

Crazy Sequential Representations: Fill the Gaps (02)

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Introduction

Others have attempted to write the natural numbers from 1 to 11111 in terms of 1 to 9 (in increasing and decreasing order) by using the operations of addition, subtraction, multiplication, division and/or potentiation (and optionally parentheses).

For example:

Number	Increasing	Decreasing
10957	$(1+2)^{(3+4)}*5-67+89$	$(9+8*7*65+4)*3-2*1$
10958		$(9+8*7*65+4)*3-2+1$
10959	$12+3+456*(7+8+9)$	$9+(8*76*(5+4)+3)*2*1$
10960	$12+(3^4+5+6)*7*(8+9)$	$9+(8*76*(5+4)+3)*2+1$
10961	$(1+2+34)*(5*6+7)*8+9$	$(9+8*7*65+4)*3+2*1$
10962	$12*3^4*5+678*9$	$9876+543*2*1$

Generally these expressions are referred to as crazy sequential representations (CSR). Interestingly, only one CSR remains to be identified, the increasing CSR for 10958.

Historic Overview

Inder Taneja published five papers on arXiv (for 1 up to 11111):

ARXIV Version	Evaluated Range	Allowed Operations	Missing Increasing	Missing Decreasing	Valid Representations
1 (06-02-2013) ¹	44 to 1000	+ * ^	2	10	1902 (of 1914)
2 (19-03-2013) ²	44 to 4444	+ * ^	50	53	8699 (of 8802)
3 (05-06-2013) ³	44 to 11111	+ * ^ ()	590	605	20941 (of 22136)
4 (05-08-2013) ⁴	0 to 11111	+ * ^ () -	449	315	21460 (of 22224)
5 (08-01-2014) ⁵	0 to 11111	+ * ^ () - /	9	10	22205 (of 22224)

Authors published three papers on Figshare/Zenodo (for -2147483647 up to 2147483647):

Date	Title
12-06-2018	Crazy Sequential Representations: Exhaustive Search ⁶
14-06-2018	Crazy Sequential Representations: Negative Integers ⁷
18-06-2018	Crazy Sequential Representations: Without Subtraction and/or Division ⁸

Inder Taneja published three papers on RGMIA (for 11112 up to 30000):

Date	Title
12-09-2018	Crazy Representations of Natural Numbers From 11112 to 20000 ⁹
10-11-2018	Crazy Representations of Natural Numbers From 20001 to 25000 ¹⁰
10-11-2018	Crazy Representations of Natural Numbers From 25001 to 30000 ¹¹

Authors published one paper on Figshare/Zenodo (comparing results for 11112 up to 30000):

Date	Title
06-12-2018	Crazy Sequential Representations: 11112 up to 30000 ¹²

Authors published two papers on Figshare/Zenodo (improving our previous work):

Date	Title
14-12-2018	Crazy Sequential Representations: Simplifications (01) ¹³
24-12-2018	Crazy Sequential Representations: Fill the Gaps (01) ¹⁴

Aim

1. Provide valid CSR/NCSR for any invalid CSR/NCSR in our previous work ^{6,7,8,12,13,14}
2. Attempt to 'fill the gaps' in our previous work ^{6,7,8,12,13,14}

Thus, within the -2147483647 up to 2147483647 range;

- Identify any increasing CSR/NCSR, for integers without any increasing CSR/NCSR.
- Identify any decreasing CSR/NCSR, for integers without any decreasing CSR/NCSR.

CSR/NCSR 'with a final negation' were ignored during the "stocktaking" process. For example: For 52388 the following increasing CSR was previously published: $-(1^2-3*(-4*-56*78-9))$
Authors now attempted to identify an increasing CSR 'without a final negation' for 52388.

Existing Definitions

Default Notation

Notation as used by most programming languages, restricted to following characters:

1	2	3	4	5	6	7	8	9	+	-	*	/	^	()
---	---	---	---	---	---	---	---	---	---	---	---	---	---	-----

Potential CSR/NCSR

Valid mathematical expression, thus well-formed interpretable syntactic construct, matching against either of the following regular expressions (using @ delimiter):

```
@^[+-*/^]*1[-+*/^]*2[-+*/^]*3[-+*/^]*4[-+*/^]*5[-+*/^]*6[-+*/^]*7[-+*/^]*8[-+*/^]*9[-+*/^]*$
```

```
@^[+-*/^]*9[-+*/^]*8[-+*/^]*7[-+*/^]*6[-+*/^]*5[-+*/^]*4[-+*/^]*3[-+*/^]*2[-+*/^]*1[-+*/^]*$
```

Ignoring evaluation result (natural, integer, real, rational, indeterminate, etc.).

In terms of 1 to 9

Digits 1 to 9 occur once and in order, either in increasing or decreasing order.
 Digits can be used as individual numbers (thus 1, 2, 3, 4, 5, 6, 7, 8 and 9).
 Digits can be concatenated into larger numbers (for example 123, 4, 5, 6 and 789).
 Negative counterparts of numbers may be used as well (also used by Inder Taneja).

Genuine CSR

Natural number (or zero) in terms of 1 to 9 (in increasing or decreasing order) by using the operations of addition, subtraction, multiplication, division and/or potentiation (and optionally parentheses).

Genuine NCSR

Negative integer (or zero) in terms of 1 to 9 (in increasing or decreasing order) by using the operations of addition, subtraction, multiplication, division and/or potentiation (and optionally parentheses).

Pseudo CSR

Potential non-genuine CSR evaluating to **natural number** (or zero).
 For example, expressions with implicit multiplication by minus one.

Pseudo NCSR

Potential non-genuine NCSR evaluating to **negative integer** (or zero).
 For example, expressions with implicit multiplication by minus one.

Final Notes

Authors consider following CSR/NCSR to be proof-of-work, as identification of CSR/NCSR is computationally expensive, while verification of CSR/NCSR is computationally inexpensive.

Authors do not guaranty:

- Published CSR/NCSR are the shortest CSR/NCSR in existence.
- Published CSR/NCSR are in their simplest form.
- Unavailable CSR/NCSR do not exists.

Identification Results

Authors identified 44048 previously unknown increasing/decreasing CSR/NCSR:

	Increasing CSR	Decreasing CSR	Increasing NCSR	Decreasing NCSR
Total	7493	10346	7908	18301

Genuine CSR/NCSR were preferred over pseudo CSR/NCSR in case multiple were identified. In total 30873 genuine CSR/NCSR and 13175 pseudo CSR/NCSR were identified:

	Increasing CSR	Decreasing CSR	Increasing NCSR	Decreasing NCSR
Genuine	6635	8367	6169	9702
Pseudo	858	1979	1739	8599

Previously, authors published various pseudo CSR/NCSR 'with a final negation'. Authors now identified 16013 alternative genuine CSR/NCSR 'without a final negation'.

	Pseudo CSR/NCSR	Genuine CSR/NCSR
-18050	$-((1-2^3)^4+5^6+7+8+9)$	$1+(2^*-3*(45*67-8)-9)$
-20442	$-(1+2+3^(4+5)-(-6-78)*9)$	$-1-2-3^(4+5)-(6+78)*9$
52388	$-(1^2-3*(-4*-56*78-9))$	$1-2-3*(4*56*-78+9)$
53373	$-(1+2-3*-4^5*(-67/8-9))$	$-1-2-3*-4^5*(67/8+9)$
25484	$-(-98*(-7-6)*-5*4-3-(2-1))$	$98*(7+6)*5*4+3+2-1$
32387	$-(98/7-(((6-54)*3)^2)-1)$	$-98/7+((6+54)*3)^2+1$

Newly identified CSR/NCSR were tabulated in the following supplements:

- Supplement 1 : Increasing CSR within the 1 up to 2147483647 range
- Supplement 2 : Decreasing CSR within the 1 up to 2147483647 range
- Supplement 3 : Increasing NCSR within the -1 down to -2147483647 range
- Supplement 4 : Decreasing NCSR within the -1 down to -2147483647 range

Validation Results

Previously published CSR/NCSR ^{6,7,8,12,13,14} were extracted from the original PDF files and checked for validity. Authors attempted to identify valid alternatives in case invalid CSR/NCSR were identified.

Within the supplements of the following manuscript...

Date	Title
12-06-2018	Crazy Sequential Representations: Exhaustive Search ⁶

Four invalid CSR were identified. Three CSR originated from the “Reference Category”. This category contains the “validated CSR as originally published by Inder Taneja”. Unfortunately, these invalid CSR were not identified during our initial validation.

Page	Integer	CSR
189	10102	$1+(2+56*(3*4))*(7+8)-9$
212	474	$9*8+7*(54+3)+2+1$
213	512	$98+7*(54+3+2)+1$

For 474 and 512, valid alternative decreasing CSR were provided in the other categories:

	Shortest Without Division	Shortest Without Potentiation	Shortest Without Concatenation
474	$98+7-6+54+321$	$98+7-6+54+321$	$9*8*7+6-5-4^3/2+1$
512	$9*8+765-4-321$	$98/7+65+432+1$	$9*8*7+6+5+4-3*2-1$

For 10102, the invalid CSR was unfortunately used for each category (being the shortest):

	Shortest Without Division	Shortest Without Potentiation	Shortest Without Concatenation
10102	$1+(2+56*(3*4))*(7+8)-9$	$1+(2+56*(3*4))*(7+8)-9$	$1+(2+56*(3*4))*(7+8)-9$

For 10102, authors identified (slightly longer) valid alternatives for each category:

	Shortest Without Division	Shortest Without Potentiation	Shortest Without Concatenation
10102	$1+(-2+3*-4*56)*(-7-8)-9$	$1+(-2+3*-4*56)*(-7-8)-9$	$-(1-2^(3+4))*(-5+6+78)+9$

The fourth invalid CSR occurred twice in the “specific operations at consecutive indexes” supplement, namely at page 16405, breaking the “addition set” and “multiply set” for 475.

$9+8+5*(7+6)*(4+3)+2+1$	$9+8+5*(7+6)*(4+3)+2+1$
-------------------------	-------------------------

Valid alternative CSR (with the correct operation at the correct index) were identified:

$9*(8*7-6^(5-4)+3)-2*1$	$9*8+7+(6+5)*4*3*(2+1)$
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The complete sets are shown on the following page.

Number: 475 Length: 21	Number: 475 Length: 21
$9+(-87-6)*(5-4-3*2)+1$	$9*8*7-6*5+((4-3)^2)^1$
$(9+8)*7^(6-5)*4-3+2^1$	$(9*8+7)*6-(5-4)^3+2^1$
$(-9+8)*-7*65-(4-3)+21$	$9*8*7-6*5+((4-3)^2)^1$
$(9*8+7)*6-(5-4)^3+2^1$	$(9*8*7-6*5*(4-3)^2)+1$
$((-98+7*6)*-5-43)*2+1$	$(9+8)*7^(6-5)*4-3+2^1$
$98+(-7+65)*(4-3/-2+1)$	$(-9+8)*-7*65-(4-3)+21$
$(-98/-7+65)*-4*-3/2+1$	$(9*8+7)*6-(5-4)^3+2^1$
$-9*(8-(7+6+5+43))-2*1$	$(9*8*7-6*5*(4-3)^2)+1$
$9*8*7-6*5+((4-3)^2)^1$	$9-(-8-76)*5+43-(-2-1)$
$-9+8+7*(65+4-(3-2)^1)$	$98+(-7+65)*(4-3/-2+1)$
$-9*(8-76)-5+4*(-32-1)$	$9+8+5*(7+6)*(4+3)+2+1$
$9+8+(-7*-6+5+4)*3^2-1$	$9*8+7+(6+5)*4*3*(2+1)$
$-9+((-8+7)*65+43)^2*1$	$(-9-(8+7+6))*-5+4+321$
$9+8+5*(7+6)*(4+3)+2+1$	$(9+8)*7^(6-5)*4-3+2^1$
$9*(8*7-6^(5-4)+3)-2*1$	$(-98/-7+65)*-4*-3/2+1$
$(-9-(8+7+6))*-5+4+321$	$9-(-87-6)*5+4-3*(2-1)$
$9+(-87-6)*(5-4*3+2)+1$	$9+(-87-6)*(5-4-3*2)+1$
$(9+8)*7^(6-5)*4-3+2^1$	$(-9+8)*(-76-54*3)*2-1$
$(-9+8)*-7*65-(4-3)+21$	$(-9+8/7-(6+54))/-3*21$
$(9*8*7-6*5*(4-3)^2)+1$	$(9+8)*7^(6-5)*4-3+2*1$

Within the supplements of the following manuscript...

Date	Title
14-06-2018	Crazy Sequential Representations: Negative Integers ⁷

One invalid NCSR was identified:

Page	Integer	NCSR
163	-10102	$-(1+(2+56*(3*4))*(7+8)-9)$

Which is identical to the previously discussed invalid CSR:

$$1+(2+56*(3*4))*(7+8)-9$$

But 'with a final negation' to invert the sign:

$$-(1+(2+56*(3*4))*(7+8)-9)$$

Authors identified a valid alternative NCSR:

$$(1-2^(3+4))*(-5+6+78)+9$$

Within the supplements of the following manuscript...

Date	Title
18-06-2018	Crazy Sequential Representations: Without Subtraction and/or Division ⁸

Four invalid CSR were identified:

Page	Integer	CSR
19	270	$9+(8+7+65)*3+21$
22	474	$9*8+7*(54+3)+2+1$
23	512	$98+7*(54+3+2)+1$
164	10102	$1+(2+56*(3*4))*(7+8)-9$

The CSR for 474, 512 and 10102 are identical to the previously discussed invalid CSR. Valid alternatives (without “-” and “/” characters) were identified for 3 integers:

Integer	CSR
270	$98+76+(5+43)*2*1$
474	$(9+8)*7+6*5+4+321$
512	$9+87+(6*(5+4^3)+2)^1$

Supplement 3 and 4 were affected by “a small bug” in the PDF generation script, resulting in various integers without any valid CSR being tabulated (just integer itself is shown). See page 281 for an increasing example and page 508 for a decreasing example:

Integer	Increasing CSR	Integer	Decreasing CSR
43046807	43046807	43046749	43046749
43046808	43046808	43046750	43046750
43046810	43046810	43046752	43046752
43046811	43046811	43046753	43046753
43046813	43046813	43046754	43046754
43046814	43046814	43046759	43046759
43046815	43046815	43046760	43046760
43046816	43046816	43046761	43046761
43046817	43046817	43046762	$9+8+(76+5)^4+3+21$
43046818	43046818	43046763	43046763

Valid alternative CSR (without “-” and “/”) were identified for the majority. Example:

Integer	Increasing CSR	Integer	Decreasing CSR
43046807		43046749	$9^8+7+6+5+4+3*2*1$
43046808	$(1+2)*3^(4+5+6)+78+9$	43046750	$9^8+7+6+5+4+3*2+1$
43046810		43046752	$9^8+7+6+5+4+3^2*1$
43046811		43046753	$9^8+7+6+5+4*3+2^1$
43046813		43046754	
43046814	$(1+2)*(3^(4+5+6)+7)+8*9$	43046759	
43046815		43046760	$9^8+7+6+5*4+3*2^1$
43046816		43046761	$9^8+7+6+5*4+3*2+1$
43046817	$(1+2)*3^(4+5+6)+7+89$	43046762	$9+8+(76+5)^4+3+21$
43046818		43046763	

See supplement 5 for the increasing series and supplement 6 for the decreasing series.

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