	1151	1154	1223	1226	1295	23346
A3	3	70	75	142	147	214	...	...	1083	1150	1155	1222	1227	1294	23346
A4	4	69	76	141	148	213	...	...	1084	1149	1156	1221	1228	1293	23346
A5	5	68	77	140	149	212	...	...	1085	1148	1157	1220	1229	1292	23346
A6	6	67	78	139	150	211	...	...	1086	1147	1158	1219	1230	1291	23346
A7	7	66	79	138	151	210	...	...	1087	1146	1159	1218	1231	1290	23346
A8	8	65	80	137	152	209	...	...	1088	1145	1160	1217	1232	1289	23346
A9	9	64	81	136	153	208	...	...	1089	1144	1161	1216	1233	1288	23346
A10	10	63	82	135	154	207	...	...	1090	1143	1162	1215	1234	1287	23346
A11	11	62	83	134	155	206	...	...	1091	1142	1163	1214	1235	1286	23346
A12	12	61	84	133	156	205	...	...	1092	1141	1164	1213	1236	1285	23346
A13	13	60	85	132	157	204	...	...	1093	1140	1165	1212	1237	1284	23346
A14	14	59	86	131	158	203	...	...	1094	1139	1166	1211	1238	1283	23346
A15	15	58	87	130	159	202	...	...	1095	1138	1167	1210	1239	1282	23346
A16	16	57	88	129	160	201	...	...	1096	1137	1168	1209	1240	1281	23346
A17	17	56	89	128	161	200	...	...	1097	1136	1169	1208	1241	1280	23346
A18	18	55	90	127	162	199	...	...	1098	1135	1170	1207	1242	1279	23346
A19	19	54	91	126	163	198	...	...	1099	1134	1171	1206	1243	1278	23346
A20	20	53	92	125	164	197	...	...	1100	1133	1172	1205	1244	1277	23346
A21	21	52	93	124	165	196	...	...	1101	1132	1173	1204	1245	1276	23346
A22	22	51	94	123	166	195	...	...	1102	1131	1174	1203	1246	1275	23346
A23	23	50	95	122	167	194	...	...	1103	1130	1175	1202	1247	1274	23346
A24	24	49	96	121	168	193	...	...	1104	1129	1176	1201	1248	1273	23346
A25	25	48	97	120	169	192	...	...	1105	1128	1177	1200	1249	1272	23346
A26	26	47	98	119	170	191	...	...	1106	1127	1178	1199	1250	1271	23346
A27	27	46	99	118	171	190	...	...	1107	1126	1179	1198	1251	1270	23346
A28	28	45	100	117	172	189	...	...	1108	1125	1180	1197	1252	1269	23346
A29	29	44	101	116	173	188	...	...	1109	1124	1181	1196	1253	1268	23346
A30	30	43	102	115	174	187	...	...	1110	1123	1182	1195	1254	1267	23346
A31	31	42	103	114	175	186	...	...	1111	1122	1183	1194	1255	1266	23346
A32	32	41	104	113	176	185	...	...	1112	1121	1184	1193	1256	1265	23346
A33	33	40	105	112	177	184	...	...	1113	1120	1185	1192	1257	1264	23346
A34	34	39	106	111	178	183	...	...	1114	1119	1186	1191	1258	1263	23346
A35	35	38	107	110	179	182	...	...	1115	1118	1187	1190	1259	1262	23346
A36	36	37	108	109	180	181	...	...	1116	1117	1188	1189	1260	1261	23346

Let's construct 36 magic squares of order 6 according to Example 14, and put them according to following structure.

Structure 6. *Let's consider 36 blocks of magic squares of order 6 as below:*

A1	*A2*	*A3*	*A4*	*A5*	*A6*
A7	*A8*	*A9*	*A10*	*A11*	*A12*
A13	*A14*	*A15*	*A16*	*A17*	*A18*
A19	*A20*	*A21*	*A22*	*A23*	*A24*
A25	*A26*	*A27*	*A28*	*A29*	*A30*
A31	*A32*	*A33*	*A34*	*A35*	*A36*

Example 19. *36 blocks of magic squares constructed according to Example 14 by using data given in Distribution 11, and putting them according to Structure 7, we get a magic square of order 36 given by*

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
(I)																	
1	1225	1224	1153	72	216	2	1226	1223	1154	71	215	3	1227	1222	1155	70	214
1080	288	1008	289	361	865	1079	287	1007	290	362	866	1078	286	1006	291	363	867
864	793	505	576	720	433	863	794	506	575	719	434	862	795	507	574	718	435
648	504	721	792	577	649	647	503	722	791	578	650	646	502	723	790	579	651
217	936	360	937	1009	432	218	935	359	938	1010	431	219	934	358	939	1011	430
1081	145	73	144	1152	1296	1082	146	74	143	1151	1295	1083	147	75	142	1150	1294
7	1231	1218	1159	66	210	8	1232	1217	1160	65	209	9	1233	1216	1161	64	208
1074	282	1002	295	367	871	1073	281	1001	296	368	872	1072	280	1000	297	369	873
858	799	511	570	714	439	857	800	512	569	713	440	856	801	513	568	712	441
642	498	727	786	583	655	641	497	728	785	584	656	640	496	729	784	585	657
223	930	354	943	1015	426	224	929	353	944	1016	425	225	928	352	945	1017	424
1087	151	79	138	1146	1290	1088	152	80	137	1145	1289	1089	153	81	136	1144	1288
13	1237	1212	1165	60	204	14	1238	1211	1166	59	203	15	1239	1210	1167	58	202
1068	276	996	301	373	877	1067	275	995	302	374	878	1066	274	994	303	375	879
852	805	517	564	708	445	851	806	518	563	707	446	850	807	519	562	706	447
636	492	733	780	589	661	635	491	734	779	590	662	634	490	735	778	591	663
229	924	348	949	1021	420	230	923	347	950	1022	419	231	922	346	951	1023	418
1093	157	85	132	1140	1284	1094	158	86	131	1139	1283	1095	159	87	130	1138	1282
19	1243	1206	1171	54	198	20	1244	1205	1172	53	197	21	1245	1204	1173	52	196
1062	270	990	307	379	883	1061	269	989	308	380	884	1060	268	988	309	381	885
846	811	523	558	702	451	845	812	524	557	701	452	844	813	525	556	700	453
630	486	739	774	595	667	629	485	740	773	596	668	628	484	741	772	597	669
235	918	342	955	1027	414	236	917	341	956	1028	413	237	916	340	957	1029	412
1099	163	91	126	1134	1278	1100	164	92	125	1133	1277	1101	165	93	124	1132	1276
25	1249	1200	1177	48	192	26	1250	1199	1178	47	191	27	1251	1198	1179	46	190
1056	264	984	313	385	889	1055	263	983	314	386	890	1054	262	982	315	387	891
840	817	529	552	696	457	839	818	530	551	695	458	838	819	531	550	694	459
624	480	745	768	601	673	623	479	746	767	602	674	622	478	747	766	603	675
241	912	336	961	1033	408	242	911	335	962	1034	407	243	910	334	963	1035	406
1105	169	97	120	1128	1272	1106	170	98	119	1127	1271	1107	171	99	118	1126	1270
31	1255	1194	1183	42	186	32	1256	1193	1184	41	185	33	1257	1192	1185	40	184
1050	258	978	319	391	895	1049	257	977	320	392	896	1048	256	976	321	393	897
834	823	535	546	690	463	833	824	536	545	689	464	832	825	537	544	688	465
618	474	751	762	607	679	617	473	752	761	608	680	616	472	753	760	609	681
247	906	330	967	1039	402	248	905	329	968	1040	401	249	904	328	969	1041	400
1111	175	103	114	1122	1266	1112	176	104	113	1121	1265	1113	177	105	112	1120	1264
23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346

19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	(II)
(II)																		23346
4	1228	1221	1156	69	213	5	1229	1220	1157	68	212	6	1230	1219	1158	67	211	23346
1077	285	1005	292	364	868	1076	284	1004	293	365	869	1075	283	1003	294	366	870	23346
861	796	508	573	717	436	860	797	509	572	716	437	859	798	510	571	715	438	23346
645	501	724	789	580	652	644	500	725	788	581	653	643	499	726	787	582	654	23346
220	933	357	940	1012	429	221	932	356	941	1013	428	222	931	355	942	1014	427	23346
1084	148	76	141	1149	1293	1085	149	77	140	1148	1292	1086	150	78	139	1147	1291	23346
10	1234	1215	1162	63	207	11	1235	1214	1163	62	206	12	1236	1213	1164	61	205	23346
1071	279	999	298	370	874	1070	278	998	299	371	875	1069	277	997	300	372	876	23346
855	802	514	567	711	442	854	803	515	566	710	443	853	804	516	565	709	444	23346
639	495	730	783	586	658	638	494	731	782	587	659	637	493	732	781	588	660	23346
226	927	351	946	1018	423	227	926	350	947	1019	422	228	925	349	948	1020	421	23346
1090	154	82	135	1143	1287	1091	155	83	134	1142	1286	1092	156	84	133	1141	1285	23346
16	1240	1209	1168	57	201	17	1241	1208	1169	56	200	18	1242	1207	1170	55	199	23346
1065	273	993	304	376	880	1064	272	992	305	377	881	1063	271	991	306	378	882	23346
849	808	520	561	705	448	848	809	521	560	704	449	847	810	522	559	703	450	23346
633	489	736	777	592	664	632	488	737	776	593	665	631	487	738	775	594	666	23346
232	921	345	952	1024	417	233	920	344	953	1025	416	234	919	343	954	1026	415	23346
1096	160	88	129	1137	1281	1097	161	89	128	1136	1280	1098	162	90	127	1135	1279	23346
22	1246	1203	1174	51	195	23	1247	1202	1175	50	194	24	1248	1201	1176	49	193	23346
1059	267	987	310	382	886	1058	266	986	311	383	887	1057	265	985	312	384	888	23346
843	814	526	555	699	454	842	815	527	554	698	455	841	816	528	553	697	456	23346
627	483	742	771	598	670	626	482	743	770	599	671	625	481	744	769	600	672	23346
238	915	339	958	1030	411	239	914	338	959	1031	410	240	913	337	960	1032	409	23346
1102	166	94	123	1131	1275	1103	167	95	122	1130	1274	1104	168	96	121	1129	1273	23346
28	1252	1197	1180	45	189	29	1253	1196	1181	44	188	30	1254	1195	1182	43	187	23346
1053	261	981	316	388	892	1052	260	980	317	389	893	1051	259	979	318	390	894	23346
837	820	532	549	693	460	836	821	533	548	692	461	835	822	534	547	691	462	23346
621	477	748	765	604	676	620	476	749	764	605	677	619	475	750	763	606	678	23346
244	909	333	964	1036	405	245	908	332	965	1037	404	246	907	331	966	1038	403	23346
1108	172	100	117	1125	1269	1109	173	101	116	1124	1268	1110	174	102	115	1123	1267	23346
34	1258	1191	1186	39	183	35	1259	1190	1187	38	182	36	1260	1189	1188	37	181	23346
1047	255	975	322	394	898	1046	254	974	323	395	899	1045	253	973	324	396	900	23346
831	826	538	543	687	466	830	827	539	542	686	467	829	828	540	541	685	468	23346
615	471	754	759	610	682	614	470	755	758	611	683	613	469	756	757	612	684	23346
250	903	327	970	1042	399	251	902	326	971	1043	398	252	901	325	972	1044	397	23346
1114	178	106	111	1119	1263	1115	179	107	110	1118	1262	1116	180	108	109	1117	1261	23346
23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346

Combining parts (I) and (II) we get the required result. In this case, the magic sum is $S_{36\times 36} = 23346$, and all the 36 blocks of order 6 are magic squares with equal magic sums $S_{6\times 6} := 3891$.

3.7 Pan Magic Square of Order 36

Block-wise construction of pan magic square of order 36 is done in two different ways. In the first way, instead working with 12x3 we shall work with 4x9, and in the second way we shall consider 9x4. This is done in two seperate subsections below.

3.7.1 81-Blocks of Order 4

A magic square of order 36 is constructed by considering 81 sub-blocks of pan magic square of order 4 with equal magic sums, resulting in a magic square of order 36. The total 1296 numbers from 1 to 296 are divided in 81 blocks of 16 each according to distribution given below.

Distribution 11. *Let's distribute the 1296 numbers from 1 to 1296 in 81 blocks resulting in equal sums:*

A1	1	162	163	324	325	486	487	648	649	810	811	972	973	1134	1135	1296	10376
A2	2	161	164	323	326	485	488	647	650	809	812	971	974	1133	1136	1295	10376
A3	3	160	165	322	327	484	489	646	651	808	813	970	975	1132	1137	1294	10376
A4	4	159	166	321	328	483	490	645	652	807	814	969	976	1131	1138	1293	10376
A5	5	158	167	320	329	482	491	644	653	806	815	968	977	1130	1139	1292	10376
A6	6	157	168	319	330	481	492	643	654	805	816	967	978	1129	1140	1291	10376
A7	7	156	169	318	331	480	493	642	655	804	817	966	979	1128	1141	1290	10376
A8	8	155	170	317	332	479	494	641	656	803	818	965	980	1127	1142	1289	10376
A9	9	154	171	316	333	478	495	640	657	802	819	964	981	1126	1143	1288	10376
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
A73	73	90	235	252	397	414	559	576	721	738	883	900	1045	1062	1207	1224	10376
A74	74	89	236	251	398	413	560	575	722	737	884	899	1046	1061	1208	1223	10376
A75	75	88	237	250	399	412	561	574	723	736	885	898	1047	1060	1209	1222	10376
A76	76	87	238	249	400	411	562	573	724	735	886	897	1048	1059	1210	1221	10376
A77	77	86	239	248	401	410	563	572	725	734	887	896	1049	1058	1211	1220	10376
A78	78	85	240	247	402	409	564	571	726	733	888	895	1050	1057	1212	1219	10376
A79	79	84	241	246	403	408	565	570	727	732	889	894	1051	1056	1213	1218	10376
A80	80	83	242	245	404	407	566	569	728	731	890	893	1052	1055	1214	1217	10376
A81	81	82	243	244	405	406	567	568	729	730	891	892	1053	1054	1215	1216	10376

Above distribution is similar to 6 or 8, where the columns are written in increasing and decreasing orders, such as, $[1,2,\ldots,81]$, $[162,161,\ldots,82]$, $[83,84,\ldots,243]$, etc. Each block results in a pan magic square of order 4 constructed according to Example **??**. To construct pan magic square of order 36, we shall use the following structure.

Structure 7. *Let's consider 81 blocks of order 4 given as below:*

A1	*A2*	*A3*	*A4*	*A5*	*A6*	*A7*	*A8*	*A9*
A10	*A11*	*A12*	*A13*	*A14*	*A15*	*A16*	*A17*	*A18*
A19	*A20*	*A21*	*A22*	*A23*	*A24*	*A25*	*A26*	*A27*
A28	*A29*	*A30*	*A31*	*A32*	*A33*	*A34*	*A35*	*A36*
A37	*A38*	*A39*	*A40*	*A41*	*A42*	*A43*	*A44*	*A45*
A46	*A47*	*A48*	*A49*	*A50*	*A51*	*A52*	*A53*	*A54*
A55	*A56*	*A57*	*A58*	*A59*	*A60*	*A61*	*A62*	*A63*
A64	*A65*	*A66*	*A67*	*A68*	*A69*	*A70*	*A71*	*A72*
A73	*A74*	*A75*	*A76*	*A77*	*A78*	*A79*	*A80*	*A81*

According to above Structure 7, where each sub-block is a perfect pan magic of order 4 constructed according to Example **??** give the following magic square of order 36.

Example 20. *A pan magic square of order 36 with each sub block of order 4 a perfect pan magic of order 4 is given by*

(I)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
		23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346
	487	972	1	1134	488	971	2	1133	489	970	3	1132	490	969	4	1131	491	968
23346	162	973	648	811	161	974	647	812	160	975	646	813	159	976	645	814	158	977
23346	1296	163	810	325	1295	164	809	326	1294	165	808	327	1293	166	807	328	1292	167
23346	649	486	1135	324	650	485	1136	323	651	484	1137	322	652	483	1138	321	653	482
23346	496	963	10	1125	497	962	11	1124	498	961	12	1123	499	960	13	1122	500	959
23346	153	982	639	820	152	983	638	821	151	984	637	822	150	985	636	823	149	986
23346	1287	172	801	334	1286	173	800	335	1285	174	799	336	1284	175	798	337	1283	176
23346	658	477	1144	315	659	476	1145	314	660	475	1146	313	661	474	1147	312	662	473
23346	505	954	19	1116	506	953	20	1115	507	952	21	1114	508	951	22	1113	509	950
23346	144	991	630	829	143	992	629	830	142	993	628	831	141	994	627	832	140	995
23346	1278	181	792	343	1277	182	791	344	1276	183	790	345	1275	184	789	346	1274	185
23346	667	468	1153	306	668	467	1154	305	669	466	1155	304	670	465	1156	303	671	464
23346	514	945	28	1107	515	944	29	1106	516	943	30	1105	517	942	31	1104	518	941
23346	135	1000	621	838	134	1001	620	839	133	1002	619	840	132	1003	618	841	131	1004
23346	1269	190	783	352	1268	191	782	353	1267	192	781	354	1266	193	780	355	1265	194
23346	676	459	1162	297	677	458	1163	296	678	457	1164	295	679	456	1165	294	680	455
23346	523	936	37	1098	524	935	38	1097	525	934	39	1096	526	933	40	1095	527	932
23346	126	1009	612	847	125	1010	611	848	124	1011	610	849	123	1012	609	850	122	1013
23346	1260	199	774	361	1259	200	773	362	1258	201	772	363	1257	202	771	364	1256	203
23346	685	450	1171	288	686	449	1172	287	687	448	1173	286	688	447	1174	285	689	446
23346	532	927	46	1089	533	926	47	1088	534	925	48	1087	535	924	49	1086	536	923
23346	117	1018	603	856	116	1019	602	857	115	1020	601	858	114	1021	600	859	113	1022
23346	1251	208	765	370	1250	209	764	371	1249	210	763	372	1248	211	762	373	1247	212
23346	694	441	1180	279	695	440	1181	278	696	439	1182	277	697	438	1183	276	698	437
23346	541	918	55	1080	542	917	56	1079	543	916	57	1078	544	915	58	1077	545	914
23346	108	1027	594	865	107	1028	593	866	106	1029	592	867	105	1030	591	868	104	1031
23346	1242	217	756	379	1241	218	755	380	1240	219	754	381	1239	220	753	382	1238	221
23346	703	432	1189	270	704	431	1190	269	705	430	1191	268	706	429	1192	267	707	428
23346	550	909	64	1071	551	908	65	1070	552	907	66	1069	553	906	67	1068	554	905
23346	99	1036	585	874	98	1037	584	875	97	1038	583	876	96	1039	582	877	95	1040
23346	1233	226	747	388	1232	227	746	389	1231	228	745	390	1230	229	744	391	1229	230
23346	712	423	1198	261	713	422	1199	260	714	421	1200	259	715	420	1201	258	716	419
23346	559	900	73	1062	560	899	74	1061	561	898	75	1060	562	897	76	1059	563	896
23346	90	1045	576	883	89	1046	575	884	88	1047	574	885	87	1048	573	886	86	1049
23346	1224	235	738	397	1223	236	737	398	1222	237	736	399	1221	238	735	400	1220	239
	721	414	1207	252	722	413	1208	251	723	412	1209	250	724	411	1210	249	725	410
	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346

19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	II
23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346
5	1130	492	967	6	1129	493	966	7	1128	494	965	8	1127	495	964	9	1126	23346
644	815	157	978	643	816	156	979	642	817	155	980	641	818	154	981	640	819	23346
806	329	1291	168	805	330	1290	169	804	331	1289	170	803	332	1288	171	802	333	23346
1139	320	654	481	1140	319	655	480	1141	318	656	479	1142	317	657	478	1143	316	23346
14	1121	501	958	15	1120	502	957	16	1119	503	956	17	1118	504	955	18	1117	23346
635	824	148	987	634	825	147	988	633	826	146	989	632	827	145	990	631	828	23346
797	338	1282	177	796	339	1281	178	795	340	1280	179	794	341	1279	180	793	342	23346
1148	311	663	472	1149	310	664	471	1150	309	665	470	1151	308	666	469	1152	307	23346
23	1112	510	949	24	1111	511	948	25	1110	512	947	26	1109	513	946	27	1108	23346
626	833	139	996	625	834	138	997	624	835	137	998	623	836	136	999	622	837	23346
788	347	1273	186	787	348	1272	187	786	349	1271	188	785	350	1270	189	784	351	23346
1157	302	672	463	1158	301	673	462	1159	300	674	461	1160	299	675	460	1161	298	23346
32	1103	519	940	33	1102	520	939	34	1101	521	938	35	1100	522	937	36	1099	23346
617	842	130	1005	616	843	129	1006	615	844	128	1007	614	845	127	1008	613	846	23346
779	356	1264	195	778	357	1263	196	777	358	1262	197	776	359	1261	198	775	360	23346
1166	293	681	454	1167	292	682	453	1168	291	683	452	1169	290	684	451	1170	289	23346
41	1094	528	931	42	1093	529	930	43	1092	530	929	44	1091	531	928	45	1090	23346
608	851	121	1014	607	852	120	1015	606	853	119	1016	605	854	118	1017	604	855	23346
770	365	1255	204	769	366	1254	205	768	367	1253	206	767	368	1252	207	766	369	23346
1175	284	690	445	1176	283	691	444	1177	282	692	443	1178	281	693	442	1179	280	23346
50	1085	537	922	51	1084	538	921	52	1083	539	920	53	1082	540	919	54	1081	23346
599	860	112	1023	598	861	111	1024	597	862	110	1025	596	863	109	1026	595	864	23346
761	374	1246	213	760	375	1245	214	759	376	1244	215	758	377	1243	216	757	378	23346
1184	275	699	436	1185	274	700	435	1186	273	701	434	1187	272	702	433	1188	271	23346
59	1076	546	913	60	1075	547	912	61	1074	548	911	62	1073	549	910	63	1072	23346
590	869	103	1032	589	870	102	1033	588	871	101	1034	587	872	100	1035	586	873	23346
752	383	1237	222	751	384	1236	223	750	385	1235	224	749	386	1234	225	748	387	23346
1193	266	708	427	1194	265	709	426	1195	264	710	425	1196	263	711	424	1197	262	23346
68	1067	555	904	69	1066	556	903	70	1065	557	902	71	1064	558	901	72	1063	23346
581	878	94	1041	580	879	93	1042	579	880	92	1043	578	881	91	1044	577	882	23346
743	392	1228	231	742	393	1227	232	741	394	1226	233	740	395	1225	234	739	396	23346
1202	257	717	418	1203	256	718	417	1204	255	719	416	1205	254	720	415	1206	253	23346
77	1058	564	895	78	1057	565	894	79	1056	566	893	80	1055	567	892	81	1054	23346
572	887	85	1050	571	888	84	1051	570	889	83	1052	569	890	82	1053	568	891	23346
734	401	1219	240	733	402	1218	241	732	403	1217	242	731	404	1216	243	730	405	23346
1211	248	726	409	1212	247	727	408	1213	246	728	407	1214	245	729	406	1215	244	23346
23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346

In this case, the magic sum is $S_{36\times 36} = 23346$. Each 4×4 block is a perfect **pan magic square** of order 4 with equal magic sums $S_{4\times 4} = 2594$.

3.7.2 16-Blocks of Order 9

First of all it is not possible write a magic square of order 36 with each block order 9 a magic square of same sums, because, the magic sum of order 36 divided by 4 give a fraction value, i.e., $\frac{23346}{4} = \frac{11673}{2}$. The only alternative is to have 16 blocks of order 9 with different sums resulting in a pan magic of order 36. Let apply the same approach as of orders 15, 18, 21, etc. Let's divide the 36 numbers 1 to 36 in four parts as given below:

Distribution 12. *Let's consider the following distribution to construct **magic square** of order 36:*

	1	2	3	4	5	6	7	8	...	...	76	77	78	79	80	81	Total
A1	1	17	33	49	65	81	97	113	...	...	1201	1217	1233	1249	1265	1281	51921
A2	2	18	34	50	66	82	98	114	...	...	1202	1218	1234	1250	1266	1282	52002
A3	3	19	35	51	67	83	99	115	...	...	1203	1219	1235	1251	1267	1283	52083
A4	4	20	36	52	68	84	100	116	...	...	1204	1220	1236	1252	1268	1284	52164
A5	5	21	37	53	69	85	101	117	...	...	1205	1221	1237	1253	1269	1285	52245
A6	6	22	38	54	70	86	102	118	...	...	1206	1222	1238	1254	1270	1286	52326
A7	7	23	39	55	71	87	103	119	...	...	1207	1223	1239	1255	1271	1287	52407
A8	8	24	40	56	72	88	104	120	...	...	1208	1224	1240	1256	1272	1288	52488
A9	9	25	41	57	73	89	105	121	...	...	1209	1225	1241	1257	1273	1289	52569
A10	10	26	42	58	74	90	106	122	...	...	1210	1226	1242	1258	1274	1290	52650
A11	11	27	43	59	75	91	107	123	...	...	1211	1227	1243	1259	1275	1291	52731
A12	12	28	44	60	76	92	108	124	...	...	1212	1228	1244	1260	1276	1292	52812
A13	13	29	45	61	77	93	109	125	...	...	1213	1229	1245	1261	1277	1293	52893
A14	14	30	46	62	78	94	110	126	...	...	1214	1230	1246	1262	1278	1294	52974
A15	15	31	47	63	79	95	111	127	...	...	1215	1231	1247	1263	1279	1295	53055
A16	16	32	48	64	80	96	112	128	...	...	1216	1232	1248	1264	1280	1296	53136

There are total 16 blocks of 81 numbers with equal difference in each row. Lets make 16 pan magic squares of order 9 and according to Example 2 and put them according to pan magic of order 4 given in Example **??**:

Structure 8. *Let's put the rows A1 to A16 according pan magic square of order 4 given in Example* **??**:

A7	A12	A1	A14
A2	A13	A8	A11
A16	A3	A10	A5
A9	A6	A15	A4

Substituting the total sum of each row block numbers in above Structure 8 we get a perfect pan magic square of order 4.

Example 21. *The pan magic square of order 4 based on the sum of each row from A1 to A16 is given by*

		23346	23346	23346	23346
	5823	5868	5769	5886	23346
23346	5778	5877	5832	5859	23346
23346	5904	5787	5850	5805	23346
23346	5841	5814	5895	5796	23346
	23346	23346	23346	23346	23346

According to above Structure 8, where each sub-block is a perfect pan magic of order 4 constructed according to Example **??** give the following magic square of order 36.

Example 22. *A pan magic square of order 36 with each sub block a pan magic square of order 9 is given by*

(I)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
		23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346
23346	7	695	1239	23	711	1207	39	679	1223	12	700	1244	28	716	1212	44	684	1228
23346	647	1191	103	663	1159	119	631	1175	135	652	1196	108	668	1164	124	636	1180	140
23346	1287	55	599	1255	71	615	1271	87	583	1292	60	604	1260	76	620	1276	92	588
23346	775	887	279	791	903	247	807	871	263	780	892	284	796	908	252	812	876	268
23346	983	231	727	999	199	743	967	215	759	988	236	732	1004	204	748	972	220	764
23346	183	823	935	151	839	951	167	855	919	188	828	940	156	844	956	172	860	924
23346	1111	359	471	1127	375	439	1143	343	455	1116	364	476	1132	380	444	1148	348	460
23346	311	567	1063	327	535	1079	295	551	1095	316	572	1068	332	540	1084	300	556	1100
23346	519	1015	407	487	1031	423	503	1047	391	524	1020	412	492	1036	428	508	1052	396
23346	2	690	1234	18	706	1202	34	674	1218	13	701	1245	29	717	1213	45	685	1229
23346	642	1186	98	658	1154	114	626	1170	130	653	1197	109	669	1165	125	637	1181	141
23346	1282	50	594	1250	66	610	1266	82	578	1293	61	605	1261	77	621	1277	93	589
23346	770	882	274	786	898	242	802	866	258	781	893	285	797	909	253	813	877	269
23346	978	226	722	994	194	738	962	210	754	989	237	733	1005	205	749	973	221	765
23346	178	818	930	146	834	946	162	850	914	189	829	941	157	845	957	173	861	925
23346	1106	354	466	1122	370	434	1138	338	450	1117	365	477	1133	381	445	1149	349	461
23346	306	562	1058	322	530	1074	290	546	1090	317	573	1069	333	541	1085	301	557	1101
23346	514	1010	402	482	1026	418	498	1042	386	525	1021	413	493	1037	429	509	1053	397
23346	16	704	1248	32	720	1216	48	688	1232	3	691	1235	19	707	1203	35	675	1219
23346	656	1200	112	672	1168	128	640	1184	144	643	1187	99	659	1155	115	627	1171	131
23346	1296	64	608	1264	80	624	1280	96	592	1283	51	595	1251	67	611	1267	83	579
23346	784	896	288	800	912	256	816	880	272	771	883	275	787	899	243	803	867	259
23346	992	240	736	1008	208	752	976	224	768	979	227	723	995	195	739	963	211	755
23346	192	832	944	160	848	960	176	864	928	179	819	931	147	835	947	163	851	915
23346	1120	368	480	1136	384	448	1152	352	464	1107	355	467	1123	371	435	1139	339	451
23346	320	576	1072	336	544	1088	304	560	1104	307	563	1059	323	531	1075	291	547	1091
23346	528	1024	416	496	1040	432	512	1056	400	515	1011	403	483	1027	419	499	1043	387
23346	9	697	1241	25	713	1209	41	681	1225	6	694	1238	22	710	1206	38	678	1222
23346	649	1193	105	665	1161	121	633	1177	137	646	1190	102	662	1158	118	630	1174	134
23346	1289	57	601	1257	73	617	1273	89	585	1286	54	598	1254	70	614	1270	86	582
23346	777	889	281	793	905	249	809	873	265	774	886	278	790	902	246	806	870	262
23346	985	233	729	1001	201	745	969	217	761	982	230	726	998	198	742	966	214	758
23346	185	825	937	153	841	953	169	857	921	182	822	934	150	838	950	166	854	918
23346	1113	361	473	1129	377	441	1145	345	457	1110	358	470	1126	374	438	1142	342	454
23346	313	569	1065	329	537	1081	297	553	1097	310	566	1062	326	534	1078	294	550	1094
	521	1017	409	489	1033	425	505	1049	393	518	1014	406	486	1030	422	502	1046	390
	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346

19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	(II)
23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346
1	689	1233	17	705	1201	33	673	1217	14	702	1246	30	718	1214	46	686	1230	23346
641	1185	97	657	1153	113	625	1169	129	654	1198	110	670	1166	126	638	1182	142	23346
1281	49	593	1249	65	609	1265	81	577	1294	62	606	1262	78	622	1278	94	590	23346
769	881	273	785	897	241	801	865	257	782	894	286	798	910	254	814	878	270	23346
977	225	721	993	193	737	961	209	753	990	238	734	1006	206	750	974	222	766	23346
177	817	929	145	833	945	161	849	913	190	830	942	158	846	958	174	862	926	23346
1105	353	465	1121	369	433	1137	337	449	1118	366	478	1134	382	446	1150	350	462	23346
305	561	1057	321	529	1073	289	545	1089	318	574	1070	334	542	1086	302	558	1102	23346
513	1009	401	481	1025	417	497	1041	385	526	1022	414	494	1038	430	510	1054	398	23346
8	696	1240	24	712	1208	40	680	1224	11	699	1243	27	715	1211	43	683	1227	23346
648	1192	104	664	1160	120	632	1176	136	651	1195	107	667	1163	123	635	1179	139	23346
1288	56	600	1256	72	616	1272	88	584	1291	59	603	1259	75	619	1275	91	587	23346
776	888	280	792	904	248	808	872	264	779	891	283	795	907	251	811	875	267	23346
984	232	728	1000	200	744	968	216	760	987	235	731	1003	203	747	971	219	763	23346
184	824	936	152	840	952	168	856	920	187	827	939	155	843	955	171	859	923	23346
1112	360	472	1128	376	440	1144	344	456	1115	363	475	1131	379	443	1147	347	459	23346
312	568	1064	328	536	1080	296	552	1096	315	571	1067	331	539	1083	299	555	1099	23346
520	1016	408	488	1032	424	504	1048	392	523	1019	411	491	1035	427	507	1051	395	23346
10	698	1242	26	714	1210	42	682	1226	5	693	1237	21	709	1205	37	677	1221	23346
650	1194	106	666	1162	122	634	1178	138	645	1189	101	661	1157	117	629	1173	133	23346
1290	58	602	1258	74	618	1274	90	586	1285	53	597	1253	69	613	1269	85	581	23346
778	890	282	794	906	250	810	874	266	773	885	277	789	901	245	805	869	261	23346
986	234	730	1002	202	746	970	218	762	981	229	725	997	197	741	965	213	757	23346
186	826	938	154	842	954	170	858	922	181	821	933	149	837	949	165	853	917	23346
1114	362	474	1130	378	442	1146	346	458	1109	357	469	1125	373	437	1141	341	453	23346
314	570	1066	330	538	1082	298	554	1098	309	565	1061	325	533	1077	293	549	1093	23346
522	1018	410	490	1034	426	506	1050	394	517	1013	405	485	1029	421	501	1045	389	23346
15	703	1247	31	719	1215	47	687	1231	4	692	1236	20	708	1204	36	676	1220	23346
655	1199	111	671	1167	127	639	1183	143	644	1188	100	660	1156	116	628	1172	132	23346
1295	63	607	1263	79	623	1279	95	591	1284	52	596	1252	68	612	1268	84	580	23346
783	895	287	799	911	255	815	879	271	772	884	276	788	900	244	804	868	260	23346
991	239	735	1007	207	751	975	223	767	980	228	724	996	196	740	964	212	756	23346
191	831	943	159	847	959	175	863	927	180	820	932	148	836	948	164	852	916	23346
1119	367	479	1135	383	447	1151	351	463	1108	356	468	1124	372	436	1140	340	452	23346
319	575	1071	335	543	1087	303	559	1103	308	564	1060	324	532	1076	292	548	1092	23346
527	1023	415	495	1039	431	511	1055	399	516	1012	404	484	1028	420	500	1044	388	23346
23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346

Combiningt Parts (I) and (II) we get the required result. In this case, the magic sum is $S_{36\times 36} = 23346$. Each 9×9 block is a **pan magic square** of order 9 with different magic sums given according to Example 21.

4 Final Comments

In this work we tried to bring block-wise equal sums pan diagonal magic squares of type 3k by use of idea of triples. Most the of the case we got satisfactory results except the cases of order 18, 27 and 30. The triples used in respective cases ares

4.1 Triples

Below are triples used to construct magic squares with equal sums.

- **Order** 9×9

①	1	6	8	15
②	3	5	7	15
③	2	4	9	15

- **Order** 12×12

①	1	6	7	12	28
②	2	5	8	11	28
③	3	4	9	10	28

- **Order** 15×15

①	1	6	8	12	13	40
②	3	5	7	11	14	40
③	2	4	9	10	15	40

- **Order** 18×18

①	1	6	7	12	13	18	57
②	2	5	8	11	14	17	57
③	3	4	9	10	15	16	57

- **Order** 21×21

①	1	6	8	12	13	182	19	77
②	3	5	7	11	14	17	20	77
③	2	4	9	10	15	16	21	77

- **Order** 24×24

①	1	6	7	12	13	18	19	24	100
②	2	5	8	11	14	17	20	23	100
③	3	4	9	10	15	16	21	22	100

- **Order** 27×27

①	1	6	8	12	13	18	19	24	25	126
②	3	5	7	11	14	17	20	23	26	126
③	2	4	9	10	15	16	21	22	27	126

- **Order** 30×30

①	1	6	7	12	13	18	19	24	25	30	155
②	2	5	8	11	14	17	20	23	26	29	155
③	3	4	9	10	15	16	21	22	27	28	155

- **Order** 33×33

①	1	6	8	12	13	18	19	24	25	30	31	187
②	3	5	7	11	14	17	20	23	26	29	32	187
③	2	4	9	10	15	16	21	22	27	28	33	187

- **Order** 36×36

①	1	6	7	12	13	18	19	24	25	30	31	36	222
②	2	5	8	11	14	17	20	23	26	29	32	35	222
③	3	4	9	10	15	16	21	22	27	28	33	34	222

During past years the author worked with magic squares in different situations. These details are given below:

- **Author's Contributions**

The author's work on magic squares done during past years is item-wise separated below:

(i) **Digital numbers** magic squares - [7, 8, 9, 10, 11, 12];
(ii) **Block-wise construction** of **bimagic squares** - [13];
(iii) Connections with **genetic tables** and **Shannon's entropy** - [14];
(iv) **Selfie** and **palindromic-type** magic squares - [15];
(v) **Intervally distributed** and **block-wise** magic squares - [16, 17, 18];
(vi) **Multi-digits** magic squares - [19];
(vii) **Perfect square sum** magic squares with uniformity and minimum sum - [20, 21];
(viii) **Pythagorean triples** to generate **perfect square sum** magic squares - [21];
(ix) **Block-wise** equal sums **pan magic squares of order 4k** - [22];
(x) **Block-wise** equal sums **magic squares of order** $3k$ - [23];
(xi) **Block-wise** unequal sums **magic squares of order** $3k$ - [25];
(xii) **Magic rectangles** in Construction of **block-wise pan magic squares** - [26].

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