Submarine Autopilot Performance Optimization with System Identification

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Synopsis

Computer simulation models play a vital role in the assessment of a ship's autopilot design. A well-tuned autopilot will contribute to reducing rudder activity, thereby minimizing wear on the actuation plant and also generally reducing fuel consumption. The equations that describe the ship motion dynamics contain a large number of hydrodynamic coefficients that must be calculated as accurately as possible to justify the use of a simulation model and its relevance to predicting the ship manoeuvring characteristics. Proper prediction of the ship performance is an essential pre-requisite in the process of tuning the autopilot.

The hydrodynamic coefficients can be calculated by using theoretical methods or by carrying out experiments on the actual ship or on a scaled model of the ship. System Identification (SI) is an experiment-based approach and in this paper the authors present an algorithm that can estimate the coefficients with great accuracy. These coefficients can classically be obtained in a towing tank using a captive model, and with a planar motion mechanism and a rotating arm. Generally, these systems are costly and entail expensive trials programs, and SI methods have been developed in an effort to obviate some of those problems and limitations. They typically process ship manoeuvring data obtained from a free-running scaled model or full-scale trials.

While similar to a surface ship, the motion dynamics of a submarine introduce additional challenges for SI methods. This is because the submarine manoeuvres in "three dimensions", which adds complexity and more hydrodynamic coefficients to the equations. The standard submarine simulation model, also referred to as the Gertler and Hagen equations, incorporates over 120 coefficients. To calculated these coefficients, the SI algorithm uses a Square-Root Unscented Kalman filter (SR-UKF). One of its appealing features is that it calculates all the coefficients by processing data from a single submarine manoeuvre that has a repeating sinusoidal pattern in both depth and course. The manoeuvre can be performed in a towing tank by a free-running scaled model of the submarine, or it can be performed at sea on the full-scale submarine as part of the sea trials schedule.

Keywords: System Identification; Dynamic simulation; Control system design; Tank testing; Sea trials

1. Introduction

The SI algorithm presented in the paper calculates the more than 120 coefficients that form part of the standard equations that describe the dynamic motion of a submarine (Gertler and Hagen, 1967). In the development process for any new class of submarine, a substantial amount of effort is dedicated to calculating those coefficients so that the maneuvering characteristics of the submarine can be assessed at the design stage. The coefficients are also required for designing, tuning and testing the depth and course autopilots, tasks which are initially carried out in a simulation environment. At the design stage, the coefficients are classically obtained by performing captive model experiments in a towing tank and by using a series of apparatus such as the rotating arm and planar motion mechanisms. Each test sequence contributes to the determination of a subset or category of coefficients. It is considered that the more important coefficients, i.e. those that most contribute to the motion dynamics of the submarine, can be calculated with an accuracy of around 5%. This category of coefficients includes, for example, the linear derivatives. In calculating the coefficients the use of a free-running scaled model of the submarine is considered to be less onerous in terms of time and cost than the classical experiment-based methods (Martinson, 2015). In the SI algorithm presented in the paper the submarine must perform a single, simple manoeuvre, whether at sea or in a towing tank on a scaled model. Submarine motion data is collected during the manoeuvre, and it is processed offline, i.e., post-manoeuvre, to calculate the submarine coefficients in a single computation run. Results presented in the document establish that, under optimal conditions, the accuracy with which the algorithm can calculate the coefficients is around 0.2%.

2. Formulation of the System Identification Algorithm

The key building block in the System Identification (SI) algorithm developed for estimating the submarine coefficients is the Square-Root (SR) Unscented Kalman Filter (UKF) for parameter estimation (van der Merwe & Wan). The particular form of the SR-UKF used in the SI algorithm is presented in Section 2.2. Before this is

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accomplished, however, the non-linear mapping which is central to the SR-UKF solution process and that represents the motion dynamics of the submarine is derived in Section 2.1.

2.1. Submarine Equations of Motion and SI Relationships

At the core of the Kalman filter described in Section 2.2 is the following non-linear mapping, expressed in discrete form:

$$\mathbf{d}_k = \mathbf{G}(\mathbf{x}_k, \mathbf{w}_\mathrm{k}) + \mathbf{e}_k \tag{1}$$

$\mathbf{G}(\mathbf{x}_k, \mathbf{w}_\mathrm{k})$ encapsulates the dynamics of the system whose state vector at the time step t_k is $\mathbf{x}_k$. The system is parametrized by the vector $\mathbf{w}_\mathrm{k}$ which, in the problem at hand, are the submarine coefficients. Further, $\mathbf{d}_k$ is the "desired" output, and $\mathbf{e}_k$ represents the error between $\mathbf{d}_k$ and the true output. The SR-UKF algorithm is effectively an iterative process in which $\mathbf{w}_\mathrm{k}$ gets adjusted over time so as to drive the error $\mathbf{e}_k$ to zero.

In the current application the non-linear mapping relates to the standard equations of motion that describe the submarine dynamics (Gertler & Hagen, 1967). These equations, which will be referred to as the G&H equations, are expressed in a reference frame which is fixed to the submarine. The reference frame will be called the B-frame, and its origin is usually chosen to be close to the centers of buoyancy and gravity.

In its present form the SI algorithm calculates the 122 coefficients listed in Table 1 below. These coefficients are non-dimensional, with the current paper using the same nomenclature as Gertler and Hagen (1967) whose report can be consulted for detail. Generally, no space is dedicated here to re-defining the nomenclature or re-writing the lengthy G&H equations, and only their salient relationships are captured in the context of the SI algorithm.

The G&H equations can be written in matrix form as follows:

$$\dot{\mathbf{x}} = \widehat{\mathbf{G}}(\mathbf{x}, \mathbf{w}) \tag{2}$$

where the "dot" denotes the time derivative and $\mathbf{x}$ is the state vector defined as

$$\mathbf{x} = [\mathbf{v}^T \quad \mathbf{r}^T]^T \tag{3}$$

with

$$\mathbf{v} = [u \quad v \quad w \quad p \quad q \quad r]^T \tag{4}$$
$$\mathbf{r} = [\phi \quad \theta \quad \psi \quad x \quad y \quad z]^T \tag{5}$$

$\mathbf{v}$ represents the linear and angular velocity of the submarine expressed in the B-frame; $\mathbf{r}$ the angular orientation and position of the B-frame relative to the inertial frame; and $\mathbf{w}$ the submarine coefficients from Table 1, all stacked up in a 122-vector. That is to say, by reference to Table 1, we have

$$\mathbf{w} = \left[X'_{qq} \quad X'_{rr} \quad \ldots \quad N'_{\delta r\eta}\right]^T \tag{6}$$

Equations (4) and (5) can be substituted into Eq. (2) to write:

$$\begin{aligned} \begin{Bmatrix} \dot{\mathbf{v}} \\ \dot{\mathbf{r}} \end{Bmatrix} &= \widehat{\mathbf{G}}(\mathbf{x}, \mathbf{w}) \\ &= \begin{Bmatrix} \mathbf{M}^{-1}\mathbf{F} \\ \mathbf{\Theta v} \end{Bmatrix} \end{aligned} \tag{7}$$

where $\mathbf{M}$ is the 6×6 generalized mass matrix that regroups the terms that are a function of $\dot{\mathbf{x}}$, whereas the 6-vector $\mathbf{F}$ contains the remaining terms in the G&H equations. Further,

$$\mathbf{\Theta} = \begin{bmatrix} \boldsymbol{\theta}_v & 0 \\ 0 & \boldsymbol{\theta}_\omega \end{bmatrix} \tag{8}$$

with

$$\boldsymbol{\theta}_v = \begin{bmatrix} \cos\theta\cos\psi & -\cos\phi\sin\psi + \sin\phi\sin\theta\cos\psi & \sin\phi\sin\psi + \cos\phi\sin\theta\cos\psi \\ \cos\theta\sin\psi & \cos\phi\cos\psi + \sin\phi\sin\theta\sin\psi & -\sin\phi\cos\psi + \cos\phi\sin\theta\sin\psi \\ -\sin\theta & \sin\phi\cos\theta & \cos\phi\cos\theta \end{bmatrix} \tag{9}$$

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$$\boldsymbol{\theta}_{\omega} = \begin{bmatrix} 1 & \sin\phi\tan\theta & \cos\phi\tan\theta \\ 0 & \cos\phi & \sin\phi \\ 0 & \sin\phi/\cos\theta & \cos\phi/\cos\theta \end{bmatrix} \tag{10}$$

The nonlinear mapping (1) is a discrete time equation. Equation (2) is transformed to discrete form by using the 1st order Euler time-differencing scheme:

$$\mathbf{x}_{k+1} = \mathbf{x}_k + \Delta t\, \widehat{\mathbf{G}}(\mathbf{x}_k, \mathbf{w}_k) \tag{11}$$

or

$$\mathbf{x}_{k+1} = \overline{\mathbf{G}}(\mathbf{x}_k, \mathbf{w}_k) \tag{12}$$

where Δt is the time step and

$$\overline{\mathbf{G}}(\mathbf{x}_k, \mathbf{w}_k) \triangleq \mathbf{x}_k + \Delta t\, \widehat{\mathbf{G}}(\mathbf{x}_k, \mathbf{w}_k) \tag{13}$$

Equation (12) for the submarine dynamics has now been couched into a form that approaches that of Eq. (1), with the state vector $\mathbf{x}_{k+1}$ at the updated time step t_{k+1} relating to the desired output $\mathbf{d}_k$. It has been found that not all the components in $\mathbf{x}_{k+1}$ need to be used for optimal results, and the expression for $\mathbf{d}_k$ can be reduced to the following subset:

$$\mathbf{d}_k \equiv [v_{k+1} \quad w_{k+1} \quad p_{k+1} \quad q_{k+1} \quad r_{k+1} \quad \phi_{k+1} \quad \theta_{k+1} \quad \psi_{k+1}]^T \tag{14}$$

These quantities represent the sway and heave velocity of the submarine, the rolling, pitching and heading rates, and the roll, pitch and heading, respectively. Equation (14) can be written as a matrix relationship by defining

$$\mathbf{C} = \begin{bmatrix} \mathbf{0}_{1\times 1} & \mathbf{0}_{1\times 8} & \mathbf{0}_{1\times 3} \\ \mathbf{0}_{8\times 1} & \mathbf{I}_{8\times 8} & \mathbf{0}_{8\times 3} \\ \mathbf{0}_{3\times 1} & \mathbf{0}_{3\times 8} & \mathbf{0}_{3\times 3} \end{bmatrix} \tag{15}$$

where $\mathbf{0}$ is a matrix of zeros and $\mathbf{I}$ a unit matrix, with the matrix dimensions specified by the subscripts. By reference to Eq. (1), the final expression for the non-linear mapping can be formalized by pre-multiplying Eq. (12) by $\mathbf{C}$ and making the following associations:

$$\mathbf{d}_k \equiv \mathbf{C}\,\mathbf{x}_{k+1}, \qquad \mathbf{G}(\mathbf{x}_k, \mathbf{w}_\mathrm{k}) \equiv \mathbf{C}\,\overline{\mathbf{G}}(\mathbf{x}_k, \mathbf{w}_k) \tag{16}$$

This completes the formulation of the non-linear mapping that is the key part in the SI algorithm. The following remarks can be made:

1. In Eq. (7) the dimensional mass matrix $\mathbf{M}$ relates to its non-dimensional counterpart $\mathbf{M}'$ through the following relation:

$$\mathbf{M} = \mathbf{D}_L \mathbf{M}' \mathbf{D}_R^{-1} \tag{17}$$

where

$$\mathbf{D}_L = \mathrm{diag}\,\{\,\rho U^2 L^2/2 \quad \rho U^2 L^2/2 \quad \rho U^2 L^2/2 \quad \rho U^2 L^3/2 \quad \rho U^2 L^3/2 \quad \rho U^2 L^3/2\,\} \tag{18}$$

$$\mathbf{D}_R = \mathrm{diag}\,\{\,U^2/L \quad U^2/L \quad U^2/L \quad U^2/L^2 \quad U^2/L^2 \quad U^2/L^2\,\} \tag{19}$$

$$\mathbf{M} = \begin{bmatrix} m' - X_{\dot u} & 0 & 0 & 0 & m'z'_g & -m'y'_g \\ 0 & m' - Y_{\dot v} & 0 & -m'z'_g - Y_{\dot p} & 0 & m'x'_g - Y_{\dot r} \\ 0 & 0 & m' - Z_{\dot w} & m'y'_g & -m'x'_g - Z_{\dot q} & 0 \\ 0 & -m'z'_g - K_{\dot v} & m'y'_g & I_x - K_{\dot p} & 0 & -K_{\dot r} \\ m'z'_g & 0 & -m'x'_g - M_{\dot w} & 0 & I_y - M_{\dot q} & 0 \\ -m'y'_g & m'x'_g - N_{\dot v} & 0 & -N_{\dot p} & 0 & I_z - N_{\dot r} \end{bmatrix} \tag{20}$$

In Eq. (20) $\mathbf{x}_g = [x'_g \quad y'_g \quad z'_g]^T$ denotes the location of the center of gravity, ρ the seawater density, U the submarine speed and L its length. diag{■} stands for a diagonal matrix.

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2. By reference to Eq. (20), the only parameters in the G&H equations that are not calculated by the SI algorithm are $m', I'_x, I'_y, I'_z, \mathbf{x}_g, \mathbf{x}_b$, which are respectively the mass and moments of inertia of the submarine and the location of the centers of gravity and buoyancy. The algorithm calculates the added moments of inertia $K_{\dot{p}}, M_{\dot{q}}$ and $N_{\dot{r}}$, and because those terms appear together with the corresponding moments of inertia in Eq. (20), any error made in estimating the moments of inertia should be picked up and corrected by the estimate of the added moments of inertia. A similar, but less forceful argument can be made about the mass of the submarine. In this case, the SI algorithm will have to assign or distribute any error in the estimate of m' to $X_{\dot{u}}, Y_{\dot{v}}$ and $Z_{\dot{w}}$, which are respectively the added mass in surge, sway and heave.
3. In Eq. (7), the mass matrix must be inverted. It has been found that to improve the stability and convergence of the algorithm, it is preferable not to perform this operation at every time step. In the current implementation of the SR-UKF, it is performed every 2,000 time steps.

2.2. Square-Root Unscented Kalman Filter for System Identification

The square-root unscented Kalman filter for parameter estimation includes a number of parameters. It is carried out in three steps (van der Merwe and Wan). In the current application, it has been implemented as follows:

1. Initialization:

$$\hat{\mathbf{w}}_0 = \mathbf{0} \qquad \mathbf{S}_{\mathbf{w}_0} = \mathrm{chol}\{E[(\boldsymbol{w} - \hat{\boldsymbol{w}}_0)(\boldsymbol{w} - \hat{\boldsymbol{w}}_0)^T]\} \tag{21}$$

For $k \in \{1, \dots, \infty\}$ do steps 2 and 3:

2. Time update and sigma point calculation:

$$\hat{\mathbf{w}}_k^- = \hat{\mathbf{w}}_{k-1}^- \tag{22}$$

$$\mathbf{S}_{\mathbf{w}_k}^- = \lambda_{\mathrm{RLS}}^{-1/2}\ \mathbf{S}_{\mathbf{w}_{k-1}} \tag{23}$$

$$\boldsymbol{\mathcal{W}}_{k|k-1} = \left[\hat{\mathbf{w}}_k^- + \sqrt{1/N}\mathbf{S}_{\mathbf{w}_k}^- \quad \hat{\mathbf{w}}_k^- - \sqrt{1/N}\mathbf{S}_{\mathbf{w}_k}^-\right] \tag{24}$$

$$\boldsymbol{\mathcal{D}}_{k|k-1} = \mathbf{G}\left[\mathbf{x}_k, \boldsymbol{\mathcal{W}}_{k|k-1}\right] \tag{25}$$

$$\hat{\mathbf{d}}_k = \frac{1}{2N}\sum_{i=1}^{2N} \mathcal{D}_{i,k|k-1} \tag{26}$$

3. Measurement update and equations:

$$\mathbf{S}_{\mathrm{d}_k} = \mathrm{qr}\{[\sqrt{1/2N}[\boldsymbol{\mathcal{D}}_{1:2L,k} - \hat{\mathbf{d}}_k] \quad \sqrt{\mathbf{R}^e}]\} \tag{27}$$

$$\mathbf{P}_{\mathbf{w}_k\mathbf{d}_k} = \frac{1}{2N}\sum_{i=1}^{2N}[\mathcal{W}_{i,k|k-1} - \hat{\mathbf{w}}_k^-]\left[\mathcal{D}_{i,k|k-1} - \hat{\mathbf{d}}_k\right]^T \tag{28}$$

$$\mathcal{K}_k = (\mathbf{P}_{\mathbf{w}_k\mathbf{d}_k}/\mathbf{S}_{\mathrm{d}_k}^T)/\mathbf{S}_{\mathrm{d}_k} \tag{29}$$

$$\hat{\mathbf{w}}_\mathrm{k} = \hat{\mathbf{w}}_k^- + \mathcal{K}_k(\mathbf{d}_k - \hat{\mathbf{d}}_k) \tag{30}$$

$$\mathbf{U} = \mathcal{K}_k\mathbf{S}_{\mathrm{d}_k} \tag{31}$$

$$\mathbf{S}_{\mathbf{w}_k} = \mathrm{cholupdate}\{\mathbf{S}_{\mathbf{w}_k}^-, \mathbf{U}, -1\} \tag{32}$$

Equation (21) initializes the submarine coefficient vector $\hat{\mathbf{w}}_0$ to zero and provides an initial value for its covariance matrix **(**E denotes the statistical inference operator**)**. $\mathbf{G}$ in Eq. (25) is the nonlinear mapping detailed in Eq. (16). Further, $\lambda_{\mathrm{RLS}}^{-1/2}$ has been set equal to 1.00016 and $\sqrt{\mathbf{R}^e}$ to $0.7\sqrt{N}$, where $N = 122$ is the number of coefficients. chol{■} denotes the Cholesky factorization and qr{■} the QR factorization. Additional detail can be found in the paper by van der Merwe and Wan.

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3. Presentation of Results

In order to put the algorithm to work, it is necessary to define a manoeuvre that will be performed by the submarine in order to generate the data $\mathbf{d}_k$ required as desired output. The data will be collected during the manoeuvre and the algorithm will be applied iteratively; the submarine coefficients $\mathbf{w}_k$ will be updated over time by the Kalman filter until they converge to their final values.

3.1. Submarine Manoeuvre

The manoeuvre that has been devised to generate the data required by the SI algorithm is a simultaneous depth and course sinusoidal variation. The results presented in the paper use data generated by a simulation model. The simulation implements the G&H equations and a set of declassified coefficients for Canada's Victoria Class Submarines. The simulated submarine maneuvers were carried out under control of a depth and course autopilot, tuned for the submarine model and integrated into the simulation environment. This type of environment provides a litmus test for an SI algorithm which, by its very nature, is designed to fit data to a model. The algorithm must as a minimum demonstrate its ability to calculate the coefficients accurately when data is generated by a simulation model that is the same as the model underlying its formulation. Figure 1 shows the main characteristics of the submarine manoeuvre:

1. Propulsor RPM is set to a value that stays constant during the manoeuvre. RPM can correspond to a relatively low value of the self-propulsion speed.
2. Depth set-point has a sinusoidal variation with an amplitude of a few meters and a period between 100s and 200s.
3. Course set-point has a sinusoidal variation with a similar period as the depth variation. Amplitude can be small, typically below 50 degrees.

Note: All figures in this paper have been unclassified by omitting the scales on the y-axis.

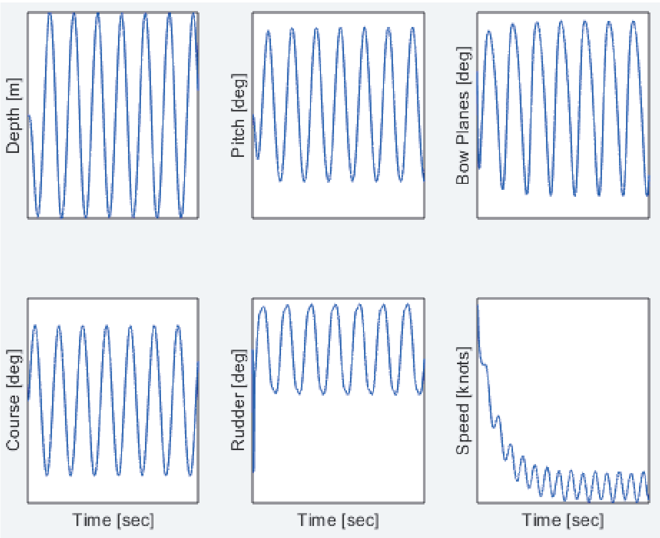

Figure 1: Motion Profile of Submarine Manoeuvre

The SI algorithm needs to work with around 60,000 data samples for proper convergence. For the results presented in the following subsection the time step in the simulation was set to 0.05s, which corresponds to the desired output data rate for $\mathbf{d}_k$. The manoeuvres are therefore all executed for a period of 3,000 seconds.

3.2. Test Results Under Perfect Conditions

In practice, either at sea or in a towing tank, the navigation system does not supply all the quantities required in Eq. (14). Most notably, v and w, which are the sway and heave velocity of the submarine, cannot be measured directly. Similarly, p, q and r, the angular rates in the B-frame, may not all be measured. The only quantities in Eq. (14) that can be assumed always to be available are ϕ, θ and ψ. The kinematic quantities that are not available must be derived from those that are measured. To that end, Eqs. (4), (5), (7) and (8) can be used, when rewritten as

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$$\mathbf{v} = \mathbf{\Theta}^{-1}\,\dot{\mathbf{r}} \tag{33}$$

Using Eq. (5), the term $\dot{\mathbf{r}}$ in Eq. (33) can be expanded as

$$\dot{\mathbf{r}} = [\dot{\phi} \quad \dot{\theta} \quad \dot{\psi} \quad \dot{x} \quad \dot{y} \quad \dot{z}]^T \tag{34}$$

In a towing tank it is possible to instrument a free-running scaled submarine model with a system that measures, collects and derives all the data in Eq. (34) (Millan, 2014). Using Eq. (33), calculation of the desired output $\mathbf{d}_k$ can then be accomplished.

In a submarine, conditions would be different. For example, the quantities supplied by the navigation system could include $\dot{\psi}, \dot{x}$ and $\dot{y}$, but not the other ones ($\dot{x}$ and $\dot{y}$ could be derived from ground-speed and course-over-ground information, making corrections as applicable to account for the presence of ocean currents). And to proceed, the other quantities could be calculated by differentiating ϕ, θ, z.

Notwithstanding the conditions at sea or in a towing tank, the paper provides results that establish the baseline accuracy of the SI algorithm under perfect conditions, that is all the data required for Eq. (14) is "collected and measured" with perfect accuracy. While these conditions cannot be achieved in the real world, they exist in the simulation environment. In addition to the quantities in Eq. (14) generated by the simulation, the SI algorithm also requires the deflection of all control surfaces and the RPM of the propulsor.

The results are presented in Table 1 and Figure 2. To keep the results unclassified, the table only compiles the percentage error defined as $(X_{\mathrm{SI}} - X_o)/X_o \times 100\%$, in which the value of the coefficient calculated by the SI algorithm is denoted by X_{SI}, and the actual value used in the G&H equations by X_o. Similarly, the y-axis is omitted in the plots.

Table 1: Percentage Error – Baseline Accuracy

Axial Force		Lateral Force		Normal Force		Rolling Moment		Pitching Moment		Yawing Moment	
X'_{qq}	0.18%	$Y'_{\dot{r}}$	0.18%	$Z'_{\dot{q}}$	0.18%	$K'_{\dot{p}}$	0.18%	$M'_{\dot{q}}$	0.18%	$N'_{\dot{r}}$	0.18%
X'_{rr}	0.18%	$Y'_{\dot{p}}$	0.18%	Z'_{pp}	0.00%	$K'_{\dot{r}}$	0.18%	M'_{pp}	0.00%	$N'_{\dot{p}}$	0.18%
X'_{rp}	0.00%	$Y'_{p\|p\|}$	0.00%	Z'_{rr}	0.24%	K'_{qr}	-0.05%	M'_{rr}	0.18%	N'_{pq}	0.18%
$X'_{\dot{u}}$	0.18%	Y'_{pq}	0.00%	Z'_{rp}	0.00%	K'_{pq}	0.00%	M'_{rp}	0.18%	N'_{qr}	0.00%
X'_{vr}	0.18%	Y'_{qr}	0.00%	$Z'_{\dot{w}}$	0.18%	$K'_{p\|p\|}$	0.17%	$M'_{q\|q\|}$	0.18%	$N'_{r\|r\|}$	0.18%
X'_{wq}	0.18%	$Y'_{\dot{v}}$	0.18%	Z'_{vr}	0.18%	K'_{p}	-0.14%	$M'_{\dot{w}}$	0.18%	$N'_{\dot{v}}$	0.18%
X'_{uu}	0.18%	Y'_{vq}	0.00%	Z'_{vp}	0.18%	K'_{r}	-28.45%	M'_{vr}	0.18%	N'_{wr}	0.00%
X'_{vv}	0.18%	Y'_{wp}	0.18%	Z'_{q}	0.18%	$K'_{\dot{v}}$	0.18%	M'_{vp}	0.00%	N'_{wp}	0.00%
$X'_{vv\eta}$	0.00%	Y'_{wr}	0.00%	$Z'_{q\eta}$	0.18%	K'_{vq}	0.00%	M'_{q}	0.18%	N'_{vq}	0.00%
X'_{ww}	0.18%	Y'_{r}	0.18%	$Z'_{\|q\|\delta s}$	0.18%	K'_{wp}	0.00%	$M'_{q\eta}$	0.18%	N'_{p}	0.18%
$X'_{ww\eta}$	0.00%	$Y'_{r\eta}$	0.18%	$Z'_{w\|q\|}$	0.18%	K'_{wr}	0.00%	$M'_{\|q\|\delta s}$	0.18%	N'_{r}	0.18%
$X'_{\delta r\delta r}$	0.18%	Y'_{p}	0.18%	Z'_{*}	-0.31%	K'_{*}	31.86%	$M'_{\|w\|q}$	0.18%	$N'_{r\eta}$	0.18%
$X'_{\delta r\delta r\eta}$	0.18%	$Y'_{\|r\|\delta r}$	0.18%	Z'_{w}	0.18%	$K'_{*\eta}$	0.00%	M'_{*}	-0.13%	$N'_{\|r\|\delta r}$	0.19%
$X'_{\delta s\delta s}$	0.18%	$Y'_{v\|r\|}$	0.18%	$Z'_{w\eta}$	0.00%	K'_{v}	0.15%	M'_{w}	0.18%	$N'_{\|v\|r}$	0.18%
$X'_{\delta s\delta s\eta}$	0.00%	Y'_{*}	0.00%	$Z'_{w\|w\|}$	0.18%	$K'_{v\eta}$	0.18%	$M'_{w\eta}$	0.00%	N'_{*}	0.00%
$X'_{\delta b\delta b}$	0.18%	Y'_{v}	0.18%	$Z'_{w\|w\|\eta}$	0.00%	K'_{vw}	0.00%	$M'_{w\|w\|}$	0.18%	N'_{v}	0.18%
		$Y'_{v\eta}$	0.18%	$Z'_{\|w\|}$	0.18%	$K'_{\delta r}$	-4.82%	$M'_{w\|w\|\eta}$	0.00%	$N'_{v\eta}$	0.00%
		$Y'_{v\|v\|}$	0.18%	Z'_{ww}	0.18%			$M'_{\|w\|}$	0.21%	$N'_{v\|v\|}$	0.18%
		$Y'_{v\|v\|\eta}$	0.00%	Z'_{vv}	0.18%			M'_{ww}	0.18%	$N'_{v\|v\|\eta}$	0.00%
		Y'_{vw}	0.00%	$Z'_{\delta s}$	0.19%			M'_{vv}	0.18%	N'_{vw}	0.00%
		$Y'_{\delta r}$	0.19%	$Z'_{\delta s\eta}$	0.18%			$M'_{\delta s}$	0.19%	$N'_{\delta r}$	0.17%
		$Y'_{\delta r\eta}$	0.18%	$Z'_{\delta b}$	0.13%			$M'_{\delta s\eta}$	0.18%	$N'_{\delta r\eta}$	0.18%
								$M'_{\delta b}$	0.20%		

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The table shows that the average error for the coefficients is around 0.2%. There are a few coefficients for which the percentage error is greater, most notably K'_r, K'_* and $K'_{\delta r}$. For these coefficients however, the absolute value is quite small, thus amplifying any small relative difference when expressed in percentage terms. In the G&H equations some coefficients were initialized with a value strictly equal to zero, in which case the calculated value remained smaller than around 10^{-5} (and in the table the corresponding percentage error was set equal to zero).

The plots in Figure 2 detail the convergence of a few selected coefficients as a function of time. These coefficients are the linear derivatives that would typically be used in the design process for the depth and course autopilots. It takes around 1,000 seconds for the SI algorithm to pick up the trend, and convergence is achieved in around 2,500 seconds. These results are observed for all coefficients.

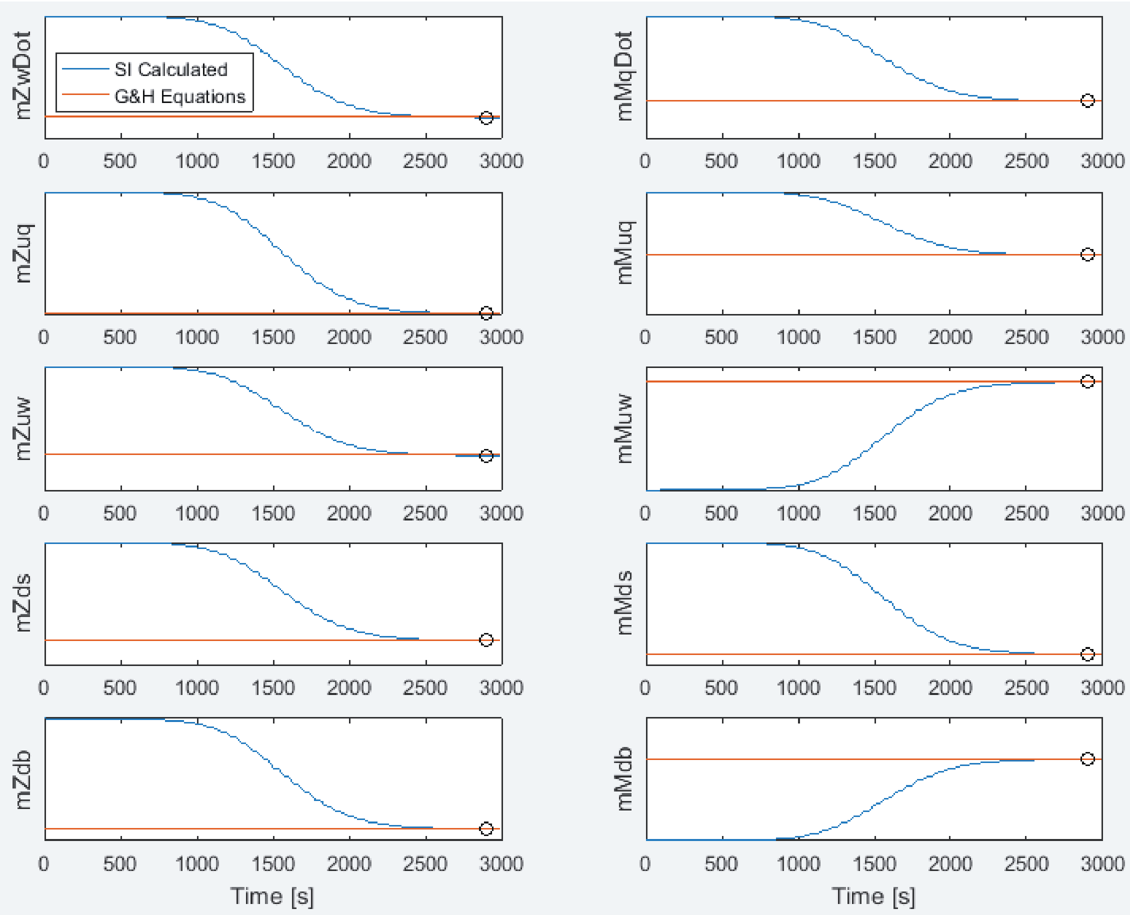

Figure 2: Convergence of Coefficients over Time

4. Conclusions

The SI algorithm presented in the paper shows good promise of being a significant help in the important task of calculating the submarine coefficients required in the standard equations of motion. The algorithm processes motion data from a single, simple submarine manoeuvre. The manoeuvre can be performed at sea by the submarine; or it can be performed in a towing tank by a free-running scaled model of the submarine. In the latter case, the algorithm has the potential of being a complement to or even a lower cost replacement for the more classical methods of calculating the coefficients at the design stage of a new class of submarine.

With minor modification, the algorithm also has the same potential of being an efficient tool for calculating the coefficients for a surface ship, a task which is classically done in very much the same way as for a submarine. The modification would consist in adapting the algorithm's non-linear mapping to represent the equations of motion of the surface ship, of which there are many variants which are generally simpler than those for a submarine.

The paper has demonstrated the performance of the algorithm under perfect conditions in a simulation environment. In practice, the information that needs to be processed by the algorithm is obtained from a navigation system whose data has finite resolution, with accuracy and noise characteristics that are not perfect,

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all of which will likely impact on the performance of the algorithm. Investigation of the impact of those factors is a work in progress.

5. References

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