Multiple-Type Patterns and Pythagorean Triples

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Abstract

The Pythagoras theorem is very famous in the literature of mathematics. This paper brings patterns obtained by multiplication by natural numbers to known Pythagorean triples resulting again in patterns. This gives many patterns of similar kind. The constructions and details of the procedures can be seen in author's work [4, 5, 6].

Contents

1 Introduction

By Pythagoras theorem it is understood that

$$a^2 + b^2 = c^2.$$

For simplicity, let's write it as (a, b, c) . If we talk of distances, then the numbers a, b and c are real positive, otherwise they can be any real numbers.

Let's consider the initial Pythagorean triple (3,4,5). This means that $3^2 + 4^2 = 5^2$. The symmetric extensions of this triple we call **patterns in Pythagorean triples**. There two obvious ways of getting **patterns in Pythagorean triples**.

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1.1   Multiplication by 10, 100, 1000, ... :

See following two examples

$$
\begin{aligned}
3^2 + 4^2 \;\;&= 5^2 \;\; := 25 \\
30^2 + 40^2 \;\;&= 50^2 := 2500 \\
300^2 + 400^2 &= 500^2 := 250000
\end{aligned}
\tag{1.1}
$$

$$
\begin{aligned}
9^2 + 40^2 \;\;&= 41^2 \;\; := 1681 \\
90^2 + 400^2 \;\;&= 410^2 \;\; := 168100 \\
900^2 + 4000^2 &= 4100^2 := 16810000
\end{aligned}
\tag{1.2}
$$

The difference in above two examples is that the first one is with single digit in each case, i.e., 3, 4 and 5, and in second example (1.2) not all the same digits are with same length, i.e., the first one is of length 1 and other two are with length 2. This process is generally true for all types of **Pythagorean triples**. Some studies on Pythagorean triples can be seen in [1, 2].

1.2   Repetition of Digits

Let's consider following examples,

$$
\begin{aligned}
3^2 + 4^2 \;\;&= 5^2 \;\; := 25 \\
33^2 + 44^2 \;\;&= 55^2 := 3025 \\
333^2 + 444^2 &= 555^2 := 308035
\end{aligned}
\tag{1.3}
$$

$$
\begin{aligned}
12^2 + 35^2 \;\;&= 37^2 \;\;\; := 1369 \\
1212^2 + 3535^2 \;\;&= 3737^2 \;\; := 13965169 \\
121212^2 + 353535^2 &= 373737^2 := 139679345169
\end{aligned}
\tag{1.4}
$$

$$
\begin{aligned}
119^2 + 120^2 \;\;&= 169^2 \;\;\; := 28561 \\
119119^2 + 120120^2 \;\;&= 169169^2 \;\; := 28618150561 \\
119119119^2 + 120120120^2 &= 169169169^2 := 28618207740150561
\end{aligned}
\tag{1.5}
$$

In the examples (1.3), (1.4) and (1.5), we observe that the repetition of digits give us patterned results when we work with same length triples. Here we have (3,4,5), (12, 35, 27) and (119, 120, 169). In the example (3, 4, 5) each digit is of length 1. In the example (12, 35, 27) each digit is of length 2, and the triple (119, 120, 169) each digit is of length 3. This theory doesn't work when the triples are with different lengths, for example, (5, 12, 13). This gives

$$
\begin{aligned}
5^2 + 12^2 = 13^2 \;\;&\Rightarrow 169 = 169 \\
55^2 + 1212^2 \neq 1313^2 &\Rightarrow 1471969 \neq 1723969
\end{aligned}
\tag{1.6}
$$

In this case, the pattern (1.6) is not extendable as in case examples (1.3), (1.4) and (1.5).

Remark 1. *Analysing the final sums, we observe that the examples* (1.1) *and* (1.2) *are good for **patterns with final sums**. The example* (1.3) *is not so good but acceptable. In case of examples* (1.4) *and* (1.5), *the patterns with final sums are not good. In this work, we shall write patterns with final sums only if they are good.*

The aim of this work is divided in two parts. First part given in Section 2 is to bring more pattern by multiplication with positive natural numbers. The part gtiven in Sections Section 3 and Section 4 is to summarize the results on **patterns in Pythagorean triples** done before by author in [4, 5]. Obviously, we can write much more patterns. Theses sections give just an idea, how beautiful we can make Pythagoran triples.

2 Multiple-Type Patterns

In paper [5, 6], we worked with different Procedures to bring patterns in Pythagorean triples. Here our aim is to write new patterns multiplying by natural numbers. Also, to observe that the numbers or lines appearing in orange color don't give good patterns. See the examples below.

Example 2.1. *Let's consider following pattern:*

$$4^2 + 3^2 = 5^2$$
$$84^2 + 13^2 = 85^2$$
$$8844^2 + 133^2 = 8845^2$$
$$888444^2 + 1333^2 = 888445^2$$
$$88884444^2 + 13333^2 = 88884445^2$$
$$8888844444^2 + 133333^2 = 8888844445^2 \qquad (2.1)$$

The Example (2.1) is the same as (3.2) given in Section 3. Let's multiply the Example (2.1) by $n = 1, 2, 3, ..., 9$, we get the following patterns:

▶ **Multiplying** (2.1) **by 2**

$$8^2 + 6^2 = 10^2$$
$$168^2 + 26^2 = 170^2$$
$$17688^2 + 266^2 = 17690^2$$
$$1776888^2 + 2666^2 = 1776890^2$$
$$177768888^2 + 26666^2 = 177768890^2$$
$$17777688888^2 + 266666^2 = 17777688890^2 \qquad (2.2)$$

▶ **Multiplying** (2.1) **by 3**

$$12^2 + 9^2 = 15^2$$
$$252^2 + 39^2 = 255^2$$
$$26532^2 + 399^2 = 26535^2$$
$$2665332^2 + 3999^2 = 2665335^2$$
$$266653332^2 + 39999^2 = 266653335^2$$
$$26666533332^2 + 399999^2 = 26666533335^2 \qquad (2.3)$$

▶ **Multiplying** (2.1) **by 4**

$$16^2 + 12^2 = 20^2$$
$$336^2 + 52^2 = 340^2$$
$$35376^2 + 532^2 = 35380^2$$
$$3553776^2 + 5332^2 = 3553780^2$$
$$355537776^2 + 53332^2 = 355537780^2$$
$$35555377776^2 + 533332^2 = 35555377780^2 \qquad (2.4)$$

▶ **Multiplying** (2.1) **by 5**

$$20^2 + 15^2 = 25^2$$
$$420^2 + 65^2 = 425^2$$
$$44220^2 + 665^2 = 44225^2$$
$$4442220^2 + 6665^2 = 4442225^2$$
$$444422220^2 + 66665^2 = 444422225^2$$
$$44444222220^2 + 666665^2 = 44444222225^2 \qquad (2.5)$$

▶ **Multiplying** (2.1) **by 6**

$$24^2 + 18^2 = 30^2$$
$$504^2 + 78^2 = 510^2$$
$$53064^2 + 798^2 = 53070^2$$
$$5330664^2 + 7998^2 = 5330670^2$$
$$533306664^2 + 79998^2 = 533306670^2$$
$$53333066664^2 + 799998^2 = 53333066670^2 \qquad (2.6)$$

▶ **Multiplying** (2.1) **by 7**

$$28^2 + 21^2 = 35^2$$
$$588^2 + 91^2 = 595^3$$
$$61908^2 + 931^2 = 61915^2$$
$$6219108^2 + 9331^2 = 6219115^2$$
$$622191108^2 + 93331^2 = 622191115^2$$
$$62221911108^2 + 933331^2 = 62221911115^2 \qquad (2.7)$$

▶ **Multiplying** (2.1) **by 8**

$$32^2 + 24^2 = 40^2$$
$$672^2 + 104^2 = 680^2$$
$$70752^2 + 1064^2 = 70760^2$$
$$7107552^2 + 10664^2 = 7107560^2$$
$$711075552^2 + 106664^2 = 711075560^2$$
$$71110755552^2 + 1066664^2 = 71110755560^2 \qquad (2.8)$$

▶ **Multiplying** (2.1) **by 9**

$$36^2 + 27^2 = 45^2$$
$$756^2 + 117^2 = 765^2$$
$$79596^2 + 1197^2 = 79605^2$$
$$7995996^2 + 11997^2 = 7996005^2$$
$$799959996^2 + 119997^2 = 799960005^2$$
$$79999599996^2 + 1199997^2 = 79999600005^2 \qquad (2.9)$$

The above expressions are obtained multiplying Example (2.1) by $n = 1, 2, 3, ..., 9$. Let's see what happen's if we multiplying (2.1) by numbers more than 10:

▶ **Multiplying** (2.1) **by 19**

$$76^2 + 57^2 = 95^2$$
$$1596^2 + 247^2 = 1615^2$$
$$168036^2 + 2527^2 = 168055^2$$
$$16880436^2 + 25327^2 = 16880455^2$$
$$1688804436^2 + 253327^2 = 1688804455^2$$
$$168888044436^2 + 2533327^2 = 168888044455^2 \qquad (2.10)$$

▶ **Multiplying** (2.1) **by 43**

$$172^2 + 129^2 = 215^2$$
$$3612^2 + 559^2 = 3655^2$$
$$380292^2 + 5719^2 = 380335^2$$
$$38203092^2 + 57319^2 = 38203135^2$$
$$3822031092^2 + 573319^2 = 3822031135^2$$
$$382220311092^2 + 5733319^2 = 382220311135^2 \tag{2.11}$$

The examples below are obtained multiplying (2.1) by power of 2, for example by 16, 32 and 64.

▶ **Multiplying** (2.1) **by 16**

$$64^2 + 48^2 = 80^2$$
$$1344^2 + 208^2 = 1360^2$$
$$141504^2 + 2128^2 = 141520^2$$
$$14215104^2 + 21328^2 = 14215120^2$$
$$1422151104^2 + 213328^2 = 1422151120^2$$
$$142221511104^2 + 2133328^2 = 142221511120^2 \tag{2.12}$$

▶ **Multiplying** (2.1) **by 32**

$$128^2 + 96^2 = 160^2$$
$$2688^2 + 416^2 = 2720^2$$
$$283008^2 + 4256^2 = 283040^2$$
$$28430208^2 + 42656^2 = 28430240^2$$
$$2844302208^2 + 426656^2 = 2844302240^2$$
$$284443022208^2 + 4266656^2 = 284443022240^2 \tag{2.13}$$

▶ **Multiplying** (2.1) **by 64**

$$256^2 + 192^2 = 320^2$$
$$5376^2 + 832^2 = 5440^2$$
$$566016^2 + 8512^2 = 566080^2$$
$$56860416^2 + 85312^2 = 56860480^2$$
$$5688604416^2 + 853312^2 = 5688604480^2$$
$$568886044416^2 + 8533312^2 = 568886044480^2 \tag{2.14}$$

Remark 2. *We observe that in some cases the first and/or second lines don't give good patterns. These lines are indicated by* **orange color**. *This process of multiplying* (2.1) *by continues. May be in some cases, non patterns lines increases from 2 to 3, 4, etc. Let's consider another example.*

Example 2.2. *Let's consider following pattern:*

$$4^2 + 3^2 = 5^2$$
$$544^2 + 33^2 = 545^2$$
$$55444^2 + 333^2 = 55445^2$$
$$5554444^2 + 3333^2 = 5554445^2$$
$$555544444^2 + 33333^2 = 555544445^2$$
$$55555444444^2 + 333333^2 = 55555444445^2 \qquad (2.15)$$

The Example (2.15) is the same as (3.3) given in Section 3. Let's multiply the Example (2.15) by $n = 1, 2, 3, ..., 9$, we get the following patterns:

▶ **Multiplying** (2.15) **by 2**

$$8^2 + 6^2 = 10^2$$
$$1088^2 + 66^2 = 1090^2$$
$$110888^2 + 666^2 = 110890^2$$
$$11108888^2 + 6666^2 = 11108890^2$$
$$1111088888^2 + 66666^2 = 1111088890^2$$
$$111110888888^2 + 666666^2 = 111110888890^2 \qquad (2.16)$$

▶ **Multiplying** (2.15) **by 3**

$$12^2 + 9^2 = 15^2$$
$$1632^2 + 99^2 = 1635^2$$
$$166332^2 + 999^2 = 166335^2$$
$$16663332^2 + 9999^2 = 16663335^2$$
$$1666633332^2 + 99999^2 = 1666633335^2$$
$$166666333332^2 + 999999^2 = 166666333335^2 \qquad (2.17)$$

▶ **Multiplying** (2.15) **by 4**

$$16^2 + 12^2 = 20^2$$
$$2176^2 + 132^2 = 2180^2$$
$$221776^2 + 1332^2 = 221780^2$$
$$22217776^2 + 13332^2 = 22217780^2$$
$$2222177776^2 + 133332^2 = 2222177780^2$$
$$222221777776^2 + 1333332^2 = 222221777780^2 \qquad (2.18)$$

▶ **Multiplying** (2.15) **by 5**

$$20^2 + 15^2 = 25^2$$
$$2720^2 + 165^2 = 2725^2$$
$$277220^2 + 1665^2 = 277225^2$$
$$27772220^2 + 16665^2 = 27772225^2$$
$$2777722220^2 + 166665^2 = 2777722225^2$$
$$277777222220^2 + 1666665^2 = 277777222225^2 \tag{2.19}$$

▶ **Multiplying** (2.15) **by 6**

$$24^2 + 18^2 = 30^2$$
$$3264^2 + 198^2 = 3270^2$$
$$332664^2 + 1998^2 = 332670^2$$
$$33326664^2 + 19998^2 = 33326670^2$$
$$3333266664^2 + 199998^2 = 3333266670^2$$
$$333332666664^2 + 1999998^2 = 333332666670^2 \tag{2.20}$$

▶ **Multiplying** (2.15) **by 7**

$$28^2 + 21^2 = 35^2$$
$$3808^2 + 231^2 = 3815^2$$
$$388108^2 + 2331^2 = 388115^2$$
$$38881108^2 + 23331^2 = 38881115^2$$
$$3888811108^2 + 233331^2 = 3888811115^2$$
$$388888111108^2 + 2333331^2 = 388888111115^2 \tag{2.21}$$

▶ **Multiplying** (2.15) **by 8**

$$32^2 + 24^2 = 40^2$$
$$4352^2 + 264^2 = 4360^2$$
$$443552^2 + 2664^2 = 443560^2$$
$$44435552^2 + 26664^2 = 44435560^2$$
$$4444355552^2 + 266664^2 = 4444355560^2$$
$$444443555552^2 + 2666664^2 = 444443555560^2 \tag{2.22}$$

▶ **Multiplying** (2.15) **by 9**

$$36^2 + 27^2 = 45^2$$
$$4896^2 + 297^2 = 4905^2$$
$$498996^2 + 2997^2 = 499005^2$$
$$49989996^2 + 29997^2 = 49990005^2$$
$$4999899996^2 + 299997^2 = 4999900005^2$$
$$499998999996^2 + 2999997^2 = 499999000005^2 \qquad (2.23)$$

The above expressions are obtained multiplying Example (2.15) by $n = 1, 2, 3, ..., 9$. Let's see what happen's if we multiplying (2.15) by numbers more than 10:

▶ **Multiplying** (2.15) **by 13**

$$52^2 + 39^2 = 65^2$$
$$7072^2 + 429^2 = 7085^2$$
$$720772^2 + 4329^2 = 720785^2$$
$$72207772^2 + 43329^2 = 72207785^2$$
$$7222077772^2 + 433329^2 = 7222077785^2$$
$$722220777772^2 + 4333329^2 = 722220777785^2 \qquad (2.24)$$

▶ **Multiplying** (2.15) **by 31**

$$124^2 + 93^2 = 155^2$$
$$16864^2 + 1023^2 = 16895^2$$
$$1718764^2 + 10323^2 = 1718795^2$$
$$172187764^2 + 103323^2 = 172187795^2$$
$$17221877764^2 + 1033323^2 = 17221877795^2$$
$$1722218777764^2 + 10333323^2 = 1722218777795^2 \qquad (2.25)$$

The examples below are obtained multiplying (2.15) by power of 2, for example by 16, 32 and 64.

▶ **Multiplying** (2.15) **by 16**

$$64^2 + 48^2 = 80^2$$
$$8704^2 + 528^2 = 8720^2$$
$$887104^2 + 5328^2 = 887120^2$$
$$88871104^2 + 53328^2 = 88871120^2$$
$$8888711104^2 + 533328^2 = 8888711120^2$$
$$888887111104^2 + 5333328^2 = 888887111120^2 \qquad (2.26)$$

▶ **Multiplying** (2.15) **by 32**

$$128^2 + 96^2 = 160^2$$
$$17408^2 + 1056^2 = 17440^2$$
$$1774208^2 + 10656^2 = 1774240^2$$
$$177742208^2 + 106656^2 = 177742240^2$$
$$17777422208^2 + 1066656^2 = 17777422240^2$$
$$1777774222208^2 + 10666656^2 = 1777774222240^2 \qquad (2.27)$$

▶ **Multiplying** (2.15) **by 64**

$$256^2 + 192^2 = 320^2$$
$$34816^2 + 2112^2 = 34880^2$$
$$3548416^2 + 21312^2 = 3548480^2$$
$$355484416^2 + 213312^2 = 355484480^2$$
$$35554844416^2 + 2133312^2 = 35554844480^2$$
$$3555548444416^2 + 21333312^2 = 3555548444480^2 \qquad (2.28)$$

Remark 3. *We observe that in some cases, the first, second, third, etc. lines don't follow the pattern. This process of multiplying* (2.15) *continues. May be in some cases, there are initial line as non pattern cases.*

Example 2.3. *Let's consider following pattern:*

$$8^2 + 15^2 = 17^2$$
$$68^2 + 1155^2 = 1157^2$$
$$668^2 + 111555^2 = 111557^2$$
$$6668^2 + 11115555^2 = 11115557^2$$
$$66668^2 + 1111155555^2 = 1111155557^2$$
$$666668^2 + 111111555555^2 = 111111555557^2 \qquad (2.29)$$

The Example (2.29) is the same as (3.7) . Let's multiply the Example (2.29) by $n = 2, 3, ..., 9$, we get the following patterns:

▶ **Multiplying** (2.29) **by 2**

$$16^2 + 30^2 = 34^2$$
$$136^2 + 2310^2 = 2314^2$$
$$1336^2 + 223110^2 = 223114^2$$
$$13336^2 + 22231110^2 = 22231114^2$$
$$133336^2 + 2222311110^2 = 2222311114^2$$
$$1333336^2 + 222223111110^2 = 222223111114^2 \qquad (2.30)$$

▶ **Multiplying** (2.29) **by 3**

$$24^2 + 45^2 = 51^2$$
$$204^2 + 3465^2 = 3471^2$$
$$2004^2 + 334665^2 = 334671^2$$
$$20004^2 + 33346665^2 = 33346671^2$$
$$200004^2 + 3333466665^2 = 3333466671^2$$
$$2000004^2 + 333334666665^2 = 333334666671^2 \tag{2.31}$$

▶ **Multiplying** (2.29) **by 4**

$$32^2 + 60^2 = 68^2$$
$$272^2 + 4620^2 = 4628^2$$
$$2672^2 + 446220^2 = 446228^2$$
$$26672^2 + 44462220^2 = 44462228^2$$
$$266672^2 + 4444622220^2 = 4444622228^2$$
$$2666672^2 + 444446222220^2 = 444446222228^2 \tag{2.32}$$

▶ **Multiplying** (2.29) **by 5**

$$40^2 + 75^2 = 85^2$$
$$340^2 + 5775^2 = 5785^2$$
$$3340^2 + 557775^2 = 557785^2$$
$$33340^2 + 55577775^2 = 55577785^2$$
$$333340^2 + 5555777775^2 = 5555777785^2$$
$$3333340^2 + 555555777775^2 = 555555777785 \tag{2.33}$$

▶ **Multiplying** (2.29) **by 6**

$$48^2 + 90^2 = 102^2$$
$$408^2 + 6930^2 = 6942^2$$
$$4008^2 + 669330^2 = 669342^2$$
$$40008^2 + 66693330^2 = 66693342^2$$
$$400008^2 + 6666933330^2 = 6666933342^2$$
$$4000008^2 + 666669333330^2 = 666669333342^2 \tag{2.34}$$

▶ **Multiplying** (2.29) **by 7**

$$56^2 + 105^2 = 119^2$$
$$476^2 + 8085^2 = 8099^2$$
$$4676^2 + 780885^2 = 780899^2$$
$$46676^2 + 77808885^2 = 77808899^2$$
$$466676^2 + 7778088885^2 = 7778088899^2$$
$$4666676^2 + 777780888885^2 = 777780888899^2 \tag{2.35}$$

▶ **Multiplying** (2.29) **by 8**

$$64^2 + 120^2 = 136^2$$
$$544^2 + 9240^2 = 9256^2$$
$$5344^2 + 892440^2 = 892456^2$$
$$53344^2 + 88924440^2 = 88924456^2$$
$$533344^2 + 8889244440^2 = 8889244456^2$$
$$5333344^2 + 888892444440^2 = 888892444456^2 \tag{2.36}$$

▶ **Multiplying** (2.29) **by 9**

$$72^2 + 135^2 = 153^2$$
$$612^2 + 10395^2 = 10413^2$$
$$6012^2 + 1003995^2 = 1004013^2$$
$$60012^2 + 100039995^2 = 100040013^2$$
$$600012^2 + 10000399995^2 = 10000400013^2$$
$$6000012^2 + 1000003999995^2 = 1000004000013^2 \tag{2.37}$$

The above expressions are obtained multiplying Example (2.29) by $n = 1, 2, 3, ..., 9$. Let's see what happen's if we multiplying (2.29) by numbers more than 10:

▶ **Multiplying** (2.29) **by 11**

$$88^2 + 165^2 = 187^2$$
$$748^2 + 12705 = 12727^2$$
$$7348^2 + 1227105 = 1227127^2$$
$$73348^2 + 122271105 = 122271127^2$$
$$733348^2 + 12222711105 = 12222711127^2$$
$$7333348^2 + 1222227111105 = 1222227111127^2 \tag{2.38}$$

▶ **Multiplying** (2.29) **by 41**

$$328^2 + 615^2 = 697^2$$
$$2788^2 + 47355 = 47437^2$$
$$27388^2 + 4573755 = 4573837^2$$
$$273388^2 + 455737755 = 455737837^2$$
$$2733388^2 + 4555377755 = 4555377837^2$$
$$27333388^2 + 455557377755 = 4555573777837^2 \qquad (2.39)$$

The examples below are obtained multiplying (2.29) by power of 2, for example by 16, 32 and 64.

▶ **Multiplying** (2.29) **by 16**

$$128^2 + 240^2 = 272^2$$
$$1088^2 + 18480^2 = 18512^2$$
$$10688^2 + 1784880^2 = 1784912^2$$
$$106688^2 + 177848880^2 = 177848912^2$$
$$1066688^2 + 17778488880^2 = 17778488912^2$$
$$10666688^2 + 1777784888880^2 = 1777784888912^2 \qquad (2.40)$$

▶ **Multiplying** (2.29) **by 32**

$$256^2 + 480^2 = 544^2$$
$$2176^2 + 36960^2 = 37024^2$$
$$21376^2 + 3569760 = 3569824^2$$
$$213376^2 + 355697760 = 355697824^2$$
$$2133376^2 + 3555697760 = 35556977824^2$$
$$21333376^2 + 3555569777760 = 3555569777824^2 \qquad (2.41)$$

▶ **Multiplying** (2.29) **by 64**

$$512^2 + 960^2 = 1088^2$$
$$4352^2 + 73920^2 = 74048^2$$
$$42752^2 + 7139520^2 = 7139648^2$$
$$426752^2 + 711395520^2 = 711395648^2$$
$$4266752^2 + 71113955520^2 = 71113955648^2$$
$$42666752^2 + 7111139555520^2 = 7111139555648^2 \qquad (2.42)$$

Remark 4. *We observe that in some cases, the first and/or terms don't follow the pattern.*

The above three examples, (2.1), (2.15) and (2.29) brings **multiple-type patterns** without consid-ering final sums. The following two examples are with final sums:

Example 2.4. *Let's consider following pattern:*

$$99^2 + 20^2 = 101^2 \qquad := 10201$$
$$9999^2 + 200^2 = 10001^2 \qquad := 100020001$$
$$999999^2 + 2000^2 = 1000001^2 \qquad := 1000002000001$$
$$99999999^2 + 20000^2 = 100000001^2 \qquad := 10000000200000001$$
$$9999999999^2 + 200000^2 = 10000000001^2 \qquad := 100000000020000000001$$
$$999999999999^2 + 2000000^2 = 1000000000001^2 := 1000000000002000000000001^2 \qquad (2.43)$$

The Example (2.43) is the same as (4.8) given in Section 4. Let's multiply the Example (2.43) by $n = 2, 3, ..., 9$, we get the following patterns:

▶ **Multiplying** (2.43) **by 2**

$$198^2 + 40^2 = 202^2 \qquad := 20402$$
$$19998^2 + 400^2 = 20002^2 \qquad := 200040002$$
$$1999998^2 + 4000^2 = 2000002^2 \qquad := 2000004000002$$
$$199999998^2 + 40000^2 = 200000002^2 \qquad := 20000000400000002$$
$$19999999998^2 + 400000^2 = 20000000002^2 \qquad := 200000000040000000002$$
$$1999999999998^2 + 4000000^2 = 2000000000002^2 := 2000000000004000000000002^2 \qquad (2.44)$$

▶ **Multiplying** (2.43) **by 3**

$$297^2 + 60^2 = 303^2 \qquad := 30603$$
$$29997^2 + 600^2 = 30003^2 \qquad := 300060003$$
$$2999997^2 + 6000^2 = 3000003^2 \qquad := 3000006000003$$
$$299999997^2 + 60000^2 = 300000003^2 \qquad := 30000000600000003$$
$$29999999997^2 + 600000^2 = 30000000003^2 \qquad := 300000000060000000003$$
$$2999999999997^2 + 6000000^2 = 3000000000003^2 := 3000000000006000000000003^2 \qquad (2.45)$$

▶ **Multiplying** (2.43) **by 4**

$$396^2 + 80^2 = 404^2 \qquad := 40804$$
$$39996^2 + 800^2 = 40004^2 \qquad := 400080004$$
$$3999996^2 + 8000^2 = 4000004^2 \qquad := 4000008000004$$
$$399999996^2 + 80000^2 = 400000004^2 \qquad := 40000000800000004$$
$$39999999996^2 + 800000^2 = 40000000004^2 \qquad := 400000000080000000004$$
$$3999999999996^2 + 8000000^2 = 4000000000004^2 := 4000000000008000000000004^2 \qquad (2.46)$$

▶ **Multiplying** (2.43) **by 5**

$$495^2 + 100^2 = 505^2 \qquad := 51005$$
$$49995^2 + 1000^2 = 50005^2 \qquad := 500100005$$
$$4999995^2 + 10000^2 = 5000005^2 \qquad := 5000010000005$$
$$499999995^2 + 100000^2 = 500000005^2 \qquad := 50000001000000005$$
$$49999999995^2 + 1000000^2 = 50000000005^2 \qquad := 500000000100000000005$$
$$4999999999995^2 + 10000000^2 = 5000000000005^2 := 5000000000010000000000005^2 \qquad (2.47)$$

▶ **Multiplying** (2.43) **by 6**

$$594^2 + 120^2 = 606^2 \qquad := 61206$$
$$59994^2 + 1200^2 = 60006^2 \qquad := 600120006$$
$$5999994^2 + 12000^2 = 6000006^2 \qquad := 6000012000006$$
$$599999994^2 + 120000^2 = 600000006^2 \qquad := 60000001200000006$$
$$59999999994^2 + 1200000^2 = 60000000006^2 \qquad := 600000000120000000006$$
$$5999999999994^2 + 12000000^2 = 6000000000006^2 := 6000000000012000000000006^2 \qquad (2.48)$$

▶ **Multiplying** (2.43) **by 7**

$$693^2 + 140^2 = 707^2 \qquad := 71407$$
$$69993^2 + 1400^2 = 70007^2 \qquad := 700140007$$
$$6999993^2 + 14000^2 = 7000007^2 \qquad := 7000014000007$$
$$699999993^2 + 140000^2 = 700000007^2 \qquad := 70000001400000007$$
$$69999999993^2 + 1400000^2 = 70000000007^2 \qquad := 700000000140000000007$$
$$6999999999993^2 + 14000000^2 = 7000000000007^2 := 7000000000014000000000007^2 \qquad (2.49)$$

▶ **Multiplying** (2.43) **by 8**

$$792^2 + 160^2 = 808^2 \qquad := 81608$$
$$79992^2 + 1600^2 = 80008^2 \qquad := 800160008$$
$$7999992^2 + 16000^2 = 8000008^2 \qquad := 8000016000008$$
$$799999992^2 + 160000^2 = 800000008^2 \qquad := 80000001600000008$$
$$79999999992^2 + 1600000^2 = 80000000008^2 \qquad := 800000000160000000008$$
$$7999999999992^2 + 16000000^2 = 8000000000008^2 := 8000000000016000000000008^2 \qquad (2.50)$$

▶ **Multiplying** (2.43) **by 9**

$$891^2 + 180^2 = 909^2 \qquad := 91809$$
$$89991^2 + 1800^2 = 90009^2 \qquad := 900180009$$
$$8999991^2 + 18000^2 = 9000009^2 \qquad := 9000018000009$$
$$899999991^2 + 180000^2 = 900000009^2 \qquad := 9000000180000009$$
$$89999999991^2 + 1800000^2 = 90000000009^2 \qquad := 900000000180000000009$$
$$8999999999991^2 + 18000000^2 = 9000000000009^2 := 90000000000018000000000009^2 \qquad (2.51)$$

The above expressions are obtained multiplying Example (2.43) by $n = 1, 2, 3, ..., 9$. Let's see what happen's if we multiplying (2.43) by numbers more than 10:

▶ **Multiplying** (2.43) **by 17**

$$1683^2 + 340^2 = 1717^2 \qquad := 173417$$
$$169983^2 + 3400^2 = 170017^2 \qquad := 1700340017$$
$$16999983^2 + 34000^2 = 17000017^2 \qquad := 17000034000017$$
$$1699999983^2 + 340000^2 = 1700000017^2 \qquad := 17000000340000017$$
$$169999999983^2 + 3400000^2 = 170000000017^2 \qquad := 1700000000340000000017$$
$$16999999999983^2 + 34000000^2 = 17000000000017^2 := 170000000000034000000000017^2 \qquad (2.52)$$

▶ **Multiplying** (2.43) **by 29**

$$2871^2 + 580^2 = 2929^2 \qquad := 295829$$
$$289971^2 + 5800^2 = 290029^2 \qquad := 2900580029$$
$$28999971^2 + 58000^2 = 29000029^2 \qquad := 29000058000029$$
$$2899999971^2 + 580000^2 = 2900000029^2 \qquad := 29000000580000029$$
$$289999999971^2 + 5800000^2 = 290000000029^2 \qquad := 2900000000580000000029$$
$$28999999999971^2 + 58000000^2 = 29000000000029^2 := 290000000000058000000000029^2 \qquad (2.53)$$

The examples below are obtained multiplying (2.43) by power of 2, for example by 16, 32 and 64.

▶ **Multiplying** (2.43) **by 16**

$$1584^2 + 320^2 = 1616^2 \qquad := 163216$$
$$159984^2 + 3200^2 = 160016^2 \qquad := 1600320016$$
$$15999984^2 + 32000^2 = 16000016^2 \qquad := 16000032000016$$
$$1599999984^2 + 320000^2 = 1600000016^2 \qquad := 16000000320000016$$
$$159999999984^2 + 3200000^2 = 160000000016^2 \qquad := 1600000000320000000016$$
$$15999999999984^2 + 32000000^2 = 16000000000016^2 := 160000000000032000000000016^2 \qquad (2.54)$$

▶ **Multiplying** (2.43) **by 32**

$$3168^2 + 640^2 = 3232^2 \qquad := 326432$$
$$319968^2 + 6400^2 = 320032^2 \qquad := 3200640032$$
$$31999968^2 + 64000^2 = 32000032^2 \qquad := 32000064000032$$
$$3199999968^2 + 640000^2 = 3200000032^2 \qquad := 32000006400000032$$
$$319999999968^2 + 6400000^2 = 320000000032^2 \qquad := 3200000000640000000032$$
$$31999999999968^2 + 64000000^2 = 32000000000032^2 := 3200000000000640000000000032^2 \qquad (2.55)$$

▶ **Multiplying** (2.43) **by 64**

$$6336^2 + 1280^2 = 6464^2 \qquad := 652864$$
$$639936^2 + 12800^2 = 640064^2 \qquad := 6401280064$$
$$63999936^2 + 128000^2 = 64000064^2 \qquad := 64000128000064$$
$$6399999936^2 + 1280000^2 = 6400000064^2 \qquad := 64000012800000064$$
$$639999999936^2 + 12800000^2 = 640000000064^2 \qquad := 6400000001280000000064$$
$$63999999999936^2 + 128000000^2 = 64000000000064^2 := 6400000000012800000000000064^2 \qquad (2.56)$$

Example 2.5. *Let's consider following pattern:*

$$220^2 + 21^2 = 221^2 \qquad := 48841$$
$$20200^2 + 201^2 = 20201^2 \qquad := 408080401$$
$$2002000^2 + 2001^2 = 2002001^2 \qquad := 4008008004001$$
$$200020000^2 + 20001^2 = 200020001^2 \qquad := 40008000800040001$$
$$20000200000^2 + 200001^2 = 20000200001^2 := 400008000080000400001 \qquad (2.57)$$

The Example (2.57) is the same as (4.10) given in Section 4. Let's multiply the Example (2.57) by $n = 2, 3, ..., 9$, we get the following patterns:

▶ **Multiplying** (2.57) **by 2**

$$440^2 + 42^2 = 442^2 \qquad := 97682$$
$$40400^2 + 402^2 = 40402^2 \qquad := 816160802$$
$$4004000^2 + 4002^2 = 4004002^2 \qquad := 8016016008002$$
$$400040000^2 + 40002^2 = 400040002^2 \qquad := 80016001600080002$$
$$40000400000^2 + 400002^2 = 40000400002^2 := 800016000160000800002^2 \qquad (2.58)$$

▶ **Multiplying** (2.57) **by 3**

17

$$660^2 + 63^2 = 663^2 \qquad := 146523$$
$$60600^2 + 603^2 = 60603^2 \qquad := 1224241203$$
$$6006000^2 + 6003^2 = 6006003^2 \qquad := 12024024012003$$
$$600060000^2 + 60003^2 = 600060003^2 \qquad := 120024002400120003$$
$$60000600000^2 + 600003^2 = 60000600003^2 := 120002400024000120003^2 \qquad (2.59)$$

▶ **Multiplying** (2.57) **by 4**

$$880^2 + 84^2 = 884^2 \qquad := 195364$$
$$80800^2 + 804^2 = 80804^2 \qquad := 1632321604$$
$$8008000^2 + 8004^2 = 8008004^2 \qquad := 16032032016004$$
$$800080000^2 + 80004^2 = 800080004^2 \qquad := 160032003200160004$$
$$80000800000^2 + 800004^2 = 80000800004^2 := 160003200032000160004^2 \qquad (2.60)$$

▶ **Multiplying** (2.57) **by 5**

$$1100^2 + 105^2 = 1105^2 \qquad := 244205$$
$$101000^2 + 1005^2 = 101005^2 \qquad := 2040402005$$
$$10010000^2 + 10005^2 = 10010005^2 \qquad := 20040040020005$$
$$1000100000^2 + 100005^2 = 1000100005^2 \qquad := 200040004000200005$$
$$100001000000^2 + 1000005^2 = 100001000005^2 := 200004000040000200005^2 \qquad (2.61)$$

▶ **Multiplying** (2.57) **by 6**

$$1320^2 + 126^2 = 1326^2 \qquad := 293046$$
$$121200^2 + 1206^2 = 121206^2 \qquad := 2448482406$$
$$12012000^2 + 12006^2 = 12012006^2 \qquad := 24048048024006$$
$$1200120000^2 + 120006^2 = 1200120006^2 \qquad := 240048004800240006$$
$$120001200000^2 + 1200006^2 = 120001200006^2 := 240004800048000240006^2 \qquad (2.62)$$

▶ **Multiplying** (2.57) **by 7**

$$1540^2 + 147^2 = 1547^2 \qquad := 341887$$
$$141400^2 + 1407^2 = 141407^2 \qquad := 2856562807$$
$$14014000^2 + 14007^2 = 14014007^2 \qquad := 28056056028007$$
$$1400140000^2 + 140007^2 = 1400140007^2 \qquad := 280056005600280007$$
$$140001400000^2 + 1400007^2 = 140001400007^2 := 280005600056000280007^2 \qquad (2.63)$$

▶ **Multiplying** (2.57) **by 8**

$$
\begin{aligned}
1760^2 + 168^2 &= 1768^2 &&:= 390728 \\
161600^2 + 1608^2 &= 161608^2 &&:= 3264643208 \\
16016000^2 + 16008^2 &= 16016008^2 &&:= 32064064032008 \\
1600160000^2 + 160008^2 &= 1600160008^2 &&:= 320064006400320008 \\
160001600000^2 + 1600008^2 &= 160001600008^2 &&:= 32000640006400032000 08^2
\end{aligned}
\tag{2.64}
$$

▶ **Multiplying** (2.57) **by 9**

$$
\begin{aligned}
1980^2 + 189^2 &= 1989^2 &&:= 439569 \\
181800^2 + 1809^2 &= 181809^2 &&:= 3672723609 \\
18018000^2 + 18009^2 &= 18018009^2 &&:= 36072072036009 \\
1800180000^2 + 180009^2 &= 1800180009^2 &&:= 3600720072003600 09 \\
180001800000^2 + 1800009^2 &= 180001800009^2 &&:= 360007200072000360000 9^2
\end{aligned}
\tag{2.65}
$$

The above expressions are obtained multiplying Example (2.57) by $n = 1, 2, 3, ..., 9$. Let's see what happen's if we multiplying (2.57) by numbers more than 10:

▶ **Multiplying** (2.57) **by 23**

$$
\begin{aligned}
5060^2 + 483^2 &= 5083^2 &&:= 1123343 \\
464600^2 + 4623^2 &= 464623^2 &&:= 9385849223 \\
46046000^2 + 46023^2 &= 46046023^2 &&:= 92184184092023 \\
4600460000^2 + 460023^2 &= 4600460023^2 &&:= 920184018400920023 \\
460004600000^2 + 4600023^2 &= 460004600023^2 &&:= 9200184001840009200023^2
\end{aligned}
\tag{2.66}
$$

▶ **Multiplying** (2.57) **by 37**

$$
\begin{aligned}
8140^2 + 777^2 &= 8177^2 &&:= 1807117 \\
747400^2 + 7437^2 &= 747437^2 &&:= 15098974837 \\
74074000^2 + 74037^2 &= 74074037^2 &&:= 148296296148037 \\
5400540000^2 + 540027^2 &= 5400540027^2 &&:= 1080216021601080027 \\
740007400000^2 + 7400037^2 &= 740007400037^2 &&:= 1480029600296001480003 7^2
\end{aligned}
\tag{2.67}
$$

The examples below are obtained multiplying (2.15) by power of 2, for example by 16, 32 and 64.

▶ **Multiplying** (2.57) **by 16**

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$$3520^2 + 336^2 = 3536^2 \qquad := 781456$$
$$323200^2 + 3216^2 = 323216^2 \qquad := 6529286416$$
$$32032000^2 + 32016^2 = 32032016^2 \qquad := 64128128064016$$
$$3200320000^2 + 320016^2 = 3200320016^2 \qquad := 640128012800640016$$
$$320003200000^2 + 3200016^2 = 320003200016^2 := 6400128001280006400016^2 \qquad (2.68)$$

▶ **Multiplying** (2.57) **by 32**

$$7040^2 + 672^2 = 7072^2 \qquad := 1562912$$
$$646400^2 + 6432^2 = 646432^2 \qquad := 13058572832$$
$$64064000^2 + 64032^2 = 64064032^2 \qquad := 128256256128032$$
$$6400640000^2 + 640032^2 = 6400640032^2 \qquad := 1280256025601280032$$
$$640006400000^2 + 6400032^2 = 640006400032^2 := 12800256002560012800032^2 \qquad (2.69)$$

▶ **Multiplying** (2.57) **by 64**

$$14080^2 + 1344^2 = 14144^2 \qquad := 3125824$$
$$1292800 + 12864 = 1292864^2 \qquad := 26117145664$$
$$128128000^2 + 128064^2 = 128128064^2 \qquad := 256512512256064$$
$$12801280000^2 + 1280064^2 = 12801280064^2 \qquad := 2560512051202560064$$
$$1280012800000^2 + 12800064^2 = 1280012800064^2 := 25600512005120025600064^2 \qquad (2.70)$$

Remark 5. *We observe that in some cases, the first and/or terms don't follow the pattern. These are specified by* **orange color**.

3 Patterns in Pythagorean Triples Without Final Sum

This section brings **patterns in Pythagorean triples**. The **pythagorean triples** considered are **primitive**. The idea of total sum is considered in Section 4. The construction way can be seen in [5, 6].

$$3^2 + 4^2 = 5^2$$
$$1023^2 + 64^2 = 1025^2$$
$$110223^2 + 664^2 = 110225^2$$
$$11102223^2 + 6664^2 = 11102225^2$$
$$1111022223^2 + 66664^2 = 1111022225^2 \qquad (3.1)$$

$$4^2 + 3^2 = 5^2$$
$$84^2 + 13^2 = 85^2$$
$$8844^2 + 133^2 = 8845^2$$
$$888444^2 + 1333^2 = 888445^2$$
$$88884444^2 + 13333^2 = 88884445^2$$
$$8888844444^2 + 133333^2 = 8888844445^2 \qquad (3.2)$$

$$4^2 + 3^2 = 5^2$$
$$544^2 + 33^2 = 545^2$$
$$55444^2 + 333^2 = 55445^2$$
$$5554444^2 + 3333^2 = 5554445^2$$
$$555544444^2 + 33333^2 = 555544445^2$$
$$55555444444^2 + 333333^2 = 55555444445^2 \qquad (3.3)$$

$$4^2 + 3^2 = 5^2$$
$$1984^2 + 63^2 = 1985^2$$
$$219784^2 + 663^2 = 219785^2$$
$$22197784^2 + 6663^2 = 22197785^2$$
$$2221977784^2 + 66663^2 = 2221977785^2 \qquad (3.4)$$

$$4^2 + 3^2 = 5^2$$
$$7564^2 + 123^2 = 7565^2$$
$$875164^2 + 1323^2 = 875165^2$$
$$88751164^2 + 13323^2 = 88751165^2$$
$$8887511164^2 + 133323^2 = 8887511165^2 \qquad (3.5)$$

$$5^2 + 12^2 = 13^2$$
$$1085^2 + 132^2 = 1093^2$$
$$110885^2 + 1332^2 = 110893^2$$
$$11108885^2 + 13332^2 = 11108893^2$$
$$1111088885,^2 + 133332^2 = 1111088893^2$$
$$111110888885^2 + 1333332^2 = 111110888893^2 \qquad (3.6)$$

$$8^2 + 15^2 = 17^2$$
$$68^2 + 1155^2 = 1157^2$$
$$668^2 + 111555,^2 = 111557^2$$
$$6668^2 + 11115555^2 = 11115557^2$$
$$66668^2 + 1111155555^2 = 1111155557^2$$
$$666668^2 + 111111555555^2 = 111111555557^2 \qquad (3.7)$$

$$8^2 + 6^2 = 10^2$$
$$1088^2 + 66^2 = 1090^2$$
$$110888^2 + 666^2 = 110890^2$$
$$11108888^2 + 6666^2 = 11108890^2$$
$$1111088888^2 + 66666^2 = 1111088890^2$$
$$111110888888^2 + 666666^2 = 111110888890^2 \tag{3.8}$$

$$9^2 + 40^2 = 41^2$$
$$3009^2 + 440^2 = 3041^2$$
$$308009^2 + 4440^2 = 308041^2$$
$$30858009^2 + 44440^2 = 30858041^2 \tag{3.9}$$

$$12^2 + 5^2 = 13^2$$
$$812^2 + 125^2 = 7813^2$$
$$877812^2 + 1325^2 = 877813^2$$
$$88777812^2 + 13325^2 = 88777813^2$$
$$8887777812^2 + 133325^2 = 8887777813^2 \tag{3.10}$$

$$12^2 + 5^2 = 13^2$$
$$2112^2 + 65^2 = 2113^2$$
$$221112^2 + 665^2 = 221113^2$$
$$22211112^2 + 6665^2 = 22211113^2$$
$$2222111112^2 + 66665^2 = 2222111113^2 \tag{3.11}$$

$$12^2 + 5^2 = 13^2$$
$$17112^2 + 185^2 = 17113^2$$
$$1970112^2 + 1985^2 = 1970113^2$$
$$199700112^2 + 19985^2 = 199700113^2$$
$$19997000112^2 + 199985^2 = 19997000113^2 \tag{3.12}$$

$$15^2 + 8^2 = 17^2$$
$$1155^2 + 68^2 = 1157^2$$
$$111555^2 + 668^2 = 111557^2$$
$$11115555^2 + 6668^2 = 11115557^2$$
$$1111155555^2 + 66668^2 = 1111155557^2$$
$$111111555555^2 + 666668^2 = 111111555557^2 \tag{3.13}$$

$$16^2 + 12^2 = 20^2$$
$$2176^2 + 132^2 = 2180^2$$
$$221776^2 + 1332^2 = 221780^2$$
$$22217776^2 + 13332^2 = 22217780^2$$
$$2222177776,^2 + 133332^2 = 2222177780^2$$
$$222221777776^2 + 1333332^2 = 222221777780^2 \tag{3.14}$$

$$24^2 + 7^2 = 25^2$$
$$2244^2 + 67^2 = 2245^2$$
$$222444^2 + 667^2 = 222445^2$$
$$22224444^2 + 6667^2 = 22224445^2$$
$$2222244444^2 + 66667^2 = 2222244445^2$$
$$222222444444^2 + 666667^2 = 222222444445^2 \tag{3.15}$$

$$24^2 + 7^2 = 25^2$$
$$2664^2 + 73^2 = 2665^2$$
$$226464^2 + 673^2 = 226465^2$$
$$22264464^2 + 6673^2 = 22264465^2$$
$$2222644464^2 + 66673^2 = 2222644465^2 \tag{3.16}$$

$$35^2 + 12^2 = 37^2$$
$$4355^2 + 132^2 = 4357^2$$
$$443555^2 + 1332^2 = 443557^2$$
$$44435555^2 + 13332^2 = 44435557^2$$
$$4444355555^2 + 133332^2 = 4444355557^2$$
$$444443555555^2 + 1333332^2 = 444443555557^2 \tag{3.17}$$

$$48^2 + 55^2 = 73^2$$
$$6148^2 + 555^2 = 6173^2$$
$$617148^2 + 5555^2 = 617173^2$$
$$61727148^2 + 55555^2 = 61727173^2 \tag{3.18}$$

$$134^2 + 4488^2 = 4490^2$$
$$1334^2 + 444888^2 = 444890^2$$
$$13334^2 + 44448888^2 = 44448890^2$$
$$133334^2 + 4444488888^2 = 4444488890^2$$
$$1333334^2 + 444444888888^2 = 444444888890^2$$
$$13333334^2 + 44444448888888^2 = 44444448888890^2 \tag{3.19}$$

$$140^2 + 1221^2 = 1229^2$$
$$1340^2 + 112221^2 = 112229^2$$
$$13340^2 + 11122221^2 = 11122229^2$$
$$133340^2 + 1111222221^2 = 1111222229^2$$
$$1333340^2 + 111112222221^2 = 111112222229^2$$
$$13333340^2 + 11111122222221^2 = 11111122222229^2 \tag{3.20}$$

$$143^2 + 24^2 = 145^2$$
$$17423^2 + 264^2 = 17425^2$$
$$1774223^2 + 2664^2 = 1774225^2$$
$$177742223^2 + 26664^2 = 177742225^2$$
$$17777422223^2 + 266664^2 = 17777422225^2 \tag{3.21}$$

$$207^2 + 2376^2 = 2385^2$$
$$2007^2 + 223776^2 = 223785^2$$
$$20007^2 + 22237776^2 = 22237785^2$$
$$200007^2 + 2222377776^2 = 2222377785^2$$
$$2000007^2 + 222223777776^2 = 222223777785^2$$
$$20000007^2 + 22222237777776^2 = 22222237777785^2 \tag{3.22}$$

$$209^2 + 120^2 = 241^2$$
$$24009^2 + 1240^2 = 24041^2$$
$$2418009^2 + 12440^2 = 2418041^2$$
$$241958009^2 + 124440^2 = 241958041^2 \tag{3.23}$$

$$221^2 + 60^2 = 229^2$$
$$24021^2 + 620^2 = 24029^2$$
$$2418021^2 + 6220^2 = 2418029^2$$
$$241958021^2 + 62220^2 = 241958029^2 \tag{3.24}$$

$$255^2 + 32^2 = 257^2$$
$$27555^2 + 332^2 = 27557^2$$
$$2775555^2 + 3332^2 = 2775557^2$$
$$277755555^2 + 33332^2 = 277755557^2 \tag{3.25}$$

$$355^2 + 2508^2 = 2533^2$$
$$3355^2 + 225108^2 = 225133^2$$
$$33355^2 + 22251108^2 = 22251133^2$$
$$333355^2 + 2222511108^2 = 2222511133^2$$
$$3333355^2 + 222225111108^2 = 222225111133^2$$
$$33333355^2 + 22222251111108^2 = 22222251111133^2 \tag{3.26}$$

$$364^2 + 27^2 = 365^2$$
$$35644^2 + 267^2 = 35645^2$$
$$3556444^2 + 2667^2 = 3556445^2$$
$$355564444^2 + 26667^2 = 355564445^2 \tag{3.27}$$

$$504^2 + 297^2 = 585^2$$
$$55404^2 + 2997^2 = 55485^2$$
$$5554404^2 + 29997^2 = 5554485^2$$
$$555544404^2 + 299997^2 = 555544485^2$$
$$55555444404^2 + 2999997^2 = 55555444485^2 \tag{3.28}$$

$$512^2 + 55^2 = 1513^2$$
$$154012^2 + 555^2 = 154013^2$$
$$15429012^2 + 5555^2 = 15429013^2$$
$$1543179012^2 + 55555^2 = 1543179013^2 \tag{3.29}$$

$$520^2 + 231^2 = 569^2$$
$$55420^2 + 2331^2 = 55469^2$$
$$5554420^2 + 23331^2 = 5554469^2$$
$$555544420^2 + 233331^2 = 555544469^2$$
$$55555444420^2 + 2333331^2 = 55555444469^2 \tag{3.30}$$

$$532^2 + 165^2 = 557^2$$
$$55432^2 + 1665^2 = 55457^2$$
$$5554432^2 + 16665^2 = 5554457^2$$
$$555544432^2 + 166665^2 = 555544457^2$$
$$55555444432^2 + 1666665^2 = 55555444457^2 \tag{3.31}$$

$$540^2 + 99^2 = 549^2$$
$$55440^2 + 999^2 = 55449^2$$
$$5554440^2 + 9999^2 = 5554449^2$$
$$555544440^2 + 99999^2 = 555544449^2$$
$$55555444440^2 + 999999^2 = 55555444449^2 \tag{3.32}$$

$$685^2 + 9372^2 = 9397^2$$
$$6685^2 + 893772^2 = 893797^2$$
$$66685^2 + 88937772^2 = 88937797^2$$
$$666685^2 + 8889377772^2 = 8889377797^2$$
$$6666685^2 + 888893777772^2 = 888893777797^2$$
$$66666685^2 + 88888937777772^2 = 88888937777797^2 \tag{3.33}$$

$$1025^2 + 528^2 = 1153^2$$
$$110825^2 + 5328^2 = 110953^2$$
$$11108825^2 + 53328^2 = 11108953^2$$
$$1111088825^2 + 533328^2 = 1111088953^2$$
$$111110888825^2 + 5333328^2 = 111110888953^2 \tag{3.34}$$

$$1053^2 + 396^2 = 1125^2$$
$$110853^2 + 3996^2 = 110925^2$$
$$11108853^2 + 39996^2 = 11108925^2$$
$$1111088853^2 + 399996^2 = 1111088925^2$$
$$111110888853^2 + 3999996^2 = 111110888925^2 \tag{3.35}$$

$$1073^2 + 264^2 = 1105^2$$
$$110873^2 + 2664^2 = 110905^2$$
$$11108873^2 + 26664^2 = 11108905^2$$
$$1111088873^2 + 266664^2 = 1111088905^2$$
$$111110888873^2 + 2666664^2 = 111110888905^2 \tag{3.36}$$

$$1221^2 + 140^2 = 1229^2$$
$$112221^2 + 1340^2 = 112229^2$$
$$11122221^2 + 13340^2 = 11122229^2$$
$$1111222221^2 + 133340^2 = 1111222229^2$$
$$111112222221^2 + 1333340^2 = 111112222229^2$$
$$11111122222221^2 + 13333340^2 = 11111122222229^2 \tag{3.37}$$

$$1419^2 + 380^2 = 1469^2$$
$$114219^2 + 3380^2 = 114269^2$$
$$11142219^2 + 33380^2 = 11142269^2$$
$$1111422219^2 + 333380^2 = 1111422269^2$$
$$111114222219^2 + 3333380^2 = 111114222269^2$$
$$11111142222219^2 + 33333380^2 = 11111142222269^2 \tag{3.38}$$

$$1508^2 + 165^2 = 1517^2$$
$$154008^2 + 1665^2 = 154017^2$$
$$15429008^2 + 16665^2 = 15429017^2$$
$$1543179008^2 + 166665^2 = 1543179017^2 \tag{3.39}$$

$$1551^2 + 560^2 = 1649^2$$
$$115551^2 + 4760^2 = 115649^2$$
$$11155551^2 + 46760^2 = 11155649^2$$
$$1111555551^2 + 466760^2 = 1111555649^2$$
$$111115555551^2 + 4666760^2 = 111115555649^2$$
$$11111155555551^2 + 46666760^2 = 11111155555649^2 \tag{3.40}$$

$$1617^2 + 656^2 = 1745^2$$
$$116217^2 + 5456^2 = 116345^2$$
$$11162217^2 + 53456^2 = 11162345^2$$
$$1111622217^2 + 533456^2 = 1111622345^2$$
$$111116222217^2 + 5333456^2 = 111116222345^2$$
$$11111162222217^2 + 53333456^2 = 11111162222345^2 \tag{3.41}$$

$$4307^2 + 924^2 = 4405^2$$
$$443507^2 + 9324^2 = 443605^2$$
$$44435507^2 + 93324^2 = 44435605^2$$
$$4444355507^2 + 933324^2 = 4444355605^2$$
$$444443555507^2 + 9333324^2 = 444443555605^2 \tag{3.42}$$

$$4331^2 + 660^2 = 4381^2$$
$$443531^2 + 6660^2 = 443581^2$$
$$44435531^2 + 66660^2 = 44435581^2$$
$$4444355531^2 + 666660^2 = 4444355581^2$$
$$444443555531^2 + 6666660^2 = 444443555581^2 \tag{3.43}$$

$$4347^2 + 396^2 = 4365^2$$
$$443547^2 + 3996^2 = 443565^2$$
$$44435547^2 + 39996^2 = 44435565^2$$
$$4444355547^2 + 399996^2 = 4444355565^2$$
$$444443555547^2 + 3999996^2 = 444443555565^2 \tag{3.44}$$

$$4275^2 + 1188^2 = 4437^2$$
$$443475^2 + 11988^2 = 443637^2$$
$$44435475^2 + 119988^2 = 44435637^2$$
$$4444355475^2 + 1199988^2 = 4444355637^2$$
$$444443555475^2 + 11999988^2 = 444443555637^2 \qquad (3.45)$$

$$6083^2 + 156^2 = 6085^2$$
$$636803^2 + 1596^2 = 636805^2$$
$$63968003^2 + 15996^2 = 63968005^2$$
$$6399680003^2 + 159996^2 = 6399680005^2 \qquad (3.46)$$

$$8064^2 + 127^2 = 8065^2$$
$$880464^2 + 1327^2 = 880465^2$$
$$88804464^2 + 13327^2 = 88804465^2$$
$$8888044464^2 + 133327^2 = 8888044465^2 \qquad (3.47)$$

$$9660^2 + 139^2 = 9661^2$$
$$896460^2 + 1339^2 = 896461^2$$
$$88964460^2 + 13339^2 = 88964461^2$$
$$8889644460^2 + 133339^2 = 8889644461^2 \qquad (3.48)$$

$$14859^2 + 1220^2 = 14909^2$$
$$1493259^2 + 12220^2 = 1493309^2$$
$$149377259^2 + 122220^2 = 149377309^2$$
$$14938217259^2 + 1222220^2 = 14938217309^2 \qquad (3.49)$$

$$17415^2 + 792^2 = 17433^2$$
$$1774215^2 + 7992^2 = 1774233^2$$
$$177742215^2 + 79992^2 = 177742233^2$$
$$17777422215^2 + 799992^2 = 17777422233^2 \qquad (3.50)$$

$$17484^2 + 187^2 = 17485^2$$
$$1974084^2 + 1987^2 = 1974085^2$$
$$199740084^2 + 19987^2 = 199740085^2$$
$$19997400084^2 + 199987^2 = 19997400085^2 \qquad (3.51)$$

$$27144^2 + 233^2 = 27145^2$$
$$2721444^2 + 2333^2 = 2721445^2$$
$$272214444^2 + 23333^2 = 272214445^2$$
$$27222144444^2 + 233333^2 = 27222144445^2$$
$$272222144444^2 + 2333333^2 = 272222144445^2 \tag{3.52}$$

$$31500^2 + 251^2 = 31501^2$$
$$3127500^2 + 2501^2 = 3127501^2$$
$$312525000^2 + 25001^2 = 312525001^2$$
$$31250250000^2 + 250001^2 = 31250250001^2$$
$$3125002500000^2 + 2500001^2 = 3125002500001^2 \tag{3.53}$$

$$44660^2 + 2691^2 = 44741$$
$$4496960^2 + 26991^2 = 4497041$$
$$449969960^2 + 269991^2 = 449970041$$
$$44999699960^2 + 2699991^2 = 44999700041 \tag{3.54}$$

$$70755^2 + 532^2 = 70757^2$$
$$7107555^2 + 5332^2 = 7107557^2$$
$$711075555^2 + 53332^2 = 711075557^2$$
$$71110755555^2 + 533332^2 = 71110755557^2 \tag{3.55}$$

$$92415^2 + 608^2 = 92417^2$$
$$9024015^2 + 6008^2 = 9024017^2$$
$$900240015^2 + 60008^2 = 900240017^2$$
$$90002400015^2 + 600008^2 = 90002400017^2 \tag{3.56}$$

$$125500^2 + 501^2 = 125501^2$$
$$12505000^2 + 5001^2 = 12505001^2$$
$$1250050000^2 + 50001^2 = 1250050001^2$$
$$125000500000^2 + 500001^2 = 125000500001^2 \tag{3.57}$$

$$184224^2 + 607^2 = 184225^2$$
$$18042024^2 + 6007^2 = 18042025^2$$
$$1800420024^2 + 60007^2 = 1800420025^2$$
$$180004200024^2 + 600007^2 = 180004200025^2 \tag{3.58}$$

$$226440^2 + 4711^2 = 226489^2$$
$$22264440^2 + 46711^2 = 22264489^2$$
$$2222644440^2 + 466711^2 = 2222644489^2$$
$$222226444440^2 + 4666711^2 = 222226444489^2$$
$$22222264444440^2 + 46666711^2 = 22222264444489^2 \qquad (3.59)$$

$$230520^2 + 679^2 = 230521^2$$
$$22304520^2 + 6679^2 = 22304521^2$$
$$2223044520^2 + 66679^2 = 2223044521^2 \qquad (3.60)$$

$$411300^2 + 6349^2 = 411349^2$$
$$40563000^2 + 63049^2 = 40563049^2$$
$$4050630000^2 + 630049^2 = 4050630049^2$$
$$405006300000^2 + 6300049^2 = 405006300049^2$$
$$40500063000000^2 + 63000049^2 = 40500063000049^2 \qquad (3.61)$$

$$535823^2 + 1464^2 = 535825^2$$
$$53758223^2 + 14664^2 = 53758225^2$$
$$5377582223^2 + 146664^2 = 5377582225^2$$
$$537775822223^2 + 1466664^2 = 537775822225^2 \qquad (3.62)$$

$$828099^2 + 1820^2 = 828101^2$$
$$81180099^2 + 18020^2 = 81180101^2$$
$$8101800099^2 + 180020^2 = 8101800101^2$$
$$810018000099^2 + 1800020^2 = 810018000101^2$$
$$81000180000099^2 + 18000020^2 = 81000180000101^2 \qquad (3.63)$$

$$978600^2 + 1399^2 = 978601$$
$$97986000^2 + 13999^2 = 97986001$$
$$9799860000^2 + 139999^2 = 9799860001$$
$$979998600000^2 + 1399999^2 = 979998600001 \qquad (3.64)$$

$$1346440^2 + 1641^2 = 1346441^2$$
$$134496400^2 + 16401^2 = 134496401^2 \qquad (3.65)$$

$$1774103^2 + 29304^2 = 1774345^2$$
$$177742103^2 + 293304^2 = 177742345^2$$
$$17777422103^2 + 2933304^2 = 17777422345^2$$
$$1777774222103^2 + 29333304^2 = 1777774222345^2 \qquad (3.66)$$

$$1774143^2 + 23976^2 = 1774305^2$$
$$177742143^2 + 239976^2 = 177742305^2$$
$$17777422143^2 + 2399976^2 = 17777422305^2$$
$$1777774222143^2 + 23999976^2 = 1777774222305^2 \qquad (3.67)$$

$$1774175^2 + 18648^2 = 1774273^2$$
$$177742175^2 + 186648^2 = 177742273^2$$
$$17777422175^2 + 1866648^2 = 17777422273^2$$
$$1777774222175^2 + 18666648^2 = 1777774222273^2 \qquad (3.68)$$

$$1774199^2 + 13320^2 = 1774249^2$$
$$177742199^2 + 133320^2 = 177742249^2$$
$$17777422199^2 + 1333320^2 = 17777422249^2$$
$$1777774222199^2 + 13333320^2 = 1777774222249^2 \qquad (3.69)$$

$$3382600^2 + 2601^2 = 3382601^2$$
$$338026000^2 + 26001^2 = 338026001^2$$
$$33800260000^2 + 260001^2 = 33800260001^2$$
$$3380002600000^2 + 2600001^2 = 3380002600001^2 \qquad (3.70)$$

$$7096887^2 + 15984^2 = 7096905^2$$
$$710968887^2 + 159984^2 = 710968905^2$$
$$71109688887^2 + 1599984^2 = 71109688905^2$$
$$7111096888887^2 + 15999984^2 = 7111096888905^2 \qquad (3.71)$$

$$7096892^2 + 10656^2 = 7096900^2$$
$$710968892^2 + 106656^2 = 710968900^2$$
$$71109688892^2 + 1066656^2 = 71109688900^2$$
$$7111096888892^2 + 10666656^2 = 7111096888900^2 \qquad (3.72)$$

$$7096895^2 + 5328^2 = 7096897^2$$
$$710968895^2 + 53328^2 = 710968897^2$$
$$71109688895^2 + 533328^2 = 71109688897^2$$
$$7111096888895^2 + 5333328^2 = 7111096888897^2 \qquad (3.73)$$

$$70968802^2 + 21312^2 = 7096912^2$$
$$7109688802^2 + 213312^2 = 710968912^2$$
$$711096888802^2 + 2133312^2 = 71109688912^2$$
$$71110968888802^2 + 21333312^2 = 7111096888912^2$$

$$\text{(3.74)}$$

$$70968712^2 + 26640^2 = 7096921^2$$
$$7109688712^2 + 266640^2 = 710968921^2$$
$$711096888712^2 + 2666640^2 = 71109688921^2$$
$$71110968888712^2 + 26666640^2 = 7111096888921^2$$

$$\text{(3.75)}$$

$$1966144^2 + 1983^2 = 1966145^2$$
$$199660144^2 + 19983^2 = 199660145^2$$
$$19996600144^2 + 199983^2 = 19996600145^2 \qquad \text{(3.76)}$$

$$24549024^2 + 7007^2 = 24549025^2$$
$$2450490024^2 + 70007^2 = 2450490025^2$$
$$245004900024^2 + 700007^2 = 245004900025^2$$
$$24500049000024^2 + 7000007^2 = 24500049000025^2 \qquad \text{(3.77)}$$

$$1468943^2 + 2424^2 = 1468945^2$$
$$144288143^2 + 24024^2 = 144288145^2$$
$$14402880143^2 + 240024^2 = 14402880145^2$$
$$1440028800143^2 + 2400024^2 = 1440028800145^2$$
$$144000288000143^2 + 24000024^2 = 144000288000145^2 \qquad \text{(3.78)}$$

$$1602755^2 + 2532^2 = 1602757^2$$
$$160427555^2 + 25332^2 = 160427557^2$$
$$16044275555^2 + 253332^2 = 16044275557^2$$
$$1604442755555^2 + 2533332^2 = 1604442755557^2 \qquad \text{(3.79)}$$

$$1954403^2 + 2796^2 = 1954405^2$$
$$195944003^2 + 27996^2 = 195944005^2$$
$$19599440003^2 + 279996^2 = 19599440005^2$$
$$1959994400003^2 + 2799996^2 = 1959994400005^2 \qquad \text{(3.80)}$$

$$3481955^2 + 3732^2 = 3481957^2$$
$$348419555^2 + 37332^2 = 348419557^2$$
$$34844195555^2 + 373332^2 = 34844195557^2$$
$$3484441955555^2 + 3733332^2 = 3484441955557^2 \tag{3.81}$$

$$3732623^2 + 3864^2 = 3732625^2$$
$$373726223^2 + 38664^2 = 373726225^2$$
$$37377262223^2 + 386664^2 = 37377262225^2$$
$$3737772622223^2 + 3866664^2 = 3737772622225^2 \tag{3.82}$$

$$40581040^2 + 9009^2 = 40581041^2$$
$$4050810040^2 + 90009^2 = 4050810041^2$$
$$405008100040^2 + 900009^2 = 405008100041^2$$
$$40500081000040^2 + 9000009^2 = 40500081000041^2 \tag{3.83}$$

$$10445823^2 + 6464^2 = 10445825^2$$
$$1026049023^2 + 64064^2 = 1026049025^2$$
$$102420481023^2 + 640064^2 = 102420481025^2$$
$$10240204801023^2 + 6400064^2 = 10240204801025^2$$
$$1024002048001023^2 + 64000064^2 = 1024002048001025^2 \tag{3.84}$$

$$13448164000^2 + 164001^2 = 13448164001^2$$
$$1344801640000^2 + 1640001^2 = 1344801640001^2$$
$$134480016400000^2 + 16400001^2 = 134480016400001^2$$
$$13448000164000000^2 + 164000001^2 = 13448000164000001^2 \tag{3.85}$$

$$\mathbf{1}\,20^2 + 22^2 = \mathbf{1}\,22^2$$
$$\mathbf{123}\,20^2 + 222^2 = \mathbf{123}\,22^2$$
$$\mathbf{12343}\,20^2 + 2222^2 = \mathbf{12343}\,22^2$$
$$\mathbf{1234543}\,20^2 + 22222^2 = \mathbf{1234543}\,22^2$$
$$\mathbf{123456543}\,20^2 + 222222^2 = \mathbf{123456543}\,22^2$$
$$\mathbf{12345676543}\,20^2 + 2222222^2 = \mathbf{12345676543}\,22^2$$
$$\mathbf{1234567876543}\,20^2 + 22222222^2 = \mathbf{1234567876543}\,22^2$$
$$\mathbf{123456789876543}\,20^2 + 222222222^2 = \mathbf{123456789876543}\,22^2$$
$$\mathbf{12345679009876543}\,20^2 + 2222222222^2 = \mathbf{12345679009876543}\,22^2 \tag{3.86}$$

$$099^2 + 20^2 = 1\,01^2$$
$$12\,099^2 + 220^2 = 121\,01^2$$
$$1232\,099^2 + 2220^2 = 12321\,01^2$$
$$123432\,099^2 + 22220^2 = 1234321\,01^2$$
$$12345432\,099^2 + 222220^2 = 123454321\,01^2$$
$$1234565432\,099^2 + 2222220^2 = 12345654321\,01^2$$
$$123456765432\,099^2 + 22222220^2 = 1234567654321\,01^2$$
$$12345678765432\,099^2 + 222222220^2 = 123456787654321\,01^2$$
$$1234567898765432\,099^2 + 2222222220^2 = 12345678987654321\,01^2 \tag{3.87}$$

$$0099^2 + 20^2 = 1\,01^2$$
$$102\,0099^2 + 2020^2 = 10201\,01^2$$
$$1020302\,0099^2 + 202020^2 = 102030201\,01^2$$
$$10203040302\,0099^2 + 20202020^2 = 1020304030201\,01^2$$
$$102030405040302\,0099^2 + 2020202020^2 = 10203040504030201\,01^2$$
$$1020304050605040302\,0099^2 + 202020202020^2 = 10203040506050403020\,101^2$$
$$10203040506070605040302\,0099^2 + 20202020202020^2 = 102030405060706050403020\,1^2$$
$$102030405060708070605040302\,0099^2 + 2020202020202020^2 = 1020304050607080706050403020\,1^2$$
$$10203040506070809080706050403020099^2 + 202020202020202020^2 = 102030405060708090807060504030201\,01^2 \tag{3.88}$$

Extended study of Examples (3.86), (3.87) and (3.88) can be seen in author's work [?, ?]

4   Patterns in Pythagorean Triples With Final Sum

We saw that the Section 3 brings the patterns without final sums. This section bring **patterns in Pythagorean triples** and also in **final sums**. The Pythagorean triple considered are primitive. The way of construction is given in [5, 6]. We observe that the patterns in final sums starts from the second, third line, onwards. There are very few examples, where we have complete patterns with final sums.

$$3^2 + 4^2 = 5^2$$
$$143^2 + 24^2 = 145^2$$
$$10403^2 + 204^2 = 10405^2 \qquad := 108264025$$
$$1004003^2 + 2004^2 = 1004005^2 \qquad := 1008026040025$$
$$100040003^2 + 20004^2 = 100040005^2 \qquad := 10008002600400025$$
$$10000400003^2 + 200004^2 = 10000400005^2 := 100008000260004000025 \tag{4.1}$$

$$4^2 + 3^2 = 5^2$$
$$264^2 + 23^2 = 265^2$$
$$20604^2 + 203^2 = 20605^2 \qquad := 424566025$$
$$2006004^2 + 2003^2 = 2006005^2 \qquad := 4024056060025$$
$$200060004^2 + 20003^2 = 200060005^2 \qquad := 40024005600600025$$
$$20000600004^2 + 200003^2 = 20000600005^2 := 400024000560006000025 \tag{4.2}$$

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$$4^2 + 3^2 = 5^2$$
$$544^2 + 33^2 = 545^2$$
$$45904^2 + 303^2 = 45905^2$$
$$4509004^2 + 3003^2 = 4509005^2$$
$$450090004^2 + 30003^2 = 450090005^2 \qquad := 2025810126009000025$$
$$45000900004^2 + 300003^2 = 45000900005^2 \qquad := 20250810012600090000025$$
$$4500009000004^2 + 3000003^2 = 4500009000005^2 := 202500810001260000900000025 \qquad (4.3)$$

$$36^2 + 27^2 = 45^2$$
$$4896^2 + 297^2 = 4905^2 \qquad := 24059025$$
$$498996^2 + 2997^2 = 499005^2 \qquad := 249005990025$$
$$49989996^2 + 29997^2 = 49990005^2 \qquad := 2499000599900025$$
$$4999899996^2 + 299997^2 = 4999900005^2 \qquad := 24999000059999000025$$
$$499998999996^2 + 2999997^2 = 499999000005^2 := 249999000005999990000025 \qquad (4.4)$$

$$40^2 + 9^2 = 41^2$$
$$4900^2 + 99^2 = 4901^2 \qquad := 24019801$$
$$499000^2 + 999^2 = 499001^2 \qquad := 249001998001$$
$$49990000^2 + 9999^2 = 49990001^2 \qquad := 2499000199980001$$
$$4999900000^2 + 99999^2 = 4999900001^2 \qquad := 24999000019999800001$$
$$499999000000^2 + 999999^2 = 499999000001^2 := 249999000001199998000001 \qquad (4.5)$$

$$77^2 + 36^2 = 85^2$$
$$9797^2 + 396^2 = 9805^2 \qquad := 96138025$$
$$997997^2 + 3996^2 = 998005^2 \qquad := 996013980025$$
$$99979997^2 + 39996^2 = 99980005^2 \qquad := 9996001399800025$$
$$9999799997^2 + 399996^2 = 9999800005^2 \qquad := 99996000139998000025$$
$$999997999997^2 + 3999996^2 = 999998000005^2 := 999996000013999980000025 \qquad (4.6)$$

$$91^2 + 60^2 = 109^2 \qquad := 11881$$
$$9991^2 + 600^2 = 10009^2 \qquad := 100180081$$
$$999991^2 + 6000^2 = 1000009^2 \qquad := 1000018000081$$
$$99999991^2 + 60000^2 = 100000009^2 := 10000001800000081 \qquad (4.7)$$

$$99^2 + 20^2 = 101^2 \qquad := 10201$$
$$9999^2 + 200^2 = 10001^2 \qquad := 100020001$$
$$999999^2 + 2000^2 = 1000001^2 \qquad := 1000002000001$$
$$99999999^2 + 20000^2 = 100000001^2 \qquad := 10000000200000001$$
$$9999999999^2 + 200000^2 = 10000000001^2 \qquad := 100000000020000000001$$
$$999999999999^2 + 2000000^2 = 1000000000001^2 := 1000000000002000000000001 \qquad (4.8)$$

$$180^2 + 19^2 = 181^2 \qquad := 32761$$
$$19800^2 + 199^2 = 19801^2 \qquad := 392079601$$
$$1998000^2 + 1999^2 = 1998001^2 \qquad := 3992007996001$$
$$199980000^2 + 19999^2 = 199980001^2 \qquad := 39992000799960001$$
$$19999800000^2 + 199999^2 = 19999800001^2 := 399992000079999600001 \qquad (4.9)$$

$$220^2 + 21^2 = 221^2 \qquad := 48841$$
$$20200^2 + 201^2 = 20201^2 \qquad := 408080401$$
$$2002000^2 + 2001^2 = 2002001^2 \qquad := 4008008004001$$
$$200020000^2 + 20001^2 = 200020001^2 \qquad := 40008000800040001$$
$$20000200000^2 + 200001^2 = 20000200001^2 := 400008000080000400001 \qquad (4.10)$$

$$264^2 + 23^2 = 265^2$$
$$20604^2 + 203^2 = 20605^2 \qquad := 424566025$$
$$2006004^2 + 2003^2 = 2006005^2 \qquad := 4024056060025$$
$$200060004^2 + 20003^2 = 200060005^2 := 40024005600600025 \qquad (4.11)$$

$$319^2 + 360^2 = 481^2$$
$$39919^2 + 3600^2 = 40081^2 \qquad := 1606486561$$
$$3999919^2 + 36000^2 = 4000081^2 \qquad := 16000648006561$$
$$399999919^2 + 360000^2 = 400000081^2 := 160000064800006561 \qquad (4.12)$$

$$323^2 + 36^2 = 325^2$$
$$39203^2 + 396^2 = 39205^2$$
$$3992003^2 + 3996^2 = 3992005^2 \qquad := 15936103920025$$
$$399920003^2 + 39996^2 = 399920005^2 \qquad := 159936010399200025$$
$$39999200003^2 + 399996^2 = 39999200005^2 := 1599936001039992000025 \qquad (4.13)$$
$$39999200003^2 + 399996^2 = 39999200005^2 := 1599936001039992000025 \qquad (4.14)$$

$$351^2 + 280^2 = 449^2$$
$$39951^2 + 2800^2 = 40049^2 \qquad := 1603922401$$
$$3999951^2 + 28000^2 = 4000049^2 \quad := 16000392002401$$
$$399999951^2 + 280000^2 = 400000049^2 := 160000039200002401 \qquad (4.15)$$

$$391^2 + 120^2 = 409^2 \qquad := 167281$$
$$39991^2 + 1200^2 = 40009^2 \qquad := 1600720081$$
$$3999991^2 + 12000^2 = 4000009^2 \quad := 16000072000081$$
$$399999991^2 + 120000^2 = 400000009^2 := 160000007200000081 \qquad (4.16)$$

$$399^2 + 40^2 = 401^2 \qquad\qquad := 160801$$
$$39999^2 + 400^2 = 40001^2 \qquad\qquad := 1600080001$$
$$3999999^2 + 4000^2 = 4000001^2 \qquad := 16000008000001$$
$$399999999^2 + 40000^2 = 400000001^2 \qquad := 16000000800000001$$
$$39999999999^2 + 400000^2 = 40000000001^2 \quad := 1600000000080000000001$$
$$3999999999999^2 + 4000000^2 = 4000000000001^2 := 160000000000008000000000001 \qquad (4.17)$$

$$399^2 + 40^2 = 401^2$$
$$48399^2 + 440^2 = 48401^2$$
$$4080399^2 + 4040^2 = 4080401^2$$
$$400800399^2 + 40040^2 = 400800401^2$$
$$40008000399^2 + 400040^2 = 40008000401^2 \qquad := 1600640096086416160801$$
$$4000080000399^2 + 4000040^2 = 4000080000401^2 \quad := 16000640009608064160160801$$
$$400000800000399^2 + 40000040^2 = 400000800000401^2 := 1600006400009608006416000160801 \quad (4.18)$$

$$480^2 + 31^2 = 481^2$$
$$45300^2 + 301^2 = 45301^2$$
$$4503000^2 + 3001^2 = 4503001^2$$
$$450030000^2 + 30001^2 = 450030001^2 \qquad := 202527001800060001$$
$$45000300000^2 + 300001^2 = 45000300001^2 \quad := 2025027000180000600001$$
$$4500003000000^2 + 3000001^2 = 4500003000001^2 := 20250027000018000006000001 \qquad (4.19)$$

$$483^2 + 44^2 = 485^2$$
$$40803^2 + 404^2 = 40805^2$$
$$4008003^2 + 4004^2 = 4008005^2 \qquad := 16064104080025$$
$$400080003^2 + 40004^2 = 400080005^2 \quad := 160064010400800025$$
$$40000800003^2 + 400004^2 = 40000800005^2 := 1600064001040008000025 \qquad (4.20)$$

$$612^2 + 10395^2 = 10413^2$$
$$6012^2 + 1003995^2 = 1004013^2 \qquad := 1008042104169$$
$$60012^2 + 100039995^2 = 100040013^2 \qquad := 10008004201040169$$
$$600012^2 + 10000399995^2 = 10000400013^2 \qquad := 100008000420010400169$$
$$6000012^2 + 1000003999995^2 = 1000004000013^2 \qquad := 1000008000042000104000169$$
$$60000012^2 + 100000039999995^2 = 100000040000013^2 := 10000008000004200001040000169 \qquad (4.21)$$

$$760^2 + 39^2 = 761^2$$
$$79600^2 + 399^2 = 79601^2 \qquad := 6336319201$$
$$7996000^2 + 3999^2 = 7996001^2 \qquad := 63936031992001$$
$$799960000^2 + 39999^2 = 799960001^2 \qquad := 639936003199920001$$
$$79999600000^2 + 399999^2 = 79999600001^2 \qquad := 6399993600031999920000001$$
$$7999996000000^2 + 3999999^2 = 7999996000001^2 := 63999993600000319999920000001 \qquad (4.22)$$

$$840^2 + 41^2 = 841^2$$
$$80400^2 + 401^2 = 80401^2 \qquad := 6464320801$$
$$8004000^2 + 4001^2 = 8004001^2 \qquad := 64064032008001$$
$$800040000^2 + 40001^2 = 800040001^2 \qquad := 640064003200080001$$
$$80000400000^2 + 400001^2 = 80000400001^2 := 6400064000320000800001 \qquad (4.23)$$

$$899^2 + 60^2 = 901^2 \qquad := 811801$$
$$89999^2 + 600^2 = 90001^2 \qquad := 8100180001$$
$$8999999^2 + 6000^2 = 9000001^2 \qquad := 81000018000001$$
$$899999999^2 + 60000^2 = 900000001^2 := 810000001800000001 \qquad (4.24)$$

$$1015^2 + 20592^2 = 20617^2$$
$$10015^2 + 2005992^2 = 2006017^2 \qquad := 4024104204289$$
$$100015^2 + 200059992^2 = 200060017^2 \qquad := 40024010402040289$$
$$1000015^2 + 20000599992^2 = 20000600017^2 \qquad := 400024001040020400289$$
$$10000015^2 + 2000005999992^2 = 2000006000017^2 \qquad := 4000024000104000204000289$$
$$100000015^2 + 200000059999992^2 = 200000060000017^2 := 40000024000010400002040000289 \qquad (4.25)$$

$$1599^2 + 80^2 = 1601^2 \qquad := 2563201$$
$$159999^2 + 800^2 = 160001^2 \qquad := 25600320001$$
$$15999999^2 + 8000^2 = 16000001^2 \qquad := 256000032000001$$
$$1599999999^2 + 80000^2 = 1600000001^2 \qquad := 2560000003200000001$$
$$159999999999^2 + 800000^2 = 160000000001^2 := 25600000000320000000001 \qquad (4.26)$$

$$1740^2 + 59^2 = 1741^2$$
$$179400^2 + 599^2 = 179401^2 \qquad := 32184718801$$
$$17994000^2 + 5999^2 = 17994001^2 \qquad := 323784071988001$$
$$1799940000^2 + 59999^2 = 1799940001^2 \quad := 3239784007199880001$$
$$179999400000^2 + 599999^2 = 179999400001^2 := 32399784000719998800001 \qquad (4.27)$$

$$1860^2 + 61^2 = 1861^2$$
$$180600^2 + 601^2 = 180601^2$$
$$18006000^2 + 6001^2 = 18006001^2 \qquad := 324216072012001$$
$$1800060000^2 + 60001^2 = 1800060001^2 \quad := 3240216007200120001$$
$$180000600000^2 + 600001^2 = 180000600001^2 := 32400216000720001200001 \qquad (4.28)$$

$$1863^2 + 21384^2 = 21465^2$$
$$18063^2 + 2013984^2 = 2014065^2$$
$$180063^2 + 200139984^2 = 200140065^2 \qquad := 40056045618204225$$
$$1800063^2 + 20001399984^2 = 20001400065^2 \qquad := 4000560045601820004225$$
$$18000063^2 + 2000013999984^2 = 2000014000065^2 \quad := 400005600045600182000042255$$
$$180000063^2 + 200000139999984^2 = 200000140000065^2 := 40000056000045600018200004225 \quad (4.29)$$

$$2484^2 + 400^2 = 2516^2$$
$$249984^2 + 4000^2 = 250016^2 \qquad := 62508000256$$
$$24999984^2 + 40000^2 = 25000016^2 \quad := 625000800000256$$
$$2499999984^2 + 400000^2 = 2500000016^2 := 6250000080000000256 \qquad (4.30)$$

$$2491^2 + 300^2 = 2509^2$$
$$249991^2 + 3000^2 = 250009^2 \qquad := 62504500081$$
$$24999991^2 + 30000^2 = 25000009^2 \quad := 625000450000081$$
$$2499999991^2 + 300000^2 = 2500000009^2 := 6250000045000000081 \qquad (4.31)$$

$$2499^2 + 100^2 = 2501^2 \qquad := 6255001$$
$$249999^2 + 1000^2 = 250001^2 \qquad := 62500500001$$
$$24999999^2 + 10000^2 = 25000001^2 \quad := 625000050000001$$
$$2499999999^2 + 100000^2 = 2500000001^2 := 6250000005000000001 \qquad (4.32)$$

$$3280^2 + 81^2 = 3281^2$$
$$320800^2 + 801^2 = 320801^2$$
$$32008000^2 + 8001^2 = 32008001^2 \qquad := 1024512128016001$$
$$3200080000^2 + 80001^2 = 3200080001^2 \quad := 10240512012800160001$$
$$320000800000^2 + 800001^2 = 320000800001^2 := 102400512001280001600001 \qquad (4.33)$$

$$3591^2 + 360^2 = 3609^2$$
$$359991^2 + 3600^2 = 360009^2 \quad := 129606480081$$
$$35999991^2 + 36000^2 = 36000009^2 \quad := 1296000648000081$$
$$3599999991^2 + 360000^2 = 3600000009^2 := 12960000064800000081 \qquad (4.34)$$

$$3599^2 + 120^2 = 3601^2 \quad := 12967201$$
$$359999^2 + 1200^2 = 360001^2 \quad := 129600720001$$
$$35999999^2 + 12000^2 = 36000001^2 \quad := 1296000072000001$$
$$3599999999^2 + 120000^2 = 3600000001^2 := 12960000007200000001 \qquad (4.35)$$

$$4860^2 + 891^2 = 4941^2$$
$$498960^2 + 8991^2 = 499041^2 \quad := 249041919681$$
$$49989960^2 + 89991^2 = 49990041^2 \quad := 2499004199181681$$
$$4999899960^2 + 899991^2 = 4999900041^2 \quad := 24999000419991801681$$
$$499998999960^2 + 8999991^2 = 499999000041^2 := 249999000041999918001681 \qquad (4.36)$$

$$4876^2 + 693^2 = 4925^2$$
$$498976^2 + 6993^2 = 499025^2 \quad := 249025950625$$
$$49989976^2 + 69993^2 = 49990025^2 \quad := 2499002599500625$$
$$4999899976^2 + 699993^2 = 4999900025^2 \quad := 24999000259995000625$$
$$499998999976^2 + 6999993^2 = 499999000025^2 := 249999000025999950000625 \qquad (4.37)$$

$$4888^2 + 495^2 = 4913^2 \quad := 24137569$$
$$498988^2 + 4995^2 = 499013^2 \quad := 249013974169$$
$$49989988^2 + 49995^2 = 49990013^2 \quad := 2499001399740169$$
$$4999899988^2 + 499995^2 = 4999900013^2 \quad := 24999000139997400169$$
$$499998999988^2 + 4999995^2 = 499999000013^2 := 249999000013999974000169 \qquad (4.38)$$

$$4896^2 + 297^2 = 4905^2 \quad := 24059025$$
$$498996^2 + 2997^2 = 499005^2 \quad := 249005990025$$
$$49989996^2 + 29997^2 = 49990005^2 \quad := 2499000599900025$$
$$4999899996^2 + 299997^2 = 4999900005^2 \quad := 24999000059999000025$$
$$499998999996^2 + 2999997^2 = 499999000005^2 := 249999000005999990000025 \qquad (4.39)$$

$$4899^2 + 140^2 = 4901^2 \quad := 24019801$$
$$489999^2 + 1400^2 = 490001^2 \quad := 240100980001$$
$$48999999^2 + 14000^2 = 49000001^2 \quad := 2401000098000001$$
$$4899999999^2 + 140000^2 = 4900000001^2 := 24010000009800000001 \qquad (4.40)$$

$$5100^2 + 101^2 = 5101^2$$
$$501000^2 + 1001^2 = 501001^2 \qquad := 251002002001$$
$$50010000^2 + 10001^2 = 50010001^2 \qquad := 2501000200020001$$
$$5000100000^2 + 100001^2 = 5000100001^2 \qquad := 25001000020000200001$$
$$500001000000^2 + 1000001^2 = 500001000001^2 := 250001000002000002000001 \qquad (4.41)$$

$$6399^2 + 160^2 = 6401^2$$
$$639999^2 + 1600^2 = 640001^2 \qquad := 409601280001$$
$$63999999^2 + 16000^2 = 64000001^2 \qquad := 4096000128000001$$
$$6399999999^2 + 160000^2 = 6400000001^2 \qquad := 40960000012800000001$$
$$639999999999^2 + 1600000^2 = 640000000001^2 := 409600000000128000000000001 \qquad (4.42)$$

$$8075^2 + 900^2 = 8125^2$$
$$809975^2 + 9000^2 = 810025^2 \qquad := 656140500625$$
$$80999975^2 + 90000^2 = 81000025^2 \qquad := 6561004050000625$$
$$8099999975^2 + 900000^2 = 8100000025^2 \qquad := 65610000405000000625$$
$$809999999975^2 + 9000000^2 = 810000000025^2 := 656100000040500000000625 \qquad (4.43)$$

$$8091^2 + 540^2 = 8109^2$$
$$809991^2 + 5400^2 = 810009^2 \qquad := 656114580081$$
$$80999991^2 + 54000^2 = 81000009^2 \qquad := 6561001458000081$$
$$8099999991^2 + 540000^2 = 8100000009^2 := 65610000145800000081 \qquad (4.44)$$

$$8099^2 + 180^2 = 8101^2$$
$$809999^2 + 1800^2 = 810001^2 \qquad := 656101620001$$
$$80999999^2 + 18000^2 = 81000001^2 \qquad := 6561000162000001$$
$$8099999999^2 + 180000^2 = 8100000001^2 := 65610000016200000001 \qquad (4.45)$$

$$9737^2 + 1584^2 = 9865^2$$
$$997937^2 + 15984^2 = 998065^2 \qquad := 996133744225$$
$$99979937^2 + 159984^2 = 99980065^2 \qquad := 9996013397404225$$
$$9999799937^2 + 1599984^2 = 9999800065^2 \qquad := 99996001339974004225$$
$$999997999937^2 + 15999984^2 = 999998000065^2 := 999996000133999740004225 \qquad (4.46)$$

$$9765^2 + 1188^2 = 9837^2$$
$$997965^2 + 11988^2 = 998037^2 \qquad := 996077853369$$
$$99979965^2 + 119988^2 = 99980037^2 \qquad := 9996007798521369$$
$$9999799965^2 + 1199988^2 = 9999800037^2 \qquad := 99996000779985201369$$
$$999997999965^2 + 11999988^2 = 999998000037^2 := 999996000077999852001369 \qquad (4.47)$$

$$9785^2 + 792^2 = 9817^2$$
$$997985^2 + 7992^2 = 998017^2 \qquad := 996037932289$$
$$99979985^2 + 79992^2 = 99980017^2 \qquad := 9996003799320289$$
$$9999799985^2 + 799992^2 = 9999800017^2 \qquad := 99996000379993200289$$
$$999997999985^2 + 7999992^2 = 999998000017^2 := 999996000037999932000289 \qquad (4.48)$$

$$10593^2 + 824^2 = 10625^2$$
$$1005993^2 + 8024^2 = 1006025^2 \qquad := 1012086300625$$
$$100059993^2 + 80024^2 = 100060025^2 \qquad := 10012008603000625$$
$$10000599993^2 + 800024^2 = 10000600025^2 \qquad := 100012000860030000625$$
$$1000005999993^2 + 8000024^2 = 1000006000025^2 \qquad := 1000012000086000300000625$$
$$100000059999993^2 + 80000024^2 = 100000060000025^2 := 10000012000008600003000000625 \qquad (4.49)$$

$$10791^2 + 1040^2 = 10841^2$$
$$1007991^2 + 10040^2 = 1008041^2 \qquad := 1016146657681$$
$$100079991^2 + 100040^2 = 100080041^2 \qquad := 10016014606561681$$
$$10000799991^2 + 1000040^2 = 10000800041^2 \qquad := 100016001460065601681$$
$$1000007999991^2 + 10000040^2 = 1000008000041^2 \qquad := 1000016000146000656001681$$
$$100000079999991^2 + 100000040^2 = 100000080000041^2 := 10000016000014600006560001681$$
$$(4.50)$$

$$11583^2 + 1944^2 = 11745^2$$
$$1015983^2 + 18144^2 = 1016145^2 \qquad := 1032550661025$$
$$100159983^2 + 180144^2 = 100160145^2 \qquad := 10032054646421025$$
$$10001599983^2 + 1800144^2 = 10001600145^2 \qquad := 100032005460464021025$$
$$1000015999983^2 + 18000144^2 = 1000016000145^2 \qquad := 1000032000546004640021025$$
$$100000159999983^2 + 180000144^2 = 100000160000145^2 := 10000032000054600046400021025 \qquad (4.51)$$

$$14399^2 + 240^2 = 14401^2$$
$$1439999^2 + 2400^2 = 1440001^2 \qquad := 2073602880001$$
$$143999999^2 + 24000^2 = 144000001^2 \qquad := 20736000288000001$$
$$14399999999^2 + 240000^2 = 14400000001^2 \qquad := 207360000028800000001$$
$$1439999999999^2 + 2400000^2 = 1440000000001^2 := 2073600000002880000000001 \qquad (4.52)$$

$$18624^2 + 193^2 = 18625^2$$
$$1986024^2 + 1993^2 = 1986025^2 \qquad := 3944295300625$$
$$199860024^2 + 19993^2 = 199860025^2 \qquad := 39944029593000625$$
$$19998600024^2 + 199993^2 = 19998600025^2 := 399944002959930000625 \qquad (4.53)$$

$$19599^2 + 280^2 = 19601^2$$
$$1959999^2 + 2800^2 = 1960001^2 \qquad := 3841603920001$$
$$195999999^2 + 28000^2 = 196000001^2 \quad := 38416000392000001$$
$$19599999999^2 + 280000^2 = 19600000001^2 := 38416000039200000001 \qquad (4.54)$$

$$22475^2 + 1500^2 = 22525^2$$
$$2249975^2 + 15000^2 = 2250025^2$$
$$224999975^2 + 150000^2 = 225000025^2 \qquad := 50625011250000625$$
$$22499999975^2 + 1500000^2 = 22500000025^2 \quad := 50625000112500000625$$
$$2249999999975^2 + 15000000^2 = 2250000000025^2 := 506250000001125000000625 \qquad (4.55)$$

$$22491^2 + 900^2 = 22509^2$$
$$2249991^2 + 9000^2 = 2250009^2 \qquad := 5062540500081$$
$$224999991^2 + 90000^2 = 225000009^2 \qquad := 50625004050000081$$
$$22499999991^2 + 900000^2 = 22500000009^2 \quad := 506250000405000000081$$
$$2249999999991^2 + 9000000^2 = 2250000000009^2 := 50625000000040500000000081 \qquad (4.56)$$

$$22499^2 + 300^2 = 22501^2$$
$$2249999^2 + 3000^2 = 2250001^2 \qquad := 5062504500001$$
$$224999999^2 + 30000^2 = 225000001^2 \quad := 50625000450000001$$
$$22499999999^2 + 300000^2 = 22500000001^2 := 506250000045000000001 \qquad (4.57)$$

$$25599^2 + 320^2 = 25601^2$$
$$2559999^2 + 3200^2 = 2560001^2 \qquad := 6553605120001$$
$$255999999^2 + 32000^2 = 256000001^2 \qquad := 65536000512000001$$
$$25599999999^2 + 320000^2 = 25600000001^2 \quad := 655360000051200000001$$
$$2559999999999^2 + 3200000^2 = 2560000000001^2 := 65536000000051200000000001 \qquad (4.58)$$

$$32399^2 + 360^2 = 32401^2$$
$$3239999^2 + 3600^2 = 3240001^2 \qquad := 10497606480001$$
$$323999999^2 + 36000^2 = 324000001^2 \quad := 104976000648000001$$
$$32399999999^2 + 360000^2 = 32400000001^2 := 1049760000064800000001 \qquad (4.59)$$

$$39179^2 + 1980^2 = 39229^2$$
$$3991979^2 + 19980^2 = 3992029^2 \qquad := 15936295536841$$
$$399919979^2 + 199980^2 = 399920029^2 \qquad := 159936029595360841$$
$$39999199979^2 + 1999980^2 = 39999200029^2 \quad := 1599936002959953600841$$
$$3999991999979^2 + 19999980^2 = 3999992000029^2 := 15999936000295999536000841 \qquad (4.60)$$

$$39155^2 + 2772^2 = 39253^2$$
$$3991955^2 + 27972^2 = 3992053^2$$
$$399919955^2 + 279972^2 = 399920053^2 \quad := 159936048791522809$$
$$39999199955^2 + 2799972^2 = 39999200053^2 \quad := 15999360048799915202809$$
$$3999991999955^2 + 27999972^2 = 3999992000053^2 := 1599993600048799915202809 \qquad (4.61)$$

$$39195^2 + 1188^2 = 39213^2$$
$$3991995^2 + 11988^2 = 3992013^2 \quad := 15936167792169$$
$$399919995^2 + 119988^2 = 399920013^2 \quad := 159936016797920169$$
$$39999199995^2 + 1199988^2 = 39999200013^2 \quad := 1599936001679979200169$$
$$3999991999995^2 + 11999988^2 = 3999992000013^2 := 15999936000167999792000169 \qquad (4.62)$$

$$44700^2 + 299^2 = 44701^2$$
$$4497000^2 + 2999^2 = 4497001^2$$
$$449970000^2 + 29999^2 = 449970001^2 \quad := 202473001799940001$$
$$44999700000^2 + 299999^2 = 44999700001^2 := 2024973000179999400001 \qquad (4.63)$$

$$45300^2 + 301^2 = 45301^2$$
$$4503000^2 + 3001^2 = 4503001^2$$
$$450030000^2 + 30001^2 = 450030001^2 \quad := 202527001800060001$$
$$45000300000^2 + 300001^2 = 45000300001^2 := 2025027000180000600001 \qquad (4.64)$$

$$57575^2 + 2400^2 = 57625^2$$
$$5759975^2 + 24000^2 = 5760025^2$$
$$575999975^2 + 240000^2 = 576000025^2 \quad := 331776028800000625$$
$$57599999975^2 + 2400000^2 = 57600000025^2 \quad := 3317760002880000000625$$
$$5759999999975^2 + 24000000^2 = 5760000000025^2 := 33177600000288000000000625 \qquad (4.65)$$

$$57584^2 + 1920^2 = 57616^2$$
$$5759984^2 + 19200^2 = 5760016^2$$
$$575999984^2 + 192000^2 = 576000016^2 \quad := 331776018432000256$$
$$57599999984^2 + 1920000^2 = 57600000016^2 \quad := 3317760001843200000256$$
$$5759999999984^2 + 19200000^2 = 5760000000016^2 := 33177600000184320000000256 \qquad (4.66)$$

$$57591^2 + 1440^2 = 57609^2$$
$$5759991^2 + 14400^2 = 5760009^2 \quad := 33177703680081$$
$$575999991^2 + 144000^2 = 576000009^2 \quad := 331776010368000081$$
$$57599999991^2 + 1440000^2 = 57600000009^2 \quad := 3317760001036800000081$$
$$5759999999991^2 + 14400000^2 = 5760000000009^2 := 33177600000103680000000081 \qquad (4.67)$$

$$57596^2 + 960^2 = 57604^2$$
$$5759996^2 + 9600^2 = 5760004^2 \qquad := 33177646080016$$
$$575999996^2 + 96000^2 = 576000004^2 \qquad := 331776004608000016$$
$$57599999996^2 + 960000^2 = 57600000004^2 \quad := 3317760000460800000016$$
$$5759999999996^2 + 9600000^2 = 5760000000004^2 := 33177600000046080000000016 \qquad (4.68)$$

$$57599^2 + 480^2 = 57601^2$$
$$5759999^2 + 4800^2 = 5760001^2 \qquad := 33177611520001$$
$$575999999^2 + 48000^2 = 576000001^2 \qquad := 331776001152000001$$
$$57599999999^2 + 480000^2 = 57600000001^2 \quad := 3317760000115200000001$$
$$5759999999999^2 + 4800000^2 = 5760000000001^2 := 33177600000011520000000001 \qquad (4.69)$$

$$78399^2 + 560^2 = 78401^2$$
$$7839999^2 + 5600^2 = 7840001^2 \qquad := 61465615680001$$
$$783999999^2 + 56000^2 = 784000001^2 \qquad := 614656001568000001$$
$$78399999999^2 + 560000^2 = 78400000001^2 \quad := 6146560000156800000001$$
$$7839999999999^2 + 5600000^2 = 7840000000001^2 := 61465600000015680000000001 \qquad (4.70)$$

$$78396^2 + 1120^2 = 78404^2$$
$$7839996^2 + 11200^2 = 7840004^2 \qquad := 61465662720016$$
$$783999996^2 + 112000^2 = 784000004^2 \qquad := 614656006272000016$$
$$78399999996^2 + 1120000^2 = 78400000004^2 \quad := 6146560000627200000016$$
$$7839999999996^2 + 11200000^2 = 7840000000004^2 := 61465600000062720000000016 \qquad (4.71)$$

$$78391^2 + 1680^2 = 78409^2$$
$$7839991^2 + 16800^2 = 7840009^2$$
$$783999991^2 + 168000^2 = 784000009^2 \qquad := 614656014112000081$$
$$78399999991^2 + 1680000^2 = 78400000009^2 \quad := 6146560001411200000081$$
$$7839999999991^2 + 16800000^2 = 7840000000009^2 := 61465600000141120000000081 \qquad (4.72)$$

$$78375^2 + 2800^2 = 78425^2$$
$$7839975^2 + 28000^2 = 7840025^2$$
$$783999975^2 + 280000^2 = 784000025^2 \qquad := 614656039200000625$$
$$78399999975^2 + 2800000^2 = 78400000025^2 \quad := 6146560003920000000625$$
$$7839999999975^2 + 28000000^2 = 7840000000025^2 := 61465600000392000000000625 \qquad (4.73)$$

$$62491^2 + 1500^2 = 62509^2$$
$$6249991^2 + 15000^2 = 6250009^2$$
$$624999991^2 + 150000^2 = 625000009^2 \qquad := 390625011250000081$$
$$62499999991^2 + 1500000^2 = 62500000009^2 \qquad := 3906250001125000000081$$
$$6249999999991^2 + 15000000^2 = 6250000000009^2 := 390625000000112500000000081 \qquad (4.74)$$

$$78375^2 + 2800^2 = 78425^2$$
$$7839975^2 + 28000^2 = 7840025^2$$
$$783999975^2 + 280000^2 = 784000025^2 \quad := 614656039200000625$$
$$78399999975^2 + 2800000^2 = 78400000025^2 := 6146560003920000000625 \qquad (4.75)$$

$$78399^2 + 560^2 = 78401^2$$
$$7839999^2 + 5600^2 = 7840001^2 \qquad := 61465615680001$$
$$783999999^2 + 56000^2 = 784000001^2 \qquad := 614656001568000001$$
$$78399999999^2 + 560000^2 = 78400000001^2 \quad := 6146560000156800000001$$
$$7839999999999^2 + 5600000^2 = 7840000000001^2 := 614656000000015680000000001 \qquad (4.76)$$

$$79600^2 + 399^2 = 79601^2 \qquad := 6336319201$$
$$7996000^2 + 3999^2 = 7996001^2 \qquad := 63936031992001$$
$$799960000^2 + 39999^2 = 799960001^2 \quad := 6399360031999200001$$
$$79999600000^2 + 399999 = 79999600001^2 := 6399936000319999200001 \qquad (4.77)$$

$$81204^2 + 403^2 = 81205^2$$
$$8012004^2 + 4003^2 = 8012005^2 \qquad := 64192224120025$$
$$800120004^2 + 40003^2 = 800120005^2 \qquad := 640192022401200025$$
$$80001200004^2 + 400003^2 = 80001200005^2 := 6400192002240012000025 \qquad (4.78)$$

$$82012^2 + 405^2 = 82013^2$$
$$8020012^2 + 4005^2 = 8020013^2 \qquad := 64320608520169$$
$$800200012^2 + 40005^2 = 800200013^2 \quad := 640320060805200169$$
$$80002000012^2 + 400005^2 = 80002000013^2 := 6400320006080052000169 \qquad (4.79)$$

$$94839^2 + 3080^2 = 94889^2$$
$$9048039^2 + 30080^2 = 9048089^2$$
$$900480039^2 + 300080^2 = 900480089^2 \qquad := 810864390685447921$$
$$90004800039^2 + 3000080^2 = 90004800089^2 \qquad := 8100864039060854407921$$
$$9000048000039^2 + 30000080^2 = 9000048000089^2 := 81000864003906008544007921 \qquad (4.80)$$

$$102399^2 + 640^2 = 102401^2$$
$$10239999^2 + 6400^2 = 10240001^2 \qquad := 104857620480001$$
$$1023999999^2 + 64000^2 = 1024000001^2 \qquad := 1048576002048000001$$
$$102399999999^2 + 640000^2 = 102400000001^2 \quad := 10485760000204800000001$$
$$10239999999999^2 + 6400000^2 = 10240000000001^2 := 1048576000000204800000000001 \qquad (4.81)$$

$$122499^2 + 700^2 = 122501^2$$
$$12249999^2 + 7000^2 = 12250001^2 \qquad := 150062524500001$$
$$1224999999^2 + 70000^2 = 1225000001^2 \qquad := 1500625002450000001$$
$$122499999999^2 + 700000^2 = 122500000001^2 \quad := 15006250000245000000001$$
$$12249999999999^2 + 7000000^2 = 12250000000001^2 := 1500625000000245000000000001 \qquad (4.82)$$

$$126500^2 + 1509^2 = 126509^2$$
$$12515000^2 + 15009^2 = 12515009^2$$
$$1250150000^2 + 150009^2 = 1250150009^2$$
$$125001500000^2 + 1500009^2 = 125001500009^2 \quad := 15625375004500027000081$$
$$12500015000000^2 + 15000009^2 = 12500015000009^2 := 156250375000450000270000081 \qquad (4.83)$$

$$129575^2 + 3600^2 = 129625^2$$
$$12959975^2 + 36000^2 = 12960025^2$$
$$1295999975^2 + 360000^2 = 1296000025^2 \qquad := 1679616064800000625$$
$$129599999975^2 + 3600000^2 = 129600000025^2 \quad := 16796160006480000000625$$
$$12959999999975^2 + 36000000^2 = 12960000000025^2 := 1679616000000648000000000625 \qquad (4.84)$$

$$129599^2 + 720^2 = 129601^2$$
$$12959999^2 + 7200^2 = 12960001^2 \qquad := 167961625920001$$
$$1295999999^2 + 72000^2 = 1296000001^2 \qquad := 1679616002592000001$$
$$129599999999^2 + 720000^2 = 129600000001^2 \quad := 16796160000259200000001$$
$$12959999999999^2 + 7200000^2 = 12960000000001^2 := 1679616000000259200000000001 \qquad (4.85)$$

$$156791^2 + 3960^2 = 156841^2$$
$$15967991^2 + 39960^2 = 15968041^2$$
$$1599679991^2 + 399960^2 = 1599680041^2 \qquad := 2558976233573761681$$
$$159996799991^2 + 3999960^2 = 159996800041^2 \qquad := 25598976023359737601681$$
$$15999967999991^2 + 39999960^2 = 15999968000041^2 := 255998976002335997376001681 \qquad (4.86)$$

$$156807^2 + 2376^2 = 156825^2$$
$$15968007^2 + 23976^2 = 15968025^2$$
$$1599680007^2 + 239976^2 = 1599680025^2 \qquad := 2558976182384000625$$
$$159996800007^2 + 2399976^2 = 159996800025^2 \qquad := 25598976018239840000625$$
$$15999968000007^2 + 23999976^2 = 15999968000025^2 := 255999897600182399840000625 \qquad (4.87)$$

$$156815^2 + 792^2 = 156817^2$$
$$15968015^2 + 7992^2 = 15968017^2$$
$$1599680015^2 + 79992^2 = 1599680017^2 \qquad := 2558976156789120289$$
$$159996800015^2 + 799992^2 = 159996800017^2 \qquad := 25598976015679891200289$$
$$15999968000015^2 + 7999992^2 = 15999968000017^2 := 255999897600156799891200289 \qquad (4.88)$$

$$163215^2 + 808^2 = 163217^2$$
$$16032015^2 + 8008^2 = 16032017^2$$
$$1600320015^2 + 80008^2 = 1600320017^2 \qquad := 2561024156810880289$$
$$160003200015^2 + 800008^2 = 160003200017^2 \qquad := 25600102401568010880028 9$$
$$16000032000015^2 + 8000008^2 = 16000032000017^2 := 256001024001568001088000289 \qquad (4.89)$$

$$181804^2 + 603^2 = 181805^2$$
$$18018004^2 + 6003^2 = 18018005^2 \qquad := 324648504180025$$
$$1800180004^2 + 60003^2 = 1800180005^2 \quad := 3240648050401800025$$
$$180001800004^2 + 600003^2 = 180001800005^2 := 32400648005040018000025 \qquad (4.90)$$

$$183012^2 + 605^2 = 183013^2$$
$$18030012^2 + 6005^2 = 18030013^2$$
$$1800300012^2 + 60005^2 = 1800300013^2 \qquad := 3241080136807800169$$
$$180003000012^2 + 600005^2 = 180003000013^2 \quad := 32401080013680078000169$$
$$18000030000012^2 + 6000005^2 = 18000030000013^2 := 324001080001368000780000169 \qquad (4.91)$$

$$184200^2 + 4249^2 = 184249^2$$
$$18042000^2 + 42049^2 = 18042049^2$$
$$1800420000^2 + 420049^2 = 1800420049^2$$
$$180004200000^2 + 4200049^2 = 180004200049^2 \qquad := 32401512035280411602401$$
$$18000042000000^2 + 42000049^2 = 18000042000049^2 := 324001512003528004116002401 \qquad (4.92)$$

$$202499^2 + 900^2 = 202501^2$$
$$20249999^2 + 9000^2 = 20250001^2 \qquad := 410062540500001$$
$$2024999999^2 + 90000^2 = 2025000001^2 \qquad := 4100625004050000001$$
$$202499999999^2 + 900000^2 = 202500000001^2 \qquad := 41006250000405000000001$$
$$20249999999999^2 + 9000000^2 = 20250000000001^2 := 410062500000040500000000001 \qquad (4.93)$$

$$308024^2 + 1110^2 = 308026^2$$
$$30802499^2 + 11100^2 = 30802501^2$$
$$3080249999^2 + 111000^2 = 3080250001^2$$
$$308024999999^2 + 1110000^2 = 308025000001^2 \quad := 94879400625616050000001$$
$$30802499999999^2 + 11100000^2 = 30802500000001^2 := 9487940062500616050000000001 \qquad (4.94)$$

$$322400^2 + 2409^2 = 322409^2$$
$$32024000^2 + 24009^2 = 32024009^2$$
$$3200240000^2 + 240009^2 = 3200240009^2 \quad := 10241536115204320081$$
$$320002400000^2 + 2400009^2 = 320002400009^2 \quad := 102401536011520043200081$$
$$32000024000000^2 + 24000009^2 = 32000024000009^2 := 1024001536001152000432000081 \qquad (4.95)$$

$$367235^2 + 1212^2 = 367237^2$$
$$36072035^2 + 12012^2 = 36072037^2$$
$$3600720035^2 + 120012^2 = 3600720037^2 \quad := 12965184784853281369$$
$$360007200035^2 + 1200012^2 = 360007200037^2 := 129605184078480532801369 \qquad (4.96)$$

$$547591^2 + 4440^2 = 547609^2$$
$$54759991^2 + 44400^2 = 54760009^2$$
$$5475999991^2 + 444000^2 = 5476000009^2 \quad := 29986576098568000081$$
$$547599999991^2 + 4440000^2 = 547600000009^2 \quad := 299865760009856800000081$$
$$54759999999991^2 + 44400000^2 = 54760000000009^2 := 2998657600000985680000000081 \qquad (4.97)$$

$$562464^2 + 9000^2 = 562536^2$$
$$56249964^2 + 90000^2 = 56250036^2$$
$$5624999964^2 + 900000^2 = 5625000036^2 \quad := 31640625405000001296$$
$$562499999964^2 + 9000000^2 = 562500000036^2 \quad := 316406250040500000001296$$
$$56249999999964^2 + 90000000^2 = 56250000000036^2 := 3164062500004050000000001296 \qquad (4.98)$$

$$562499^2 + 1500^2 = 562501^2$$
$$56249999^2 + 15000^2 = 56250001^2 \quad := 3164062612500001$$
$$5624999999^2 + 150000^2 = 5625000001^2 \quad := 31640625011250000001$$
$$562499999999^2 + 1500000^2 = 562500000001^2 \quad := 316406250001125000000001$$
$$56249999999999^2 + 15000000^2 = 56250000000001^2 := 3164062500000112500000000001 \qquad (4.99)$$

$$652863^2 + 1616^2 = 652865^2$$
$$64128063^2 + 16016^2 = 64128065^2$$
$$6401280063^2 + 160016^2 = 6401280065^2$$
$$640012800063^2 + 1600016^2 = 640012800065^2 \quad := 409616384247041664004225$$
$$64000128000063^2 + 16000016^2 = 64000128000065^2 := 4096016384024704016640004225 \qquad (4.100)$$

$$721200^2 + 1201^2 = 721201^2$$
$$72012000^2 + 12001^2 = 72012001^2$$
$$7200120000^2 + 120001^2 = 7200120001^2 \quad := 51841728028800240001$$
$$720001200000^2 + 1200001^2 = 720001200001^2 \quad := 518401728002880002400001$$
$$72000012000000^2 + 12000001^2 = 72000012000001^2 := 5184001728000288000024000001 \qquad (4.101)$$

$$981400^2 + 1401^2 = 981401^2$$
$$98014000^2 + 14001^2 = 98014001^2$$
$$9800140000^2 + 140001^2 = 9800140001^2 \quad := 96042744039200280001$$
$$980001400000^2 + 1400001^2 = 980001400001^2 \quad := 960402744003920002800001$$
$$98000014000000^2 + 14000001^2 = 98000014000001^2 := 9604002744000392000028000001 \qquad (4.102)$$

$$1281600^2 + 1601^2 = 1281601^2$$
$$128016000^2 + 16001^2 = 128016001^2$$
$$12800160000^2 + 160001^2 = 12800160001^2 \quad := 163844096051200320001$$
$$1280001600000^2 + 1600001^2 = 1280001600001^2 \quad := 1638404096005120003200001$$
$$128000016000000^2 + 16000001^2 = 128000016000001^2 := 16384004096000512000032000001 \qquad (4.103)$$

$$1562499^2 + 2500^2 \quad = 1562501^2$$
$$156249999^2 + 25000^2 \quad = 156250001^2$$
$$15624999999^2 + 250000^2 \quad = 15625000001^2 \quad := 244140625031250000001$$
$$1562499999999^2 + 2500000^2 = 1562500000001^2 := 2441406250003125000000001 \qquad (4.104)$$

$$1621800^2 + 1801^2 = 1621801^2$$
$$162018000^2 + 18001^2 = 162018001^2$$
$$16200180000^2 + 180001^2 = 16200180001^2 \quad := 262445832064800360001$$
$$1620001800000^2 + 1800001^2 = 1620001800001^2 \quad := 2624405832006480003600001$$
$$162000018000000^2 + 18000001^2 = 162000018000001^2 := 26244005832000648000036000001 \qquad (4.105)$$

$$12525012^2 + 5005^2 = 12525013^2$$
$$1250250012^2 + 50005^2 = 1250250013^2$$
$$125002500012^2 + 500005^2 = 125002500013^2 \quad := 15625625009500065000169$$
$$12500025000012^2 + 5000005^2 = 12500025000013^2 := 156250625000950000650000169 \qquad (4.106)$$

$$64128063^2 + 16016^2 = 64128065^2$$
$$6401280063^2 + 160016^2 = 6401280065^2$$
$$640012800063^2 + 1600016^2 = 640012800065^2 \quad := 409616384247041664004225$$
$$64000128000063^2 + 16000016^2 = 64000128000065^2 := 4096016384024704016640004225 \quad (4.107)$$

$$18078060^2 + 42091^2 = 18078109^2$$
$$1800780060^2 + 420091^2 = 1800780109^2$$
$$180007800060^2 + 4200091^2 = 180007800109^2 \quad := 32402808100081700411881$$
$$18000078000060^2 + 42000091^2 = 18000078000109^2 := 3240028080100080170004011881 \quad (4.108)$$

$$81306225^2 + 144272^2 = 81306353^2$$
$$8103060225^2 + 1440272^2 = 8103060353^2$$
$$810030600225^2 + 14400272^2 = 810030600353^2 \quad := 656149573508241603724609$$
$$81000306000225^2 + 144000272^2 = 81000306000353^2 := 6561049572150822216036124609 \quad (4.109)$$

$$5062499^2 + 4500^2 = 5062501^2$$
$$506249999^2 + 45000^2 = 506250001^2$$
$$50624999999^2 + 450000^2 = 50625000001^2 \quad := 2562890625101250000001$$
$$5062499999999^2 + 4500000^2 = 5062500000001^2 \quad := 25628906250010125000000001$$
$$506249999999999^2 + 45000000^2 = 506250000000001^2 := 256289062500001012500000000001 \quad (4.110)$$

$$4928399^2 + 4440^2 = 4928401^2$$
$$492839999^2 + 44400^2 = 492840001^2$$
$$49283999999^2 + 444000^2 = 49284000001^2 \quad := 2428912656098568000001$$
$$4928399999999^2 + 4440000^2 = 4928400000001^2 \quad := 24289126560009856800000001$$
$$492839999999999^2 + 44400000^2 = 492840000000001^2 := 242891265600000985680000000001 \quad (4.111)$$

$$3422499^2 + 3700^2 = 3422501^2$$
$$342249999^2 + 37000^2 = 342250001^2$$
$$34224999999^2 + 370000^2 = 34225000001^2 \quad := 1171350625068450000001$$
$$3422499999999^2 + 3700000^2 = 3422500000001^2 \quad := 11713506250006845000000001$$
$$342249999999999^2 + 37000000^2 = 342250000000001^2 := 117135062500000684500000000001 \quad (4.112)$$

$$3991923^2 + 35964^2 = 3992085^2$$
$$399919923^2 + 359964^2 = 399920085^2 \quad := 159936074386407225$$
$$39999199923^2 + 3599964^2 = 39999200085^2 \quad := 1599936007439864007225$$
$$3999991999923^2 + 35999964^2 = 3999992000085^2 := 15999936000743998640007225 \quad (4.113)$$

$$3991835^2 + 51948^2 = 3992173^2$$
$$399919835^2 + 519948^2 = 399920173^2$$
$$39999199835^2 + 5199948^2 = 39999200173^2 := 1599936014479723229929$$
$$3999991999835^2 + 51999948^2 = 3999992000173^2 := 15999993600144799723202929 \qquad (4.114)$$

$$3991883^2 + 43956^2 = 3992125^2$$
$$399919883^2 + 439956^2 = 399920125^2 := 159936106380015625$$
$$39999199883^2 + 4399956^2 = 39999200125^2 := 1599936010639800015625$$
$$3999991999883^2 + 43999956^2 = 3999992000125^2 := 15999993600106399800000015625$$
$$(4.115)$$

$$3991779^2 + 59940^2 = 3992229^2$$
$$399919779^2 + 599940^2 = 399920229^2$$
$$39999199779^2 + 5999940^2 = 39999200229^2 := 1599936018959633652441$$
$$3999991999779^2 + 59999940^2 = 3999992000229^2 := 15999993600189599633605241 \qquad (4.116)$$

$$3991715^2 + 67932^2 = 3992293^2$$
$$399919715^2 + 679932^2 = 399920293^2$$
$$39999199715^2 + 6799932^2 = 39999200293^2 := 1599936024079531285849$$
$$3999991999715^2 + 67999932^2 = 3999992000293^2 := 15999993600240799531208584 9 \qquad (4.117)$$

$$14062491^2 + 22500^2 = 14062509^2$$
$$1406249991^2 + 225000^2 = 1406250009^2$$
$$140624999991^2 + 2250000^2 = 140625000009^2 := 39550781255062500000162$$
$$14062499999991^2 + 22500000^2 = 14062500000009^2 := 395507812500506250000000162 \qquad (4.118)$$

$$76562499^2 + 17500^2 = 76562501^2$$
$$7656249999^2 + 175000^2 = 7656250001^2$$
$$765624999999^2 + 1750000^2 = 765625000001^2 := 586181640626531250000001$$
$$76562499999999^2 + 17500000^2 = 76562500000001^2 := 5861816406250153125000000001$$
$$7656249999999999^2 + 175000000^2 = 7656250000000001^2 := 58618164062500015312500000000001$$
$$(4.119)$$

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