

Generating Pythagorean Triples, Patterns, and Magic Squares

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Abstract

*This paper brings simplified and symmetric procedure to generate **Pythagorean triples**. These triples are obtained in different procedures. First procedure is given in three blocks. The second procedure is the extension of first procedure, but in little different way. These triples are applied to generate **perfect square sums magic squares of consecutive odd numbers**, and **patterned Pythagorean triples**. The patterned Pythagorean triples are obtained in two different way. One way is a general way, and the second procedure give us **Palindromic-Type Pandigital Pythagorean Triples** in two different forms. As examples, the magic squares of orders 3 to 20 are given. The sum of entries of magic squares always give a perfect square resulting in a Pythagorean triples.*

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1 Essence of Work

This work is concentrated on Pythagorean triples in different aspects. For example, let’s consider a Pythagorean triple:

$$75^2 + 100^2 = 125^2. \tag{1.1}$$

The aim in this work on on generation of Pythagorean triples in two different ways. Then apply Pythagorean triples to generate patterns in three different ways and magic squares:

(i) General patterns;

(ii) Pandigital palindromic type patterns:

$$1, 121, 12321, \dots, 12345678987654321;$$

(iii) Pandigital palindromic type patterns with 0 in between:

$$1, 10201, 102030201, \dots, 102030405060708090807060504030201;$$

(iv) The applications are extended to Magic squares, where entries are consecutive odd numbers, where sum of all entries a perfect square sum resulting in a Pythagorean triple.

Below are examples of these aspects based on a triple given in (1.1)

(i) General patterns

$$\begin{aligned}
 75^2 + 100^2 = 125^2 & \quad := 15625 \\
 9975^2 + 1000^2 = 10025^2 & \quad := 100500625 \\
 999975^2 + 10000^2 = 1000025^2 & \quad := 1000050000625 \\
 99999975^2 + 100000^2 = 100000025^2 & \quad := 10000005000000625 \\
 9999999975^2 + 1000000^2 = 1000000025^2 & \quad := 100000000500000000625
 \end{aligned}$$

(ii) Palindromic-Type Pandigital Pattern: First Type

$$\begin{aligned}
 075^2 + 100^2 &= 1\ 25^2 \\
 12\ 075^2 + 1100^2 &= 121\ 25^2 \\
 1232\ 075^2 + 11100^2 &= 12321\ 25^2 \\
 123432\ 075^2 + 111100^2 &= 1234321\ 25^2 \\
 12345432\ 075^2 + 1111100^2 &= 123454321\ 25^2 \\
 1234565432\ 075^2 + 11111100^2 &= 12345654321\ 25^2 \\
 123456765432\ 075^2 + 111111100^2 &= 1234567654321\ 25^2 \\
 12345678765432\ 075^2 + 1111111100^2 &= 123456787654321\ 25^2 \\
 1234567898765432\ 075^2 + 11111111100^2 &= 12345678987654321\ 25^2
 \end{aligned}$$

(iii) Palindromic-Type Pandigital Pattern: Second Type

$$\begin{aligned}
 075^2 + 100^2 &= 1\ 25^2 \\
 1020\ 075^2 + 10100^2 &= 10201\ 25^2 \\
 10203020\ 075^2 + 1010100^2 &= 102030201\ 25^2 \\
 102030403020\ 075^2 + 101010100^2 &= 1020304030201\ 25^2 \\
 1020304050403020\ 075^2 + 10101010100^2 &= 10203040504030201\ 25^2 \\
 10203040506050403020\ 075^2 + 1010101010100^2 &= 102030405060504030201\ 25^2 \\
 102030405060706050403020\ 075^2 + 101010101010100^2 &= 1020304050607060504030201\ 25^2 \\
 1020304050607080706050403020\ 075^2 + 10101010101010100^2 &= 10203040506070807060504030201\ 25^2 \\
 10203040506070809080706050403020\ 075^2 + 1010101010101010100^2 &= 102030405060708090807060504030201\ 25^2
 \end{aligned}$$

(iv) Magic Square of Order 5

		1125	1125	1125	1125	1125
	201	213	225	237	249	1125
1125	235	247	209	211	223	1125
1125	219	221	233	245	207	1125
1125	243	205	217	229	231	1125
1125	227	239	241	203	215	1125
	1125	1125	1125	1125	1125	1125

The above magic square is of magic sum $S_{5 \times 5} = 1125$, and total number of entries is a perfect square sum, i.e.,

$$1125 \times 5 = 5625 := 75^2 = 125^2 - 100^2.$$

In this work, we have studied the above aspects in extended way. The idea of generating Pythagorean triples is done in two different ways. Magic squares are also generated based on Pythagorean triples. Examples of magic squares of orders 3 to 20. are also given. Many examples of general patterns and palindromic-type patterns are also studied.

More work on Pythagorean triples, patterns, magic square etc. refer to Taneja [3]-[9].

2 Introduction

Pythagoras Theorem is well-known in the literature. By Pythagoras theorem it is understood that

$$a^2 + b^2 = c^2.$$

For simplicity, let's write it as (a, b, c) . If we talk of distances, then the numbers a , b and c are real positive numbers, otherwise they can be any real number. The **Pythagorean triple** $(3, 4, 5)$ is understood as $3^2 + 4^2 = 5^2$. Let's consider the following well-known procedure [1] to write Pythagorean triples:

$$\begin{aligned} F(m, n) &:= m^2 - n^2 \\ G(m, n) &:= 2mn \\ H(m, n) &:= m^2 + n^2 \end{aligned} \tag{2.1}$$

Then we can easily check that

$$\begin{aligned} F(m, n)^2 + G(m, n)^2 &= (m^2 - n^2)^2 + (2mn)^2 \\ &= m^4 - 2m^2n^2 + n^4 + 4m^2n^2 \\ &= m^4 + 2m^2n^2 + n^4 \\ &= (m^2 + n^2)^2 = H(m, n)^2. \end{aligned}$$

We shall use frequently the formula (2.1) to find Pythagorean triples in subsequent sections. For extensive study on Pythagorean triples refer to online work by Knott [1]. In this paper, our aim is to write procedures to bring Pythagorean triples in a symmetrical way. Also to generate magic squares from the Pythagorean triples.

3 First Procedure: First Step

This section give simplified procedure to write Pythagorean triples. It is divided in different blocks.

Let's write a number 100 as following 10 sums:

1. $100 = 00 + 100$
2. $100 = 19 + 81$
3. $100 = 36 + 64$
4. $100 = 51 + 49$
5. $100 = 64 + 36$
6. $100 = 75 + 25$
7. $100 = 84 + 16$
8. $100 = 91 + 09$
9. $100 = 96 + 04$
10. $100 = 99 + 01$

(3.1)

We observe that the right hand sides of the expression (3.1) are formed by two sums, where the last two digits are perfect squares written in decreasing order. Even though, 100 can also be written as $100 + 00 + 100$. Let's consider a difference of squares between the terms of *r.h.s.* with 1 in the

front of perfect square term. Then this difference is again a perfect square multiple of 20 resulting in Pythagorean triples. See below:

$$\begin{aligned}
 181^2 - 019^2 &= 180^2 \Rightarrow 019^2 + 180^2 = 181^2 \Rightarrow (019, 180, 181) \\
 164^2 - 036^2 &= 160^2 \Rightarrow 036^2 + 160^2 = 164^2 \Rightarrow (036, 160, 164) \\
 149^2 - 051^2 &= 140^2 \Rightarrow 051^2 + 140^2 = 149^2 \Rightarrow (051, 140, 149) \\
 136^2 - 064^2 &= 120^2 \Rightarrow 064^2 + 120^2 = 136^2 \Rightarrow (064, 120, 136) \\
 125^2 - 075^2 &= 100^2 \Rightarrow 075^2 + 100^2 = 125^2 \Rightarrow (075, 100, 125) \\
 116^2 - 084^2 &= 080^2 \Rightarrow 084^2 + 080^2 = 116^2 \Rightarrow (084, 080, 116) \\
 109^2 - 091^2 &= 060^2 \Rightarrow 091^2 + 060^2 = 109^2 \Rightarrow (091, 060, 109) \\
 104^2 - 096^2 &= 040^2 \Rightarrow 096^2 + 040^2 = 104^2 \Rightarrow (096, 040, 104) \\
 101^2 - 099^2 &= 020^2 \Rightarrow 099^2 + 020^2 = 101^2 \Rightarrow (099, 020, 101)
 \end{aligned} \tag{3.2}$$

Above procedure is only for 1 and 0. Let's extend it for further squares. The first line with (300, 400, 500) is added to have symmetry in results. See below

$$\begin{aligned}
 500^2 - 300^2 &= 400^2 \Rightarrow 300^2 + 400^2 = 500^2 \Rightarrow (300, 400, 500) \\
 481^2 - 319^2 &= 360^2 \Rightarrow 319^2 + 360^2 = 481^2 \Rightarrow (319, 360, 481) \\
 464^2 - 336^2 &= 320^2 \Rightarrow 336^2 + 320^2 = 464^2 \Rightarrow (336, 320, 464) \\
 449^2 - 351^2 &= 280^2 \Rightarrow 351^2 + 280^2 = 449^2 \Rightarrow (351, 280, 449) \\
 436^2 - 364^2 &= 240^2 \Rightarrow 364^2 + 240^2 = 436^2 \Rightarrow (364, 240, 436) \\
 425^2 - 375^2 &= 200^2 \Rightarrow 375^2 + 200^2 = 425^2 \Rightarrow (375, 200, 425) \\
 416^2 - 384^2 &= 160^2 \Rightarrow 384^2 + 160^2 = 416^2 \Rightarrow (384, 160, 416) \\
 409^2 - 391^2 &= 120^2 \Rightarrow 391^2 + 120^2 = 409^2 \Rightarrow (391, 120, 409) \\
 404^2 - 396^2 &= 080^2 \Rightarrow 396^2 + 080^2 = 404^2 \Rightarrow (396, 080, 404) \\
 401^2 - 399^2 &= 040^2 \Rightarrow 399^2 + 040^2 = 401^2 \Rightarrow (399, 040, 401)
 \end{aligned} \tag{3.3}$$

$$\begin{aligned}
 1000^2 - 800^2 &= 600^2 \Rightarrow 800^2 + 600^2 = 1000^2 \Rightarrow (800, 600, 1000) \\
 981^2 - 819^2 &= 540^2 \Rightarrow 819^2 + 540^2 = 981^2 \Rightarrow (819, 540, 981) \\
 964^2 - 836^2 &= 480^2 \Rightarrow 836^2 + 480^2 = 964^2 \Rightarrow (836, 480, 964) \\
 949^2 - 851^2 &= 420^2 \Rightarrow 851^2 + 420^2 = 949^2 \Rightarrow (851, 420, 949) \\
 936^2 - 864^2 &= 360^2 \Rightarrow 864^2 + 360^2 = 936^2 \Rightarrow (864, 360, 936) \\
 925^2 - 875^2 &= 300^2 \Rightarrow 875^2 + 300^2 = 925^2 \Rightarrow (875, 300, 925) \\
 916^2 - 884^2 &= 240^2 \Rightarrow 884^2 + 240^2 = 916^2 \Rightarrow (884, 240, 916) \\
 909^2 - 891^2 &= 180^2 \Rightarrow 891^2 + 180^2 = 909^2 \Rightarrow (891, 180, 909) \\
 904^2 - 896^2 &= 120^2 \Rightarrow 896^2 + 120^2 = 904^2 \Rightarrow (896, 120, 904) \\
 901^2 - 899^2 &= 060^2 \Rightarrow 899^2 + 060^2 = 901^2 \Rightarrow (899, 060, 901)
 \end{aligned} \tag{3.4}$$

$$\begin{aligned}
 1700^2 - 1500^2 &= 800^2 \Rightarrow 1500^2 + 800^2 = 1700^2 \Rightarrow (1500, 800, 1700) \\
 1681^2 - 1519^2 &= 720^2 \Rightarrow 1519^2 + 720^2 = 1681^2 \Rightarrow (1519, 720, 1681)
 \end{aligned}$$

$$\begin{aligned}
1664^2 - 1536^2 &= 640^2 \Rightarrow 1536^2 + 640^2 = 1664^2 \Rightarrow (1536, 640, 1664) \\
1649^2 - 1551^2 &= 560^2 \Rightarrow 1551^2 + 560^2 = 1649^2 \Rightarrow (1551, 560, 1649) \\
1636^2 - 1564^2 &= 480^2 \Rightarrow 1564^2 + 480^2 = 1636^2 \Rightarrow (1564, 480, 1636) \\
1625^2 - 1575^2 &= 400^2 \Rightarrow 1575^2 + 400^2 = 1625^2 \Rightarrow (1575, 400, 1625) \\
1616^2 - 1584^2 &= 320^2 \Rightarrow 1584^2 + 320^2 = 1616^2 \Rightarrow (1584, 320, 1616) \\
1609^2 - 1591^2 &= 240^2 \Rightarrow 1591^2 + 240^2 = 1609^2 \Rightarrow (1591, 240, 1609) \\
1604^2 - 1596^2 &= 160^2 \Rightarrow 1596^2 + 160^2 = 1604^2 \Rightarrow (1596, 160, 1604) \\
1601^2 - 1599^2 &= 080^2 \Rightarrow 1599^2 + 080^2 = 1601^2 \Rightarrow (1599, 080, 1601)
\end{aligned} \tag{3.5}$$

$$\begin{aligned}
2600^2 - 2400^2 &= 1000^2 \Rightarrow 2400^2 + 1000^2 = 2600^2 \Rightarrow (2400, 1000, 2600) \\
2581^2 - 2419^2 &= 0900^2 \Rightarrow 2419^2 + 0900^2 = 2581^2 \Rightarrow (2419, 0900, 2581) \\
2564^2 - 2436^2 &= 0800^2 \Rightarrow 2436^2 + 0800^2 = 2564^2 \Rightarrow (2436, 0800, 2564) \\
2549^2 - 2451^2 &= 0700^2 \Rightarrow 2451^2 + 0700^2 = 2549^2 \Rightarrow (2451, 0700, 2549) \\
2536^2 - 2464^2 &= 0600^2 \Rightarrow 2464^2 + 0600^2 = 2536^2 \Rightarrow (2464, 0600, 2536) \\
2525^2 - 2475^2 &= 0500^2 \Rightarrow 2475^2 + 0500^2 = 2525^2 \Rightarrow (2475, 0500, 2525) \\
2516^2 - 2484^2 &= 0400^2 \Rightarrow 2484^2 + 0400^2 = 2516^2 \Rightarrow (2484, 0400, 2516) \\
2509^2 - 2491^2 &= 0300^2 \Rightarrow 2491^2 + 0300^2 = 2509^2 \Rightarrow (2491, 0300, 2509) \\
2504^2 - 2496^2 &= 0200^2 \Rightarrow 2496^2 + 0200^2 = 2504^2 \Rightarrow (2496, 0200, 2504) \\
2501^2 - 2499^2 &= 0100^2 \Rightarrow 2499^2 + 0100^2 = 2501^2 \Rightarrow (2499, 0100, 2501)
\end{aligned} \tag{3.6}$$

$$\begin{aligned}
3700^2 - 3500^2 &= 1000^2 \Rightarrow 3500^2 + 1000^2 = 3700^2 \Rightarrow (3500, 1000, 3700) \\
3681^2 - 3519^2 &= 1080^2 \Rightarrow 3519^2 + 1080^2 = 3681^2 \Rightarrow (3519, 1080, 3681) \\
3664^2 - 3536^2 &= 0960^2 \Rightarrow 3536^2 + 0960^2 = 3664^2 \Rightarrow (3536, 0960, 3664) \\
3649^2 - 3551^2 &= 0840^2 \Rightarrow 3551^2 + 0840^2 = 3649^2 \Rightarrow (3551, 0840, 3649) \\
3636^2 - 3564^2 &= 0720^2 \Rightarrow 3564^2 + 0720^2 = 3636^2 \Rightarrow (3564, 0720, 3636) \\
3625^2 - 3575^2 &= 0600^2 \Rightarrow 3575^2 + 0600^2 = 3625^2 \Rightarrow (3575, 0600, 3625) \\
3616^2 - 3584^2 &= 0480^2 \Rightarrow 3584^2 + 0480^2 = 3616^2 \Rightarrow (3584, 0480, 3616) \\
3609^2 - 3591^2 &= 0360^2 \Rightarrow 3591^2 + 0360^2 = 3609^2 \Rightarrow (3591, 0360, 3609) \\
3604^2 - 3596^2 &= 0240^2 \Rightarrow 3596^2 + 0240^2 = 3604^2 \Rightarrow (3596, 0240, 3604) \\
3601^2 - 3599^2 &= 0120^2 \Rightarrow 3599^2 + 0120^2 = 3601^2 \Rightarrow (3599, 0120, 3601)
\end{aligned} \tag{3.7}$$

$$\begin{aligned}
5000^2 - 4800^2 &= 1400^2 \Rightarrow 4800^2 + 1400^2 = 5000^2 \Rightarrow (4800, 1400, 5000) \\
4981^2 - 4819^2 &= 1260^2 \Rightarrow 4819^2 + 1260^2 = 4981^2 \Rightarrow (4819, 1260, 4981) \\
4964^2 - 4836^2 &= 1120^2 \Rightarrow 4836^2 + 1120^2 = 4964^2 \Rightarrow (4836, 1120, 4964) \\
4949^2 - 4851^2 &= 0980^2 \Rightarrow 4851^2 + 0980^2 = 4949^2 \Rightarrow (4851, 0980, 4949) \\
4936^2 - 4864^2 &= 0840^2 \Rightarrow 4864^2 + 0840^2 = 4936^2 \Rightarrow (4864, 0840, 4936) \\
4925^2 - 4875^2 &= 0700^2 \Rightarrow 4875^2 + 0700^2 = 4925^2 \Rightarrow (4875, 0700, 4925) \\
4916^2 - 4884^2 &= 0560^2 \Rightarrow 4884^2 + 0560^2 = 4916^2 \Rightarrow (4884, 0560, 4916)
\end{aligned}$$

$$\begin{aligned}
4909^2 - 4891^2 &= 0420^2 \Rightarrow 4891^2 + 0420^2 = 4909^2 \Rightarrow (4891, 0420, 4909) \\
4904^2 - 4896^2 &= 0280^2 \Rightarrow 4896^2 + 0280^2 = 4904^2 \Rightarrow (4896, 0280, 4904) \\
4901^2 - 4899^2 &= 0140^2 \Rightarrow 4899^2 + 0140^2 = 4901^2 \Rightarrow (4899, 0140, 4901)
\end{aligned} \tag{3.8}$$

$$\begin{aligned}
6500^2 - 6300^2 &= 1600^2 \Rightarrow 6300^2 + 1600^2 = 6500^2 \Rightarrow (6300, 1600, 6500) \\
6481^2 - 6319^2 &= 1440^2 \Rightarrow 6319^2 + 1440^2 = 6481^2 \Rightarrow (6319, 1440, 6481) \\
6464^2 - 6336^2 &= 1280^2 \Rightarrow 6336^2 + 1280^2 = 6464^2 \Rightarrow (6336, 1280, 6464) \\
6449^2 - 6351^2 &= 1120^2 \Rightarrow 6351^2 + 1120^2 = 6449^2 \Rightarrow (6351, 1120, 6449) \\
6436^2 - 6364^2 &= 0960^2 \Rightarrow 6364^2 + 0960^2 = 6436^2 \Rightarrow (6364, 0960, 6436) \\
6425^2 - 6375^2 &= 0800^2 \Rightarrow 6375^2 + 0800^2 = 6425^2 \Rightarrow (6375, 0800, 6425) \\
6416^2 - 6384^2 &= 0640^2 \Rightarrow 6384^2 + 0640^2 = 6416^2 \Rightarrow (6384, 0640, 6416) \\
6409^2 - 6391^2 &= 0480^2 \Rightarrow 6391^2 + 0480^2 = 6409^2 \Rightarrow (6391, 0480, 6409) \\
6404^2 - 6396^2 &= 0320^2 \Rightarrow 6396^2 + 0320^2 = 6404^2 \Rightarrow (6396, 0320, 6404) \\
6401^2 - 6399^2 &= 0160^2 \Rightarrow 6399^2 + 0160^2 = 6401^2 \Rightarrow (6399, 0160, 6401)
\end{aligned} \tag{3.9}$$

$$\begin{aligned}
8200^2 - 8000^2 &= 1800^2 \Rightarrow 8000^2 + 1800^2 = 8200^2 \Rightarrow (8000, 1800, 8200) \\
8181^2 - 8019^2 &= 1620^2 \Rightarrow 8019^2 + 1620^2 = 8181^2 \Rightarrow (8019, 1620, 8181) \\
8164^2 - 8036^2 &= 1440^2 \Rightarrow 8036^2 + 1440^2 = 8164^2 \Rightarrow (8036, 1440, 8164) \\
8149^2 - 8051^2 &= 1260^2 \Rightarrow 8051^2 + 1260^2 = 8149^2 \Rightarrow (8051, 1260, 8149) \\
8136^2 - 8064^2 &= 1080^2 \Rightarrow 8064^2 + 1080^2 = 8136^2 \Rightarrow (8064, 1080, 8136) \\
8125^2 - 8075^2 &= 0900^2 \Rightarrow 8075^2 + 0900^2 = 8125^2 \Rightarrow (8075, 0900, 8125) \\
8116^2 - 8084^2 &= 0720^2 \Rightarrow 8084^2 + 0720^2 = 8116^2 \Rightarrow (8084, 0720, 8116) \\
8109^2 - 8091^2 &= 0540^2 \Rightarrow 8091^2 + 0540^2 = 8109^2 \Rightarrow (8091, 0540, 8109) \\
8104^2 - 8096^2 &= 0360^2 \Rightarrow 8096^2 + 0360^2 = 8104^2 \Rightarrow (8096, 0360, 8104) \\
8101^2 - 8099^2 &= 0180^2 \Rightarrow 8099^2 + 0180^2 = 8101^2 \Rightarrow (8099, 0180, 8101)
\end{aligned} \tag{3.10}$$

$$\begin{aligned}
10100^2 - 9900^2 &= 2000^2 \Rightarrow 9900^2 + 2000^2 = 10100^2 \Rightarrow (9900, 2000, 10100) \\
10081^2 - 9919^2 &= 1800^2 \Rightarrow 9919^2 + 1800^2 = 10081^2 \Rightarrow (9919, 1800, 10081) \\
10064^2 - 9936^2 &= 1600^2 \Rightarrow 9936^2 + 1600^2 = 10064^2 \Rightarrow (9936, 1600, 10064) \\
10049^2 - 9951^2 &= 1400^2 \Rightarrow 9951^2 + 1400^2 = 10049^2 \Rightarrow (9951, 1400, 10049) \\
10036^2 - 9964^2 &= 1200^2 \Rightarrow 9964^2 + 1200^2 = 10036^2 \Rightarrow (9964, 1200, 10036) \\
10025^2 - 9975^2 &= 1000^2 \Rightarrow 9975^2 + 1000^2 = 10025^2 \Rightarrow (9975, 1000, 10025) \\
10016^2 - 9984^2 &= 0800^2 \Rightarrow 9984^2 + 0800^2 = 10016^2 \Rightarrow (9984, 0800, 10016) \\
10009^2 - 9991^2 &= 0600^2 \Rightarrow 9991^2 + 0600^2 = 10009^2 \Rightarrow (9991, 0600, 10009) \\
10004^2 - 9996^2 &= 0400^2 \Rightarrow 9996^2 + 0400^2 = 10004^2 \Rightarrow (9996, 0400, 10004) \\
10001^2 - 9999^2 &= 0200^2 \Rightarrow 9999^2 + 0200^2 = 10001^2 \Rightarrow (9999, 0200, 10001)
\end{aligned} \tag{3.11}$$

We observe that the first block, i.e., (3.2) is with 9 triples, while other 9 blocks are with 10 triples in each case. Below is an alternative way to get these triples by use of the formula given in (2.1).

Remark 1. Analyzing the 10 blocks, (3.2) to (3.11), we observe the following:

- (i) The last column in each case is constant as a square, i.e, 01, 04, 09, 16, 25, 36, 49, 64, 81 and 100. The fixed element in each block is 1 minus of square element of third columns, i.e., 0, 3, 8, 15, 24, 35, 48, 63, 80 and 99.
- (ii) The numbers given in item (i) can be understood as formula: $a01^2 - (a - 1)99^2$, where a is a perfect square, for examples, $1601^2 - 1599^2$, $4901^2 - 4899^2$, etc.

3.1 Alternative Approach

The triples given in (3.2)-(3.11) can also be obtained by use of formula given in (2.1). These are given below in blocks.

- Let's fix $m = 10$ and vary n , i.e., $n = 9, 8, 7, 6, 5, 4, 3, 2$ and $n = 1$ in (2.1), we get respectively, the Pythagorean triples given (3.2).
- Let's fix $m = 20$ and vary n , i.e., $n = 10, 9, 8, 7, 6, 5, 4, 3, 2$ and $n = 1$ in (2.1), we get respectively, the Pythagorean triples given (3.3).
- Let's fix $m = 30$ and vary n , i.e., $n = 10, 9, 8, 7, 6, 5, 4, 3, 2$ and $n = 1$ in (2.1), we get respectively, the Pythagorean triples given (3.4).
- Let's fix $m = 40$ and vary n , i.e., $n = 10, 9, 8, 7, 6, 5, 4, 3, 2$ and $n = 1$ in (2.1), we get respectively, the Pythagorean triples given (3.5).
- Let's fix $m = 50$ and vary n , i.e., $n = 10, 9, 8, 7, 6, 5, 4, 3, 2$ and $n = 1$ in (2.1), we get respectively, the Pythagorean triples given (3.6).
- Let's fix $m = 60$ and vary n , i.e., $n = 10, 9, 8, 7, 6, 5, 4, 3, 2$ and $n = 1$ in (2.1), we get respectively, the Pythagorean triples given (3.7).
- Let's fix $m = 70$ and vary n , i.e., $n = 10, 9, 8, 7, 6, 5, 4, 3, 2$ and $n = 1$ in (2.1), we get respectively, the Pythagorean triples given (3.8).
- Let's fix $m = 80$ and vary n , i.e., $n = 10, 9, 8, 7, 6, 5, 4, 3, 2$ and $n = 1$ in (2.1), we get respectively, the Pythagorean triples given (3.9).
- Let's fix $m = 90$ and vary n , i.e., $n = 10, 9, 8, 7, 6, 5, 4, 3, 2$ and $n = 1$ in (2.1), we get respectively, the Palindromic triples given (3.10).
- Let's fix $m = 100$ and vary n , i.e., $n = 10, 9, 8, 7, 6, 5, 4, 3, 2$ and $n = 1$ in (2.1), we get respectively, the Palindromic triples given (3.11)

Remark 2. The choice of considering n in decreasing order shall help us in generating magic squares in increasing order. It is shown in Section 4.

3.2 Patterns in Pythagorean Triples

Based on the triples given in (3.2)-(3.11), let's construct patterns in Pythagorean triples with final sums. This is done for each triple separately. It is divided in 10 blocks as given in (3.2)-(3.11). The first block is with 9 patterns, while other 9 blocks are with 10 patterns each.

► 1st Block

Let's fix the values of m , i.e., $m = 10, 100, 1000, 10000, \dots$, and change the values of n as $n = 9, 8, 7, 6, 5, 4, 3, 2$ and $n = 1$, we get respectively the following 9 patterns in Pythagorean Triple:

$$\begin{aligned}
 19^2 + 180^2 &= 181^2 && := 32761 \\
 9919^2 + 1800^2 &= 10081^2 && := 101626561 \\
 999919^2 + 18000^2 &= 1000081^2 && := 1000162006561 \\
 99999919^2 + 180000^2 &= 100000081^2 && := 10000016200006561 \\
 9999999919^2 + 1800000^2 &= 10000000081^2 && := 100000001620000006561
 \end{aligned} \tag{3.12}$$

$$\begin{aligned}
 36^2 + 160^2 &= 164^2 && := 26896 \\
 9936^2 + 1600^2 &= 10064^2 && := 101284096 \\
 999936^2 + 16000^2 &= 1000064^2 && := 1000128004096 \\
 99999936^2 + 160000^2 &= 100000064^2 && := 10000012800004096 \\
 9999999936^2 + 1600000^2 &= 10000000064^2 && := 100000001280000004096
 \end{aligned} \tag{3.13}$$

$$\begin{aligned}
 51^2 + 140^2 &= 149^2 && := 22201 \\
 9951^2 + 1400^2 &= 10049^2 && := 100982401 \\
 999951^2 + 14000^2 &= 1000049^2 && := 1000098002401 \\
 99999951^2 + 140000^2 &= 100000049^2 && := 10000009800002401 \\
 9999999951^2 + 1400000^2 &= 10000000049^2 && := 100000000980000002401
 \end{aligned} \tag{3.14}$$

$$\begin{aligned}
 64^2 + 120^2 &= 136^2 && := 18496 \\
 9964^2 + 1200^2 &= 10036^2 && := 100721296 \\
 999964^2 + 12000^2 &= 1000036^2 && := 1000072001296 \\
 99999964^2 + 120000^2 &= 100000036^2 && := 10000007200001296 \\
 9999999964^2 + 1200000^2 &= 10000000036^2 && := 100000000720000001296
 \end{aligned} \tag{3.15}$$

$$\begin{aligned}
 75^2 + 100^2 &= 125^2 && := 15625 \\
 9975^2 + 1000^2 &= 10025^2 && := 100500625 \\
 999975^2 + 10000^2 &= 1000025^2 && := 1000050000625 \\
 99999975^2 + 100000^2 &= 100000025^2 && := 10000005000000625 \\
 9999999975^2 + 1000000^2 &= 10000000025^2 && := 100000000500000000625
 \end{aligned} \tag{3.16}$$

$$\begin{aligned}
 84^2 + 80^2 &= 116^2 && := 13456 \\
 9984^2 + 800^2 &= 10016^2 && := 100320256 \\
 999984^2 + 8000^2 &= 1000016^2 && := 1000032000256 \\
 99999984^2 + 80000^2 &= 100000016^2 && := 10000003200000256 \\
 9999999984^2 + 800000^2 &= 10000000016^2 && := 100000000320000000256
 \end{aligned} \tag{3.17}$$

$$\begin{aligned}
 91^2 + 60^2 &= 109^2 && := 11881 \\
 9991^2 + 600^2 &= 10009^2 && := 100180081 \\
 999991^2 + 6000^2 &= 1000009^2 && := 1000018000081 \\
 99999991^2 + 60000^2 &= 100000009^2 && := 10000001800000081 \\
 9999999991^2 + 600000^2 &= 10000000009^2 && := 100000000180000000081
 \end{aligned} \tag{3.18}$$

$$\begin{aligned}
 96^2 + 40^2 &= 104^2 && := 10816 \\
 9996^2 + 400^2 &= 10004^2 && := 100080016 \\
 999996^2 + 4000^2 &= 1000004^2 && := 1000008000016 \\
 99999996^2 + 40000^2 &= 100000004^2 && := 10000000800000016 \\
 9999999996^2 + 400000^2 &= 10000000004^2 && := 100000000080000000016
 \end{aligned} \tag{3.19}$$

$$\begin{aligned}
 99^2 + 20^2 &= 101^2 && := 10201 \\
 9999^2 + 200^2 &= 10001^2 && := 100020001 \\
 999999^2 + 2000^2 &= 1000001^2 && := 1000002000001 \\
 99999999^2 + 20000^2 &= 100000001^2 && := 10000000200000001 \\
 9999999999^2 + 200000^2 &= 10000000001^2 && := 100000000020000000001
 \end{aligned} \tag{3.20}$$

Remark 3. In some cases, the final sum don't obey the regular pattern. These lines are given in different colors. The extension of above 9 patterns to 99 patterns is given in subsection ??.

► 2nd Block

Let's fix the values of m , i.e., $m = 20, 200, 2000, 20000, \dots$, and change the values of n as $n = 10, 9, 8, 7, 6, 5, 4, 3, 2$ and $n = 1$, we get respectively the following 10 patterns in Pythagorean Triple:

$$\begin{aligned}
 300^2 + 400^2 &= 500^2 && := 250000 \\
 39900^2 + 4000^2 &= 40100^2 && := 1608010000 \\
 3999900^2 + 40000^2 &= 4000100^2 && := 16000800010000 \\
 399999900^2 + 400000^2 &= 400000100^2 && := 160000080000010000 \\
 39999999900^2 + 4000000^2 &= 40000000100^2 && := 1600000008000000010000
 \end{aligned} \tag{3.21}$$

$$\begin{aligned}
 319^2 + 360^2 &= 481^2 && := 231361 \\
 39919^2 + 3600^2 &= 40081^2 && := 1606486561 \\
 3999919^2 + 36000^2 &= 4000081^2 && := 16000648006561 \\
 399999919^2 + 360000^2 &= 400000081^2 && := 160000064800006561 \\
 39999999919^2 + 3600000^2 &= 40000000081^2 && := 1600000006480000006561
 \end{aligned} \tag{3.22}$$

$$\begin{aligned}
336^2 + 320^2 &= 464^2 && := 215296 \\
39936^2 + 3200^2 &= 40064^2 && := 1605124096 \\
3999936^2 + 32000^2 &= 4000064^2 && := 16000512004096 \\
399999936^2 + 320000^2 &= 400000064^2 && := 160000051200004096 \\
39999999936^2 + 3200000^2 &= 40000000064^2 && := 1600000005120000004096
\end{aligned} \tag{3.23}$$

$$\begin{aligned}
351^2 + 280^2 &= 449^2 && := 201601 \\
39951^2 + 2800^2 &= 40049^2 && := 1603922401 \\
3999951^2 + 28000^2 &= 4000049^2 && := 16000392002401 \\
399999951^2 + 280000^2 &= 400000049^2 && := 160000039200002401 \\
39999999951^2 + 2800000^2 &= 40000000049^2 && := 1600000003920000002401
\end{aligned} \tag{3.24}$$

$$\begin{aligned}
364^2 + 240^2 &= 436^2 && := 190096 \\
39964^2 + 2400^2 &= 40036^2 && := 1602881296 \\
3999964^2 + 24000^2 &= 4000036^2 && := 16000288001296 \\
399999964^2 + 240000^2 &= 400000036^2 && := 160000028800001296 \\
39999999964^2 + 2400000^2 &= 40000000036^2 && := 1600000002880000001296
\end{aligned} \tag{3.25}$$

$$\begin{aligned}
375^2 + 200^2 &= 425^2 && := 180625 \\
39975^2 + 2000^2 &= 40025^2 && := 1602000625 \\
3999975^2 + 20000^2 &= 4000025^2 && := 16000200000625 \\
399999975^2 + 200000^2 &= 400000025^2 && := 160000020000000625 \\
39999999975^2 + 2000000^2 &= 40000000025^2 && := 1600000002000000000625
\end{aligned} \tag{3.26}$$

$$\begin{aligned}
384^2 + 160^2 &= 416^2 && := 173056 \\
39984^2 + 1600^2 &= 40016^2 && := 1601280256 \\
3999984^2 + 16000^2 &= 4000016^2 && := 16000128000256 \\
399999984^2 + 160000^2 &= 400000016^2 && := 160000012800000256 \\
39999999984^2 + 1600000^2 &= 40000000016^2 && := 1600000001280000000256
\end{aligned} \tag{3.27}$$

$$\begin{aligned}
391^2 + 120^2 &= 409^2 && := 167281 \\
39991^2 + 1200^2 &= 40009^2 && := 1600720081 \\
3999991^2 + 12000^2 &= 4000009^2 && := 16000072000081 \\
399999991^2 + 120000^2 &= 400000009^2 && := 160000007200000081 \\
39999999991^2 + 1200000^2 &= 40000000009^2 && := 1600000000720000000081
\end{aligned} \tag{3.28}$$

$$\begin{aligned}
 396^2 + 80^2 &= 404^2 && := 163216 \\
 39996^2 + 800^2 &= 40004^2 && := 1600320016 \\
 399996^2 + 8000^2 &= 4000004^2 && := 16000032000016 \\
 3999996^2 + 80000^2 &= 400000004^2 && := 160000003200000016 \\
 39999996^2 + 800000^2 &= 40000000004^2 && := 1600000000320000000016 \tag{3.29}
 \end{aligned}$$

$$\begin{aligned}
 399^2 + 40^2 &= 401^2 && := 160801 \\
 3999^2 + 400^2 &= 40001^2 && := 1600080001 \\
 39999^2 + 4000^2 &= 4000001^2 && := 16000008000001 \\
 399999^2 + 40000^2 &= 400000001^2 && := 160000000800000001 \\
 3999999^2 + 400000^2 &= 40000000001^2 && := 1600000000080000000001 \tag{3.30}
 \end{aligned}$$

► 3rd Block

Let's fix the values of m , i.e., $m = 30, 300, 3000, 30000, \dots$, and change the values of n as $n = 10, 9, 8, 7, 6, 5, 4, 3, 2$ and $n = 1$, we get respectively the following 10 patterns in Pythagorean Triple:

$$\begin{aligned}
 800^2 + 600^2 &= 1000^2 && := 1000000 \\
 89900^2 + 6000^2 &= 90100^2 && := 8118010000 \\
 899900^2 + 60000^2 &= 9000100^2 && := 81001800010000 \\
 8999900^2 + 600000^2 &= 900000100^2 && := 810000180000010000 \\
 89999900^2 + 6000000^2 &= 90000000100^2 && := 8100000018000000010000 \tag{3.31}
 \end{aligned}$$

$$\begin{aligned}
 819^2 + 540^2 &= 981^2 && := 962361 \\
 89919^2 + 5400^2 &= 90081^2 && := 8114586561 \\
 899919^2 + 54000^2 &= 9000081^2 && := 81001458006561 \\
 8999919^2 + 540000^2 &= 900000081^2 && := 810000145800006561 \\
 89999919^2 + 5400000^2 &= 90000000081^2 && := 8100000014580000006561 \tag{3.32}
 \end{aligned}$$

$$\begin{aligned}
 836^2 + 480^2 &= 964^2 && := 929296 \\
 89936^2 + 4800^2 &= 90064^2 && := 8111524096 \\
 899936^2 + 48000^2 &= 9000064^2 && := 81001152004096 \\
 8999936^2 + 480000^2 &= 900000064^2 && := 810000115200004096 \\
 89999936^2 + 4800000^2 &= 90000000064^2 && := 8100000011520000004096 \tag{3.33}
 \end{aligned}$$

$$\begin{aligned}
 851^2 + 420^2 &= 949^2 && := 900601 \\
 89951^2 + 4200^2 &= 90049^2 && := 8108822401 \\
 899951^2 + 42000^2 &= 9000049^2 && := 81000882002401 \\
 8999951^2 + 420000^2 &= 900000049^2 && := 810000088200002401 \\
 89999951^2 + 4200000^2 &= 90000000049^2 && := 8100000008820000002401 \tag{3.34}
 \end{aligned}$$

$$\begin{aligned}
864^2 + 360^2 &= 936^2 && := 876096 \\
89964^2 + 3600^2 &= 90036^2 && := 8106481296 \\
8999964^2 + 36000^2 &= 9000036^2 && := 81000648001296 \\
899999964^2 + 360000^2 &= 900000036^2 && := 810000064800001296 \\
89999999964^2 + 3600000^2 &= 90000000036^2 && := 8100000006480000001296
\end{aligned} \tag{3.35}$$

$$\begin{aligned}
875^2 + 300^2 &= 925^2 && := 855625 \\
89975^2 + 3000^2 &= 90025^2 && := 8104500625 \\
8999975^2 + 30000^2 &= 9000025^2 && := 81000450000625 \\
899999975^2 + 300000^2 &= 900000025^2 && := 810000045000000625 \\
89999999975^2 + 3000000^2 &= 90000000025^2 && := 8100000004500000000625
\end{aligned} \tag{3.36}$$

$$\begin{aligned}
884^2 + 240^2 &= 916^2 && := 839056 \\
89984^2 + 2400^2 &= 90016^2 && := 8102880256 \\
8999984^2 + 24000^2 &= 9000016^2 && := 81000288000256 \\
899999984^2 + 240000^2 &= 900000016^2 && := 810000028800000256 \\
89999999984^2 + 2400000^2 &= 90000000016^2 && := 8100000002880000000256
\end{aligned} \tag{3.37}$$

$$\begin{aligned}
891^2 + 180^2 &= 909^2 && := 826281 \\
89991^2 + 1800^2 &= 90009^2 && := 8101620081 \\
8999991^2 + 18000^2 &= 9000009^2 && := 81000162000081 \\
899999991^2 + 180000^2 &= 900000009^2 && := 810000016200000081 \\
89999999991^2 + 1800000^2 &= 90000000009^2 && := 8100000001620000000081
\end{aligned} \tag{3.38}$$

$$\begin{aligned}
896^2 + 120^2 &= 904^2 && := 817216 \\
89996^2 + 1200^2 &= 90004^2 && := 8100720016 \\
8999996^2 + 12000^2 &= 9000004^2 && := 81000072000016 \\
899999996^2 + 120000^2 &= 900000004^2 && := 810000007200000016 \\
89999999996^2 + 1200000^2 &= 90000000004^2 && := 8100000000720000000016
\end{aligned} \tag{3.39}$$

$$\begin{aligned}
899^2 + 60^2 &= 901^2 && := 811801 \\
89999^2 + 600^2 &= 90001^2 && := 8100180001 \\
899999^2 + 6000^2 &= 9000001^2 && := 81000018000001 \\
89999999^2 + 60000^2 &= 900000001^2 && := 810000001800000001 \\
8999999999^2 + 600000^2 &= 90000000001^2 && := 8100000000180000000001
\end{aligned} \tag{3.40}$$

► 4th Block

Let's fix the values of m , i.e., $m = 40, 400, 4000, 40000, \dots$, and change the values of n as $n = 10, 9, 8, 7, 6, 5, 4, 3, 2$ and $n = 1$, we get respectively the following 10 patterns in Pythagorean Triple:

$$\begin{aligned}
 1500^2 + 800^2 &= 1700^2 & := 2890000 \\
 159900^2 + 8000^2 &= 160100^2 & := 25632010000 \\
 15999900^2 + 80000^2 &= 16000100^2 & := 256003200010000 \\
 1599999900^2 + 800000^2 &= 1600000100^2 & := 2560000320000010000 \\
 159999999900^2 + 8000000^2 &= 160000000100^2 & := 25600000032000000010000
 \end{aligned} \tag{3.41}$$

$$\begin{aligned}
 1519^2 + 720^2 &= 1681^2 & := 2825761 \\
 159919^2 + 7200^2 &= 160081^2 & := 25625926561 \\
 15999919^2 + 72000^2 &= 16000081^2 & := 256002592006561 \\
 1599999919^2 + 720000^2 &= 1600000081^2 & := 2560000259200006561 \\
 159999999919^2 + 7200000^2 &= 160000000081^2 & := 25600000025920000006561
 \end{aligned} \tag{3.42}$$

$$\begin{aligned}
 1536^2 + 640^2 &= 1664^2 & := 2768896 \\
 159936^2 + 6400^2 &= 160064^2 & := 25620484096 \\
 15999936^2 + 64000^2 &= 16000064^2 & := 256002048004096 \\
 1599999936^2 + 640000^2 &= 1600000064^2 & := 2560000204800004096 \\
 159999999936^2 + 6400000^2 &= 160000000064^2 & := 25600000020480000004096
 \end{aligned} \tag{3.43}$$

$$\begin{aligned}
 1551^2 + 560^2 &= 1649^2 & := 2719201 \\
 159951^2 + 5600^2 &= 160049^2 & := 25615682401 \\
 15999951^2 + 56000^2 &= 16000049^2 & := 256001568002401 \\
 1599999951^2 + 560000^2 &= 1600000049^2 & := 2560000156800002401 \\
 159999999951^2 + 5600000^2 &= 160000000049^2 & := 25600000015680000002401
 \end{aligned} \tag{3.44}$$

$$\begin{aligned}
 1564^2 + 480^2 &= 1636^2 & := 2676496 \\
 159964^2 + 4800^2 &= 160036^2 & := 25611521296 \\
 15999964^2 + 48000^2 &= 16000036^2 & := 256001152001296 \\
 1599999964^2 + 480000^2 &= 1600000036^2 & := 2560000115200001296 \\
 159999999964^2 + 4800000^2 &= 160000000036^2 & := 25600000011520000001296
 \end{aligned} \tag{3.45}$$

$$\begin{aligned}
 1575^2 + 400^2 &= 1625^2 & := 2640625 \\
 159975^2 + 4000^2 &= 160025^2 & := 25608000625 \\
 15999975^2 + 40000^2 &= 16000025^2 & := 256000800000625 \\
 1599999975^2 + 400000^2 &= 1600000025^2 & := 2560000080000000625 \\
 159999999975^2 + 4000000^2 &= 160000000025^2 & := 25600000008000000000625
 \end{aligned} \tag{3.46}$$

$$\begin{aligned}
 1584^2 + 320^2 &= 1616^2 & := 2611456 \\
 159984^2 + 3200^2 &= 160016^2 & := 25605120256 \\
 15999984^2 + 32000^2 &= 16000016^2 & := 256000512000256 \\
 1599999984^2 + 320000^2 &= 1600000016^2 & := 2560000051200000256 \\
 159999999984^2 + 3200000^2 &= 160000000016^2 & := 25600000005120000000256
 \end{aligned} \tag{3.47}$$

$$\begin{aligned}
 1591^2 + 240^2 &= 1609^2 & := 2588881 \\
 159991^2 + 2400^2 &= 160009^2 & := 25602880081 \\
 15999991^2 + 24000^2 &= 16000009^2 & := 256000288000081 \\
 1599999991^2 + 240000^2 &= 1600000009^2 & := 2560000028800000081 \\
 159999999991^2 + 2400000^2 &= 160000000009^2 & := 25600000002880000000081
 \end{aligned} \tag{3.48}$$

$$\begin{aligned}
 1596^2 + 160^2 &= 1604^2 & := 2572816 \\
 159996^2 + 1600^2 &= 160004^2 & := 25601280016 \\
 15999996^2 + 16000^2 &= 16000004^2 & := 256000128000016 \\
 1599999996^2 + 160000^2 &= 1600000004^2 & := 2560000012800000016 \\
 159999999996^2 + 1600000^2 &= 160000000004^2 & := 25600000001280000000016
 \end{aligned} \tag{3.49}$$

$$\begin{aligned}
 1599^2 + 80^2 &= 1601^2 & := 2563201 \\
 159999^2 + 800^2 &= 160001^2 & := 25600320001 \\
 15999999^2 + 8000^2 &= 16000001^2 & := 256000032000001 \\
 1599999999^2 + 80000^2 &= 1600000001^2 & := 2560000003200000001 \\
 159999999999^2 + 800000^2 &= 160000000001^2 & := 25600000000320000000001
 \end{aligned} \tag{3.50}$$

► 5th Block

Let's fix the values of m , i.e., $m = 50, 500, 5000, 50000, \dots$, and change the values of n as $n = 10, 9, 8, 7, 6, 5, 4, 3, 2$ and $n = 1$, we get respectively the following 10 patterns in Pythagorean Triple:

$$\begin{aligned}
 2400^2 + 1000^2 &= 2600^2 & := 6760000 \\
 249900^2 + 10000^2 &= 250100^2 & := 62550010000 \\
 24999900^2 + 100000^2 &= 25000100^2 & := 625005000010000 \\
 2499999900^2 + 1000000^2 &= 2500000100^2 & := 6250000500000010000 \\
 249999999900^2 + 10000000^2 &= 250000000100^2 & := 62500000050000000010000
 \end{aligned} \tag{3.51}$$

$$\begin{aligned}
 2419^2 + 900^2 &= 2581^2 & := 6661561 \\
 249919^2 + 9000^2 &= 250081^2 & := 62540506561 \\
 24999919^2 + 90000^2 &= 25000081^2 & := 625004050006561 \\
 2499999919^2 + 900000^2 &= 2500000081^2 & := 6250000405000006561 \\
 249999999919^2 + 9000000^2 &= 250000000081^2 & := 62500000040500000006561
 \end{aligned} \tag{3.52}$$

$$\begin{aligned}
2436^2 + 800^2 &= 2564^2 & := 6574096 \\
249936^2 + 8000^2 &= 250064^2 & := 62532004096 \\
24999936^2 + 80000^2 &= 25000064^2 & := 625003200004096 \\
249999936^2 + 800000^2 &= 2500000064^2 & := 6250000320000004096 \\
24999999936^2 + 8000000^2 &= 250000000064^2 := 62500000032000000004096 & (3.53)
\end{aligned}$$

$$\begin{aligned}
2451^2 + 700^2 &= 2549^2 & := 6497401 \\
249951^2 + 7000^2 &= 250049^2 & := 62524502401 \\
24999951^2 + 70000^2 &= 25000049^2 & := 625002450002401 \\
249999951^2 + 700000^2 &= 2500000049^2 & := 6250000245000002401 \\
24999999951^2 + 7000000^2 &= 250000000049^2 := 62500000024500000002401 & (3.54)
\end{aligned}$$

$$\begin{aligned}
2464^2 + 600^2 &= 2536^2 & := 6431296 \\
249964^2 + 6000^2 &= 250036^2 & := 62518001296 \\
24999964^2 + 60000^2 &= 25000036^2 & := 625001800001296 \\
249999964^2 + 600000^2 &= 2500000036^2 & := 6250000180000001296 \\
24999999964^2 + 6000000^2 &= 250000000036^2 := 62500000018000000001296 & (3.55)
\end{aligned}$$

$$\begin{aligned}
2475^2 + 500^2 &= 2525^2 & := 6375625 \\
249975^2 + 5000^2 &= 250025^2 & := 62512500625 \\
24999975^2 + 50000^2 &= 25000025^2 & := 625001250000625 \\
249999975^2 + 500000^2 &= 2500000025^2 & := 6250000125000000625 \\
24999999975^2 + 5000000^2 &= 250000000025^2 := 62500000012500000000625 & (3.56)
\end{aligned}$$

$$\begin{aligned}
2484^2 + 400^2 &= 2516^2 & := 6330256 \\
249984^2 + 4000^2 &= 250016^2 & := 62508000256 \\
24999984^2 + 40000^2 &= 25000016^2 & := 625000800000256 \\
249999984^2 + 400000^2 &= 2500000016^2 & := 6250000080000000256 \\
24999999984^2 + 4000000^2 &= 250000000016^2 := 62500000008000000000256 & (3.57)
\end{aligned}$$

$$\begin{aligned}
2491^2 + 300^2 &= 2509^2 & := 6295081 \\
249991^2 + 3000^2 &= 250009^2 & := 62504500081 \\
24999991^2 + 30000^2 &= 25000009^2 & := 625000450000081 \\
2499999991^2 + 300000^2 &= 2500000009^2 & := 6250000045000000081 \\
249999999991^2 + 3000000^2 &= 250000000009^2 := 62500000004500000000081 & (3.58)
\end{aligned}$$

$$\begin{aligned}
 2496^2 + 200^2 &= 2504^2 & := 6270016 \\
 249996^2 + 2000^2 &= 250004^2 & := 62502000016 \\
 24999996^2 + 20000^2 &= 25000004^2 & := 625000200000016 \\
 2499999996^2 + 200000^2 &= 2500000004^2 & := 6250000020000000016 \\
 249999999996^2 + 2000000^2 &= 250000000004^2 & := 62500000002000000000016
 \end{aligned} \tag{3.59}$$

$$\begin{aligned}
 2499^2 + 100^2 &= 2501^2 & := 6255001 \\
 249999^2 + 1000^2 &= 250001^2 & := 62500500001 \\
 24999999^2 + 10000^2 &= 25000001^2 & := 625000050000001 \\
 2499999999^2 + 100000^2 &= 2500000001^2 & := 6250000005000000001 \\
 249999999999^2 + 1000000^2 &= 250000000001^2 & := 62500000000500000000001
 \end{aligned} \tag{3.60}$$

► 6th Block

Let's fix the values of m , i.e., $m = 60, 600, 6000, 60000, \dots$, and change the values of n as $n = 10, 9, 8, 7, 6, 5, 4, 3, 2$ and $n = 1$, we get respectively the following 10 patterns in Pythagorean Triple:

$$\begin{aligned}
 3500^2 + 1200^2 &= 3700^2 & := 13690000 \\
 359900^2 + 12000^2 &= 360100^2 & := 129672010000 \\
 35999900^2 + 120000^2 &= 36000100^2 & := 1296007200010000 \\
 3599999900^2 + 1200000^2 &= 3600000100^2 & := 12960000720000010000 \\
 359999999900^2 + 12000000^2 &= 360000000100^2 & := 129600000072000000010000
 \end{aligned} \tag{3.61}$$

$$\begin{aligned}
 3519^2 + 1080^2 &= 3681^2 & := 13549761 \\
 359919^2 + 10800^2 &= 360081^2 & := 129658326561 \\
 35999919^2 + 108000^2 &= 36000081^2 & := 1296005832006561 \\
 3599999919^2 + 1080000^2 &= 3600000081^2 & := 12960000583200006561 \\
 359999999919^2 + 10800000^2 &= 360000000081^2 & := 129600000058320000006561
 \end{aligned} \tag{3.62}$$

$$\begin{aligned}
 3536^2 + 960^2 &= 3664^2 & := 13424896 \\
 359936^2 + 9600^2 &= 360064^2 & := 129646084096 \\
 35999936^2 + 96000^2 &= 36000064^2 & := 1296004608004096 \\
 3599999936^2 + 960000^2 &= 3600000064^2 & := 12960000460800004096 \\
 359999999936^2 + 9600000^2 &= 360000000064^2 & := 129600000046080000004096
 \end{aligned} \tag{3.63}$$

$$\begin{aligned}
 3551^2 + 840^2 &= 3649^2 & := 13315201 \\
 359951^2 + 8400^2 &= 360049^2 & := 129635282401 \\
 35999951^2 + 84000^2 &= 36000049^2 & := 1296003528002401 \\
 3599999951^2 + 840000^2 &= 3600000049^2 & := 12960000352800002401 \\
 359999999951^2 + 8400000^2 &= 360000000049^2 & := 129600000035280000002401
 \end{aligned} \tag{3.64}$$

$$\begin{aligned}
3564^2 + 720^2 &= 3636^2 & := & 13220496 \\
359964^2 + 7200^2 &= 360036^2 & := & 129625921296 \\
35999964^2 + 72000^2 &= 36000036^2 & := & 1296002592001296 \\
359999964^2 + 720000^2 &= 3600000036^2 & := & 12960000259200001296 \\
35999999964^2 + 7200000^2 &= 360000000036^2 & := & 129600000025920000001296
\end{aligned} \tag{3.65}$$

$$\begin{aligned}
3575^2 + 600^2 &= 3625^2 & := & 13140625 \\
359975^2 + 6000^2 &= 360025^2 & := & 129618000625 \\
35999975^2 + 60000^2 &= 36000025^2 & := & 1296001800000625 \\
359999975^2 + 600000^2 &= 3600000025^2 & := & 12960000180000000625 \\
35999999975^2 + 6000000^2 &= 360000000025^2 & := & 129600000018000000000625
\end{aligned} \tag{3.66}$$

$$\begin{aligned}
3584^2 + 480^2 &= 3616^2 & := & 13075456 \\
359984^2 + 4800^2 &= 360016^2 & := & 129611520256 \\
35999984^2 + 48000^2 &= 36000016^2 & := & 1296001152000256 \\
359999984^2 + 480000^2 &= 3600000016^2 & := & 12960000115200000256 \\
35999999984^2 + 4800000^2 &= 360000000016^2 & := & 129600000011520000000256
\end{aligned} \tag{3.67}$$

$$\begin{aligned}
3591^2 + 360^2 &= 3609^2 & := & 13024881 \\
359991^2 + 3600^2 &= 360009^2 & := & 129606480081 \\
35999991^2 + 36000^2 &= 36000009^2 & := & 1296000648000081 \\
3599999991^2 + 360000^2 &= 3600000009^2 & := & 12960000064800000081 \\
359999999991^2 + 3600000^2 &= 360000000009^2 & := & 129600000006480000000081
\end{aligned} \tag{3.68}$$

$$\begin{aligned}
3596^2 + 240^2 &= 3604^2 & := & 12988816 \\
359996^2 + 2400^2 &= 360004^2 & := & 129602880016 \\
35999996^2 + 24000^2 &= 36000004^2 & := & 1296000288000016 \\
3599999996^2 + 240000^2 &= 3600000004^2 & := & 12960000028800000016 \\
359999999996^2 + 2400000^2 &= 360000000004^2 & := & 129600000002880000000016
\end{aligned} \tag{3.69}$$

$$\begin{aligned}
3599^2 + 120^2 &= 3601^2 & := & 12967201 \\
359999^2 + 1200^2 &= 360001^2 & := & 129600720001 \\
35999999^2 + 12000^2 &= 36000001^2 & := & 1296000072000001 \\
3599999999^2 + 120000^2 &= 3600000001^2 & := & 12960000007200000001 \\
359999999999^2 + 1200000^2 &= 360000000001^2 & := & 129600000000720000000001
\end{aligned} \tag{3.70}$$

► 7th Block

Let's fix the values of m , i.e., $m = 70, 700, 7000, 70000, \dots$, and change the values of n as $n = 10, 9, 8, 7, 6, 5, 4, 3, 2$ and $n = 1$, we get respectively the following 10 patterns in Pythagorean Triple:

$$\begin{aligned}
 4800^2 + 1400^2 &= 5000^2 & := 25000000 \\
 489900^2 + 14000^2 &= 490100^2 & := 240198010000 \\
 48999900^2 + 140000^2 &= 49000100^2 & := 2401009800010000 \\
 489999900^2 + 1400000^2 &= 4900000100^2 & := 24010000980000010000 \\
 4899999900^2 + 14000000^2 &= 490000000100^2 & := 240100000098000000010000
 \end{aligned} \tag{3.71}$$

$$\begin{aligned}
 4819^2 + 1260^2 &= 4981^2 & := 24810361 \\
 489919^2 + 12600^2 &= 490081^2 & := 240179386561 \\
 48999919^2 + 126000^2 &= 49000081^2 & := 2401007938006561 \\
 489999919^2 + 1260000^2 &= 4900000081^2 & := 24010000793800006561 \\
 4899999919^2 + 12600000^2 &= 490000000081^2 & := 240100000079380000006561
 \end{aligned} \tag{3.72}$$

$$\begin{aligned}
 4836^2 + 1120^2 &= 4964^2 & := 24641296 \\
 489936^2 + 11200^2 &= 490064^2 & := 240162724096 \\
 48999936^2 + 112000^2 &= 49000064^2 & := 2401006272004096 \\
 489999936^2 + 1120000^2 &= 4900000064^2 & := 24010000627200004096 \\
 4899999936^2 + 11200000^2 &= 490000000064^2 & := 240100000062720000004096
 \end{aligned} \tag{3.73}$$

$$\begin{aligned}
 4851^2 + 980^2 &= 4949^2 & := 24492601 \\
 489951^2 + 9800^2 &= 490049^2 & := 240148022401 \\
 48999951^2 + 98000^2 &= 49000049^2 & := 2401004802002401 \\
 489999951^2 + 980000^2 &= 4900000049^2 & := 24010000480200002401 \\
 4899999951^2 + 9800000^2 &= 490000000049^2 & := 240100000048020000002401
 \end{aligned} \tag{3.74}$$

$$\begin{aligned}
 4864^2 + 840^2 &= 4936^2 & := 24364096 \\
 489964^2 + 8400^2 &= 490036^2 & := 240135281296 \\
 48999964^2 + 84000^2 &= 49000036^2 & := 2401003528001296 \\
 489999964^2 + 840000^2 &= 4900000036^2 & := 24010000352800001296 \\
 4899999964^2 + 8400000^2 &= 490000000036^2 & := 240100000035280000001296
 \end{aligned} \tag{3.75}$$

$$\begin{aligned}
 4875^2 + 700^2 &= 4925^2 & := 24255625 \\
 489975^2 + 7000^2 &= 490025^2 & := 240124500625 \\
 48999975^2 + 70000^2 &= 49000025^2 & := 2401002450000625 \\
 489999975^2 + 700000^2 &= 4900000025^2 & := 24010000245000000625 \\
 4899999975^2 + 7000000^2 &= 490000000025^2 & := 240100000024500000000625
 \end{aligned} \tag{3.76}$$

$$\begin{aligned}
 4884^2 + 560^2 &= 4916^2 & := 24167056 \\
 489984^2 + 5600^2 &= 490016^2 & := 240115680256 \\
 48999984^2 + 56000^2 &= 49000016^2 & := 2401001568000256 \\
 489999984^2 + 560000^2 &= 4900000016^2 & := 24010000156800000256 \\
 4899999984^2 + 5600000^2 &= 490000000016^2 & := 240100000015680000000256 \tag{3.77}
 \end{aligned}$$

$$\begin{aligned}
 4891^2 + 420^2 &= 4909^2 & := 24098281 \\
 489991^2 + 4200^2 &= 490009^2 & := 240108820081 \\
 48999991^2 + 42000^2 &= 49000009^2 & := 2401000882000081 \\
 489999991^2 + 420000^2 &= 4900000009^2 & := 24010000088200000081 \\
 4899999991^2 + 4200000^2 &= 490000000009^2 & := 240100000008820000000081 \tag{3.78}
 \end{aligned}$$

$$\begin{aligned}
 4896^2 + 280^2 &= 4904^2 & := 24049216 \\
 489996^2 + 2800^2 &= 490004^2 & := 240103920016 \\
 48999996^2 + 28000^2 &= 49000004^2 & := 2401000392000016 \\
 489999996^2 + 280000^2 &= 4900000004^2 & := 24010000039200000016 \\
 4899999996^2 + 2800000^2 &= 490000000004^2 & := 240100000003920000000016 \tag{3.79}
 \end{aligned}$$

$$\begin{aligned}
 4899^2 + 140^2 &= 4901^2 & := 24019801 \\
 489999^2 + 1400^2 &= 490001^2 & := 240100980001 \\
 4899999^2 + 14000^2 &= 49000001^2 & := 2401000098000001 \\
 48999999^2 + 140000^2 &= 4900000001^2 & := 24010000009800000001 \\
 489999999^2 + 1400000^2 &= 490000000001^2 & := 240100000000980000000001 \tag{3.80}
 \end{aligned}$$

► 8th Block

Let's fix the values of m , i.e., $m = 80, 800, 8000, 80000, \dots$, and change the values of n as $n = 10, 9, 8, 7, 6, 5, 4, 3, 2$ and $n = 1$, we get respectively the following 10 patterns in Pythagorean Triple:

$$\begin{aligned}
 6300^2 + 1600^2 &= 6500^2 & := 42250000 \\
 639900^2 + 16000^2 &= 640100^2 & := 409728010000 \\
 63999900^2 + 160000^2 &= 64000100^2 & := 4096012800010000 \\
 639999900^2 + 1600000^2 &= 6400000100^2 & := 40960001280000010000 \\
 6399999900^2 + 16000000^2 &= 640000000100^2 & := 409600000128000000010000 \tag{3.81}
 \end{aligned}$$

$$\begin{aligned}
 6319^2 + 1440^2 &= 6481^2 & := 42003361 \\
 639919^2 + 14400^2 &= 640081^2 & := 409703686561 \\
 63999919^2 + 144000^2 &= 64000081^2 & := 4096010368006561 \\
 639999919^2 + 1440000^2 &= 6400000081^2 & := 40960001036800006561 \\
 6399999919^2 + 14400000^2 &= 640000000081^2 & := 409600000103680000006561 \tag{3.82}
 \end{aligned}$$

$$\begin{aligned}
6336^2 + 1280^2 &= 6464^2 & := 41783296 \\
639936^2 + 12800^2 &= 640064^2 & := 409681924096 \\
63999936^2 + 128000^2 &= 64000064^2 & := 4096008192004096 \\
639999936^2 + 1280000^2 &= 6400000064^2 & := 40960000819200004096 \\
63999999936^2 + 12800000^2 &= 640000000064^2 & := 409600000081920000004096
\end{aligned} \tag{3.83}$$

$$\begin{aligned}
6351^2 + 1120^2 &= 6449^2 & := 41589601 \\
639951^2 + 11200^2 &= 640049^2 & := 409662722401 \\
63999951^2 + 112000^2 &= 64000049^2 & := 4096006272002401 \\
639999951^2 + 1120000^2 &= 6400000049^2 & := 40960000627200002401 \\
63999999951^2 + 11200000^2 &= 640000000049^2 & := 409600000062720000002401
\end{aligned} \tag{3.84}$$

$$\begin{aligned}
6364^2 + 960^2 &= 6436^2 & := 41422096 \\
639964^2 + 9600^2 &= 640036^2 & := 409646081296 \\
63999964^2 + 96000^2 &= 64000036^2 & := 4096004608001296 \\
639999964^2 + 960000^2 &= 6400000036^2 & := 40960000460800001296 \\
63999999964^2 + 9600000^2 &= 640000000036^2 & := 409600000046080000001296
\end{aligned} \tag{3.85}$$

$$\begin{aligned}
6375^2 + 800^2 &= 6425^2 & := 41280625 \\
639975^2 + 8000^2 &= 640025^2 & := 409632000625 \\
63999975^2 + 80000^2 &= 64000025^2 & := 4096003200000625 \\
639999975^2 + 800000^2 &= 6400000025^2 & := 40960000320000000625 \\
63999999975^2 + 8000000^2 &= 640000000025^2 & := 409600000032000000000625
\end{aligned} \tag{3.86}$$

$$\begin{aligned}
6384^2 + 640^2 &= 6416^2 & := 41165056 \\
639984^2 + 6400^2 &= 640016^2 & := 409620480256 \\
63999984^2 + 64000^2 &= 64000016^2 & := 4096002048000256 \\
639999984^2 + 640000^2 &= 6400000016^2 & := 40960000204800000256 \\
63999999984^2 + 6400000^2 &= 640000000016^2 & := 409600000020480000000256
\end{aligned} \tag{3.87}$$

$$\begin{aligned}
6391^2 + 480^2 &= 6409^2 & := 41075281 \\
639991^2 + 4800^2 &= 640009^2 & := 409611520081 \\
63999991^2 + 48000^2 &= 64000009^2 & := 4096001152000081 \\
6399999991^2 + 480000^2 &= 6400000009^2 & := 40960000115200000081 \\
639999999991^2 + 4800000^2 &= 640000000009^2 & := 409600000011520000000081
\end{aligned} \tag{3.88}$$

$$\begin{aligned}
 6396^2 + 320^2 &= 6404^2 & := 41011216 \\
 639996^2 + 3200^2 &= 640004^2 & := 409605120016 \\
 63999996^2 + 32000^2 &= 64000004^2 & := 4096000512000016 \\
 6399999996^2 + 320000^2 &= 6400000004^2 & := 40960000051200000016 \\
 639999999996^2 + 3200000^2 &= 640000000004^2 & := 409600000005120000000016 \tag{3.89}
 \end{aligned}$$

$$\begin{aligned}
 6399^2 + 160^2 &= 6401^2 & := 40972801 \\
 639999^2 + 1600^2 &= 640001^2 & := 409601280001 \\
 63999999^2 + 16000^2 &= 64000001^2 & := 4096000128000001 \\
 6399999999^2 + 160000^2 &= 6400000001^2 & := 40960000012800000001 \\
 639999999999^2 + 1600000^2 &= 640000000001^2 & := 409600000001280000000001 \tag{3.90}
 \end{aligned}$$

► 9th Block

Let's fix the values of m , i.e., $m = 90, 900, 9000, 90000, \dots$, and change the values of n as $n = 10, 9, 8, 7, 6, 5, 4, 3, 2$ and $n = 1$, we get respectively the following 10 patterns in Pythagorean Triple:

$$\begin{aligned}
 8000^2 + 1800^2 &= 8200^2 & := 67240000 \\
 809900^2 + 18000^2 &= 810100^2 & := 656262010000 \\
 80999900^2 + 180000^2 &= 81000100^2 & := 6561016200010000 \\
 8099999900^2 + 1800000^2 &= 8100000100^2 & := 65610001620000010000 \\
 809999999900^2 + 18000000^2 &= 810000000100^2 & := 656100000162000000010000 \tag{3.91}
 \end{aligned}$$

$$\begin{aligned}
 8019^2 + 1620^2 &= 8181^2 & := 66928761 \\
 809919^2 + 16200^2 &= 810081^2 & := 656231226561 \\
 80999919^2 + 162000^2 &= 81000081^2 & := 6561013122006561 \\
 8099999919^2 + 1620000^2 &= 8100000081^2 & := 65610001312200006561 \\
 809999999919^2 + 16200000^2 &= 810000000081^2 & := 656100000131220000006561 \tag{3.92}
 \end{aligned}$$

$$\begin{aligned}
 8036^2 + 1440^2 &= 8164^2 & := 66650896 \\
 809936^2 + 14400^2 &= 810064^2 & := 656203684096 \\
 80999936^2 + 144000^2 &= 81000064^2 & := 6561010368004096 \\
 8099999936^2 + 1440000^2 &= 8100000064^2 & := 65610001036800004096 \\
 809999999936^2 + 14400000^2 &= 810000000064^2 & := 656100000103680000004096 \tag{3.93}
 \end{aligned}$$

$$\begin{aligned}
 8051^2 + 1260^2 &= 8149^2 & := 66406201 \\
 809951^2 + 12600^2 &= 810049^2 & := 656179382401 \\
 80999951^2 + 126000^2 &= 81000049^2 & := 6561007938002401 \\
 8099999951^2 + 1260000^2 &= 8100000049^2 & := 65610000793800002401 \\
 809999999951^2 + 12600000^2 &= 810000000049^2 & := 656100000079380000002401 \tag{3.94}
 \end{aligned}$$

$$\begin{aligned}
8064^2 + 1080^2 &= 8136^2 && := 66194496 \\
809964^2 + 10800^2 &= 810036^2 && := 656158321296 \\
80999964^2 + 108000^2 &= 81000036^2 && := 6561005832001296 \\
809999964^2 + 1080000^2 &= 8100000036^2 && := 65610000583200001296 \\
80999999964^2 + 10800000^2 &= 810000000036^2 && := 656100000058320000001296 \quad (3.95)
\end{aligned}$$

$$\begin{aligned}
8075^2 + 900^2 &= 8125^2 && := 66015625 \\
809975^2 + 9000^2 &= 810025^2 && := 656140500625 \\
80999975^2 + 90000^2 &= 81000025^2 && := 6561004050000625 \\
809999975^2 + 900000^2 &= 8100000025^2 && := 65610000405000000625 \\
80999999975^2 + 9000000^2 &= 810000000025^2 && := 656100000040500000000625 \quad (3.96)
\end{aligned}$$

$$\begin{aligned}
8084^2 + 720^2 &= 8116^2 && := 65869456 \\
809984^2 + 7200^2 &= 810016^2 && := 656125920256 \\
80999984^2 + 72000^2 &= 81000016^2 && := 6561002592000256 \\
809999984^2 + 720000^2 &= 8100000016^2 && := 65610000259200000256 \\
80999999984^2 + 7200000^2 &= 810000000016^2 && := 656100000025920000000256 \quad (3.97)
\end{aligned}$$

$$\begin{aligned}
8091^2 + 540^2 &= 8109^2 && := 65755881 \\
809991^2 + 5400^2 &= 810009^2 && := 656114580081 \\
80999991^2 + 54000^2 &= 81000009^2 && := 6561001458000081 \\
8099999991^2 + 540000^2 &= 8100000009^2 && := 65610000145800000081 \\
809999999991^2 + 5400000^2 &= 810000000009^2 && := 656100000014580000000081 \quad (3.98)
\end{aligned}$$

$$\begin{aligned}
8096^2 + 360^2 &= 8104^2 && := 65674816 \\
809996^2 + 3600^2 &= 810004^2 && := 656106480016 \\
80999996^2 + 36000^2 &= 81000004^2 && := 6561000648000016 \\
8099999996^2 + 360000^2 &= 8100000004^2 && := 65610000064800000016 \\
809999999996^2 + 3600000^2 &= 810000000004^2 && := 656100000006480000000016 \quad (3.99)
\end{aligned}$$

$$\begin{aligned}
8099^2 + 180^2 &= 8101^2 && := 65626201 \\
809999^2 + 1800^2 &= 810001^2 && := 656101620001 \\
8099999^2 + 18000^2 &= 81000001^2 && := 6561000162000001 \\
809999999^2 + 180000^2 &= 8100000001^2 && := 65610000016200000001 \\
80999999999^2 + 1800000^2 &= 810000000001^2 && := 656100000001620000000001 \quad (3.100)
\end{aligned}$$

► 10th Block

Let's fix the values of m , i.e., $m = 100, 1000, 10000, 100000, \dots$ and change the values of n as $n = 10, 9, 8, 7, 6, 5, 4, 3, 2$ and $n = 1$, we get respectively the following 10 patterns in Pythagorean Triple:

$$\begin{aligned}
 9900^2 + 2000^2 &= 10100^2 & := 102010000 \\
 999900^2 + 20000^2 &= 1000100^2 & := 1000200010000 \\
 99999900^2 + 200000^2 &= 100000100^2 & := 10000020000010000 \\
 9999999900^2 + 2000000^2 &= 10000000100^2 & := 100000002000000010000 \\
 999999999900^2 + 20000000^2 &= 1000000000100^2 & := 1000000000200000000010000 \quad (3.101)
 \end{aligned}$$

$$\begin{aligned}
 9919^2 + 1800^2 &= 10081^2 & := 101626561 \\
 999919^2 + 18000^2 &= 1000081^2 & := 1000162006561 \\
 99999919^2 + 180000^2 &= 100000081^2 & := 10000016200006561 \\
 9999999919^2 + 1800000^2 &= 10000000081^2 & := 100000001620000006561 \\
 999999999919^2 + 18000000^2 &= 1000000000081^2 & := 1000000000162000000006561 \quad (3.102)
 \end{aligned}$$

$$\begin{aligned}
 9936^2 + 1600^2 &= 10064^2 & := 101284096 \\
 999936^2 + 16000^2 &= 1000064^2 & := 1000128004096 \\
 99999936^2 + 160000^2 &= 100000064^2 & := 10000012800004096 \\
 9999999936^2 + 1600000^2 &= 10000000064^2 & := 100000001280000004096 \\
 999999999936^2 + 16000000^2 &= 1000000000064^2 & := 1000000000128000000004096 \quad (3.103)
 \end{aligned}$$

$$\begin{aligned}
 9951^2 + 1400^2 &= 10049^2 & := 100982401 \\
 999951^2 + 14000^2 &= 1000049^2 & := 1000098002401 \\
 99999951^2 + 140000^2 &= 100000049^2 & := 10000009800002401 \\
 9999999951^2 + 1400000^2 &= 10000000049^2 & := 100000000980000002401 \\
 999999999951^2 + 14000000^2 &= 1000000000049^2 & := 1000000000098000000002401 \quad (3.104)
 \end{aligned}$$

$$\begin{aligned}
 9964^2 + 1200^2 &= 10036^2 & := 100721296 \\
 999964^2 + 12000^2 &= 1000036^2 & := 1000072001296 \\
 99999964^2 + 120000^2 &= 100000036^2 & := 10000007200001296 \\
 9999999964^2 + 1200000^2 &= 10000000036^2 & := 100000000720000001296 \\
 999999999964^2 + 12000000^2 &= 1000000000036^2 & := 1000000000072000000001296 \quad (3.105)
 \end{aligned}$$

$$\begin{aligned}
 9975^2 + 1000^2 &= 10025^2 & := 100500625 \\
 999975^2 + 10000^2 &= 1000025^2 & := 1000050000625 \\
 99999975^2 + 100000^2 &= 100000025^2 & := 10000005000000625 \\
 9999999975^2 + 1000000^2 &= 10000000025^2 & := 100000000500000000625 \\
 999999999975^2 + 10000000^2 &= 1000000000025^2 & := 1000000000050000000000625 \quad (3.106)
 \end{aligned}$$

$$\begin{aligned}
9984^2 + 800^2 &= 10016^2 & := & 100320256 \\
999984^2 + 8000^2 &= 1000016^2 & := & 1000032000256 \\
99999984^2 + 80000^2 &= 100000016^2 & := & 10000003200000256 \\
999999984^2 + 800000^2 &= 10000000016^2 & := & 100000000320000000256 \\
9999999984^2 + 8000000^2 &= 1000000000016^2 & := & 1000000000032000000000256 \quad (3.107)
\end{aligned}$$

$$\begin{aligned}
9991^2 + 600^2 &= 10009^2 & := & 100180081 \\
999991^2 + 6000^2 &= 1000009^2 & := & 1000018000081 \\
99999991^2 + 60000^2 &= 100000009^2 & := & 10000001800000081 \\
999999991^2 + 600000^2 &= 10000000009^2 & := & 100000000180000000081 \\
9999999991^2 + 6000000^2 &= 1000000000009^2 & := & 1000000000018000000000081 \quad (3.108)
\end{aligned}$$

$$\begin{aligned}
9996^2 + 400^2 &= 10004^2 & := & 100080016 \\
999996^2 + 4000^2 &= 1000004^2 & := & 1000008000016 \\
99999996^2 + 40000^2 &= 100000004^2 & := & 10000000800000016 \\
999999996^2 + 400000^2 &= 10000000004^2 & := & 100000000080000000016 \\
9999999996^2 + 4000000^2 &= 1000000000004^2 & := & 1000000000008000000000016 \quad (3.109)
\end{aligned}$$

$$\begin{aligned}
9999^2 + 200^2 &= 10001^2 & := & 100020001 \\
99999^2 + 2000^2 &= 1000001^2 & := & 1000002000001 \\
9999999^2 + 20000^2 &= 100000001^2 & := & 10000000200000001 \\
999999999^2 + 200000^2 &= 10000000001^2 & := & 100000000020000000001 \\
9999999999^2 + 2000000^2 &= 1000000000001^2 & := & 1000000000002000000000001 \quad (3.110)
\end{aligned}$$

3.3 Palindromic-Type Pandigital Patterns

In the previous subsection we have general pattern for 99 Pythagorean triples. Below are 9 **palindromic-type pandigital patterns** based on Pythagorean triples given in (3.2). For the triples given in (3.3)-(3.11), we don't have similar kind of patterns. There are two ways to write these patterns. One is normal palindromic-type, i.e., $1, 121, 12321, 1234321, \dots, 12345678987654321$ and second is palindromic-type with 0 between each digit, i.e., $1, 10201, 102030201, \dots, 102030405060708090807060504030201$. Let's write the Pythagorean triples given in (3.2) in reverse order, i.e.,

$$\begin{aligned}
 &(99, 020, 101) \\
 &(96, 040, 104) \\
 &(91, 060, 109) \\
 &(84, 080, 116) \\
 &(75, 100, 125) \\
 &(64, 120, 136) \\
 &(51, 140, 149) \\
 &(36, 160, 164) \\
 &(19, 180, 181)
 \end{aligned} \tag{3.111}$$

3.3.1 First Way

Below are 9 **palindromic-type pandigital patterns** based on Pythagorean triples given in (3.2), but in reverse order, i.e., according to order given in (3.111). For this, let's consider

$$m = 10, 110, 1110, 11110, 111110, 1111110, 11111110, 111111110$$

in (2.1), then for each value of $n = 1, 2, 3, 4, 5, 6, 7, 8, 9$; there are 9 **palindromic-type pandigital Pythagorean patterns** given below:

$$\begin{aligned}
 &099^2 + 20^2 &&= 101^2 \\
 &12099^2 + 220^2 &&= 12101^2 \\
 &1232099^2 + 2220^2 &&= 1232101^2 \\
 &123432099^2 + 22220^2 &&= 123432101^2 \\
 &12345432099^2 + 222220^2 &&= 12345432101^2 \\
 &1234565432099^2 + 2222220^2 &&= 1234565432101^2 \\
 &123456765432099^2 + 22222220^2 &&= 123456765432101^2 \\
 &12345678765432099^2 + 222222220^2 &&= 12345678765432101^2 \\
 &1234567898765432099^2 + 2222222220^2 &&= 1234567898765432101^2
 \end{aligned} \tag{3.112}$$

$$\begin{aligned}
 &096^2 + 40^2 &&= 104^2 \\
 &12096^2 + 440^2 &&= 12104^2 \\
 &1232096^2 + 4440^2 &&= 1232104^2 \\
 &123432096^2 + 44440^2 &&= 123432104^2 \\
 &12345432096^2 + 444440^2 &&= 12345432104^2 \\
 &1234565432096^2 + 4444440^2 &&= 1234565432104^2 \\
 &123456765432096^2 + 44444440^2 &&= 123456765432104^2 \\
 &12345678765432096^2 + 444444440^2 &&= 12345678765432104^2 \\
 &1234567898765432096^2 + 4444444440^2 &&= 1234567898765432104^2
 \end{aligned} \tag{3.113}$$

$$\begin{aligned}
091^2 + 60^2 &= 109^2 \\
12091^2 + 660^2 &= 12109^2 \\
1232091^2 + 6660^2 &= 1232109^2 \\
123432091^2 + 66660^2 &= 123432109^2 \\
12345432091^2 + 666660^2 &= 12345432109^2 \\
1234565432091^2 + 6666660^2 &= 1234565432109^2 \\
123456765432091^2 + 66666660^2 &= 123456765432109^2 \\
12345678765432091^2 + 666666660^2 &= 12345678765432109^2 \\
1234567898765432091^2 + 6666666660^2 &= 1234567898765432109^2 \quad (3.114)
\end{aligned}$$

$$\begin{aligned}
084^2 + 80^2 &= 116^2 \\
12084^2 + 880^2 &= 12116^2 \\
1232084^2 + 8880^2 &= 1232116^2 \\
123432084^2 + 88880^2 &= 123432116^2 \\
12345432084^2 + 888880^2 &= 12345432116^2 \\
1234565432084^2 + 8888880^2 &= 1234565432116^2 \\
123456765432084^2 + 88888880^2 &= 123456765432116^2 \\
12345678765432084^2 + 888888880^2 &= 12345678765432116^2 \\
1234567898765432084^2 + 8888888880^2 &= 1234567898765432116^2 \quad (3.115)
\end{aligned}$$

$$\begin{aligned}
075^2 + 100^2 &= 125^2 \\
12075^2 + 1100^2 &= 12125^2 \\
1232075^2 + 11100^2 &= 1232125^2 \\
123432075^2 + 111100^2 &= 123432125^2 \\
12345432075^2 + 1111100^2 &= 12345432125^2 \\
1234565432075^2 + 11111100^2 &= 1234565432125^2 \\
123456765432075^2 + 111111100^2 &= 123456765432125^2 \\
12345678765432075^2 + 1111111100^2 &= 12345678765432125^2 \\
1234567898765432075^2 + 11111111100^2 &= 1234567898765432125^2 \quad (3.116)
\end{aligned}$$

$$\begin{aligned}
064^2 + 120^2 &= 136^2 \\
12064^2 + 1320^2 &= 12136^2 \\
1232064^2 + 13320^2 &= 1232136^2 \\
123432064^2 + 133320^2 &= 123432136^2 \\
12345432064^2 + 1333320^2 &= 12345432136^2 \\
1234565432064^2 + 13333320^2 &= 1234565432136^2 \\
123456765432064^2 + 133333320^2 &= 123456765432136^2 \\
12345678765432064^2 + 1333333320^2 &= 12345678765432136^2 \\
1234567898765432064^2 + 13333333320^2 &= 1234567898765432136^2 \quad (3.117)
\end{aligned}$$

$$\begin{aligned}
051^2 + 140^2 &= 149^2 \\
12051^2 + 1540^2 &= 12149^2 \\
1232051^2 + 15540^2 &= 1232149^2 \\
123432051^2 + 155540^2 &= 123432149^2 \\
12345432051^2 + 1555540^2 &= 12345432149^2 \\
1234565432051^2 + 15555540^2 &= 1234565432149^2 \\
123456765432051^2 + 155555540^2 &= 123456765432149^2 \\
12345678765432051^2 + 1555555540^2 &= 12345678765432149^2 \\
1234567898765432051^2 + 15555555540^2 &= 1234567898765432149^2 \quad (3.118)
\end{aligned}$$

$$\begin{aligned}
036^2 + 160^2 &= 164^2 \\
12036^2 + 1760^2 &= 12164^2 \\
1232036^2 + 17760^2 &= 1232164^2 \\
123432036^2 + 177760^2 &= 123432164^2 \\
12345432036^2 + 1777760^2 &= 12345432164^2 \\
1234565432036^2 + 17777760^2 &= 1234565432164^2 \\
123456765432036^2 + 177777760^2 &= 123456765432164^2 \\
12345678765432036^2 + 1777777760^2 &= 12345678765432164^2 \\
1234567898765432036^2 + 17777777760^2 &= 1234567898765432164^2 \quad (3.119)
\end{aligned}$$

$$\begin{aligned}
019^2 + 180^2 &= 181^2 \\
12019^2 + 1980^2 &= 12181^2 \\
1232019^2 + 19980^2 &= 1232181^2 \\
123432019^2 + 199980^2 &= 123432181^2 \\
12345432019^2 + 1999980^2 &= 12345432181^2 \\
1234565432019^2 + 19999980^2 &= 1234565432181^2 \\
123456765432019^2 + 199999980^2 &= 123456765432181^2 \\
12345678765432019^2 + 1999999980^2 &= 12345678765432181^2 \\
1234567898765432019^2 + 19999999980^2 &= 1234567898765432181^2 \quad (3.120)
\end{aligned}$$

For more examples of similar kind refer [9].

3.3.2 Second Way

Below are 9 **palindromic-type pandigital patterns** based on Pythagorean triples considered in reverse order of (3.2), i.e., according to order of (3.111). These patterns are different from the one given in subsection 3.3. Here we have 0 in between each digit. For this, let's consider

$$m = 10, 1010, 101010, 10101010, 1010101010, 101010101010, 10101010101010, 1010101010101010, 101010101010101010$$

in (2.1), then for each value of $n = 1, 2, 3, \dots, 7, 8, 9$; there are 9 palindromic-type pandigital Pythagorean patterns given below:

$$\begin{aligned}
 &099^2 + 20^2 &&= 101^2 \\
 &102099^2 + 2020^2 &&= 1020101^2 \\
 &1020302099^2 + 202020^2 &&= 10203020101^2 \\
 &10203040302099^2 + 20202020^2 &&= 102030403020101^2 \\
 &102030405040302099^2 + 2020202020^2 &&= 1020304050403020101^2 \\
 &1020304050605040302099^2 + 202020202020^2 &&= 10203040506050403020101^2 \\
 &10203040506070605040302099^2 + 20202020202020^2 &&= 102030405060706050403020101^2 \\
 &102030405060708070605040302099^2 + 2020202020202020^2 &&= 1020304050607080706050403020101^2 \\
 &1020304050607080908070605040302099^2 + 202020202020202020^2 &&= 10203040506070809080706050403020101^2
 \end{aligned}
 \tag{3.121}$$

$$\begin{aligned}
 &096^2 + 40^2 &&= 104^2 \\
 &102096^2 + 4040^2 &&= 1020104^2 \\
 &1020302096^2 + 404040^2 &&= 10203020104^2 \\
 &10203040302096^2 + 40404040^2 &&= 102030403020104^2 \\
 &102030405040302096^2 + 4040404040^2 &&= 1020304050403020104^2 \\
 &1020304050605040302096^2 + 404040404040^2 &&= 10203040506050403020104^2 \\
 &10203040506070605040302096^2 + 40404040404040^2 &&= 102030405060706050403020104^2 \\
 &102030405060708070605040302096^2 + 4040404040404040^2 &&= 1020304050607080706050403020104^2 \\
 &1020304050607080908070605040302096^2 + 404040404040404040^2 &&= 10203040506070809080706050403020104^2
 \end{aligned}
 \tag{3.122}$$

$$\begin{aligned}
 &091^2 + 60^2 &&= 109^2 \\
 &102091^2 + 6060^2 &&= 1020109^2 \\
 &1020302091^2 + 606060^2 &&= 10203020109^2 \\
 &10203040302091^2 + 60606060^2 &&= 102030403020109^2 \\
 &102030405040302091^2 + 6060606060^2 &&= 1020304050403020109^2 \\
 &1020304050605040302091^2 + 606060606060^2 &&= 10203040506050403020109^2 \\
 &10203040506070605040302091^2 + 60606060606060^2 &&= 102030405060706050403020109^2 \\
 &102030405060708070605040302091^2 + 6060606060606060^2 &&= 1020304050607080706050403020109^2 \\
 &1020304050607080908070605040302091^2 + 606060606060606060^2 &&= 10203040506070809080706050403020109^2
 \end{aligned}
 \tag{3.123}$$

$$\begin{aligned}
 &084^2 + 80^2 &&= 116^2 \\
 &102084^2 + 8080^2 &&= 1020116^2 \\
 &1020302084^2 + 808080^2 &&= 10203020116^2 \\
 &10203040302084^2 + 80808080^2 &&= 102030403020116^2 \\
 &102030405040302084^2 + 8080808080^2 &&= 1020304050403020116^2 \\
 &1020304050605040302084^2 + 808080808080^2 &&= 10203040506050403020116^2 \\
 &10203040506070605040302084^2 + 80808080808080^2 &&= 102030405060706050403020116^2 \\
 &102030405060708070605040302084^2 + 8080808080808080^2 &&= 1020304050607080706050403020116^2 \\
 &1020304050607080908070605040302084^2 + 808080808080808080^2 &&= 10203040506070809080706050403020116^2
 \end{aligned}
 \tag{3.124}$$

$$\begin{aligned}
 &075^2 + 100^2 = 1\ 25^2 \\
 &1020\ 075^2 + 10100^2 = 10201\ 25^2 \\
 &10203020\ 075^2 + 1010100^2 = 102030201\ 25^2 \\
 &102030403020\ 075^2 + 101010100^2 = 1020304030201\ 25^2 \\
 &1020304050403020\ 075^2 + 10101010100^2 = 10203040504030201\ 25^2 \\
 &10203040506050403020\ 075^2 + 1010101010100^2 = 102030405060504030201\ 25^2 \\
 &102030405060706050403020\ 075^2 + 101010101010100^2 = 1020304050607060504030201\ 25^2 \\
 &1020304050607080706050403020\ 075^2 + 10101010101010100^2 = 10203040506070807060504030201\ 25^2 \\
 &10203040506070809080706050403020\ 075^2 + 1010101010101010100^2 = 102030405060708090807060504030201\ 25^2 \\
 & \hspace{30em} (3.125)
 \end{aligned}$$

$$\begin{aligned}
 &064^2 + 120^2 = 1\ 36^2 \\
 &1020\ 064^2 + 12120^2 = 10201\ 36^2 \\
 &10203020\ 064^2 + 1212120^2 = 102030201\ 36^2 \\
 &102030403020\ 064^2 + 121212120^2 = 1020304030201\ 36^2 \\
 &1020304050403020\ 064^2 + 12121212120^2 = 10203040504030201\ 36^2 \\
 &10203040506050403020\ 064^2 + 1212121212120^2 = 102030405060504030201\ 36^2 \\
 &102030405060706050403020\ 064^2 + 121212121212120^2 = 1020304050607060504030201\ 36^2 \\
 &1020304050607080706050403020\ 064^2 + 12121212121212120^2 = 10203040506070807060504030201\ 36^2 \\
 &10203040506070809080706050403020\ 064^2 + 1212121212121212120^2 = 102030405060708090807060504030201\ 36^2 \\
 & \hspace{30em} (3.126)
 \end{aligned}$$

$$\begin{aligned}
 &051^2 + 140^2 = 1\ 49^2 \\
 &1020\ 051^2 + 14140^2 = 10201\ 49^2 \\
 &10203020\ 051^2 + 1414140^2 = 102030201\ 49^2 \\
 &102030403020\ 051^2 + 141414140^2 = 1020304030201\ 49^2 \\
 &1020304050403020\ 051^2 + 14141414140^2 = 10203040504030201\ 49^2 \\
 &10203040506050403020\ 051^2 + 1414141414140^2 = 102030405060504030201\ 49^2 \\
 &102030405060706050403020\ 051^2 + 141414141414140^2 = 1020304050607060504030201\ 49^2 \\
 &1020304050607080706050403020\ 051^2 + 14141414141414140^2 = 10203040506070807060504030201\ 49^2 \\
 &10203040506070809080706050403020\ 051^2 + 1414141414141414140^2 = 102030405060708090807060504030201\ 49^2 \\
 & \hspace{30em} (3.127)
 \end{aligned}$$

$$\begin{aligned}
 &036^2 + 160^2 = 1\ 64^2 \\
 &1020\ 036^2 + 16160^2 = 10201\ 64^2 \\
 &10203020\ 036^2 + 1616160^2 = 102030201\ 64^2 \\
 &102030403020\ 036^2 + 161616160^2 = 1020304030201\ 64^2 \\
 &1020304050403020\ 036^2 + 16161616160^2 = 10203040504030201\ 64^2 \\
 &10203040506050403020\ 036^2 + 1616161616160^2 = 102030405060504030201\ 64^2 \\
 &102030405060706050403020\ 036^2 + 161616161616160^2 = 1020304050607060504030201\ 64^2 \\
 &1020304050607080706050403020\ 036^2 + 16161616161616160^2 = 10203040506070807060504030201\ 64^2 \\
 &10203040506070809080706050403020\ 036^2 + 1616161616161616160^2 = 102030405060708090807060504030201\ 64^2 \\
 & \hspace{30em} (3.128)
 \end{aligned}$$

$$\begin{aligned}
 019^2 + 180^2 &= 181^2 \\
 1020019^2 + 18180^2 &= 1020181^2 \\
 10203020019^2 + 1818180^2 &= 10203020181^2 \\
 102030403020019^2 + 181818180^2 &= 102030403020181^2 \\
 1020304050403020019^2 + 18181818180^2 &= 1020304050403020181^2 \\
 10203040506050403020019^2 + 1818181818180^2 &= 10203040506050403020181^2 \\
 102030405060706050403020019^2 + 181818181818180^2 &= 102030405060706050403020181^2 \\
 1020304050607080706050403020019^2 + 18181818181818180^2 &= 1020304050607080706050403020181^2 \\
 10203040506070809080706050403020019^2 + 1818181818181818180^2 &= 10203040506070809080706050403020181^2
 \end{aligned}
 \tag{3.129}$$

For more examples of similar kind refer [?].

Remark 4. In Section 4, there are total 99 Pythagorean triples written in (3.2)- (3.11), but in Subsection 3.3.1 and 3.3.2, there are only 9 **palindromic-type pandigital patterns**. The reason is that we are unable to bring similar kind of patterns for the Pythagorean triples given in (3.3)- (3.11). In Sections 5 and 6, there are an extensions of the sum given in (3.1) leading us to 99, 999, etc. **palindromic-type pandigital patterns**. In compensation, the 99 Pythagorean triples given in Section 4 are good for the construction magic squares, while the Pythagorean triples given in Sections 5 and 6 lead us to magic square but of very high number entries. The subsection below give magic square constructed based on the Pythagorean triples given in (3.2)- (3.11).

4 Magic Squares

Let’s write a Procedure studied by author in [3, 4] to generate magic squares from the Pythagorean triples:

Result 4.1. In a Pythagorean triple, (a, b, c) if any of the difference

$$c - b \text{ or } c - a, \quad c > b, \quad c > a,$$

is a perfect square greater than or equal to 9, then we can always write a **perfect square sum magic square**.

Let us consider

$$(a, b, c) \Rightarrow \{\text{order of magic square, first member of sequence, last member of sequence, magic sum, sum of all members of a magic square}\}. \tag{4.1}$$

For example,

$$\{15, 8, 17\} \Rightarrow \{3, 17, 33, 75, 225\}. \tag{4.2}$$

The above representation (4.2) is understood as:

- 3 \Rightarrow Order of a magic square;
- 17 \Rightarrow First member of the sequence;
- 33 \Rightarrow Last member of the sequence;
- 75 \Rightarrow Magic sum;
- 225 \Rightarrow Sum of all members of the magic square, is a perfect square, $225 = 15^2$.

In this case, the 9 consecutive odd numbers generating magic square are 17, 19, 21, 23, 25, 27, 29, 31 and 33.

Result 4.2. To reach the result appearing in equation (4.2) we used the following formula:

$$(a, b, c) \Rightarrow \left\{ \left\{ \frac{c^2 - a^2}{\sqrt{c - a}}, c^2 - a^2, 2a + 1, 2c - 1, \sqrt{c - a} \right\}, \left\{ \frac{c^2 - b^2}{\sqrt{c - b}}, c^2 - b^2, 2b + 1, 2c - 1, \sqrt{c - b} \right\} \right\} \quad (4.3)$$

Result 4.1 is good for testing the existence of magic square, whilst Result 4.2 is useful for calculating the details of magic square. Using the Procedure given in 4.3, we have following distributions for the construction of magic squares based on 99 Pythagorean triples given in (3.2)-(3.11):

$$\begin{aligned}
 (51, 140, 149) &\Rightarrow \{3, 281, 297, 867, 2601 = 51^2\} \\
 (64, 120, 136) &\Rightarrow \{4, 241, 271, 1024, 4096 = 64^2\} \\
 (75, 100, 125) &\Rightarrow \{5, 201, 249, 1125, 5625 = 75^2\} \\
 (84, 80, 116) &\Rightarrow \{6, 161, 231, 1176, 7056 = 84^2\} \\
 (91, 60, 109) &\Rightarrow \{7, 121, 217, 1183, 8281 = 91^2\} \\
 (96, 40, 104) &\Rightarrow \{8, 81, 207, 1152, 9216 = 96^2\} \\
 (99, 20, 101) &\Rightarrow \{9, 41, 201, 1089, 9801 = 99^2\} \\
 (300, 400, 500) &\Rightarrow \{10, 801, 999, 9000, 90000 = 300^2\} \\
 (319, 360, 481) &\Rightarrow \{11, 721, 961, 9251, 101761 = 319^2\} \\
 (336, 320, 464) &\Rightarrow \{12, 641, 927, 9408, 112896 = 336^2\} \\
 (351, 280, 449) &\Rightarrow \{13, 561, 897, 9477, 123201 = 351^2\} \\
 (364, 240, 436) &\Rightarrow \{14, 481, 871, 9464, 132496 = 364^2\} \\
 (375, 200, 425) &\Rightarrow \{15, 401, 849, 9375, 140625 = 375^2\} \\
 (384, 160, 416) &\Rightarrow \{16, 321, 831, 9216, 147456 = 384^2\} \\
 (391, 120, 409) &\Rightarrow \{17, 241, 817, 8993, 152881 = 391^2\} \\
 (396, 80, 404) &\Rightarrow \{18, 161, 807, 8712, 156816 = 396^2\} \\
 (399, 40, 401) &\Rightarrow \{19, 81, 801, 8379, 159201 = 399^2\} \\
 (800, 600, 1000) &\Rightarrow \{20, 1201, 1999, 32000, 640000 = 800^2\} \\
 (819, 540, 981) &\Rightarrow \{21, 1081, 1961, 31941, 670761 = 819^2\} \\
 (836, 480, 964) &\Rightarrow \{22, 961, 1927, 31768, 698896 = 836^2\} \\
 (851, 420, 949) &\Rightarrow \{23, 841, 1897, 31487, 724201 = 851^2\} \\
 (864, 360, 936) &\Rightarrow \{24, 721, 1871, 31104, 746496 = 864^2\} \\
 (875, 300, 925) &\Rightarrow \{25, 601, 1849, 30625, 765625 = 875^2\} \\
 (884, 240, 916) &\Rightarrow \{26, 481, 1831, 30056, 781456 = 884^2\} \\
 (891, 180, 909) &\Rightarrow \{27, 361, 1817, 29403, 793881 = 891^2\} \\
 (896, 120, 904) &\Rightarrow \{28, 241, 1807, 28672, 802816 = 896^2\} \\
 (899, 60, 901) &\Rightarrow \{29, 121, 1801, 27869, 808201 = 899^2\} \\
 (1500, 800, 1700) &\Rightarrow \{30, 1601, 3399, 75000, 2250000 = 1500^2\} \\
 (1519, 720, 1681) &\Rightarrow \{31, 1441, 3361, 74431, 2307361 = 1519^2\} \\
 (1536, 640, 1664) &\Rightarrow \{32, 1281, 3327, 73728, 2359296 = 1536^2\} \\
 (1551, 560, 1649) &\Rightarrow \{33, 1121, 3297, 72897, 2405601 = 1551^2\}
 \end{aligned}$$

$$\begin{aligned}
(1564, 480, 1636) &\Rightarrow \{34, 961, 3271, 71944, 2446096 = 1564^2\} \\
(1575, 400, 1625) &\Rightarrow \{35, 801, 3249, 70875, 2480625 = 1575^2\} \\
(1584, 320, 1616) &\Rightarrow \{36, 641, 3231, 69696, 2509056 = 1584^2\} \\
(1591, 240, 1609) &\Rightarrow \{37, 481, 3217, 68413, 2531281 = 1591^2\} \\
(1596, 160, 1604) &\Rightarrow \{38, 321, 3207, 67032, 2547216 = 1596^2\} \\
(1599, 80, 1601) &\Rightarrow \{39, 161, 3201, 65559, 2556801 = 1599^2\} \\
(2400, 1000, 2600) &\Rightarrow \{40, 2001, 5199, 144000, 5760000 = 2400^2\} \\
(2419, 900, 2581) &\Rightarrow \{41, 1801, 5161, 142721, 5851561 = 2419^2\} \\
(2436, 800, 2564) &\Rightarrow \{42, 1601, 5127, 141288, 5934096 = 2436^2\} \\
(2451, 700, 2549) &\Rightarrow \{43, 1401, 5097, 139707, 6007401 = 2451^2\} \\
(2464, 600, 2536) &\Rightarrow \{44, 1201, 5071, 137984, 6071296 = 2464^2\} \\
(2475, 500, 2525) &\Rightarrow \{45, 1001, 5049, 136125, 6125625 = 2475^2\} \\
(2484, 400, 2516) &\Rightarrow \{46, 801, 5031, 134136, 6170256 = 2484^2\} \\
(2491, 300, 2509) &\Rightarrow \{47, 601, 5017, 132023, 6205081 = 2491^2\} \\
(2496, 200, 2504) &\Rightarrow \{48, 401, 5007, 129792, 6230016 = 2496^2\} \\
(2499, 100, 2501) &\Rightarrow \{49, 201, 5001, 127449, 6245001 = 2499^2\} \\
(3500, 1200, 3700) &\Rightarrow \{50, 2401, 7399, 245000, 12250000 = 3500^2\} \\
(3519, 1080, 3681) &\Rightarrow \{51, 2161, 7361, 242811, 12383361 = 3519^2\} \\
(3536, 960, 3664) &\Rightarrow \{52, 1921, 7327, 240448, 12503296 = 3536^2\} \\
(3551, 840, 3649) &\Rightarrow \{53, 1681, 7297, 237917, 12609601 = 3551^2\} \\
(3564, 720, 3636) &\Rightarrow \{54, 1441, 7271, 235224, 12702096 = 3564^2\} \\
(3575, 600, 3625) &\Rightarrow \{55, 1201, 7249, 232375, 12780625 = 3575^2\} \\
(3584, 480, 3616) &\Rightarrow \{56, 961, 7231, 229376, 12845056 = 3584^2\} \\
(3591, 360, 3609) &\Rightarrow \{57, 721, 7217, 226233, 12895281 = 3591^2\} \\
(3596, 240, 3604) &\Rightarrow \{58, 481, 7207, 222952, 12931216 = 3596^2\} \\
(3599, 120, 3601) &\Rightarrow \{59, 241, 7201, 219539, 12952801 = 3599^2\} \\
(4800, 1400, 5000) &\Rightarrow \{60, 2801, 9999, 384000, 23040000 = 4800^2\} \\
(4819, 1260, 4981) &\Rightarrow \{61, 2521, 9961, 380701, 23222761 = 4819^2\} \\
(4836, 1120, 4964) &\Rightarrow \{62, 2241, 9927, 377208, 23386896 = 4836^2\} \\
(4851, 980, 4949) &\Rightarrow \{63, 1961, 9897, 373527, 23532201 = 4851^2\} \\
(4864, 840, 4936) &\Rightarrow \{64, 1681, 9871, 369664, 23658496 = 4864^2\} \\
(4875, 700, 4925) &\Rightarrow \{65, 1401, 9849, 365625, 23765625 = 4875^2\} \\
(4884, 560, 4916) &\Rightarrow \{66, 1121, 9831, 361416, 23853456 = 4884^2\} \\
(4891, 420, 4909) &\Rightarrow \{67, 841, 9817, 357043, 23921881 = 4891^2\} \\
(4896, 280, 4904) &\Rightarrow \{68, 561, 9807, 352512, 23970816 = 4896^2\} \\
(4899, 140, 4901) &\Rightarrow \{69, 281, 9801, 347829, 24000201 = 4899^2\} \\
(6300, 1600, 6500) &\Rightarrow \{70, 3201, 12999, 567000, 39690000 = 6300^2\}
\end{aligned}$$

$$\begin{aligned}
(6319, 1440, 6481) &\Rightarrow \{71, 2881, 12961, 562391, 39929761 = 6319^2\} \\
(6336, 1280, 6464) &\Rightarrow \{72, 2561, 12927, 557568, 40144896 = 6336^2\} \\
(6351, 1120, 6449) &\Rightarrow \{73, 2241, 12897, 552537, 40335201 = 6351^2\} \\
(6364, 960, 6436) &\Rightarrow \{74, 1921, 12871, 547304, 40500496 = 6364^2\} \\
(6375, 800, 6425) &\Rightarrow \{75, 1601, 12849, 541875, 40640625 = 6375^2\} \\
(6384, 640, 6416) &\Rightarrow \{76, 1281, 12831, 536256, 40755456 = 6384^2\} \\
(6391, 480, 6409) &\Rightarrow \{77, 961, 12817, 530453, 40844881 = 6391^2\} \\
(6396, 320, 6404) &\Rightarrow \{78, 641, 12807, 524472, 40908816 = 6396^2\} \\
(6399, 160, 6401) &\Rightarrow \{79, 321, 12801, 518319, 40947201 = 6399^2\} \\
(8000, 1800, 8200) &\Rightarrow \{80, 3601, 16399, 800000, 64000000 = 8000^2\} \\
(8019, 1620, 8181) &\Rightarrow \{81, 3241, 16361, 793881, 64304361 = 8019^2\} \\
(8036, 1440, 8164) &\Rightarrow \{82, 2881, 16327, 787528, 64577296 = 8036^2\} \\
(8051, 1260, 8149) &\Rightarrow \{83, 2521, 16297, 780947, 64818601 = 8051^2\} \\
(8064, 1080, 8136) &\Rightarrow \{84, 2161, 16271, 774144, 65028096 = 8064^2\} \\
(8075, 900, 8125) &\Rightarrow \{85, 1801, 16249, 767125, 65205625 = 8075^2\} \\
(8084, 720, 8116) &\Rightarrow \{86, 1441, 16231, 759896, 65351056 = 8084^2\} \\
(8091, 540, 8109) &\Rightarrow \{87, 1081, 16217, 752463, 65464281 = 8091^2\} \\
(8096, 360, 8104) &\Rightarrow \{88, 721, 16207, 744832, 65545216 = 8096^2\} \\
(8099, 180, 8101) &\Rightarrow \{89, 361, 16201, 737009, 65593801 = 8099^2\} \\
(9900, 2000, 10100) &\Rightarrow \{90, 4001, 20199, 1089000, 98010000 = 9900^2\} \\
(9919, 1800, 10081) &\Rightarrow \{91, 3601, 20161, 1081171, 98386561 = 9919^2\} \\
(9936, 1600, 10064) &\Rightarrow \{92, 3201, 20127, 1073088, 98724096 = 9936^2\} \\
(9951, 1400, 10049) &\Rightarrow \{93, 2801, 20097, 1064757, 99022401 = 9951^2\} \\
(9964, 1200, 10036) &\Rightarrow \{94, 2401, 20071, 1056184, 99281296 = 9964^2\} \\
(9975, 1000, 10025) &\Rightarrow \{95, 2001, 20049, 1047375, 99500625 = 9975^2\} \\
(9984, 800, 10016) &\Rightarrow \{96, 1601, 20031, 1038336, 99680256 = 9984^2\} \\
(9991, 600, 10009) &\Rightarrow \{97, 1201, 20017, 1029073, 99820081 = 9991^2\} \\
(9996, 400, 10004) &\Rightarrow \{98, 801, 20007, 1019592, 99920016 = 9996^2\} \\
(9999, 200, 10001) &\Rightarrow \{99, 401, 20001, 1009899, 99980001 = 9999^2\} \tag{4.4}
\end{aligned}$$

The the first 97 lines of (4.4) give magic square of orders 3 to 99 of consecutive odd numbers with final sum a perfect square. The subsection below give examples of magic squares of order 3 to 20 constructed based on above distribution given in (4.4).

4.1 Magic Squares of Orders 3 to 20

This subsection brings magic squares of orders 3 to 20 constructed according to values given in (4.4).

► Magic Square of Order 3

According to distribution

$$(51, 140, 149) \Rightarrow \{3, 281, 297, 867, 2601 = 51^2\},$$

given in 3rd line of (4.4), we can construct a magic square of order 3 of consecutive odd numbers, (281, 283, ..., 295, 297). See below:

			867
291	281	295	867
293	289	285	867
283	297	287	867
867	867	867	867

The above magic square is of magic sum $S_{3 \times 3} = 867$, and total number of entries is a perfect square sum, i.e., $T_9 := 2601 = 51^2 = 149^2 - 140^2$.

► Magic Square of Order 4

According to distribution

$$(64, 120, 136) \Rightarrow \{4, 241, 271, 1024, 4096 = 64^2\},$$

given in 4th line of (4.4), we can construct a **pandiagonal** magic square of order 4 using consecutive odd numbers, (241, 243, ..., 269, 271). See below:

		1024	1024	1024	1024
	253	263	241	267	1024
1024	243	265	255	261	1024
1024	271	245	259	249	11024
1024	257	251	269	247	1024
	1024	1024	1024	1024	1024

The above magic square is of magic sum $S_{4 \times 4} = 1024$, and total number of entries is a perfect square sum, i.e., $T_{16} := 4096 = 64^2 = 136^2 - 120^2$.

► Magic Square of Order 5

According to distribution

$$(75, 100, 125) \Rightarrow \{5, 201, 249, 1125, 5625 = 75^2\},$$

given in 5th line of (4.4), we can construct a **pandiagonal** magic square of order 5 using consecutive odd numbers, (201, 203, ..., 247, 249). See below:

		1125	1125	1125	1125	1125
	201	213	225	237	249	1125
1125	235	247	209	211	223	1125
1125	219	221	233	245	207	1125
1125	243	205	217	229	231	1125
1125	227	239	241	203	215	1125
	1125	1125	1125	1125	1125	1125

The above magic square is of magic sum $S_{5 \times 5} = 1125$, and total number of entries is a perfect square sum, i.e., $T_{25} := 5625 = 75^2 = 125^2 - 100^2$.

► Magic Square of Order 6

According to distribution

$$(84, 80, 116) \Rightarrow \{6, 161, 231, 1176, 7056 = 84^2\},$$

given in 6th line of (4.4), we can construct a magic square of order 6 using consecutive odd numbers, (161, 263, ..., 229, 231). See below:

						1176
161	205	215	227	193	175	1176
217	173	229	187	201	169	1176
183	171	185	213	221	203	1176
223	191	167	207	179	209	1176
197	225	181	165	219	189	1176
195	211	199	177	163	231	1176
1176	1176	1176	1176	1176	1176	1176

The above magic square is of magic sum $S_{6 \times 6} = 1176$, and total number of entries is a perfect square sum, i.e., $T_{36} := 7056 = 84^2 = 116^2 - 80^2$.

► Magic Square of Order 7

According to distribution

$$(91, 60, 109) \Rightarrow \{7, 121, 217, 1183, 8281 = 91^2\},$$

given in 7th line of (4.4), a **pandiagonal** square of order 7 using consecutive odd numbers (121, 123, ..., 215, 217) is given by

		1183	1183	1183	1183	1183	1183	1183	1183
	121	137	153	169	185	201	217	1183	
1183	199	215	133	135	151	167	183	1183	
1183	165	181	197	213	131	147	149	1183	
1183	145	161	163	179	195	211	129	1183	
1183	209	127	143	159	1183	177	193	1183	
1183	189	191	207	125	141	157	173	1183	
1183	155	171	187	203	205	123	139	1183	
	1183	1183	1183	1183	1183	1183	1183	1183	1183

The above magic square is of magic sum $S_{7 \times 7} = 1183$, and total number of entries is a perfect square sum, i.e., $T_{49} := 8281 = 91^2 = 109^2 - 60^2$.

► Magic Square of Order 8

According to distribution

$$(96, 40, 104) \Rightarrow \{8, 81, 207, 1152, 9216 = 96^2\},$$

given in 8th line of (4.4), a **pandiagonal** square of order 8 using consecutive odd numbers (81, 83, ..., 205, 207) is given by

		1152	1152	1152	1152	1152	1152	1152	1152
	81	201	135	159	97	185	119	175	1152
1152	143	151	89	193	127	167	105	177	1152
1152	153	129	207	87	169	113	191	103	1152
1152	199	95	145	137	183	111	161	121	1152
1152	83	203	133	157	99	187	117	173	1152
1152	141	149	91	195	125	165	107	179	1152
1152	155	131	205	85	171	115	189	101	1152
1152	197	93	147	139	181	109	163	123	1152
	1152	1152	1152	1152	1152	1152	1152	1152	1152

The above magic square is of magic sum $S_{8 \times 8} = 1152$, and total number of entries is a perfect square sum, i.e., $T_{64} := 9216 = 96^2 = 104^2 - 40^2$.

► Magic Square of Order 9

According to distribution

$$(99, 20, 101) \Rightarrow \{9, 41, 201, 1089, 9801 = 99^2\},$$

given in 9th line of (4.4), a **pandiagonal** square of order 8 using consecutive odd numbers (41, 43, ..., 199, 201) is given by

		1089	1089	1089	1089	1089	1089	1089	1089	1089
	41	65	89	99	23	147	151	175	199	1089
1089	97	121	145	149	173	197	45	69	93	1089
1089	153	177	201	43	67	91	95	119	143	1089
1089	83	53	59	141	111	117	193	163	169	1089
1089	139	109	115	191	161	167	87	57	63	1089
1089	195	165	171	85	55	61	137	107	113	1089
1089	71	77	47	129	135	105	181	187	157	1089
1089	127	133	103	179	185	155	75	81	51	1089
1089	183	189	159	73	79	49	125	131	101	1089
	1089	1089	1089	1089	1089	1089	1089	1089	1089	1089

The above magic square is of magic sum $S_{9 \times 9} = 1089$, and total number of entries is a perfect square sum, i.e., $T_{81} := 9801 = 99^2 = 101^2 - 20^2$.

► **Magic Square of Order 10**

According to distribution

$$(300, 400, 500) \Rightarrow \{10, 801, 999, 9000, 90000 = 300^2\},$$

given in (4.4), we can construct a magic square of order 10 of consecutive odd numbers, 801, 803, ..., 997, 999. See below:

										9000
801	959	929	993	877	843	895	971	905	827	9000
995	823	817	931	979	947	909	865	881	853	9000
893	961	845	957	831	869	987	919	923	815	9000
939	913	975	867	803	981	857	829	951	885	9000
967	997	903	821	889	935	945	813	859	871	9000
825	875	887	819	953	911	963	841	989	937	9000
949	891	879	965	855	837	933	983	807	901	9000
917	847	991	883	921	805	839	955	873	969	9000
851	809	833	915	985	899	861	927	977	943	9000
863	925	941	849	907	973	811	897	835	999	9000
9000	9000	9000	9000	9000	9000	9000	9000	9000	9000	9000

Above magic square if with magic sum, $S_{10 \times 10} := 9000$ with sum of entries a perfect square sum, i.e., $T_{100} := 90000 = 300^2 = 500^2 - 300^2$.

► Magic Square of Order 11

According to distribution

$$(319, 360, 481) \Rightarrow \{11, 721, 961, 9251, 101761 = 319^2\},$$

given in (4.4), we can construct a **pandiagonal** magic square of order 11 of consecutive odd numbers, 721, 723, ..., 959, 961. See below:

		9251	9251	9251	9251	9251	9251	9251	9251	9251	9251	9251
	721	761	779	797	815	833	873	891	909	927	945	9251
9251	921	961	737	755	773	791	809	849	867	885	903	9251
9251	879	897	937	955	731	749	767	807	825	843	861	9251
9251	837	855	895	913	931	949	725	743	783	801	819	9251
9251	795	813	831	871	889	907	925	943	741	759	777	9251
9251	753	771	789	829	847	865	883	901	919	959	735	9251
9251	953	729	747	765	805	823	841	859	877	917	935	9251
9251	911	929	947	723	763	781	799	817	835	853	893	9251
9251	869	887	905	923	941	739	757	775	793	811	851	9251
9251	827	845	863	881	899	939	957	733	751	769	787	9251
9251	785	803	821	839	857	875	915	933	951	727	745	9251
	9251	9251	9251	9251	9251	9251	9251	9251	9251	9251	9251	9251

Above magic square if with magic sum, $S_{11 \times 11} := 9251$ with sum of entries a perfect square sum, i.e., $T_{121} := 101761 = 319^2 = 481^2 - 360^2$.

► Magic Square of Order 12

According to distribution

$$(336, 320, 464) \Rightarrow \{12, 641, 927, 9408, 112896 = 336^2\},$$

given in (4.4), we can construct a **pandiagonal** magic square of order 12 of consecutive odd numbers, 641, 643, ..., 925, 927. See below:

		9408	9408	9408	9408	9408	9408	9408	9408	9408	9408	9408	9408
	749	855	641	891	751	853	643	889	753	851	645	887	9408
9408	675	857	783	821	673	859	781	823	671	861	779	825	9408
9408	927	677	819	713	925	679	817	715	923	681	815	717	9408
9408	785	747	893	711	787	745	895	709	789	743	897	707	9408
9408	755	849	647	885	757	847	649	883	759	845	651	881	9408
9408	669	863	777	827	667	865	775	829	665	867	773	831	9408
9408	921	683	813	719	919	685	811	721	917	687	809	723	9408
9408	791	741	899	705	793	739	901	703	795	737	903	701	9408
9408	761	843	653	879	763	841	655	877	765	839	657	875	9408
9408	663	869	771	833	661	871	769	835	659	873	767	837	9408
9408	915	689	807	725	913	691	805	727	911	693	803	729	9408
9408	797	735	905	699	799	733	907	697	801	731	909	695	9408
	9408	9408	9408	9408	9408	9408	9408	9408	9408	9408	9408	9408	9408

Above magic square if with magic sum, $S_{12 \times 12} := 9408$ with sum of entries a perfect square sum, i.e., $T_{144} := 112896 = 336^2 = 464^2 - 320^2$. Moreover, each block of order 4 is magic square with magic sum, $S_{4 \times 4} := 3136$.

► Magic Square of Order 13

According to distribution

$$(351, 280, 449) \Rightarrow \{13, 561, 897, 9477, 123201 = 351^2\},$$

given in (4.4), we can construct a magic square of order 13 of consecutive odd numbers, 561, 563, ..., 895, 897. See below:

		9477	9477	9477	9477	9477	9477	9477	9477	9477	9477	9477	9477	9477
	561	609	631	653	675	697	719	767	789	811	833	855	877	9477
9477	849	897	581	603	625	647	669	691	739	761	783	805	827	9477
9477	799	821	869	891	575	597	619	641	689	711	733	755	777	9477
9477	749	771	819	841	863	885	569	591	613	661	683	705	727	9477
9477	699	721	743	791	813	835	857	879	563	611	633	655	677	9477
9477	649	671	693	741	763	785	807	829	851	873	583	605	627	9477
9477	599	621	643	665	713	735	757	779	801	823	871	893	577	9477
9477	887	571	593	615	663	685	707	729	751	773	795	843	865	9477
9477	837	859	881	565	587	635	657	679	701	723	745	793	815	9477
9477	787	809	831	853	875	585	607	629	651	673	695	717	765	9477
9477	737	759	781	803	825	847	895	579	601	623	645	667	715	9477
9477	687	709	731	753	775	797	845	867	889	573	595	617	639	9477
9477	637	659	681	703	725	747	769	817	839	861	883	567	589	9477
	9477	9477	9477	9477	9477	9477	9477	9477	9477	9477	9477	9477	9477	9477

Above magic square if with magic sum, $S_{13 \times 13} := 9477$ with sum of entries a perfect square sum, i.e., $T_{169} := 123201 = 351^2 = 449^2 - 280^2$.

► **Magic Square of Order 14**

According to distribution

$$(364, 240, 436) \Rightarrow \{14, 481, 871, 9464, 132496 = 364^2\},$$

given in (4.4), we can construct a magic square of order 14 of consecutive odd numbers, 481, 483, ..., 869, 871. See below:

														9464
481	669	861	703	755	635	595	543	513	797	771	835	729	577	9464
773	511	699	831	855	733	653	713	487	585	619	645	555	805	9464
731	803	541	753	823	605	639	491	573	531	673	677	777	847	9464
859	701	779	571	789	709	489	633	547	819	525	671	759	613	9464
807	647	589	503	721	651	537	681	745	603	825	529	851	775	9464
687	499	657	553	521	811	837	787	869	627	567	747	597	705	9464
517	549	743	593	485	785	871	809	839	663	723	581	623	683	9464
569	609	621	659	551	865	783	841	815	717	739	483	685	527	9464
539	565	599	625	667	843	813	867	781	693	493	715	523	741	9464
615	767	791	533	629	575	689	665	719	751	845	563	501	821	9464
641	853	829	707	617	515	749	583	649	507	691	765	799	559	9464
757	827	495	801	763	695	711	509	637	849	557	601	587	675	9464
833	737	725	863	591	545	519	607	679	761	643	795	661	505	9464
655	727	535	769	697	497	579	735	611	561	793	857	817	631	9464
9464	9464	9464	9464	9464	9464	9464	9464	9464	9464	9464	9464	9464	9464	9464

Above magic square if with magic sum, $S_{14 \times 14} := 9464$ with sum of entries a perfect square sum, i.e., $T_{196} := 132496 = 364^2 = 436^2 - 240^2$.

► Magic Square of Order 15

According to distribution

$$(375, 200, 425) \Rightarrow \{15, 401, 849, 9375, 140625 = 375^2\},$$

given in (4.4), we can construct a **pandiagonal** magic square of order 15 of consecutive odd numbers, 401, 403, ..., 847, 849. See below:

		9375	9375	9375	9375	9375	9375	9375	9375	9375	9375	9375	9375	9375	9375	9375
	401	573	621	755	775	405	571	619	757	773	403	569	617	759	777	9375
9375	741	785	415	551	633	739	787	413	555	631	737	789	417	553	629	9375
9375	565	611	753	771	425	563	615	751	769	427	567	613	749	767	429	9375
9375	783	411	575	625	731	781	409	577	623	735	779	407	579	627	733	9375
9375	635	745	761	423	561	637	743	765	421	559	639	747	763	419	557	9375
9375	461	543	591	725	805	465	541	589	727	803	463	539	587	729	807	9375
9375	711	815	475	521	603	709	817	473	525	601	707	819	477	523	599	9375
9375	535	581	723	801	485	533	585	721	799	487	537	583	719	797	489	9375
9375	813	471	545	595	701	811	469	547	593	705	809	467	549	597	703	9375
9375	605	715	791	483	531	607	713	795	481	529	609	717	793	479	527	9375
9375	431	513	651	695	835	435	511	649	697	833	433	509	647	699	837	9375
9375	681	845	445	491	663	679	847	443	495	661	677	849	447	493	659	9375
9375	505	641	693	831	455	503	645	691	829	457	507	643	689	827	459	9375
9375	843	441	515	655	671	841	439	517	653	675	839	437	519	657	673	9375
9375	665	685	821	453	501	667	683	825	451	499	669	687	823	449	497	9375
	9375	9375	9375	9375	9375	9375	9375	9375	9375	9375	9375	9375	9375	9375	9375	9375

Above magic square if with magic sum, $S_{15 \times 15} := 9375$ with sum of entries a perfect square sum, i.e., $T_{225} := 140625 = 375^2 = 425^2 - 375^2$. Moreover, each block of order 5 is a **pandiagonal** magic square with magic sum, $S_{5 \times 5} := 3125$.

► **Magic Square of Order 16**

According to distribution

$$(384, 160, 416) \Rightarrow \{16, 321, 831, 9216, 147456 = 384^2\},$$

given in (4.4), we can construct a magic square of order 16 of consecutive odd numbers, 321, 323, ..., 829, 831. See below:

● **First Approach: Pandiagonal Magic Square of Order 16**

		9216	9216	9216	9216	9216	9216	9216	9216	9216	9216	9216	9216	9216	9216	9216	9216
	533	635	357	779	503	665	391	745	561	607	321	815	467	701	419	717	9216
9216	363	773	539	629	393	743	505	663	335	801	575	593	429	707	477	691	9216
9216	795	373	619	517	761	407	649	487	831	337	591	545	733	435	685	451	9216
9216	613	523	789	379	647	489	759	409	577	559	817	351	675	461	723	445	9216
9216	563	605	323	813	465	703	417	719	535	633	359	777	501	667	389	747	9216
9216	333	803	573	595	431	705	479	689	361	775	537	631	395	741	507	661	9216
9216	829	339	589	547	735	433	687	449	793	375	617	519	763	405	651	485	9216
9216	579	557	819	349	673	463	721	447	615	521	791	377	645	491	757	411	9216
9216	471	697	423	713	565	603	325	811	499	669	387	749	529	639	353	783	9216
9216	425	711	473	695	331	805	571	597	397	739	509	659	367	769	543	625	9216
9216	729	439	681	455	827	341	587	549	765	403	653	483	799	369	623	513	9216
9216	679	457	727	441	581	555	821	347	643	493	755	413	609	527	785	383	9216
9216	497	671	385	751	531	637	355	781	469	699	421	715	567	601	327	809	9216
9216	399	737	511	657	365	771	541	627	427	709	475	693	329	807	569	599	9216
9216	767	401	655	481	797	371	621	515	731	437	683	453	825	343	585	551	9216
9216	641	495	753	415	611	525	787	381	677	459	725	443	583	553	823	345	9216
	9216	9216	9216	9216	9216	9216	9216	9216	9216	9216	9216	9216	9216	9216	9216	9216	9216

Above magic square is **pandiagonal** with magic sum, $S_{16 \times 16} := 9216$ with sum of entries a perfect square sum, i.e., $T_{256} := 147456 = 384^2 = 416^2 - 384^2$. All the 4×4 blocks are **pandiagonal** magic square of order 4 with equal magic sums, $S_{4 \times 4} = 2304$.

• **Second Approach: Bimagic Square of Order 16**

																	9216
321	627	797	559	365	607	817	515	407	677	715	505	443	649	743	469		9216
783	573	339	609	803	529	383	589	729	491	389	695	757	455	425	667		9216
563	769	623	349	543	813	579	369	485	727	697	395	457	763	661	423		9216
637	335	545	787	593	355	525	831	683	409	503	709	647	437	475	745		9216
411	681	711	501	439	645	747	473	333	639	785	547	353	595	829	527		9216
725	487	393	699	761	459	421	663	771	561	351	621	815	541	371	577		9216
489	731	693	391	453	759	665	427	575	781	611	337	531	801	591	381		9216
679	405	507	713	651	441	471	741	625	323	557	799	605	367	513	819		9216
429	671	753	451	385	691	733	495	379	585	807	533	343	613	779	569		9216
739	465	447	653	719	509	403	673	821	519	361	603	793	555	325	631		9216
479	749	643	433	499	705	687	413	521	827	597	359	549	791	633	331		9216
657	419	461	767	701	399	481	723	583	373	539	809	619	345	567	773		9216
375	581	811	537	347	617	775	565	417	659	765	463	397	703	721	483		9216
825	523	357	599	789	551	329	635	751	477	435	641	707	497	415	685		9216
517	823	601	363	553	795	629	327	467	737	655	445	511	717	675	401		9216
587	377	535	805	615	341	571	777	669	431	449	755	689	387	493	735		9216
9216	9216	9216	9216	9216	9216	9216	9216	9216	9216	9216	9216	9216	9216	9216	9216	9216	9216

Above magic square is with **bimagic** with magic sums, $S_{16 \times 16} := 9216$ and $Sb_{16 \times 16} := 5657936$. The sum of entries a perfect square sum, i.e., $T_{256} := 147456 = 384^2 = 416^2 - 384^2$. All the 4×4 blocks are equal of sums entries, i.e., $T_{16} = 9216$.

► **Magic Square of Order 17**

According to distribution

$$(391, 120, 409) \Rightarrow \{17, 241, 817, 8993, 152881 = 391^2\},$$

given in (4.4), we can construct a **pandiagonal** magic square of order 17 of consecutive odd numbers, 241, 243, ..., 815, 817. See below:

		8993	8993	8993	8993	8993	8993	8993	8993	8993	8993	8993	8993	8993	8993	8993	8993	8993	8993
	241	305	335	365	395	425	455	485	515	579	609	639	669	699	729	759	789	8993	
8993	753	817	269	299	329	359	389	419	449	479	543	573	603	633	663	693	723	8993	
8993	687	717	781	811	263	293	323	353	383	413	477	507	537	567	597	627	657	8993	
8993	621	651	715	745	775	805	257	287	317	347	377	441	471	501	531	561	591	8993	
8993	555	585	615	679	709	739	769	799	251	281	311	375	405	435	465	495	525	8993	
8993	489	519	549	613	643	673	703	733	763	793	245	275	339	369	399	429	459	8993	
8993	423	453	483	513	577	607	637	667	697	727	757	787	273	303	333	363	393	8993	
8993	357	387	417	447	511	541	571	601	631	661	691	721	751	815	267	297	327	8993	
8993	291	321	351	381	411	475	505	535	565	595	625	655	685	749	779	809	261	8993	
8993	803	255	285	315	345	409	439	469	499	529	559	589	619	649	713	743	773	8993	
8993	737	767	797	249	279	309	373	403	433	463	493	523	553	583	647	677	707	8993	
8993	671	701	731	761	791	243	307	337	367	397	427	457	487	517	547	611	641	8993	
8993	605	635	665	695	725	755	785	271	301	331	361	391	421	451	481	545	575	8993	
8993	539	569	599	629	659	689	719	783	813	265	295	325	355	385	415	445	509	8993	
8993	473	503	533	563	593	623	653	683	747	777	807	259	289	319	349	379	443	8993	
8993	407	437	467	497	527	557	587	617	681	711	741	771	801	253	283	313	343	8993	
8993	341	371	401	431	461	491	521	551	581	645	675	705	735	765	795	247	277	8993	
	8993	8993	8993	8993	8993	8993	8993	8993	8993	8993	8993	8993	8993	8993	8993	8993	8993	8993	

Above magic square if with magic sum, $S_{17 \times 17} := 8993$ with sum of entries a perfect square sum, i.e., $T_{289} := 152881 = 391^2 = 409^2 - 120^2$.

► **Magic Square of Order 18**

According to distribution

$$(396, 80, 404) \Rightarrow \{18, 161, 807, 8712, 156816 = 396^2\},$$

given in (4.4), we can construct a magic square of order 18 of consecutive odd numbers, 161, 163, ..., 805, 807. See below:

																		8712
161	773	771	737	195	267	163	775	769	739	193	265	165	777	767	741	191	263	8712
699	303	663	305	341	593	697	301	661	307	343	595	695	299	659	309	345	597	8712
591	557	413	447	519	377	589	559	415	445	517	379	587	561	417	443	515	381	8712
483	411	521	555	449	485	481	409	523	553	451	487	479	407	525	551	453	489	8712
269	627	339	629	665	375	271	625	337	631	667	373	273	623	335	633	669	371	8712
701	233	197	231	735	807	703	235	199	229	733	805	705	237	201	227	731	803	8712
167	779	765	743	189	261	169	781	763	745	187	259	171	783	761	747	185	257	8712
693	297	657	311	347	599	691	295	655	313	349	601	689	293	653	315	351	603	8712
585	563	419	441	513	383	583	565	421	439	511	385	581	567	423	437	509	387	8712
477	405	527	549	455	491	475	403	529	547	457	493	473	401	531	545	459	495	8712
275	621	333	635	671	369	277	619	331	637	673	367	279	617	329	639	675	365	8712
707	239	203	225	729	801	709	241	205	223	727	799	711	243	207	221	725	797	8712
173	785	759	749	183	255	175	787	757	751	181	253	177	789	755	753	179	251	8712
687	291	651	317	353	605	685	289	649	319	355	607	683	287	647	321	357	609	8712
579	569	425	435	507	389	577	571	427	433	505	391	575	573	429	431	503	393	8712
471	399	533	543	461	497	469	397	535	541	463	499	467	395	537	539	465	501	8712
281	615	327	641	677	363	283	613	325	643	679	361	285	611	323	645	681	359	8712
713	245	209	219	723	795	715	247	211	217	721	793	717	249	213	215	719	791	8712
8712	8712	8712	8712	8712	8712	8712	8712	8712	8712	8712	8712	8712	8712	8712	8712	8712	8712	8712

Above magic square if with magic sum, $S_{18 \times 18} := 8712$ with sum of entries a perfect square sum, i.e., $T_{324} := 156816 = 396^2 = 404^2 - 80^2$. Moreover each block of order 6 is a magic square with magic sum, $S_{6 \times 6} := 2904$.

► Magic Square of Order 19

According to distribution

$$(399, 40, 401) \Rightarrow \{19, 81, 801, 8379, 159201 = 399^2\},$$

given in (4.4), we can construct a **pandiagonal** magic square of order 19 of consecutive odd numbers, 81, 83, ..., 799, 801. See below:

		8379	8379	8379	8379	8379	8379	8379	8379	8379	8379	8379	8379	8379	8379	8379	8379	8379	8379	8379
	81	153	187	221	255	289	323	357	391	425	497	531	565	599	633	667	701	735	769	8379
8379	729	801	113	147	181	215	249	283	317	351	385	457	491	525	559	593	627	661	695	8379
8379	655	689	761	795	107	141	175	209	243	277	311	383	417	451	485	519	553	587	621	8379
8379	581	615	687	721	755	789	101	135	169	203	237	271	343	377	411	445	479	513	547	8379
8379	507	541	575	647	681	715	749	783	95	129	163	197	269	303	337	371	405	439	473	8379
8379	433	467	501	573	607	641	675	709	743	777	89	123	157	229	263	297	331	365	399	8379
8379	359	393	427	461	533	567	601	635	669	703	737	771	83	155	189	223	257	291	325	8379
8379	285	319	353	387	459	493	527	561	595	629	663	697	731	765	115	149	183	217	251	8379
8379	211	245	279	313	347	419	453	487	521	555	589	623	657	691	763	797	109	143	177	8379
8379	137	171	205	239	273	345	379	413	447	481	515	549	583	617	651	723	757	791	103	8379
8379	785	97	131	165	199	233	305	339	373	407	441	475	509	543	577	649	683	717	751	8379
8379	711	745	779	91	125	159	231	265	299	333	367	401	435	469	503	537	609	643	677	8379
8379	637	671	705	739	773	85	119	191	225	259	293	327	361	395	429	463	535	569	603	8379
8379	563	597	631	665	699	733	767	117	151	185	219	253	287	321	355	389	423	495	529	8379
8379	489	523	557	591	625	659	693	727	799	111	145	179	213	247	281	315	349	421	455	8379
8379	415	449	483	517	551	585	619	653	725	759	793	105	139	173	207	241	275	309	381	8379
8379	341	375	409	443	477	511	545	579	613	685	719	753	787	99	133	167	201	235	307	8379
8379	267	301	335	369	403	437	471	505	539	611	645	679	713	747	781	93	127	161	195	8379
8379	193	227	261	295	329	363	397	431	465	499	571	605	639	673	707	741	775	87	121	8379
	8379	8379	8379	8379	8379	8379	8379	8379	8379	8379	8379	8379	8379	8379	8379	8379	8379	8379	8379	8379

Above magic square if with magic sum, $S_{19 \times 19} := 8379$ with sum of entries a perfect square sum, i.e., $T_{361} := 159201 = 399^2 = 401^2 - 40^2$.

► **Magic Square of Order 20**

According to distribution

$$(800, 600, 1000) \Rightarrow \{20, 1201, 1999, 32000, 640000 = 800^2\},$$

given in (4.4), we can construct a magic square of order 20 of consecutive odd numbers, 1201, 1203, ..., 1997, 1999. See below:

		32000	32000	32000	32000	32000	32000	32000	32000	32000	32000	32000	32000	32000	32000	32000	32000	32000	32000	32000	32000
	1579	1601	1239	1981	1533	1647	1273	1947	1497	1683	1317	1903	1451	1729	1351	1869	1415	1765	1395	1825	32000
32000	1221	1999	1561	1619	1267	1953	1527	1653	1303	1917	1483	1697	1349	1871	1449	1731	1385	1835	1405	1775	32000
32000	1961	1219	1621	1599	1927	1253	1667	1553	1883	1297	1703	1517	1849	1331	1749	1471	1805	1375	1785	1435	32000
32000	1639	1581	1979	1201	1673	1547	1933	1247	1717	1503	1897	1283	1751	1469	1851	1329	1795	1425	1815	1365	32000
32000	1457	1723	1357	1863	1411	1769	1391	1829	1575	1605	1235	1985	1539	1641	1279	1941	1493	1687	1313	1907	32000
32000	1343	1877	1443	1737	1389	1831	1409	1771	1225	1995	1565	1615	1261	1959	1521	1659	1307	1913	1487	1693	32000
32000	1843	1337	1743	1477	1809	1371	1789	1431	1965	1215	1625	1595	1921	1259	1661	1559	1887	1293	1707	1513	32000
32000	1757	1463	1857	1323	1791	1429	1811	1369	1635	1585	1975	1205	1679	1541	1939	1241	1713	1507	1893	1287	32000
32000	1535	1645	1275	1945	1499	1681	1319	1901	1453	1727	1353	1867	1417	1763	1397	1823	1571	1609	1231	1989	32000
32000	1265	1955	1525	1655	1301	1919	1481	1699	1347	1873	1447	1733	1383	1837	1403	1777	1229	1991	1569	1611	32000
32000	1925	1255	1665	1555	1881	1299	1701	1519	1847	1333	1747	1473	1803	1377	1783	1437	1969	1211	1629	1591	32000
32000	1675	1545	1935	1245	1719	1501	1899	1281	1753	1467	1853	1327	1797	1423	1817	1363	1631	1589	1971	1209	32000
32000	1413	1767	1393	1827	1577	1603	1237	1983	1531	1649	1271	1949	1495	1685	1315	1905	1459	1721	1359	1861	32000
32000	1387	1833	1407	1773	1223	1997	1563	1617	1269	1951	1529	1651	1305	1915	1485	1695	1341	1879	1441	1739	32000
32000	1807	1373	1787	1433	1963	1217	1623	1597	1929	1251	1669	1551	1885	1295	1705	1515	1841	1339	1741	1479	32000
32000	1793	1427	1813	1367	1637	1583	1977	1203	1671	1549	1931	1249	1715	1505	1895	1285	1759	1461	1859	1321	32000
32000	1491	1689	1311	1909	1455	1725	1355	1865	1419	1761	1399	1821	1573	1607	1233	1987	1537	1643	1277	1943	32000
32000	1309	1911	1489	1691	1345	1875	1445	1735	1381	1839	1401	1779	1227	1993	1567	1613	1263	1957	1523	1657	32000
32000	1889	1291	1709	1511	1845	1335	1745	1475	1801	1379	1781	1439	1967	1213	1627	1593	1923	1257	1663	1557	32000
32000	1711	1509	1891	1289	1755	1465	1855	1325	1799	1421	1819	1361	1633	1587	1973	1207	1677	1543	1937	1243	32000
	32000	32000	32000	32000	32000	32000	32000	32000	32000	32000	32000	32000	32000	32000	32000	32000	32000	32000	32000	32000	32000

Above magic square is **pandiagonal** with magic sum, $S_{20 \times 20} := 32000$ with sum of entries a perfect square sum, i.e., $T_{400} := 640000 = 800^2 = 1000^2 - 600^2$. All the 4×4 blocks are also **pandiagonal** magic squares of order 4 with the equal magic sums, $S_{4 \times 4} = 6400$.

5 First Procedure: Second Step

Let's write a number 10000 as following 99 sums:

1. $10000 = 9999 + 0001$
2. $10000 = 9996 + 0004$
3. $10000 = 9991 + 0009$
4. $10000 = 9984 + 0016$
5. $10000 = 9975 + 0025$
- ...
96. $10000 = 0784 + 9216$
97. $10000 = 0591 + 9409$
98. $10000 = 0396 + 9604$
99. $10000 = 0199 + 9801$ (5.1)

We observe that the second terms in the r.h.s. are perfect squares. Consider a difference of squares between the terms of r.h.s. with 1 in the front perfect square term. Then this difference is again a perfect square multiple of 200. See below:

$$\begin{aligned}
 10001^2 - 9999^2 = 00200^2 &\Rightarrow 9999^2 + 00200^2 = 10001^2 \Rightarrow (9999, 00200, 10001) \\
 10004^2 - 9996^2 = 00400^2 &\Rightarrow 9996^2 + 00400^2 = 10004^2 \Rightarrow (9996, 00400, 10004) \\
 10009^2 - 9991^2 = 00600^2 &\Rightarrow 9991^2 + 00600^2 = 10009^2 \Rightarrow (9991, 00600, 10009)
 \end{aligned}$$

$$\begin{aligned}
 &10016^2 - 9984^2 = 00800^2 \Rightarrow 9984^2 + 00800^2 = 10016^2 \Rightarrow (9984, 00800, 10016) \\
 &10025^2 - 9975^2 = 01000^2 \Rightarrow 9975^2 + 01000^2 = 10025^2 \Rightarrow (9975, 01000, 10025) \\
 &\dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \\
 &19216^2 - 0784^2 = 19200^2 \Rightarrow 0784^2 + 19200^2 = 19216^2 \Rightarrow (0784, 19200, 19216) \\
 &19409^2 - 0591^2 = 19400^2 \Rightarrow 0591^2 + 19400^2 = 19409^2 \Rightarrow (0591, 19400, 19409) \\
 &19604^2 - 0396^2 = 19600^2 \Rightarrow 0396^2 + 19600^2 = 19604^2 \Rightarrow (0396, 19600, 19604) \\
 &19801^2 - 0199^2 = 19800^2 \Rightarrow 0199^2 + 19800^2 = 19801^2 \Rightarrow (0199, 19800, 19801) \tag{5.2}
 \end{aligned}$$

5.1 Alternative Approach

By considering $m = 100$ and varying the values of $n = 1, 2, 3, 4, \dots, 99$ in (2.1), we get respectively the following 99 Pythagorean triples:

(9999, 0200, 10001)	(9324, 5200, 10676)	(7399, 10200, 12601)	(4224, 15200, 15776)
(9996, 0400, 10004)	(9271, 5400, 10729)	(7296, 10400, 12704)	(4071, 15400, 15929)
(9991, 0600, 10009)	(9216, 5600, 10784)	(7191, 10600, 12809)	(3916, 15600, 16084)
(9984, 0800, 10016)	(9159, 5800, 10841)	(7084, 10800, 12916)	(3759, 15800, 16241)
(9975, 1000, 10025)	(9100, 6000, 10900)	(6975, 11000, 13025)	(3600, 16000, 16400)
(9964, 1200, 10036)	(9039, 6200, 10961)	(6864, 11200, 13136)	(3439, 16200, 16561)
(9951, 1400, 10049)	(8976, 6400, 11024)	(6751, 11400, 13249)	(3276, 16400, 16724)
(9936, 1600, 10064)	(8911, 6600, 11089)	(6636, 11600, 13364)	(3111, 16600, 16889)
(9919, 1800, 10081)	(8844, 6800, 11156)	(6519, 11800, 13481)	(2944, 16800, 17056)
(9900, 2000, 10100)	(8775, 7000, 11225)	(6400, 12000, 13600)	(2775, 17000, 17225)
(9879, 2200, 10121)	(8704, 7200, 11296)	(6279, 12200, 13721)	(2604, 17200, 17396)
(9856, 2400, 10144)	(8631, 7400, 11369)	(6156, 12400, 13844)	(2431, 17400, 17569)
(9831, 2600, 10169)	(8556, 7600, 11444)	(6031, 12600, 13969)	(2256, 17600, 17744)
(9804, 2800, 10196)	(8479, 7800, 11521)	(5904, 12800, 14096)	(2079, 17800, 17921)
(9775, 3000, 10225)	(8400, 8000, 11600)	(5775, 13000, 14225)	(1900, 18000, 18100)
(9744, 3200, 10256)	(8319, 8200, 11681)	(5644, 13200, 14356)	(1719, 18200, 18281)
(9711, 3400, 10289)	(8236, 8400, 11764)	(5511, 13400, 14489)	(1536, 18400, 18464)
(9676, 3600, 10324)	(8151, 8600, 11849)	(5376, 13600, 14624)	(1351, 18600, 18649)
(9639, 3800, 10361)	(8064, 8800, 11936)	(5239, 13800, 14761)	(1164, 18800, 18836)
(9600, 4000, 10400)	(7975, 9000, 12025)	(5100, 14000, 14900)	(975, 19000, 19025)
(9559, 4200, 10441)	(7884, 9200, 12116)	(4959, 14200, 15041)	(784, 19200, 19216)
(9516, 4400, 10484)	(7791, 9400, 12209)	(4816, 14400, 15184)	(591, 19400, 19409)
(9471, 4600, 10529)	(7696, 9600, 12304)	(4671, 14600, 15329)	(396, 19600, 19604)
(9424, 4800, 10576)	(7599, 9800, 12401)	(4524, 14800, 15476)	(199, 19800, 19801)
(9375, 5000, 10625)	(7500, 10000, 12500)	(4375, 15000, 15625)	

(5.3)

The triples given in (5.2) and (5.3) are the same, but obtained in different ways.

5.2 Patterns in Pythagorean Triples

Based on the triples given in (5.2) or (5.3), let's construct patterns with final sums. This is done for each triple separately. Let's fix the values of m , i.e., $m = 100, 1000, 10000, \dots$ and changing the values of n as $n = 1, 2, 3, \dots, 97, 98, 99$ in (2.1), we get respectively the following 99 patterns:

$$\begin{aligned}
 9999^2 + 200^2 &= 10001^2 & := & 100020001 \\
 99999^2 + 2000^2 &= 100001^2 & := & 10000200001 \\
 999999^2 + 20000^2 &= 10000001^2 & := & 100000020000001 \\
 9999999^2 + 200000^2 &= 1000000001^2 & := & 1000000002000000001 \\
 99999999^2 + 2000000^2 &= 100000000001^2 & := & 10000000000200000000001 \tag{5.4}
 \end{aligned}$$

$$\begin{aligned}
 9996^2 + 400^2 &= 10004^2 & := & 100080016 \\
 99996^2 + 4000^2 &= 100004^2 & := & 100008000016 \\
 999996^2 + 40000^2 &= 10000004^2 & := & 1000000800000016 \\
 9999996^2 + 400000^2 &= 1000000004^2 & := & 10000000080000000016 \\
 99999996^2 + 4000000^2 &= 100000000004^2 & := & 100000000008000000000016 \tag{5.5}
 \end{aligned}$$

$$\begin{aligned}
 9991^2 + 600^2 &= 10009^2 & := & 100180081 \\
 99991^2 + 6000^2 &= 100009^2 & := & 1000018000081 \\
 999991^2 + 60000^2 &= 10000009^2 & := & 10000001800000081 \\
 9999991^2 + 600000^2 &= 1000000009^2 & := & 100000000180000000081 \\
 99999991^2 + 6000000^2 &= 100000000009^2 & := & 1000000000018000000000081 \tag{5.6}
 \end{aligned}$$

$$\begin{aligned}
 9984^2 + 800^2 &= 10016^2 & := & 100320256 \\
 99984^2 + 8000^2 &= 100016^2 & := & 1000032000256 \\
 999984^2 + 80000^2 &= 10000016^2 & := & 10000003200000256 \\
 9999984^2 + 800000^2 &= 1000000016^2 & := & 100000000320000000256 \\
 99999984^2 + 8000000^2 &= 100000000016^2 & := & 1000000000032000000000256 \tag{5.7}
 \end{aligned}$$

$$\begin{aligned}
 9975^2 + 1000^2 &= 10025^2 & := & 100500625 \\
 99975^2 + 10000^2 &= 100025^2 & := & 1000050000625 \\
 999975^2 + 100000^2 &= 10000025^2 & := & 10000005000000625 \\
 9999975^2 + 1000000^2 &= 1000000025^2 & := & 100000000500000000625 \\
 99999975^2 + 10000000^2 &= 100000000025^2 & := & 1000000000050000000000625 \tag{5.8}
 \end{aligned}$$

$$\begin{aligned}
 9964^2 + 1200^2 &= 10036^2 & := & 100721296 \\
 99964^2 + 12000^2 &= 100036^2 & := & 1000072001296 \\
 999964^2 + 120000^2 &= 10000036^2 & := & 10000007200001296 \\
 9999964^2 + 1200000^2 &= 1000000036^2 & := & 100000000720000001296 \\
 99999964^2 + 12000000^2 &= 100000000036^2 & := & 1000000000072000000001296 \tag{5.9}
 \end{aligned}$$

$$\begin{aligned}
9951^2 + 1400^2 &= 10049^2 & := & 100982401 \\
999951^2 + 14000^2 &= 1000049^2 & := & 1000098002401 \\
99999951^2 + 140000^2 &= 100000049^2 & := & 10000009800002401 \\
999999951^2 + 1400000^2 &= 10000000049^2 & := & 100000000980000002401 \\
99999999951^2 + 14000000^2 &= 1000000000049^2 & := & 1000000000098000000002401 \quad (5.10)
\end{aligned}$$

$$\begin{aligned}
9936^2 + 1600^2 &= 10064^2 & := & 101284096 \\
999936^2 + 16000^2 &= 1000064^2 & := & 1000128004096 \\
99999936^2 + 160000^2 &= 100000064^2 & := & 10000012800004096 \\
999999936^2 + 1600000^2 &= 10000000064^2 & := & 100000001280000004096 \\
99999999936^2 + 16000000^2 &= 1000000000064^2 & := & 1000000000128000000004096 \quad (5.11)
\end{aligned}$$

$$\begin{aligned}
9919^2 + 1800^2 &= 10081^2 & := & 101626561 \\
999919^2 + 18000^2 &= 1000081^2 & := & 1000162006561 \\
99999919^2 + 180000^2 &= 100000081^2 & := & 10000016200006561 \\
999999919^2 + 1800000^2 &= 10000000081^2 & := & 100000001620000006561 \\
99999999919^2 + 18000000^2 &= 1000000000081^2 & := & 1000000000162000000006561 \quad (5.12)
\end{aligned}$$

$$\begin{aligned}
9900^2 + 2000^2 &= 10100^2 & := & 102010000 \\
999900^2 + 20000^2 &= 1000100^2 & := & 1000200010000 \\
99999900^2 + 200000^2 &= 100000100^2 & := & 10000020000010000 \\
999999900^2 + 2000000^2 &= 10000000100^2 & := & 100000002000000010000 \\
99999999900^2 + 20000000^2 &= 1000000000100^2 & := & 1000000000200000000010000 \quad (5.13)
\end{aligned}$$

$$\begin{aligned}
9879^2 + 2200^2 &= 10121^2 & := & 102434641 \\
999879^2 + 22000^2 &= 1000121^2 & := & 1000242014641 \\
99999879^2 + 220000^2 &= 100000121^2 & := & 10000024200014641 \\
9999999879^2 + 2200000^2 &= 10000000121^2 & := & 100000002420000014641 \\
999999999879^2 + 22000000^2 &= 1000000000121^2 & := & 1000000000242000000014641 \quad (5.14)
\end{aligned}$$

$$\begin{aligned}
9856^2 + 2400^2 &= 10144^2 & := & 102900736 \\
999856^2 + 24000^2 &= 1000144^2 & := & 1000288020736 \\
99999856^2 + 240000^2 &= 100000144^2 & := & 10000028800020736 \\
9999999856^2 + 2400000^2 &= 10000000144^2 & := & 100000002880000020736 \\
999999999856^2 + 24000000^2 &= 1000000000144^2 & := & 1000000000288000000020736 \quad (5.15)
\end{aligned}$$

$$\begin{aligned}
9831^2 + 2600^2 &= 10169^2 & := & 103408561 \\
999831^2 + 26000^2 &= 1000169^2 & := & 1000338028561 \\
9999831^2 + 260000^2 &= 100000169^2 & := & 10000033800028561 \\
99999831^2 + 2600000^2 &= 10000000169^2 & := & 100000003380000028561 \\
999999831^2 + 26000000^2 &= 1000000000169^2 & := & 1000000000338000000028561
\end{aligned} \tag{5.16}$$

$$\begin{aligned}
9804^2 + 2800^2 &= 10196^2 & := & 103958416 \\
999804^2 + 28000^2 &= 1000196^2 & := & 1000392038416 \\
9999804^2 + 280000^2 &= 100000196^2 & := & 10000039200038416 \\
99999804^2 + 2800000^2 &= 10000000196^2 & := & 100000003920000038416 \\
999999804^2 + 28000000^2 &= 1000000000196^2 & := & 1000000000392000000038416
\end{aligned} \tag{5.17}$$

$$\begin{aligned}
9775^2 + 3000^2 &= 10225^2 & := & 104550625 \\
999775^2 + 30000^2 &= 1000225^2 & := & 1000450050625 \\
9999775^2 + 300000^2 &= 100000225^2 & := & 10000045000050625 \\
99999775^2 + 3000000^2 &= 10000000225^2 & := & 100000004500000050625 \\
999999775^2 + 30000000^2 &= 1000000000225^2 & := & 1000000000450000000050625
\end{aligned} \tag{5.18}$$

$$\begin{aligned}
9744^2 + 3200^2 &= 10256^2 & := & 105185536 \\
999744^2 + 32000^2 &= 1000256^2 & := & 1000512065536 \\
9999744^2 + 320000^2 &= 100000256^2 & := & 10000051200065536 \\
99999744^2 + 3200000^2 &= 10000000256^2 & := & 100000005120000065536 \\
999999744^2 + 32000000^2 &= 1000000000256^2 & := & 1000000000512000000065536
\end{aligned} \tag{5.19}$$

$$\begin{aligned}
9711^2 + 3400^2 &= 10289^2 & := & 105863521 \\
999711^2 + 34000^2 &= 1000289^2 & := & 1000578083521 \\
9999711^2 + 340000^2 &= 100000289^2 & := & 10000057800083521 \\
99999711^2 + 3400000^2 &= 10000000289^2 & := & 100000005780000083521 \\
999999711^2 + 34000000^2 &= 1000000000289^2 & := & 1000000000578000000083521
\end{aligned} \tag{5.20}$$

$$\begin{aligned}
9676^2 + 3600^2 &= 10324^2 & := & 106584976 \\
999676^2 + 36000^2 &= 1000324^2 & := & 1000648104976 \\
9999676^2 + 360000^2 &= 100000324^2 & := & 10000064800104976 \\
99999676^2 + 3600000^2 &= 10000000324^2 & := & 100000006480000104976 \\
999999676^2 + 36000000^2 &= 1000000000324^2 & := & 1000000000648000000104976
\end{aligned} \tag{5.21}$$

$$\begin{aligned}
9639^2 + 3800^2 &= 10361^2 & := & 107350321 \\
999639^2 + 38000^2 &= 1000361^2 & := & 1000722130321 \\
9999639^2 + 380000^2 &= 100000361^2 & := & 10000072200130321 \\
99999639^2 + 3800000^2 &= 10000000361^2 & := & 100000007220000130321 \\
999999639^2 + 38000000^2 &= 1000000000361^2 & := & 1000000000722000000130321
\end{aligned} \tag{5.22}$$

$$\begin{aligned}
9600^2 + 4000^2 &= 10400^2 & := & 108160000 \\
999600^2 + 40000^2 &= 1000400^2 & := & 1000800160000 \\
9999600^2 + 400000^2 &= 100000400^2 & := & 10000080000160000 \\
99999600^2 + 4000000^2 &= 10000000400^2 & := & 100000008000000160000 \\
999999600^2 + 40000000^2 &= 1000000000400^2 & := & 1000000000800000000160000
\end{aligned} \tag{5.23}$$

$$\begin{aligned}
9559^2 + 4200^2 &= 10441^2 & := & 109014481 \\
999559^2 + 42000^2 &= 1000441^2 & := & 1000882194481 \\
9999559^2 + 420000^2 &= 100000441^2 & := & 10000088200194481 \\
99999559^2 + 4200000^2 &= 10000000441^2 & := & 100000008820000194481 \\
999999559^2 + 42000000^2 &= 1000000000441^2 & := & 1000000000882000000194481
\end{aligned} \tag{5.24}$$

$$\begin{aligned}
9516^2 + 4400^2 &= 10484^2 & := & 109914256 \\
999516^2 + 44000^2 &= 1000484^2 & := & 1000968234256 \\
9999516^2 + 440000^2 &= 100000484^2 & := & 10000096800234256 \\
99999516^2 + 4400000^2 &= 10000000484^2 & := & 100000009680000234256 \\
999999516^2 + 44000000^2 &= 1000000000484^2 & := & 1000000000968000000234256
\end{aligned} \tag{5.25}$$

$$\begin{aligned}
9471^2 + 4600^2 &= 10529^2 & := & 110859841 \\
999471^2 + 46000^2 &= 1000529^2 & := & 1001058279841 \\
9999471^2 + 460000^2 &= 100000529^2 & := & 10000105800279841 \\
99999471^2 + 4600000^2 &= 10000000529^2 & := & 100000010580000279841 \\
999999471^2 + 46000000^2 &= 1000000000529^2 & := & 1000000001058000000279841
\end{aligned} \tag{5.26}$$

$$\begin{aligned}
9424^2 + 4800^2 &= 10576^2 & := & 111851776 \\
999424^2 + 48000^2 &= 1000576^2 & := & 1001152331776 \\
9999424^2 + 480000^2 &= 100000576^2 & := & 10000115200331776 \\
99999424^2 + 4800000^2 &= 10000000576^2 & := & 100000011520000331776 \\
999999424^2 + 48000000^2 &= 1000000000576^2 & := & 1000000001152000000331776
\end{aligned} \tag{5.27}$$

$$\begin{aligned}
9375^2 + 5000^2 &= 10625^2 & := 112890625 \\
999375^2 + 50000^2 &= 1000625^2 & := 1001250390625 \\
9999375^2 + 500000^2 &= 10000625^2 & := 10000125000390625 \\
99999375^2 + 5000000^2 &= 1000000625^2 & := 100000012500000390625 \\
9999999375^2 + 50000000^2 &= 100000000625^2 & := 1000000001250000000390625
\end{aligned} \tag{5.28}$$

$$\begin{aligned}
9324^2 + 5200^2 &= 10676^2 & := 113976976 \\
999324^2 + 52000^2 &= 1000676^2 & := 1001352456976 \\
9999324^2 + 520000^2 &= 10000676^2 & := 10000135200456976 \\
99999324^2 + 5200000^2 &= 1000000676^2 & := 100000013520000456976 \\
9999999324^2 + 52000000^2 &= 100000000676^2 & := 1000000001352000000456976
\end{aligned} \tag{5.29}$$

$$\begin{aligned}
9271^2 + 5400^2 &= 10729^2 & := 115111441 \\
999271^2 + 54000^2 &= 1000729^2 & := 1001458531441 \\
9999271^2 + 540000^2 &= 10000729^2 & := 10000145800531441 \\
99999271^2 + 5400000^2 &= 1000000729^2 & := 100000014580000531441 \\
9999999271^2 + 54000000^2 &= 100000000729^2 & := 1000000001458000000531441
\end{aligned} \tag{5.30}$$

$$\begin{aligned}
9216^2 + 5600^2 &= 10784^2 & := 116294656 \\
999216^2 + 56000^2 &= 1000784^2 & := 1001568614656 \\
9999216^2 + 560000^2 &= 10000784^2 & := 10000156800614656 \\
99999216^2 + 5600000^2 &= 1000000784^2 & := 100000015680000614656 \\
9999999216^2 + 56000000^2 &= 100000000784^2 & := 1000000001568000000614656
\end{aligned} \tag{5.31}$$

$$\begin{aligned}
9159^2 + 5800^2 &= 10841^2 & := 117527281 \\
999159^2 + 58000^2 &= 1000841^2 & := 1001682707281 \\
9999159^2 + 580000^2 &= 10000841^2 & := 10000168200707281 \\
99999159^2 + 5800000^2 &= 1000000841^2 & := 100000016820000707281 \\
9999999159^2 + 58000000^2 &= 100000000841^2 & := 1000000001682000000707281
\end{aligned} \tag{5.32}$$

$$\begin{aligned}
9100^2 + 6000^2 &= 10900^2 & := 118810000 \\
999100^2 + 60000^2 &= 1000900^2 & := 1001800810000 \\
9999100^2 + 600000^2 &= 10000900^2 & := 10000180000810000 \\
99999100^2 + 6000000^2 &= 1000000900^2 & := 100000018000000810000 \\
9999999100^2 + 60000000^2 &= 100000000900^2 & := 1000000001800000000810000
\end{aligned} \tag{5.33}$$

$$\begin{aligned}
9039^2 + 6200^2 &= 10961^2 & := 120143521 \\
999039^2 + 62000^2 &= 1000961^2 & := 1001922923521 \\
99999039^2 + 620000^2 &= 100000961^2 & := 10000192200923521 \\
9999999039^2 + 6200000^2 &= 10000000961^2 & := 100000019220000923521 \\
999999999039^2 + 62000000^2 &= 1000000000961^2 & := 1000000001922000000923521
\end{aligned} \tag{5.34}$$

$$\begin{aligned}
8976^2 + 6400^2 &= 11024^2 & := 121528576 \\
998976^2 + 64000^2 &= 1001024^2 & := 1002049048576 \\
99998976^2 + 640000^2 &= 100001024^2 & := 10000204801048576 \\
9999998976^2 + 6400000^2 &= 10000001024^2 & := 100000020480001048576 \\
999999998976^2 + 64000000^2 &= 1000000001024^2 & := 1000000002048000001048576
\end{aligned} \tag{5.35}$$

$$\begin{aligned}
8911^2 + 6600^2 &= 11089^2 & := 122965921 \\
998911^2 + 66000^2 &= 1001089^2 & := 1002179185921 \\
99998911^2 + 660000^2 &= 100001089^2 & := 10000217801185921 \\
9999998911^2 + 6600000^2 &= 10000001089^2 & := 100000021780001185921 \\
999999998911^2 + 66000000^2 &= 1000000001089^2 & := 1000000002178000001185921
\end{aligned} \tag{5.36}$$

$$\begin{aligned}
8844^2 + 6800^2 &= 11156^2 & := 124456336 \\
998844^2 + 68000^2 &= 1001156^2 & := 1002313336336 \\
99998844^2 + 680000^2 &= 100001156^2 & := 10000231201336336 \\
9999998844^2 + 6800000^2 &= 10000001156^2 & := 100000023120001336336 \\
999999998844^2 + 68000000^2 &= 1000000001156^2 & := 1000000002312000001336336
\end{aligned} \tag{5.37}$$

$$\begin{aligned}
8775^2 + 7000^2 &= 11225^2 & := 126000625 \\
998775^2 + 70000^2 &= 1001225^2 & := 1002451500625 \\
99998775^2 + 700000^2 &= 100001225^2 & := 10000245001500625 \\
9999998775^2 + 7000000^2 &= 10000001225^2 & := 100000024500001500625 \\
999999998775^2 + 70000000^2 &= 1000000001225^2 & := 1000000002450000001500625
\end{aligned} \tag{5.38}$$

$$\begin{aligned}
8704^2 + 7200^2 &= 11296^2 & := 127599616 \\
998704^2 + 72000^2 &= 1001296^2 & := 1002593679616 \\
99998704^2 + 720000^2 &= 100001296^2 & := 10000259201679616 \\
9999998704^2 + 7200000^2 &= 10000001296^2 & := 100000025920001679616 \\
999999998704^2 + 72000000^2 &= 1000000001296^2 & := 1000000002592000001679616
\end{aligned} \tag{5.39}$$

$$\begin{aligned}
8631^2 + 7400^2 &= 11369^2 & := & 129254161 \\
998631^2 + 74000^2 &= 1001369^2 & := & 1002739874161 \\
99998631^2 + 740000^2 &= 100001369^2 & := & 10000273801874161 \\
9999998631^2 + 7400000^2 &= 10000001369^2 & := & 100000027380001874161 \\
999999998631^2 + 74000000^2 &= 1000000001369^2 & := & 1000000002738000001874161
\end{aligned} \tag{5.40}$$

$$\begin{aligned}
8556^2 + 7600^2 &= 11444^2 & := & 130965136 \\
998556^2 + 76000^2 &= 1001444^2 & := & 1002890085136 \\
99998556^2 + 760000^2 &= 100001444^2 & := & 10000288802085136 \\
9999998556^2 + 7600000^2 &= 10000001444^2 & := & 100000028880002085136 \\
999999998556^2 + 76000000^2 &= 1000000001444^2 & := & 1000000002888000002085136
\end{aligned} \tag{5.41}$$

$$\begin{aligned}
8479^2 + 7800^2 &= 11521^2 & := & 132733441 \\
998479^2 + 78000^2 &= 1001521^2 & := & 1003044313441 \\
99998479^2 + 780000^2 &= 100001521^2 & := & 10000304202313441 \\
9999998479^2 + 7800000^2 &= 10000001521^2 & := & 100000030420002313441 \\
999999998479^2 + 78000000^2 &= 1000000001521^2 & := & 1000000003042000002313441
\end{aligned} \tag{5.42}$$

$$\begin{aligned}
8400^2 + 8000^2 &= 11600^2 & := & 134560000 \\
998400^2 + 80000^2 &= 1001600^2 & := & 1003202560000 \\
99998400^2 + 800000^2 &= 100001600^2 & := & 10000320002560000 \\
9999998400^2 + 8000000^2 &= 10000001600^2 & := & 100000032000002560000 \\
999999998400^2 + 80000000^2 &= 1000000001600^2 & := & 1000000003200000002560000
\end{aligned} \tag{5.43}$$

$$\begin{aligned}
8319^2 + 8200^2 &= 11681^2 & := & 136445761 \\
998319^2 + 82000^2 &= 1001681^2 & := & 1003364825761 \\
99998319^2 + 820000^2 &= 100001681^2 & := & 10000336202825761 \\
9999998319^2 + 8200000^2 &= 10000001681^2 & := & 100000033620002825761 \\
999999998319^2 + 82000000^2 &= 1000000001681^2 & := & 1000000003362000002825761
\end{aligned} \tag{5.44}$$

$$\begin{aligned}
8236^2 + 8400^2 &= 11764^2 & := & 138391696 \\
998236^2 + 84000^2 &= 1001764^2 & := & 1003531111696 \\
99998236^2 + 840000^2 &= 100001764^2 & := & 10000352803111696 \\
9999998236^2 + 8400000^2 &= 10000001764^2 & := & 100000035280003111696 \\
999999998236^2 + 84000000^2 &= 1000000001764^2 & := & 1000000003528000003111696
\end{aligned} \tag{5.45}$$

$$\begin{aligned}
8151^2 + 8600^2 &= 11849^2 & := & 140398801 \\
998151^2 + 86000^2 &= 1001849^2 & := & 1003701418801 \\
99998151^2 + 860000^2 &= 100001849^2 & := & 10000369803418801 \\
9999998151^2 + 8600000^2 &= 10000001849^2 & := & 100000036980003418801 \\
999999998151^2 + 86000000^2 &= 1000000001849^2 & := & 1000000003698000003418801
\end{aligned} \tag{5.46}$$

$$\begin{aligned}
8064^2 + 8800^2 &= 11936^2 & := & 142468096 \\
998064^2 + 88000^2 &= 1001936^2 & := & 1003875748096 \\
99998064^2 + 880000^2 &= 100001936^2 & := & 10000387203748096 \\
9999998064^2 + 8800000^2 &= 10000001936^2 & := & 100000038720003748096 \\
999999998064^2 + 88000000^2 &= 1000000001936^2 & := & 1000000003872000003748096
\end{aligned} \tag{5.47}$$

$$\begin{aligned}
7975^2 + 9000^2 &= 12025^2 & := & 144600625 \\
997975^2 + 90000^2 &= 1002025^2 & := & 1004054100625 \\
99997975^2 + 900000^2 &= 100002025^2 & := & 10000405004100625 \\
9999997975^2 + 9000000^2 &= 10000002025^2 & := & 100000040500004100625 \\
999999997975^2 + 90000000^2 &= 1000000002025^2 & := & 1000000004050000004100625
\end{aligned} \tag{5.48}$$

$$\begin{aligned}
7884^2 + 9200^2 &= 12116^2 & := & 146797456 \\
997884^2 + 92000^2 &= 1002116^2 & := & 1004236477456 \\
99997884^2 + 920000^2 &= 100002116^2 & := & 10000423204477456 \\
9999997884^2 + 9200000^2 &= 10000002116^2 & := & 100000042320004477456 \\
999999997884^2 + 92000000^2 &= 1000000002116^2 & := & 1000000004232000004477456
\end{aligned} \tag{5.49}$$

$$\begin{aligned}
7791^2 + 9400^2 &= 12209^2 & := & 149059681 \\
997791^2 + 94000^2 &= 1002209^2 & := & 1004422879681 \\
99997791^2 + 940000^2 &= 100002209^2 & := & 10000441804879681 \\
9999997791^2 + 9400000^2 &= 10000002209^2 & := & 100000044180004879681 \\
999999997791^2 + 94000000^2 &= 1000000002209^2 & := & 1000000004418000004879681
\end{aligned} \tag{5.50}$$

$$\begin{aligned}
7696^2 + 9600^2 &= 12304^2 & := & 151388416 \\
997696^2 + 96000^2 &= 1002304^2 & := & 1004613308416 \\
99997696^2 + 960000^2 &= 100002304^2 & := & 10000460805308416 \\
9999997696^2 + 9600000^2 &= 10000002304^2 & := & 100000046080005308416 \\
999999997696^2 + 96000000^2 &= 1000000002304^2 & := & 1000000004608000005308416
\end{aligned} \tag{5.51}$$

$$\begin{aligned}
7599^2 + 9800^2 &= 12401^2 & := & 153784801 \\
997599^2 + 98000^2 &= 1002401^2 & := & 1004807764801 \\
99997599^2 + 980000^2 &= 100002401^2 & := & 10000480205764801 \\
9999997599^2 + 9800000^2 &= 10000002401^2 & := & 100000048020005764801 \\
999999997599^2 + 98000000^2 &= 1000000002401^2 & := & 1000000004802000005764801
\end{aligned} \tag{5.52}$$

$$\begin{aligned}
7500^2 + 10000^2 &= 12500^2 & := & 156250000 \\
997500^2 + 100000^2 &= 1002500^2 & := & 1005006250000 \\
99997500^2 + 1000000^2 &= 100002500^2 & := & 10000500006250000 \\
9999997500^2 + 10000000^2 &= 10000002500^2 & := & 100000050000006250000 \\
999999997500^2 + 100000000^2 &= 1000000002500^2 & := & 1000000005000000006250000
\end{aligned} \tag{5.53}$$

$$\begin{aligned}
7399^2 + 10200^2 &= 12601^2 & := & 158785201 \\
997399^2 + 102000^2 &= 1002601^2 & := & 1005208765201 \\
99997399^2 + 1020000^2 &= 100002601^2 & := & 10000520206765201 \\
9999997399^2 + 10200000^2 &= 10000002601^2 & := & 100000052020006765201 \\
999999997399^2 + 102000000^2 &= 1000000002601^2 & := & 1000000005202000006765201
\end{aligned} \tag{5.54}$$

$$\begin{aligned}
7296^2 + 10400^2 &= 12704^2 & := & 161391616 \\
997296^2 + 104000^2 &= 1002704^2 & := & 1005415311616 \\
99997296^2 + 1040000^2 &= 100002704^2 & := & 10000540807311616 \\
9999997296^2 + 10400000^2 &= 10000002704^2 & := & 100000054080007311616 \\
999999997296^2 + 104000000^2 &= 1000000002704^2 & := & 1000000005408000007311616
\end{aligned} \tag{5.55}$$

$$\begin{aligned}
7191^2 + 10600^2 &= 12809^2 & := & 164070481 \\
997191^2 + 106000^2 &= 1002809^2 & := & 1005625890481 \\
99997191^2 + 1060000^2 &= 100002809^2 & := & 10000561807890481 \\
9999997191^2 + 10600000^2 &= 10000002809^2 & := & 100000056180007890481 \\
999999997191^2 + 106000000^2 &= 1000000002809^2 & := & 1000000005618000007890481
\end{aligned} \tag{5.56}$$

$$\begin{aligned}
7084^2 + 10800^2 &= 12916^2 & := & 166823056 \\
997084^2 + 108000^2 &= 1002916^2 & := & 1005840503056 \\
99997084^2 + 1080000^2 &= 100002916^2 & := & 10000583208503056 \\
9999997084^2 + 10800000^2 &= 10000002916^2 & := & 100000058320008503056 \\
999999997084^2 + 108000000^2 &= 1000000002916^2 & := & 1000000005832000008503056
\end{aligned} \tag{5.57}$$

$$\begin{aligned}
6975^2 + 11000^2 &= 13025^2 & := & 169650625 \\
996975^2 + 110000^2 &= 1003025^2 & := & 1006059150625 \\
99996975^2 + 1100000^2 &= 100003025^2 & := & 10000605009150625 \\
999996975^2 + 11000000^2 &= 10000003025^2 & := & 100000060500009150625 \\
99999996975^2 + 110000000^2 &= 1000000003025^2 & := & 100000000605000009150625
\end{aligned} \tag{5.58}$$

$$\begin{aligned}
6864^2 + 11200^2 &= 13136^2 & := & 172554496 \\
996864^2 + 112000^2 &= 1003136^2 & := & 1006281834496 \\
99996864^2 + 1120000^2 &= 100003136^2 & := & 10000627209834496 \\
999996864^2 + 11200000^2 &= 10000003136^2 & := & 100000062720009834496 \\
99999996864^2 + 112000000^2 &= 1000000003136^2 & := & 100000000627200009834496
\end{aligned} \tag{5.59}$$

$$\begin{aligned}
6751^2 + 11400^2 &= 13249^2 & := & 175536001 \\
996751^2 + 114000^2 &= 1003249^2 & := & 1006508556001 \\
99996751^2 + 1140000^2 &= 100003249^2 & := & 10000649810556001 \\
999996751^2 + 11400000^2 &= 10000003249^2 & := & 100000064980010556001 \\
99999996751^2 + 114000000^2 &= 1000000003249^2 & := & 1000000006498000010556001
\end{aligned} \tag{5.60}$$

$$\begin{aligned}
6636^2 + 11600^2 &= 13364^2 & := & 178596496 \\
996636^2 + 116000^2 &= 1003364^2 & := & 1006739316496 \\
99996636^2 + 1160000^2 &= 100003364^2 & := & 10000672811316496 \\
999996636^2 + 11600000^2 &= 10000003364^2 & := & 100000067280011316496 \\
99999996636^2 + 116000000^2 &= 1000000003364^2 & := & 1000000006728000011316496
\end{aligned} \tag{5.61}$$

$$\begin{aligned}
6519^2 + 11800^2 &= 13481^2 & := & 181737361 \\
996519^2 + 118000^2 &= 1003481^2 & := & 1006974117361 \\
99996519^2 + 1180000^2 &= 100003481^2 & := & 10000696212117361 \\
999996519^2 + 11800000^2 &= 10000003481^2 & := & 100000069620012117361 \\
99999996519^2 + 118000000^2 &= 1000000003481^2 & := & 1000000006962000012117361
\end{aligned} \tag{5.62}$$

$$\begin{aligned}
6400^2 + 12000^2 &= 13600^2 & := & 184960000 \\
996400^2 + 120000^2 &= 1003600^2 & := & 1007212960000 \\
99996400^2 + 1200000^2 &= 100003600^2 & := & 10000720012960000 \\
999996400^2 + 12000000^2 &= 10000003600^2 & := & 100000072000012960000 \\
99999996400^2 + 120000000^2 &= 1000000003600^2 & := & 1000000007200000012960000
\end{aligned} \tag{5.63}$$

$$\begin{aligned}
6279^2 + 12200^2 &= 13721^2 & := & 188265841 \\
996279^2 + 122000^2 &= 1003721^2 & := & 1007455845841 \\
99996279^2 + 1220000^2 &= 100003721^2 & := & 10000744213845841 \\
9999996279^2 + 12200000^2 &= 10000003721^2 & := & 100000074420013845841 \\
999999996279^2 + 122000000^2 &= 1000000003721^2 & := & 1000000007442000013845841
\end{aligned} \tag{5.64}$$

$$\begin{aligned}
6156^2 + 12400^2 &= 13844^2 & := & 191656336 \\
996156^2 + 124000^2 &= 1003844^2 & := & 1007702776336 \\
99996156^2 + 1240000^2 &= 100003844^2 & := & 10000768814776336 \\
9999996156^2 + 12400000^2 &= 10000003844^2 & := & 100000076880014776336 \\
999999996156^2 + 124000000^2 &= 1000000003844^2 & := & 1000000007688000014776336
\end{aligned} \tag{5.65}$$

$$\begin{aligned}
6031^2 + 12600^2 &= 13969^2 & := & 195132961 \\
996031^2 + 126000^2 &= 1003969^2 & := & 1007953752961 \\
99996031^2 + 1260000^2 &= 100003969^2 & := & 10000793815752961 \\
9999996031^2 + 12600000^2 &= 10000003969^2 & := & 100000079380015752961 \\
999999996031^2 + 126000000^2 &= 1000000003969^2 & := & 1000000007938000015752961
\end{aligned} \tag{5.66}$$

$$\begin{aligned}
5904^2 + 12800^2 &= 14096^2 & := & 198697216 \\
995904^2 + 128000^2 &= 1004096^2 & := & 1008208777216 \\
99995904^2 + 1280000^2 &= 100004096^2 & := & 10000819216777216 \\
9999995904^2 + 12800000^2 &= 10000004096^2 & := & 100000081920016777216 \\
999999995904^2 + 128000000^2 &= 1000000004096^2 & := & 1000000008192000016777216
\end{aligned} \tag{5.67}$$

$$\begin{aligned}
5775^2 + 13000^2 &= 14225^2 & := & 202350625 \\
995775^2 + 130000^2 &= 1004225^2 & := & 1008467850625 \\
99995775^2 + 1300000^2 &= 100004225^2 & := & 10000845017850625 \\
9999995775^2 + 13000000^2 &= 10000004225^2 & := & 100000084500017850625 \\
999999995775^2 + 130000000^2 &= 1000000004225^2 & := & 1000000008450000017850625
\end{aligned} \tag{5.68}$$

$$\begin{aligned}
5644^2 + 13200^2 &= 14356^2 & := & 206094736 \\
995644^2 + 132000^2 &= 1004356^2 & := & 1008730974736 \\
99995644^2 + 1320000^2 &= 100004356^2 & := & 10000871218974736 \\
9999995644^2 + 13200000^2 &= 10000004356^2 & := & 100000087120018974736 \\
999999995644^2 + 132000000^2 &= 1000000004356^2 & := & 1000000008712000018974736
\end{aligned} \tag{5.69}$$

$$\begin{aligned}
5511^2 + 13400^2 &= 14489^2 & := 209931121 \\
995511^2 + 134000^2 &= 1004489^2 & := 1008998151121 \\
99995511^2 + 1340000^2 &= 100004489^2 & := 10000897820151121 \\
9999995511^2 + 13400000^2 &= 10000004489^2 & := 100000089780020151121 \\
999999995511^2 + 134000000^2 &= 1000000004489^2 & := 1000000008978000020151121
\end{aligned} \tag{5.70}$$

$$\begin{aligned}
5376^2 + 13600^2 &= 14624^2 & := 213861376 \\
995376^2 + 136000^2 &= 1004624^2 & := 1009269381376 \\
99995376^2 + 1360000^2 &= 100004624^2 & := 10000924821381376 \\
9999995376^2 + 13600000^2 &= 10000004624^2 & := 100000092480021381376 \\
999999995376^2 + 136000000^2 &= 1000000004624^2 & := 1000000009248000021381376
\end{aligned} \tag{5.71}$$

$$\begin{aligned}
5239^2 + 13800^2 &= 14761^2 & := 217887121 \\
995239^2 + 138000^2 &= 1004761^2 & := 1009544667121 \\
99995239^2 + 1380000^2 &= 100004761^2 & := 10000952222667121 \\
9999995239^2 + 13800000^2 &= 10000004761^2 & := 100000095220022667121 \\
999999995239^2 + 138000000^2 &= 1000000004761^2 & := 1000000009522000022667121
\end{aligned} \tag{5.72}$$

$$\begin{aligned}
5100^2 + 14000^2 &= 14900^2 & := 222010000 \\
995100^2 + 140000^2 &= 1004900^2 & := 1009824010000 \\
99995100^2 + 1400000^2 &= 100004900^2 & := 10000980024010000 \\
9999995100^2 + 14000000^2 &= 10000004900^2 & := 100000098000024010000 \\
999999995100^2 + 140000000^2 &= 1000000004900^2 & := 1000000009800000024010000
\end{aligned} \tag{5.73}$$

$$\begin{aligned}
4959^2 + 14200^2 &= 15041^2 & := 226231681 \\
994959^2 + 142000^2 &= 1005041^2 & := 1010107411681 \\
99994959^2 + 1420000^2 &= 100005041^2 & := 10001008225411681 \\
9999994959^2 + 14200000^2 &= 10000005041^2 & := 100000100820025411681 \\
999999994959^2 + 142000000^2 &= 1000000005041^2 & := 1000000010082000025411681
\end{aligned} \tag{5.74}$$

$$\begin{aligned}
4816^2 + 14400^2 &= 15184^2 & := 230553856 \\
994816^2 + 144000^2 &= 1005184^2 & := 1010394873856 \\
99994816^2 + 1440000^2 &= 100005184^2 & := 10001036826873856 \\
9999994816^2 + 14400000^2 &= 10000005184^2 & := 100000103680026873856 \\
999999994816^2 + 144000000^2 &= 1000000005184^2 & := 1000000010368000026873856
\end{aligned} \tag{5.75}$$

$$\begin{aligned}
4671^2 + 14600^2 &= 15329^2 & := & 234978241 \\
994671^2 + 146000^2 &= 1005329^2 & := & 1010686398241 \\
99994671^2 + 1460000^2 &= 100005329^2 & := & 10001065828398241 \\
9999994671^2 + 14600000^2 &= 10000005329^2 & := & 100000106580028398241 \\
999999994671^2 + 146000000^2 &= 1000000005329^2 & := & 1000000010658000028398241
\end{aligned} \tag{5.76}$$

$$\begin{aligned}
4524^2 + 14800^2 &= 15476^2 & := & 239506576 \\
994524^2 + 148000^2 &= 1005476^2 & := & 1010981986576 \\
99994524^2 + 1480000^2 &= 100005476^2 & := & 10001095229986576 \\
9999994524^2 + 14800000^2 &= 10000005476^2 & := & 100000109520029986576 \\
999999994524^2 + 148000000^2 &= 1000000005476^2 & := & 1000000010952000029986576
\end{aligned} \tag{5.77}$$

$$\begin{aligned}
4375^2 + 15000^2 &= 15625^2 & := & 244140625 \\
994375^2 + 150000^2 &= 1005625^2 & := & 1011281640625 \\
99994375^2 + 1500000^2 &= 100005625^2 & := & 10001125031640625 \\
9999994375^2 + 15000000^2 &= 10000005625^2 & := & 100000112500031640625 \\
999999994375^2 + 150000000^2 &= 1000000005625^2 & := & 1000000011250000031640625
\end{aligned} \tag{5.78}$$

$$\begin{aligned}
4224^2 + 15200^2 &= 15776^2 & := & 248882176 \\
994224^2 + 152000^2 &= 1005776^2 & := & 1011585362176 \\
99994224^2 + 1520000^2 &= 100005776^2 & := & 10001155233362176 \\
9999994224^2 + 15200000^2 &= 10000005776^2 & := & 100000115520033362176 \\
999999994224^2 + 152000000^2 &= 1000000005776^2 & := & 1000000011552000033362176
\end{aligned} \tag{5.79}$$

$$\begin{aligned}
4071^2 + 15400^2 &= 15929^2 & := & 253733041 \\
994071^2 + 154000^2 &= 1005929^2 & := & 1011893153041 \\
99994071^2 + 1540000^2 &= 100005929^2 & := & 10001185835153041 \\
9999994071^2 + 15400000^2 &= 10000005929^2 & := & 100000118580035153041 \\
999999994071^2 + 154000000^2 &= 1000000005929^2 & := & 1000000011858000035153041
\end{aligned} \tag{5.80}$$

$$\begin{aligned}
3916^2 + 15600^2 &= 16084^2 & := & 258695056 \\
993916^2 + 156000^2 &= 1006084^2 & := & 1012205015056 \\
99993916^2 + 1560000^2 &= 100006084^2 & := & 10001216837015056 \\
9999993916^2 + 15600000^2 &= 10000006084^2 & := & 100000121680037015056 \\
999999993916^2 + 156000000^2 &= 1000000006084^2 & := & 1000000012168000037015056
\end{aligned} \tag{5.81}$$

$$\begin{aligned}
3759^2 + 15800^2 &= 16241^2 & := & 263770081 \\
993759^2 + 158000^2 &= 1006241^2 & := & 1012520950081 \\
99993759^2 + 1580000^2 &= 100006241^2 & := & 10001248238950081 \\
9999993759^2 + 15800000^2 &= 10000006241^2 & := & 100000124820038950081 \\
999999993759^2 + 158000000^2 &= 1000000006241^2 & := & 1000000012482000038950081
\end{aligned} \tag{5.82}$$

$$\begin{aligned}
3600^2 + 16000^2 &= 16400^2 & := & 268960000 \\
993600^2 + 160000^2 &= 1006400^2 & := & 1012840960000 \\
99993600^2 + 1600000^2 &= 100006400^2 & := & 10001280040960000 \\
9999993600^2 + 16000000^2 &= 10000006400^2 & := & 100000128000040960000 \\
999999993600^2 + 160000000^2 &= 1000000006400^2 & := & 1000000012800000040960000
\end{aligned} \tag{5.83}$$

$$\begin{aligned}
3439^2 + 16200^2 &= 16561^2 & := & 274266721 \\
993439^2 + 162000^2 &= 1006561^2 & := & 1013165046721 \\
99993439^2 + 1620000^2 &= 100006561^2 & := & 10001312243046721 \\
9999993439^2 + 16200000^2 &= 10000006561^2 & := & 100000131220043046721 \\
999999993439^2 + 162000000^2 &= 1000000006561^2 & := & 1000000013122000043046721
\end{aligned} \tag{5.84}$$

$$\begin{aligned}
3276^2 + 16400^2 &= 16724^2 & := & 279692176 \\
993276^2 + 164000^2 &= 1006724^2 & := & 1013493212176 \\
99993276^2 + 1640000^2 &= 100006724^2 & := & 10001344845212176 \\
9999993276^2 + 16400000^2 &= 10000006724^2 & := & 100000134480045212176 \\
999999993276^2 + 164000000^2 &= 1000000006724^2 & := & 1000000013448000045212176
\end{aligned} \tag{5.85}$$

$$\begin{aligned}
3111^2 + 16600^2 &= 16889^2 & := & 285238321 \\
993111^2 + 166000^2 &= 1006889^2 & := & 1013825458321 \\
99993111^2 + 1660000^2 &= 100006889^2 & := & 10001377847458321 \\
9999993111^2 + 16600000^2 &= 10000006889^2 & := & 100000137780047458321 \\
999999993111^2 + 166000000^2 &= 1000000006889^2 & := & 1000000013778000047458321
\end{aligned} \tag{5.86}$$

$$\begin{aligned}
2944^2 + 16800^2 &= 17056^2 & := & 290907136 \\
992944^2 + 168000^2 &= 1007056^2 & := & 1014161787136 \\
99992944^2 + 1680000^2 &= 100007056^2 & := & 10001411249787136 \\
9999992944^2 + 16800000^2 &= 10000007056^2 & := & 100000141120049787136 \\
999999992944^2 + 168000000^2 &= 1000000007056^2 & := & 1000000014112000049787136
\end{aligned} \tag{5.87}$$

$$\begin{aligned}
2775^2 + 17000^2 &= 17225^2 & := & 296700625 \\
992775^2 + 170000^2 &= 1007225^2 & := & 1014502200625 \\
99992775^2 + 1700000^2 &= 100007225^2 & := & 10001445052200625 \\
9999992775^2 + 17000000^2 &= 10000007225^2 & := & 100000144500052200625 \\
999999992775^2 + 170000000^2 &= 1000000007225^2 & := & 1000000014450000052200625
\end{aligned} \tag{5.88}$$

$$\begin{aligned}
2604^2 + 17200^2 &= 17396^2 & := & 302620816 \\
992604^2 + 172000^2 &= 1007396^2 & := & 1014846700816 \\
99992604^2 + 1720000^2 &= 100007396^2 & := & 10001479254700816 \\
9999992604^2 + 17200000^2 &= 10000007396^2 & := & 100000147920054700816 \\
999999992604^2 + 172000000^2 &= 1000000007396^2 & := & 1000000014792000054700816
\end{aligned} \tag{5.89}$$

$$\begin{aligned}
2431^2 + 17400^2 &= 17569^2 & := & 308669761 \\
992431^2 + 174000^2 &= 1007569^2 & := & 1015195289761 \\
99992431^2 + 1740000^2 &= 100007569^2 & := & 10001513857289761 \\
9999992431^2 + 17400000^2 &= 10000007569^2 & := & 100000151380057289761 \\
999999992431^2 + 174000000^2 &= 1000000007569^2 & := & 1000000015138000057289761
\end{aligned} \tag{5.90}$$

$$\begin{aligned}
2256^2 + 17600^2 &= 17744^2 & := & 314849536 \\
992256^2 + 176000^2 &= 1007744^2 & := & 1015547969536 \\
99992256^2 + 1760000^2 &= 100007744^2 & := & 10001548859969536 \\
9999992256^2 + 17600000^2 &= 10000007744^2 & := & 100000154880059969536 \\
999999992256^2 + 176000000^2 &= 1000000007744^2 & := & 1000000015488000059969536
\end{aligned} \tag{5.91}$$

$$\begin{aligned}
2079^2 + 17800^2 &= 17921^2 & := & 321162241 \\
992079^2 + 178000^2 &= 1007921^2 & := & 1015904742241 \\
99992079^2 + 1780000^2 &= 100007921^2 & := & 10001584262742241 \\
9999992079^2 + 17800000^2 &= 10000007921^2 & := & 100000158420062742241 \\
999999992079^2 + 178000000^2 &= 1000000007921^2 & := & 1000000015842000062742241
\end{aligned} \tag{5.92}$$

$$\begin{aligned}
1900^2 + 18000^2 &= 18100^2 & := & 327610000 \\
991900^2 + 180000^2 &= 1008100^2 & := & 1016265610000 \\
99991900^2 + 1800000^2 &= 100008100^2 & := & 10001620065610000 \\
9999991900^2 + 18000000^2 &= 10000008100^2 & := & 100000162000065610000 \\
999999991900^2 + 180000000^2 &= 1000000008100^2 & := & 1000000016200000065610000
\end{aligned} \tag{5.93}$$

$$\begin{aligned}
1719^2 + 18200^2 &= 18281^2 & := 334194961 \\
991719^2 + 182000^2 &= 1008281^2 & := 1016630574961 \\
99991719^2 + 1820000^2 &= 100008281^2 & := 10001656268574961 \\
9999991719^2 + 18200000^2 &= 10000008281^2 & := 100000165620068574961 \\
999999991719^2 + 182000000^2 &= 1000000008281^2 & := 1000000016562000068574961
\end{aligned} \tag{5.94}$$

$$\begin{aligned}
1536^2 + 18400^2 &= 18464^2 & := 340919296 \\
991536^2 + 184000^2 &= 1008464^2 & := 1016999639296 \\
99991536^2 + 1840000^2 &= 100008464^2 & := 10001692871639296 \\
9999991536^2 + 18400000^2 &= 10000008464^2 & := 100000169280071639296 \\
999999991536^2 + 184000000^2 &= 1000000008464^2 & := 1000000016928000071639296
\end{aligned} \tag{5.95}$$

$$\begin{aligned}
1351^2 + 18600^2 &= 18649^2 & := 347785201 \\
991351^2 + 186000^2 &= 1008649^2 & := 1017372805201 \\
99991351^2 + 1860000^2 &= 100008649^2 & := 10001729874805201 \\
9999991351^2 + 18600000^2 &= 10000008649^2 & := 100000172980074805201 \\
999999991351^2 + 186000000^2 &= 1000000008649^2 & := 1000000017298000074805201
\end{aligned} \tag{5.96}$$

$$\begin{aligned}
1164^2 + 18800^2 &= 18836^2 & := 354794896 \\
991164^2 + 188000^2 &= 1008836^2 & := 1017750074896 \\
99991164^2 + 1880000^2 &= 100008836^2 & := 10001767278074896 \\
9999991164^2 + 18800000^2 &= 10000008836^2 & := 100000176720078074896 \\
999999991164^2 + 188000000^2 &= 1000000008836^2 & := 1000000017672000078074896
\end{aligned} \tag{5.97}$$

$$\begin{aligned}
975^2 + 19000^2 &= 19025^2 & := 361950625 \\
990975^2 + 190000^2 &= 1009025^2 & := 1018131450625 \\
99990975^2 + 1900000^2 &= 100009025^2 & := 10001805081450625 \\
9999990975^2 + 19000000^2 &= 10000009025^2 & := 100000180500081450625 \\
999999990975^2 + 190000000^2 &= 1000000009025^2 & := 1000000018050000081450625
\end{aligned} \tag{5.98}$$

$$\begin{aligned}
784^2 + 19200^2 &= 19216^2 & := 369254656 \\
990784^2 + 192000^2 &= 1009216^2 & := 1018516934656 \\
99990784^2 + 1920000^2 &= 100009216^2 & := 10001843284934656 \\
9999990784^2 + 19200000^2 &= 10000009216^2 & := 100000184320084934656 \\
999999990784^2 + 192000000^2 &= 1000000009216^2 & := 1000000018432000084934656
\end{aligned} \tag{5.99}$$

$$\begin{aligned}
591^2 + 19400^2 &= 19409^2 & := & 376709281 \\
990591^2 + 194000^2 &= 1009409^2 & := & 1018906529281 \\
99990591^2 + 1940000^2 &= 100009409^2 & := & 10001881888529281 \\
9999990591^2 + 19400000^2 &= 10000009409^2 & := & 100000188180088529281 \\
999999990591^2 + 194000000^2 &= 1000000009409^2 & := & 1000000018818000088529281 \quad (5.100)
\end{aligned}$$

$$\begin{aligned}
396^2 + 19600^2 &= 19604^2 & := & 384316816 \\
990396^2 + 196000^2 &= 1009604^2 & := & 1019300236816 \\
99990396^2 + 1960000^2 &= 100009604^2 & := & 10001920892236816 \\
9999990396^2 + 19600000^2 &= 10000009604^2 & := & 100000192080092236816 \\
999999990396^2 + 196000000^2 &= 1000000009604^2 & := & 1000000019208000092236816 \quad (5.101)
\end{aligned}$$

$$\begin{aligned}
199^2 + 19800^2 &= 19801^2 & := & 392079601 \\
990199^2 + 198000^2 &= 1009801^2 & := & 1019698059601 \\
99990199^2 + 1980000^2 &= 100009801^2 & := & 10001960296059601 \\
9999990199^2 + 19800000^2 &= 10000009801^2 & := & 100000196020096059601 \\
999999990199^2 + 198000000^2 &= 1000000009801^2 & := & 1000000019602000096059601 \quad (5.102)
\end{aligned}$$

Remark 5. The first 9 patterns are the same as given in 1st block of subsection 3.3, starting from second line. Some initial lines in the final sums don't obey the regular pattern. They are specified with different colors.

5.3 Palindromic-Type Pandigital Patterns

Below are 99 examples of **palindromic-type pandigital patterns** with 9 digits from 1 to 9 written as **palindromic-type**. This subsection is divided in two parts. The first part give first 19 patterns are complete. The other 80 are written only with first four lines. The other five lines follows similar lines. There are two ways to write these patterns. One is normal palindromic-type, i.e.,

$$1, 121, 12321, 1234321, \dots, 12345678987654321$$

and second is palindromic-type with 0 between each digit, i.e.,

$$1, 10201, 102030201, 1020304030201, \dots, 102030405060708090807060504030201.$$

5.3.1 First Way

For $n = 1, 2, 3, \dots, 17, 18, 19$:

Let's consider

$$m = 100, 1100, 11100, 111100, 1111100, 11111100, 111111100, 1111111100$$

in (2.1), then for each value of $n = 1, 2, 3, \dots, 17, 18, 19$; there are 19 **palindromic-type pandigital Pythagorean patterns** given below:

$$\begin{aligned}
 09999^2 + 200^2 &= 1\ 0001^2 \\
 12\ 09999^2 + 2200^2 &= 121\ 0001^2 \\
 1232\ 09999^2 + 22200^2 &= 12321\ 0001^2 \\
 123432\ 09999^2 + 222200^2 &= 1234321\ 0001^2 \\
 12345432\ 09999^2 + 2222200^2 &= 123454321\ 0001^2 \\
 1234565432\ 09999^2 + 22222200^2 &= 12345654321\ 0001^2 \\
 123456765432\ 09999^2 + 222222200^2 &= 1234567654321\ 0001^2 \\
 12345678765432\ 09999^2 + 2222222200^2 &= 123456787654321\ 0001^2 \\
 1234567898765432\ 09999^2 + 22222222200^2 &= 12345678987654321\ 0001^2 \quad (5.103)
 \end{aligned}$$

$$\begin{aligned}
 09996^2 + 400^2 &= 1\ 0004^2 \\
 12\ 09996^2 + 4400^2 &= 121\ 0004^2 \\
 1232\ 09996^2 + 44400^2 &= 12321\ 0004^2 \\
 123432\ 09996^2 + 444400^2 &= 1234321\ 0004^2 \\
 12345432\ 09996^2 + 4444400^2 &= 123454321\ 0004^2 \\
 1234565432\ 09996^2 + 44444400^2 &= 12345654321\ 0004^2 \\
 123456765432\ 09996^2 + 444444400^2 &= 1234567654321\ 0004^2 \\
 12345678765432\ 09996^2 + 4444444400^2 &= 123456787654321\ 0004^2 \\
 1234567898765432\ 09996^2 + 44444444400^2 &= 12345678987654321\ 0004^2 \quad (5.104)
 \end{aligned}$$

$$\begin{aligned}
 09991^2 + 600^2 &= 1\ 0009^2 \\
 12\ 09991^2 + 6600^2 &= 121\ 0009^2 \\
 1232\ 09991^2 + 66600^2 &= 12321\ 0009^2 \\
 123432\ 09991^2 + 666600^2 &= 1234321\ 0009^2 \\
 12345432\ 09991^2 + 6666600^2 &= 123454321\ 0009^2 \\
 1234565432\ 09991^2 + 66666600^2 &= 12345654321\ 0009^2 \\
 123456765432\ 09991^2 + 666666600^2 &= 1234567654321\ 0009^2 \\
 12345678765432\ 09991^2 + 6666666600^2 &= 123456787654321\ 0009^2 \\
 1234567898765432\ 09991^2 + 66666666600^2 &= 12345678987654321\ 0009^2 \quad (5.105)
 \end{aligned}$$

$$\begin{aligned}
09984^2 + 800^2 &= 1\ 0016^2 \\
12\ 09984^2 + 8800^2 &= 121\ 0016^2 \\
1232\ 09984^2 + 88800^2 &= 12321\ 0016^2 \\
123432\ 09984^2 + 888800^2 &= 1234321\ 0016^2 \\
12345432\ 09984^2 + 8888800^2 &= 123454321\ 0016^2 \\
1234565432\ 09984^2 + 88888800^2 &= 12345654321\ 0016^2 \\
123456765432\ 09984^2 + 888888800^2 &= 1234567654321\ 0016^2 \\
12345678765432\ 09984^2 + 8888888800^2 &= 123456787654321\ 0016^2 \\
1234567898765432\ 09984^2 + 88888888800^2 &= 12345678987654321\ 0016^2 \quad (5.106)
\end{aligned}$$

$$\begin{aligned}
09975^2 + 1000^2 &= 1\ 0025^2 \\
12\ 09975^2 + 11000^2 &= 121\ 0025^2 \\
1232\ 09975^2 + 111000^2 &= 12321\ 0025^2 \\
123432\ 09975^2 + 1111000^2 &= 1234321\ 0025^2 \\
12345432\ 09975^2 + 11111000^2 &= 123454321\ 0025^2 \\
1234565432\ 09975^2 + 111111000^2 &= 12345654321\ 0025^2 \\
123456765432\ 09975^2 + 1111111000^2 &= 1234567654321\ 0025^2 \\
12345678765432\ 09975^2 + 11111111000^2 &= 123456787654321\ 0025^2 \\
1234567898765432\ 09975^2 + 111111111000^2 &= 12345678987654321\ 0025^2 \quad (5.107)
\end{aligned}$$

$$\begin{aligned}
09964^2 + 1200^2 &= 1\ 0036^2 \\
12\ 09964^2 + 13200^2 &= 121\ 0036^2 \\
1232\ 09964^2 + 133200^2 &= 12321\ 0036^2 \\
123432\ 09964^2 + 1333200^2 &= 1234321\ 0036^2 \\
12345432\ 09964^2 + 13333200^2 &= 123454321\ 0036^2 \\
1234565432\ 09964^2 + 133333200^2 &= 12345654321\ 0036^2 \\
123456765432\ 09964^2 + 1333333200^2 &= 1234567654321\ 0036^2 \\
12345678765432\ 09964^2 + 13333333200^2 &= 123456787654321\ 0036^2 \\
1234567898765432\ 09964^2 + 133333333200^2 &= 12345678987654321\ 0036^2 \quad (5.108)
\end{aligned}$$

$$\begin{aligned}
09951^2 + 1400^2 &= 1\ 49^2 \\
12\ 09951^2 + 15400^2 &= 121\ 0049^2 \\
1232\ 09951^2 + 155400^2 &= 12321\ 0049^2 \\
123432\ 09951^2 + 1555400^2 &= 1234321\ 0049^2 \\
12345432\ 09951^2 + 15555400^2 &= 123454321\ 0049^2 \\
1234565432\ 09951^2 + 155555400^2 &= 12345654321\ 0049^2 \\
123456765432\ 09951^2 + 1555555400^2 &= 1234567654321\ 0049^2 \\
12345678765432\ 09951^2 + 15555555400^2 &= 123456787654321\ 0049^2 \\
1234567898765432\ 09951^2 + 155555555400^2 &= 12345678987654321\ 0049^2 \quad (5.109)
\end{aligned}$$

$$\begin{aligned}
09936^2 + 1600^2 &= 1\ 0064^2 \\
12\ 09936^2 + 17600^2 &= 121\ 0064^2 \\
1232\ 09936^2 + 177600^2 &= 12321\ 0064^2 \\
123432\ 09936^2 + 1777600^2 &= 1234321\ 0064^2 \\
12345432\ 09936^2 + 17777600^2 &= 123454321\ 0064^2 \\
1234565432\ 09936^2 + 177777600^2 &= 12345654321\ 0064^2 \\
123456765432\ 09936^2 + 1777777600^2 &= 1234567654321\ 0064^2 \\
12345678765432\ 09936^2 + 17777777600^2 &= 123456787654321\ 0064^2 \\
1234567898765432\ 09936^2 + 177777777600^2 &= 12345678987654321\ 0064^2 \quad (5.110)
\end{aligned}$$

$$\begin{aligned}
09919^2 + 1800^2 &= 1\ 81^2 \\
12\ 09919^2 + 19800^2 &= 121\ 0081^2 \\
1232\ 09919^2 + 199800^2 &= 12321\ 0081^2 \\
123432\ 09919^2 + 1999800^2 &= 1234321\ 0081^2 \\
12345432\ 09919^2 + 19999800^2 &= 123454321\ 0081^2 \\
1234565432\ 09919^2 + 199999800^2 &= 12345654321\ 0081^2 \\
123456765432\ 09919^2 + 1999999800^2 &= 1234567654321\ 0081^2 \\
12345678765432\ 09919^2 + 19999999800^2 &= 123456787654321\ 0081^2 \\
1234567898765432\ 09919^2 + 199999999800^2 &= 12345678987654321\ 0081^2 \quad (5.111)
\end{aligned}$$

$$\begin{aligned}
09900^2 + 2000^2 &= 1\ 0100^2 \\
12\ 09900^2 + 22000^2 &= 121\ 0100^2 \\
1232\ 09900^2 + 222000^2 &= 12321\ 0100^2 \\
123432\ 09900^2 + 2222000^2 &= 1234321\ 0100^2 \\
12345432\ 09900^2 + 22222000^2 &= 123454321\ 0100^2 \\
1234565432\ 09900^2 + 222222000^2 &= 12345654321\ 0100^2 \\
123456765432\ 09900^2 + 2222222000^2 &= 1234567654321\ 0100^2 \\
12345678765432\ 09900^2 + 22222222000^2 &= 123456787654321\ 0100^2 \\
1234567898765432\ 09900^2 + 222222222000^2 &= 12345678987654321\ 0100^2 \quad (5.112)
\end{aligned}$$

$$\begin{aligned}
09879^2 + 2200^2 &= 10121^2 \\
1209879^2 + 24200^2 &= 1210121^2 \\
123209879^2 + 244200^2 &= 123210121^2 \\
12343209879^2 + 2444200^2 &= 12343210121^2 \\
1234543209879^2 + 24444200^2 &= 1234543210121^2 \\
123456543209879^2 + 244444200^2 &= 123456543210121^2 \\
12345676543209879^2 + 2444444200^2 &= 12345676543210121^2 \\
1234567876543209879^2 + 24444444200^2 &= 1234567876543210121^2 \\
123456789876543209879^2 + 244444444200^2 &= 123456789876543210121^2 \quad (5.113)
\end{aligned}$$

$$\begin{aligned}
09856^2 + 2400^2 &= 10144^2 \\
1209856^2 + 26400^2 &= 1210144^2 \\
123209856^2 + 266400^2 &= 123210144^2 \\
12343209856^2 + 2666400^2 &= 12343210144^2 \\
1234543209856^2 + 26666400^2 &= 1234543210144^2 \\
123456543209856^2 + 266666400^2 &= 123456543210144^2 \\
12345676543209856^2 + 2666666400^2 &= 12345676543210144^2 \\
1234567876543209856^2 + 26666666400^2 &= 1234567876543210144^2 \\
123456789876543209856^2 + 266666666400^2 &= 123456789876543210144^2 \quad (5.114)
\end{aligned}$$

$$\begin{aligned}
09831^2 + 2600^2 &= 10169^2 \\
1209831^2 + 28600^2 &= 1210169^2 \\
123209831^2 + 288600^2 &= 123210169^2 \\
12343209831^2 + 2888600^2 &= 12343210169^2 \\
1234543209831^2 + 28888600^2 &= 1234543210169^2 \\
123431543209831^2 + 288888600^2 &= 123431543210169^2 \\
12343176543209831^2 + 2888888600^2 &= 12343176543210169^2 \\
1234317876543209831^2 + 28888888600^2 &= 1234317876543210169^2 \\
123431789876543209831^2 + 288888888600^2 &= 123431789876543210169^2 \quad (5.115)
\end{aligned}$$

$$\begin{aligned}
09804^2 + 2800^2 &= 10196^2 \\
1209804^2 + 30800^2 &= 1210196^2 \\
123209804^2 + 310800^2 &= 123210196^2 \\
12343209804^2 + 3110800^2 &= 12343210196^2 \\
1234543209804^2 + 31110800^2 &= 1234543210196^2 \\
123456543209804^2 + 311110800^2 &= 123404543210196^2 \\
12345676543209804^2 + 3111110800^2 &= 12340476543210196^2 \\
1234567876543209804^2 + 31111110800^2 &= 1234047876543210196^2 \\
123456789876543209804^2 + 311111110800^2 &= 123404789876543210196^2 \quad (5.116)
\end{aligned}$$

$$\begin{aligned}
09775^2 + 3300^2 &= 1\ 0225^2 \\
12\ 09775^2 + 33000^2 &= 121\ 0225^2 \\
1232\ 09775^2 + 333000^2 &= 12321\ 0225^2 \\
123432\ 09775^2 + 3333000^2 &= 1234321\ 0225^2 \\
12345432\ 09775^2 + 33333000^2 &= 123454321\ 0225^2 \\
1234045432\ 09775^2 + 333333000^2 &= 12340454321\ 0225^2 \\
123404765432\ 09775^2 + 3333333000^2 &= 1234047654321\ 0225^2 \\
12340478765432\ 09775^2 + 33333333000^2 &= 123404787654321\ 0225^2 \\
1234047898765432\ 09775^2 + 333333333000^2 &= 12340478987654321\ 0225^2 \quad (5.117)
\end{aligned}$$

$$\begin{aligned}
09744^2 + 3200^2 &= 1\ 0256^2 \\
12\ 09744^2 + 35200^2 &= 121\ 0256^2 \\
1232\ 09744^2 + 355200^2 &= 12321\ 0256^2 \\
123432\ 09744^2 + 3555200^2 &= 1234321\ 0256^2 \\
12345432\ 09744^2 + 35555200^2 &= 123454321\ 0256^2 \\
1234045432\ 09744^2 + 355555200^2 &= 12340454321\ 0256^2 \\
123404765432\ 09744^2 + 3555555200^2 &= 1234047654321\ 0256^2 \\
12340478765432\ 09744^2 + 35555555200^2 &= 123404787654321\ 0256^2 \\
1234047898765432\ 09744^2 + 355555555200^2 &= 12340478987654321\ 0256^2 \quad (5.118)
\end{aligned}$$

$$\begin{aligned}
09711^2 + 3400^2 &= 1\ 0289^2 \\
12\ 09711^2 + 37400^2 &= 121\ 0289^2 \\
1232\ 09711^2 + 377400^2 &= 12321\ 0289^2 \\
123432\ 09711^2 + 3777400^2 &= 1234321\ 0289^2 \\
12345432\ 09711^2 + 37777400^2 &= 123454321\ 0289^2 \\
1234045432\ 09711^2 + 377777400^2 &= 12340454321\ 0289^2 \\
123404765432\ 09711^2 + 3777777400^2 &= 1234047654321\ 0289^2 \\
12340478765432\ 09711^2 + 37777777400^2 &= 123404787654321\ 0289^2 \\
1234047898765432\ 09711^2 + 377777777400^2 &= 12340478987654321\ 0289^2 \quad (5.119)
\end{aligned}$$

$$\begin{aligned}
 09676^2 + 3600^2 &= 1\ 0324^2 \\
 12\ 09676^2 + 39600^2 &= 121\ 0324^2 \\
 1232\ 09676^2 + 399600^2 &= 12321\ 0324^2 \\
 123432\ 09676^2 + 3999600^2 &= 1234321\ 0324^2 \\
 12345432\ 09676^2 + 39999600^2 &= 123454321\ 0324^2 \\
 1234565432\ 09676^2 + 399999600^2 &= 12340454321\ 0324^2 \\
 123456765432\ 09676^2 + 3999999600^2 &= 1234047654321\ 0324^2 \\
 12345678765432\ 09676^2 + 39999999600^2 &= 123404787654321\ 0324^2 \\
 1234567898765432\ 09676^2 + 399999999600^2 &= 12340478987654321\ 0324^2 \quad (5.120)
 \end{aligned}$$

$$\begin{aligned}
 09639^2 + 3800^2 &= 1\ 0361^2 \\
 12\ 09639^2 + 41800^2 &= 121\ 0361^2 \\
 1232\ 09639^2 + 421800^2 &= 12321\ 0361^2 \\
 123432\ 09639^2 + 4221800^2 &= 1234321\ 0361^2 \\
 12345432\ 09639^2 + 42221800^2 &= 123454321\ 0361^2 \\
 12345645432\ 09639^2 + 422221800^2 &= 12340454321\ 0361^2 \\
 123456765432\ 09639^2 + 4222221800^2 &= 1234047654321\ 0361^2 \\
 12345678765432\ 09639^2 + 42222221800^2 &= 123404787654321\ 0361^2 \\
 1234567898765432\ 09639^2 + 422222221800^2 &= 12340478987654321\ 0361^2 \quad (5.121)
 \end{aligned}$$

• For $n = 20, 21, 23, \dots, 97, 98, 99$:

Let's consider

$$m = 100, 1100, 11100, 111100, 1111100, 11111100, 111111100, 1111111100, 11111111100$$

in (2.1), then for each value of $n = 20, 21, 23, \dots, 97, 98, 99$; there are 80 **palindromic-type pandigital Pythagorean patterns** given below. In each case, only first four values are written. The other 5 values can be written on similar lines as done in subsection 5.3.1.

$$\begin{aligned}
 09600^2 + 4000^2 &= 1\ 0400^2 & 09516^2 + 4400^2 &= 1\ 0484^2 \\
 12\ 09600^2 + 44000^2 &= 121\ 0400^2 & 12\ 09516^2 + 48400^2 &= 121\ 0484^2 \\
 1232\ 09600^2 + 444000^2 &= 12321\ 0400^2 & 1232\ 09516^2 + 488400^2 &= 12321\ 0484^2 \\
 123432\ 09600^2 + 4444000^2 &= 1234321\ 0400^2 \quad (5.122) & 123432\ 09516^2 + 4888400^2 &= 1234321\ 0484^2 \quad (5.124)
 \end{aligned}$$

$$\begin{aligned}
 09559^2 + 4200^2 &= 1\ 0441^2 & 09471^2 + 4600^2 &= 1\ 0529^2 \\
 12\ 09559^2 + 46200^2 &= 121\ 0441^2 & 12\ 09471^2 + 50600^2 &= 121\ 0529^2 \\
 1232\ 09559^2 + 466200^2 &= 12321\ 0441^2 & 1232\ 09471^2 + 510600^2 &= 12321\ 0529^2 \\
 123432\ 09559^2 + 4666200^2 &= 1234321\ 0441^2 \quad (5.123) & 123432\ 09471^2 + 5110600^2 &= 1234321\ 0529^2 \quad (5.125)
 \end{aligned}$$

$$\begin{aligned}
 09424^2 + 4800^2 &= 1\ 0576^2 \\
 12\ 09424^2 + 52800^2 &= 121\ 0576^2 \\
 1232\ 09424^2 + 532800^2 &= 12321\ 0576^2 \\
 123432\ 09424^2 + 5332800^2 &= 1234321\ 0576^2 \quad (5.126)
 \end{aligned}$$

$$\begin{aligned}
 09375^2 + 5000^2 &= 1\ 0625^2 \\
 12\ 09375^2 + 55000^2 &= 121\ 0625^2 \\
 1232\ 09375^2 + 555000^2 &= 12321\ 0625^2 \\
 123432\ 09375^2 + 5555000^2 &= 1234321\ 0625^2 \quad (5.127)
 \end{aligned}$$

$$\begin{aligned}
 09324^2 + 5200^2 &= 1\ 0676^2 \\
 12\ 09324^2 + 57200^2 &= 121\ 0676^2 \\
 1232\ 09324^2 + 577200^2 &= 12321\ 0676^2 \\
 123432\ 09324^2 + 5777200^2 &= 1234321\ 0676^2 \quad (5.128)
 \end{aligned}$$

$$\begin{aligned}
 09271^2 + 5400^2 &= 1\ 0729^2 \\
 12\ 09271^2 + 59400^2 &= 121\ 0729^2 \\
 1232\ 09271^2 + 599400^2 &= 12321\ 0729^2 \\
 123432\ 09271^2 + 5999400^2 &= 1234321\ 0729^2 \quad (5.129)
 \end{aligned}$$

$$\begin{aligned}
 09216^2 + 5600^2 &= 1\ 0784^2 \\
 12\ 09216^2 + 61600^2 &= 121\ 0784^2 \\
 1232\ 09216^2 + 621600^2 &= 12321\ 0784^2 \\
 123432\ 09216^2 + 6221600^2 &= 1234321\ 0784^2 \quad (5.130)
 \end{aligned}$$

$$\begin{aligned}
 09159^2 + 5800^2 &= 1\ 0841^2 \\
 12\ 09159^2 + 63800^2 &= 121\ 0841^2 \\
 1232\ 09159^2 + 643800^2 &= 12321\ 0841^2 \\
 123432\ 09159^2 + 6443800^2 &= 1234321\ 0841^2 \quad (5.131)
 \end{aligned}$$

$$\begin{aligned}
 09100^2 + 6000^2 &= 1\ 0900^2 \\
 12\ 09100^2 + 66000^2 &= 121\ 0900^2 \\
 1232\ 09100^2 + 666000^2 &= 12321\ 0900^2 \\
 123432\ 09100^2 + 6666000^2 &= 1234321\ 0900^2 \quad (5.132)
 \end{aligned}$$

$$\begin{aligned}
 09039^2 + 6200^2 &= 1\ 0961^2 \\
 12\ 09039^2 + 68200^2 &= 121\ 0961^2 \\
 1232\ 09039^2 + 688200^2 &= 12321\ 0961^2 \\
 123432\ 09039^2 + 6888200^2 &= 1234321\ 0961^2 \quad (5.133)
 \end{aligned}$$

$$\begin{aligned}
 08976^2 + 6400^2 &= 1\ 1024^2 \\
 12\ 08976^2 + 70400^2 &= 121\ 1024^2 \\
 1232\ 08976^2 + 710400^2 &= 12321\ 1024^2 \\
 123432\ 08976^2 + 7110400^2 &= 1234321\ 1024^2 \quad (5.134)
 \end{aligned}$$

$$\begin{aligned}
 08911^2 + 6600^2 &= 1\ 1089^2 \\
 12\ 08911^2 + 72600^2 &= 121\ 1089^2 \\
 1232\ 08911^2 + 732600^2 &= 12321\ 1089^2 \\
 123432\ 08911^2 + 7332600^2 &= 1234321\ 1089^2 \quad (5.135)
 \end{aligned}$$

$$\begin{aligned}
 08844^2 + 6800^2 &= 1\ 1156^2 \\
 12\ 08844^2 + 74800^2 &= 121\ 1156^2 \\
 1232\ 08844^2 + 754800^2 &= 12321\ 1156^2 \\
 123432\ 08844^2 + 7554800^2 &= 1234321\ 1156^2 \quad (5.136)
 \end{aligned}$$

$$\begin{aligned}
 08775^2 + 7000^2 &= 1\ 1225^2 \\
 12\ 08775^2 + 77000^2 &= 121\ 1225^2 \\
 1232\ 08775^2 + 777000^2 &= 12321\ 1225^2 \\
 123432\ 08775^2 + 7777000^2 &= 1234321\ 1225^2 \quad (5.137)
 \end{aligned}$$

$$\begin{aligned}
 08704^2 + 7200^2 &= 1\ 1296^2 \\
 12\ 08704^2 + 79200^2 &= 121\ 1296^2 \\
 1232\ 08704^2 + 799200^2 &= 12321\ 1296^2 \\
 123432\ 08704^2 + 7999200^2 &= 1234321\ 1296^2 \quad (5.138)
 \end{aligned}$$

$$\begin{aligned}
 08631^2 + 7400^2 &= 1\ 1369^2 \\
 12\ 08631^2 + 81400^2 &= 121\ 1369^2 \\
 1232\ 08631^2 + 821400^2 &= 12321\ 1369^2 \\
 123432\ 08631^2 + 8221400^2 &= 1234321\ 1369^2 \quad (5.139)
 \end{aligned}$$

$$\begin{aligned}
 08556^2 + 7600^2 &= 1\ 1444^2 \\
 12\ 08556^2 + 83600^2 &= 121\ 1444^2 \\
 1232\ 08556^2 + 843600^2 &= 12321\ 1444^2 \\
 123432\ 08556^2 + 8443600^2 &= 1234321\ 1444^2 \quad (5.140)
 \end{aligned}$$

$$\begin{aligned}
 08479^2 + 7800^2 &= 1\ 1521^2 \\
 12\ 08479^2 + 85800^2 &= 121\ 1521^2 \\
 1232\ 08479^2 + 865800^2 &= 12321\ 1521^2 \\
 123432\ 08479^2 + 8665800^2 &= 1234321\ 1521^2 \quad (5.141)
 \end{aligned}$$

$$\begin{aligned}
 08400^2 + 8000^2 &= 1\ 1600^2 \\
 12\ 08400^2 + 88000^2 &= 121\ 1600^2 \\
 1232\ 08400^2 + 888000^2 &= 12321\ 1600^2 \\
 123432\ 08400^2 + 8888000^2 &= 1234321\ 1600^2 \quad (5.142)
 \end{aligned}$$

$$\begin{aligned}
 08319^2 + 8200^2 &= 1\ 1681^2 \\
 12\ 08319^2 + 90200^2 &= 121\ 1681^2 \\
 1232\ 08319^2 + 910200^2 &= 12321\ 1681^2 \\
 123432\ 08319^2 + 9110200^2 &= 1234321\ 1681^2 \quad (5.143)
 \end{aligned}$$

$$\begin{aligned}
 08236^2 + 8400^2 &= 1\ 1764^2 \\
 12\ 08236^2 + 92400^2 &= 121\ 1764^2 \\
 1232\ 08236^2 + 932400^2 &= 12321\ 1764^2 \\
 123432\ 08236^2 + 9332400^2 &= 1234321\ 1764^2 \quad (5.144)
 \end{aligned}$$

$$\begin{aligned}
 08151^2 + 8600^2 &= 1\ 1849^2 \\
 12\ 08151^2 + 94600^2 &= 121\ 1849^2 \\
 1232\ 08151^2 + 954600^2 &= 12321\ 1849^2 \\
 123432\ 08151^2 + 9554600^2 &= 1234321\ 1849^2 \quad (5.145)
 \end{aligned}$$

$$\begin{aligned}
 08064^2 + 8800^2 &= 1\ 1936^2 \\
 12\ 08064^2 + 96800^2 &= 121\ 1936^2 \\
 1232\ 08064^2 + 976800^2 &= 12321\ 1936^2 \\
 123432\ 08064^2 + 9776800^2 &= 1234321\ 1936^2 \quad (5.146)
 \end{aligned}$$

$$\begin{aligned}
 07975^2 + 9000^2 &= 1\ 2025^2 \\
 12\ 07975^2 + 99000^2 &= 121\ 2025^2 \\
 1232\ 07975^2 + 999000^2 &= 12321\ 2025^2 \\
 123432\ 07975^2 + 9999000^2 &= 1234321\ 2025^2 \quad (5.147)
 \end{aligned}$$

$$\begin{aligned}
 07884^2 + 9200^2 &= 1\ 2116^2 \\
 12\ 07884^2 + 101200^2 &= 121\ 2116^2 \\
 1232\ 07884^2 + 1021200^2 &= 12321\ 2116^2 \\
 123432\ 07884^2 + 10221200^2 &= 1234321\ 2116^2 \quad (5.148)
 \end{aligned}$$

$$\begin{aligned}
 07791^2 + 9400^2 &= 1\ 2209^2 \\
 12\ 07791^2 + 103400^2 &= 121\ 2209^2 \\
 1232\ 07791^2 + 1043400^2 &= 12321\ 2209^2 \\
 123432\ 07791^2 + 10443400^2 &= 1234321\ 2209^2 \quad (5.149)
 \end{aligned}$$

$$\begin{aligned}
 07696^2 + 9600^2 &= 1\ 2304^2 \\
 12\ 07696^2 + 105600^2 &= 121\ 2304^2 \\
 1232\ 07696^2 + 1065600^2 &= 12321\ 2304^2 \\
 123432\ 07696^2 + 10665600^2 &= 1234321\ 2304^2 \quad (5.150)
 \end{aligned}$$

$$\begin{aligned}
 07599^2 + 9800^2 &= 1\ 2401^2 \\
 12\ 07599^2 + 107800^2 &= 121\ 2401^2 \\
 1232\ 07599^2 + 1087800^2 &= 12321\ 2401^2 \\
 123432\ 07599^2 + 10887800^2 &= 1234321\ 2401^2 \quad (5.151)
 \end{aligned}$$

$$\begin{aligned}
 07500^2 + 10000^2 &= 1\ 2500^2 \\
 12\ 07500^2 + 110000^2 &= 121\ 2500^2 \\
 1232\ 07500^2 + 1110000^2 &= 12321\ 2500^2 \\
 123432\ 07500^2 + 11110000^2 &= 1234321\ 2500^2 \quad (5.152)
 \end{aligned}$$

$$\begin{aligned}
 07399^2 + 10200^2 &= 1\ 2601^2 \\
 12\ 07399^2 + 112200^2 &= 121\ 2601^2 \\
 1232\ 07399^2 + 1132200^2 &= 12321\ 2601^2 \\
 123432\ 07399^2 + 11332200^2 &= 1234321\ 2601^2 \quad (5.153)
 \end{aligned}$$

$$\begin{aligned}
 07296^2 + 10400^2 &= 1\ 2704^2 \\
 12\ 07296^2 + 114400^2 &= 121\ 2704^2 \\
 1232\ 07296^2 + 1154400^2 &= 12321\ 2704^2 \\
 123432\ 07296^2 + 11554400^2 &= 1234321\ 2704^2 \quad (5.154)
 \end{aligned}$$

$$\begin{aligned}
 07191^2 + 10600^2 &= 1\ 2809^2 \\
 12\ 07191^2 + 116600^2 &= 121\ 2809^2 \\
 1232\ 07191^2 + 1176600^2 &= 12321\ 2809^2 \\
 123432\ 07191^2 + 11776600^2 &= 1234321\ 2809^2 \quad (5.155)
 \end{aligned}$$

$$\begin{aligned}
 07084^2 + 10800^2 &= 1\ 2916^2 \\
 12\ 07084^2 + 118800^2 &= 121\ 2916^2 \\
 1232\ 07084^2 + 1198800^2 &= 12321\ 2916^2 \\
 123432\ 07084^2 + 11998800^2 &= 1234321\ 2916^2 \quad (5.156)
 \end{aligned}$$

$$\begin{aligned}
 06975^2 + 11000^2 &= 1\ 3025^2 \\
 12\ 06975^2 + 121000^2 &= 121\ 3025^2 \\
 1232\ 06975^2 + 1221000^2 &= 12321\ 3025^2 \\
 123432\ 06975^2 + 12221000^2 &= 1234321\ 3025^2 \quad (5.157)
 \end{aligned}$$

$$\begin{aligned}
 06864^2 + 11200^2 &= 1\ 3136^2 \\
 12\ 06864^2 + 123200^2 &= 121\ 3136^2 \\
 1232\ 06864^2 + 1243200^2 &= 12321\ 3136^2 \\
 123432\ 06864^2 + 12443200^2 &= 1234321\ 3136^2 \quad (5.158)
 \end{aligned}$$

$$\begin{aligned}
 06751^2 + 11400^2 &= 1\ 3249^2 \\
 12\ 06751^2 + 125400^2 &= 121\ 3249^2 \\
 1232\ 06751^2 + 1265400^2 &= 12321\ 3249^2 \\
 123432\ 06751^2 + 12665400^2 &= 1234321\ 3249^2 \quad (5.159)
 \end{aligned}$$

$$\begin{aligned}
06636^2 + 11600^2 &= 1\ 3364^2 \\
12\ 06636^2 + 127600^2 &= 121\ 3364^2 \\
1232\ 06636^2 + 1287600^2 &= 12321\ 3364^2 \\
123432\ 06636^2 + 12887600^2 &= 1234321\ 3364^2 \quad (5.160)
\end{aligned}$$

$$\begin{aligned}
06519^2 + 11800^2 &= 1\ 3481^2 \\
12\ 06519^2 + 129800^2 &= 121\ 3481^2 \\
1232\ 06519^2 + 1309800^2 &= 12321\ 3481^2 \\
123432\ 06519^2 + 13109800^2 &= 1234321\ 3481^2 \quad (5.161)
\end{aligned}$$

$$\begin{aligned}
06400^2 + 12000^2 &= 1\ 3600^2 \\
12\ 06400^2 + 132000^2 &= 121\ 3600^2 \\
1232\ 06400^2 + 1332000^2 &= 12321\ 3600^2 \\
123432\ 06400^2 + 13332000^2 &= 1234321\ 3600^2 \quad (5.162)
\end{aligned}$$

$$\begin{aligned}
06279^2 + 12200^2 &= 1\ 3721^2 \\
12\ 06279^2 + 134200^2 &= 121\ 3721^2 \\
1232\ 06279^2 + 1354200^2 &= 12321\ 3721^2 \\
123432\ 06279^2 + 13554200^2 &= 1234321\ 3721^2 \quad (5.163)
\end{aligned}$$

$$\begin{aligned}
06156^2 + 12400^2 &= 1\ 3844^2 \\
12\ 06156^2 + 136400^2 &= 121\ 3844^2 \\
1232\ 06156^2 + 1376400^2 &= 12321\ 3844^2 \\
123432\ 06156^2 + 13776400^2 &= 1234321\ 3844^2 \quad (5.164)
\end{aligned}$$

$$\begin{aligned}
06031^2 + 12600^2 &= 1\ 3969^2 \\
12\ 06031^2 + 138600^2 &= 121\ 3969^2 \\
1232\ 06031^2 + 1398600^2 &= 12321\ 3969^2 \\
123432\ 06031^2 + 13998600^2 &= 1234321\ 3969^2 \quad (5.165)
\end{aligned}$$

$$\begin{aligned}
05904^2 + 12800^2 &= 1\ 4096^2 \\
12\ 05904^2 + 140800^2 &= 121\ 4096^2 \\
1232\ 05904^2 + 1420800^2 &= 12321\ 4096^2 \\
123432\ 05904^2 + 14220800^2 &= 1234321\ 4096^2 \quad (5.166)
\end{aligned}$$

$$\begin{aligned}
05775^2 + 13000^2 &= 1\ 4225^2 \\
12\ 05775^2 + 143000^2 &= 121\ 4225^2 \\
1232\ 05775^2 + 1443000^2 &= 12321\ 4225^2 \\
123432\ 05775^2 + 14443000^2 &= 1234321\ 4225^2 \quad (5.167)
\end{aligned}$$

$$\begin{aligned}
05644^2 + 13200^2 &= 1\ 4356^2 \\
12\ 05644^2 + 145200^2 &= 121\ 4356^2 \\
1232\ 05644^2 + 1465200^2 &= 12321\ 4356^2 \\
123432\ 05644^2 + 14665200^2 &= 1234321\ 4356^2 \quad (5.168)
\end{aligned}$$

$$\begin{aligned}
05511^2 + 13400^2 &= 1\ 4489^2 \\
12\ 05511^2 + 147400^2 &= 121\ 4489^2 \\
1232\ 05511^2 + 1487400^2 &= 12321\ 4489^2 \\
123432\ 05511^2 + 14887400^2 &= 1234321\ 4489^2 \quad (5.169)
\end{aligned}$$

$$\begin{aligned}
05376^2 + 13600^2 &= 1\ 4624^2 \\
12\ 05376^2 + 149600^2 &= 121\ 4624^2 \\
1232\ 05376^2 + 1509600^2 &= 12321\ 4624^2 \\
123432\ 05376^2 + 15109600^2 &= 1234321\ 4624^2 \quad (5.170)
\end{aligned}$$

$$\begin{aligned}
05239^2 + 13800^2 &= 1\ 4761^2 \\
12\ 05239^2 + 151800^2 &= 121\ 4761^2 \\
1232\ 05239^2 + 1531800^2 &= 12321\ 4761^2 \\
123432\ 05239^2 + 15331800^2 &= 1234321\ 4761^2 \quad (5.171)
\end{aligned}$$

$$\begin{aligned}
05100^2 + 14000^2 &= 1\ 4900^2 \\
12\ 05100^2 + 154000^2 &= 121\ 4900^2 \\
1232\ 05100^2 + 1554000^2 &= 12321\ 4900^2 \\
123432\ 05100^2 + 15554000^2 &= 1234321\ 4900^2 \quad (5.172)
\end{aligned}$$

$$\begin{aligned}
04959^2 + 14200^2 &= 1\ 5041^2 \\
12\ 04959^2 + 156200^2 &= 121\ 5041^2 \\
1232\ 04959^2 + 1576200^2 &= 12321\ 5041^2 \\
123432\ 04959^2 + 15776200^2 &= 1234321\ 5041^2 \quad (5.173)
\end{aligned}$$

$$\begin{aligned}
04816^2 + 14400^2 &= 1\ 5184^2 \\
12\ 04816^2 + 158400^2 &= 121\ 5184^2 \\
1232\ 04816^2 + 1598400^2 &= 12321\ 5184^2 \\
123432\ 04816^2 + 15998400^2 &= 1234321\ 5184^2 \quad (5.174)
\end{aligned}$$

$$\begin{aligned}
04671^2 + 14600^2 &= 1\ 5329^2 \\
12\ 04671^2 + 160600^2 &= 121\ 5329^2 \\
1232\ 04671^2 + 1620600^2 &= 12321\ 5329^2 \\
123432\ 04671^2 + 16220600^2 &= 1234321\ 5329^2 \quad (5.175)
\end{aligned}$$

$$\begin{aligned}
04524^2 + 14800^2 &= 1\ 5476^2 \\
12\ 04524^2 + 162800^2 &= 121\ 5476^2 \\
1232\ 04524^2 + 1642800^2 &= 12321\ 5476^2 \\
123432\ 04524^2 + 16442800^2 &= 1234321\ 5476^2 \quad (5.176)
\end{aligned}$$

$$\begin{aligned}
04375^2 + 15000^2 &= 1\ 5625^2 \\
12\ 04375^2 + 165000^2 &= 121\ 5625^2 \\
1232\ 04375^2 + 1665000^2 &= 12321\ 5625^2 \\
123432\ 04375^2 + 16665000^2 &= 1234321\ 5625^2 \quad (5.177)
\end{aligned}$$

$$\begin{aligned}
04224^2 + 15200^2 &= 1\ 5776^2 \\
12\ 04224^2 + 167200^2 &= 121\ 5776^2 \\
1232\ 04224^2 + 1687200^2 &= 12321\ 5776^2 \\
123432\ 04224^2 + 16887200^2 &= 1234321\ 5776^2 \quad (5.178)
\end{aligned}$$

$$\begin{aligned}
04071^2 + 15400^2 &= 1\ 5929^2 \\
12\ 04071^2 + 169400^2 &= 121\ 5929^2 \\
1232\ 04071^2 + 1709400^2 &= 12321\ 5929^2 \\
123432\ 04071^2 + 17109400^2 &= 1234321\ 5929^2 \quad (5.179)
\end{aligned}$$

$$\begin{aligned}
03916^2 + 15600^2 &= 1\ 6084^2 \\
12\ 03916^2 + 171600^2 &= 121\ 6084^2 \\
1232\ 03916^2 + 1731600^2 &= 12321\ 6084^2 \\
123432\ 03916^2 + 17331600^2 &= 1234321\ 6084^2 \quad (5.180)
\end{aligned}$$

$$\begin{aligned}
03759^2 + 15800^2 &= 1\ 6241^2 \\
12\ 03759^2 + 173800^2 &= 121\ 6241^2 \\
1232\ 03759^2 + 1753800^2 &= 12321\ 6241^2 \\
123432\ 03759^2 + 17553800^2 &= 1234321\ 6241^2 \quad (5.181)
\end{aligned}$$

$$\begin{aligned}
03600^2 + 16000^2 &= 1\ 6400^2 \\
12\ 03600^2 + 176000^2 &= 121\ 6400^2 \\
1232\ 03600^2 + 1776000^2 &= 12321\ 6400^2 \\
123432\ 03600^2 + 17776000^2 &= 1234321\ 6400^2 \quad (5.182)
\end{aligned}$$

$$\begin{aligned}
03439^2 + 16200^2 &= 1\ 6561^2 \\
12\ 03439^2 + 178200^2 &= 121\ 6561^2 \\
1232\ 03439^2 + 1798200^2 &= 12321\ 6561^2 \\
123432\ 03439^2 + 17998200^2 &= 1234321\ 6561^2 \quad (5.183)
\end{aligned}$$

$$\begin{aligned}
03276^2 + 16400^2 &= 1\ 6724^2 \\
12\ 03276^2 + 180400^2 &= 121\ 6724^2 \\
1232\ 03276^2 + 1820400^2 &= 12321\ 6724^2 \\
123432\ 03276^2 + 18220400^2 &= 1234321\ 6724^2 \quad (5.184)
\end{aligned}$$

$$\begin{aligned}
03111^2 + 16600^2 &= 1\ 6889^2 \\
12\ 03111^2 + 182600^2 &= 121\ 6889^2 \\
1232\ 03111^2 + 1842600^2 &= 12321\ 6889^2 \\
123432\ 03111^2 + 18442600^2 &= 1234321\ 6889^2 \quad (5.185)
\end{aligned}$$

$$\begin{aligned}
02944^2 + 16800^2 &= 1\ 7056^2 \\
12\ 02944^2 + 184800^2 &= 121\ 7056^2 \\
1232\ 02944^2 + 1864800^2 &= 12321\ 7056^2 \\
123432\ 02944^2 + 18664800^2 &= 1234321\ 7056^2 \quad (5.186)
\end{aligned}$$

$$\begin{aligned}
02775^2 + 17000^2 &= 1\ 7225^2 \\
12\ 02775^2 + 187000^2 &= 121\ 7225^2 \\
1232\ 02775^2 + 1887000^2 &= 12321\ 7225^2 \\
123432\ 02775^2 + 18887000^2 &= 1234321\ 7225^2 \quad (5.187)
\end{aligned}$$

$$\begin{aligned}
02604^2 + 17200^2 &= 1\ 7396^2 \\
12\ 02604^2 + 189200^2 &= 121\ 7396^2 \\
1232\ 02604^2 + 1909200^2 &= 12321\ 7396^2 \\
123432\ 02604^2 + 19109200^2 &= 1234321\ 7396^2 \quad (5.188)
\end{aligned}$$

$$\begin{aligned}
02431^2 + 17400^2 &= 1\ 7569^2 \\
12\ 02431^2 + 191400^2 &= 121\ 7569^2 \\
1232\ 02431^2 + 1931400^2 &= 12321\ 7569^2 \\
123432\ 02431^2 + 19331400^2 &= 1234321\ 7569^2 \quad (5.189)
\end{aligned}$$

$$\begin{aligned}
02256^2 + 17600^2 &= 1\ 7744^2 \\
12\ 02256^2 + 193600^2 &= 121\ 7744^2 \\
1232\ 02256^2 + 1953600^2 &= 12321\ 7744^2 \\
123432\ 02256^2 + 19553600^2 &= 1234321\ 7744^2 \quad (5.190)
\end{aligned}$$

$$\begin{aligned}
02079^2 + 17800^2 &= 1\ 7921^2 \\
12\ 02079^2 + 195800^2 &= 121\ 7921^2 \\
1232\ 02079^2 + 1975800^2 &= 12321\ 7921^2 \\
123432\ 02079^2 + 19775800^2 &= 1234321\ 7921^2 \quad (5.191)
\end{aligned}$$

$$\begin{aligned}
01900^2 + 18000^2 &= 1\ 8100^2 \\
12\ 01900^2 + 198000^2 &= 121\ 8100^2 \\
1232\ 01900^2 + 1998000^2 &= 12321\ 8100^2 \\
123432\ 01900^2 + 19998000^2 &= 1234321\ 8100^2 \quad (5.192)
\end{aligned}$$

$$\begin{aligned}
01719^2 + 18200^2 &= 1\ 8281^2 \\
12\ 01719^2 + 200200^2 &= 121\ 8281^2 \\
1232\ 01719^2 + 2020200^2 &= 12321\ 8281^2 \\
123432\ 01719^2 + 20220200^2 &= 1234321\ 8281^2 \quad (5.193)
\end{aligned}$$

$$\begin{aligned}
 01536^2 + 18400^2 &= 18464^2 & 00784^2 + 19200^2 &= 19216^2 \\
 1201536^2 + 202400^2 &= 1218464^2 & 1200784^2 + 211200^2 &= 1219216^2 \\
 123201536^2 + 2042400^2 &= 123218464^2 & 123200784^2 + 2131200^2 &= 123219216^2 \\
 12343201536^2 + 20442400^2 &= 12343218464^2 & 12343200784^2 + 21331200^2 &= 12343219216^2 \quad (5.198)
 \end{aligned} \quad (5.194)$$

$$\begin{aligned}
 01351^2 + 18600^2 &= 18649^2 & 00591^2 + 19400^2 &= 19409^2 \\
 1201351^2 + 204600^2 &= 1218649^2 & 1200591^2 + 213400^2 &= 1219409^2 \\
 123201351^2 + 2064600^2 &= 123218649^2 & 123200591^2 + 2153400^2 &= 123219409^2 \\
 12343201351^2 + 20664600^2 &= 12343218649^2 & 12343200591^2 + 21553400^2 &= 12343219409^2 \quad (5.199)
 \end{aligned} \quad (5.195)$$

$$\begin{aligned}
 01164^2 + 18800^2 &= 18836^2 & 00396^2 + 19600^2 &= 19604^2 \\
 1201164^2 + 206800^2 &= 1218836^2 & 1200396^2 + 215600^2 &= 1219604^2 \\
 123201164^2 + 2086800^2 &= 123218836^2 & 123200396^2 + 2175600^2 &= 123219604^2 \\
 12343201164^2 + 20886800^2 &= 12343218836^2 & 12343200396^2 + 21775600^2 &= 12343219604^2 \quad (5.200)
 \end{aligned} \quad (5.196)$$

$$\begin{aligned}
 00975^2 + 19000^2 &= 19025^2 & 00199^2 + 19800^2 &= 19801^2 \\
 1200975^2 + 209000^2 &= 1219025^2 & 1200199^2 + 217800^2 &= 1219801^2 \\
 123200975^2 + 2109000^2 &= 123219025^2 & 123200199^2 + 2197800^2 &= 123219801^2 \\
 12343200975^2 + 21109000^2 &= 12343219025^2 & 12343200199^2 + 21997800^2 &= 12343219801^2 \quad (5.201)
 \end{aligned} \quad (5.197)$$

5.3.2 Second Way

• For $n = 1, 2, 3, \dots, 17, 18, 19$:

Let's consider

$$m = 100, 10100, 1010100, 101010100, 10101010100, 1010101010100, 101010101010100, 10101010101010100, 1010101010101010100$$

in (2.1), then for each value of $n = 1, 2, 3, \dots, 17, 18, 19$; there are 19 **palindromic-type pandigital Pythagorean patterns** given below:

$$\begin{aligned}
 09999^2 + 200^2 &= 10001^2 \\
 102009999^2 + 20200^2 &= 102010001^2 \\
 1020302009999^2 + 2020200^2 &= 1020302010001^2 \\
 10203040302009999^2 + 202020200^2 &= 10203040302010001^2 \\
 102030405040302009999^2 + 20202020200^2 &= 102030405040302010001^2 \\
 1020304050605040302009999^2 + 2020202020200^2 &= 1020304050605040302010001^2 \\
 10203040506070605040302009999^2 + 202020202020200^2 &= 10203040506070605040302010001^2 \\
 102030405060708070605040302009999^2 + 20202020202020200^2 &= 102030405060708070605040302010001^2 \\
 1020304050607080908070605040302009999^2 + 2020202020202020200^2 &= 1020304050607080908070605040302010001^2 \quad (5.202)
 \end{aligned}$$

$$\begin{aligned}
 &09996^2 + 400^2 = 1\ 0004^2 \\
 &1020\ 09996^2 + 40400^2 = 10201\ 0004^2 \\
 &10203020\ 09996^2 + 4040400^2 = 102030201\ 0004^2 \\
 &102030403020\ 09996^2 + 404040400^2 = 1020304030201\ 0004^2 \\
 &1020304050403020\ 09996^2 + 40404040400^2 = 10203040504030201\ 0004^2 \\
 &10203040506050403020\ 09996^2 + 4040404040400^2 = 102030405060504030201\ 0004^2 \\
 &102030405060706050403020\ 09996^2 + 404040404040400^2 = 1020304050607060504030201\ 0004^2 \\
 &1020304050607080706050403020\ 09996^2 + 40404040404040400^2 = 10203040506070807060504030201\ 0004^2 \\
 &10203040506070809080706050403020\ 09996^2 + 4040404040404040400^2 = 102030405060708090807060504030201\ 0004^2 \\
 & \hspace{15em} (5.203)
 \end{aligned}$$

$$\begin{aligned}
 &09991^2 + 600^2 = 1\ 0009^2 \\
 &1020\ 09991^2 + 60600^2 = 10201\ 0009^2 \\
 &10203020\ 09991^2 + 6060600^2 = 102030201\ 0009^2 \\
 &102030403020\ 09991^2 + 606060600^2 = 1020304030201\ 0009^2 \\
 &1020304050403020\ 09991^2 + 60606060600^2 = 10203040504030201\ 0009^2 \\
 &10203040506050403020\ 09991^2 + 6060606060600^2 = 102030405060504030201\ 0009^2 \\
 &102030405060706050403020\ 09991^2 + 606060606060600^2 = 1020304050607060504030201\ 0009^2 \\
 &1020304050607080706050403020\ 09991^2 + 60606060606060600^2 = 10203040506070807060504030201\ 0009^2 \\
 &10203040506070809080706050403020\ 09991^2 + 6060606060606060600^2 = 102030405060708090807060504030201\ 0009^2 \\
 & \hspace{15em} (5.204)
 \end{aligned}$$

$$\begin{aligned}
 &09984^2 + 800^2 = 1\ 0016^2 \\
 &1020\ 09984^2 + 80800^2 = 10201\ 0016^2 \\
 &10203020\ 09984^2 + 8080800^2 = 102030201\ 0016^2 \\
 &102030403020\ 09984^2 + 808080800^2 = 1020304030201\ 0016^2 \\
 &1020304050403020\ 09984^2 + 80808080800^2 = 10203040504030201\ 0016^2 \\
 &10203040506050403020\ 09984^2 + 8080808080800^2 = 102030405060504030201\ 0016^2 \\
 &102030405060706050403020\ 09984^2 + 808080808080800^2 = 1020304050607060504030201\ 0016^2 \\
 &1020304050607080706050403020\ 09984^2 + 80808080808080800^2 = 10203040506070807060504030201\ 0016^2 \\
 &10203040506070809080706050403020\ 09984^2 + 8080808080808080800^2 = 102030405060708090807060504030201\ 0016^2 \\
 & \hspace{15em} (5.205)
 \end{aligned}$$

$$\begin{aligned}
 &09975^2 + 1000^2 = 1\ 0025^2 \\
 &1020\ 09975^2 + 101000^2 = 10201\ 0025^2 \\
 &10203020\ 09975^2 + 10101000^2 = 102030201\ 0025^2 \\
 &102030403020\ 09975^2 + 1010101000^2 = 1020304030201\ 0025^2 \\
 &1020304050403020\ 09975^2 + 101010101000^2 = 10203040504030201\ 0025^2 \\
 &10203040506050403020\ 09975^2 + 10101010101000^2 = 102030405060504030201\ 0025^2 \\
 &102030405060706050403020\ 09975^2 + 1010101010101000^2 = 1020304050607060504030201\ 0025^2 \\
 &1020304050607080706050403020\ 09975^2 + 101010101010101000^2 = 10203040506070807060504030201\ 0025^2 \\
 &10203040506070809080706050403020\ 09975^2 + 10101010101010101000^2 = 102030405060708090807060504030201\ 0025^2 \\
 & \hspace{15em} (5.206)
 \end{aligned}$$

$$\begin{aligned}
 & 09964^2 + 1200^2 = 10036^2 \\
 & 1020\ 09964^2 + 121200^2 = 10201\ 0036^2 \\
 & 10203020\ 09964^2 + 12121200^2 = 102030201\ 0036^2 \\
 & 102030403020\ 09964^2 + 1212121200^2 = 1020304030201\ 0036^2 \\
 & 1020304050403020\ 09964^2 + 121212121200^2 = 10203040504030201\ 0036^2 \\
 & 10203040506050403020\ 09964^2 + 12121212121200^2 = 102030405060504030201\ 0036^2 \\
 & 102030405060706050403020\ 09964^2 + 1212121212121200^2 = 1020304050607060504030201\ 0036^2 \\
 & 1020304050607080706050403020\ 09964^2 + 121212121212121200^2 = 10203040506070807060504030201\ 0036^2 \\
 & 10203040506070809080706050403020\ 09964^2 + 12121212121212121200^2 = 102030405060708090807060504030201\ 0036^2 \\
 & \hspace{16em} (5.207)
 \end{aligned}$$

$$\begin{aligned}
 & 09951^2 + 1400^2 = 10049^2 \\
 & 1020\ 09951^2 + 141400^2 = 10201\ 0049^2 \\
 & 10203020\ 09951^2 + 14141400^2 = 102030201\ 0049^2 \\
 & 102030403020\ 09951^2 + 1414141400^2 = 1020304030201\ 0049^2 \\
 & 1020304050403020\ 09951^2 + 141414141400^2 = 10203040504030201\ 0049^2 \\
 & 10203040506050403020\ 09951^2 + 14141414141400^2 = 102030405060504030201\ 0049^2 \\
 & 102030405060706050403020\ 09951^2 + 1414141414141400^2 = 1020304050607060504030201\ 0049^2 \\
 & 1020304050607080706050403020\ 09951^2 + 141414141414141400^2 = 10203040506070807060504030201\ 0049^2 \\
 & 10203040506070809080706050403020\ 09951^2 + 14141414141414141400^2 = 102030405060708090807060504030201\ 0049^2 \\
 & \hspace{16em} (5.208)
 \end{aligned}$$

$$\begin{aligned}
 & 09936^2 + 1600^2 = 10064^2 \\
 & 1020\ 09936^2 + 161600^2 = 10201\ 0064^2 \\
 & 10203020\ 09936^2 + 16161600^2 = 102030201\ 0064^2 \\
 & 102030403020\ 09936^2 + 1616161600^2 = 1020304030201\ 0064^2 \\
 & 1020304050403020\ 09936^2 + 161616161600^2 = 10203040504030201\ 0064^2 \\
 & 10203040506050403020\ 09936^2 + 16161616161600^2 = 102030405060504030201\ 0064^2 \\
 & 102030405060706050403020\ 09936^2 + 1616161616161600^2 = 1020304050607060504030201\ 0064^2 \\
 & 1020304050607080706050403020\ 09936^2 + 161616161616161600^2 = 10203040506070807060504030201\ 0064^2 \\
 & 10203040506070809080706050403020\ 09936^2 + 16161616161616161600^2 = 102030405060708090807060504030201\ 0064^2 \\
 & \hspace{16em} (5.209)
 \end{aligned}$$

$$\begin{aligned}
 & 09919^2 + 1800^2 = 10081^2 \\
 & 1020\ 09919^2 + 181800^2 = 10201\ 0081^2 \\
 & 10203020\ 09919^2 + 18181800^2 = 102030201\ 0081^2 \\
 & 102030403020\ 09919^2 + 1818181800^2 = 1020304030201\ 0081^2 \\
 & 1020304050403020\ 09919^2 + 181818181800^2 = 10203040504030201\ 0081^2 \\
 & 10203040506050403020\ 09919^2 + 18181818181800^2 = 102030405060504030201\ 0081^2 \\
 & 102030405060706050403020\ 09919^2 + 1818181818181800^2 = 1020304050607060504030201\ 0081^2 \\
 & 1020304050607080706050403020\ 09919^2 + 181818181818181800^2 = 10203040506070807060504030201\ 0081^2 \\
 & 10203040506070809080706050403020\ 09919^2 + 18181818181818181800^2 = 102030405060708090807060504030201\ 0081^2 \\
 & \hspace{16em} (5.210)
 \end{aligned}$$

$$\begin{aligned}
 & 09900^2 + 2000^2 &= 10100^2 \\
 & 102009900^2 + 202000^2 &= 102010100^2 \\
 & 1020302009900^2 + 20202000^2 &= 1020302010100^2 \\
 & 10203040302009900^2 + 2020202000^2 &= 10203040302010100^2 \\
 & 102030405040302009900^2 + 202020202000^2 &= 102030405040302010100^2 \\
 & 1020304050605040302009900^2 + 20202020202000^2 &= 1020304050605040302010100^2 \\
 & 10203040506070605040302009900^2 + 2020202020202000^2 &= 10203040506070605040302010100^2 \\
 & 102030405060708070605040302009900^2 + 202020202020202000^2 &= 102030405060708070605040302010100^2 \\
 & 1020304050607080908070605040302009900^2 + 20202020202020202000^2 &= 1020304050607080908070605040302010100^2 \\
 & & \text{(5.211)}
 \end{aligned}$$

$$\begin{aligned}
 & 09879^2 + 2200^2 &= 10121^2 \\
 & 102009879^2 + 222200^2 &= 102010121^2 \\
 & 1020302009879^2 + 2222200^2 &= 1020302010121^2 \\
 & 10203040302009879^2 + 22222200^2 &= 10203040302010121^2 \\
 & 102030405040302009879^2 + 222222200^2 &= 102030405040302010121^2 \\
 & 1020304050605040302009879^2 + 2222222200^2 &= 1020304050605040302010121^2 \\
 & 10203040506070605040302009879^2 + 22222222200^2 &= 10203040506070605040302010121^2 \\
 & 102030405060708070605040302009879^2 + 222222222200^2 &= 102030405060708070605040302010121^2 \\
 & 1020304050607080908070605040302009879^2 + 2222222222200^2 &= 1020304050607080908070605040302010121^2 \\
 & & \text{(5.212)}
 \end{aligned}$$

$$\begin{aligned}
 & 09856^2 + 2400^2 &= 10144^2 \\
 & 102009856^2 + 242400^2 &= 102010144^2 \\
 & 1020302009856^2 + 24242400^2 &= 1020302010144^2 \\
 & 10203040302009856^2 + 2424242400^2 &= 10203040302010144^2 \\
 & 102030405040302009856^2 + 242424242400^2 &= 102030405040302010144^2 \\
 & 1020304050605040302009856^2 + 24242424242400^2 &= 1020304050605040302010144^2 \\
 & 10203040506070605040302009856^2 + 2424242424242400^2 &= 10203040506070605040302010144^2 \\
 & 102030405060708070605040302009856^2 + 242424242424242400^2 &= 102030405060708070605040302010144^2 \\
 & 1020304050607080908070605040302009856^2 + 24242424242424242400^2 &= 1020304050607080908070605040302010144^2 \\
 & & \text{(5.213)}
 \end{aligned}$$

$$\begin{aligned}
 & 09831^2 + 2600^2 &= 10169^2 \\
 & 102009831^2 + 262600^2 &= 102010169^2 \\
 & 1020302009831^2 + 26262600^2 &= 1020302010169^2 \\
 & 10203040302009831^2 + 2626262600^2 &= 10203040302010169^2 \\
 & 102030405040302009831^2 + 262626262600^2 &= 102030405040302010169^2 \\
 & 1020304050605040302009831^2 + 26262626262600^2 &= 1020304050605040302010169^2 \\
 & 10203040506070605040302009831^2 + 2626262626262600^2 &= 10203040506070605040302010169^2 \\
 & 102030405060708070605040302009831^2 + 262626262626262600^2 &= 102030405060708070605040302010169^2 \\
 & 1020304050607080908070605040302009831^2 + 26262626262626262600^2 &= 1020304050607080908070605040302010169^2 \\
 & & \text{(5.214)}
 \end{aligned}$$

$$\begin{aligned}
 &09804^2 + 2800^2 = 10196^2 \\
 &102009804^2 + 282800^2 = 102010196^2 \\
 &1020302009804^2 + 28282800^2 = 1020302010196^2 \\
 &10203040302009804^2 + 2828282800^2 = 10203040302010196^2 \\
 &102030405040302009804^2 + 282828282800^2 = 102030405040302010196^2 \\
 &1020304050605040302009804^2 + 28282828282800^2 = 1020304050605040302010196^2 \\
 &10203040506070605040302009804^2 + 2828282828282800^2 = 10203040506070605040302010196^2 \\
 &102030405060708070605040302009804^2 + 282828282828282800^2 = 102030405060708070605040302010196^2 \\
 &1020304050607080908070605040302009804^2 + 28282828282828282800^2 = 1020304050607080908070605040302010196^2 \\
 & \hspace{10em} (5.215)
 \end{aligned}$$

$$\begin{aligned}
 &09775^2 + 3000^2 = 10225^2 \\
 &102009775^2 + 303000^2 = 102010225^2 \\
 &1020302009775^2 + 30303000^2 = 1020302010225^2 \\
 &10203040302009775^2 + 3030303000^2 = 10203040302010225^2 \\
 &102030405040302009775^2 + 303030303000^2 = 102030405040302010225^2 \\
 &1020304050605040302009775^2 + 30303030303000^2 = 1020304050605040302010225^2 \\
 &10203040506070605040302009775^2 + 3030303030303000^2 = 10203040506070605040302010225^2 \\
 &102030405060708070605040302009775^2 + 303030303030303000^2 = 102030405060708070605040302010225^2 \\
 &1020304050607080908070605040302009775^2 + 30303030303030303000^2 = 1020304050607080908070605040302010225^2 \\
 & \hspace{10em} (5.216)
 \end{aligned}$$

$$\begin{aligned}
 &09744^2 + 3200^2 = 10256^2 \\
 &102009744^2 + 323200^2 = 102010256^2 \\
 &1020302009744^2 + 32323200^2 = 1020302010256^2 \\
 &10203040302009744^2 + 3232323200^2 = 10203040302010256^2 \\
 &102030405040302009744^2 + 323232323200^2 = 102030405040302010256^2 \\
 &1020304050605040302009744^2 + 32323232323200^2 = 1020304050605040302010256^2 \\
 &10203040506070605040302009744^2 + 3232323232323200^2 = 10203040506070605040302010256^2 \\
 &102030405060708070605040302009744^2 + 323232323232323200^2 = 102030405060708070605040302010256^2 \\
 &1020304050607080908070605040302009744^2 + 32323232323232323200^2 = 1020304050607080908070605040302010256^2 \\
 & \hspace{10em} (5.217)
 \end{aligned}$$

$$\begin{aligned}
 &09711^2 + 3400^2 = 10289^2 \\
 &102009711^2 + 343400^2 = 102010289^2 \\
 &1020302009711^2 + 34343400^2 = 1020302010289^2 \\
 &10203040302009711^2 + 3434343400^2 = 10203040302010289^2 \\
 &102030405040302009711^2 + 343434343400^2 = 102030405040302010289^2 \\
 &1020304050605040302009711^2 + 34343434343400^2 = 1020304050605040302010289^2 \\
 &10203040506070605040302009711^2 + 3434343434343400^2 = 10203040506070605040302010289^2 \\
 &102030405060708070605040302009711^2 + 343434343434343400^2 = 102030405060708070605040302010289^2 \\
 &1020304050607080908070605040302009711^2 + 34343434343434343400^2 = 1020304050607080908070605040302010289^2 \\
 & \hspace{10em} (5.218)
 \end{aligned}$$

$$\begin{aligned}
 &09676^2 + 3600^2 = 10324^2 \\
 &102009676^2 + 363600^2 = 102010324^2 \\
 &1020302009676^2 + 36363600^2 = 1020302010324^2 \\
 &10203040302009676^2 + 3636363600^2 = 10203040302010324^2 \\
 &102030405040302009676^2 + 363636363600^2 = 102030405040302010324^2 \\
 &1020304050605040302009676^2 + 36363636363600^2 = 1020304050605040302010324^2 \\
 &10203040506070605040302009676^2 + 3636363636363600^2 = 10203040506070605040302010324^2 \\
 &102030405060708070605040302009676^2 + 363636363636363600^2 = 102030405060708070605040302010324^2 \\
 &1020304050607080908070605040302009676^2 + 36363636363636363600^2 = 1020304050607080908070605040302010324^2 \\
 & \hspace{15em} (5.219)
 \end{aligned}$$

$$\begin{aligned}
 &09639^2 + 3800^2 = 10361^2 \\
 &102009639^2 + 383800^2 = 102010361^2 \\
 &1020302009639^2 + 38383800^2 = 1020302010361^2 \\
 &10203040302009639^2 + 3838383800^2 = 10203040302010361^2 \\
 &102030405040302009639^2 + 383838383800^2 = 102030405040302010361^2 \\
 &1020304050605040302009639^2 + 38383838383800^2 = 1020304050605040302010361^2 \\
 &10203040506070605040302009639^2 + 3838383838383800^2 = 10203040506070605040302010361^2 \\
 &102030405060708070605040302009639^2 + 383838383838383800^2 = 102030405060708070605040302010361^2 \\
 &1020304050607080908070605040302009639^2 + 38383838383838383800^2 = 1020304050607080908070605040302010361^2 \\
 & \hspace{15em} (5.220)
 \end{aligned}$$

- For $n = 20, 21, 23, \dots, 97, 98, 99$:

Let's consider

$$m = 100, 10100, 1010100, 101010100, 10101010100, 1010101010100, 101010101010100, 10101010101010100, 1010101010101010100$$

in (2.1), then for each value of $n = 20, 21, 23, \dots, 97, 98, 99$; there are 80 palindromic-type pandigital Pythagorean patterns given below. In each case, only first four values are written. The other 5 values can be written on similar lines as done in subsection 5.3.2.

$$\begin{aligned}
 &09600^2 + 4000^2 = 10400^2 & 09471^2 + 4600^2 = 10529^2 \\
 &102009600^2 + 404000^2 = 102010400^2 & 102009471^2 + 464600^2 = 102010529^2 \\
 &1020302009600^2 + 40404000^2 = 1020302010400^2 & 1020302009471^2 + 46464600^2 = 1020302010529^2 \\
 &10203040302009600^2 + 4040404000^2 = 10203040302010400^2 & 10203040302009471^2 + 4646464600^2 = 10203040302010529^2 \\
 & \hspace{15em} (5.221) & \hspace{15em} (5.224)
 \end{aligned}$$

$$\begin{aligned}
 &09559^2 + 4200^2 = 10441^2 & 09424^2 + 4800^2 = 10576^2 \\
 &102009559^2 + 424200^2 = 102010441^2 & 102009424^2 + 484800^2 = 102010576^2 \\
 &1020302009559^2 + 42424200^2 = 1020302010441^2 & 1020302009424^2 + 48484800^2 = 1020302010576^2 \\
 &10203040302009559^2 + 4242424200^2 = 10203040302010441^2 & 10203040302009424^2 + 4848484800^2 = 10203040302010576^2 \\
 & \hspace{15em} (5.222) & \hspace{15em} (5.225)
 \end{aligned}$$

$$\begin{aligned}
 &09516^2 + 4400^2 = 10484^2 & 09375^2 + 5000^2 = 10625^2 \\
 &102009516^2 + 444400^2 = 102010484^2 & 102009375^2 + 505000^2 = 102010625^2 \\
 &1020302009516^2 + 44444400^2 = 1020302010484^2 & 1020302009375^2 + 50505000^2 = 1020302010625^2 \\
 &10203040302009516^2 + 4444444400^2 = 10203040302010484^2 & 10203040302009375^2 + 5050505000^2 = 10203040302010625^2 \\
 & \hspace{15em} (5.223) & \hspace{15em} (5.226)
 \end{aligned}$$

$$\begin{aligned}
 09324^2 + 5200^2 &= 10676^2 \\
 102009324^2 + 525200^2 &= 102010676^2 \\
 1020302009324^2 + 52525200^2 &= 1020302010676^2 \\
 10203040302009324^2 + 5252525200^2 &= 10203040302010676^2 \quad (5.227)
 \end{aligned}$$

$$\begin{aligned}
 09271^2 + 5400^2 &= 10729^2 \\
 102009271^2 + 545400^2 &= 102010729^2 \\
 1020302009271^2 + 54545400^2 &= 1020302010729^2 \\
 10203040302009271^2 + 5454545400^2 &= 10203040302010729^2 \quad (5.228)
 \end{aligned}$$

$$\begin{aligned}
 09216^2 + 5600^2 &= 10784^2 \\
 102009216^2 + 565600^2 &= 102010784^2 \\
 1020302009216^2 + 56565600^2 &= 1020302010784^2 \\
 10203040302009216^2 + 5656565600^2 &= 10203040302010784^2 \quad (5.229)
 \end{aligned}$$

$$\begin{aligned}
 09159^2 + 5800^2 &= 10841^2 \\
 102009159^2 + 585800^2 &= 102010841^2 \\
 1020302009159^2 + 58585800^2 &= 1020302010841^2 \\
 10203040302009159^2 + 5858585800^2 &= 10203040302010841^2 \quad (5.230)
 \end{aligned}$$

$$\begin{aligned}
 09100^2 + 6000^2 &= 10900^2 \\
 102009100^2 + 606000^2 &= 102010900^2 \\
 1020302009100^2 + 60606000^2 &= 1020302010900^2 \\
 10203040302009100^2 + 6060606000^2 &= 10203040302010900^2 \quad (5.231)
 \end{aligned}$$

$$\begin{aligned}
 09039^2 + 6200^2 &= 10961^2 \\
 102009039^2 + 626200^2 &= 102010961^2 \\
 1020302009039^2 + 62626200^2 &= 1020302010961^2 \\
 10203040302009039^2 + 6262626200^2 &= 10203040302010961^2 \quad (5.232)
 \end{aligned}$$

$$\begin{aligned}
 08976^2 + 6400^2 &= 11024^2 \\
 102008976^2 + 646400^2 &= 102011024^2 \\
 1020302008976^2 + 64646400^2 &= 1020302011024^2 \\
 10203040302008976^2 + 6464646400^2 &= 10203040302011024^2 \quad (5.233)
 \end{aligned}$$

$$\begin{aligned}
 08911^2 + 6600^2 &= 11089^2 \\
 102008911^2 + 666600^2 &= 102011089^2 \\
 1020302008911^2 + 66666600^2 &= 1020302011089^2 \\
 10203040302008911^2 + 6666666600^2 &= 10203040302011089^2 \quad (5.234)
 \end{aligned}$$

$$\begin{aligned}
 08844^2 + 6800^2 &= 11156^2 \\
 102008844^2 + 686800^2 &= 102011156^2 \\
 1020302008844^2 + 68686800^2 &= 1020302011156^2 \\
 10203040302008844^2 + 6868686800^2 &= 10203040302011156^2 \quad (5.235)
 \end{aligned}$$

$$\begin{aligned}
 08775^2 + 7000^2 &= 11225^2 \\
 102008775^2 + 707000^2 &= 102011225^2 \\
 1020302008775^2 + 70707000^2 &= 1020302011225^2 \\
 10203040302008775^2 + 7070707000^2 &= 10203040302011225^2 \quad (5.236)
 \end{aligned}$$

$$\begin{aligned}
 08704^2 + 7200^2 &= 11296^2 \\
 102008704^2 + 727200^2 &= 102011296^2 \\
 1020302008704^2 + 72727200^2 &= 1020302011296^2 \\
 10203040302008704^2 + 7272727200^2 &= 10203040302011296^2 \quad (5.237)
 \end{aligned}$$

$$\begin{aligned}
 08631^2 + 7400^2 &= 11369^2 \\
 102008631^2 + 747400^2 &= 102011369^2 \\
 1020302008631^2 + 74747400^2 &= 1020302011369^2 \\
 10203040302008631^2 + 7474747400^2 &= 10203040302011369^2 \quad (5.238)
 \end{aligned}$$

$$\begin{aligned}
 08556^2 + 7600^2 &= 11444^2 \\
 102008556^2 + 767600^2 &= 102011444^2 \\
 1020302008556^2 + 76767600^2 &= 1020302011444^2 \\
 10203040302008556^2 + 7676767600^2 &= 10203040302011444^2 \quad (5.239)
 \end{aligned}$$

$$\begin{aligned}
 08479^2 + 7800^2 &= 11521^2 \\
 102008479^2 + 787800^2 &= 102011521^2 \\
 1020302008479^2 + 78787800^2 &= 1020302011521^2 \\
 10203040302008479^2 + 7878787800^2 &= 10203040302011521^2 \quad (5.240)
 \end{aligned}$$

$$\begin{aligned}
 08400^2 + 8000^2 &= 11600^2 \\
 102008400^2 + 808000^2 &= 102011600^2 \\
 1020302008400^2 + 80808000^2 &= 1020302011600^2 \\
 10203040302008400^2 + 8080808000^2 &= 10203040302011600^2 \quad (5.241)
 \end{aligned}$$

$$\begin{aligned}
 08319^2 + 8200^2 &= 11681^2 \\
 102008319^2 + 828200^2 &= 102011681^2 \\
 1020302008319^2 + 82828200^2 &= 1020302011681^2 \\
 10203040302008319^2 + 8282828200^2 &= 10203040302011681^2 \quad (5.242)
 \end{aligned}$$

$$\begin{aligned}
 08236^2 + 8400^2 &= 11764^2 \\
 102008236^2 + 848400^2 &= 102011764^2 \\
 1020302008236^2 + 84848400^2 &= 1020302011764^2 \\
 10203040302008236^2 + 8484848400^2 &= 10203040302011764^2 \quad (5.243)
 \end{aligned}$$

$$\begin{aligned}
 08151^2 + 8600^2 &= 11849^2 \\
 102008151^2 + 868600^2 &= 102011849^2 \\
 1020302008151^2 + 86868600^2 &= 1020302011849^2 \\
 10203040302008151^2 + 8686868600^2 &= 10203040302011849^2 \quad (5.244)
 \end{aligned}$$

$$\begin{aligned}
 08064^2 + 8800^2 &= 11936^2 \\
 102008064^2 + 888800^2 &= 102011936^2 \\
 1020302008064^2 + 88888800^2 &= 1020302011936^2 \\
 10203040302008064^2 + 8888888800^2 &= 10203040302011936^2 \quad (5.245)
 \end{aligned}$$

$$\begin{aligned}
 07975^2 + 9000^2 &= 1\ 2025^2 \\
 1020\ 07975^2 + 909000^2 &= 10201\ 2025^2 \\
 10203020\ 07975^2 + 90909000^2 &= 102030201\ 2025^2 \\
 102030403020\ 07975^2 + 9090909000^2 &= 1020304030201\ 2025^2 \\
 &(5.246)
 \end{aligned}$$

$$\begin{aligned}
 07884^2 + 9200^2 &= 1\ 2116^2 \\
 1020\ 07884^2 + 929200^2 &= 10201\ 2116^2 \\
 10203020\ 07884^2 + 92929200^2 &= 102030201\ 2116^2 \\
 102030403020\ 07884^2 + 9292929200^2 &= 1020304030201\ 2116^2 \\
 &(5.247)
 \end{aligned}$$

$$\begin{aligned}
 07791^2 + 9400^2 &= 1\ 2209^2 \\
 1020\ 07791^2 + 949400^2 &= 10201\ 2209^2 \\
 10203020\ 07791^2 + 94949400^2 &= 102030201\ 2209^2 \\
 102030403020\ 07791^2 + 9494949400^2 &= 1020304030201\ 2209^2 \\
 &(5.248)
 \end{aligned}$$

$$\begin{aligned}
 07696^2 + 9600^2 &= 1\ 2304^2 \\
 1020\ 07696^2 + 969600^2 &= 10201\ 2304^2 \\
 10203020\ 07696^2 + 96969600^2 &= 102030201\ 2304^2 \\
 102030403020\ 07696^2 + 9696969600^2 &= 1020304030201\ 2304^2 \\
 &(5.249)
 \end{aligned}$$

$$\begin{aligned}
 07599^2 + 9800^2 &= 1\ 2401^2 \\
 1020\ 07599^2 + 989800^2 &= 10201\ 2401^2 \\
 10203020\ 07599^2 + 98989800^2 &= 102030201\ 2401^2 \\
 102030403020\ 07599^2 + 9898989800^2 &= 1020304030201\ 2401^2 \\
 &(5.250)
 \end{aligned}$$

$$\begin{aligned}
 07500^2 + 10000^2 &= 1\ 2500^2 \\
 1020\ 07500^2 + 1010000^2 &= 10201\ 2500^2 \\
 10203020\ 07500^2 + 101010000^2 &= 102030201\ 2500^2 \\
 102030403020\ 07500^2 + 10101010000^2 &= 1020304030201\ 2500^2 \\
 &(5.251)
 \end{aligned}$$

$$\begin{aligned}
 07399^2 + 10200^2 &= 1\ 2601^2 \\
 1020\ 07399^2 + 1030200^2 &= 10201\ 2601^2 \\
 10203020\ 07399^2 + 103030200^2 &= 102030201\ 2601^2 \\
 102030403020\ 07399^2 + 10303030200^2 &= 1020304030201\ 2601^2 \\
 &(5.252)
 \end{aligned}$$

$$\begin{aligned}
 07296^2 + 10400^2 &= 1\ 2704^2 \\
 1020\ 07296^2 + 1050400^2 &= 10201\ 2704^2 \\
 10203020\ 07296^2 + 105050400^2 &= 102030201\ 2704^2 \\
 102030403020\ 07296^2 + 10505050400^2 &= 1020304030201\ 2704^2 \\
 &(5.253)
 \end{aligned}$$

$$\begin{aligned}
 07191^2 + 10600^2 &= 1\ 2809^2 \\
 1020\ 07191^2 + 1070600^2 &= 10201\ 2809^2 \\
 10203020\ 07191^2 + 107070600^2 &= 102030201\ 2809^2 \\
 102030403020\ 07191^2 + 10707070600^2 &= 1020304030201\ 2809^2 \\
 &(5.254)
 \end{aligned}$$

$$\begin{aligned}
 07084^2 + 10800^2 &= 1\ 2916^2 \\
 1020\ 07084^2 + 1090800^2 &= 10201\ 2916^2 \\
 10203020\ 07084^2 + 109090800^2 &= 102030201\ 2916^2 \\
 102030403020\ 07084^2 + 10909090800^2 &= 1020304030201\ 2916^2 \\
 &(5.255)
 \end{aligned}$$

$$\begin{aligned}
 06975^2 + 11000^2 &= 1\ 3025^2 \\
 1020\ 06975^2 + 1111000^2 &= 10201\ 3025^2 \\
 10203020\ 06975^2 + 11111000^2 &= 102030201\ 3025^2 \\
 102030403020\ 06975^2 + 111111000^2 &= 1020304030201\ 3025^2 \\
 &(5.256)
 \end{aligned}$$

$$\begin{aligned}
 06864^2 + 11200^2 &= 1\ 3136^2 \\
 1020\ 06864^2 + 1131200^2 &= 10201\ 3136^2 \\
 10203020\ 06864^2 + 113131200^2 &= 102030201\ 3136^2 \\
 102030403020\ 06864^2 + 11313131200^2 &= 1020304030201\ 3136^2 \\
 &(5.257)
 \end{aligned}$$

$$\begin{aligned}
 06751^2 + 11400^2 &= 1\ 3249^2 \\
 1020\ 06751^2 + 1151400^2 &= 10201\ 3249^2 \\
 10203020\ 06751^2 + 115151400^2 &= 102030201\ 3249^2 \\
 102030403020\ 06751^2 + 11515151400^2 &= 1020304030201\ 3249^2 \\
 &(5.258)
 \end{aligned}$$

$$\begin{aligned}
 06636^2 + 11600^2 &= 1\ 3364^2 \\
 1020\ 06636^2 + 1171600^2 &= 10201\ 3364^2 \\
 10203020\ 06636^2 + 117171600^2 &= 102030201\ 3364^2 \\
 102030403020\ 06636^2 + 11717171600^2 &= 1020304030201\ 3364^2 \\
 &(5.259)
 \end{aligned}$$

$$\begin{aligned}
 06519^2 + 11800^2 &= 1\ 3481^2 \\
 1020\ 06519^2 + 1191800^2 &= 10201\ 3481^2 \\
 10203020\ 06519^2 + 119191800^2 &= 102030201\ 3481^2 \\
 102030403020\ 06519^2 + 11919191800^2 &= 1020304030201\ 3481^2 \\
 &(5.260)
 \end{aligned}$$

$$\begin{aligned}
 06400^2 + 12000^2 &= 1\ 3600^2 \\
 1020\ 06400^2 + 1212000^2 &= 10201\ 3600^2 \\
 10203020\ 06400^2 + 121212000^2 &= 102030201\ 3600^2 \\
 102030403020\ 06400^2 + 12121212000^2 &= 1020304030201\ 3600^2 \\
 &(5.261)
 \end{aligned}$$

$$\begin{aligned}
 06279^2 + 12200^2 &= 1\ 3721^2 \\
 1020\ 06279^2 + 1232200^2 &= 10201\ 3721^2 \\
 10203020\ 06279^2 + 123232200^2 &= 102030201\ 3721^2 \\
 102030403020\ 06279^2 + 12323232200^2 &= 1020304030201\ 3721^2 \\
 &(5.262)
 \end{aligned}$$

$$\begin{aligned}
 06156^2 + 12400^2 &= 1\ 3844^2 \\
 1020\ 06156^2 + 1252400^2 &= 10201\ 3844^2 \\
 10203020\ 06156^2 + 125252400^2 &= 102030201\ 3844^2 \\
 102030403020\ 06156^2 + 12525252400^2 &= 1020304030201\ 3844^2 \\
 &(5.263)
 \end{aligned}$$

$$\begin{aligned}
 06031^2 + 12600^2 &= 1\ 3969^2 \\
 1020\ 06031^2 + 1272600^2 &= 10201\ 3969^2 \\
 10203020\ 06031^2 + 127272600^2 &= 102030201\ 3969^2 \\
 102030403020\ 06031^2 + 12727272600^2 &= 1020304030201\ 3969^2 \\
 &(5.264)
 \end{aligned}$$

$$\begin{aligned}
 05904^2 + 12800^2 &= 1\ 4096^2 \\
 1020\ 05904^2 + 1292800^2 &= 10201\ 4096^2 \\
 10203020\ 05904^2 + 129292800^2 &= 102030201\ 4096^2 \\
 102030403020\ 05904^2 + 12929292800^2 &= 1020304030201\ 4096^2 \quad (5.265)
 \end{aligned}$$

$$\begin{aligned}
 05775^2 + 13000^2 &= 1\ 4225^2 \\
 1020\ 05775^2 + 1313000^2 &= 10201\ 4225^2 \\
 10203020\ 05775^2 + 131313000^2 &= 102030201\ 4225^2 \\
 102030403020\ 05775^2 + 13131313000^2 &= 1020304030201\ 4225^2 \quad (5.266)
 \end{aligned}$$

$$\begin{aligned}
 05644^2 + 13200^2 &= 1\ 4356^2 \\
 1020\ 05644^2 + 1333200^2 &= 10201\ 4356^2 \\
 10203020\ 05644^2 + 133333200^2 &= 102030201\ 4356^2 \\
 102030403020\ 05644^2 + 13333333200^2 &= 1020304030201\ 4356^2 \quad (5.267)
 \end{aligned}$$

$$\begin{aligned}
 05511^2 + 13400^2 &= 1\ 4489^2 \\
 1020\ 05511^2 + 1353400^2 &= 10201\ 4489^2 \\
 10203020\ 05511^2 + 135353400^2 &= 102030201\ 4489^2 \\
 102030403020\ 05511^2 + 13535353400^2 &= 1020304030201\ 4489^2 \quad (5.268)
 \end{aligned}$$

$$\begin{aligned}
 05376^2 + 13600^2 &= 1\ 4624^2 \\
 1020\ 05376^2 + 1373600^2 &= 10201\ 4624^2 \\
 10203020\ 05376^2 + 137373600^2 &= 102030201\ 4624^2 \\
 102030403020\ 05376^2 + 13737373600^2 &= 1020304030201\ 4624^2 \quad (5.269)
 \end{aligned}$$

$$\begin{aligned}
 05239^2 + 13800^2 &= 1\ 4761^2 \\
 1020\ 05239^2 + 1393800^2 &= 10201\ 4761^2 \\
 10203020\ 05239^2 + 139393800^2 &= 102030201\ 4761^2 \\
 102030403020\ 05239^2 + 13939393800^2 &= 1020304030201\ 4761^2 \quad (5.270)
 \end{aligned}$$

$$\begin{aligned}
 05100^2 + 14000^2 &= 1\ 4900^2 \\
 1020\ 05100^2 + 1414000^2 &= 10201\ 4900^2 \\
 10203020\ 05100^2 + 141414000^2 &= 102030201\ 4900^2 \\
 102030403020\ 05100^2 + 14141414000^2 &= 1020304030201\ 4900^2 \quad (5.271)
 \end{aligned}$$

$$\begin{aligned}
 04959^2 + 14200^2 &= 1\ 5041^2 \\
 1020\ 04959^2 + 1434200^2 &= 10201\ 5041^2 \\
 10203020\ 04959^2 + 143434200^2 &= 102030201\ 5041^2 \\
 102030403020\ 04959^2 + 14343434200^2 &= 1020304030201\ 5041^2 \quad (5.272)
 \end{aligned}$$

$$\begin{aligned}
 04816^2 + 14400^2 &= 1\ 5184^2 \\
 1020\ 04816^2 + 1454400^2 &= 10201\ 5184^2 \\
 10203020\ 04816^2 + 145454400^2 &= 102030201\ 5184^2 \\
 102030403020\ 04816^2 + 14545454400^2 &= 1020304030201\ 5184^2 \quad (5.273)
 \end{aligned}$$

$$\begin{aligned}
 04671^2 + 14600^2 &= 1\ 5329^2 \\
 1020\ 04671^2 + 1474600^2 &= 10201\ 5329^2 \\
 10203020\ 04671^2 + 147474600^2 &= 102030201\ 5329^2 \\
 102030403020\ 04671^2 + 14747474600^2 &= 1020304030201\ 5329^2 \quad (5.274)
 \end{aligned}$$

$$\begin{aligned}
 04524^2 + 14800^2 &= 1\ 5476^2 \\
 1020\ 04524^2 + 1494800^2 &= 10201\ 5476^2 \\
 10203020\ 04524^2 + 149494800^2 &= 102030201\ 5476^2 \\
 102030403020\ 04524^2 + 14949494800^2 &= 1020304030201\ 5476^2 \quad (5.275)
 \end{aligned}$$

$$\begin{aligned}
 04375^2 + 15000^2 &= 1\ 5625^2 \\
 1020\ 04375^2 + 1515000^2 &= 10201\ 5625^2 \\
 10203020\ 04375^2 + 151515000^2 &= 102030201\ 5625^2 \\
 102030403020\ 04375^2 + 15151515000^2 &= 1020304030201\ 5625^2 \quad (5.276)
 \end{aligned}$$

$$\begin{aligned}
 04224^2 + 15200^2 &= 1\ 5776^2 \\
 1020\ 04224^2 + 1535200^2 &= 10201\ 5776^2 \\
 10203020\ 04224^2 + 153535200^2 &= 102030201\ 5776^2 \\
 102030403020\ 04224^2 + 15353535200^2 &= 1020304030201\ 5776^2 \quad (5.277)
 \end{aligned}$$

$$\begin{aligned}
 04071^2 + 15400^2 &= 1\ 5929^2 \\
 1020\ 04071^2 + 1555400^2 &= 10201\ 5929^2 \\
 10203020\ 04071^2 + 155555400^2 &= 102030201\ 5929^2 \\
 102030403020\ 04071^2 + 15555555400^2 &= 1020304030201\ 5929^2 \quad (5.278)
 \end{aligned}$$

$$\begin{aligned}
 03916^2 + 15600^2 &= 1\ 6084^2 \\
 1020\ 03916^2 + 1575600^2 &= 10201\ 6084^2 \\
 10203020\ 03916^2 + 157575600^2 &= 102030201\ 6084^2 \\
 102030403020\ 03916^2 + 15757575600^2 &= 1020304030201\ 6084^2 \quad (5.279)
 \end{aligned}$$

$$\begin{aligned}
 03759^2 + 15800^2 &= 1\ 6241^2 \\
 1020\ 03759^2 + 1595800^2 &= 10201\ 6241^2 \\
 10203020\ 03759^2 + 159595800^2 &= 102030201\ 6241^2 \\
 102030403020\ 03759^2 + 15959595800^2 &= 1020304030201\ 6241^2 \quad (5.280)
 \end{aligned}$$

$$\begin{aligned}
 03600^2 + 16000^2 &= 1\ 6400^2 \\
 1020\ 03600^2 + 1616000^2 &= 10201\ 6400^2 \\
 10203020\ 03600^2 + 161616000^2 &= 102030201\ 6400^2 \\
 102030403020\ 03600^2 + 16161616000^2 &= 1020304030201\ 6400^2 \quad (5.281)
 \end{aligned}$$

$$\begin{aligned}
 03439^2 + 16200^2 &= 1\ 6561^2 \\
 1020\ 03439^2 + 1636200^2 &= 10201\ 6561^2 \\
 10203020\ 03439^2 + 163636200^2 &= 102030201\ 6561^2 \\
 102030403020\ 03439^2 + 16363636200^2 &= 1020304030201\ 6561^2 \quad (5.282)
 \end{aligned}$$

$$\begin{aligned}
 03276^2 + 16400^2 &= 1\ 6724^2 \\
 1020\ 03276^2 + 1656400^2 &= 10201\ 6724^2 \\
 10203020\ 03276^2 + 165656400^2 &= 102030201\ 6724^2 \\
 102030403020\ 03276^2 + 16565656400^2 &= 1020304030201\ 6724^2 \quad (5.283)
 \end{aligned}$$

$$\begin{aligned}
 03111^2 + 16600^2 &= 1\ 6889^2 \\
 1020\ 03111^2 + 1676600^2 &= 10201\ 6889^2 \\
 10203020\ 03111^2 + 167676600^2 &= 102030201\ 6889^2 \\
 102030403020\ 03111^2 + 16767676600^2 &= 1020304030201\ 6889^2 \quad (5.284)
 \end{aligned}$$

$$\begin{aligned}
 02944^2 + 16800^2 &= 1\ 7056^2 \\
 1020\ 02944^2 + 1696800^2 &= 10201\ 7056^2 \\
 10203020\ 02944^2 + 169696800^2 &= 102030201\ 7056^2 \\
 102030403020\ 02944^2 + 16969696800^2 &= 1020304030201\ 7056^2 \quad (5.285)
 \end{aligned}$$

$$\begin{aligned}
 02775^2 + 17000^2 &= 1\ 7225^2 \\
 1020\ 02775^2 + 1717000^2 &= 10201\ 7225^2 \\
 10203020\ 02775^2 + 171717000^2 &= 102030201\ 7225^2 \\
 102030403020\ 02775^2 + 17171717000^2 &= 1020304030201\ 7225^2 \quad (5.286)
 \end{aligned}$$

$$\begin{aligned}
 02604^2 + 17200^2 &= 1\ 7396^2 \\
 1020\ 02604^2 + 1737200^2 &= 10201\ 7396^2 \\
 10203020\ 02604^2 + 173737200^2 &= 102030201\ 7396^2 \\
 102030403020\ 02604^2 + 17373737200^2 &= 1020304030201\ 7396^2 \quad (5.287)
 \end{aligned}$$

$$\begin{aligned}
 02431^2 + 17400^2 &= 1\ 7569^2 \\
 1020\ 02431^2 + 1757400^2 &= 10201\ 7569^2 \\
 10203020\ 02431^2 + 175757400^2 &= 102030201\ 7569^2 \\
 102030403020\ 02431^2 + 17575757400^2 &= 1020304030201\ 7569^2 \quad (5.288)
 \end{aligned}$$

$$\begin{aligned}
 02256^2 + 17600^2 &= 1\ 7744^2 \\
 1020\ 02256^2 + 1777600^2 &= 10201\ 7744^2 \\
 10203020\ 02256^2 + 177777600^2 &= 102030201\ 7744^2 \\
 102030403020\ 02256^2 + 17777777600^2 &= 1020304030201\ 7744^2 \quad (5.289)
 \end{aligned}$$

$$\begin{aligned}
 02079^2 + 17800^2 &= 1\ 7921^2 \\
 1020\ 02079^2 + 1797800^2 &= 10201\ 7921^2 \\
 10203020\ 02079^2 + 179797800^2 &= 102030201\ 7921^2 \\
 102030403020\ 02079^2 + 17979797800^2 &= 1020304030201\ 7921^2 \quad (5.290)
 \end{aligned}$$

$$\begin{aligned}
 01900^2 + 18000^2 &= 1\ 8100^2 \\
 1020\ 01900^2 + 1818000^2 &= 10201\ 8100^2 \\
 10203020\ 01900^2 + 181818000^2 &= 102030201\ 8100^2 \\
 102030403020\ 01900^2 + 18181818000^2 &= 1020304030201\ 8100^2 \quad (5.291)
 \end{aligned}$$

$$\begin{aligned}
 01719^2 + 18200^2 &= 1\ 8281^2 \\
 1020\ 01719^2 + 1838200^2 &= 10201\ 8281^2 \\
 10203020\ 01719^2 + 183838200^2 &= 102030201\ 8281^2 \\
 102030403020\ 01719^2 + 18383838200^2 &= 1020304030201\ 8281^2 \quad (5.292)
 \end{aligned}$$

$$\begin{aligned}
 01536^2 + 18400^2 &= 1\ 8464^2 \\
 1020\ 01536^2 + 1858400^2 &= 10201\ 8464^2 \\
 10203020\ 01536^2 + 185858400^2 &= 102030201\ 8464^2 \\
 102030403020\ 01536^2 + 18585858400^2 &= 1020304030201\ 8464^2 \quad (5.293)
 \end{aligned}$$

$$\begin{aligned}
 01351^2 + 18600^2 &= 1\ 8649^2 \\
 1020\ 01351^2 + 1878600^2 &= 10201\ 8649^2 \\
 10203020\ 01351^2 + 187878600^2 &= 102030201\ 8649^2 \\
 102030403020\ 01351^2 + 18787878600^2 &= 1020304030201\ 8649^2 \quad (5.294)
 \end{aligned}$$

$$\begin{aligned}
 01164^2 + 18800^2 &= 1\ 8836^2 \\
 1020\ 01164^2 + 1898800^2 &= 10201\ 8836^2 \\
 10203020\ 01164^2 + 189898800^2 &= 102030201\ 8836^2 \\
 102030403020\ 01164^2 + 18989898800^2 &= 1020304030201\ 8836^2 \quad (5.295)
 \end{aligned}$$

$$\begin{aligned}
 00975^2 + 19000^2 &= 1\ 9025^2 \\
 1020\ 00975^2 + 1919000^2 &= 10201\ 9025^2 \\
 10203020\ 00975^2 + 191919000^2 &= 102030201\ 9025^2 \\
 102030403020\ 00975^2 + 19191919000^2 &= 1020304030201\ 9025^2 \quad (5.296)
 \end{aligned}$$

$$\begin{aligned}
 00784^2 + 19200^2 &= 1\ 9216^2 \\
 1020\ 00784^2 + 1939200^2 &= 10201\ 9216^2 \\
 10203020\ 00784^2 + 193939200^2 &= 102030201\ 9216^2 \\
 102030403020\ 00784^2 + 19393939200^2 &= 1020304030201\ 9216^2 \quad (5.297)
 \end{aligned}$$

$$\begin{aligned}
 00591^2 + 19400^2 &= 1\ 9409^2 \\
 1020\ 00591^2 + 1959400^2 &= 10201\ 9409^2 \\
 10203020\ 00591^2 + 195959400^2 &= 102030201\ 9409^2 \\
 102030403020\ 00591^2 + 19595959400^2 &= 1020304030201\ 9409^2 \quad (5.298)
 \end{aligned}$$

$$\begin{aligned}
 00396^2 + 19600^2 &= 1\ 9604^2 \\
 1020\ 00396^2 + 1979600^2 &= 10201\ 9604^2 \\
 10203020\ 00396^2 + 197979600^2 &= 102030201\ 9604^2 \\
 102030403020\ 00396^2 + 19797979600^2 &= 1020304030201\ 9604^2 \quad (5.299)
 \end{aligned}$$

$$\begin{aligned}
 00199^2 + 19800^2 &= 1\ 9801^2 \\
 1020\ 00199^2 + 1999800^2 &= 10201\ 9801^2 \\
 10203020\ 00199^2 + 199999800^2 &= 102030201\ 9801^2 \\
 102030403020\ 00199^2 + 19999999800^2 &= 1020304030201\ 9801^2 \quad (5.300)
 \end{aligned}$$

5.4 Magic Square

According notations given in (4.1) and procedure (4.3), below are 99 distributions resulting in magic squares. For calculations procedure see author's work [3, 4]. Out of these 99 Pythagorean triples, only the first 97 generates magic square in decreasing order, i.e., starting from order 99 to order 3 of consecutive odd numbers. The last two lines are with 2 and 1, that don't give magic squares. See below:

$$\begin{aligned}
 (9999, 200, 10001) &\Rightarrow \{99, 401, 20001, 1009899, 99980001 = 9999^2\} \\
 (9996, 400, 10004) &\Rightarrow \{98, 801, 20007, 1019592, 99920016 = 9996^2\} \\
 (9991, 600, 10009) &\Rightarrow \{97, 1201, 20017, 1029073, 99820081 = 9991^2\} \\
 (9984, 800, 10016) &\Rightarrow \{96, 1601, 20031, 1038336, 99680256 = 9984^2\} \\
 (9975, 1000, 10025) &\Rightarrow \{95, 2001, 20049, 1047375, 99500625 = 9975^2\} \\
 (9964, 1200, 10036) &\Rightarrow \{94, 2401, 20071, 1056184, 99281296 = 9964^2\} \\
 (9951, 1400, 10049) &\Rightarrow \{93, 2801, 20097, 1064757, 99022401 = 9951^2\} \\
 (9936, 1600, 10064) &\Rightarrow \{92, 3201, 20127, 1073088, 98724096 = 9936^2\} \\
 (9919, 1800, 10081) &\Rightarrow \{91, 3601, 20161, 1081171, 98386561 = 9919^2\} \\
 (9900, 2000, 10100) &\Rightarrow \{90, 4001, 20199, 1089000, 98010000 = 9900^2\} \\
 (9879, 2200, 10121) &\Rightarrow \{89, 4401, 20241, 1096569, 97594641 = 9879^2\} \\
 (9856, 2400, 10144) &\Rightarrow \{88, 4801, 20287, 1103872, 97140736 = 9856^2\} \\
 (9831, 2600, 10169) &\Rightarrow \{87, 5201, 20337, 1110903, 96648561 = 9831^2\} \\
 (9804, 2800, 10196) &\Rightarrow \{86, 5601, 20391, 1117656, 96118416 = 9804^2\} \\
 (9775, 3000, 10225) &\Rightarrow \{85, 6001, 20449, 1124125, 95550625 = 9775^2\} \\
 (9744, 3200, 10256) &\Rightarrow \{84, 6401, 20511, 1130304, 94945536 = 9744^2\} \\
 (9711, 3400, 10289) &\Rightarrow \{83, 6801, 20577, 1136187, 94303521 = 9711^2\} \\
 (9676, 3600, 10324) &\Rightarrow \{82, 7201, 20647, 1141768, 93624976 = 9676^2\} \\
 (9639, 3800, 10361) &\Rightarrow \{81, 7601, 20721, 1147041, 92910321 = 9639^2\} \\
 (9600, 4000, 10400) &\Rightarrow \{80, 8001, 20799, 1152000, 92160000 = 9600^2\} \\
 (9559, 4200, 10441) &\Rightarrow \{79, 8401, 20881, 1156639, 91374481 = 9559^2\} \\
 (9516, 4400, 10484) &\Rightarrow \{78, 8801, 20967, 1160952, 90554256 = 9516^2\} \\
 (9471, 4600, 10529) &\Rightarrow \{77, 9201, 21057, 1164933, 89699841 = 9471^2\} \\
 (9424, 4800, 10576) &\Rightarrow \{76, 9601, 21151, 1168576, 88811776 = 9424^2\} \\
 (9375, 5000, 10625) &\Rightarrow \{75, 10001, 21249, 1171875, 87890625 = 9375^2\} \\
 (9324, 5200, 10676) &\Rightarrow \{74, 10401, 21351, 1174824, 86936976 = 9324^2\} \\
 (9271, 5400, 10729) &\Rightarrow \{73, 10801, 21457, 1177417, 85951441 = 9271^2\} \\
 (9216, 5600, 10784) &\Rightarrow \{72, 11201, 21567, 1179648, 84934656 = 9216^2\} \\
 (9159, 5800, 10841) &\Rightarrow \{71, 11601, 21681, 1181511, 83887281 = 9159^2\} \\
 (9100, 6000, 10900) &\Rightarrow \{70, 12001, 21799, 1183000, 82810000 = 9100^2\} \\
 (9039, 6200, 10961) &\Rightarrow \{69, 12401, 21921, 1184109, 81703521 = 9039^2\} \\
 (8976, 6400, 11024) &\Rightarrow \{68, 12801, 22047, 1184832, 80568576 = 8976^2\}
 \end{aligned}$$

$$\begin{aligned}
(8911, 6600, 11089) &\Rightarrow \{67, 13201, 22177, 1185163, 79405921 = 8911^2\} \\
(8844, 6800, 11156) &\Rightarrow \{66, 13601, 22311, 1185096, 78216336 = 8844^2\} \\
(8775, 7000, 11225) &\Rightarrow \{65, 14001, 22449, 1184625, 77000625 = 8775^2\} \\
(8704, 7200, 11296) &\Rightarrow \{64, 14401, 22591, 1183744, 75759616 = 8704^2\} \\
(8631, 7400, 11369) &\Rightarrow \{63, 14801, 22737, 1182447, 74494161 = 8631^2\} \\
(8556, 7600, 11444) &\Rightarrow \{62, 15201, 22887, 1180728, 73205136 = 8556^2\} \\
(8479, 7800, 11521) &\Rightarrow \{61, 15601, 23041, 1178581, 71893441 = 8479^2\} \\
(8400, 8000, 11600) &\Rightarrow \{60, 16001, 23199, 1176000, 70560000 = 8400^2\} \\
(8319, 8200, 11681) &\Rightarrow \{59, 16401, 23361, 1172979, 69205761 = 8319^2\} \\
(8236, 8400, 11764) &\Rightarrow \{58, 16801, 23527, 1169512, 67831696 = 8236^2\} \\
(8151, 8600, 11849) &\Rightarrow \{57, 17201, 23697, 1165593, 66438801 = 8151^2\} \\
(8064, 8800, 11936) &\Rightarrow \{56, 17601, 23871, 1161216, 65028096 = 8064^2\} \\
(7975, 9000, 12025) &\Rightarrow \{55, 18001, 24049, 1156375, 63600625 = 7975^2\} \\
(7884, 9200, 12116) &\Rightarrow \{54, 18401, 24231, 1151064, 62157456 = 7884^2\} \\
(7791, 9400, 12209) &\Rightarrow \{53, 18801, 24417, 1145277, 60699681 = 7791^2\} \\
(7696, 9600, 12304) &\Rightarrow \{52, 19201, 24607, 1139008, 59228416 = 7696^2\} \\
(7599, 9800, 12401) &\Rightarrow \{51, 19601, 24801, 1132251, 57744801 = 7599^2\} \\
(7500, 10000, 12500) &\Rightarrow \{50, 20001, 24999, 1125000, 56250000 = 7500^2\} \\
(7399, 10200, 12601) &\Rightarrow \{49, 20401, 25201, 1117249, 54745201 = 7399^2\} \\
(7296, 10400, 12704) &\Rightarrow \{48, 20801, 25407, 1108992, 53231616 = 7296^2\} \\
(7191, 10600, 12809) &\Rightarrow \{47, 21201, 25617, 1100223, 51710481 = 7191^2\} \\
(7084, 10800, 12916) &\Rightarrow \{46, 21601, 25831, 1090936, 50183056 = 7084^2\} \\
(6975, 11000, 13025) &\Rightarrow \{45, 22001, 26049, 1081125, 48650625 = 6975^2\} \\
(6864, 11200, 13136) &\Rightarrow \{44, 22401, 26271, 1070784, 47114496 = 6864^2\} \\
(6751, 11400, 13249) &\Rightarrow \{43, 22801, 26497, 1059907, 45576001 = 6751^2\} \\
(6636, 11600, 13364) &\Rightarrow \{42, 23201, 26727, 1048488, 44036496 = 6636^2\} \\
(6519, 11800, 13481) &\Rightarrow \{41, 23601, 26961, 1036521, 42497361 = 6519^2\} \\
(6400, 12000, 13600) &\Rightarrow \{40, 24001, 27199, 1024000, 40960000 = 6400^2\} \\
(6279, 12200, 13721) &\Rightarrow \{39, 24401, 27441, 1010919, 39425841 = 6279^2\} \\
(6156, 12400, 13844) &\Rightarrow \{38, 24801, 27687, 997272, 37896336 = 6156^2\} \\
(6031, 12600, 13969) &\Rightarrow \{37, 25201, 27937, 983053, 36372961 = 6031^2\} \\
(5904, 12800, 14096) &\Rightarrow \{36, 25601, 28191, 968256, 34857216 = 5904^2\} \\
(5775, 13000, 14225) &\Rightarrow \{35, 26001, 28449, 952875, 33350625 = 5775^2\} \\
(5644, 13200, 14356) &\Rightarrow \{34, 26401, 28711, 936904, 31854736 = 5644^2\} \\
(5511, 13400, 14489) &\Rightarrow \{33, 26801, 28977, 920337, 30371121 = 5511^2\} \\
(5376, 13600, 14624) &\Rightarrow \{32, 27201, 29247, 903168, 28901376 = 5376^2\} \\
(5239, 13800, 14761) &\Rightarrow \{31, 27601, 29521, 885391, 27447121 = 5239^2\}
\end{aligned}$$

$$\begin{aligned}
(5100, 14000, 14900) &\Rightarrow \{30, 28001, 29799, 867000, 26010000 = 5100^2\} \\
(4959, 14200, 15041) &\Rightarrow \{29, 28401, 30081, 847989, 24591681 = 4959^2\} \\
(4816, 14400, 15184) &\Rightarrow \{28, 28801, 30367, 828352, 23193856 = 4816^2\} \\
(4671, 14600, 15329) &\Rightarrow \{27, 29201, 30657, 808083, 21818241 = 4671^2\} \\
(4524, 14800, 15476) &\Rightarrow \{26, 29601, 30951, 787176, 20466576 = 4524^2\} \\
(4375, 15000, 15625) &\Rightarrow \{25, 30001, 31249, 765625, 19140625 = 4375^2\} \\
(4224, 15200, 15776) &\Rightarrow \{24, 30401, 31551, 743424, 17842176 = 4224^2\} \\
(4071, 15400, 15929) &\Rightarrow \{23, 30801, 31857, 720567, 16573041 = 4071^2\} \\
(3916, 15600, 16084) &\Rightarrow \{22, 31201, 32167, 697048, 15335056 = 3916^2\} \\
(3759, 15800, 16241) &\Rightarrow \{21, 31601, 32481, 672861, 14130081 = 3759^2\} \\
(3600, 16000, 16400) &\Rightarrow \{20, 32001, 32799, 648000, 12960000 = 3600^2\} \\
(3439, 16200, 16561) &\Rightarrow \{19, 32401, 33121, 622459, 11826721 = 3439^2\} \\
(3276, 16400, 16724) &\Rightarrow \{18, 32801, 33447, 596232, 10732176 = 3276^2\} \\
(3111, 16600, 16889) &\Rightarrow \{17, 33201, 33777, 569313, 9678321 = 3111^2\} \\
(2944, 16800, 17056) &\Rightarrow \{16, 33601, 34111, 541696, 8667136 = 2944^2\} \\
(2775, 17000, 17225) &\Rightarrow \{15, 34001, 34449, 513375, 7700625 = 2775^2\} \\
(2604, 17200, 17396) &\Rightarrow \{14, 34401, 34791, 484344, 6780816 = 2604^2\} \\
(2431, 17400, 17569) &\Rightarrow \{13, 34801, 35137, 454597, 5909761 = 2431^2\} \\
(2256, 17600, 17744) &\Rightarrow \{12, 35201, 35487, 424128, 5089536 = 2256^2\} \\
(2079, 17800, 17921) &\Rightarrow \{11, 35601, 35841, 392931, 4322241 = 2079^2\} \\
(1900, 18000, 18100) &\Rightarrow \{10, 36001, 36199, 361000, 3610000 = 1900^2\} \\
(1719, 18200, 18281) &\Rightarrow \{9, 36401, 36561, 328329, 2954961 = 1719^2\} \\
(1536, 18400, 18464) &\Rightarrow \{8, 36801, 36927, 294912, 2359296 = 1536^2\} \\
(1351, 18600, 18649) &\Rightarrow \{7, 37201, 37297, 260743, 1825201 = 1351^2\} \\
(1164, 18800, 18836) &\Rightarrow \{6, 37601, 37671, 225816, 1354896 = 1164^2\} \\
(0975, 19000, 19025) &\Rightarrow \{5, 38001, 38049, 190125, 950625 = 975^2\} \\
(0784, 19200, 19216) &\Rightarrow \{4, 38401, 38431, 153664, 614656 = 784^2\} \\
(0591, 19400, 19409) &\Rightarrow \{3, 38801, 38817, 116427, 349281 = 591^2\} \tag{5.301}
\end{aligned}$$

Remark 6. *There are 97 perfect square sum magic squares of consecutive odd orders of orders 99 to 3. We observe that, the order of magic squares of (4.4) and (5.301) are same, but with different entries.*

6 First Procedure: Third Step

Let's write a number 1000000 as following 999 sums:

1. $1000000 = 999999 + 000001$
2. $1000000 = 999996 + 000004$

$$\begin{aligned}
 3. \quad & 1000000 = 999991 + 000009 \\
 4. \quad & 1000000 = 999984 + 000016 \\
 5. \quad & 1000000 = 999975 + 000025 \\
 & \dots \quad \dots \quad \dots \quad \dots \quad \dots \\
 996. & 1000000 = 007984 + 992016 \\
 997. & 1000000 = 005991 + 994009 \\
 998. & 1000000 = 003996 + 996004 \\
 999. & 1000000 = 001999 + 998001
 \end{aligned} \tag{6.1}$$

We observe that the second terms in the r.h.s. are perfect squares. Consider a difference of squares between the terms of r.h.s. with 1 in the front perfect square term. Then this difference is again a perfect square multiple of 2000. See below:

$$\begin{aligned}
 1000001^2 - 999999^2 &= 0002000^2 \Rightarrow 999999^2 + 0002000^2 = 1000001^2 \Rightarrow (999999, 0002000, 1000001) \\
 1000004^2 - 999996^2 &= 0004000^2 \Rightarrow 999996^2 + 0004000^2 = 1000004^2 \Rightarrow (999996, 0004000, 1000004) \\
 1000009^2 - 999991^2 &= 0006000^2 \Rightarrow 999991^2 + 0006000^2 = 1000009^2 \Rightarrow (999991, 0006000, 1000009) \\
 1000016^2 - 999984^2 &= 0008000^2 \Rightarrow 999984^2 + 0008000^2 = 1000016^2 \Rightarrow (999984, 0008000, 1000016) \\
 1000025^2 - 999975^2 &= 0010000^2 \Rightarrow 999975^2 + 0010000^2 = 1000025^2 \Rightarrow (999975, 0010000, 1000025) \\
 & \dots \quad \dots \quad \dots \quad \dots \quad \dots \\
 1992016^2 - 007984^2 &= 1992000^2 \Rightarrow 007984^2 + 1992000^2 = 1992016^2 \Rightarrow (007984, 1992000, 1992016) \\
 1994009^2 - 005991^2 &= 1994000^2 \Rightarrow 005991^2 + 1994000^2 = 1994009^2 \Rightarrow (005991, 1994000, 1994009) \\
 1996004^2 - 003996^2 &= 1996000^2 \Rightarrow 003996^2 + 1996000^2 = 1996004^2 \Rightarrow (003996, 1996000, 1996004) \\
 1998001^2 - 001999^2 &= 1998000^2 \Rightarrow 001999^2 + 1998000^2 = 1998001^2 \Rightarrow (001999, 1998000, 1998001)
 \end{aligned} \tag{6.2}$$

Thus we have a 999 Pythagorean triples given in (6.2). Let’s see how to generate them using the formula (2.1).

6.1 Alternative Approach

By considering $m = 1000$ and varying the values of $n = 1, 2, 3, 4, \dots, 997, 998, 999$ in (2.1), we get respectively the following 999 Pythagorean triples:

(999999, 2000, 1000001)	(999744, 32000, 1000256)	(999039, 62000, 1000961)	(997884, 92000, 1002116)
(999996, 4000, 1000004)	(999711, 34000, 1000289)	(998976, 64000, 1001024)	(997791, 94000, 1002209)
(999991, 6000, 1000009)	(999676, 36000, 1000324)	(998911, 66000, 1001089)	(997696, 96000, 1002304)
(999984, 8000, 1000016)	(999639, 38000, 1000361)	(998844, 68000, 1001156)	(997599, 98000, 1002401)
(999975, 10000, 1000025)	(999600, 40000, 1000400)	(998775, 70000, 1001225)	(997500, 100000, 1002500)
(999964, 12000, 1000036)	(999559, 42000, 1000441)	(998704, 72000, 1001296)	(997399, 102000, 1002601)
(999951, 14000, 1000049)	(999516, 44000, 1000484)	(998631, 74000, 1001369)	(997296, 104000, 1002704)
(999936, 16000, 1000064)	(999471, 46000, 1000529)	(998556, 76000, 1001444)	(997191, 106000, 1002809)
(999919, 18000, 1000081)	(999424, 48000, 1000576)	(998479, 78000, 1001521)	(997084, 108000, 1002916)
(999900, 20000, 1000100)	(999375, 50000, 1000625)	(998400, 80000, 1001600)	(996975, 110000, 1003025)
(999879, 22000, 1000121)	(999324, 52000, 1000676)	(998319, 82000, 1001681)	(996864, 112000, 1003136)
(999856, 24000, 1000144)	(999271, 54000, 1000729)	(998236, 84000, 1001764)	(996751, 114000, 1003249)
(999831, 26000, 1000169)	(999216, 56000, 1000784)	(998151, 86000, 1001849)	(996636, 116000, 1003364)
(999804, 28000, 1000196)	(999159, 58000, 1000841)	(998064, 88000, 1001936)	(996519, 118000, 1003481)
(999775, 30000, 1000225)	(999100, 60000, 1000900)	(997975, 90000, 1002025)	(996400, 120000, 1003600)

(996279, 122000, 1003721)	(988119, 218000, 1011881)	(975351, 314000, 1024649)	(957975, 410000, 1042025)
(996156, 124000, 1003844)	(987900, 220000, 1012100)	(975036, 316000, 1024964)	(957564, 412000, 1042436)
(996031, 126000, 1003969)	(987679, 222000, 1012321)	(974719, 318000, 1025281)	(957151, 414000, 1042849)
(995904, 128000, 1004096)	(987456, 224000, 1012544)	(974400, 320000, 1025600)	(956736, 416000, 1043264)
(995775, 130000, 1004225)	(987231, 226000, 1012769)	(974079, 322000, 1025921)	(956319, 418000, 1043681)
(995644, 132000, 1004356)	(987004, 228000, 1012996)	(973756, 324000, 1026244)	(955900, 420000, 1044100)
(995511, 134000, 1004489)	(986775, 230000, 1013225)	(973431, 326000, 1026569)	(955479, 422000, 1044521)
(995376, 136000, 1004624)	(986544, 232000, 1013456)	(973104, 328000, 1026896)	(955056, 424000, 1044944)
(995239, 138000, 1004761)	(986311, 234000, 1013689)	(972775, 330000, 1027225)	(954631, 426000, 1045369)
(995100, 140000, 1004900)	(986076, 236000, 1013924)	(972444, 332000, 1027556)	(954204, 428000, 1045796)
(994959, 142000, 1005041)	(985839, 238000, 1014161)	(972111, 334000, 1027889)	(953775, 430000, 1046225)
(994816, 144000, 1005184)	(985600, 240000, 1014400)	(971776, 336000, 1028224)	(953344, 432000, 1046656)
(994671, 146000, 1005329)	(985359, 242000, 1014641)	(971439, 338000, 1028561)	(952911, 434000, 1047089)
(994524, 148000, 1005476)	(985116, 244000, 1014884)	(971100, 340000, 1028900)	(952476, 436000, 1047524)
(994375, 150000, 1005625)	(984871, 246000, 1015129)	(970759, 342000, 1029241)	(952039, 438000, 1047961)
(994224, 152000, 1005776)	(984624, 248000, 1015376)	(970416, 344000, 1029584)	(951600, 440000, 1048400)
(994071, 154000, 1005929)	(984375, 250000, 1015625)	(970071, 346000, 1029929)	(951159, 442000, 1048841)
(993916, 156000, 1006084)	(984124, 252000, 1015876)	(969724, 348000, 1030276)	(950716, 444000, 1049284)
(993759, 158000, 1006241)	(983871, 254000, 1016129)	(969375, 350000, 1030625)	(950271, 446000, 1049729)
(993600, 160000, 1006400)	(983616, 256000, 1016384)	(969024, 352000, 1030976)	(949824, 448000, 1050176)
(993439, 162000, 1006561)	(983359, 258000, 1016641)	(968671, 354000, 1031329)	(949375, 450000, 1050625)
(993276, 164000, 1006724)	(983100, 260000, 1016900)	(968316, 356000, 1031684)	(948924, 452000, 1051076)
(993111, 166000, 1006889)	(982839, 262000, 1017161)	(967959, 358000, 1032041)	(948471, 454000, 1051529)
(992944, 168000, 1007056)	(982576, 264000, 1017424)	(967600, 360000, 1032400)	(948016, 456000, 1051984)
(992775, 170000, 1007225)	(982311, 266000, 1017689)	(967239, 362000, 1032761)	(947559, 458000, 1052441)
(992604, 172000, 1007396)	(982044, 268000, 1017956)	(966876, 364000, 1033124)	(947100, 460000, 1052900)
(992431, 174000, 1007569)	(981775, 270000, 1018225)	(966511, 366000, 1033489)	(946639, 462000, 1053361)
(992256, 176000, 1007744)	(981504, 272000, 1018496)	(966144, 368000, 1033856)	(946176, 464000, 1053824)
(992079, 178000, 1007921)	(981231, 274000, 1018769)	(965775, 370000, 1034225)	(945711, 466000, 1054289)
(991900, 180000, 1008100)	(980956, 276000, 1019044)	(965404, 372000, 1034596)	(945244, 468000, 1054756)
(991719, 182000, 1008281)	(980679, 278000, 1019321)	(965031, 374000, 1034969)	(944775, 470000, 1055225)
(991536, 184000, 1008464)	(980400, 280000, 1019600)	(964656, 376000, 1035344)	(944304, 472000, 1055696)
(991351, 186000, 1008649)	(980119, 282000, 1019881)	(964279, 378000, 1035721)	(943831, 474000, 1056169)
(991164, 188000, 1008836)	(979836, 284000, 1020164)	(963900, 380000, 1036100)	(943356, 476000, 1056644)
(990975, 190000, 1009025)	(979551, 286000, 1020449)	(963519, 382000, 1036481)	(942879, 478000, 1057121)
(990784, 192000, 1009216)	(979264, 288000, 1020736)	(963136, 384000, 1036864)	(942400, 480000, 1057600)
(990591, 194000, 1009409)	(978975, 290000, 1021025)	(962751, 386000, 1037249)	(941919, 482000, 1058081)
(990396, 196000, 1009604)	(978684, 292000, 1021316)	(962364, 388000, 1037636)	(941436, 484000, 1058564)
(990199, 198000, 1009801)	(978391, 294000, 1021609)	(961975, 390000, 1038025)	(940951, 486000, 1059049)
(990000, 200000, 1010000)	(978096, 296000, 1021904)	(961584, 392000, 1038416)	(940464, 488000, 1059536)
(989799, 202000, 1010201)	(977799, 298000, 1022201)	(961191, 394000, 1038809)	(939975, 490000, 1060025)
(989596, 204000, 1010404)	(977500, 300000, 1022500)	(960796, 396000, 1039204)	(939484, 492000, 1060516)
(989391, 206000, 1010609)	(977199, 302000, 1022801)	(960399, 398000, 1039601)	(938991, 494000, 1061009)
(989184, 208000, 1010816)	(976896, 304000, 1023104)	(960000, 400000, 1040000)	(938496, 496000, 1061504)
(988975, 210000, 1011025)	(976591, 306000, 1023409)	(959599, 402000, 1040401)	(937999, 498000, 1062001)
(988764, 212000, 1011236)	(976284, 308000, 1023716)	(959196, 404000, 1040804)	(937500, 500000, 1062500)
(988551, 214000, 1011449)	(975975, 310000, 1024025)	(958791, 406000, 1041209)	(936999, 502000, 1063001)
(988336, 216000, 1011664)	(975664, 312000, 1024336)	(958384, 408000, 1041616)	(936496, 504000, 1063504)

(935991, 506000, 1064009)	(909399, 602000, 1090601)	(878199, 698000, 1121801)	(842391, 794000, 1157609)
(935484, 508000, 1064516)	(908796, 604000, 1091204)	(877500, 700000, 1122500)	(841596, 796000, 1158404)
(934975, 510000, 1065025)	(908191, 606000, 1091809)	(876799, 702000, 1123201)	(840799, 798000, 1159201)
(934464, 512000, 1065536)	(907584, 608000, 1092416)	(876096, 704000, 1123904)	(840000, 800000, 1160000)
(933951, 514000, 1066049)	(906975, 610000, 1093025)	(875391, 706000, 1124609)	(839199, 802000, 1160801)
(933436, 516000, 1066564)	(906364, 612000, 1093636)	(874684, 708000, 1125316)	(838396, 804000, 1161604)
(932919, 518000, 1067081)	(905751, 614000, 1094249)	(873975, 710000, 1126025)	(837591, 806000, 1162409)
(932400, 520000, 1067600)	(905136, 616000, 1094864)	(873264, 712000, 1126736)	(836784, 808000, 1163216)
(931879, 522000, 1068121)	(904519, 618000, 1095481)	(872551, 714000, 1127449)	(835975, 810000, 1164025)
(931356, 524000, 1068644)	(903900, 620000, 1096100)	(871836, 716000, 1128164)	(835164, 812000, 1164836)
(930831, 526000, 1069169)	(903279, 622000, 1096721)	(871119, 718000, 1128881)	(834351, 814000, 1165649)
(930304, 528000, 1069696)	(902656, 624000, 1097344)	(870400, 720000, 1129600)	(833536, 816000, 1166464)
(929775, 530000, 1070225)	(902031, 626000, 1097969)	(869679, 722000, 1130321)	(832719, 818000, 1167281)
(929244, 532000, 1070756)	(901404, 628000, 1098596)	(868956, 724000, 1131044)	(831900, 820000, 1168100)
(928711, 534000, 1071289)	(900775, 630000, 1099225)	(868231, 726000, 1131769)	(831079, 822000, 1168921)
(928176, 536000, 1071824)	(900144, 632000, 1099856)	(867504, 728000, 1132496)	(830256, 824000, 1169744)
(927639, 538000, 1072361)	(899511, 634000, 1100489)	(866775, 730000, 1133225)	(829431, 826000, 1170569)
(927100, 540000, 1072900)	(898876, 636000, 1101124)	(866044, 732000, 1133956)	(828604, 828000, 1171396)
(926559, 542000, 1073441)	(898239, 638000, 1101761)	(865311, 734000, 1134689)	(827775, 830000, 1172225)
(926016, 544000, 1073984)	(897600, 640000, 1102400)	(864576, 736000, 1135424)	(826944, 832000, 1173056)
(925471, 546000, 1074529)	(896959, 642000, 1103041)	(863839, 738000, 1136161)	(826111, 834000, 1173889)
(924924, 548000, 1075076)	(896316, 644000, 1103684)	(863100, 740000, 1136900)	(825276, 836000, 1174724)
(924375, 550000, 1075625)	(895671, 646000, 1104329)	(862359, 742000, 1137641)	(824439, 838000, 1175561)
(923824, 552000, 1076176)	(895024, 648000, 1104976)	(861616, 744000, 1138384)	(823600, 840000, 1176400)
(923271, 554000, 1076729)	(894375, 650000, 1105625)	(860871, 746000, 1139129)	(822759, 842000, 1177241)
(922716, 556000, 1077284)	(893724, 652000, 1106276)	(860124, 748000, 1139876)	(821916, 844000, 1178084)
(922159, 558000, 1077841)	(893071, 654000, 1106929)	(859375, 750000, 1140625)	(821071, 846000, 1178929)
(921600, 560000, 1078400)	(892416, 656000, 1107584)	(858624, 752000, 1141376)	(820224, 848000, 1179776)
(921039, 562000, 1078961)	(891759, 658000, 1108241)	(857871, 754000, 1142129)	(819375, 850000, 1180625)
(920476, 564000, 1079524)	(891100, 660000, 1108900)	(857116, 756000, 1142884)	(818524, 852000, 1181476)
(919911, 566000, 1080089)	(890439, 662000, 1109561)	(856359, 758000, 1143641)	(817671, 854000, 1182329)
(919344, 568000, 1080656)	(889776, 664000, 1110224)	(855600, 760000, 1144400)	(816816, 856000, 1183184)
(918775, 570000, 1081225)	(889111, 666000, 1110889)	(854839, 762000, 1145161)	(815959, 858000, 1184041)
(918204, 572000, 1081796)	(888444, 668000, 1111556)	(854076, 764000, 1145924)	(815100, 860000, 1184900)
(917631, 574000, 1082369)	(887775, 670000, 1112225)	(853311, 766000, 1146689)	(814239, 862000, 1185761)
(917056, 576000, 1082944)	(887104, 672000, 1112896)	(852544, 768000, 1147456)	(813376, 864000, 1186624)
(916479, 578000, 1083521)	(886431, 674000, 1113569)	(851775, 770000, 1148225)	(812511, 866000, 1187489)
(915900, 580000, 1084100)	(885756, 676000, 1114244)	(851004, 772000, 1148996)	(811644, 868000, 1188356)
(915319, 582000, 1084681)	(885079, 678000, 1114921)	(850231, 774000, 1149769)	(810775, 870000, 1189225)
(914736, 584000, 1085264)	(884400, 680000, 1115600)	(849456, 776000, 1150544)	(809904, 872000, 1190096)
(914151, 586000, 1085849)	(883719, 682000, 1116281)	(848679, 778000, 1151321)	(809031, 874000, 1190969)
(913564, 588000, 1086436)	(883036, 684000, 1116964)	(847900, 780000, 1152100)	(808156, 876000, 1191844)
(912975, 590000, 1087025)	(882351, 686000, 1117649)	(847119, 782000, 1152881)	(807279, 878000, 1192721)
(912384, 592000, 1087616)	(881664, 688000, 1118336)	(846336, 784000, 1153664)	(806400, 880000, 1193600)
(911791, 594000, 1088209)	(880975, 690000, 1119025)	(845551, 786000, 1154449)	(805519, 882000, 1194481)
(911196, 596000, 1088804)	(880284, 692000, 1119716)	(844764, 788000, 1155236)	(804636, 884000, 1195364)
(910599, 598000, 1089401)	(879591, 694000, 1120409)	(843975, 790000, 1156025)	(803751, 886000, 1196249)
(910000, 600000, 1090000)	(878896, 696000, 1121104)	(843184, 792000, 1156816)	(802864, 888000, 1197136)

(801975, 890000, 1198025)	(756951, 986000, 1243049)	(707319, 1082000, 1292681)	(653079, 1178000, 1346921)
(801084, 892000, 1198916)	(755964, 988000, 1244036)	(706236, 1084000, 1293764)	(651900, 1180000, 1348100)
(800191, 894000, 1199809)	(754975, 990000, 1245025)	(705151, 1086000, 1294849)	(650719, 1182000, 1349281)
(799296, 896000, 1200704)	(753984, 992000, 1246016)	(704064, 1088000, 1295936)	(649536, 1184000, 1350464)
(798399, 898000, 1201601)	(752991, 994000, 1247009)	(702975, 1090000, 1297025)	(648351, 1186000, 1351649)
(797500, 900000, 1202500)	(751996, 996000, 1248004)	(701884, 1092000, 1298116)	(647164, 1188000, 1352836)
(796599, 902000, 1203401)	(750999, 998000, 1249001)	(700791, 1094000, 1299209)	(645975, 1190000, 1354025)
(795696, 904000, 1204304)	(750000, 1000000, 1250000)	(699696, 1096000, 1300304)	(644784, 1192000, 1355216)
(794791, 906000, 1205209)	(748999, 1002000, 1251001)	(698599, 1098000, 1301401)	(643591, 1194000, 1356409)
(793884, 908000, 1206116)	(747996, 1004000, 1252004)	(697500, 1100000, 1302500)	(642396, 1196000, 1357604)
(792975, 910000, 1207025)	(746991, 1006000, 1253009)	(696399, 1102000, 1303601)	(641199, 1198000, 1358801)
(792064, 912000, 1207936)	(745984, 1008000, 1254016)	(695296, 1104000, 1304704)	(640000, 1200000, 1360000)
(791151, 914000, 1208849)	(744975, 1010000, 1255025)	(694191, 1106000, 1305809)	(638799, 1202000, 1361201)
(790236, 916000, 1209764)	(743964, 1012000, 1256036)	(693084, 1108000, 1306916)	(637596, 1204000, 1362404)
(789319, 918000, 1210681)	(742951, 1014000, 1257049)	(691975, 1110000, 1308025)	(636391, 1206000, 1363609)
(788400, 920000, 1211600)	(741936, 1016000, 1258064)	(690864, 1112000, 1309136)	(635184, 1208000, 1364816)
(787479, 922000, 1212521)	(740919, 1018000, 1259081)	(689751, 1114000, 1310249)	(633975, 1210000, 1366025)
(786556, 924000, 1213444)	(739900, 1020000, 1260100)	(688636, 1116000, 1311364)	(632764, 1212000, 1367236)
(785631, 926000, 1214369)	(738879, 1022000, 1261121)	(687519, 1118000, 1312481)	(631551, 1214000, 1368449)
(784704, 928000, 1215296)	(737856, 1024000, 1262144)	(686400, 1120000, 1313600)	(630336, 1216000, 1369664)
(783775, 930000, 1216225)	(736831, 1026000, 1263169)	(685279, 1122000, 1314721)	(629119, 1218000, 1370881)
(782844, 932000, 1217156)	(735804, 1028000, 1264196)	(684156, 1124000, 1315844)	(627900, 1220000, 1372100)
(781911, 934000, 1218089)	(734775, 1030000, 1265225)	(683031, 1126000, 1316969)	(626679, 1222000, 1373321)
(780976, 936000, 1219024)	(733744, 1032000, 1266256)	(681904, 1128000, 1318096)	(625456, 1224000, 1374544)
(780039, 938000, 1219961)	(732711, 1034000, 1267289)	(680775, 1130000, 1319225)	(624231, 1226000, 1375769)
(779100, 940000, 1220900)	(731676, 1036000, 1268324)	(679644, 1132000, 1320356)	(623004, 1228000, 1376996)
(778159, 942000, 1221841)	(730639, 1038000, 1269361)	(678511, 1134000, 1321489)	(621775, 1230000, 1378225)
(777216, 944000, 1222784)	(729600, 1040000, 1270400)	(677376, 1136000, 1322624)	(620544, 1232000, 1379456)
(776271, 946000, 1223729)	(728559, 1042000, 1271441)	(676239, 1138000, 1323761)	(619311, 1234000, 1380689)
(775324, 948000, 1224676)	(727516, 1044000, 1272484)	(675100, 1140000, 1324900)	(618076, 1236000, 1381924)
(774375, 950000, 1225625)	(726471, 1046000, 1273529)	(673959, 1142000, 1326041)	(616839, 1238000, 1383161)
(773424, 952000, 1226576)	(725424, 1048000, 1274576)	(672816, 1144000, 1327184)	(615600, 1240000, 1384400)
(772471, 954000, 1227529)	(724375, 1050000, 1275625)	(671671, 1146000, 1328329)	(614359, 1242000, 1385641)
(771516, 956000, 1228484)	(723324, 1052000, 1276676)	(670524, 1148000, 1329476)	(613116, 1244000, 1386884)
(770559, 958000, 1229441)	(722271, 1054000, 1277729)	(669375, 1150000, 1330625)	(611871, 1246000, 1388129)
(769600, 960000, 1230400)	(721216, 1056000, 1278784)	(668224, 1152000, 1331776)	(610624, 1248000, 1389376)
(768639, 962000, 1231361)	(720159, 1058000, 1279841)	(667071, 1154000, 1332929)	(609375, 1250000, 1390625)
(767676, 964000, 1232324)	(719100, 1060000, 1280900)	(665916, 1156000, 1334084)	(608124, 1252000, 1391876)
(766711, 966000, 1233289)	(718039, 1062000, 1281961)	(664759, 1158000, 1335241)	(606871, 1254000, 1393129)
(765744, 968000, 1234256)	(716976, 1064000, 1283024)	(663600, 1160000, 1336400)	(605616, 1256000, 1394384)
(764775, 970000, 1235225)	(715911, 1066000, 1284089)	(662439, 1162000, 1337561)	(604359, 1258000, 1395641)
(763804, 972000, 1236196)	(714844, 1068000, 1285156)	(661276, 1164000, 1338724)	(603100, 1260000, 1396900)
(762831, 974000, 1237169)	(713775, 1070000, 1286225)	(660111, 1166000, 1339889)	(601839, 1262000, 1398161)
(761856, 976000, 1238144)	(712704, 1072000, 1287296)	(658944, 1168000, 1341056)	(600576, 1264000, 1399424)
(760879, 978000, 1239121)	(711631, 1074000, 1288369)	(657775, 1170000, 1342225)	(599311, 1266000, 1400689)
(759900, 980000, 1240100)	(710556, 1076000, 1289444)	(656604, 1172000, 1343396)	(598044, 1268000, 1401956)
(758919, 982000, 1241081)	(709479, 1078000, 1290521)	(655431, 1174000, 1344569)	(596775, 1270000, 1403225)
(757936, 984000, 1242064)	(708400, 1080000, 1291600)	(654256, 1176000, 1345744)	(595504, 1272000, 1404496)

(594231, 1274000, 1405769)	(530775, 1370000, 1469225)	(462711, 1466000, 1537289)	(390039, 1562000, 1609961)
(592956, 1276000, 1407044)	(529404, 1372000, 1470596)	(461244, 1468000, 1538756)	(388476, 1564000, 1611524)
(591679, 1278000, 1408321)	(528031, 1374000, 1471969)	(459775, 1470000, 1540225)	(386911, 1566000, 1613089)
(590400, 1280000, 1409600)	(526656, 1376000, 1473344)	(458304, 1472000, 1541696)	(385344, 1568000, 1614656)
(589119, 1282000, 1410881)	(525279, 1378000, 1474721)	(456831, 1474000, 1543169)	(383775, 1570000, 1616225)
(587836, 1284000, 1412164)	(523900, 1380000, 1476100)	(455356, 1476000, 1544644)	(382204, 1572000, 1617796)
(586551, 1286000, 1413449)	(522519, 1382000, 1477481)	(453879, 1478000, 1546121)	(380631, 1574000, 1619369)
(585264, 1288000, 1414736)	(521136, 1384000, 1478864)	(452400, 1480000, 1547600)	(379056, 1576000, 1620944)
(583975, 1290000, 1416025)	(519751, 1386000, 1480249)	(450919, 1482000, 1549081)	(377479, 1578000, 1622521)
(582684, 1292000, 1417316)	(518364, 1388000, 1481636)	(449436, 1484000, 1550564)	(375900, 1580000, 1624100)
(581391, 1294000, 1418609)	(516975, 1390000, 1483025)	(447951, 1486000, 1552049)	(374319, 1582000, 1625681)
(580096, 1296000, 1419904)	(515584, 1392000, 1484416)	(446464, 1488000, 1553536)	(372736, 1584000, 1627264)
(578799, 1298000, 1421201)	(514191, 1394000, 1485809)	(444975, 1490000, 1555025)	(371151, 1586000, 1628849)
(577500, 1300000, 1422500)	(512796, 1396000, 1487204)	(443484, 1492000, 1556516)	(369564, 1588000, 1630436)
(576199, 1302000, 1423801)	(511399, 1398000, 1488601)	(441991, 1494000, 1558009)	(367975, 1590000, 1632025)
(574896, 1304000, 1425104)	(510000, 1400000, 1490000)	(440496, 1496000, 1559504)	(366384, 1592000, 1633616)
(573591, 1306000, 1426409)	(508599, 1402000, 1491401)	(438999, 1498000, 1561001)	(364791, 1594000, 1635209)
(572284, 1308000, 1427716)	(507196, 1404000, 1492804)	(437500, 1500000, 1562500)	(363196, 1596000, 1636804)
(570975, 1310000, 1429025)	(505791, 1406000, 1494209)	(435999, 1502000, 1564001)	(361599, 1598000, 1638401)
(569664, 1312000, 1430336)	(504384, 1408000, 1495616)	(434496, 1504000, 1565504)	(360000, 1600000, 1640000)
(568351, 1314000, 1431649)	(502975, 1410000, 1497025)	(432991, 1506000, 1567009)	(358399, 1602000, 1641601)
(567036, 1316000, 1432964)	(501564, 1412000, 1498436)	(431484, 1508000, 1568516)	(356796, 1604000, 1643204)
(565719, 1318000, 1434281)	(500151, 1414000, 1499849)	(429975, 1510000, 1570025)	(355191, 1606000, 1644809)
(564400, 1320000, 1435600)	(498736, 1416000, 1501264)	(428464, 1512000, 1571536)	(353584, 1608000, 1646416)
(563079, 1322000, 1436921)	(497319, 1418000, 1502681)	(426951, 1514000, 1573049)	(351975, 1610000, 1648025)
(561756, 1324000, 1438244)	(495900, 1420000, 1504100)	(425436, 1516000, 1574564)	(350364, 1612000, 1649636)
(560431, 1326000, 1439569)	(494479, 1422000, 1505521)	(423919, 1518000, 1576081)	(348751, 1614000, 1651249)
(559104, 1328000, 1440896)	(493056, 1424000, 1506944)	(422400, 1520000, 1577600)	(347136, 1616000, 1652864)
(557775, 1330000, 1442225)	(491631, 1426000, 1508369)	(420879, 1522000, 1579121)	(345519, 1618000, 1654481)
(556444, 1332000, 1443556)	(490204, 1428000, 1509796)	(419356, 1524000, 1580644)	(343900, 1620000, 1656100)
(555111, 1334000, 1444889)	(488775, 1430000, 1511225)	(417831, 1526000, 1582169)	(342279, 1622000, 1657721)
(553776, 1336000, 1446224)	(487344, 1432000, 1512656)	(416304, 1528000, 1583696)	(340656, 1624000, 1659344)
(552439, 1338000, 1447561)	(485911, 1434000, 1514089)	(414775, 1530000, 1585225)	(339031, 1626000, 1660969)
(551100, 1340000, 1448900)	(484476, 1436000, 1515524)	(413244, 1532000, 1586756)	(337404, 1628000, 1662596)
(549759, 1342000, 1450241)	(483039, 1438000, 1516961)	(411711, 1534000, 1588289)	(335775, 1630000, 1664225)
(548416, 1344000, 1451584)	(481600, 1440000, 1518400)	(410176, 1536000, 1589824)	(334144, 1632000, 1665856)
(547071, 1346000, 1452929)	(480159, 1442000, 1519841)	(408639, 1538000, 1591361)	(332511, 1634000, 1667489)
(545724, 1348000, 1454276)	(478716, 1444000, 1521284)	(407100, 1540000, 1592900)	(330876, 1636000, 1669124)
(544375, 1350000, 1455625)	(477271, 1446000, 1522729)	(405559, 1542000, 1594441)	(329239, 1638000, 1670761)
(543024, 1352000, 1456976)	(475824, 1448000, 1524176)	(404016, 1544000, 1595984)	(327600, 1640000, 1672400)
(541671, 1354000, 1458329)	(474375, 1450000, 1525625)	(402471, 1546000, 1597529)	(325959, 1642000, 1674041)
(540316, 1356000, 1459684)	(472924, 1452000, 1527076)	(400924, 1548000, 1599076)	(324316, 1644000, 1675684)
(538959, 1358000, 1461041)	(471471, 1454000, 1528529)	(399375, 1550000, 1600625)	(322671, 1646000, 1677329)
(537600, 1360000, 1462400)	(470016, 1456000, 1529984)	(397824, 1552000, 1602176)	(321024, 1648000, 1678976)
(536239, 1362000, 1463761)	(468559, 1458000, 1531441)	(396271, 1554000, 1603729)	(319375, 1650000, 1680625)
(534876, 1364000, 1465124)	(467100, 1460000, 1532900)	(394716, 1556000, 1605284)	(317724, 1652000, 1682276)
(533511, 1366000, 1466489)	(465639, 1462000, 1534361)	(393159, 1558000, 1606841)	(316071, 1654000, 1683929)
(532144, 1368000, 1467856)	(464176, 1464000, 1535824)	(391600, 1560000, 1608400)	(314416, 1656000, 1685584)

(312759, 1658000, 1687241)	(239616, 1744000, 1760384)	(162775, 1830000, 1837225)	(82236, 1916000, 1917764)
(311100, 1660000, 1688900)	(237871, 1746000, 1762129)	(160944, 1832000, 1839056)	(80319, 1918000, 1919681)
(309439, 1662000, 1690561)	(236124, 1748000, 1763876)	(159111, 1834000, 1840889)	(78400, 1920000, 1921600)
(307776, 1664000, 1692224)	(234375, 1750000, 1765625)	(157276, 1836000, 1842724)	(76479, 1922000, 1923521)
(306111, 1666000, 1693889)	(232624, 1752000, 1767376)	(155439, 1838000, 1844561)	(74556, 1924000, 1925444)
(304444, 1668000, 1695556)	(230871, 1754000, 1769129)	(153600, 1840000, 1846400)	(72631, 1926000, 1927369)
(302775, 1670000, 1697225)	(229116, 1756000, 1770884)	(151759, 1842000, 1848241)	(70704, 1928000, 1929296)
(301104, 1672000, 1698896)	(227359, 1758000, 1772641)	(149916, 1844000, 1850084)	(68775, 1930000, 1931225)
(299431, 1674000, 1700569)	(225600, 1760000, 1774400)	(148071, 1846000, 1851929)	(66844, 1932000, 1933156)
(297756, 1676000, 1702244)	(223839, 1762000, 1776161)	(146224, 1848000, 1853776)	(64911, 1934000, 1935089)
(296079, 1678000, 1703921)	(222076, 1764000, 1777924)	(144375, 1850000, 1855625)	(62976, 1936000, 1937024)
(294400, 1680000, 1705600)	(220311, 1766000, 1779689)	(142524, 1852000, 1857476)	(61039, 1938000, 1938961)
(292719, 1682000, 1707281)	(218544, 1768000, 1781456)	(140671, 1854000, 1859329)	(59100, 1940000, 1940900)
(291036, 1684000, 1708964)	(216775, 1770000, 1783225)	(138816, 1856000, 1861184)	(57159, 1942000, 1942841)
(289351, 1686000, 1710649)	(215004, 1772000, 1784996)	(136959, 1858000, 1863041)	(55216, 1944000, 1944784)
(287664, 1688000, 1712336)	(213231, 1774000, 1786769)	(135100, 1860000, 1864900)	(53271, 1946000, 1946729)
(285975, 1690000, 1714025)	(211456, 1776000, 1788544)	(133239, 1862000, 1866761)	(51324, 1948000, 1948676)
(284284, 1692000, 1715716)	(209679, 1778000, 1790321)	(131376, 1864000, 1868624)	(49375, 1950000, 1950625)
(282591, 1694000, 1717409)	(207900, 1780000, 1792100)	(129511, 1866000, 1870489)	(47424, 1952000, 1952576)
(280896, 1696000, 1719104)	(206119, 1782000, 1793881)	(127644, 1868000, 1872356)	(45471, 1954000, 1954529)
(279199, 1698000, 1720801)	(204336, 1784000, 1795664)	(125775, 1870000, 1874225)	(43516, 1956000, 1956484)
(277500, 1700000, 1722500)	(202551, 1786000, 1797449)	(123904, 1872000, 1876096)	(41559, 1958000, 1958441)
(275799, 1702000, 1724201)	(200764, 1788000, 1799236)	(122031, 1874000, 1877969)	(39600, 1960000, 1960400)
(274096, 1704000, 1725904)	(198975, 1790000, 1801025)	(120156, 1876000, 1879844)	(37639, 1962000, 1962361)
(272391, 1706000, 1727609)	(197184, 1792000, 1802816)	(118279, 1878000, 1881721)	(35676, 1964000, 1964324)
(270684, 1708000, 1729316)	(195391, 1794000, 1804609)	(116400, 1880000, 1883600)	(33711, 1966000, 1966289)
(268975, 1710000, 1731025)	(193596, 1796000, 1806404)	(114519, 1882000, 1885481)	(31744, 1968000, 1968256)
(267264, 1712000, 1732736)	(191799, 1798000, 1808201)	(112636, 1884000, 1887364)	(29775, 1970000, 1970225)
(265551, 1714000, 1734449)	(190000, 1800000, 1810000)	(110751, 1886000, 1889249)	(27804, 1972000, 1972196)
(263836, 1716000, 1736164)	(188199, 1802000, 1811801)	(108864, 1888000, 1891136)	(25831, 1974000, 1974169)
(262119, 1718000, 1737881)	(186396, 1804000, 1813604)	(106975, 1890000, 1893025)	(23856, 1976000, 1976144)
(260400, 1720000, 1739600)	(184591, 1806000, 1815409)	(105084, 1892000, 1894916)	(21879, 1978000, 1978121)
(258679, 1722000, 1741321)	(182784, 1808000, 1817216)	(103191, 1894000, 1896809)	(19900, 1980000, 1980100)
(256956, 1724000, 1743044)	(180975, 1810000, 1819025)	(101296, 1896000, 1898704)	(17919, 1982000, 1982081)
(255231, 1726000, 1744769)	(179164, 1812000, 1820836)	(99399, 1898000, 1900601)	(15936, 1984000, 1984064)
(253504, 1728000, 1746496)	(177351, 1814000, 1822649)	(97500, 1900000, 1902500)	(13951, 1986000, 1986049)
(251775, 1730000, 1748225)	(175536, 1816000, 1824464)	(95599, 1902000, 1904401)	(11964, 1988000, 1988036)
(250044, 1732000, 1749956)	(173719, 1818000, 1826281)	(93696, 1904000, 1906304)	(9975, 1990000, 1990025)
(248311, 1734000, 1751689)	(171900, 1820000, 1828100)	(91791, 1906000, 1908209)	(7984, 1992000, 1992016)
(246576, 1736000, 1753424)	(170079, 1822000, 1829921)	(89884, 1908000, 1910116)	(5991, 1994000, 1994009)
(244839, 1738000, 1755161)	(168256, 1824000, 1831744)	(87975, 1910000, 1912025)	(3996, 1996000, 1996004)
(243100, 1740000, 1756900)	(166431, 1826000, 1833569)	(86064, 1912000, 1913936)	(1999, 1998000, 1998001)
(241359, 1742000, 1758641)	(164604, 1828000, 1835396)	(84151, 1914000, 1915849)	(6.3)

For more studies refer Taneja [3, 4]. For magic square generations refer Taneja [5].

6.2 Palindromic-Type Pandigital Patterns

6.2.1 First Way

Let's consider

$$m = 1000, 11000, 111000, 1111000, 11111000, 111111000, 1111111000, 11111111000, 111111111000$$

in (2.1), then for $n = 1$ and $n = 999$, we have the first and the last **palindromic-type pandigital Pythagorean patterns** given below:

► First Pattern -1

$$\begin{aligned}
 0999999^2 + 2000^2 &= 1\ 000001^2 \\
 12\ 0999999^2 + 22000^2 &= 121\ 000001^2 \\
 1232\ 0999999^2 + 222000^2 &= 12321\ 000001^2 \\
 123432\ 0999999^2 + 2222000^2 &= 1234321\ 000001^2 \\
 12345432\ 0999999^2 + 22222000^2 &= 123454321\ 000001^2 \\
 1234565432\ 0999999^2 + 222222000^2 &= 12345654321\ 000001^2 \\
 123456765432\ 0999999^2 + 2222222000^2 &= 1234567654321\ 000001^2 \\
 12345678765432\ 0999999^2 + 22222222000^2 &= 123456787654321\ 000001^2 \\
 1234567898765432\ 0999999^2 + 222222222000^2 &= 12345678987654321\ 000001^2 \quad (6.4)
 \end{aligned}$$

► Last Pattern - 999

$$\begin{aligned}
 0001999^2 + 1998000^2 &= 1\ 998001^2 \\
 12\ 0001999^2 + 21978000^2 &= 121\ 998001^2 \\
 1232\ 0001999^2 + 221778000^2 &= 12321\ 998001^2 \\
 123432\ 0001999^2 + 2219778000^2 &= 1234321\ 998001^2 \\
 12345432\ 0001999^2 + 22199778000^2 &= 123454321\ 998001^2 \\
 1234565432\ 0001999^2 + 221999778000^2 &= 12345654321\ 998001^2 \\
 123456765432\ 0001999^2 + 2219999778000^2 &= 1234567654321\ 998001^2 \\
 12345678765432\ 0001999^2 + 22199999778000^2 &= 123456787654321\ 998001^2 \\
 1234567898765432\ 0001999^2 + 221999999778000^2 &= 12345678987654321\ 998001^2 \quad (6.5)
 \end{aligned}$$

In this case, we observe that the second term in the first line of pattern (6.5) doesn't give good pattern. It happens with many results. Above we have written on two patterns. The complete list can be seen in author's work [9].

6.2.2 Second Way

Let's consider

$$m = 1000, 101000, 10101000, 1010101000, 101010101000, 10101010101000, 1010101010101000, 101010101010101000, 10101010101010101000$$

in (2.1), then for each value of $n = 1, 2, 3, \dots, 997, 998, 999$; there are 999 **palindromic-type pandigital Pythagorean patterns** given below:

► **First Pattern - 1**

$$\begin{aligned}
 &0999999^2 + 2000^2 &= &1\ 000001^2 \\
 &1020\ 0999999^2 + 202000^2 &= &10201\ 000001^2 \\
 &10203020\ 0999999^2 + 20202000^2 &= &102030201\ 000001^2 \\
 &102030403020\ 0999999^2 + 2020202000^2 &= &1020304030201\ 000001^2 \\
 &1020304050403020\ 0999999^2 + 202020202000^2 &= &10203040504030201\ 000001^2 \\
 &10203040506050403020\ 0999999^2 + 20202020202000^2 &= &102030405060504030201\ 000001^2 \\
 &102030405060706050403020\ 0999999^2 + 2020202020202000^2 &= &1020304050607060504030201\ 000001^2 \\
 &1020304050607080706050403020\ 0999999^2 + 202020202020202000^2 &= &10203040506070807060504030201\ 000001^2 \\
 &10203040506070809080706050403020\ 0999999^2 + 20202020202020202000^2 &= &102030405060708090807060504030201\ 000001^2 \tag{6.6}
 \end{aligned}$$

► **Last Pattern - 999**

$$\begin{aligned}
 &00001999^2 + 1998000^2 &= &1\ 998001^2 \\
 &102\ 00001999^2 + 201798000^2 &= &10201\ 998001^2 \\
 &1020302\ 00001999^2 + 20181798000^2 &= &102030201\ 998001^2 \\
 &10203040302\ 00001999^2 + 2018181798000^2 &= &1020304030201\ 998001^2 \\
 &102030405040302\ 00001999^2 + 201818181798000^2 &= &10203040504030201\ 998001^2 \\
 &1020304050605040302\ 00001999^2 + 20181818181798000^2 &= &102030405060504030201\ 998001^2 \\
 &10203040506070605040302\ 00001999^2 + 2018181818181798000^2 &= &1020304050607060504030201\ 998001^2 \\
 &102030405060708070605040302\ 00001999^2 + 201818181818181798000^2 &= &10203040506070807060504030201\ 998001^2 \tag{6.7}
 \end{aligned}$$

In this case, we observe that the second term in the first line of pattern (6.7) doesn't give good pattern. It happens with many results. Above we have written on two patterns. The complete list can be seen in author's work [9].

6.3 Magic Squares

Applying the procedure given in (4.3) over the triples given in (6.3), we can construct magic squares of order 999 to 3. See below few examples:

$$\begin{aligned}
 (999999, 2000, 1000001) &\Rightarrow \{999, 4001, 2000001, 1000998999, 999998000001\} \\
 (999996, 4000, 1000004) &\Rightarrow \{998, 8001, 2000007, 1001995992, 999992000016\} \\
 (999991, 6000, 1000009) &\Rightarrow \{997, 12001, 2000017, 1002990973, 999982000081\} \\
 (999984, 8000, 1000016) &\Rightarrow \{996, 16001, 2000031, 1003983936, 999968000256\} \\
 (999975, 10000, 1000025) &\Rightarrow \{995, 20001, 2000049, 1004974875, 999950000625\} \\
 &\dots \quad \dots \quad \dots \quad \dots \quad \dots \\
 (754975, 990000, 1245025) &\Rightarrow \{505, 1980001, 2490049, 1128687625, 569987250625\} \\
 (753984, 992000, 1246016) &\Rightarrow \{504, 1984001, 2492031, 1127960064, 568491872256\} \\
 (752991, 994000, 1247009) &\Rightarrow \{503, 1988001, 2494017, 1127227527, 566995446081\} \\
 (751996, 996000, 1248004) &\Rightarrow \{502, 1992001, 2496007, 1126490008, 565497984016\}
 \end{aligned}$$

$$\begin{aligned}
 (750999, 998000, 1249001) &\Rightarrow \{501, 1996001, 2498001, 1125747501, 563999498001\} \\
 \dots \quad \dots &\quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \\
 (011964, 1988000, 1988036) &\Rightarrow \{6, 3976001, 3976071, 23856216, 143137296\} \\
 (009975, 1990000, 1990025) &\Rightarrow \{5, 3980001, 3980049, 19900125, 99500625\} \\
 (007984, 1992000, 1992016) &\Rightarrow \{4, 3984001, 3984031, 15936064, 63744256\} \\
 (005991, 1994000, 1994009) &\Rightarrow \{3, 3988001, 3988017, 11964027, 35892081\} \tag{6.8}
 \end{aligned}$$

For more details refer Taneja [3, 4].

7 Second Procedure

This procedure is little different from the one given in (3.2). Here we shall consider only the first line given in (3.1), i.e., we shall use only the 01 and 99 as constant, the other values are increased by square numbers in one column and minus one of the same square number in the in second column, i.e., $a01^2 - (a - 1)99^2$, where a is a perfect square, for examples, $1601^2 - 1599^2$, $4901^2 - 4899^2$, etc. This difference is always a perfect square. See below these cases, for the values from 1^2 to 99^2 giving 99 Pythagorean triples:

$$\begin{aligned}
 000101^2 - 000099^2 = 0020^2 &\Rightarrow 000099^2 + 0020^2 = 000101^2 \\
 000401^2 - 000399^2 = 0040^2 &\Rightarrow 000399^2 + 0040^2 = 000401^2 \\
 000901^2 - 000899^2 = 0060^2 &\Rightarrow 000899^2 + 0060^2 = 000901^2 \\
 001601^2 - 001599^2 = 0080^2 &\Rightarrow 001599^2 + 0080^2 = 001601^2 \\
 002501^2 - 002499^2 = 0100^2 &\Rightarrow 002499^2 + 0100^2 = 002501^2 \\
 003601^2 - 003599^2 = 0120^2 &\Rightarrow 003599^2 + 0120^2 = 003601^2 \\
 004901^2 - 004899^2 = 0140^2 &\Rightarrow 004899^2 + 0140^2 = 004901^2 \\
 006401^2 - 006399^2 = 0160^2 &\Rightarrow 006399^2 + 0160^2 = 006401^2 \\
 008101^2 - 008099^2 = 0180^2 &\Rightarrow 008099^2 + 0180^2 = 008101^2 \\
 \dots \quad \dots \quad \dots &\quad \dots \quad \dots \quad \dots \\
 940901^2 - 940899^2 = 1940^2 &\Rightarrow 940899^2 + 1940^2 = 940901^2 \\
 960401^2 - 960399^2 = 1960^2 &\Rightarrow 960399^2 + 1960^2 = 960401^2 \\
 980101^2 - 980099^2 = 1980^2 &\Rightarrow 980099^2 + 1980^2 = 980101^2 \tag{7.1}
 \end{aligned}$$

Remark 7. The first four digits of first column are perfect squares, and then just put 01^2 . In the second column the first four digits are 1 less than a perfect square of first column and then put 99^2 at the end. Taking the difference, we get Pythagorean triples with multiple of 20. The same process can be extended as long as we want. This procedure extend the one given in (3.1) but only the last line. Each line of (3.1) can be extended in a similar way resulting in lot of Pythagorean triples.

7.1 Alternative Approach

The above triples can also be obtained by use of formula given in (2.1). In the previous case we fixed the values of m and vary the values of n . Now varying the values of m and fixing the value of n ,

in (2.1), we can get the triples as given above in (7.1). For this, let's consider 99 values of m as $m = 10, 20, 30, \dots, 100, 110, 120, \dots, 990$ and fix n as $n = 1$, we get respectively the same triples as written above in (7.1). See below:

(99, 20, 101)	(67599, 520, 67601)	(260099, 1020, 260101)	(577599, 1520, 577601)
(399, 40, 401)	(72899, 540, 72901)	(270399, 1040, 270401)	(592899, 1540, 592901)
(899, 60, 901)	(78399, 560, 78401)	(280899, 1060, 280901)	(608399, 1560, 608401)
(1599, 80, 1601)	(84099, 580, 84101)	(291599, 1080, 291601)	(624099, 1580, 624101)
(2499, 100, 2501)	(89999, 600, 90001)	(302499, 1100, 302501)	(639999, 1600, 640001)
(3599, 120, 3601)	(96099, 620, 96101)	(313599, 1120, 313601)	(656099, 1620, 656101)
(4899, 140, 4901)	(102399, 640, 102401)	(324899, 1140, 324901)	(672399, 1640, 672401)
(6399, 160, 6401)	(108899, 660, 108901)	(336399, 1160, 336401)	(688899, 1660, 688901)
(8099, 180, 8101)	(115599, 680, 115601)	(348099, 1180, 348101)	(705599, 1680, 705601)
(9999, 200, 10001)	(122499, 700, 122501)	(359999, 1200, 360001)	(722499, 1700, 722501)
(12099, 220, 12101)	(129599, 720, 129601)	(372099, 1220, 372101)	(739599, 1720, 739601)
(14399, 240, 14401)	(136899, 740, 136901)	(384399, 1240, 384401)	(756899, 1740, 756901)
(16899, 260, 16901)	(144399, 760, 144401)	(396899, 1260, 396901)	(774399, 1760, 774401)
(19599, 280, 19601)	(152099, 780, 152101)	(409599, 1280, 409601)	(792099, 1780, 792101)
(22499, 300, 22501)	(159999, 800, 160001)	(422499, 1300, 422501)	(809999, 1800, 810001)
(25599, 320, 25601)	(168099, 820, 168101)	(435599, 1320, 435601)	(828099, 1820, 828101)
(28899, 340, 28901)	(176399, 840, 176401)	(448899, 1340, 448901)	(846399, 1840, 846401)
(32399, 360, 32401)	(184899, 860, 184901)	(462399, 1360, 462401)	(864899, 1860, 864901)
(36099, 380, 36101)	(193599, 880, 193601)	(476099, 1380, 476101)	(883599, 1880, 883601)
(39999, 400, 40001)	(202499, 900, 202501)	(489999, 1400, 490001)	(902499, 1900, 902501)
(44099, 420, 44101)	(211599, 920, 211601)	(504099, 1420, 504101)	(921599, 1920, 921601)
(48399, 440, 48401)	(220899, 940, 220901)	(518399, 1440, 518401)	(940899, 1940, 940901)
(52899, 460, 52901)	(230399, 960, 230401)	(532899, 1460, 532901)	(960399, 1960, 960401)
(57599, 480, 57601)	(240099, 980, 240101)	(547599, 1480, 547601)	(980099, 1980, 980101)
(62499, 500, 62501)	(249999, 1000, 250001)	(562499, 1500, 562501)	

Remark 8. The first triple of (7.2) are the same as the last triple of each block of (3.2)-(3.11). In the similar way we can extend the other triples given eqrefeq3-(3.11) resulting in lot of Pythagorean triples.

7.2 Patterns in Pythagorean Triples

In this subsection, we shall give two types of patterns. One when the value of n is fixed as $n = 1$. Second type is when n is varying, i.e., $n = 2, 3, \dots, 9$. In the first case, we have written 100 values. In the second situation there are only 9 values for each value of n .

7.2.1 First Type

Let's consider

$$k_1 := \{m = 10, 100, 1000, \dots\},$$

$$k_2 := \{m = 20, 200, 2000, \dots\},$$

... ..

$$k_{98} := \{m = 980, 9800, 98000, \dots\},$$

$$k_{99} := \{m = 990, 9900, 99000, \dots\}$$

• For $n = 1$ and $k_1, k_2, \dots, k_{98}, k_{99}$ in (2.1):

Below are 99 patterns with final sums. In some cases the final sums don't give good pattern. These are written in **different color**.

$$\begin{aligned} 99^2 + 20^2 &= 101^2 && := 10201 \\ 9999^2 + 200^2 &= 10001^2 && := 100020001 \\ 999999^2 + 2000^2 &= 1000001^2 && := 1000002000001 \\ 99999999^2 + 20000^2 &= 100000001^2 && := 10000000200000001 \\ 9999999999^2 + 200000^2 &= 10000000001^2 && := 100000000020000000001 \end{aligned} \tag{7.2}$$

$$\begin{aligned} 399^2 + 40^2 &= 401^2 && := 160801 \\ 39999^2 + 400^2 &= 40001^2 && := 1600080001 \\ 3999999^2 + 4000^2 &= 4000001^2 && := 16000008000001 \\ 399999999^2 + 40000^2 &= 400000001^2 && := 160000000800000001 \\ 39999999999^2 + 400000^2 &= 40000000001^2 && := 1600000000080000000001 \end{aligned} \tag{7.3}$$

$$\begin{aligned} 899^2 + 60^2 &= 901^2 && := 811801 \\ 89999^2 + 600^2 &= 90001^2 && := 8100180001 \\ 8999999^2 + 6000^2 &= 9000001^2 && := 81000018000001 \\ 899999999^2 + 60000^2 &= 900000001^2 && := 810000001800000001 \\ 89999999999^2 + 600000^2 &= 90000000001^2 && := 8100000000180000000001 \end{aligned} \tag{7.4}$$

$$\begin{aligned} 1599^2 + 80^2 &= 1601^2 && := 2563201 \\ 159999^2 + 800^2 &= 160001^2 && := 25600320001 \\ 15999999^2 + 8000^2 &= 16000001^2 && := 256000032000001 \\ 1599999999^2 + 80000^2 &= 1600000001^2 && := 2560000003200000001 \\ 159999999999^2 + 800000^2 &= 160000000001^2 && := 25600000000320000000001 \end{aligned} \tag{7.5}$$

$$\begin{aligned} 2499^2 + 100^2 &= 2501^2 && := 6255001 \\ 249999^2 + 1000^2 &= 250001^2 && := 62500500001 \\ 24999999^2 + 10000^2 &= 25000001^2 && := 625000050000001 \\ 2499999999^2 + 100000^2 &= 2500000001^2 && := 6250000005000000001 \\ 249999999999^2 + 1000000^2 &= 250000000001^2 && := 62500000000500000000001 \end{aligned} \tag{7.6}$$

$$\begin{aligned}
 3599^2 + 120^2 &= 3601^2 & := 12967201 \\
 35999^2 + 1200^2 &= 360001^2 & := 129600720001 \\
 359999^2 + 12000^2 &= 36000001^2 & := 1296000072000001 \\
 3599999^2 + 120000^2 &= 3600000001^2 & := 12960000007200000001 \\
 35999999^2 + 1200000^2 &= 360000000001^2 & := 129600000000720000000001 \quad (7.7)
 \end{aligned}$$

$$\begin{aligned}
 4899^2 + 140^2 &= 4901^2 & := 24019801 \\
 48999^2 + 1400^2 &= 490001^2 & := 240100980001 \\
 489999^2 + 14000^2 &= 49000001^2 & := 2401000098000001 \\
 4899999^2 + 140000^2 &= 4900000001^2 & := 24010000009800000001 \\
 48999999^2 + 1400000^2 &= 490000000001^2 & := 240100000000980000000001 \quad (7.8)
 \end{aligned}$$

$$\begin{aligned}
 6399^2 + 160^2 &= 6401^2 & := 40972801 \\
 63999^2 + 1600^2 &= 640001^2 & := 409601280001 \\
 639999^2 + 16000^2 &= 64000001^2 & := 4096000128000001 \\
 6399999^2 + 160000^2 &= 6400000001^2 & := 40960000012800000001 \\
 63999999^2 + 1600000^2 &= 640000000001^2 & := 409600000001280000000001 \quad (7.9)
 \end{aligned}$$

$$\begin{aligned}
 8099^2 + 180^2 &= 8101^2 & := 65626201 \\
 80999^2 + 1800^2 &= 810001^2 & := 656101620001 \\
 809999^2 + 18000^2 &= 81000001^2 & := 6561000162000001 \\
 8099999^2 + 180000^2 &= 8100000001^2 & := 65610000016200000001 \\
 80999999^2 + 1800000^2 &= 810000000001^2 & := 656100000001620000000001 \quad (7.10)
 \end{aligned}$$

$$\begin{aligned}
 9999^2 + 200^2 &= 10001^2 & := 100020001 \\
 99999^2 + 2000^2 &= 1000001^2 & := 1000002000001 \\
 999999^2 + 20000^2 &= 100000001^2 & := 10000000200000001 \\
 9999999^2 + 200000^2 &= 10000000001^2 & := 100000000020000000001 \\
 99999999^2 + 2000000^2 &= 1000000000001^2 & := 1000000000002000000000001 \quad (7.11)
 \end{aligned}$$

$$\begin{aligned}
 12099^2 + 220^2 &= 12101^2 & := 146434201 \\
 120999^2 + 2200^2 &= 1210001^2 & := 1464102420001 \\
 1209999^2 + 22000^2 &= 121000001^2 & := 14641000242000001 \\
 12099999^2 + 220000^2 &= 12100000001^2 & := 146410000024200000001 \\
 120999999^2 + 2200000^2 &= 1210000000001^2 & := 1464100000002420000000001 \quad (7.12)
 \end{aligned}$$

$$\begin{aligned}
14399^2 + 240^2 &= 14401^2 & := 207388801 \\
143999^2 + 2400^2 &= 1440001^2 & := 2073602880001 \\
14399999^2 + 24000^2 &= 144000001^2 & := 20736000288000001 \\
143999999^2 + 240000^2 &= 14400000001^2 & := 207360000028800000001 \\
1439999999^2 + 2400000^2 &= 144000000001^2 & := 207360000002880000000001 \quad (7.13)
\end{aligned}$$

$$\begin{aligned}
16899^2 + 260^2 &= 16901^2 & := 285643801 \\
168999^2 + 2600^2 &= 1690001^2 & := 2856103380001 \\
16899999^2 + 26000^2 &= 169000001^2 & := 28561000338000001 \\
168999999^2 + 260000^2 &= 16900000001^2 & := 285610000033800000001 \\
1689999999^2 + 2600000^2 &= 169000000001^2 & := 2856100000003380000000001 \quad (7.14)
\end{aligned}$$

$$\begin{aligned}
19599^2 + 280^2 &= 19601^2 & := 384199201 \\
195999^2 + 2800^2 &= 1960001^2 & := 3841603920001 \\
19599999^2 + 28000^2 &= 196000001^2 & := 38416000392000001 \\
195999999^2 + 280000^2 &= 19600000001^2 & := 384160000039200000001 \\
1959999999^2 + 2800000^2 &= 196000000001^2 & := 3841600000003920000000001 \quad (7.15)
\end{aligned}$$

$$\begin{aligned}
22499^2 + 300^2 &= 22501^2 & := 506295001 \\
224999^2 + 3000^2 &= 2250001^2 & := 5062504500001 \\
22499999^2 + 30000^2 &= 225000001^2 & := 50625000450000001 \\
224999999^2 + 300000^2 &= 22500000001^2 & := 506250000045000000001 \\
2249999999^2 + 3000000^2 &= 225000000001^2 & := 5062500000004500000000001 \quad (7.16)
\end{aligned}$$

$$\begin{aligned}
25599^2 + 320^2 &= 25601^2 & := 655411201 \\
255999^2 + 3200^2 &= 2560001^2 & := 6553605120001 \\
25599999^2 + 32000^2 &= 256000001^2 & := 65536000512000001 \\
255999999^2 + 320000^2 &= 25600000001^2 & := 655360000051200000001 \\
2559999999^2 + 3200000^2 &= 256000000001^2 & := 6553600000005120000000001 \quad (7.17)
\end{aligned}$$

$$\begin{aligned}
28899^2 + 340^2 &= 28901^2 & := 835267801 \\
288999^2 + 3400^2 &= 2890001^2 & := 8352105780001 \\
28899999^2 + 34000^2 &= 289000001^2 & := 83521000578000001 \\
288999999^2 + 340000^2 &= 28900000001^2 & := 835210000057800000001 \\
2889999999^2 + 3400000^2 &= 289000000001^2 & := 8352100000005780000000001 \quad (7.18)
\end{aligned}$$

$$\begin{aligned}
32399^2 + 360^2 &= 32401^2 & := & 1049824801 \\
323999^2 + 3600^2 &= 3240001^2 & := & 10497606480001 \\
32399999^2 + 36000^2 &= 324000001^2 & := & 104976000648000001 \\
323999999^2 + 360000^2 &= 32400000001^2 & := & 1049760000064800000001 \\
3239999999^2 + 3600000^2 &= 3240000000001^2 & := & 10497600000006480000000001
\end{aligned} \tag{7.19}$$

$$\begin{aligned}
36099^2 + 380^2 &= 36101^2 & := & 1303282201 \\
360999^2 + 3800^2 &= 3610001^2 & := & 13032107220001 \\
3609999^2 + 38000^2 &= 361000001^2 & := & 130321000722000001 \\
36099999^2 + 380000^2 &= 36100000001^2 & := & 1303210000072200000001 \\
360999999^2 + 3800000^2 &= 3610000000001^2 & := & 13032100000007220000000001
\end{aligned} \tag{7.20}$$

$$\begin{aligned}
39999^2 + 400^2 &= 40001^2 & := & 1600080001 \\
399999^2 + 4000^2 &= 4000001^2 & := & 16000008000001 \\
3999999^2 + 40000^2 &= 400000001^2 & := & 160000000800000001 \\
39999999^2 + 400000^2 &= 40000000001^2 & := & 1600000000080000000001 \\
399999999^2 + 4000000^2 &= 4000000000001^2 & := & 16000000000008000000000001
\end{aligned} \tag{7.21}$$

$$\begin{aligned}
44099^2 + 420^2 &= 44101^2 & := & 1944898201 \\
440999^2 + 4200^2 &= 4410001^2 & := & 19448108820001 \\
4409999^2 + 42000^2 &= 441000001^2 & := & 194481000882000001 \\
44099999^2 + 420000^2 &= 44100000001^2 & := & 1944810000088200000001 \\
440999999^2 + 4200000^2 &= 4410000000001^2 & := & 19448100000008820000000001
\end{aligned} \tag{7.22}$$

$$\begin{aligned}
48399^2 + 440^2 &= 48401^2 & := & 2342656801 \\
483999^2 + 4400^2 &= 4840001^2 & := & 23425609680001 \\
4839999^2 + 44000^2 &= 484000001^2 & := & 234256000968000001 \\
48399999^2 + 440000^2 &= 48400000001^2 & := & 2342560000096800000001 \\
483999999^2 + 4400000^2 &= 4840000000001^2 & := & 23425600000009680000000001
\end{aligned} \tag{7.23}$$

$$\begin{aligned}
52899^2 + 460^2 &= 52901^2 & := & 2798515801 \\
528999^2 + 4600^2 &= 5290001^2 & := & 27984110580001 \\
5289999^2 + 46000^2 &= 529000001^2 & := & 279841001058000001 \\
52899999^2 + 460000^2 &= 52900000001^2 & := & 2798410000105800000001 \\
528999999^2 + 4600000^2 &= 5290000000001^2 & := & 27984100000010580000000001
\end{aligned} \tag{7.24}$$

$$\begin{aligned}
57599^2 + 480^2 &= 57601^2 & := 3317875201 \\
575999^2 + 4800^2 &= 5760001^2 & := 33177611520001 \\
57599999^2 + 48000^2 &= 576000001^2 & := 331776001152000001 \\
575999999^2 + 480000^2 &= 57600000001^2 & := 3317760000115200000001 \\
5759999999^2 + 4800000^2 &= 5760000000001^2 & := 33177600000011520000000001 \quad (7.25)
\end{aligned}$$

$$\begin{aligned}
62499^2 + 500^2 &= 62501^2 & := 3906375001 \\
624999^2 + 5000^2 &= 6250001^2 & := 39062512500001 \\
62499999^2 + 50000^2 &= 625000001^2 & := 390625001250000001 \\
624999999^2 + 500000^2 &= 62500000001^2 & := 3906250000125000000001 \\
6249999999^2 + 5000000^2 &= 6250000000001^2 & := 39062500000012500000000001 \quad (7.26)
\end{aligned}$$

$$\begin{aligned}
67599^2 + 520^2 &= 67601^2 & := 4569895201 \\
675999^2 + 5200^2 &= 6760001^2 & := 45697613520001 \\
67599999^2 + 52000^2 &= 676000001^2 & := 456976001352000001 \\
675999999^2 + 520000^2 &= 67600000001^2 & := 4569760000135200000001 \\
6759999999^2 + 5200000^2 &= 6760000000001^2 & := 45697600000013520000000001 \quad (7.27)
\end{aligned}$$

$$\begin{aligned}
72899^2 + 540^2 &= 72901^2 & := 5314555801 \\
728999^2 + 5400^2 &= 7290001^2 & := 53144114580001 \\
72899999^2 + 54000^2 &= 729000001^2 & := 531441001458000001 \\
728999999^2 + 540000^2 &= 72900000001^2 & := 5314410000145800000001 \\
7289999999^2 + 5400000^2 &= 7290000000001^2 & := 53144100000014580000000001 \quad (7.28)
\end{aligned}$$

$$\begin{aligned}
78399^2 + 560^2 &= 78401^2 & := 6146716801 \\
783999^2 + 5600^2 &= 7840001^2 & := 61465615680001 \\
78399999^2 + 56000^2 &= 784000001^2 & := 614656001568000001 \\
783999999^2 + 560000^2 &= 78400000001^2 & := 6146560000156800000001 \\
7839999999^2 + 5600000^2 &= 7840000000001^2 & := 61465600000015680000000001 \quad (7.29)
\end{aligned}$$

$$\begin{aligned}
84099^2 + 580^2 &= 84101^2 & := 7072978201 \\
840999^2 + 5800^2 &= 8410001^2 & := 70728116820001 \\
84099999^2 + 58000^2 &= 841000001^2 & := 707281001682000001 \\
840999999^2 + 580000^2 &= 84100000001^2 & := 7072810000168200000001 \\
8409999999^2 + 5800000^2 &= 8410000000001^2 & := 70728100000016820000000001 \quad (7.30)
\end{aligned}$$

$$\begin{aligned}
89999^2 + 600^2 &= 90001^2 & := & 8100180001 \\
899999^2 + 6000^2 &= 9000001^2 & := & 81000018000001 \\
89999999^2 + 60000^2 &= 900000001^2 & := & 810000001800000001 \\
899999999^2 + 600000^2 &= 90000000001^2 & := & 8100000000180000000001 \\
8999999999^2 + 6000000^2 &= 9000000000001^2 & := & 81000000000018000000000001
\end{aligned} \tag{7.31}$$

$$\begin{aligned}
96099^2 + 620^2 &= 96101^2 & := & 9235402201 \\
960999^2 + 6200^2 &= 9610001^2 & := & 92352119220001 \\
9609999^2 + 62000^2 &= 961000001^2 & := & 923521001922000001 \\
96099999^2 + 620000^2 &= 96100000001^2 & := & 9235210000192200000001 \\
960999999^2 + 6200000^2 &= 9610000000001^2 & := & 92352100000019220000000001
\end{aligned} \tag{7.32}$$

$$\begin{aligned}
102399^2 + 640^2 &= 102401^2 & := & 10485964801 \\
1023999^2 + 6400^2 &= 10240001^2 & := & 104857620480001 \\
10239999^2 + 64000^2 &= 1024000001^2 & := & 1048576002048000001 \\
102399999^2 + 640000^2 &= 102400000001^2 & := & 10485760000204800000001 \\
1023999999^2 + 6400000^2 &= 10240000000001^2 & := & 104857600000020480000000001
\end{aligned} \tag{7.33}$$

$$\begin{aligned}
108899^2 + 660^2 &= 108901^2 & := & 11859427801 \\
1088999^2 + 6600^2 &= 10890001^2 & := & 118592121780001 \\
10889999^2 + 66000^2 &= 1089000001^2 & := & 1185921002178000001 \\
108899999^2 + 660000^2 &= 108900000001^2 & := & 11859210000217800000001 \\
1088999999^2 + 6600000^2 &= 10890000000001^2 & := & 118592100000021780000000001
\end{aligned} \tag{7.34}$$

$$\begin{aligned}
115599^2 + 680^2 &= 115601^2 & := & 13363591201 \\
1155999^2 + 6800^2 &= 11560001^2 & := & 133633623120001 \\
11559999^2 + 68000^2 &= 1156000001^2 & := & 1336336002312000001 \\
115599999^2 + 680000^2 &= 115600000001^2 & := & 13363360000231200000001 \\
1155999999^2 + 6800000^2 &= 11560000000001^2 & := & 133633600000023120000000001
\end{aligned} \tag{7.35}$$

$$\begin{aligned}
122499^2 + 700^2 &= 122501^2 & := & 15006495001 \\
1224999^2 + 7000^2 &= 12250001^2 & := & 150062524500001 \\
12249999^2 + 70000^2 &= 1225000001^2 & := & 1500625002450000001 \\
122499999^2 + 700000^2 &= 122500000001^2 & := & 15006250000245000000001 \\
1224999999^2 + 7000000^2 &= 12250000000001^2 & := & 150062500000024500000000001
\end{aligned} \tag{7.36}$$

$$\begin{aligned}
129599^2 + 720^2 &= 129601^2 & := & 16796419201 \\
1295999^2 + 7200^2 &= 12960001^2 & := & 167961625920001 \\
12959999^2 + 72000^2 &= 1296000001^2 & := & 1679616002592000001 \\
129599999^2 + 720000^2 &= 129600000001^2 & := & 16796160000259200000001 \\
1295999999^2 + 7200000^2 &= 12960000000001^2 & := & 167961600000025920000000001 \quad (7.37)
\end{aligned}$$

$$\begin{aligned}
136899^2 + 740^2 &= 136901^2 & := & 18741883801 \\
1368999^2 + 7400^2 &= 13690001^2 & := & 187416127380001 \\
13689999^2 + 74000^2 &= 1369000001^2 & := & 1874161002738000001 \\
136899999^2 + 740000^2 &= 136900000001^2 & := & 18741610000273800000001 \\
1368999999^2 + 7400000^2 &= 13690000000001^2 & := & 187416100000027380000000001 \quad (7.38)
\end{aligned}$$

$$\begin{aligned}
144399^2 + 760^2 &= 144401^2 & := & 20851648801 \\
1443999^2 + 7600^2 &= 14440001^2 & := & 208513628880001 \\
14439999^2 + 76000^2 &= 1444000001^2 & := & 2085136002888000001 \\
144399999^2 + 760000^2 &= 144400000001^2 & := & 20851360000288800000001 \\
1443999999^2 + 7600000^2 &= 14440000000001^2 & := & 208513600000028880000000001 \quad (7.39)
\end{aligned}$$

$$\begin{aligned}
152099^2 + 780^2 &= 152101^2 & := & 23134714201 \\
1520999^2 + 7800^2 &= 15210001^2 & := & 231344130420001 \\
15209999^2 + 78000^2 &= 1521000001^2 & := & 2313441003042000001 \\
152099999^2 + 780000^2 &= 152100000001^2 & := & 23134410000304200000001 \\
1520999999^2 + 7800000^2 &= 15210000000001^2 & := & 231344100000030420000000001 \quad (7.40)
\end{aligned}$$

$$\begin{aligned}
159999^2 + 800^2 &= 160001^2 & := & 25600320001 \\
1599999^2 + 8000^2 &= 16000001^2 & := & 256000032000001 \\
15999999^2 + 80000^2 &= 1600000001^2 & := & 2560000003200000001 \\
159999999^2 + 800000^2 &= 160000000001^2 & := & 25600000000320000000001 \\
1599999999^2 + 8000000^2 &= 16000000000001^2 & := & 256000000000032000000000001 \quad (7.41)
\end{aligned}$$

$$\begin{aligned}
168099^2 + 820^2 &= 168101^2 & := & 28257946201 \\
1680999^2 + 8200^2 &= 16810001^2 & := & 282576133620001 \\
16809999^2 + 82000^2 &= 1681000001^2 & := & 2825761003362000001 \\
168099999^2 + 820000^2 &= 168100000001^2 & := & 28257610000336200000001 \\
1680999999^2 + 8200000^2 &= 16810000000001^2 & := & 282576100000033620000000001 \quad (7.42)
\end{aligned}$$

$$\begin{aligned}
176399^2 + 840^2 &= 176401^2 & := & 31117312801 \\
17639999^2 + 8400^2 &= 17640001^2 & := & 311169635280001 \\
1763999999^2 + 84000^2 &= 1764000001^2 & := & 3111696003528000001 \\
176399999999^2 + 840000^2 &= 176400000001^2 & := & 31116960000352800000001 \\
17639999999999^2 + 8400000^2 &= 17640000000001^2 & := & 311169600000035280000000001 \quad (7.43)
\end{aligned}$$

$$\begin{aligned}
184899^2 + 860^2 &= 184901^2 & := & 34188379801 \\
18489999^2 + 8600^2 &= 18490001^2 & := & 341880136980001 \\
1848999999^2 + 86000^2 &= 1849000001^2 & := & 3418801003698000001 \\
184899999999^2 + 860000^2 &= 184900000001^2 & := & 34188010000369800000001 \\
18489999999999^2 + 8600000^2 &= 18490000000001^2 & := & 341880100000036980000000001 \quad (7.44)
\end{aligned}$$

$$\begin{aligned}
193599^2 + 880^2 &= 193601^2 & := & 37481347201 \\
19359999^2 + 8800^2 &= 19360001^2 & := & 374809638720001 \\
1935999999^2 + 88000^2 &= 1936000001^2 & := & 3748096003872000001 \\
193599999999^2 + 880000^2 &= 193600000001^2 & := & 37480960000387200000001 \\
19359999999999^2 + 8800000^2 &= 19360000000001^2 & := & 374809600000038720000000001 \quad (7.45)
\end{aligned}$$

$$\begin{aligned}
202499^2 + 900^2 &= 202501^2 & := & 41006655001 \\
20249999^2 + 9000^2 &= 20250001^2 & := & 410062540500001 \\
2024999999^2 + 90000^2 &= 2025000001^2 & := & 4100625004050000001 \\
202499999999^2 + 900000^2 &= 202500000001^2 & := & 41006250000405000000001 \\
20249999999999^2 + 9000000^2 &= 20250000000001^2 & := & 410062500000040500000000001 \quad (7.46)
\end{aligned}$$

$$\begin{aligned}
211599^2 + 920^2 &= 211601^2 & := & 44774983201 \\
21159999^2 + 9200^2 &= 21160001^2 & := & 447745642320001 \\
2115999999^2 + 92000^2 &= 2116000001^2 & := & 4477456004232000001 \\
211599999999^2 + 920000^2 &= 211600000001^2 & := & 44774560000423200000001 \\
21159999999999^2 + 9200000^2 &= 21160000000001^2 & := & 447745600000042320000000001 \quad (7.47)
\end{aligned}$$

$$\begin{aligned}
220899^2 + 940^2 &= 220901^2 & := & 48797251801 \\
22089999^2 + 9400^2 &= 22090001^2 & := & 487968144180001 \\
2208999999^2 + 94000^2 &= 2209000001^2 & := & 4879681004418000001 \\
220899999999^2 + 940000^2 &= 220900000001^2 & := & 48796810000441800000001 \\
22089999999999^2 + 9400000^2 &= 22090000000001^2 & := & 487968100000044180000000001 \quad (7.48)
\end{aligned}$$

$$\begin{aligned}
230399^2 + 960^2 &= 230401^2 & := 53084620801 \\
23039999^2 + 9600^2 &= 23040001^2 & := 530841646080001 \\
2303999999^2 + 96000^2 &= 2304000001^2 & := 5308416004608000001 \\
230399999999^2 + 960000^2 &= 230400000001^2 & := 53084160000460800000001 \\
23039999999999^2 + 9600000^2 &= 23040000000001^2 & := 530841600000046080000000001 \quad (7.49)
\end{aligned}$$

$$\begin{aligned}
240099^2 + 980^2 &= 240101^2 & := 57648490201 \\
24009999^2 + 9800^2 &= 24010001^2 & := 576480148020001 \\
2400999999^2 + 98000^2 &= 2401000001^2 & := 5764801004802000001 \\
240099999999^2 + 980000^2 &= 240100000001^2 & := 57648010000480200000001 \\
24009999999999^2 + 9800000^2 &= 24010000000001^2 & := 576480100000048020000000001 \quad (7.50)
\end{aligned}$$

$$\begin{aligned}
249999^2 + 1000^2 &= 250001^2 & := 62500500001 \\
24999999^2 + 10000^2 &= 25000001^2 & := 625000050000001 \\
2499999999^2 + 100000^2 &= 2500000001^2 & := 6250000005000000001 \\
249999999999^2 + 1000000^2 &= 250000000001^2 & := 62500000000500000000001 \\
24999999999999^2 + 10000000^2 &= 25000000000001^2 & := 625000000000050000000000001 \quad (7.51)
\end{aligned}$$

$$\begin{aligned}
260099^2 + 1020^2 &= 260101^2 & := 67652530201 \\
26009999^2 + 10200^2 &= 26010001^2 & := 676520152020001 \\
2600999999^2 + 102000^2 &= 2601000001^2 & := 6765201005202000001 \\
260099999999^2 + 1020000^2 &= 260100000001^2 & := 67652010000520200000001 \\
26009999999999^2 + 10200000^2 &= 26010000000001^2 & := 676520100000052020000000001 \quad (7.52)
\end{aligned}$$

$$\begin{aligned}
270399^2 + 1040^2 &= 270401^2 & := 73116700801 \\
27039999^2 + 10400^2 &= 27040001^2 & := 731161654080001 \\
2703999999^2 + 104000^2 &= 2704000001^2 & := 7311616005408000001 \\
270399999999^2 + 1040000^2 &= 270400000001^2 & := 73116160000540800000001 \\
27039999999999^2 + 10400000^2 &= 27040000000001^2 & := 731161600000054080000000001 \quad (7.53)
\end{aligned}$$

$$\begin{aligned}
280899^2 + 1060^2 &= 280901^2 & := 78905371801 \\
28089999^2 + 10600^2 &= 28090001^2 & := 789048156180001 \\
2808999999^2 + 106000^2 &= 2809000001^2 & := 7890481005618000001 \\
280899999999^2 + 1060000^2 &= 280900000001^2 & := 78904810000561800000001 \\
28089999999999^2 + 10600000^2 &= 28090000000001^2 & := 789048100000056180000000001 \quad (7.54)
\end{aligned}$$

$$\begin{aligned}
 291599^2 + 1080^2 &= 291601^2 & := & 85031143201 \\
 2915999^2 + 10800^2 &= 29160001^2 & := & 850305658320001 \\
 29159999^2 + 108000^2 &= 2916000001^2 & := & 8503056005832000001 \\
 291599999^2 + 1080000^2 &= 291600000001^2 & := & 85030560000583200000001 \\
 2915999999^2 + 10800000^2 &= 29160000000001^2 & := & 850305600000058320000000001
 \end{aligned} \tag{7.55}$$

$$\begin{aligned}
 302499^2 + 1100^2 &= 302501^2 & := & 91506855001 \\
 3024999^2 + 11000^2 &= 30250001^2 & := & 915062560500001 \\
 30249999^2 + 110000^2 &= 3025000001^2 & := & 9150625006050000001 \\
 302499999^2 + 1100000^2 &= 302500000001^2 & := & 91506250000605000000001 \\
 3024999999^2 + 11000000^2 &= 30250000000001^2 & := & 915062500000060500000000001
 \end{aligned} \tag{7.56}$$

$$\begin{aligned}
 313599^2 + 1120^2 &= 313601^2 & := & 98345587201 \\
 3135999^2 + 11200^2 &= 31360001^2 & := & 983449662720001 \\
 31359999^2 + 112000^2 &= 3136000001^2 & := & 9834496006272000001 \\
 313599999^2 + 1120000^2 &= 313600000001^2 & := & 98344960000627200000001 \\
 3135999999^2 + 11200000^2 &= 31360000000001^2 & := & 983449600000062720000000001
 \end{aligned} \tag{7.57}$$

$$\begin{aligned}
 324899^2 + 1140^2 &= 324901^2 & := & 105560659801 \\
 3248999^2 + 11400^2 &= 32490001^2 & := & 1055600164980001 \\
 32489999^2 + 114000^2 &= 3249000001^2 & := & 10556001006498000001 \\
 324899999^2 + 1140000^2 &= 324900000001^2 & := & 105560010000649800000001 \\
 3248999999^2 + 11400000^2 &= 32490000000001^2 & := & 1055600100000064980000000001
 \end{aligned} \tag{7.58}$$

$$\begin{aligned}
 336399^2 + 1160^2 &= 336401^2 & := & 113165632801 \\
 3363999^2 + 11600^2 &= 33640001^2 & := & 1131649667280001 \\
 33639999^2 + 116000^2 &= 3364000001^2 & := & 11316496006728000001 \\
 336399999^2 + 1160000^2 &= 336400000001^2 & := & 113164960000672800000001 \\
 3363999999^2 + 11600000^2 &= 33640000000001^2 & := & 1131649600000067280000000001
 \end{aligned} \tag{7.59}$$

$$\begin{aligned}
 348099^2 + 1180^2 &= 348101^2 & := & 121174306201 \\
 3480999^2 + 11800^2 &= 34810001^2 & := & 1211736169620001 \\
 34809999^2 + 118000^2 &= 3481000001^2 & := & 12117361006962000001 \\
 348099999^2 + 1180000^2 &= 348100000001^2 & := & 121173610000696200000001 \\
 3480999999^2 + 11800000^2 &= 34810000000001^2 & := & 1211736100000069620000000001
 \end{aligned} \tag{7.60}$$

$$\begin{aligned}
 359999^2 + 1200^2 &= 360001^2 & := & 129600720001 \\
 3599999^2 + 12000^2 &= 3600001^2 & := & 1296000072000001 \\
 35999999^2 + 120000^2 &= 360000001^2 & := & 12960000007200000001 \\
 359999999^2 + 1200000^2 &= 36000000001^2 & := & 129600000000720000000001 \\
 3599999999^2 + 12000000^2 &= 360000000001^2 & := & 1296000000000072000000000001 \quad (7.61)
 \end{aligned}$$

$$\begin{aligned}
 372099^2 + 1220^2 &= 372101^2 & := & 138459154201 \\
 3720999^2 + 12200^2 &= 37210001^2 & := & 1384584174420001 \\
 37209999^2 + 122000^2 &= 3721000001^2 & := & 13845841007442000001 \\
 372099999^2 + 1220000^2 &= 372100000001^2 & := & 138458410000744200000001 \\
 3720999999^2 + 12200000^2 &= 3721000000001^2 & := & 1384584100000074420000000001 \quad (7.62)
 \end{aligned}$$

$$\begin{aligned}
 384399^2 + 1240^2 &= 384401^2 & := & 147764128801 \\
 3843999^2 + 12400^2 &= 38440001^2 & := & 1477633676880001 \\
 38439999^2 + 124000^2 &= 3844000001^2 & := & 14776336007688000001 \\
 384399999^2 + 1240000^2 &= 384400000001^2 & := & 147763360000768800000001 \\
 3843999999^2 + 12400000^2 &= 3844000000001^2 & := & 1477633600000076880000000001 \quad (7.63)
 \end{aligned}$$

$$\begin{aligned}
 396899^2 + 1260^2 &= 396901^2 & := & 157530403801 \\
 3968999^2 + 12600^2 &= 39690001^2 & := & 1575296179380001 \\
 39689999^2 + 126000^2 &= 3969000001^2 & := & 15752961007938000001 \\
 396899999^2 + 1260000^2 &= 396900000001^2 & := & 157529610000793800000001 \\
 3968999999^2 + 12600000^2 &= 3969000000001^2 & := & 1575296100000079380000000001 \quad (7.64)
 \end{aligned}$$

$$\begin{aligned}
 409599^2 + 1280^2 &= 409601^2 & := & 167772979201 \\
 4095999^2 + 12800^2 &= 40960001^2 & := & 1677721681920001 \\
 40959999^2 + 128000^2 &= 4096000001^2 & := & 16777216008192000001 \\
 409599999^2 + 1280000^2 &= 409600000001^2 & := & 167772160000819200000001 \\
 4095999999^2 + 12800000^2 &= 4096000000001^2 & := & 1677721600000081920000000001 \quad (7.65)
 \end{aligned}$$

$$\begin{aligned}
 422499^2 + 1300^2 &= 422501^2 & := & 178507095001 \\
 4224999^2 + 13000^2 &= 42250001^2 & := & 1785062584500001 \\
 42249999^2 + 130000^2 &= 4225000001^2 & := & 17850625008450000001 \\
 422499999^2 + 1300000^2 &= 422500000001^2 & := & 178506250000845000000001 \\
 4224999999^2 + 13000000^2 &= 4225000000001^2 & := & 1785062500000084500000000001 \quad (7.66)
 \end{aligned}$$

$$\begin{aligned}
435599^2 + 1320^2 &= 435601^2 & := & 189748231201 \\
43559999^2 + 13200^2 &= 43560001^2 & := & 1897473687120001 \\
4355999999^2 + 132000^2 &= 4356000001^2 & := & 18974736008712000001 \\
435599999999^2 + 1320000^2 &= 435600000001^2 & := & 189747360000871200000001 \\
43559999999999^2 + 13200000^2 &= 43560000000001^2 & := & 1897473600000087120000000001 \quad (7.67)
\end{aligned}$$

$$\begin{aligned}
448899^2 + 1340^2 &= 448901^2 & := & 201512107801 \\
44889999^2 + 13400^2 &= 44890001^2 & := & 2015112189780001 \\
4488999999^2 + 134000^2 &= 4489000001^2 & := & 20151121008978000001 \\
448899999999^2 + 1340000^2 &= 448900000001^2 & := & 201511210000897800000001 \\
44889999999999^2 + 13400000^2 &= 44890000000001^2 & := & 2015112100000089780000000001 \quad (7.68)
\end{aligned}$$

$$\begin{aligned}
462399^2 + 1360^2 &= 462401^2 & := & 213814684801 \\
46239999^2 + 13600^2 &= 46240001^2 & := & 2138137692480001 \\
4623999999^2 + 136000^2 &= 4624000001^2 & := & 21381376009248000001 \\
462399999999^2 + 1360000^2 &= 462400000001^2 & := & 213813760000924800000001 \\
46239999999999^2 + 13600000^2 &= 46240000000001^2 & := & 2138137600000092480000000001 \quad (7.69)
\end{aligned}$$

$$\begin{aligned}
476099^2 + 1380^2 &= 476101^2 & := & 226672162201 \\
47609999^2 + 13800^2 &= 47610001^2 & := & 2266712195220001 \\
4760999999^2 + 138000^2 &= 4761000001^2 & := & 22667121009522000001 \\
476099999999^2 + 1380000^2 &= 476100000001^2 & := & 226671210000952200000001 \\
47609999999999^2 + 13800000^2 &= 47610000000001^2 & := & 2266712100000095220000000001 \quad (7.70)
\end{aligned}$$

$$\begin{aligned}
489999^2 + 1400^2 &= 490001^2 & := & 240100980001 \\
48999999^2 + 14000^2 &= 49000001^2 & := & 2401000098000001 \\
4899999999^2 + 140000^2 &= 4900000001^2 & := & 24010000009800000001 \\
489999999999^2 + 1400000^2 &= 490000000001^2 & := & 240100000000980000000001 \\
48999999999999^2 + 14000000^2 &= 49000000000001^2 & := & 2401000000000098000000000001 \quad (7.71)
\end{aligned}$$

$$\begin{aligned}
504099^2 + 1420^2 &= 504101^2 & := & 254117818201 \\
50409999^2 + 14200^2 &= 50410001^2 & := & 2541168200820001 \\
5040999999^2 + 142000^2 &= 5041000001^2 & := & 25411681010082000001 \\
504099999999^2 + 1420000^2 &= 504100000001^2 & := & 254116810001008200000001 \\
50409999999999^2 + 14200000^2 &= 50410000000001^2 & := & 2541168100000100820000000001 \quad (7.72)
\end{aligned}$$

$$\begin{aligned}
518399^2 + 1440^2 &= 518401^2 & := & 268739596801 \\
51839999^2 + 14400^2 &= 51840001^2 & := & 2687385703680001 \\
5183999999^2 + 144000^2 &= 5184000001^2 & := & 26873856010368000001 \\
518399999999^2 + 1440000^2 &= 518400000001^2 & := & 268738560001036800000001 \\
51839999999999^2 + 14400000^2 &= 51840000000001^2 & := & 2687385600000103680000000001 \quad (7.73)
\end{aligned}$$

$$\begin{aligned}
532899^2 + 1460^2 &= 532901^2 & := & 283983475801 \\
53289999^2 + 14600^2 &= 53290001^2 & := & 2839824206580001 \\
5328999999^2 + 146000^2 &= 5329000001^2 & := & 28398241010658000001 \\
532899999999^2 + 1460000^2 &= 532900000001^2 & := & 283982410001065800000001 \\
53289999999999^2 + 14600000^2 &= 53290000000001^2 & := & 2839824100000106580000000001 \quad (7.74)
\end{aligned}$$

$$\begin{aligned}
547599^2 + 1480^2 &= 547601^2 & := & 299866855201 \\
54759999^2 + 14800^2 &= 54760001^2 & := & 2998657709520001 \\
5475999999^2 + 148000^2 &= 5476000001^2 & := & 29986576010952000001 \\
547599999999^2 + 1480000^2 &= 547600000001^2 & := & 299865760001095200000001 \\
54759999999999^2 + 14800000^2 &= 54760000000001^2 & := & 2998657600000109520000000001 \quad (7.75)
\end{aligned}$$

$$\begin{aligned}
562499^2 + 1500^2 &= 562501^2 & := & 316407375001 \\
56249999^2 + 15000^2 &= 56250001^2 & := & 3164062612500001 \\
5624999999^2 + 150000^2 &= 5625000001^2 & := & 31640625011250000001 \\
562499999999^2 + 1500000^2 &= 562500000001^2 & := & 316406250001125000000001 \\
56249999999999^2 + 15000000^2 &= 56250000000001^2 & := & 3164062500000112500000000001 \quad (7.76)
\end{aligned}$$

$$\begin{aligned}
577599^2 + 1520^2 &= 577601^2 & := & 333622915201 \\
57759999^2 + 15200^2 &= 57760001^2 & := & 3336217715520001 \\
5775999999^2 + 152000^2 &= 5776000001^2 & := & 33362176011552000001 \\
577599999999^2 + 1520000^2 &= 577600000001^2 & := & 333621760001155200000001 \\
57759999999999^2 + 15200000^2 &= 57760000000001^2 & := & 3336217600000115520000000001 \quad (7.77)
\end{aligned}$$

$$\begin{aligned}
592899^2 + 1540^2 &= 592901^2 & := & 351531595801 \\
59289999^2 + 15400^2 &= 59290001^2 & := & 3515304218580001 \\
5928999999^2 + 154000^2 &= 5929000001^2 & := & 35153041011858000001 \\
592899999999^2 + 1540000^2 &= 592900000001^2 & := & 351530410001185800000001 \\
59289999999999^2 + 15400000^2 &= 59290000000001^2 & := & 3515304100000118580000000001 \quad (7.78)
\end{aligned}$$

$$\begin{aligned}
608399^2 + 1560^2 &= 608401^2 & := & 370151776801 \\
60839999^2 + 15600^2 &= 60840001^2 & := & 3701505721680001 \\
6083999999^2 + 156000^2 &= 6084000001^2 & := & 37015056012168000001 \\
608399999999^2 + 1560000^2 &= 608400000001^2 & := & 370150560001216800000001 \\
60839999999999^2 + 15600000^2 &= 60840000000001^2 & := & 3701505600000121680000000001 \quad (7.79)
\end{aligned}$$

$$\begin{aligned}
624099^2 + 1580^2 &= 624101^2 & := & 389502058201 \\
62409999^2 + 15800^2 &= 62410001^2 & := & 3895008224820001 \\
6240999999^2 + 158000^2 &= 6241000001^2 & := & 38950081012482000001 \\
624099999999^2 + 1580000^2 &= 624100000001^2 & := & 389500810001248200000001 \\
62409999999999^2 + 15800000^2 &= 62410000000001^2 & := & 3895008100000124820000000001 \quad (7.80)
\end{aligned}$$

$$\begin{aligned}
639999^2 + 1600^2 &= 640001^2 & := & 409601280001 \\
63999999^2 + 16000^2 &= 64000001^2 & := & 4096000128000001 \\
6399999999^2 + 160000^2 &= 6400000001^2 & := & 40960000012800000001 \\
639999999999^2 + 1600000^2 &= 640000000001^2 & := & 409600000001280000000001 \\
63999999999999^2 + 16000000^2 &= 64000000000001^2 & := & 4096000000000128000000000001 \quad (7.81)
\end{aligned}$$

$$\begin{aligned}
656099^2 + 1620^2 &= 656101^2 & := & 430468522201 \\
65609999^2 + 16200^2 &= 65610001^2 & := & 4304672231220001 \\
6560999999^2 + 162000^2 &= 6561000001^2 & := & 43046721013122000001 \\
656099999999^2 + 1620000^2 &= 656100000001^2 & := & 430467210001312200000001 \\
65609999999999^2 + 16200000^2 &= 65610000000001^2 & := & 4304672100000131220000000001 \quad (7.82)
\end{aligned}$$

$$\begin{aligned}
672399^2 + 1640^2 &= 672401^2 & := & 452123104801 \\
67239999^2 + 16400^2 &= 67240001^2 & := & 4521217734480001 \\
6723999999^2 + 164000^2 &= 6724000001^2 & := & 45212176013448000001 \\
672399999999^2 + 1640000^2 &= 672400000001^2 & := & 452121760001344800000001 \\
67239999999999^2 + 16400000^2 &= 67240000000001^2 & := & 4521217600000134480000000001 \quad (7.83)
\end{aligned}$$

$$\begin{aligned}
688899^2 + 1660^2 &= 688901^2 & := & 474584587801 \\
68889999^2 + 16600^2 &= 68890001^2 & := & 4745832237780001 \\
6888999999^2 + 166000^2 &= 6889000001^2 & := & 47458321013778000001 \\
688899999999^2 + 1660000^2 &= 688900000001^2 & := & 474583210001377800000001 \\
68889999999999^2 + 16600000^2 &= 68890000000001^2 & := & 4745832100000137780000000001 \quad (7.84)
\end{aligned}$$

$$\begin{aligned}
705599^2 + 1680^2 &= 705601^2 & := & 497872771201 \\
7055999^2 + 16800^2 &= 70560001^2 & := & 4978713741120001 \\
70559999^2 + 168000^2 &= 7056000001^2 & := & 49787136014112000001 \\
705599999^2 + 1680000^2 &= 705600000001^2 & := & 497871360001411200000001 \\
7055999999^2 + 16800000^2 &= 70560000000001^2 & := & 4978713600000141120000000001 \quad (7.85)
\end{aligned}$$

$$\begin{aligned}
722499^2 + 1700^2 &= 722501^2 & := & 522007695001 \\
7224999^2 + 17000^2 &= 72250001^2 & := & 5220062644500001 \\
72249999^2 + 170000^2 &= 7225000001^2 & := & 52200625014450000001 \\
722499999^2 + 1700000^2 &= 722500000001^2 & := & 522006250001445000000001 \\
7224999999^2 + 17000000^2 &= 72250000000001^2 & := & 5220062500000144500000000001 \quad (7.86)
\end{aligned}$$

$$\begin{aligned}
739599^2 + 1720^2 &= 739601^2 & := & 547009639201 \\
7395999^2 + 17200^2 &= 73960001^2 & := & 5470081747920001 \\
73959999^2 + 172000^2 &= 7396000001^2 & := & 54700816014792000001 \\
739599999^2 + 1720000^2 &= 739600000001^2 & := & 547008160001479200000001 \\
7395999999^2 + 17200000^2 &= 73960000000001^2 & := & 5470081600000147920000000001 \quad (7.87)
\end{aligned}$$

$$\begin{aligned}
756899^2 + 1740^2 &= 756901^2 & := & 572899123801 \\
7568999^2 + 17400^2 &= 75690001^2 & := & 5728976251380001 \\
75689999^2 + 174000^2 &= 7569000001^2 & := & 57289761015138000001 \\
756899999^2 + 1740000^2 &= 756900000001^2 & := & 572897610001513800000001 \\
7568999999^2 + 17400000^2 &= 75690000000001^2 & := & 5728976100000151380000000001 \quad (7.88)
\end{aligned}$$

$$\begin{aligned}
774399^2 + 1760^2 &= 774401^2 & := & 599696908801 \\
7743999^2 + 17600^2 &= 77440001^2 & := & 5996953754880001 \\
77439999^2 + 176000^2 &= 7744000001^2 & := & 59969536015488000001 \\
774399999^2 + 1760000^2 &= 774400000001^2 & := & 599695360001548800000001 \\
7743999999^2 + 17600000^2 &= 77440000000001^2 & := & 5996953600000154880000000001 \quad (7.89)
\end{aligned}$$

$$\begin{aligned}
792099^2 + 1780^2 &= 792101^2 & := & 627423994201 \\
7920999^2 + 17800^2 &= 79210001^2 & := & 6274224258420001 \\
79209999^2 + 178000^2 &= 7921000001^2 & := & 62742241015842000001 \\
792099999^2 + 1780000^2 &= 792100000001^2 & := & 627422410001584200000001 \\
7920999999^2 + 17800000^2 &= 79210000000001^2 & := & 6274224100000158420000000001 \quad (7.90)
\end{aligned}$$

$$\begin{aligned}
 809999^2 + 1800^2 &= 810001^2 & := 656101620001 \\
 8099999^2 + 18000^2 &= 8100001^2 & := 6561000162000001 \\
 80999999^2 + 180000^2 &= 810000001^2 & := 65610000016200000001 \\
 809999999^2 + 1800000^2 &= 81000000001^2 & := 656100000001620000000001 \\
 8099999999^2 + 18000000^2 &= 810000000001^2 & := 6561000000000162000000000001 \quad (7.91)
 \end{aligned}$$

$$\begin{aligned}
 828099^2 + 1820^2 &= 828101^2 & := 685751266201 \\
 8280999^2 + 18200^2 &= 82810001^2 & := 6857496265620001 \\
 82809999^2 + 182000^2 &= 8281000001^2 & := 68574961016562000001 \\
 828099999^2 + 1820000^2 &= 828100000001^2 & := 685749610001656200000001 \\
 8280999999^2 + 18200000^2 &= 8281000000001^2 & := 6857496100000165620000000001 \quad (7.92)
 \end{aligned}$$

$$\begin{aligned}
 846399^2 + 1840^2 &= 846401^2 & := 716394652801 \\
 8463999^2 + 18400^2 &= 84640001^2 & := 7163929769280001 \\
 84639999^2 + 184000^2 &= 8464000001^2 & := 71639296016928000001 \\
 846399999^2 + 1840000^2 &= 846400000001^2 & := 716392960001692800000001 \\
 8463999999^2 + 18400000^2 &= 8464000000001^2 & := 7163929600000169280000000001 \quad (7.93)
 \end{aligned}$$

$$\begin{aligned}
 864899^2 + 1860^2 &= 864901^2 & := 748053739801 \\
 8648999^2 + 18600^2 &= 86490001^2 & := 7480520272980001 \\
 86489999^2 + 186000^2 &= 8649000001^2 & := 74805201017298000001 \\
 864899999^2 + 1860000^2 &= 864900000001^2 & := 748052010001729800000001 \\
 8648999999^2 + 18600000^2 &= 8649000000001^2 & := 7480520100000172980000000001 \quad (7.94)
 \end{aligned}$$

$$\begin{aligned}
 883599^2 + 1880^2 &= 883601^2 & := 780750727201 \\
 8835999^2 + 18800^2 &= 88360001^2 & := 7807489776720001 \\
 88359999^2 + 188000^2 &= 8836000001^2 & := 78074896017672000001 \\
 883599999^2 + 1880000^2 &= 883600000001^2 & := 780748960001767200000001 \\
 8835999999^2 + 18800000^2 &= 8836000000001^2 & := 7807489600000176720000000001 \quad (7.95)
 \end{aligned}$$

$$\begin{aligned}
 902499^2 + 1900^2 &= 902501^2 & := 814508055001 \\
 9024999^2 + 19000^2 &= 90250001^2 & := 8145062680500001 \\
 90249999^2 + 190000^2 &= 9025000001^2 & := 81450625018050000001 \\
 902499999^2 + 1900000^2 &= 902500000001^2 & := 814506250001805000000001 \\
 9024999999^2 + 19000000^2 &= 9025000000001^2 & := 8145062500000180500000000001 \quad (7.96)
 \end{aligned}$$

$$\begin{aligned}
 921599^2 + 1920^2 &= 921601^2 & := & 849348403201 \\
 9215999^2 + 19200^2 &= 92160001^2 & := & 8493465784320001 \\
 92159999^2 + 192000^2 &= 9216000001^2 & := & 84934656018432000001 \\
 921599999^2 + 1920000^2 &= 921600000001^2 & := & 849346560001843200000001 \\
 9215999999^2 + 19200000^2 &= 92160000000001^2 & := & 8493465600000184320000000001 \quad (7.97)
 \end{aligned}$$

$$\begin{aligned}
 940899^2 + 1940^2 &= 940901^2 & := & 885294691801 \\
 9408999^2 + 19400^2 &= 94090001^2 & := & 8852928288180001 \\
 94089999^2 + 194000^2 &= 9409000001^2 & := & 88529281018818000001 \\
 940899999^2 + 1940000^2 &= 940900000001^2 & := & 885292810001881800000001 \\
 9408999999^2 + 19400000^2 &= 94090000000001^2 & := & 8852928100000188180000000001 \quad (7.98)
 \end{aligned}$$

$$\begin{aligned}
 960399^2 + 1960^2 &= 960401^2 & := & 922370080801 \\
 9603999^2 + 19600^2 &= 96040001^2 & := & 9223681792080001 \\
 96039999^2 + 196000^2 &= 9604000001^2 & := & 92236816019208000001 \\
 960399999^2 + 1960000^2 &= 960400000001^2 & := & 922368160001920800000001 \\
 9603999999^2 + 19600000^2 &= 96040000000001^2 & := & 9223681600000192080000000001 \quad (7.99)
 \end{aligned}$$

$$\begin{aligned}
 980099^2 + 1980^2 &= 980101^2 & := & 960597970201 \\
 9800999^2 + 19800^2 &= 98010001^2 & := & 9605960296020001 \\
 98009999^2 + 198000^2 &= 9801000001^2 & := & 96059601019602000001 \\
 980099999^2 + 1980000^2 &= 980100000001^2 & := & 960596010001960200000001 \\
 9800999999^2 + 19800000^2 &= 98010000000001^2 & := & 9605960100000196020000000001 \quad (7.100)
 \end{aligned}$$

7.3 Magic Squares

Based on the Pythagorean triples given in subsection 7.2.1, below are magic square distributions, calculated according to formula given in Taneja [3]. The representations are as given in equation (4.4).

$$\begin{aligned}
 (99, 20, 101) &\Rightarrow \{9, 41, 201, 1089, 9801 = 99^2\} \\
 (399, 40, 401) &\Rightarrow \{19, 81, 801, 8379, 159201 = 399^2\} \\
 (899, 60, 901) &\Rightarrow \{29, 121, 1801, 27869, 808201 = 899^2\} \\
 (1599, 80, 1601) &\Rightarrow \{39, 161, 3201, 65559, 2556801 = 1599^2\} \\
 (2499, 100, 2501) &\Rightarrow \{49, 201, 5001, 127449, 6245001 = 2499^2\} \\
 (3599, 120, 3601) &\Rightarrow \{59, 241, 7201, 219539, 12952801 = 3599^2\} \\
 (4899, 140, 4901) &\Rightarrow \{69, 281, 9801, 347829, 24000201 = 4899^2\} \\
 (6399, 160, 6401) &\Rightarrow \{79, 321, 12801, 518319, 40947201 = 6399^2\} \\
 (8099, 180, 8101) &\Rightarrow \{89, 361, 16201, 737009, 65593801 = 8099^2\}
 \end{aligned}$$

$$\begin{aligned}
(9999, 200, 10001) &\Rightarrow \{99, 401, 20001, 1009899, 99980001 = 9999^2\} \\
(12099, 220, 12101) &\Rightarrow \{109, 441, 24201, 1342989, 146385801 = 12099^2\} \\
(14399, 240, 14401) &\Rightarrow \{119, 481, 28801, 1742279, 207331201 = 14399^2\} \\
(16899, 260, 16901) &\Rightarrow \{129, 521, 33801, 2213769, 285576201 = 16899^2\} \\
(19599, 280, 19601) &\Rightarrow \{139, 561, 39201, 2763459, 384120801 = 19599^2\} \\
(22499, 300, 22501) &\Rightarrow \{149, 601, 45001, 3397349, 506205001 = 22499^2\} \\
(25599, 320, 25601) &\Rightarrow \{159, 641, 51201, 4121439, 655308801 = 25599^2\} \\
(28899, 340, 28901) &\Rightarrow \{169, 681, 57801, 4941729, 835152201 = 28899^2\} \\
(32399, 360, 32401) &\Rightarrow \{179, 721, 64801, 5864219, 1049695201 = 32399^2\} \\
(36099, 380, 36101) &\Rightarrow \{189, 761, 72201, 6894909, 1303137801 = 36099^2\} \\
(39999, 400, 40001) &\Rightarrow \{199, 801, 80001, 8039799, 1599920001 = 39999^2\} \\
(44099, 420, 44101) &\Rightarrow \{209, 841, 88201, 9304889, 1944721801 = 44099^2\} \\
(48399, 440, 48401) &\Rightarrow \{219, 881, 96801, 10696179, 2342463201 = 48399^2\} \\
(52899, 460, 52901) &\Rightarrow \{229, 921, 105801, 12219669, 2798304201 = 52899^2\} \\
(57599, 480, 57601) &\Rightarrow \{239, 961, 115201, 13881359, 3317644801 = 57599^2\} \\
(62499, 500, 62501) &\Rightarrow \{249, 1001, 125001, 15687249, 3906125001 = 62499^2\} \\
(67599, 520, 67601) &\Rightarrow \{259, 1041, 135201, 17643339, 4569624801 = 67599^2\} \\
(72899, 540, 72901) &\Rightarrow \{269, 1081, 145801, 19755629, 5314264201 = 72899^2\} \\
(78399, 560, 78401) &\Rightarrow \{279, 1121, 156801, 22030119, 6146403201 = 78399^2\} \\
(84099, 580, 84101) &\Rightarrow \{289, 1161, 168201, 24472809, 7072641801 = 84099^2\} \\
(89999, 600, 90001) &\Rightarrow \{299, 1201, 180001, 27089699, 8099820001 = 89999^2\} \\
(96099, 620, 96101) &\Rightarrow \{309, 1241, 192201, 29886789, 9235017801 = 96099^2\} \\
(102399, 640, 102401) &\Rightarrow \{319, 1281, 204801, 32870079, 10485555201 = 102399^2\} \\
(108899, 660, 108901) &\Rightarrow \{329, 1321, 217801, 36045569, 11858992201 = 108899^2\} \\
(115599, 680, 115601) &\Rightarrow \{339, 1361, 231201, 39419259, 13363128801 = 115599^2\} \\
(122499, 700, 122501) &\Rightarrow \{349, 1401, 245001, 42997149, 15006005001 = 122499^2\} \\
(129599, 720, 129601) &\Rightarrow \{359, 1441, 259201, 46785239, 16795900801 = 129599^2\} \\
(136899, 740, 136901) &\Rightarrow \{369, 1481, 273801, 50789529, 18741336201 = 136899^2\} \\
(144399, 760, 144401) &\Rightarrow \{379, 1521, 288801, 55016019, 20851071201 = 144399^2\} \\
(152099, 780, 152101) &\Rightarrow \{389, 1561, 304201, 59470709, 23134105801 = 152099^2\} \\
(159999, 800, 160001) &\Rightarrow \{399, 1601, 320001, 64159599, 25599680001 = 159999^2\} \\
(168099, 820, 168101) &\Rightarrow \{409, 1641, 336201, 69088689, 28257273801 = 168099^2\} \\
(176399, 840, 176401) &\Rightarrow \{419, 1681, 352801, 74263979, 31116607201 = 176399^2\} \\
(184899, 860, 184901) &\Rightarrow \{429, 1721, 369801, 79691469, 34187640201 = 184899^2\} \\
(193599, 880, 193601) &\Rightarrow \{439, 1761, 387201, 85377159, 37480572801 = 193599^2\} \\
(202499, 900, 202501) &\Rightarrow \{449, 1801, 405001, 91327049, 41005845001 = 202499^2\} \\
(211599, 920, 211601) &\Rightarrow \{459, 1841, 423201, 97547139, 44774136801 = 211599^2\}
\end{aligned}$$

$$\begin{aligned}
(220899, 940, 220901) &\Rightarrow \{469, 1881, 441801, 104043429, 48796368201 = 220899^2\} \\
(230399, 960, 230401) &\Rightarrow \{479, 1921, 460801, 110821919, 53083699201 = 230399^2\} \\
(240099, 980, 240101) &\Rightarrow \{489, 1961, 480201, 117888609, 57647529801 = 240099^2\} \\
(249999, 1000, 250001) &\Rightarrow \{499, 2001, 500001, 125249499, 62499500001 = 249999^2\} \\
(260099, 1020, 260101) &\Rightarrow \{509, 2041, 520201, 132910589, 67651489801 = 260099^2\} \\
(270399, 1040, 270401) &\Rightarrow \{519, 2081, 540801, 140877879, 73115619201 = 270399^2\} \\
(280899, 1060, 280901) &\Rightarrow \{529, 2121, 561801, 149157369, 78904248201 = 280899^2\} \\
(291599, 1080, 291601) &\Rightarrow \{539, 2161, 583201, 157755059, 85029976801 = 291599^2\} \\
(302499, 1100, 302501) &\Rightarrow \{549, 2201, 605001, 166676949, 91505645001 = 302499^2\} \\
(313599, 1120, 313601) &\Rightarrow \{559, 2241, 627201, 175929039, 98344332801 = 313599^2\} \\
(324899, 1140, 324901) &\Rightarrow \{569, 2281, 649801, 185517329, 105559360201 = 324899^2\} \\
(336399, 1160, 336401) &\Rightarrow \{579, 2321, 672801, 195447819, 113164287201 = 336399^2\} \\
(348099, 1180, 348101) &\Rightarrow \{589, 2361, 696201, 205726509, 121172913801 = 348099^2\} \\
(359999, 1200, 360001) &\Rightarrow \{599, 2401, 720001, 216359399, 129599280001 = 359999^2\} \\
(372099, 1220, 372101) &\Rightarrow \{609, 2441, 744201, 227352489, 138457665801 = 372099^2\} \\
(384399, 1240, 384401) &\Rightarrow \{619, 2481, 768801, 238711779, 147762591201 = 384399^2\} \\
(396899, 1260, 396901) &\Rightarrow \{629, 2521, 793801, 250443269, 157528816201 = 396899^2\} \\
(409599, 1280, 409601) &\Rightarrow \{639, 2561, 819201, 262552959, 167771340801 = 409599^2\} \\
(422499, 1300, 422501) &\Rightarrow \{649, 2601, 845001, 275046849, 178505405001 = 422499^2\} \\
(435599, 1320, 435601) &\Rightarrow \{659, 2641, 871201, 287930939, 189746488801 = 435599^2\} \\
(448899, 1340, 448901) &\Rightarrow \{669, 2681, 897801, 301211229, 201510312201 = 448899^2\} \\
(462399, 1360, 462401) &\Rightarrow \{679, 2721, 924801, 314893719, 213812835201 = 462399^2\} \\
(476099, 1380, 476101) &\Rightarrow \{689, 2761, 952201, 328984409, 226670257801 = 476099^2\} \\
(489999, 1400, 490001) &\Rightarrow \{699, 2801, 980001, 343489299, 240099020001 = 489999^2\} \\
(504099, 1420, 504101) &\Rightarrow \{709, 2841, 1008201, 358414389, 254115801801 = 504099^2\} \\
(518399, 1440, 518401) &\Rightarrow \{719, 2881, 1036801, 373765679, 268737523201 = 518399^2\} \\
(532899, 1460, 532901) &\Rightarrow \{729, 2921, 1065801, 389549169, 283981344201 = 532899^2\} \\
(547599, 1480, 547601) &\Rightarrow \{739, 2961, 1095201, 405770859, 299864664801 = 547599^2\} \\
(562499, 1500, 562501) &\Rightarrow \{749, 3001, 1125001, 422436749, 316405125001 = 562499^2\} \\
(577599, 1520, 577601) &\Rightarrow \{759, 3041, 1155201, 439552839, 333620604801 = 577599^2\} \\
(592899, 1540, 592901) &\Rightarrow \{769, 3081, 1185801, 457125129, 351529224201 = 592899^2\} \\
(608399, 1560, 608401) &\Rightarrow \{779, 3121, 1216801, 475159619, 370149343201 = 608399^2\} \\
(624099, 1580, 624101) &\Rightarrow \{789, 3161, 1248201, 493662309, 389499561801 = 624099^2\} \\
(639999, 1600, 640001) &\Rightarrow \{799, 3201, 1280001, 512639199, 409598720001 = 639999^2\} \\
(656099, 1620, 656101) &\Rightarrow \{809, 3241, 1312201, 532096289, 430465897801 = 656099^2\} \\
(672399, 1640, 672401) &\Rightarrow \{819, 3281, 1344801, 552039579, 452120415201 = 672399^2\} \\
(688899, 1660, 688901) &\Rightarrow \{829, 3321, 1377801, 572475069, 474581832201 = 688899^2\}
\end{aligned}$$

$$\begin{aligned}
(705599, 1680, 705601) &\Rightarrow \{839, 3361, 1411201, 593408759, 497869948801 = 705599^2\} \\
(722499, 1700, 722501) &\Rightarrow \{849, 3401, 1445001, 614846649, 522004805001 = 722499^2\} \\
(739599, 1720, 739601) &\Rightarrow \{859, 3441, 1479201, 636794739, 547006680801 = 739599^2\} \\
(756899, 1740, 756901) &\Rightarrow \{869, 3481, 1513801, 659259029, 572896096201 = 756899^2\} \\
(774399, 1760, 774401) &\Rightarrow \{879, 3521, 1548801, 682245519, 599693811201 = 774399^2\} \\
(792099, 1780, 792101) &\Rightarrow \{889, 3561, 1584201, 705760209, 627420825801 = 792099^2\} \\
(809999, 1800, 810001) &\Rightarrow \{899, 3601, 1620001, 729809099, 656098380001 = 809999^2\} \\
(828099, 1820, 828101) &\Rightarrow \{909, 3641, 1656201, 754398189, 685747953801 = 828099^2\} \\
(846399, 1840, 846401) &\Rightarrow \{919, 3681, 1692801, 779533479, 716391267201 = 846399^2\} \\
(864899, 1860, 864901) &\Rightarrow \{929, 3721, 1729801, 805220969, 748050280201 = 864899^2\} \\
(883599, 1880, 883601) &\Rightarrow \{939, 3761, 1767201, 831466659, 780747192801 = 883599^2\} \\
(902499, 1900, 902501) &\Rightarrow \{949, 3801, 1805001, 858276549, 814504445001 = 902499^2\} \\
(921599, 1920, 921601) &\Rightarrow \{959, 3841, 1843201, 885656639, 849344716801 = 921599^2\} \\
(940899, 1940, 940901) &\Rightarrow \{969, 3881, 1881801, 913612929, 885290928201 = 940899^2\} \\
(960399, 1960, 960401) &\Rightarrow \{979, 3921, 1920801, 942151419, 922366239201 = 960399^2\} \\
(980099, 1980, 980101) &\Rightarrow \{989, 3961, 1960201, 971278109, 960594049801 = 980099^2\} \quad (7.101)
\end{aligned}$$

Here the magic squares generated are not in a decreasing order as in case of (5.3). The orders of magic squares are with difference of 10, i.e., orders are 9, 19, 29, 39, ..., 989.

7.3.1 Comparison

The details below give difference among the magic squares generated in Section 4 and Subsection 7.3.

- (i) According to first procedure Section 4 the magic square generated in (4.4) for the orders 9, 19, ...99 are given by

$$\begin{aligned}
(99, 20, 101) &\Rightarrow \{9, 41, 201, 1089, 9801 = 99^2\} \\
(399, 40, 401) &\Rightarrow \{19, 81, 801, 8379, 159201 = 399^2\} \\
(899, 60, 901) &\Rightarrow \{29, 121, 1801, 27869, 808201 = 899^2\} \\
(1599, 80, 1601) &\Rightarrow \{39, 161, 3201, 65559, 2556801 = 1599^2\} \\
(2499, 100, 2501) &\Rightarrow \{49, 201, 5001, 127449, 6245001 = 2499^2\} \\
(3599, 120, 3601) &\Rightarrow \{59, 241, 7201, 219539, 12952801 = 3599^2\} \\
(4899, 140, 4901) &\Rightarrow \{69, 281, 9801, 347829, 24000201 = 4899^2\} \\
(6399, 160, 6401) &\Rightarrow \{79, 321, 12801, 518319, 40947201 = 6399^2\} \\
(8099, 180, 8101) &\Rightarrow \{89, 361, 16201, 737009, 65593801 = 8099^2\} \\
(9999, 200, 10001) &\Rightarrow \{99, 401, 20001, 1009899, 99980001 = 9999^2\} \quad (7.102)
\end{aligned}$$

- (i) According to first procedure subsection 5.4 the magic square generated in (5.301) for the orders 9, 19, ...99 are given by

$$\begin{aligned}
(1719, 18200, 18281) &\Rightarrow \{9, 36401, 36561, 328329, 2954961 = 1719^2\} \\
(3439, 16200, 16561) &\Rightarrow \{19, 32401, 33121, 622459, 11826721 = 3439^2\} \\
(4959, 14200, 15041) &\Rightarrow \{29, 28401, 30081, 847989, 24591681 = 4959^2\} \\
(6279, 12200, 13721) &\Rightarrow \{39, 24401, 27441, 1010919, 39425841 = 6279^2\} \\
(7399, 10200, 12601) &\Rightarrow \{49, 20401, 25201, 1117249, 54745201 = 7399^2\} \\
(8319, 8200, 11681) &\Rightarrow \{59, 16401, 23361, 1172979, 69205761 = 8319^2\} \\
(9039, 6200, 10961) &\Rightarrow \{69, 12401, 21921, 1184109, 81703521 = 9039^2\} \\
(9559, 4200, 10441) &\Rightarrow \{79, 8401, 20881, 1156639, 91374481 = 9559^2\} \\
(9879, 2200, 10121) &\Rightarrow \{89, 4401, 20241, 1096569, 97594641 = 9879^2\} \\
(9999, 200, 10001) &\Rightarrow \{99, 401, 20001, 1009899, 99980001 = 9999^2\} \quad (7.103)
\end{aligned}$$

(i) According to first procedure subsection 7.3 the magic square generated in (7.101) for the orders 9, 19, ...99 are given by

$$\begin{aligned}
(99, 20, 101) &\Rightarrow \{9, 41, 201, 1089, 9801 = 99^2\} \\
(399, 40, 401) &\Rightarrow \{19, 81, 801, 8379, 159201 = 399^2\} \\
(899, 60, 901) &\Rightarrow \{29, 121, 1801, 27869, 808201 = 899^2\} \\
(1599, 80, 1601) &\Rightarrow \{39, 161, 3201, 65559, 2556801 = 1599^2\} \\
(2499, 100, 2501) &\Rightarrow \{49, 201, 5001, 127449, 6245001 = 2499^2\} \\
(3599, 120, 3601) &\Rightarrow \{59, 241, 7201, 219539, 12952801 = 3599^2\} \\
(4899, 140, 4901) &\Rightarrow \{69, 281, 9801, 347829, 24000201 = 4899^2\} \\
(6399, 160, 6401) &\Rightarrow \{79, 321, 12801, 518319, 40947201 = 6399^2\} \\
(8099, 180, 8101) &\Rightarrow \{89, 361, 16201, 737009, 65593801 = 8099^2\} \\
(9999, 200, 10001) &\Rightarrow \{99, 401, 20001, 1009899, 99980001 = 9999^2\} \quad (7.104)
\end{aligned}$$

From (7.102) and (7.103), we see that there is only case where we have the same magic square, i.e., of order 99. The (7.102) and (7.104) are the same, but the results in (7.103) goes up to orders 989 with gap of 10 each, i.e., order , 9, 19, ..., 979, 989. Thus, the results appearing in (7.102) and (7.103) are totally different. Moreover, the results (7.103) are with much higher entries that those given in (7.102).

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