

Crazy Sequential Representations: Fill the Gaps (01)

A.E. Bras
V.H.J. van der Velden

Laboratory Medical Immunology (LMI), Department of Immunology, Erasmus Medical Center, Erasmus University, Rotterdam, the Netherlands
Correspondence: a.e.bras@gmail.com / a.bras@erasmusmc.nl / v.h.j.vandervelden@erasmusmc.nl

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Introduction

Others have attempted to write the natural numbers from 1 to 11111 in terms of 1 to 9 (in increasing and decreasing order) by using the operations of addition, subtraction, multiplication, division and/or potentiation (and optionally parentheses).

For example:

Number	Increasing	Decreasing
10957	$(1+2)^{(3+4)}*5-67+89$	$(9+8*7*65+4)*3-2*1$
10958		$(9+8*7*65+4)*3-2+1$
10959	$12+3+456*(7+8+9)$	$9+(8*76*(5+4)+3)*2*1$
10960	$12+(3^4+5+6)*7*(8+9)$	$9+(8*76*(5+4)+3)*2+1$
10961	$(1+2+34)*(5*6+7)*8+9$	$(9+8*7*65+4)*3+2*1$
10962	$12*3^4*5+678*9$	$9876+543*2*1$

Generally these expressions are referred to as crazy sequential representations (CSR). Interestingly, only one CSR remains to be identified, the increasing CSR for 10958.

Historic Overview

Inder Taneja published five papers on arXiv (for 1 up to 11111):

ARXIV Version	Evaluated Range	Allowed Operations	Missing Increasing	Missing Decreasing	Valid Representations
1 (06-02-2013) ¹	44 to 1000	+ * ^	2	10	1902 (of 1914)
2 (19-03-2013) ²	44 to 4444	+ * ^	50	53	8699 (of 8802)
3 (05-06-2013) ³	44 to 11111	+ * ^ ()	590	605	20941 (of 22136)
4 (05-08-2013) ⁴	0 to 11111	+ * ^ () -	449	315	21460 (of 22224)
5 (08-01-2014) ⁵	0 to 11111	+ * ^ () - /	9	10	22205 (of 22224)

Authors published three papers on Figshare/Zenodo (for -2147483647 up to 2147483647):

Date	Title
12-06-2018	Crazy Sequential Representations: Exhaustive Search ⁶
14-06-2018	Crazy Sequential Representations: Negative Integers ⁷
18-06-2018	Crazy Sequential Representations: Without Subtraction and/or Division ⁸

Inder Taneja published three papers on RGMIA (for 11112 up to 30000):

Date	Title
12-09-2018	Crazy Representations of Natural Numbers From 11112 to 20000 ⁹
10-11-2018	Crazy Representations of Natural Numbers From 20001 to 25000 ¹⁰
10-11-2018	Crazy Representations of Natural Numbers From 25001 to 30000 ¹¹

Authors published one paper on Figshare/Zenodo (comparing results for 11112 up to 30000):

Date	Title
06-12-2018	Crazy Sequential Representations: 11112 up to 30000 ¹²

Authors published one paper on Figshare/Zenodo (simplifications for our previous work^{6,7}):

Date	Title
14-12-2018	Crazy Sequential Representations: Simplifications (01) ¹³

Aim

Attempt to ‘fill the gaps’ in our previous work, thus;

Identify any CSR/NCSR for the integers without any CSR/NCSR within the -2147483647 up to 2147483647 range.

Existing Definitions

Default Notation

Notation as used by most programming languages, restricted to following characters:

1	2	3	4	5	6	7	8	9	+	-	*	/	^	()
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

Potential CSR/NCSR

Valid mathematical expression, thus well-formed interpretable syntactic construct, matching against either of the following regular expressions (using @ delimiter):

@^[+-*/^]*1[-+*/^]*2[-+*/^]*3[-+*/^]*4[-+*/^]*5[-+*/^]*6[-+*/^]*7[-+*/^]*8[-+*/^]*9[-+*/^]*\$
@^[+-*/^]*9[-+*/^]*8[-+*/^]*7[-+*/^]*6[-+*/^]*5[-+*/^]*4[-+*/^]*3[-+*/^]*2[-+*/^]*1[-+*/^]*\$

Ignoring evaluation result (natural, integer, real, rational, indeterminate, etc.).

In terms of 1 to 9

Digits 1 to 9 occur once and in order, either in increasing or decreasing order.
 Digits can be used as individual numbers (thus 1, 2, 3, 4, 5, 6, 7, 8 and 9).
 Digits can be concatenated into larger numbers (for example 123, 4, 5, 6 and 789).
 Negative counterparts of numbers may be used as well (also used by Inder Taneja).

Genuine CSR

Natural number (or zero) in terms of 1 to 9 (in increasing or decreasing order) by using the operations of addition, subtraction, multiplication, division and/or potentiation (and optionally parentheses).

Genuine NCSR

Negative integer (or zero) in terms of 1 to 9 (in increasing or decreasing order) by using the operations of addition, subtraction, multiplication, division and/or potentiation (and optionally parentheses).

Pseudo CSR

Potential non-genuine CSR evaluating to **natural number** (or zero).
 For example, expressions with implicit multiplication by minus one.

Pseudo NCSR

Potential non-genuine NCSR evaluating to **negative integer** (or zero).
 For example, expressions with implicit multiplication by minus one.

Methods

Available CSR/NCSR were extracted from the following supplements:

Date	Title
12-06-2018	Crazy Sequential Representations: Exhaustive Search ⁶ - Supplement 1 : Increasing CSR for 0 up to 11111 - Supplement 2 : Decreasing CSR for 0 up to 11111 - Supplement 3 : Increasing CSR for 11112 up to 2147483647 - Supplement 4 : Decreasing CSR for 11112 up to 2147483647
14-06-2018	Crazy Sequential Representations: Negative Integers ⁷ - Supplement 1 : Increasing NCSR for -1 down to -11111 - Supplement 2 : Decreasing NCSR for -1 down to -11111 - Supplement 3 : Increasing NCSR for -11112 down to -2147483647 - Supplement 4 : Decreasing NCSR for -11112 down to -2147483647
06-12-2018	Crazy Sequential Representations: 11112 up to 30000 ¹² - Supplement 1 : Decreasing and increasing CSR for 11112 up to 30000

The ‘integers without any CSR/NCSR within the -2147483647 up to 2147483647 range’ were identified, and authors attempted to identify any CSR/NCSR for these integers.

For example, within the 78560 up to 78570 range...

	Increasing CSR	Decreasing CSR
78560	$(1^{23}+4)*(5^6+78+9)$	$9*8765-4-321$
78561	$(-12+3)*(-4^5-67)*8+9$	$9*(8765-4-32)^1$
78562	$1+(-2+3+(4^5+67)*8)*9$	$9-8*7+(6*5+4)^3*2+1$
78563		
78564	$(-1/(-2*-3)-(4^5+67))*-8*9$	
78565		$9-(-((8+7)^6)+5)/((-4*3)^2+1)$
78566	$12^3/4+5^(6-7+8)+9$	$-9*-8*-(7+6)-(5-43^(2+1))$
78567	$-12-(-3+(-4^5-67)*8)*9$	
78568		$9*8765+4-321$
78569	$-1+(-23*-4+5)*-6*(-7-8)*9$	$-9-(-8-7+(6*5+4)^3)*-2/1$
78570	$(1*-23*-4+5)*-6*(-7-8)*9$	$9+87^(6-5)*43*21$

Authors attempted to identify a CSR for 78563...

- Either increasing and/or decreasing.
- Preferably genuine, otherwise pseudo.

Final Notes

Authors consider following CSR/NCSR to be proof-of-work, as identification of CSR/NCSR is computationally expensive, while verification of CSR/NCSR is computationally inexpensive.

Authors do not guaranty:

- Published CSR/NCSR are the shortest CSR/NCSR in existence.
- Published CSR/NCSR are in their simplest form.
- Unavailable CSR/NCSR do not exists.

Results

For the 'integers without any CSR/NCSR within the -2147483647 up to 2147483647 range'...

Authors identified 84440 increasing/decreasing CSR/NCSR:

	Increasing CSR	Decreasing CSR	Increasing NCSR	Decreasing NCSR
Total	18139	28595	17855	19851

Authors identified 80301 genuine CSR/NCSR and 4139 pseudo CSR/NCSR:

	Increasing CSR	Decreasing CSR	Increasing NCSR	Decreasing NCSR
Genuine	17832	28019	17497	16953
Pseudo	307	576	358	2898

For 15692 integers, increasing CSR and increasing 'counterpart NCSR' were identified:

	CSR		NCSR
8343127	$((1+23)/4*5^6-7)*89$	-8343127	$((1^2-3-4)*5^6+7)*89$
38999992	$1+2^3*4*5^6*78-9$	-38999992	$-1+(2-34)/(5^6-6/78)+9$
51885627	$(12*34-5)*6+7^8*9$	-51885627	$12-3^4*5*6-7^8*9$

For 17301 integers, decreasing CSR and decreasing 'counterpart NCSR' were identified:

	CSR		NCSR
259999	$-9+8+(7+6)*5^4*32*1$	-259999	$(9-87)/6*5^4*32+1$
470739	$9*8*(7^6/(54/3)+2)-1$	-470739	$9-8*(7^6-5+43)/2^1$
813678	$-9*8+7*6/(5^4-(32-1))$	-813678	$9-((8*7+6)*5^4-3)*21$

Sometimes, the shortest CSR and shortest 'counterpart NCSR' were nearly identical:

	CSR		NCSR
207470336	$1+2-3-4*(5^6-7^8*9)$	-207470336	$1+2-3+4*(5^6-7^8*9)$
207595336	$1+2-3+4*(5^6+7^8*9)$	-207595336	$1+2-3-4*(5^6+7^8*9)$
95006	$9-8+(76*5^4+3)*2-1$	-95006	$9-8-(76*5^4+3)*2-1$
142348	$9-8+76*(5^4*3-2)-1$	-142348	$9-8-76*(5^4*3-2)-1$
19216820	$98*(7^6+5)*(4/3*2-1)$	-19216820	$98*(7^6+5)*(4/3-2-1)$

Frequently, the shortest CSR and shortest 'counterpart NCSR' were completely different:

	CSR		NCSR
233132	$(1+23^4*5)/6-78+9$	-233132	$1234-5^6*(7+8)+9$
265510	$(1-2)^3*-4+(5^6-7)*(8+9)$	-265510	$1*234-(5^6+7)*(8+9)$
232500	$(9+8+76)*5^4*(3+2-1)$	-232500	$(9-8-7-6)/(5^4-(32-1))$
470451	$9*8*(7^6/(54/3)-2)-1$	-470451	$98-(7^6-5)*4+3^(2+1)$
259999	$-9+8+(7+6)*5^4*32*1$	-259999	$(9-87)/6*5^4*32+1$

For 23 integers, increasing and decreasing CSR were identified:

	Increasing CSR	Decreasing CSR
71321	$(12 \cdot 3^4 - 5) \cdot (6 - 7 - 8 \cdot 9)$	$9 \cdot 8 + 76 \cdot 5^4 \cdot 3 / 2 - 1$
94989	$(1 + 2) \cdot (3^4 \cdot 56 \cdot 7 - 89)$	$-9 - 8 + (76 \cdot 5^4 + 3) \cdot 2 \cdot 1$
95474	$12^3 - 4 + 5^6 \cdot (7 + 8 - 9)$	$9 + 8 + 76 \cdot (5^4 + 3) \cdot 2 + 1$

For 18 integers, increasing and decreasing NCSR were identified:

	Increasing NCSR	Decreasing NCSR
-94989	$(1 + 2) \cdot (3^4 \cdot 56 \cdot 7 + 89)$	$9 + 8 - (76 \cdot 5^4 + 3) \cdot 2 \cdot 1$
-216349	$1 \cdot 2 - 3^4 \cdot (5 \cdot 6 \cdot 7 \cdot 8 - 9)$	$(-9 + (8 \cdot 7 - 6 \cdot 5) \cdot 4 / 3) \cdot 2 \cdot 1$
-232500	$12 \cdot ((3^4 \cdot 5 \cdot 6 - 7) \cdot -8 + 9)$	$(9 - 8 - 7 - 6) / (5^4 - 4 / (32 - 1))$

Newly identified CSR/NCSR were tabulated in the following supplements:

- Supplement 1 : Increasing CSR within the 1 up to 2147483647 range
- Supplement 2 : Decreasing CSR within the 1 up to 2147483647 range
- Supplement 3 : Increasing NCSR within the -1 down to -2147483647 range
- Supplement 4 : Decreasing NCSR within the -1 down to -2147483647 range

Discussion

For various CSR/NCSR no 'counterpart CSR/NCSR' was identified. For example:

CSR		NCSR	
31811	$1 - (2 - 3^4 \cdot 56) \cdot 7 + 8 \cdot 9$	-35732	$1 - (2 + (3^4 - 5) \cdot 6) \cdot 78 - 9$
31835	$(12 + 3^4 \cdot 56) \cdot 7 + 8 - 9$	-36332	$1 + 2 - (3^4 \cdot 56 + 7) \cdot 8 + 9$
31844	$1 + 2 + 3^4 \cdot 56 \cdot 7 + 89$	-40412	$1 - (2 + (3^4 + 5) \cdot 6) \cdot 78 - 9$
31696	$9 \cdot 8 \cdot 7 \cdot (-6 / 54 + 3 \cdot 21)$	-71177	$9 \cdot 8 - 76 \cdot 5^4 \cdot 3 / 2 + 1$
35483	$(9 + 8 \cdot (7 + 6)) \cdot (5^4 + 3) / 2 + 1$	-77686	$98 \cdot 7 - 6 \cdot (5^4 - 3) \cdot 21$
46472	$(9 \cdot 8 + 76) \cdot (5^4 + 3) / 2 \cdot 1$	-78362	$9 + 8 - 7 - 6 \cdot (5^4 - 3) \cdot 21$

Obviously the 'counterpart CSR/NCSR' can be easily derived by applying a final negation:

NCSR		Counterpart CSR	
-31811	$-(1 - (2 - 3^4 \cdot 56) \cdot 7 + 8 \cdot 9)$	35732	$-(1 - (2 + (3^4 - 5) \cdot 6) \cdot 78 - 9)$
-31835	$-((12 + 3^4 \cdot 56) \cdot 7 + 8 - 9)$	36332	$-(1 + 2 - (3^4 \cdot 56 + 7) \cdot 8 + 9)$
-31844	$-(1 + 2 + 3^4 \cdot 56 \cdot 7 + 89)$	40412	$-(1 - (2 + (3^4 + 5) \cdot 6) \cdot 78 - 9)$
-31696	$-(9 \cdot 8 \cdot 7 \cdot (-6 / 54 + 3 \cdot 21))$	71177	$-(9 \cdot 8 - 76 \cdot 5^4 \cdot 3 / 2 + 1)$
-35483	$-((9 + 8 \cdot (7 + 6)) \cdot (5^4 + 3) / 2 + 1)$	77686	$-(98 \cdot 7 - 6 \cdot (5^4 - 3) \cdot 21)$
-46472	$-((9 \cdot 8 + 76) \cdot (5^4 + 3) / 2 \cdot 1)$	78362	$-(9 + 8 - 7 - 6 \cdot (5^4 - 3) \cdot 21)$

However, authors were only interested in CSR/NCSR without a final negation!

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