

## A COMPARISON OF THE FECHNER AND MUNSELL SCALES OF LUMINOUS SENSATION VALUE

BY ELLIOT Q. ADAMS

A critical study of the Munsell scale of (luminous sensation) value has been made by Priest, Gibson, and McNicholas.<sup>1</sup> Their results "verify in a remarkable manner the consistency of the Munsell values for different hues . . . considering the uncertainties of heterochromatic photometry which were necessarily involved in Munsell's work." They establish that "the squares of the Munsell value numbers are directly proportional to the reflection of sunlight"—as might be expected from the construction of the Munsell photometer, which employs an Aubert diaphragm,<sup>2</sup>—and object that "the implication that values, read directly as the diagonal of the shutter, are proportional to sensation in the sense of Fechner's law is quite wrong." Since it is well known<sup>3</sup> that Fechner's law is only an approximation to the law relating brightness and sensation, and an equation which constitutes a closer approximation has recently been published,<sup>4</sup> it will be well to examine, in the light of this newly-found relation, both the Fechner and the Munsell (or Stefanini)<sup>5</sup> scales.

Before proceeding to a quantitative comparison, it will be well to point out anew<sup>6</sup> the difference between the range of valid-

<sup>1</sup> I. G. Priest, K. S. Gibson, and H. J. McNicholas, Technological Paper No. 167, Bur. Stds. (Sept. 1920) "An Examination of the Munsell Color System. I. Spectral and Total Reflection and the Munsell Scale of Value."

<sup>2</sup> H. Aubert, *Grundzüge der physiologischen Optik*, pp. 489, 547. Leipzig, W. Englemann, 1876.

<sup>3</sup> And is explicitly conceded by the authors on page 29 of the reference in footnote 1.

<sup>4</sup> E. Q. Adams and P. W. Cobb, *J. Exp. Psych.*, 5, pp. 39-45; 1922.

<sup>5</sup> A. Stefanini, *Nuov. Cim.* (3) 22, p. 97; 1887 (for sound). *Atti della R. Acc. Lucc. di Sc. Lett. ed Arti.*, 25, pp. 383-400 (for light and weight). The Stefanini formula is the special case of Plateau's formula  $E = kR^\epsilon$ , in which  $\epsilon = 0.5$ .

<sup>6</sup> See for example, A. Elsas, *Wundt's Philos. Stud.* 4, 162-79 (1888), also E. B. Titchener, *Experimental Psychology, Instructor's Manual, Quantitative*, pp. 210-32. §29. New York, Macmillan, 1905. The method of equal sense distances: historical and critical.

ity of Weber's law, and the range over which Fechner's sensation law holds. This comparison can be made more concrete by the analogy of the measurement of current by a tangent galvanometer<sup>7</sup> provided with an assortment of shunts. Such an instrument gives greatest percentage precision when the scale reading is in the neighborhood of  $45^\circ$ , and the percentage precision at that (or any other constant) angular deflection is the same whenever the shunt is selected so that the current measured gives that deflection. Hence with a sufficiently varied supply of shunts the percentage precision may be made constant over the range covered by the shunts, that is:

$$\Delta I = \frac{\Delta s}{b} I = cI \quad (1)$$

where  $\Delta I$  is the least detectable increase in current,  $I$ ;  $b$  and  $c$  constants, and  $\Delta s$  the least perceptible change in scale reading,  $s$ . At the same time *with any one shunt the scale reading,  $s$ , is related to the current by the equation:*

$$I = I_m \tan s \quad (2)$$

where  $I_m$  is the current which for that shunt gives a scale reading of  $45^\circ$  and hence maximum precision. Yet the equation obtained by integrating (1), in the form  $Ids = b dI$ ;

$$s = b \ln I \quad (3)$$

does not hold at all, for over the range of validity of (1), the scale reading is always near to  $45^\circ$ . Now if  $I$  represent light intensity and  $s$  sensation, (1) and (3) become Weber's and Fechner's Laws respectively.

The analogy is, of course, not perfect, for while equation (1) in the form

$$\Delta B = cB \quad (4)$$

holds over a considerable range of brightness, the equation<sup>8</sup>

<sup>7</sup> This instrument is chosen because its deflections follow a simple mathematical law and remain finite as the current is indefinitely increased. For the sake of continuous variation of the shunt resistance a slide wire might be used.

<sup>8</sup> Equation (2) of the article referred to in footnote (4), based on the assumption that visual impressions are transmitted along each fiber of the optic nerve by a series of impulses whose effect depends only on their frequency,—the All-or-None hypothesis of Keith Lucas, "The Conduction of the Nervous Impulse," p. 9, London; 1917. Cf. also L. T. Troland, J. Opt. Soc. Am., 4, p. 160; 1920.

which has been found to relate sensation to brightness *at constant adaptation*, (the analog of galvanometer readings with a given shunt) is not of the form of equation (2), but is

$$s = \frac{B}{B+k} \quad (5)$$

where  $k$  is a constant dependent on the state of adaptation, being equal to the brightness at which photometric precision is a maximum (for the given state of adaptation).

$s$  expresses sensation as a fraction of the maximum possible range of sensation; if it is desired to express it in other units, a coefficient,  $a$ , must be inserted in equation (5). Similarly if sensation is to be measured from any other point of reference than the sensation corresponding to (physically) complete blackness, a term,  $s_0$ , for that sensation, must be introduced into equation (5) which thus becomes

$$s = s_0 + a \frac{B}{B+k} \quad (6)$$

The relation between sensation and brightness thus assumes an infinite number of forms according to the value of  $k$ . Since in equation (6)  $s$  becomes independent of  $B$  at both limits,  $k \doteq 0$  and  $k \doteq \infty$ , the law of variation in these limiting cases may be found as follows. For  $k \doteq 0$ , i.e., for dark adaptation

$$s = s_0 + a \frac{B}{B+k} = s_0 + a \frac{(1)}{1+k/B} \doteq s_0 + a(1 - \frac{k}{B}) = (s_0 + a) - \frac{ak}{B} \quad (7)$$

while for adaptation to infinite brightness,  $k \doteq \infty$ ,

$$s = s_0 + a \frac{B}{B+k} \doteq s_0 + \left(\frac{a}{k}\right) B \quad (8)$$

that is, the sensation approaches in the first case a linear function of the reciprocal of brightness, in the second a linear function of brightness itself.

Many of the other formulas which have been found empirically are special cases of the Plateau equation

$$s = k'B^\epsilon \quad (9)$$

where  $\epsilon$  is an exponent lying between 0 and 1, and  $k'$  a constant.

If  $\epsilon$  be given the value  $\frac{1}{2}$ , equation (9) becomes the Stefanini<sup>5</sup> equation, on which the Munsell scale is based:

$$s = k' \sqrt{B} \quad (10)$$

If sensation be not measured from the sensation produced by the physical absence of light, a term,  $s_o$ , must be added in equation (9) as in the case of equations (5) and (6), giving

$$s = s_o + k' B^\epsilon \quad (11)$$

If  $\epsilon$  be made equal to unity, this becomes the Merkel<sup>9</sup> proportionality law:

$$s = s_o + k' B \quad (12)$$

which is identical with (8).

When  $\epsilon$  approaches 0, (11) becomes

$$s = s_o + k' e^{\epsilon \ln B} \doteq s_o + k' (1 + \epsilon \ln B) = (s_o + k') + k' \epsilon \ln B \quad (13)$$

that is, Fechner's law, which is, therefore, the other limit of the Plateau equation.

It will be noted that equations (5) and (9),—and hence also the equations derived from them,—retain their form if  $B$  be measured in other units, provided the appropriate changes are made in the constants of the equations. Since for any constant illumination the relation between sensation and test object *albedo*<sup>10</sup> (the brightness relative to that of a perfect diffusely reflecting surface similarly illuminated) will depend upon the albedo of the surroundings but will be independent of the illumination,—within the range of brightnesses for which Weber's law holds,—equations (4) to (13) may be made the same for the relation between sensation,  $s$ , and test object *albedo*, as for the relation between sensation and brightness. In what follows the symbol,  $B$ , and the term "albedo" will both signify *test object albedo*.

Priest, Gibson, and McNicholas give in their Fig. 16, (on p. 31), a comparison of the Merkel, Munsell, and Fechner scales, made to coincide for Nos. 1 and 9 of the Munsell scale.

<sup>9</sup> Julius Merkel, Wundt's Philos. Stud., 4, pp. 117-60, 251-91, 541-94, 1888; 5, pp. 245-91, 499-557, 1889; 10, pp. 140-59, 203-48, 369-92, 507-22 especially p. 517; 1894.

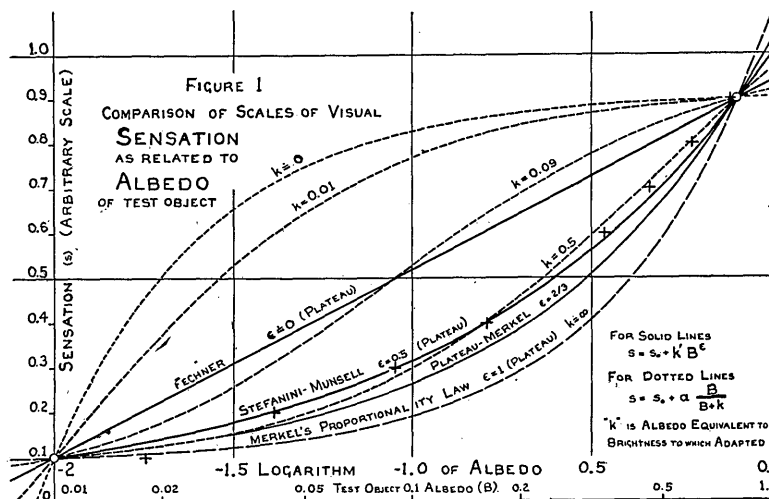
<sup>10</sup> This term is used habitually by astronomers in stating the reflecting powers of the planets.

TABLE 1. Relation Between Sensation and Test Object Albedo, According to Various Scales and Theories

Sensation No.	Scale No.	Albedo according to:									
		Munsell Scale		Merkel $\epsilon=1$	Fechner $\epsilon=0$	Plateau $\epsilon=2.3$	Adams and Cobb				
		meas. by Priest <i>et al</i>	Theoreti- cal (Stefani- ni)				k=0	k=0.01	k=0.09	k=0.50	
0.0	0	....	.00	.....	.0058	....	.0089	.0078	0	.....	.....
0.1	1	.018	.01	≡ .01	≡ .01	≡ .01	≡ .01	≡ .01	≡ .01	≡ .01	≡ .01
0.2	2	.041	.04	.11	.0173	.0578	.0114	.0128	.0225	.0522	.0522
0.3	3	.090	.09	.21	.0300	.126	.0133	.0165	.0386	.102	.102
0.4	4	.161	.16	.31	.0520	.211	.0159	.0215	.0600	.162	.162
0.5	5	.234	.25	.41	.0900	.310	.0198	.0290	.09	.234	.234
0.6	6	.343	.36	.51	.156	.420	.0261	.0412	.135	.325	.325
0.7	7	.465	.49	.61	.270	.542	.0386	.0656	.210	.441	.441
0.8	8	.602	.64	.71	.468	.670	.0736	.1267	.360	.595	.595
0.9	9	.772	.81	≡ .81	≡ .81	≡ .81	≡ .81	≡ .81	≡ .81	≡ .81	≡ .81
1.0	10	....	1.00	.91	1.403	.958	.....	.....	.....	1.130	1.130
Equation No. ....		10		8,12	13	11	7	6	6	6	6
s <sub>0</sub> ....		0		.09	.938 <sup>b</sup>	.0549	.91 <sup>b</sup>	-.72	0	0	.0738
Coefficient <sup>a</sup> .....		1		1	2.147 <sup>a</sup>	1.972	-.0081	1.64	1	1	1.336

<sup>a</sup> The coefficient, in the appropriate equation, of B, log B or 1/B respectively. For Fechner's Scale the coefficient is that for common logarithms.<sup>b</sup> These values are that for s<sub>1</sub>, the sensation for unit albedo, in the case of the Fechner scale, and that for S<sub>∞</sub>, the sensation for infinite relative brightness, for the limiting case k = 0, since in both s<sub>1</sub> is infinite.

A similar comparison is given in Table 1 and Fig. 1 of this paper. The figure differs from that of Priest *et al.*,—besides giving curves for several other formulas than the three named,—in two respects; sensation (or value) has been plotted against log albedo, instead of log albedo against sensation, and the theoretical curves have been made to agree at numbers 1 and 9 of the *theoretical* Munsell scale, (equation 10) while the *albedo* of the Munsell papers, as measured by Priest *et al.*, is indicated by crosses. Solid lines indicate formulas derived from that of Plateau (equations 9 to 13) for the indicated values of the exponent  $\epsilon$ , dotted lines show the relations given by the equation of Adams and Cobb



(equations 4 to 8) for various values of  $k$ , the brightness to which the eye is adapted, (expressed in *albedo* units). The Merkel proportionality law, being a limiting case of both the foregoing, is indicated by a dashed line.

From the figure it can be seen that all the other curves lie within those representing the limiting cases of constant adaptation to zero and infinite brightness, respectively; hence, by an appropriate constancy or variation in the state of adaptation during the measurements, any of the relations shown could be obtained experimentally. Again, it will be noted that with adaptation to the

geometric mean ( $k=0.09$ ) of scale numbers 1 and 9 (of the theoretical Munsell scale) the relation between sensation and albedo approximates the scale of Fechner, whereas with adaptation to the arithmetic mean<sup>11</sup> of the same brightnesses ( $k=0.41$ ), it agrees well with the Stefanini equation on which the Munsell scale is based. It is noteworthy that the actual Munsell scale agrees fully as well with the equation of Adams and Cobb (for  $k=0.5$ ), as with the Stefanini equation.

In view of the criticisms by Priest, Gibson, and McNicholas it may be well to point out the *physical basis* of the Munsell scale. Its numbers represent the *amplitude* of the light waves from a diffusely reflecting surface relative to that of the waves from a *perfect* diffuse reflector, similarly illuminated.

In view of the marked dependence of the subjective scale of (luminous sensation) value on the state of adaptation, it is doubtful if the axiom of Priest, Gibson and McNicholas (p. 29): "It will probably be agreed by all who are interested in the subject and consider it carefully, that the steps in the value scale should be apparently equal; that is, the visual contrast between the cards of any two adjacent numbers should equal that between any other adjacent two,"—can be applied to the grading of the series of grays. It may well be preferable to use the actual albedo of the surfaces, since this scale is one of the *limits* of the subjective scale, namely that approached as the adaptation brightness is increased.

#### SUMMARY

1. Only if the state of adaptation of the eye is maintained constant, is it proper to speak of luminous sensation as a function of brightness.

2. With constant adaptation,  $k$ , the functional relation between sensation,  $s$ , and brightness,  $B$ , is well represented by the equation of Adams and Cobb.

$$s = \frac{B}{B+k} \quad (5)$$

<sup>11</sup> The curve for  $k=0.41$  in the equation of Adams and Cobb has not been represented on Fig. 1, but it can easily be seen that it would lie only slightly above that for  $k=0.50$ .

3. All the equations connecting sensation and brightness are of such a form that,—within the range of validity of Weber's law,—the relation between sensation and test object *albedo* may be made independent of the absolute level of brightness (for any constant illumination of test object and surroundings) but will depend on the albedo of the surroundings to which the eye is adapted.

4. Between numbers 1 and 9 of the Munsell scale of (luminous sensation) value, the sensations of an eye adapted to a brightness corresponding to the arithmetic mean<sup>11</sup> of the albedo of those scale numbers (i.e., 0.41) approximate the values of the Munsell scale.

5. Within the same limits, the sensations of an eye adapted to a brightness corresponding to an albedo of 0.09,—the *geometric* mean of the albedos corresponding to Munsell scale numbers 1 and 9,—approximate the values of the Fechner scale.

6. In view of the marked dependence of subjective value on the state of adaptation of the eye, grays should be rated according to their *albedo*, which is a physically determinate property.

NELA RESEARCH LABORATORIES  
CLEVELAND, OHIO  
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