THE GRAPHICAL METHOD VERSUS THAT BY LEAST SQUARES.

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In some recent articles upon the Method of Least Squares, prepared for the JOURNAL OF THE FRANKLIN INSTITUTE by Professor Merriman, of the Sheffield Scientific School, he has selected, as an example, the mean curve of observation given in "The Physics and Hydraulics of the Mississippi," showing the law of change in velocity from surface to bottom in a vertical plane parallel to the current. By applying the method of least squares to the observations, he has deduced new values for the constants of the formula, which give a rather closer accordance than those determined by the graphical method in the original Report. The absolute differences in the two curves, however, are trifling; the largest, which is more than double any of the rest, being only five-thousandths of a foot per second—a quantity too small to be detected by measurement.

Every mathematician, of course, will admit that the method of least squares is the most accurate of any for fixing the values of the constants in such cases, but I cannot agree with Prof. Merriman in characterizing the graphical method as "very unscientific." In my judgment, each of the two methods has its advantages and its disadvantages, and true science requires that the best selection shall be made for the particular case in hand.

The method of least squares is, so to speak, mechanical. It leaves no scope for judgment on the part of the investigator, and any numerical mistake in the long and tedious computation is not readily detected, and may often vitiate the final result. By the graphical method the mind grasps the whole problem; the eye perceives how each given point of the curve deviates from the general law, and if any of them are more doubtful than the others, their weight can be intelligently regulated. In general, I consider that where the data are exact and sufficient, and the labor of computation is warranted by the importance of the result, the method of least squares should have the preference. In other cases, I should use the graphical method.

In this particular case of the grand mean curve of velocity, the method of least squares can advantageously be applied; and if this

curve had been the only problem of the kind under discussion at the time, preference would naturally have been given to the more exact method.

It is not stated in the report, and it probably did not suggest itself to Prof. Merriman in considering the matter, that with so many similar curves to discuss, we could greatly abbreviate the labor of the graphical method. The different curves were all plotted on the same scale on accurate section paper, printed from an engraved stone. One general set of parabolas, which, perhaps, required a couple of hours to compute, was plotted on a piece of tracing paper, with a common vertex. By placing this over a plotted set of observations, a parameter, very closely according with the data, could be taken off at sight. Starting with this, the labor of fitting the curve to the observations was neither excessive nor tedious, and any error of computation was at once detected. Having become habituated to this method, it was naturally used in framing the final equation of the grand mean curve; although in this case it might probably have been better to use the method of least squares. The two results, however, as already stated, do not differ within the limits of measurement.

That we derived great assistance from the general use of the graphical method is incontestable, and I think this will be the experience of any investigator who gives it a fair trial. By no other method are the results so vividly presented to the mind at every step of the study. Indeed, a foreign writer, after discussing many of the old observations made upon the rivers of Europe, and showing graphically that they accord, in a surprising degree, with our new subsurface theory, attributes the failure to discover the law from them to the fact that they had been discussed by a purely mathematical, and not by the graphical, method. The expression, "very unscientific," seems, therefore, rather too sweeping.

I will add that it is a matter of regret that the slight numerical error, in the formula pointed out by Prof. Merriman, had not been discovered before the second edition of "The Physics and Hydraulics of the Mississippi" passed through the press. It should be, as he suggests: $V = -0{\cdot}79222\ d_{11}^2 + 3{\cdot}2600$,

instead of $V = -0{\cdot}79222\ d_{11}^2 + 3{\cdot}2611$.