PROCEEDINGS

OF THE

AMERICAN PHYSICAL SOCIETY.

MINUTES OF THE STANFORD UNIVERSITY MEETING, JANUARY 22, 1921.

THE 107th meeting of the American Physical Society was held in the lecture room of the Department of Physics, Stanford University, January 22, 1921, at two o'clock. Professor Fernando Sanford presided. About fifty persons were in attendance.

The following program was presented:

The Balmer Series of Hydrogen and the Quantum Theory of Line Spectra. R. T. BIRGE.

Charts as an Aid in Teaching Electricity. F. J. ROGERS.

Damped Electrical Oscillations. R. B. ABBOTT.

A Thermoelectric Motor. PAUL KIRKPATRICK.

A Wave Composition Model. JOSEPH G. BROWN.

Diurnal Variations in Terrestrial Magnetism and in the Electrical Potential of the Earth. FERNANDO SANFORD.

Sunspots and Terrestrial Magnetic Storms. FERNANDO SANFORD.

Note on the Fahrenheit Scale. FLORIAN CAJORI.

Nature of Ionization in the Point to Plate Discharge. EVALD ANDERSON.

An Explanation of X-Ray Diffraction Patterns from Rolled Metals. C. T. DOZIER.

The Concentration of Monochromatic X-Rays by Crystal Reflection. ELMER DERSHEM and C. T. DOZIER.

At the conclusion of this program, Professor D. L. Webster showed and explained his high potential X-ray outfit.

Thirty-six persons were present at the dinner served at Stanford Union at six-thirty o'clock.

E. P. LEWIS,
Local Secretary for the Pacific Coast.

THE BALMER SERIES OF HYDROGEN, AND THE QUANTUM THEORY OF LINE SPECTRA.

BY RAYMOND T. BIRGE.

THE purpose of this paper is to make a rigid comparison between the experimental results for the Balmer series of hydrogen, and the theoretical developments of the quantum theory of line spectra. In particular, Merton's experimental results for the separation, half-width, and relative intensity of the H_α and H_β doublets are examined in the light of the recent Bohr-Kramers' work on the intensity of the fine structure components, as estimated on the basis of the "selective" principle.

It is concluded that, by assuming a general electric field of 100 volts per cm., all of Merton's experimental results check quantitatively with theory. The frequencies of all the lines of the Balmer series are then fitted into the Bohr relativity formula, after the fine structure has been analyzed and the "circle" line computed, using the Sommerfeld theory to determine the position of the fine structure components, and the Bohr-Kramers' theory to determine the relative intensities. The Rydberg constant for hydrogen is calculated to be 109,677.7 ± 0.2, and the constant for a nucleus of infinite mass 109,736.9 ± 0.2.

The paper concluded with a brief discussion of Merton and Nicholson's observations on the appearance of the Balmer series in mixtures of hydrogen and helium, at relatively high pressure.

University of California.

Lecture Room Wall Charts.

By F. J. Rogers.

LECTURES in physics, particularly in electricity and mganetism, demand more or less profuse illustration by plots and diagrams. This may be secured by blackboard diagrams, lantern slides, or wall charts. The latter have marked advantages, chief of which is the fact that they continue silently to proclaim their messages so long as they hang on the wall.

I have had constructed about fifty charts illustrating all branches of electricity and magnetism. They are uniformly four feet square, and when stored are neither folded nor rolled but are hung compactly on a rack one behind another so that any one can be removed sideways without disturbing any of the others. One or two colors besides black are used when this will contribute to clearness of exposition. To the same end, explanatory words and phrases are printed on the charts so that the meaning of the chart may be conveyed with the minimum of explanation. It has been found that letters an inch square or somewhat less, are amply large enough to be readily visible to a class of a hundred.

The content of the message conveyed by a chart varies more or less continuously between two extremes. In the case of one extreme the chart is expected to present a single fact or principle rather concretely, precisely, and with reference to a real experiment. As an example consider the chart, "Lines of Force of a Magnet." A long slim magnet was laid on a sheet of paper and its field carefully plotted over the whole area, four feet square. The effect of the earth's field was cancelled. The result at first glance is quite familiar, but a second glance shows that many lines near the poles have a double curvature, which I have never seen represented in any published diagram. As a second illustration consider the chart representing "Electric Flux-densities for Equal Voltage Gradients" and "Voltage Gradients Required to Produce Equal Flux Densities." Along with the diagrams representing flux densities for condensers in parallel and voltage gradients for condensers in series are given numerical data for common dielectrics.

Charts representing the other extreme are intended to illustrate a series of related facts, principles, and experiments, as fully as may be done in a single chart. As an example let us consider the chart, "The Effect of Temperature on Resistance." This chart represents graphically the resistance of ten metals, three alloys, carbon, and electrolytes at temperatures from $-180°$ C. to $500°$ C. All are assumed to have a resistance of 100 at $0°$ C. Most pure solid metals are represented by lines very close together, with iron and nickel increasing in resistance considerably faster with temperature than the others. The alloys and carbon are represented by lines which are nearly horizontal while electrolytes are shown to change at a very much greater rate in the opposite direction. The changes of resistance upon melting of Hg., Sn., Pb., Bi., and Zn. are shown. On the charts are the following brief statements: "Melting a metal doubles its resistance, Bi and Sb exceptions." "Glass and many salts become conductors at red heat or below." "All insulators become conductors above $2000°$ C."

The chart "Vacuum Tube Rays" may be taken as another example of charts with a multiple message. A diagram of tube with a pierced cathode has properly placed a single loop carrying a current, beyond the cathode is a "magnetic screen" and beyond that an electromagnet. The "cathode rays" represented by a series of dots, "Electrons" are curved by the "weak magnetic field" and strike the "Anti-cathode," from which radiate "X-rays" represented as a series of concentric wave crests called "ether waves." Passing through the pierced cathode and slightly deflected by the "strong magnetic field" on the "canal rays" or "positive atoms" which are represented by a series of dots much larger than those representing the cathode rays. On the same chart is a diagram of a "Coolidge tube" showing the "hot cathode" in series with a battery and the tungsten anti-cathode. "Cathode rays" are represented as striking the anti-cathode in a "highest vacuum" and "X-rays" are represented as radiating from the anti-cathode and through the glass walls of the tube.

Stanford University.

Damped Electrical Oscillations.

By R. B. Abbott.

THE subject of "Damped Electric Oscillations" can be treated by the method of analogy. The parallelism between the dynamical and electrical equations for simple harmonic motion and logarithmically damped motion is well known. A parallelism exists between the electrical and dynamical equations for linearly damped motion and for combined linearly and logarithmically damped motion. The solutions in the last two cases named above are easily obtained, by analogy and a greater simplification is obtained in the solution of the latter case than has hitherto been obtained.

Oscillatory motion is most easily described in terms of the projection of a radius vector r, rotating with constant angular velocity w, about a center c. Let the radius vector make an angle θ with the X-axis, passing through the center c such that $\theta = wt$, where t is the time from the instant when the radius vector coincides with the positive X-axis. The motion may be described as taking place in either X or Y, thus: the displacement $x = r \cos wt$ and $y = r \sin wt$.

The point P at the end of r traces out some curve such as a circle, ellipse or some form of spiral. If there is no dissipation of energy during periodic motion, the auxiliary curves will be of the spiral family: the auxiliary curve for damping proportional to the speed being the logarithmic spiral and for constant damping, the rectilinear (Archimedes') spiral.

The differential equations expressing the acceleration in free periodic motion such as the cases above mentioned, are of the second order with coefficients which are either constants or expressible as functions of the time. If p and q are such coefficients, the general equation is written as follows:

$$d^2y/dt^2 + pdy/dt + qy = 0.$$

By reference to the auxiliary curves, the values of p and q can be determined for all classes of periodic motion, and their differential equations can be written.

In the case of logarithmic damping, r decreases at a rate proportional to itself.

$$\frac{dr}{dt} = -kr,$$

from which $\log r/r_0 = kt$ and $r = r_0e^{-kt}$. Therefore the auxiliary curve is a logarithmic spiral. In the case of rectilinear damping, r decreases at a constant rate,

$$dr/dt = -k_1,$$

from which $r = r_0 - k_1t$.

In the case of combined logarithmic and rectilinear damping, it is easily shown that $r = r_0 - k_1te^{-kt}$.

In each case substitute $y = r \sin \theta$ in the general equation. By comparison with the electrical cases, the constants can be determined in terms of C, L, and R.

Washington State College.

A Thermo Electric Motor.

By Paul Kirkpatrick.

THE reaction of thermo-electric currents to a magnetic field is familiarly shown in the radio-micrometer. If the suspended system of the radio-micrometer were free to turn without constraint and the radiant impulses communicated to its juncture intermittently the coil might be caused to rotate continuously. Such an instrument, which may be called a thermo electric motor, has been constructed.

The armature possesses a number of thermo couples disposed in a circle about the intended axis of rotation. An opaque shade screens half of these while the remainder are exposed to a radiation, say of the sun. Currents traverse a net of wires connecting the couples and, being mounted in the field of a permanent magnet, the system starts to rotate. The successive eclipses of the junctures serve the purpose of commutator and brushes and the forward rotation is maintained.

In the present model there are six silver-German silver junctions and six conductors in the drum armature. The axis of rotation is vertical and the armature carried on a single jewel. The magnet poles are shaped closely about the armature and a fixed iron core protrudes into the latter. The motor will operate under the action of sunshine or any artificial radiation of proper intensity.

UNIVERSITY OF CALIFORNIA.

A WAVE COMPOSITION MODEL.

BY J. G. BROWN.

A MODEL designed to show the simultaneous production of transverse and longitudinal waves was shown at a previous meeting of the Physical Society. The present model, based upon the same principle, shows the composition of two transverse waves in the same plane and also in planes at right angles.

Each component wave is produced by the rotation of a shaft having cranks set at uniform distances and with constant difference of phase along the entire shaft. By acting upon rods, these shafts convey their motions to two rows of balls, the motions being combined in the same direction in the upper row and at right angles in the lower row.

The wave-length of the back component is fixed at 18 inches, but the front component can be varied by changing shafts. Wave-lengths of 18 inches, 12 inches and 9 inches are provided, giving ratios of 1 : 1, 1 : 2 and 2 : 3.

The shafts are connected by chain and sprockets and the relative frequencies are determined by the size of the sprockets. Ratios of 1 : 1, 1 : 2, 2 : 3 and 3 : 4 are provided.

Relative velocities are determined by the combination of wave-lengths and frequencies. The direction of the wave components may be made the same or opposite by means of a reversing gear.

The phase of the wave components can be fixed by the positions of the shafts when the gears are enmeshed.

The model is especially designed for laboratory use, although it will show certain things very well to large groups.

Some of the phenomena which can be illustrated are the following:

1. Variation of amplitude with phase in the interference of similar plane polarized waves.
2. Standing waves. Reflection at a dense medium and at a rare medium.
3. Elliptically and circularly polarized waves.
4. Beats between similar waves.
5. Plane polarized white light in a vacuum.
6. Unpolarized or ordinary white light in a vacuum.
7. Plane polarized white light in glass. Ordinary refraction and dispersion.
8. Ordinary monochromatic light in calcite. Double refraction.

A trace of the motion of any ball can be taken at any time so that the motion of the parts of the medium as well as the distortion of the medium can be studied in detail.

STANFORD UNIVERSITY.

DIURNAL VARIATIONS IN TERRESTRIAL MAGNETISM AND IN THE ELECTRICAL POTENTIAL OF THE EARTH.

BY FERNANDO SANFORD.

IN an article entitled "The Electrostatic Charges of the Earth and Sun and their Relation to Terrestrial Magnetism" which was presented at the Cleveland meeting of the American Physical Society on November 27, 1920, were given some curves showing the diurnal variation of the earth's potential at Palo Alto, California, for the month of August, 1920, and a curve showing the

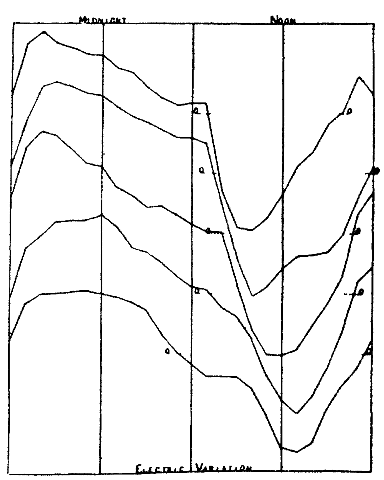

Fig. 1.

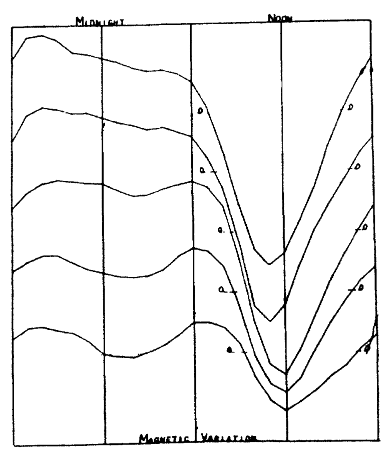

Fig. 2.

mean diurnal variation of the N component of terrestrial magnetism at the Falmouth, England, magnetic observatory for the month of August for the twelve years, 1891–1902. The measurements of the potential change at Palo Alto have been continued, and in Table I. are given the data expressed in millivolts of the observed change for the months August, September, October, November and December, 1920. Along with these, for purposes of comparison, are given Chree's values of the mean diurnal variation of the *N* magnetic component for the corresponding months for the twelve years above mentioned. The actual numbers given by Chree have all been divided by 5, thus making the magnetic force unit used $5 \cdot 10^{-6}$ C.G.S. unit, instead of $1 \cdot 10^{-6}$ C.G.S.

TABLE I.

Diurnal Variation in Millivolts in the Negative Charge of the Earth at Palo Alto, California, Compared with the Diurnal Variation in the N Component of Terrestrial Magnetism for the Corresponding Months at Falmouth, England, Expressed in $5 \cdot 10^{-6}$ C.G.S. Units.

The positive sign before the quantities in the table of electrical variations does not mean an increase of the positive potential of the earth, but an increase in the negative potential. It is used in this manner because an increase in the negative potential at a given place should cause an increase in the N magnetic component at that place.

Electric Variation.					Hr.	Magnetic Variation.				
Aug.	Sept.	Oct.	Nov.	Dec.		Aug.	Sept.	Oct.	Nov.	Dec.
+ 6.	+ 2.2	+11.8	− 1.2	+ 4.6	6 p.m.	+18.4	+11.4	+ 8.8	8.0	+ 5.6
+23.3	+17.3	+28.3	+14.8	+16.5	7 "	+26.0	+19.8	+13.8	+10.8	+ 9.0
+27.3	+28.7	+33.8	+19.3	+19.9	8 "	+17.2	+22.4	+16.6	+13.0	+ 9.6
+23.3	+30.2	+32.6	+23.8	+20.0	9 "	+25.4	+22.0	+17.8	+13.0	+ 8.2
+21.7	+28.7	+27.7	+24.0	+20.4	10 "	+21.2	+20.4	+17.4	+10.6	+ 7.4
+19.5	+26.7	+23.2	+24.0	+21.0	11 "	+20.6	+20.6	+17.0	+ 8.4	+ 4.6
+19.2	+25.7	+22.0	+25.8	+19.9	12	+19.2	+19.2	+16.6	+ 7.2	+ 0.4
+14.7	+22.2	+15.1	+22.0	+18.9	1 a.m.	+17.8	+17.6	+14.4	+ 5.8	− 0.8
+13.7	+19.2	+12.5	+15.3	+17.2	2 "	+15.8	+16.2	+12.6	+ 5.6	− 1.0
+ 8.7	+16.8	+ 8.5	+11.8	+14.2	3 "	+14.8	+14.8	+12.8	+ 6.0	+ 0.6
+ 5.0	+14.2	+ 9.0	+10.3	+ 6.6	4 "	+14.8	+15.6	+14.6	+ 8.0	+ 2.8
+ 3.0	+12.0	+ 6.2	+ 6.0	+ 0.8	5 "	+13.8	+14.0	+16.0	+12.4	+ 6.0
+ 3.5	+11.7	+ 3.0	+ 2.5	− 3.3	6 "	+11.0	+12.6	+17.0	+15.0	+10.2
+ 3.5	+10.0	+ 1.0	+ 1.5	− 7.3	7 "	+ 2.6	+ 5.2	+15.2	+14.0	+10.2
−26.0	−10.0	+ 0.6	− 4.4	− 7.4	8 "	−12.2	− 7.0	+ 8.6	+ 8.8	+ 8.4
−38.0	−29.3	−17.7	− 7.4	− 7.7	9 "	−30.4	−27.0	− 8.8	− 4.4	+ 2.8
−39.0	−41.0	−32.1	−14.7	−11.9	10 "	−45.4	−44.4	−30.0	−21.0	− 8.0
−34.8	−37.7	−40.3	−27.2	−20.5	11 "	−51.0	−49.8	−44.0	−30.6	−16.2
−27.3	−31.7	−40.8	−35.9	−31.9	12	−46.8	−44.2	−47.2	−33.2	−19.6
−17.5	−27.8	−39.0	−40.2	−33.2	1 p.m.	−36.8	−31.4	−39.2	−28.6	−16.6
−13.0	−27.4	−31.0	−34.4	−30.1	2 "	−26.2	−19.6	−27.4	−19.0	−12.6
− 3.7	−26.3	−23.8	−24.9	−19.3	3 "	−11.6	−10.8	−16.6	−10.4	− 7.8
+ 0.7	−20.2	−14.0	−12.5	−10.6	4 "	+ 1.2	− 3.0	− 7.4	− 3.0	− 4.2
+12.3	− 7.7	+ 6.6	+ 4.0	− 4.7	5 "	+10.8	+ 5.4	+ 1.4	+ 3.8	+ 1.6

In the table, an increase in the negative potential of the earth is marked + and a decrease −, in order to bring them into correspondence with the magnetic data, since an increase in negative potential should cause an increase in the *N* magnetic component.

The two sets of data are also expressed graphically in Fig. 1 and Fig. 2.

PALO ALTO, CALIFORNIA.

Sun Spots and Terrestrial Magnetic Storms.

By Fernando Sanford.

It has long been known that a close relation exists between sun spots and the irregular magnetic disturbances known as magnetic storms, but no satisfactory reason for this relation has been shown.

The work of Hale and others, at Mt. Wilson, has shown that sun spots are the visible manifestation of tremendous cyclonic disturbances upon the sun. It has also been shown that powerful magnetic fields are produced over large areas in the centers of these cyclonic vortices, which fields are apparently due to convection currents set up by the rotation of negative ions around the storm centers.

Störmer's calculation of the electric and magnetic forces involved in a given sunspot show them to be great enough to produce strong induced currents at the distance of the earth.

It has frequently been shown that magnetic storms have the characteristics which would follow from vortex currents in the earth or in the upper atmosphere. It has been assumed that these currents are due to electrons expelled from the sun and forced into spiral paths by the earth's magnetic field.

The fact that both clockwise and counter-clockwise vortices are shown by magnetic storms in the same regions indicate that their rotational motion is not caused by the earth's magnetic field. Also, it is known that the sun has a magnetic field about 50 times as strong as the earth's field, hence no electrons can be expelled from the sun except in the direction of its magnetic axis, and such electrons would not reach the earth.

The diurnal variations in the earth's electrical potential which have been recorded at Palo Alto seem to show that the earth, as well as the sun, is highly electrified, and with a negative charge. This gives plausibility to the assumption that the other planets are similarly electrified, and furnishes a theoretical basis for the conclusion, hitherto based upon empirical data, that the periodicity of sun spots depends upon the relative positions of the planets. Reference is made to the evidence furnished by La Rue, Stewart and Loewy that the sun spot area varies with the angular distance between Mercury and Venus and to the fact that the mean sun spot period of a little more than 11 years is closely related to an integral number of revolutions of each of the nearer planets.

Thus: Mercury	makes 45	revolutions in	11	years;
Venus	makes 18	revolutions in	11.08	years;
Earth	makes 11	revolutions in	11	years;
Mars	makes 6	revolutions in	11.25	years;
Jupiter	makes 1	revolution in	11.8	years.

Attention is also called to the fact that the diurnal variation of the N component of terrestrial magnetism has been shown by Chree to be about 70 per cent. greater in the four-year period including sunspot maximum than in the corresponding period at sunspot minimum. This phenomenon can not be due to the sun spots, themselves, since it is shown by the records of days of least magnetic disturbance. From the point of view of the present writer, such a variation could be caused by the sun only if the total electric charge of the sun varied periodically with the sunspots. Apparently, both the disturbances in the sun's electric field which gives rise to sun spots and the increase in the diurnal variation of the earth's magnetic field must be due to electrical induction by other charges in the solar system.

STANFORD UNIVERSITY.

Note on the Fahrenheit Scale.

BY FLORIAN CAJORI.

QUOTATIONS were read from Fahrenheit's five papers in the Philosophical Transactions of London, for the year 1724, the three parts describing the thermometers Fahrenheit had used for the ten preceding years. Conclusions: (1) From the first quotation it follows that Fahrenheit used as fixed points the ice-water-salt or sal-ammoniac temperature and blood temperature; the interval was divided into 4 × 24 or 96 steps; (2) from the second quotation it follows that Fahrenheit (unlike other experimenters) used also a third point, the ice-water temperature, presumably for more perfect checking; (3) from all three quotations it follows that previous to 1724 the boiling point of water was not used by Fahrenheit as a fixed point in the graduation of his thermometers, that continuing his scale upward by equal steps the boiling point of water fell at 212°. On his scale the freezing point of water chanced to come at 32°. It is not known whether or not Fahrenheit changed his fixed points to the freezing and boiling points of water after the year 1724. A disregard of Fahrenheit's own statements has led to many false and contradictory statements, such as: (1) Before 1724 he used 32° and 212° as fixed points in his graduations; (2) the interval between ice-water-salt and ice-water was subdivided by continued bisection into 32° and the scale built up from these degrees; (3) he borrowed the number 96 of his subdivisions from Florentine thermometers; (4) he used as fixed points ice-water-salt and boiling mercury, subdividing the interval into 600 degrees; (5) he regarded his zero as an absolute zero (very probably untrue).

Corroboration of the conclusions of this paper is found in two contemporary writers, namely, C. Wolf in Acta Eruditorum for 1714 and C. Kirch in Miscellanea Berolinensia, T. VI, printed 1740. Both of them owned thermometers made by Fahrenheit.

Nature of Ionization in the Point to Plate Discharge.

BY EVALD ANDERSON.

IT has long been known that in the so-called point to plate, electrical unidirectional discharge the ions are principally of the same sign as the point. In 1911 Zeleny tried some experiments, the results of which indicated that at moderate currents, and gas pressures above half an atmosphere, very few if any ions of the opposite sign to the point, were present, while at lower pressures, these ions became more and more numerous. Field exploration experiments by H. T. Booth, in 1917 indicated the presence of ions of the opposite kind, but an investigation by Tolman and Carrer gave results which were not in accord with this latter conclusion. However, certain phenomena in connection with so-called electrical precipitation seem to show that at least at the higher voltages ions of the opposite sign are present.

Experiments were made for the purpose of throwing further light on this point. The method consisted in measuring the potential acquired by an insulated wire probe between the point and plate and determining how this potential varied with the potential of the point. The distance between the point and the plate was in all cases 25 mm. The probe was usually a fine wire bent in a small loop. It was carefully insulated from both the point and and plate. The potential was measured by a Braun type voltmeter reading to 5,000 volts, and by an attraction disk voltmeter, reading to 50,000 volts. Since both the potential of the point and the probe were measured on the same instruments the comparative values are correct, and independent of the absolute calibration of the instruments. The gas was air in all the experiments, and the pressure was either 10 or 76 cm. In the former case the air was dried over P_2O_5, but in the latter no drying agent was used.

For the low pressure experiments, a battery of ten 600 volts D.C. generators, connected in series, supplied current at a potential up to 6,000 volts, while for the other experiments a 50,000 volt D.C. generator was used.

At a pressure of 10 cm. the potential of the probe varied directly as that of the point up to the maximum potential tried, 4,800 volts. The arcing potential for this pressure and electrode setting was about 5,600 volts, for either positive or negative point. It is evident from the plotted results that the sign of the point does not markedly influence the character of the curve. The current passing, was, however, approximately three times as large when the point was negative.

The curves for atmospheric pressure are markedly different. In the first place, the arcing potential is approximately twice as high when the point is negative as when it is positive, although the potential at which ionization begins is nearly the same in both cases. Thus the curves for the point negative all show a break at about 30,000 volts, indicating that at this point the relative number of positive ions is suddenly increased. This could be explained by the assumption that at this potential the negative ions had sufficient energy to ionize the air at or in the surface of the plate. This explanation is supported by the phenomena noted when a thin plate of mica was placed on the plate so that its edge was directly under the point and probe. It has long been known that the presence of such a dielectric at the positive terminal causes ionization at this terminal. Such ionization is easily detectable by the characteristic light given off where positive ions are formed. In this case the break in the curve with the mica on the plate occurred at 24,000 volts, instead of at 30,000 volts, without the mica. The arcing voltage was also correspondingly lowered.

Curves showing the current, and the probe potential as functions of the potential of the point for one position of the probe, with and without mica on the plate indicate that at low potentials, the mica simply acts as an insulator, raising the potential of the probe and lowering the current. At higher potentials when ionization sets in at the edge of the mica, the potential of the probe is lowered, due to the positive ions, and the current is greatly increased. Sharp points, on the plate, as iron filings, have effects similar to an insulator, in that they lower both the arcing potential and the potential of the probe.

With the point positive none of the above effects are noticed. The data taken were not sufficient to definitely prove the shape of the curves in this case, due largely to the small range between the ionization and arcing potentials, but it was definitely proven that neither small points nor insulators at the plate affected the field in such cases. A test was then made on the effect on the arcing potentials of such materials, when the point was positive, and it was proven that not only this was entirely unaffected, but that also the arcing potential for a point and plate with the point positive was exactly the same as for two points the same distance apart. This shows that the phenomena at the positive terminal determined the maximum potential in such a system. It is probable, therefore, that the effect of dielectrics at the positive terminal is due to a modification of the plate into a point or edge, and local disturbance of the field due to the dielectric capacity and charges on its surface.

The behavior at 10 cm. pressure was quite different. Not only was the arcing voltage about the same for either sign, but a thin sheet of mica on the plate had no reproducible effect. It is probable, therefore, that the nature of the ions in a point to plate discharge is markedly different at low pressures, although it is not yet known whether the change is gradual or is abrupt at someone pressure.

These experiments have also shown that the potential acquired by a probe in a point to plate discharge probably is not simply and directly related to the concentration of ions at that point, when the probe is not there. Thus, the arcing voltage between the point and plate in these tests was only 35 kv., with the probe 5 mm. from the plate, while with the probe absent, the arcing voltage was 45 kv. showing that the probe had a decidedly disturbing influence. The probe, in fact, acts as a separate ionization center in the field, which is shown by silent discharge, from the probe to the plate, or sparks to or from the probe at the higher voltages.

UNIVERSITY OF CALIFORNIA.

An Explanation of X-Ray Diffraction Patterns from Rolled Metals

By C. T. Dozier.

When X-rays are passed through a thin sheet of rolled metal a diffraction pattern of two-fold symmetry is formed on a photographic plate set a few centimeters back of the sheet of metal. The diffraction pattern is weaker parallel to the direction of rolling than it is in the direction perpendicular to this. It is shown by photographs that this effect can be duplicated by parallel laying of elongated isometric crystals. The laterally diffracted rays are doubtless less absorbed than those emerging longitudinally. The effect in rolled metals is explained by assuming that the rolling causes the longer dimensions of embedded crystals to be forced into parallel alignment and the X-ray diffraction pattern thus affords a means of determining the direction of rolling in a metal specimen.

University of California.

The Concentration of Monochromatic X-Rays by Crystal Reflection.

By Elmer Dershem and C. T. Dozier.

It has previously been shown, Proc. Am. Phys. Soc., Phys. Rev., Vol. XI. p. 244, that a crystal surface if deformed into the shape of a logarithmic spiral of revolution will bring X-rays of a certain wave-length to a focus provided the source of X-rays is at the origin of the spiral.

Such a reflecting surface has now been formed by the use of small crystals of halite. The intensity of the monochromatic X-rays obtained in this way is 10 per cent. of the intensity of the entire radiation of the X-ray tube at the same distance, this intensity being measured with an air-filled ionization chamber.

University of California.

Minutes of the New York Meeting, February 26, 1921.

The 108th meeting of the American Physical Society was held in Room 305 Schermerhorn Hall, Columbia University, New York City, on Saturday, February 26, 1921. There was a single session beginning at ten o'clock a.m. with an attendance of about 225. President Theodore Lyman presided.

There was a meeting of the Council held at 2:30 o'clock p.m. on February 26, 1921, in the office of Dean Pegram, of Columbia University. The following elections to membership were made: *Elected to ordinary membership*: Frederick Barry, Richard T. Cox, T. M. Dahm, Marion Eppley, Roy Y. Ferner, Twao Fukushima, Warren K. Green, L. O. Heath, Usaku Kakinuma, Paul M. Mueller, Alexander McLean Nicolson, Simon Sonkin, W. Ewart Williams; *transferred from membership to fellowship*: Alfred H. Bucherer and Frank W. Ham.

At the annual meeting of the Society held in Chicago on December 30, 1920, amendments in the wording of the Constitution and By-Laws were adopted, the sole purpose and effect of which is to change the designations of the two classes of membership formerly named "regular members" and "associate members" to "fellows" and "members," respectively. All persons now designated regular members become fellows without further action, and all associate members become members. There is no change in the conditions for future election to either class of membership nor for transfer. The grade of honorary fellow replaces that of honorary member.

The following program of eighteen papers was presented, three papers being ready by title only:

Reflection of X-Rays from Crystals. W. M. STEMPEL.

The Reflection of X-Rays from Calcite. BERGEN DAVIS and W. M. STEMPEL.

Soft X-Rays of Characteristic Type. E. H. KURTH, introduced by K. T. Compton.

On the Absorption of X-Rays by Chromium, Manganese and Iron. WILLIAM DUANE and HUGO FRICKE.

The Piezo-Electric Resonator. W. G. CADY.

The Binaural Location of Pure Tones. R. V. L. HARTLEY and THORNTON C. FRY.

Note on the Characteristics of the New Singing Tube. CHAS. T. KNIPP. (Read by title.)

Broken Tone From Reed Instruments. JOHN B. TAYLOR.

Electron-Tube Drive for Tuning Forks. E. A. ECKHARDT, J. C. KARCHER and M. KEISER.

A High-Speed Oscillograph Camera. E. A. ECKHARDT.

A Method of Measuring Surface Tension of Liquids. HARRY CLARK.

The Spectral Structure of the Luminescence Excited by the Hydrogen Flame. HORACE L. HOWES.

Photoelectric Phenomena in Coated-Filament Audion Bulbs. ERNEST MERRITT.

Polarization Capacity and Polarization Resistance as Dependent upon Frequency. ERNEST MERRITT.

The Motion of Electrons Between Coaxial Cylinders in a Uniform Magnetic Field. ALBERT W. HULL.

An Electrical Doublet Theory of the Nature of the Molecular Forces of Chemical and Physical Interaction. R. D. KLEEMAN.

The Copernican Atom Radiating Energy.—A Physical Interpretation of Planck's Quantum Rule. ALBERT C. CREHORE. (Read by title.)

A Copernican Atomic Model Based on Electromagnetic Theory. ALBERT C. CREHORE. (Read by title.)

Reflection of X-Rays from Crystals.

By W. M. Stempel.

The reflection of X-rays from crystals was treated theoretically by C. G. Darwin in 1914 on the assumption that the electrons were clustered so closely about the nucleus of the atom that their distances from the nucleus were small compared to the distance between the planes of atoms in the crystals.

Three years later (1917) A. H. Compton attempted to take into account, theoretically, the effect of assuming that the distances of the electrons from the nucleus was comparable with the distance of the reflecting planes, and arrived at the conclusion, after attempting to derive a general distribution function, that electrons arranged as in the Bohr atom or at least in rings was the only arrangement that would satisfy experimental facts. Both the above authors had to resort to plane waves or their equivalent to get their final workable equations.

The present writer starts with the rings of electrons and plane waves, and in addition carries out the integration in a somewhat different order as follows: A plane is considered in the first or surface atomic layer. This plane is chosen at random and the succeeding planes are taken equal distances apart from this, say D plus E, where D is the distance between the atomic planes of the crystal. The reflection from this series of planes is obtained. The initial plane is then changed and the process repeated. This integration is carried out for all possible positions of the initial plane. The result of this work is as follows:

The intensity of the beam of monochromatic X-rays reflected from a perfect crystal is

$$I_m = I_0 D \left(\frac{e^2}{m_e c^2} \right)^2 \frac{1 + \cos^2 2\theta}{2} \frac{1}{E^2} \frac{1}{1 + \frac{\beta^2}{E^2} \delta^2} \cdot e^{-Bn^2} \times \sum_s \left\{ N_s \left(1 + 2m - m(m+1) \frac{D \sin\theta}{2a_s} \right) \left(\frac{\sin \beta a_s}{\beta a_s} \right)^2 \right\},$$

where I_0 is the intensity of the incident beam,

e is the unit charge on the electron (in e^2/mc^2),

e in e^{-Bn^2} in base of Naperian log,

m_e is the mass of the electron,

m is an integer defined by $2a/mD > \sin\theta > 2a/(m+1)D$,

c is the velocity of light,

D is the grating space of the crystal used,

θ is the grazing angle,

E is the fractional part of the energy extracted from the incident beam per atomic layer that it passes through. Here the entire energy is being taken and includes as part both the pure absorption and that part re-radiated by the electron.

β is a constant and equal to $2\pi n/D$,

e^{-Bn^2} is Debye heat factor,

n is the order of the spectrum under consideration,

N is the number of electrons per c.c. belonging to one sized ring,

s refers to the particular electron orbit under consideration, and the summation is understood to be extended to each of the different orbits present no matter to what atom they may belong, the radius of the orbit is all that counts so long as the atom is in the reflecting plane, if not a correction must be made on account of this displacement.

It is to be observed that the intensity of the reflected beam given in equation one is in general not the maximum. Reflection can take place from a system of mathematical planes only when their distance apart is so adjusted as to satisfy the equation, $n\lambda = 2(D + \mathcal{E}) \sin \theta$. The reflected intensity given by equation (1) is thus the intensity to be expected for any arbitrary wave-length and any arbitrary grazing angle. The maximum value will be reached only when the proper distance between reflecting planes becomes equal to the distance between the atomic planes of the crystal, or $\mathcal{E} = 0$. The values of the intensity fall off very rapidly on either side of $\mathcal{E} = 0$.

When a heterogeneous beam is used a small range of wave-lengths will be reflected. That wave-length for which $\mathcal{E} = 0$ will be most intense, but on either side energy will still be reflected, the intensity falling off rapidly. The range of waves which would have any appreciable value is however very small and (I_0) for such a range is easily considered constant. To get the total energy reflected in the case of a heterogeneous beam we may integrate the equation (1) with respect to $\mathcal{E}$ and get

$$I_{HT} = I_0 D \left(\frac{e^2}{m_e c^2}\right)^2 \frac{1 + \cos^2 2\theta}{2} \frac{\pi}{E\beta} e^{-Bn^2} \times \sum_s \left\{ N_s \left(1 + 2m - m[m+1] \frac{D \sin\theta}{2a_s} \right) \left(\frac{\sin \beta a_s}{\beta a_s} \right)^2 \right\}.$$

In all these equations it is to be remembered that the first bracket under the summation sign is never less than one as will be noted on inspecting the condition which determines the value of (m).

If the beam reflected from the one crystal is allowed to fall upon another, placed with its atomic planes parallel with the first, the condition for reflection is correct for every element of the beam and thus serves an ideal method of studying reflection phenomena.

If for two parallel crystals, monochromatic light is allowed to fall upon the first crystal, the intensity of the beam coming from the second is

$$I_{m2} = I_0 D^2 \left(\frac{e^2}{m_e c^2}\right)^4 \left\{ \frac{1 + \cos^2 2\theta}{2} \right\}^x \frac{1}{E^4} \frac{1}{1 + \frac{\beta^2}{E^2} \mathcal{E}_1^2} \frac{1}{1 + \frac{\beta^2}{E^2} (\mathcal{E}_1 - \mathcal{E}_2)^2} e^{-2Bn^2} \times \sum_s \left\{ N_s^2 \, 1 + 2m - m[m+1] \frac{D \sin\theta}{2a_s} \right)^2 \left(\frac{\sin \beta a_s}{\beta a_s} \right)^4 \right\},$$

where (x), the exponent of the polarization factor, will lie somewhere between

1 and 2, probably not far from 1. Its value will depend upon how completely the beam is polarized by the first reflection. $\mathcal{E}_1$ refers to the first crystal and $\mathcal{E}_2$ to the second. We will think of $\mathcal{E}_1$ being required because the wave-length is not correct for the grazing angle, and $\mathcal{E}_2$ because the second crystal is not quite parallel with the first.

When heterogeneous X-rays fall upon the first crystal, the total energy reflected from the second for any value of $\mathcal{E}_2$ will be obtained by integrating this last equation with respect to $\mathcal{E}_1$. The following equation is obtained,

$$I_{H2} = I_0 D^2 \left(\frac{e^2}{m_e c^2} \right)^4 \left\{ \frac{1 + \cos^2 2\theta}{2} \right\}^x \cdot \frac{f(\mathcal{E}_2)}{\beta E^3} \cdot e^{-2Bn^2}$$
$$\times \sum_s \left\{ N_s^2 \left(1 + 2m - m[m+1] \frac{D \sin\theta}{2a_s} \right)^2 \left(\frac{\sin \beta a_s}{\beta a_s} \right)^4 \right\},$$

and the values of the $f(\mathcal{E}_2)$ are given below for various values of $\mathcal{E}_2$ where $A = E/\beta$.

$\mathcal{E}_2$	0	.2A	.4A	.6A	.8A	1.0A	1.4A	1.8A
$f(\mathcal{E}_2)$	1.571	1.555	1.510	1.441	1.354	1.257	1.054	.868
$\mathcal{E}_2$	2.0A	2.4A	2.8A	3.2A	3.6A	4.0A	5.0A	8.0A
$f(\mathcal{E})_2$	.785	.644	.531	.443	.371	.306	.218	.090

To get the fractional part of the beam that is reflected at the second crystal we have only to divide the last equation by the second and we get

$$D \left(\frac{e^2}{m_e c^2} \right)^2 \left\{ \frac{1 + \cos^2 2\theta}{2} \right\}^{x-1} \frac{f(\mathcal{E}_2)}{\pi E^2} e^{-Bn^2}$$
$$\times \sum_s \left\{ N_s \left(1 + 2m - m[m+1] \frac{D \sin\theta}{2a_s} \right) \left(\frac{\sin \beta a_s}{\beta a_s} \right)^2 \right\}$$

The fractional part of the beam reflected may be easily obtained for all the other cases.

The theory has the advantage of disclosing the following facts which are new to X-ray reflection theory:

The reflected energy does not approach infinity for either large electron concentration or small grazing angles.

The theory definitely discloses the variation of the intensity of the reflected beam for conditions deviating from those for perfect reflection, which variation measures the sharpness of interference patterns. This sharpness of interference patterns is independent of electron distribution, a fact shown some years ago by Debye.

The theory discloses many other interesting facts too complex to point out here. One may be stated because of its extreme importance, and that is that for polished or irregular crystals the intensities of successive orders vary inversely as the square of the order, a fact shown experimentally to be true by the Braggs (they no doubt worked with polished crystals).

The possibilities of the theory together with some interesting experimental conformation will appear later when the details of this work are published.

POLARIZATION CAPACITY AND POLARIZATION RESISTANCE AS DEPENDENT UPON FREQUENCY.

BY ERNEST MERRITT.

THE success of the experiments described by Gen. Squier, in which radio frequency signals were sent along bare wires immersed in water, makes it reasonably certain that the electrolytic capacity of such wires is much less at high frequencies than for a slowly changing charging potential, and preliminary measurements at the Bureau of Standards have confirmed this conclusion. I have recently made determinations of the electrolytic capacity of platinum in sulphuric acid through a wide range of frequencies and find that in this case also the capacity depends upon the frequency. Platinum electrodes in sulphuric acid were chosen for test in order to make possible a quantitative comparison of the results with those predicted by theory. The electrodes were of wire 0.41 mm. in diameter and 18 mm. long. As measured by ordinary charge and discharge the charging potential being 0.1 volt or less, the capacity of the cell was approximately 5 microfarads.

In determining the capacity at different frequencies an alternating current of the frequency desired was developed by an oscillating audion, and a loosely coupled secondary circuit was tuned to resonance by its adjustable condenser. When the electrolytic cell was placed in series with the variable condenser it was necessary to make a small change in the setting in order to restore resonance; from this change and the original capacity the electrolytic capacity could be computed. When the cell was not used its place in the circuit was taken by a "dummy" having the same inductance and resistance. For measurements in the audible range a standard mica condenser was used. Frequencies were determined by a wave meter in the case of radio frequencies and in the case of lower frequencies by comparison with tuning forks. Current measurements were made with a crystal detector in an untuned circuit loosely coupled to the secondary.

The results obtained are in the main in agreement with the simpler form of the Warburg diffusion theory. For example, the capacity of the cell (two electrodes in series) was found to be 2.37 mf. for $n = 1{,}810$ and 0.12 mf. at $n = 1{,}000{,}000$. If the capacity were proportional to $1/\sqrt{n}$ throughout this whole range as predicted by the Warburg theory, the value for $n = 1{,}000{,}000$ should be 0.10 mf. In the range $n = 4{,}000{,}000$ to $n = 500{,}000$ the capacity appears however to be proportional to $1/n$ instead of $1/\sqrt{n}$. There is some reason to expect such a change in the law at high frequencies, since the theory is based upon the assumption that the ions are so numerous that equilibrium between the ionic concentration in the liquid and that in the electrode can be established instantly and without appreciable change in the concentration in the liquid. At sufficiently high frequencies this assumption must cease to be justified.

Cornell University.

Photoelectric Phenomena in Coated Filament Audion Bulbs.

By Ernest Merritt.

In audion bulbs having oxide-coated filaments a film often forms on the plate, sometimes appearing as an irregular discoloration of the polished surface, but often scarcely visible. It has been pointed out by Case that these films show photoelectric activity. In repeating the experiment described by Case I find that the activity, although smaller than that of a sodium or potassium cell of the Elster and Geitel type, is sufficient to permit of convenient measurement by means of a moderately sensitive galvanometer. The active rays appear to be at the violet end of the spectrum and in the near ultraviolet. The current produced by light from an open arc is reduced by about 20 per cent. by a piece of glass 5 mm. thick. Owing to the high vacuum the voltage current curve is remarkably flat; complete saturation is reached at about 10 volts and increase in voltage to 120 volts produces no further change in current.

Since the active films are produced by the evaporation of the oxide coating on the filament it seemed reasonable to expect that the filament would also show photoelectric activity I find this to be true in the case of all the tubes tested. With the plate and grid connected and maintained at a positive potential the current obtained by illuminating the filament is much smaller than when the plate is made negative with reference to grid and filament. In one case, for example, it was found that $i = 5 \times 10^{-9}$ for filament negative and $i = 4 \times 10^{-8}$ for plate negative. But the difference is not as great as the difference in area would lead one to expect.

The current obtained when the filament is illuminated increases greatly when the temperature of the filament is raised. To study this temperature change the effect of illumination was measured for different values of the filament current, the thermionic current being first measured and then the increase due to illumination. The photoelectric current, *i.e.*, the increase in current due to illumination, was found to increase steadily with the temperature, although less rapidly than the thermionic current. At a dull red heat the photoelectric current was in one case found to be 1,400 times as great as that from the same filament when cold. The photoelectric current measured was in this case 7 microamperes. With the same illumination and applied E.M.F. an Elster and Geitel potassium hydride cell, set at the angle of incidence showing maximum effect, gave a current of 0.34 microamperes. This result is all the more remarkable when it is remembered that the area illuminated in the case of the potassium cell must have been at least a hundred times greater than the area of the filament. Increased photoelectric activity at high temperatures has also been observed by Case with barium cells. While the results may prove to be due to secondary causes there is every indication that we have to deal with a real increase in photoelectric activity.

When polarized light was used there was no indication of any difference in the magnitude of the effect for different positions of the plane of polarization, either with the filament or plate.

Coated filament tubes, probably because of the active film referred to above, have been found to show readily the phenomena that result from secondary emission from the plate. With the grid maintained at a positive potential of 120 volts with respect to the filament the curve of plate potential-plate current is the typical dynatron characteristic described by Hull. In one of the tubes used in these experiments (a Western Electric "V" tube) the plate current reaches a positive maximum at about 70 volts, falls again to zero at 94 v., reverses and reaches a negative maximum at 97 v., passes through zero a second time at 107 v. and then continues on the positive side of the axis. The emission of secondary electrons from the plate is especially strong in the range between the positive and the negative maximum. In order to determine whether the secondary emission was influenced by light the plate voltage was set at the point corresponding to the first zero, *i.e.*, on the "negative resistance" part of the characteristic, and the plate was then illuminated. The effects observed were at first erratic, the deflection observed being sometimes positive and sometimes negative, depending upon the tube used, and occasionally changing from positive to negative with the same tube. This erratic behavior was found to be due to the fact that the effect of illuminating the plate is opposite to that of illuminating the filament, the former being negative, the latter positive. The two effects could be separated by using a narrow beam of light.

The effect of illuminating the plate is in the direction corresponding to increased electron emission from the plate. In some cases however the effect appears to be greater than would result from the simple addition of the photoelectric effect to the secondary emission.

When the filament alone is illuminated the deflection is positive, indicating a diminished secondary emission from the plate. The effect is probably due to the increased space charge near the filament, which reduces the velocity of the electrons reaching the plate. Experiments intended to test this explanation are not yet completed.

CORNELL UNIVERSITY.

THE REFLECTION OF X-RAYS FROM CALCITE.

BY BERGEN DAVIS AND W. M. STEMPEL.

THE reflection of X-rays of various degrees of homogeneity from calcite was measured by means of a double X-ray spectrometer. The experiments covered a wave-length range from 3 to .8 Å. The radiation from a Coolidge tube under accurate control after passing through the slits of the spectrometer fell on the first crystal designated by A. A portion of the general radiation ($\Delta\lambda$) reflected from crystal A fell on a second crystal designated by B. Crystal B was placed so that its reflecting planes were parallel to those of A. A very great reflectivity was observed at this position of parallelism. The reflectivity was measured by comparing the ionization produced by the bean ($\Delta\lambda$) of X-rays before and after reflection from Crystal B.

The reflection obtained from B depended on the degree of homogeneity of the beam reflected from A. The reflectivity of B thus depended on the character of crystal A. The crystals were consequently investigated in combinations or pairs. The reflectivity depended also on the similarity of the crystals and on the perfection of the crystals and on the state of polish of their surfaces.

The pairs of crystals investigated are designated as follows:

($A_1 - B_1$): A_1 was a specimen of Montant calcite, not quite clear. Defects were visible in the interior. B_1 was a clear specimen of Iceland spar. The surfaces of both crystals were polished. ($A_2 - B_2$): A clear specimen of Iceland spar was split along a cleavage plane. The two surfaces were polished. The crystals were so mounted that the reflecting surfaces were the surfaces that had been contiguous before splitting. ($A_3 - B_3$): A clear specimen of Iceland spar was split along a cleavage plane. The surfaces were not polished. The crystals were so mounted that the reflection took place from the planes that were contiguous before cleavage. Maximum reflection was obtained when the several pairs of crystals were mounted with the reflecting planes parallel. Some reflection was obtained when crystal B was turned slightly out of parallelism with A. Interesting curves of energy distribution were obtained by rocking crystal B through small angles each side the position of maximum reflection. These curves differed in width. The dissimilar pair ($A_1 - B_1$) gave much wider rocking curves than the others. The curves were quite narrow for the more perfect pair ($A_3 - B_3$). The width at half maximum in this case was about 16″ of arc. A table is given of the per cent. of the energy reflected from crystal B for each pair. The results indicate that the reflection of homogeneous radiation from a perfect crystal of calcite (were such possible) would be greater than 50 per cent.

Angle.		Percent Reflection.		
		$A_1 - B_1$.	$A_2 - B_2$.	$A_3 - B_3$.
3	.317	25.8	37.3	
3–30	.369	24.6		
4	.422	23.4	33.6	44.4
5	.527	21.5	30.5	
5–30	.580	20.8		43.1
6	.632	19.8	27.9	
6–30	.686			42.8
7	.737	16.7	26.	
7-30	.790			42.8
8	.842	17.3	24.6	

Phoenix Physical Laboratory,
Columbia University.

SOFT X-RAYS OF CHARACTERISTIC TYPE.

BY E. H. KURTH.

THE following investigation was undertaken in the hope that, if special care were exercised to secure a high vacuum, troublesome ion effects experienced by earlier investigators, making the use of a window imperative or the interpretation of results uncertain, would disappear.

The apparatus used was of the type generally employed, the radiation being studied through its photo-electric action upon a metal plate. The target and filament were sealed into the small end of a large conical glass tube and a platinum detecting disk was supported at the other end. In the body of the cone proper a series of plates were arranged diverging from the target to the disk, and they were connected alternately together.

In order to secure as high a vacuum as practicable, the apparatus was joined by means of large bore tubing to a condensation pump. An appendix containing a small quantity of charcoal was also attached to the tube. Electric heaters were employed to bake out the apparatus for several hours at about 400 degrees before making a run. Finally at the end of this period the vapor traps and charcoal were immersed in liquid air.

The detecting disk was connected to an electrometer with a sensitivity of about 1,700 millimeters per volt.

The intensity of the measured effect was found to be essentially independent of potentials up to 800 volts which were applied to the diverging plates to prevent the passage of ions from one end of the tube to the other. Variations in the strength of a magnetic field applied perpendicularly to these plates likewise had no effect. However a potential of 135 volts on the plates and a weak magnetic field were applied during all the runs.

Three elements have thus far been tested, aluminium, iron and carbon. The radiation was found to be entirely characteristic of the particular element used, and was in every sense reproducible.

With aluminium very definite breaks were found in the intensity curve at 38 and 120 volts, indicating three types of radiation corresponding to 326 Å.U. and 103 Å.U., respectively. These values may be uncertain within possibly 5 per cent. It is likely that there is additional radiation shorter than 133, but, if so, it is not strong enough to be detected when superimposed on the effects of the longer wave-lengths.

It seems possible that 103 may be the L series, and 326 may be the M series.

The radiation curves of iron show very definite breaks at 62.8 Å.U., 48.4 Å.U. and 16.3 Å.U. The two long wave-lengths seem to represent the extremes of the M series for iron while the 16.3 break probably corresponds with the L series lines of the element. The radiation effect set in at approximately 25 volts and there was some indication of a break in the curve at 50 volts corresponding to 247 Å.U.

With the element carbon a break at 43.6 Å.U. was very pronounced. This agrees exactly with the extrapolated value for the K series within the limits of error of the extrapolation. No certain indication of any further breaks which might belong to a longer wave-length series have yet been observed. The radiation set in initially at approximately 20 volts.

This work is at present being extended to cover some of the other elements including beryllium.

Palmer Physical Laboratory,
Princeton, New Jersey.

On the Absorption of X-Rays by Chromium, Manganese and Iron.

By William Duane and Hugo Fricke.

In the experiments reported in this paper we have measured the K critical absorption wave-lengths for chromium, manganese and iron. The object of the research has been not only to obtain accurate values for these particular absorption wave-lengths, but also to collect evidence bearing on the following general questions:

(*a*) Has the nature of the chemical compound containing a chemical element any influence on the critical absorption wave-lengths characteristic of that element?

(*b*) Does a critical absorption wave-length represent a single sharp boundary separating wave-lengths of X-rays that are more absorbed from those that are less absorbed, or does it consist of several such boundary lines lying close together?

Evidence has been obtained by means of photographic spectrometers indicating that black phosphorus has a K critical absorption differing slightly from that of yellow phosphorus.[1] Further, on general theoretical grounds, we would expect energy changes due to transfers of electrons in atoms to depend to some extent upon the forces binding the atoms together; and, since the frequencies of vibration of X-rays are proportional (theoretically) to the energy changes, the X-ray wave-lengths should vary, therefore, with the nature of the chemical bonds.

In measuring critical absorption of light elements by means of photographic spectrometers, photographs have been obtained[2] that show certain irregularities between the light and the dark areas. These irregularities were interpreted as indicating a certain "structure" in the absorption limit. Attention was called to the fact that they could be explained by supposing that the critical absorption corresponds to the transfer of an electron from an inner orbit to any one of a number of possible orbits outside of the periphery of the atom, each one of these outside orbits representing an absorption line.

In our measurements we have used an ionization spectrometer with a calcite reflecting crystal. The general arrangement of the apparatus and the mode of

[1] J. Bergensen, Comptes Rendus, Oct. 4, 1920, p. 624.

[2] Hugo Fricke, Phys. Rev., Sept., 1920, p. 202; and G. Hertz, Zeits. für Physik, Vol. 3, 1920, p. 19.

procedure have been explained in several publications from this laboratory.[1] We have made only one change in the instruments. The X-rays now emerge from the X-ray tube through a very thin mica window fastened at the end of a long side tube, which projects out from the X-ray tube very near to the crystal. This reduces the absorption of the X-rays by the walls of the tube and by the air, and greatly increases the intensity of the X-rays in the long wave-length region of the spectrum. The rays leave the target making a very small angle with its surface, and the width of the slit is such as to define a beam of rays that has angular breadths ranging from 5′ to 9′ of arc in different experiments. These breadths are of the same order of magnitude as those used in the photographic work.

The absorbing screens are made by dipping a very thin sheet of paper into a solution containing the chemical element to be investigated, and then allowing it to dry. The deposit of salt obtained in this way is not very uniform. The screens are placed in the path of the X-rays before they strike the crystal. The potential on the tube is so chosen that we do not have the spectrum of second order.

The curves in the lantern slide shown represent the ionization current as a function of the angle that fixes the position of the crystal. A peak appears on one of these curves, which represents the longest L emission line (Ll) in the X-ray spectrum of the tungsten target. This line is quite faint and appears to be complex.

The sharp drops in the curves indicate the positions of the critical absorption. It is possible to estimate the values of the critical absorption wave-lengths to within considerably less than 1/20 of one per cent.

The curves represent the critical absorption of iron, manganese and chromium. The values of the wave-lengths are:

$$\begin{array}{lccc} & \text{Fe} & \text{Mn} & \text{Cr} \\ \lambda \times 10^8 \text{ cm.} = & 1.7377 = & 1.8893 = & 2.0623. \end{array}$$

We used two salts of iron, Fe_2O_3 and $FeSO_4$ $(7H_2O)$, and could detect no difference between their critical absorption wave-lengths. This result indicates that the energy changes corresponding to the critical absorption must be sensibly the same for the iron atoms in the two states of chemical combination respectively.

All of the curves have sharp drops corresponding to the K critical absorption wave-lengths, but none of them show more than one such drop, nor do they indicate any irregularity in the neighborhood. In other words, there appears to be no evidence for a "structure of the absorption limit."

The failure of the ionization method to indicate an absorption structure can not be attributed to a lack of resolving power, for, as stated above, the breadth of the absorption drop is about the same as in the photographic measurements.

Irregularities in the thickness of the absorbing screen would tend to obliterate evidence as to the structure of the limit. Our screens, however, were about as

[1] PHYS. REV., Dec., 1917, p. 624; ibid., Dec., 1919, p. 516; ibid., Dec., 1920, p. 526.

uniform as those used in some of the photographic experiments. They were usually just thick enough to cut down the intensity on the short wave-length side of the drop to about one half that on the long wave-length side.

The Piezo-Electric Resonator.

By W. G. Cady.

A PLATE or rod suitably prepared from a piezo-electric crystal, and provided with metallic coatings, can be brought into a state of vigorous resonant longitudinal vibration when the coatings are connected to a source of alternating E.M.F. of the right frequency. Under these conditions, the plate reacts upon the electric circuit in a remarkable manner.

The vibrations are a consequence of the so-called "converse" piezo-electric effect, *i.e.*, the deformation resulting from an electric stress; while the periodically strained condition of the plate, through the action of the "direct" effect, sets up a periodic component of polarization which causes the reaction referred to.

On the theoretical side, the phase and magnitude of the counter-polarization in the plate are derived, as well as expressions for the current flowing to the plate and for the total current in the oscillatory circuit. The plate is assumed to be in parallel with the circuit capacity. It is shown that, owing to the piezo-electric polarization, and to the absorption of energy in the plate, the apparent electrostatic capacity and resistance of the plate are not constant, but depend upon the frequency somewhat as does the motional impedance of a telephone receiver. Over a certain range in frequency the capacity may even become negative.

A graphical method is developed for presenting the results of the theory. By an application of this method, it is possible, after making a series of purely electrical observations, to deduce the coefficient of viscosity of the material, even though the absolute value of the piezo-electric constant is not known.

Methods are described for mounting a small plate of piezo-electric crystal upon a rod of any solid elastic substance in such a manner as to excite longitudinal vibrations in the entire rod. The high-frequency viscosity of the rod can then be found, subject, however, to more or less error due to losses in the cement which attaches the crystal plate to the rod.

Experiments on longitudinal vibrations in rods or plates of steel, quartz, and Rochelle salt are described.

The possibility is discussed of using the piezo-electric resonator for a standard of high frequency; for excluding from a circuit oscillations of a given frequency; and as a coupling device to transfer small amounts of power from one circuit to another at a particular frequency.

Wesleyan University, Middletown, Conn.,
February 3, 1921.

The Binaural Location of Pure Tones.

By R. V. L. Hartley and Thornton G. Fry.

The theoretical explanation of experiments on the apparent location of sounds has been very largely confined to the relatively simple case of pure tones where the phenomena are almost entirely binaural. The theories advanced have been quite successful in explaining the results of that type of experiment in which the intensities at the two ears are kept equal and the direction of the image is observed for various phase differences. For those experiments in which the phases are kept equal and the intensity ratio is varied no satisfactory explanation has so far been offered.

The present paper presents a theory which when applied to the latter type of experiment explains why the observer has difficulty in assigning a definite direction to the image and why different observers get widely different results. The results given by Stewart[1] for this type of experiment are shown to be in entire agreement with the present theory. When the theory is applied to experiments with equal intensities and varying phase difference it indicates why the results of different observers are in substantial agreement with one another and why the theories advanced have explained them successfully even though certain factors have been neglected.

Experiments are suggested in which the phase difference and intensity ratio are adjusted simultaneously in such a manner as more nearly to approach the conditions of every day experience. To facilitate such experiments computed curves connecting the polar coördinates of a source with the phase difference and intensity ratio of the resulting stimulus are given for sounds of representative frequencies. Emphasis is placed upon the importance of recording the apparent distance as well as the direction of the sound image.

Research Laboratories of the American Telephone & Telegraph Co.
and the Western Electric Company, Inc.,
February 10, 1921.

Note on the Characteristics of the New Singing Tube.

By Charles T. Knipp.

The temperature difference that is necessary to cause the new singing tube to emit a tone, when the portion B (Fig. 1) is kept at room temperature while the tip A is heated, was observed to be about 400° Centigrade. If,

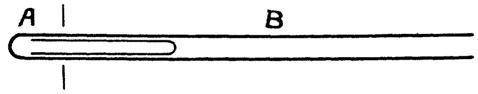

Fig. 1.
New singing tube.

however, B is cooled to the temperature of liquid air the temperature difference

[1] Physical Review, May, 1920, p. 425.

necessary is greatly reduced, being about 200° Centigrade.[1] The pitch is also considerably lowered.

It was deemed desirable to make quantitative measurements of these temperature differences and also of the corresponding pitch of the tone emitted. To this end B was held successively at different temperatures, ranging from that of liquid air to values considerably above room temperature, while A was heated electrically in each case to a temperature where a tone was emitted continuously. The two portions A and B were each housed within a separate copper tube of about 1 cm. wall thickness and heavily insulated with asbestos. The temperatures were accurately measured by means of thermo-junctions,—three being attached to A and three to B. The temperature control of each section was good and could be held fairly constant at will.

TABLE I.

No.	A Average Temp. in Degrees C.	B Average Temp. in Degrees C.	Total Temp. Difference in Degrees C.	Temp. of B in Absolute Measure.	Vibrations per Second.
1.........	1	−181	182	92	213
2.........	204	− 88	292	185	300
3.........	355	− 16	371	257	378
4.........	448	+ 26	422	299	425
5.........	524	+ 57	464	330	450

The results from the only run thus far made are contained in Table I. In observation No. 1 the part B was placed within a glass jacket heavily wrapped

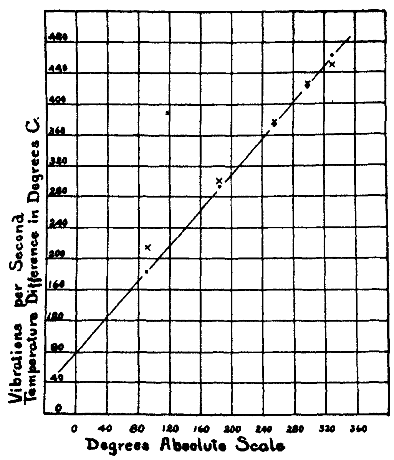

Fig. 2.

Characteristics of the new singing tube.

with aspestos and cooled directly to − 181° C. by means of liquid air. The temperature of A was allowed to fall until the tone emitted was just main-

[1] PHYS. REV., N.S., Vol. XV., p. 336.

tained. This by repeated trials was found to be at 1° C. Thus the temperature difference when B was cooled to − 181° C., for this particular tube, was found to be 182° C. In observations 2 and 3 the part B was placed in a special copper tube designed by the author[1] some years back for the determination of intermediate temperatures. Observation 4 was for B at room temperature (note that it was now necessary to heat A to 448° C.), while in No. 5 the part B was warmed up to 57° C., and the tip heated electrically to 527° C. before the tube responded.

The absolute temperatures of the part B are given in the second last column, while the corresponding vibrations per second are listed in the last column. These data are represented graphically in Fig. 2, in which the total temperature differences as ordinates are plotted against absolute temperatures. The relation is strictly linear except possibly for the last reading at 330° absolute temperature. By extending the straight line to the left we are able to determine the temperature difference that should maintain the tone when B is cooled to absolute zero. For this particular tube the graph shows this temperature difference to be about 80° C.

The same figure also shows the corresponding vibration frequencies (indicated by crosses) plotted to the same scale against absolute temperatures. This relation also seems to be linear except for the point taken at 91° absolute. The pitch was determined by means of a tone variator.

Lastly the vibration frequencies and temperature differences in degrees centigrade, as shown by the graphs, are nearly equal numerically. This, however, should be considered as a coincidence.

Observations with tubes of different pitches and extending over wider temperature ranges are under way.

UNIVERSITY OF ILLINOIS,
February 10, 1921.

BROKEN TONE FROM REED INSTRUMENT.

BY JOHN B. TAYLOR.

IN musical instruments of the wood wind group, the pitch of the tone is generally considered as determined by the distance from the mouthpiece to the nearest of the side openings or vents which are uncovered. The opening or closing of vents below the first has some influence in modifying the pitch and quality.

If the vent hole nearest the mouthpiece is small, and located in a proper proportional relation to the total length of the air column, a harmonic tone is determined, in which case there may be considerable variation in the size and location of the harmonic vent without appreciably affecting the tone.

If the harmonic vent occupies a position intermediate to those positions proper for two harmonics, other factors may determine which one of the two

[1] PHYS. REV., Vol. XV., p. 125.

harmonics will be heard—but, generally, whichever one starts to sound will continue so long as there is no change in the manner of blowing.

The bassoon differs from the other wood wind instruments in a large degree because the theoretical locations of the ventages are much more widely separated than the spread of the finger tips. This difficulty was met in the early days before the development of key mechanisms by leaving an unusually thick wall on the wooden tube and boring the long ventages of small size at an angle. As a result, the upper and lower portions of the instrument are more "closely coupled" than is the case with the other wood wind instruments such as flute, oboe and clarinet, and the uncovering of a single vent does not approximate an open-end condition of the air column.

While experimenting with unusual cross fingerings, several combinations have been found producing tones quite foreign to the normal characteristic voice of the instrument. The term "broken tone" has been selected as best describing these unusual sounds.

One of these broken tones has been selected for illustration and wave form records have been made. The particular tone is that produced by sounding the low F (approximately 87 cycles). If while this F is sounding, the third finger of the left hand is raised, uncovering the vent which is ordinarily considered as that through which the tone d (approximately 145 cycles) speaks, the complex tone results.

A skilled observer judged the complex tone to be composed of B^b (116 cycles), f (174 cycles) and a differential resultant tone of BB^b (58 cycles).

Such tones would result if the third finger left hand vent (called d) is regarded as being near enough to the octave position for the low F (87 cycles), tending to produce the octave f (174 cycles.) Also, if the d vent is sufficiently long and restricted to produce B^b at 116 cycles, when the air column is regarded as ending at the outside of the third finger d vent, the second member of the complex tone may be explained.

The broken or somewhat rattling character of the tone was judged to indicate that the mode of vibration alternated rapidly between that producing B^b as a fundamental, and that giving f as a second harmonic. The rapid alternation of the two produces the low pitched differential tone BB^b.

The photographic records of the sound wave are considered to be in agreement with this suggested explanation. The complex or broken tone is not easy to decipher because the normal single tones of the bassoon are rich in harmonics and of complicated shape.

Schenectady, N. Y.,
February 11, 1921.

Electron Tube Drive for Tuning Forks.

By E. A. Eckhardt, J. C. Karcher and M. Keiser.

Eccles and Jordan and Abraham and Bloch have recently described methods for maintaining the vibrations of tuning forks by the use of electron tube circuits. Regenerative circuits are used, the coupling between the grid and plate circuits being provided by the tuning fork. There is no appreciable electrical or magnetic coupling.

It was found by the authors that this method of driving a tuning fork does not yield large amplitudes unless the circuit is carefully designed. If the circuit is properly adjusted for one electron tube it will frequently work less well with another of the same type and sometimes not at all.

A more flexible driving arrangement was devised by the authors which permits the driving of a given fork with any tube of a given type, provision having been made for the slight readjustments which are necessary when the tubes are changed. The adjustments are such that even the transition from one type of tube to another can be made in a few minutes.

In our driving arrangement the tuning fork does not provide the coupling between the two branches of the regenerative circuits. When the circuits are closed the fork driving magnet is energized and the fork is displaced from its equilibrium position. This sets up a small initial vibration of the fork which induces in the plate circuit a small E.M.F. of the fork frequency and hence an alternating component of the plate current. The plate circuit is coupled loosely to the grid circuit. If the mutual inductance between the grid and plate circuits is of the proper sign the alternating current of the fork frequency will be regeneratively amplified and the periodic driving forces on the fork will increase giving rise to larger fork amplitudes. The limit is set primarily by the operating characteristics of the electron tube.

A transformer may be inserted in the plate circuit and the arrangement may then be used as a generator of feeble alternating currents. By proper design of the circuits a very satisfactory wave-form can be obtained. The tuning fork acts as an automatic frequency control of high precision.

We believe that the apparatus is likely to be quite useful for the precise measurement of electrical quantities which are a function of the frequency.

BUREAU OF STANDARDS,
WASHINGTON, D. C.
February 11, 1921.

A HIGH-SPEED OSCILLOGRAPH CAMERA.

BY E. A. ECKHARDT.

A HIGH-SPEED oscillograph camera was developed in our Sound Laboratory which has the advantage of being available for use in connection with any type of oscillograph. It has been used with G. E. oscillographs, an Einthoven galvanometer, and to record various purely mechanical vibrations.

An aluminum drum 5 feet in circumference and 4 inches wide is arranged to have a film of similar dimensions stretched over its convex surface. This drum is mounted in a light tight housing, which is something over 8 inches wide, to which access is obtained by means of a sliding door. The shaft protrudes from the housing on both sides. One end of it carries the driving pulley which is belted to a motor, the other carries a phosphor bronze worm which engages an idling cast iron wheel. In contact with the flat face of this wheel are the pole faces of an electromagnet which is energized only when an exposure is being made. During the exposure this electromagnet grips the cast iron wheel and prevents its rotation. Consequently the worm feeds along the wheel and the drum executes a spiral motion, the pitch of which is determined by that of the worm and the wheel. With a spiral of ½ inch per revolution nearly eight full revolutions are obtained on the film. This gives a film record 40 feet long. Several worm and wheel combinations can be mounted interchangeably on the drum, which permits the obtaining of longer and narrower records.

The shutter consists of a thin steel blade which covers the shutter opening. A plunger attached to the blade is pulled into a controlling solenoid against a spring when exposure is to be made. When the shutter solenoid is deënergized the shutter is closed by the action of this spring.

The shutter and spiral initiating solenoids are connected in series. They are energized simultaneously therefore and the shutter opens and the drum begins to spiral at the same moment. This operation can be controlled by a relay and the performance may therefore be subjected to remote control.

When a record is to be made the drum is brought to the desired speed. By closing the control key the shutter is opened and the drum begins its spiral motion. A contact is whipped open by the driving pulley when the drum has spiralled the desired distance. The shutter closes and the gripping magnet releases the cast-iron wheel which again begins to idle. A spring then restores the drum to its initial position in the housing.

Records have been obtained in which the 40 feet of film covered a time period of less than one second. With suitable time index marks and a steady rotational speed of the drum, time intervals may be read to one millionth of a second.

Bureau of Standards,
Washington, D.C.,
February 11, 1921.

A Method of Measuring Surface Tension of Liquids.

By Harry Clark.

Many good methods of measuring surface tension require either such elaborate mathematical treatment or such expensive apparatus that they cannot be used in elementary laboratory courses, and none of the other methods are quite satisfactory. The method here described requires little apparatus, is simple mathematically, and gives fairly accurate results.

The apparatus consists of a glass tube, perhaps 2 cm. in diameter and 10 cm. long, one end of which is provided with a metal plug held in place by a bit of rubber tubing. A fine hole is drilled axially through the plug, which is turned down so that it terminates externally in a small tube about 3 mm. long and of diameter appropriate to the liquid used—about 0.75 mm. for water.

If liquid is poured into the glass tube, held vertically in a clamp, a drop will form on the end of the metal tube, the horizontal diameter of the drop being equal to the external diameter of the tube. As more water is added, the drop increases in size vertically until it becomes hemispherical (to an excellent degree of approximation) beyond which it becomes unstable and falls off. The surface tension is given by

$$S = rdg\left[\frac{h}{2} + \frac{r}{3}\right],$$

where h is the maximum height of the liquid column above the end of the metal tube; d, the density of the liquid; r, the radius of the metal tube; and g, the acceleration of gravity.

The liquid is introduced slowly and continuously through a funnel, the tube of which is drawn down very fine and lies against the inner surface of the large glass tube. Heights are measured on a paper scale behind the tube, or better still, on graduations etched on opposite sides of the tube. The diameter of the metal tube is measured with a micrometer gauge. No cleaning is required beyond wiping the external edge of the metal tube with a cloth. The error in the results is proportional to that of the measurements, which need not exceed 2 per cent.

ROCKEFELLER INSTITUTE FOR MEDICAL RESEARCH.

THE SPECTRAL STRUCTURE OF THE LUMINESCENCE EXCITED BY THE HYDROGEN FLAME.

BY HORACE L. HOWES.

THE discovery of the luminescence of many substances when partially bathed in the hydrogen flame but at temperatures below red heat has recently been made by Prof. E. L. Nichols. The present paper represents only one of several tests which have recently been developed to prove that this new type of luminescence exhibits properties similar to that excited at room temperatures by the iron spark, the cathode rays, etc. To this end calcium oxide under hydrogen flame excitation was found to yield the same series of bands as calcite under iron spark excitation and three samples of the Lenard and Klatt phosphorescent sulphides under the two distinct modes of excitation yielded similar series but the distribution of intensity was very different. A new Hilger spectro-photometer was employed with the specimen mounted on a rotating disk of brass gauze and the analysis of the spectra was rather difficult because of very extensive overlapping of the bands.

NEW HAMPSHIRE College,
February 9, 1921.

The Motion of Electrons between Coaxial Cylinders in a Uniform Magnetic Field.

By Albert W. Hull.

The thermionic current between a straight filament and a concentric cylindrical anode is unaffected by a uniform magnetic field parallel to the filament so long as this field is smaller than a critical value, but falls abruptly to zero when the field is increased beyond this value. The same is true if the outer cylinder is cathode and the inner anode, except that the fall is less abrupt.

The relation between these quantities is very simple in practical cases.

The equations of motion of the electrons, using cylindrical coördinates r, θ, and z, are:

$$\frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2 = F\frac{e}{m} - H\frac{e}{m}r\frac{d\theta}{dt}, \quad (1)$$

$$\frac{1}{r}\frac{d}{dt}\left(r^2\frac{d\theta}{dt}\right) = H\frac{e}{m}\frac{dr}{dt}, \quad (2)$$

$$\frac{d^2z}{dt^2} = Z\frac{e}{m}, \quad (3)$$

where F and Z are the electric intensities in the radial and axial directions respectively, and H the magnetic field, which is uniform and parallel to the filament. F is a function of r only.

The first integrals of these equations are readily obtained, giving the velocity components dr/dt and $r(d\theta/dt)$. The critical magnetic field, at which the electrons are just able to reach the cylinder, is found either by placing the radial velocity component equal to zero; or by equating the square of the tangential velocity component, multiplied by half the mass, to the total work done on the electron by the radial electric force, *i.e.*,

$$\frac{dr}{dt} = 0,$$

or

$$\frac{1}{2}m\left(r\frac{d\theta}{dt}\right)^2 = eV. \quad (4)$$

This gives

$$V = H^2\frac{e}{8m}\left(R^2 - 2r_0^2 + \frac{r_0^4}{R^2}\right) + H\frac{r_0v_0}{2}\left(1 - \frac{r_0^2}{R^2}\right) - \frac{m}{2e}\left(u_0^2 + v_0^2\left[1 - \frac{r_0^2}{R^2}\right]\right),$$

where R and r_0 are the radii of outer and inner cylinders respectively, V the potential difference between them, u_0 and v_0 are the radial and tangential components, respectively, of the initial velocities of the electrons, and H the magnetic field that is just sufficient to prevent electrons reaching the anode.

If the electrons start from a filament of radius r_0 small compared with R, and the initial energies $\frac{1}{2}(m/e)u_0^2$ and $\frac{1}{2}(m/e)v_0^2$ are small compared with V, this becomes:

$$H = \sqrt{8\frac{m}{e}}\frac{V^{1/2}}{R}. \tag{5}$$

This is the case of a straight filament cathode in the axis of a cylindrical anode.

If the outer cylinder is cathode and the inner anode, eq. (4) becomes:

$$H = \frac{r_0}{R^2}\sqrt{\frac{8m}{e}V} + 2\frac{mv_0}{eR}. \tag{6}$$

This applies to a helical filament with anode wire in its axis. It applies also to the motion of positive ions produced by electrons from a straight filament in the axis of a cylindrical anode. In this case R is the distance from the axis at which the ions are produced, and e/m the ratio of charge to mass of the ion.

If the radii of both cylinders are large compared with their distance d apart Eq. (4) becomes:

$$V = H^2\frac{e}{2m}d^2 + Hv_0d - \frac{u_0^2}{2\frac{e}{m}}. \tag{7}$$

This is the case of plane parallel plates with electric field normal and magnetic field parallel to the planes.

The approximate paths of the electrons can be calculated for the case of the straight filament and external concentric anode, on the assumption that the space charge distribution is the same with magnetic field as without. This assumption is justified by the experimental fact that the positive ionization, as measured by a central negatively charged electrode (the filament is a small helix in this case, with collecting electrode in its axes) is unchanged by a magnetic field, even when the current to the anode is cut down to less than 1 per cent. of its maximum value by the field. The equation of the path on this assumption is:

$$r = R\left[\sin\frac{2}{3}\theta\right]^{3/2}. \tag{8}$$

RESEARCH LABORATORY,
GENERAL ELECTRIC CO.,
SCHENECTADY, N. Y.,
February 10, 1920.

AN ELECTRICAL DOUBLET THEORY OF THE NATURE OF THE MOLECULAR FORCES OF CHEMICAL AND PHYSICAL INTERACTION.

BY R. D. KLEEMAN.

THE forces of interaction between molecules which give rise to the internal heat of evaporation of liquids, their surface tension, etc., are explained by means of the electric forces associated with each atom on account of the electrons and positive electrical charges it contains. Whatever the arrangement of electrical charges their effect at a point some distance from the atom could be represented by an electrical doublet. Now we may arrange a number of doublets in space so that they attract each other in all directions, by placing them at the points of intersection of three sets of parallel equidistant planes which are at right angles to each other, so that opposite charges face each other in a row of doublets, and the order of the charges is different in two adjacent rows. When the doublets are in motion it would follow from the principle that *the potential energy of a system tends to a minimum* that the doublets would undergo motion of translation and rotation in such a manner that on the average attraction between the doublets would result, and that it would be as large as possible. Thus though any direction of the axis of a doublet in space might be equally probable, attraction would result through an association of the directions of the various doublets. The attraction between two molecules in a liquid would accordingly vary inversely as the fourth power of their distance of separation, and vary directly as the product of the moments of the molecular doublets, multiplied by a factor which would depend on the nature of the interaction of the molecules, and hence depend on the temperature of the liquid, the distance of separation of the molecules, etc.

It can be shown that it follows from the stopping power of the α particle that the moment of the doublet of an atom is proportional to the square root of its atomic weight, and that the moment of a molecule is equal to the sum of the moments of its atoms. This is confirmed by the results of previous investigations by the writer on the law of attraction giving rise to the internal heat of evaporation of liquids, etc., according to which the attraction an atom exerts is proportional to the square root of its atomic weight.

Union College,
February, 1921.

A Copernican Atomic Model Based on Electromagnetic Theory.

By Albert C. Crehore.

The atomic model applied to the hydrogen atom is represented in meridian section in Fig. 1. The nucleus, *C*, is relatively enlarged, it being impractical to represent nucleus and electron to the same scale. The effective nucleus consists of two charges, namely the nucleus, *C*, with positive charge $2e$ and one negative electron, *A* or *B*, of charge e which are inseparable, thus making an effective nuclear charge of plus e. Thus from the neutral atom only one electron is detachable. Both electrons,

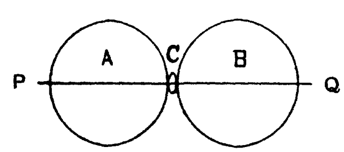

Fig. 1.

A and *B*, in the normal state of the atom at the zero of temperature are in contact with the positive charge, *C*, one at each pole. All three charges rotate about a common axis, *PQ*, but the electron has no tendency to describe an orbit revolving around the nucleus on the Saha theory. The rotating nucleus, so far as great distances are concerned as I have shown elsewhere,[1] may be replaced by an equivalent closed circular charged rotating ring.

[1] Abstract, Phys. Rev., Feb., 1921, p. 252.

The precise state of motion of rotation of the nucleus has not been rigorously solved on the basis of Saha's fundamental equations, but a state of rotation of the nuclear charge, which is supposed uniformly distributed throughout the volume as in the "solid" Lorentz electron, is not incompatible with the theory. It is not necessary to suppose that the angular velocity of an element of charge is constant for all radii; it may increase with the radius. It follows from the theory that two coaxial rings revolving in the same direction exert an accelerating effect upon each other when the distance between centers is several diameters of the rings. This problem is managable because the series employed remains convergent. The convergence disappears, however, as the rings are brought together to within a fraction of the radius, and an exact solution without resorting to these series must apparently be found before the mutual effect of two elementary rings within the body of the nucleus becomes known.

Assuming that a state of rotation meets the theoretical requirements for equilibrium, the spherical charge at rest is unstable, and becomes a pure fiction. This consideration makes possible an approximation to the shape of the positive charge in advance of an exact solution. The potential energy of a uniformly distributed spherical charge, E, at rest is[1]

$$W = 3E^2/5ak. \tag{1}$$

The mass[2] of the charge is

$$m = \frac{4}{5ak}\left(\frac{E}{c}\right)^2. \tag{2}$$

Eliminating the quantity $E^2/5ak$ between (1) and (2) gives the potential energy of the charged sphere at rest as

$$W = \frac{3}{4}mc^2. \tag{3}$$

Einstein and others have adopted mc^2 rather than $mc^2/2$ as the total energy associated with a mass m. In view of (3) the figure $mc^2/2$ cannot represent the total energy, and allowing for an equilibrium rotation the $3mc^2/4$ should be increased by the energy due to the rotation. Why is not the $mc^2/4$ the energy due to the rotation? For, together with (3) the total energy becomes mc^2. Let it be assumed that this is the case, and also that the shape of the sphere is distorted into an oblate spheroid when in equilibrium rotation. The pressure per unit area of surface of the sphere at rest is given by Schott[3]

$$P = 3E^2/8\pi a^4 k.$$

Surface pressure and a tension along the surface are mathematically equivalent and convertible by the formula

$$T = P/\frac{1}{r}\left(+\frac{1}{r^1}\right) = \frac{a}{2}P,$$

T being the tension and r and r' the two radii of curvature of the surface, which

[1] Schott, Electromagnetic Radiation, Eq. (381).

[2] Loc. cit., Eqs. (379), (380).

[3] Loc. cit., Art. 257.

merge into a when the surface is spherical. Let it now be supposed that the sphere is distorted by the rotation into an oblate spheroid, both the volume of it and the tension remaining constant. The work required is that to stretch the surface over a larger area, say A. The work is, therefore, AT, and this is to be equated to $mc^2/4$, giving

$$AT = A\frac{a}{2}P = 3AE^2/16\pi a^3k = mc^2/4. \tag{4}$$

Whence, by (2) and (4)

$$A = \frac{16}{15}\pi a^2.$$

The area of a sphere, being four great circles, must be stretched according to this result 16/15 of a great circle. If the shape is an oblate spheroid, this fact determines the eccentricity and the major and minor axes. The result is the eccentricity

$$e = 0.945,$$

and the ratio of the axes

$$a/b = 3.058.$$

Numerically, setting $E = 2e = 2 \times 4.762\ 10^{-10}$ E.S.U., and $m = 1.659 \times 10^{-24}$ grams, (2) gives the radius of the sphere of the nucleus at rest as

$$r = 4.86 \times 10^{-16} \text{ cm.}$$

The equatorial radius of the spheroid is, therefore, 7.22×10^{-16} cm., and the polar radius 2.20×10^{-16} cm.

In a former communication[1] the attraction between two such atoms as in Fig. 1 was given when the rotating nucleus is replaced by an equivalent ring of radius a_H, the frequency of rotation being $2K$, twice the Rydberg constant. When the atoms are given all orientations the force comes out an attraction equal to the known gravitational attraction, provided the ring has a radius

$$a = 6.4 \times 10^{-16} \text{ cm.}$$

This ring falls within the body of the rotating nucleus, being less than the equatorial radius 7.22×10^{-16} above. It is not possible to say how to calculate a ring equivalent to the spheroid above until the angular velocities of the elements at different radii are known or obtained from the Saha theory.

The ring equivalent to the negative electron, also a rotating body, is much larger in diameter, and has a linear speed about 1/30.4 part of the ring of the nucleus, and on this account has a small almost negligible gravitational effect.

The natural free period of the nucleus, when suddenly deformed in shape, as by a collision with one of it's own electrons, probably corresponds to the greatest frequency of any known body, as it is also the body of smallest known dimensions. Its natural frequency may be given by setting the whole energy content equal to $h\nu$, giving

$$h\nu = mc^2 \text{ or } \nu = 2.27 \times 10^{23} \text{ frequency.}$$

[1] Loc. cit.

This corresponds to a wave-length of

$$\lambda = c/\nu = 13.2 \times 10^{-18} \text{ cm.}$$

This appears to be the order of magnitude of the wave-length to be expected. The vibration implies a periodical interchange between the rotational energy and the potential energy due to the changing shape of the nucleus.

NELA RESEARCH LABORATORIES,
CLEVELAND, OHIO,
February, 1921.

THE COPERNICAN ATOM RADIATING ENERGY. A PHYSICAL INTERPRETATION OF PLANCK'S QUANTUM RULE.

BY ALBERT C. CREHORE.

LET Fig. 1 of the preceding abstract represent an atom of hydrogen gas subjected to bombardment by a stream of electrons. To study the effect of a passing electron, begin by neglecting the rotation of the parts of the atom. The mechanical force upon the atom due to the moving electron may be divided into two parts, (1) the electrostatic part, and (2) the part arising from the velocity. The electrostatic force upon one electron in the atom due to the passing electron, barring an actual collision which is not necessary to account for the radiation, can never be as great as the electrostatic force exerted by the rest of the atom upon its own electron, because of the comparatively great distance to the passing electron. Hence, the electrostatic part (1) of the mechanical force alone can never pull an electron away from the atom.

The part (2) due to the velocity of the passing electron is very much less than the electrostatic force, unless the electron moves with a velocity very close to that of light. The experimental velocity observed to produce radiation is of the order 2×10^8 cm./sec. (11 volts) in hydrogen, a speed less than one hundredth that of light. Hence, no such low speed electron will ever succeed in separating an electron from the atom according to theory so long as the rotations of the parts of the atom are neglected. The chief effect of these forces (1) and (2) is to increase the pressure of one electron against the nucleus and diminish that of the other, thus rendering one more readily detachable than the other.

The key to the behavior of this atom is the rotation of the nucleus coupled with its elongated[1] shape. The passing electron exerts a couple on the nucleus tending to turn the direction of its axis of rotation. This effect is due to the term $(\mathbf{q}_1 \cdot \mathbf{q}_2)\mathbf{R}$ in the Saha ponderomotive force equation. Referring to Fig. 2, let PQR be the path of the electron passing the atom H. At the nearest point Q, where the effect is a maximum, this term gives a force shown by the arrow, A, directed toward Q, the vector $\mathbf{R}$ joining the two points. This being the force upon an element in the upper side, the force upon a corresponding element

[1] See abstract, "A Copernican Atomic Model Based on Electromagnetic Theory."

of the rotating nucleus diametrically opposite is reversed in direction and represented by the arrow, *B*, directed away from *Q*, the two forces forming a couple tending to turn the axis of rotation.

When the effect of this couple is small, the nucleus may turn a little and roll smoothly without losing contact with the electrons at any time. Then, as the passing electron leaves, this motion rights itself thus restoring all of the energy received back to the passing electron again, which departs with as much energy as it had on arrival. It is considered that the energy of the passing electron must be sufficient to cause the nucleus to roll over at a rate sufficient to make it leave contact with and bump against or collide with one of its own electrons before energy may be said to be received by the atom.

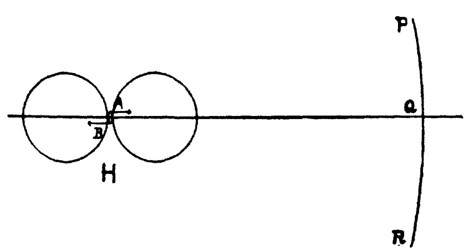

Fig. 2.

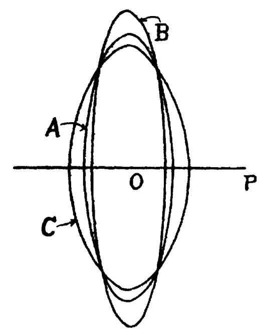

Fig. 3.

This process may be likened to the rolling of an elliptical wheel along a horizontal table. When the velocity of the wheel exceeds a certain critical value, the wheel leaves contact with the table and bumps along each half revolution. This analogy also points to the existence of a critical velocity.

These collisions set up the high frequency vibrations characteristic of the nucleus,[1] and the energy thus absorbed is not returnable to the passing electron as it was before, being transformed in character. It is postulated in advance of a rigid proof that this high frequency energy is absorbed by the nearest charges and is effectually imprisoned within the atom, very little, if any, of it being radiated into space as high frequency waves. The only way open to the atom under these circumstances to get rid of this energy is to eject an electron. How this takes place is made clear by electromagnetic theory.

Picture the nucleus thus set into high frequency vibration something like Fig. 3. The axis of rotation, *OP*, is supposed to remain fixed while the meridian section normally represented by the curve, *A*, becomes elongated into *B* and foreshortened into *C* as extreme distortions during a complete cycle. The motion is probably not as simple as this, but any complexity in the vibration does not affect the argument, for the best that can be expected is to obtain an approximation to the effect that such a vibration may produce. The area of the surface undergoes a periodical variation, and there must occur a corresponding periodical exchange of rotational energy with the potential energy

[1] Loc. cit.

due to the shape. But, the whole energy content of the nucleus takes part in this exchange, and the available energy is, therefore, large.

To approximate the force upon an electron in the immediate vicinity due to this vibration, divide the nucleus into two halves by the equatorial plane, and replace one half by a single equivalent point charge on the axis, say at the center of volume. This point will then oscillate along the axis OP during a complete cycle, since the center of volume moves along this line. The whole nucleus may thus be replaced by two point charges vibrating along the axis, first from and then toward each other, let us say harmonically. The average mechanical force that two such vibrating point charges exert upon an electron moving in its vicinity, together with the force exerted by the second electron inseparably associated with the nucleus, may represent the whole average force that the remainder of the atom exerts upon an electron ejected from itself. This force derived from the Saha theory is

$$F = \frac{e^2}{kr^2(1 - \beta_1^2)^{1/2}}[1 - 2\beta_2^2(1 - \tfrac{1}{2}\sin^2\alpha)]. \tag{1}$$

Let E_1, Fig. 4, represent the position of the ejected electron experiencing this force, and E_2 the position of the nucleus from which E_1 has become detached.

Fig. 4.

The broken line, P_1P_2, represents the direction of the axis, OP of Fig. 3, and the two point charges equivalent to the nucleus are supposed to move back and forth along these lines, P_1 and P_2. When the axis is directed toward the electron E_1, $\sin\alpha = 1$, and (1) reduces to

$$F = \frac{e^2}{kr^2}\frac{1 - \beta_2^2}{(1 - \beta_1^2)^{1/2}}, \quad \text{polar force.} \tag{2}$$

When E_1 is in the equatorial plane of E_2, $\sin\alpha = 0$, and (1) becomes

$$F = \frac{e^2}{kr^2}\frac{1 - 2\beta_2^2}{(1 - \beta_1^2)^{1/2}}, \quad \text{equatorial force.} \tag{3}$$

In these equations $\beta_1 c$ is the velocity of the electron E_1, and $\beta_2 c$ the maximum velocity of the two harmonically vibrating points substituted for the nucleus. When the expression is positive the electron E_1 is attracted toward the nucleus. The whole force acts along a radial line, there being no component perpendicular to the line joining the nucleus to the electron.

Equation (2) states that the force is always an attraction when the nucleus points its pole toward the electron E_1, for it may be assumed that β_2 cannot exceed unity. Equation (3) states that the force may be a repulsion provided β_2 exceeds $\frac{1}{2}\sqrt{2}$. Moreover, this repulsion may be very great indeed if the speed of the electron, $\beta_1 c$, approaches the velocity of light, because of the factor

$(1 - \beta_1^2)^{1/2}$ in the denominator. Although the mass increases with the speed, so also does the force in a similar fashion, and it should not be surprising if velocities approaching that of light are obtained. In fact, the force due to such a vibrating nucleus more than counterbalances the immense electrostatic force normally holding the electron in contact with the nucleus, and will in some cases succeed in ejecting an electron from the atom.

It also appears from (1) that there exists a radius at an angle, α, along which the force is zero, provided β_2 exceeds $\frac{1}{2}\sqrt{2}$. Equating (1) to zero gives this angle

$$\sin^2 \alpha = 2 - 1/\beta_2^2. \tag{4}$$

If the nucleus could turn its axis to this critical direction, the force upon the electron would be zero, and it would consequently move away with a uniform velocity. There would then occur no loss of energy by radiation, because no energy is radiated by uniform motion in a straight line. Under these circumstances the electron would carry away from the atom the whole of its own kinetic energy, and by this means the atom would lose far more energy than it normally does.

Regarding the nucleus and its electron, namely the effective nucleus, as a single physical system, like all physical systems left to themselves the entropy tends toward a maximum. In this instance the system strives to get rid of energy at a maximum rate, and would succeed in increasing the rate if the axis of the nucleus could turn immediately to the critical angle expressed by (4). The mutual forces between it and the electron govern the motion, and it must be expected that the pole of the nucleus will oscillate about this critical angle of zero force, being damped because of the loss of energy by radiation into space from the moving electron. The acceleration of the electron at the distance, x, should, therefore, be expressed by an equation of the type

$$\frac{d^2x}{dt^2} = k_1 \nu e^{-\nu t} \cos \nu t, \tag{5}$$

which multiplied by the mass is the force upon the electron. For, this force oscillates harmonically about a zero value, changing from repulsion to attraction, and decreasing in amplitude as the energy of the high frequency vibration is spent in the secondary process of radiation from the moving electron. The frequency $\nu/2\pi$ expressed by (5) refers to the oscillation of the pole of the nucleus. The whole motion is determined by this equation. A first integral with respect to time gives the velocity.

$$\frac{dx}{dt} = \tfrac{1}{2} k_1 e^{-\nu t}(\sin \nu t - \cos \nu t) + k_2. \tag{6}$$

Now the total energy radiated by an electron moving with this velocity may be calculated from the Liénard expression for the rate of radiation, which for straight line motion is

$$\dot{R} = \frac{\frac{2e^2}{3kc^3}\left(\frac{d^2x}{dt^2}\right)^2}{(1-\beta^2)^3}. \tag{7}$$

The Saha expression for the rate of radiation is not yet known to the writer. Assuming that the electron goes out and back along a radius, the constant of integration becomes zero, because the velocity is zero when the time is infinite. The whole energy lost by radiation in one excursion out and back is then

$$E = \int_0^\infty \dot{R}dt = \frac{\pi e^2 k_1^2}{2kc^3}\left(1 + \frac{23}{90}\frac{k_1^2}{c^2} + \cdots\right)\nu', \tag{8}$$

where ν' is the frequency corresponding to the angular velocity ν in (5), and k_1 is constant by hypothesis. Consequently the whole coefficient of the frequency is a constant, and (8) exactly expresses Planck's quantum rule, radiated energy is equal to a constant times the frequency.

The value of Planck's constant, h, cannot be determined from this, but an estimate of the initial velocity of the electron may be obtained required to make (8) equal to $h\nu'$. The series in the parenthesis may have any value from 1 to infinity according to the value of k_1, and by (6) $k_1/2$ represents the initial velocity. Denoting the sum of this series by S, its value must be $S = 138.2$ to give the proper value of h, this value being

$$h = 2\pi e^2 S/ck. \tag{9}$$

This sum shows that the initial velocity must be very close indeed to that of light.

To obtain an estimate of the relative energies, the radiated energy and the kinetic energy of the electron at some point, consider the maximum velocity possessed by the electron, for example, on its return trip to the nucleus. This velocity is 3.23×10^9 cm./sec., not far from one tenth the velocity of light. The kinetic energy is approximately $47. \times 10^{-10}$ ergs. The maximum energy ever radiated from a hydrogen atom is $h\nu = hK = 0.2154 \times 10^{-10}$ ergs, and the kinetic energy just computed is about 220 times this maximum radiated energy.

The energy lost by the whole atomic system, however, is no greater than the transformed energy received from the passing electron, since the kinetic energy of its own electron is all returned again to the atom. Thus it becomes manifest that the internal energy of the nucleus must have been drawn upon temporarily to provide this large amount of kinetic energy for one of its own electrons. The available energy in the nucleus is $mc^2 = 1.49 \times 10^{-3}$ ergs. This conception of the process involved in radiation illustrates the trigger action, which becomes possible because the energy received from the passing electron is entirely transformed in character, the atom itself impressing upon all subsequent events its characteristic properties.

NELA RESEARCH LABORATORIES,
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