

STUDIES ON SOIL PHYSICS.

PART II.—THE PERMEABILITY OF AN IDEAL SOIL TO
AIR AND WATER.

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Introduction.

§ 1. In Part I. (vol. IV. pp. 1—24) it was shown that the intrinsic permeability of a soil when measured with water was less than when measured with air as the experimental fluid, and that the ratio of the two values varied with the amount of colloidal matter present. It was desirable to determine accurately whether the values would be identical for soils composed of pure sand or other uniform and non-colloidal particles.

It should also be possible to calculate the permeability of a soil from a knowledge of the sizes of its component particles and of the specific pore-space; and this problem is one that has engaged the attention of several authors both from the experimental and mathematical standpoints.

Previous Investigations.

§ 2. Allen Hazen (*Ann. Rep. State Board of Health, Mass., U.S.A., 1892*), in an investigation of filter-bed sands and gravels, used an experimental filter filled with various grades of sands, and deduced the formula

$$v = cd^3 \frac{h}{l} (0.70 + 0.03\theta),$$

where c is a constant, approximately 1000; d is the effective diameter of sand grain; h is the head of water pressure; l is the length of sand column; θ is the temperature; and v is the velocity of water-flow in metres daily.

§ 3. King and Slichter (*Nineteenth Annual Report*, 1899, Part 2, of the U.S. Geol. Survey), in a very complete and masterly research, have not only experimentally measured the permeability of sands, soils and rocks to both air and water, but the latter has developed a mathematical solution of the problem.

Slichter's mathematical argument may be summarized as follows:—

If we consider a soil composed of spherical particles, then the lines joining any eight contiguous spheres will be found to outline a parallelepiped varying in form from a cube to a rhombohedron, according as the particles are packed in the loosest or closest manner. This unit element of volume with length of side d will contain the equivalent of a single sphere of diameter d .

"If the grains of soil are arranged in the most compact manner possible, each grain will touch surrounding grains at twelve points, and the elements of volume will be a rhombohedron having face angles equal to 60° and 120° ." "If the grains are not arranged in the most compact manner, the rhombohedron will have its face angles greater than 60° and each sphere will touch other spheres in but six points and *nearly* touch in six other points." "The most open arrangement of the soil grains, which is possible with the grains in contact, is had when the rhombohedron is a cube."

For a rhombohedron whose side is of unit length the volume is given by $(1 - \cos \theta)(\sqrt{1 + 2 \cos \theta})$, where θ is the face angle; and consequently the pore-space S , not occupied by the enclosed sphere, is given by

$$S = 1 - \frac{\frac{\pi}{6}}{(1 - \cos \theta)\sqrt{1 + 2 \cos \theta}} \dots \dots \dots (1).$$

The values of S for different values of θ are thus readily calculable and are given in Table I.

Slichter finds, that if l be the length of the soil column under consideration and d the diameter of each particle, then the length of the pore-space capillary may be taken as equal to $\frac{l(1 - \cos \theta)}{\sin \theta \sqrt{1 + 2 \cos \theta}}$, and the area of its minimum cross-section as equal to

$$\frac{\sin \theta - \frac{\pi}{4}}{2} \cdot d^2.$$

A mathematical and experimental investigation showed that the error involved in assuming the pore to be *circular and equal in area*

to the minimum cross-section of the actually triangular pore is almost balanced (to within one per cent.) by the assumption that the pore is straight instead of curved.

Hence, using the same nomenclature as in Part I. (*loc. cit.* § 6), we may write Poiseuille's equation

$$\frac{v}{t} = \frac{\pi}{8\eta} \cdot \frac{ghs}{l} \cdot \Sigma r^4 \dots\dots\dots(2)$$

in the form

$$\begin{aligned} \frac{v}{t} &= \frac{\pi}{8\eta} \cdot \frac{ghs \sin \theta \sqrt{1+2 \cos \theta}}{l(1+\cos \theta)} \cdot \frac{\left(\sin \theta - \frac{\pi}{4}\right)^2}{4\pi^2} \cdot d^4 \\ &= \frac{ghsd^4}{32\eta l} \cdot \frac{\sin \theta \sqrt{1+2 \cos \theta}}{1+\cos \theta} \cdot \left(\sin \theta - \frac{\pi}{4}\right)^2 \dots\dots\dots(3), \end{aligned}$$

for each of the two pores penetrating a unit element, i.e. the rate of flow per area $\frac{\sin \theta}{2} \cdot d^2$ which is half the area of cross-section occupied by each rhombohedron.

Then if the cylinder containing the column of soil be of area A ,

$$\begin{aligned} \frac{v}{t} \text{ (for the whole area)} &= \frac{ghsAd^3}{16\pi\eta l} \cdot \frac{\sqrt{1+2 \cos \theta}}{1+\cos \theta} \cdot \left(\sin \theta - \frac{\pi}{4}\right)^2 \\ &= \frac{ghsAd^3}{16\pi\eta l} \cdot \frac{(1-\cos \theta)(\sqrt{1+2 \cos \theta})\left(\sin \theta - \frac{\pi}{4}\right)^2}{\sin^2 \theta} \dots\dots\dots(4). \end{aligned}$$

But from equation (1)

$$1-S = \frac{\frac{\pi}{6}}{(1-\cos \theta)(\sqrt{1+2 \cos \theta})};$$

therefore

$$\begin{aligned} \frac{v}{t} &= \frac{ghsAd^3}{96\eta l} \cdot \frac{\left(\sin \theta - \frac{\pi}{4}\right)^2}{(1-S) \sin^2 \theta} \\ &= \frac{ghsAd^3}{96\eta l} \cdot \frac{\left(1 - \frac{\pi}{4} \operatorname{cosec} \theta\right)^2}{1-S} \dots\dots\dots(5). \end{aligned}$$

Put $B = 1 - \frac{\pi}{4} \operatorname{cosec} \theta$, and take $g = 980$ and $s = 1$, then

$$\eta \cdot \frac{v}{th} \cdot \frac{l}{A} = 10.2d^3 \cdot \frac{B^2}{1-S} \dots\dots\dots(6),$$

and putting

$$k = \frac{1 - S}{B^2},$$

we get

$$\eta P = 10.2 \frac{d^3}{k} \dots \dots \dots (7).$$

The values of S and the mathematically derived constants B and k were calculated by Slichter for various values of θ and are given in Table I. It will be seen that k varies from 11.37 for the loosest packing to 85.43 for the most compact arrangement possible with perfect spheres.

TABLE I.

Values of the "permeability constant" (k), as calculated by Slichter, for all possible variations of pore-space (S) when dealing with a soil composed of spherical particles.

S Pore-space	θ Angle of packing	B $= 1 - \frac{\pi}{4} \operatorname{cosec} \theta$	k $= \frac{1 - S}{B}$
.2595	60° 00'	.0934	85.43
.30	62° 36'	.1155	52.45
.31	63° 18'	.1210	47.1
.32	64° 3'	.1266	42.45
.33	64° 49'	.1322	38.45
.34	65° 37'	.1378	34.75
.35	66° 27'	.1434	31.6
.36	67° 21'	.1491	28.8
.37	68° 18'	.1549	26.26
.38	69° 17'	.1605	24.08
.39	70° 20'	.1661	22.1
.40	71° 28'	.1719	20.3
.41	72° 43'	.1775	18.75
.42	74° 3'	.1832	17.3
.43	75° 32'	.1890	15.95
.44	77° 10'	.1946	14.75
.45	79° 6'	.2003	13.70
.46	81° 25'	.2057	12.75
.47	84° 59'	.2117	11.83
.4764	90° 00'	.2146	11.37

§ 4. The desirability of testing the formula

$$\eta P = 10.2 \frac{d^3}{k}$$

experimentally for both air and water is self-evident and King has carried out, in conjunction with Slichter, an elaborate series of measurements with sands of various sized grains.

A summary of the results obtained in one series [*Fifteenth Annual Report*, Agr. Expt. Stn. Univ. Wisconsin, p. 127] is given in Table II.

TABLE II.

Summary of experiments, by F. H. King, on the Permeability of Sands to Air and Water.

Measured diameter of sand grains	Permeability to water calculated from			Average percentage error
	known diameter of grains	observed flow of air	observed flow of water	
	$P_w = c \frac{d^2}{\eta_w k}$	$P_w = P_a \frac{\eta_w}{\eta_a}$	$P_w = \frac{v}{t}$	
2.755	2680	2277	2296	- 17.5
1.998	1372	1132	1080	- 24
1.588	909.1	757	756	- 20
1.345	688.6	522	542	- 20
1.157	499.6	453.2	504.6	- 4.5
1.106	326.6	297.5	329.2	- 4
.802	194.0	193.0	210.0	+ 3.5
.665	106.2	122.0	138.6	+ 22.5
.582	75.7	80.6	94.8	+ 16
.489	59.8	66.8	72.3	+ 16

They confirm the formula to within a little more than experimental error, but it must be borne in mind that King's apparatus allowed a very considerable margin for this factor. King himself admits these discrepancies and goes on to say,—“Search has been made thus far in vain for some medium consisting of spherical grains of perfectly uniform diameter, with which to secure a rigid test of the method, but, as yet, none has been found.”

Shot are unsuitable, partly on account of their weight, but also because the finest dust shot the present authors can obtain have a diameter of about 1 mm. and much finer sizes are required.

Preparation of Material.

§ 5. *A suitable material has however been found* in the “glistening dew” of the picture post-card artist. This is composed of almost perfectly spherical grains or beads of glass ranging in diameter from 0.25 mm. upwards. Enquiries have been made for “beads” of still smaller sizes, but such are apparently not manufactured; if obtainable

they would be extremely valuable for an extension of this and similar researches on soil physics and the phenomena of adsorption.

Two varieties were used; the smaller of which (0.5 mm. or less) were composed of colourless glass, often surface stained with aniline dyes, whilst the larger (up to 1 mm.) were coloured with various metallic oxides.

Sifting was soon found to be a tedious method of grading them, for the sieves became almost immediately choked and were with difficulty freed from beads which were but little larger than the perforations.

§ 6. Finally an elutriation method was devised, based on the ordinary processes used for the mechanical analysis of soils.

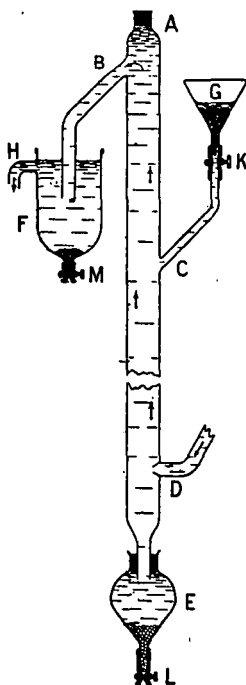


FIG. 1. Elutriator for grading beads and sands.

The apparatus¹ consisted of a vertical tube *ABCDE* about six feet long and one inch in diameter, provided with an inlet tube at *D* with a constricted jet for the water supplied from a constant level reservoir.

The water passed upwards and out at *B* to the cylinder *F*, finally escaping to the waste by overflowing at *H*. After they have been

¹ We are indebted to Mr Radcliff of the Bairnsdale School of Mines for suggesting this shape of elutriating tube.

cleaned by successive treatment with caustic soda and aqua regia, the beads to be graded are poured into the funnel *G* and flow in a stream, regulated by the clip *K*, through *C* into the main elutriation tube where they meet the upflowing current of water.

Those beads having a diameter greater than the "critical diameter" for the water current will sink past *D* into *E* and can be drawn off through *L* as required: the lighter beads will rise and be carried over into *F* where they will settle and may be removed by opening the clip *M*.

Many precautions were found necessary, as Hilgard has pointed out for his soil elutriator, and even under the best conditions it is impossible to obtain a perfect separation. When many beads are present in the tube *BC* it will no longer have the same efficient area as when empty of beads, and so the velocity of upward flow of the water becomes materially increased and beads having more than the critical diameter will be carried over into *F*. Another cause of error is the variation in stream velocity with the distance from the walls of the tube and often beads may be observed travelling upwards for several inches in mid-stream, and then, after approaching the wall, tumbling downwards again.

Occasionally beads have hovered about in the tube for two or three days, but even these on microscopical examination exhibit a considerable variation in their diameters.

§ 7. The separations obtained were the results of many repetitions of this elutriation process, care being taken to put each sample through slowly. The original mixture was thus classified into thirteen grades, of which five were selected for the final experimentation.

§ 8. The average diameter of the beads in each grade was determined by counting at least two thousand beads and taking weighings at regular intervals. As each bead was separately picked out with a small pair of forceps and transferred to a weighing bottle the process became very tedious, especially in the case of the smaller sizes. In all, some 40,000 beads were counted out in this way—one by one.

The density of each grade was determined by displacement of water in a specific gravity bottle—it was found to vary considerably with the size. The two larger sizes being coloured with metallic oxides showed a corresponding increase of density.

The density of these beads was found to be 3.117 and their average diameter was therefore 0.9374 mm.

The discrepancy between the weights of the separate lots of beads

was greater than would have been expected either from the method employed in sorting them or from their appearance under the microscope (see Pl. I, figs. 8 and 9). The discrepancy was less in the case of the smaller beads but greater for the sand grains used in the later part of this work. The large numbers counted and weighed must however have eliminated any perceptible error due to this cause.

TABLE III.

Weight of large coloured glass beads.

Weight of first thousand=1.3319		Weight of sixth thousand=1.3861
„ second „ =1.3946		„ seventh „ =1.3149
„ third „ =1.3428		„ eighth „ =1.2835
„ fourth „ =1.3467		„ ninth „ =1.4070
„ fifth „ =1.3027		
Average weight per thousand=1.3456		

Standardization of Apparatus for measuring Permeability.

§ 9. As the tubes filled with beads had a very much larger permeability than the soils previously dealt with, the rates of flow for both air and water no longer conformed to Poiseuille's simple capillary tube law¹ and were sufficiently high to make the correction for loss of kinetic energy of the moving fluid an appreciable factor.

In order to determine the value of this and other corrections, and to test the accuracy of the method, the special apparatus employed was first used to measure the permeability of a straight capillary tube whose constants could be exactly obtained from its dimensions.

This capillary tube was carefully calibrated with a thread of clean mercury: the variations in diameter amounted to less than one per cent.

Length of capillary = 54.90 cm.

Average area of cross-section = 0.00745 sq. cms.

Then using the same nomenclature as in Part I. of this paper [*vide* this *Journal*, vol. IV. pt I. pp. 3—6], from Poiseuille's equation

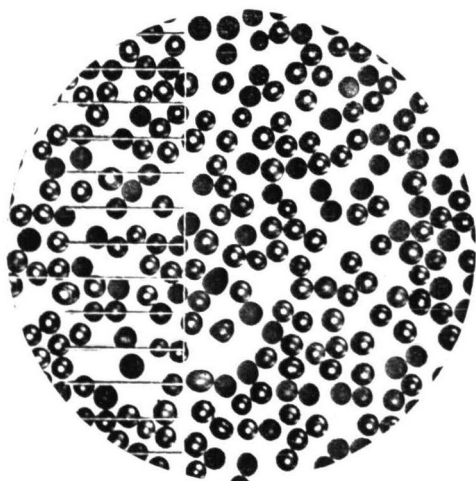
$$\eta = \frac{r^4 \pi g}{8} \cdot \frac{hst}{lv},$$

and taking

$$P = \frac{v}{t} \cdot \frac{l}{hs},$$

$$\eta P = \frac{\pi g}{8} \cdot r^4.$$

¹ See Vol. IV. p. 3, § 6.



Grade B: diameter=0.709 mm.

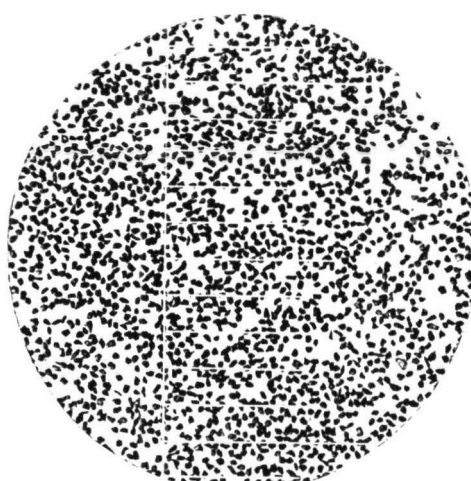


Grade E: diameter=0.250 mm.

Fig. 8.



Grade a: diameter=0.825 mm.



Grade c: diameter=0.186 mm.

Fig. 9.

Therefore, for the tube in question,

$$\eta P = 0.002153.$$

§ 10. *Permeability to air.* The corrections applied were:—

(i) *For loss of kinetic energy.* The well-known Couette-Finkener formula is

$$\eta = \frac{\pi}{8} \cdot \frac{ghst}{vl} \cdot r^4 - \frac{\rho}{8\pi} \cdot \frac{v}{lt},$$

where ρ is the density of the fluid (air); then as

$$\begin{aligned} P_{\text{obs.}} &= \frac{vl}{th}, \\ P_{\text{corr.}} &= P_{\text{obs.}} \left(1 + \frac{\rho}{8\pi} \cdot \frac{v}{\eta lt} \right) \\ &= P_{\text{obs.}} \left\{ 1 + \frac{\rho P_{\text{obs.}}^2}{\pi^2 l^2 g r^4} \cdot h \right\}, \end{aligned}$$

which for the capillary tube in question simplifies to

$$P_{\text{corr.}} = P_{\text{obs.}} (1 + 0.00107h).$$

(ii) *For compressibility of the air.* Meyer, Breitenbach and others have shown that Poiseuille's law when applied to gases should take the form

$$\eta = \frac{\pi}{8} \cdot \frac{r^4 t}{vl} \cdot \frac{p_1^2 - p_2^2}{2p_0},$$

where p_0 is the pressure under which the air is measured;

p_1 " " of air at the *high* pressure end of the capillary;

p_2 " " " " *low* " " " "

As pressures were measured in centimetres of water in the gauge, this correction reduced to

$$v_{\text{corr.}} = v_{\text{obs.}} \left(1 + \frac{2h_0 - h}{2B + h} \right)$$

for experiments in which a "head" of pressure was employed, and to

$$v_{\text{corr.}} = v_{\text{obs.}} \left(1 + \frac{h - 2h_f}{2B - h} \right)$$

for experiments in which a "tail" of pressure was employed, *i.e.* where the pressure in the bulb *C* was less than atmospheric.

B = barometric pressure expressed in cm. of water;

h = mean working pressure;

h_0 = initial pressure in measuring bulb;

h_f = final pressure in measuring bulb.

(iii) Corrections were also applied for the "dead space" in the connecting tubes between the measuring bulb and the capillary tube, and for the slight alteration in volume due to movement of the water in the pressure gauge.

All these corrections were summarized for our apparatus in the following equation, which, though complicated in appearance, was found to be simple in application :

$$P_{\text{corr.}} = P_{\text{obs.}} (1 + 0.00107h) \left[1 + \frac{2h_0 - h}{2B + h} \text{ or } 1 + \frac{h - 2h_f}{2B - h} \right] \left\{ 1 + \frac{1}{v} \left(\frac{h_0 - h_f}{20} + \frac{h_0 - h_f}{14} \right) \right\}.$$

§ 11. The apparatus, shown in Fig. 2, consisted of a measuring bulb *C*, of known capacity between the marks *B* and *D*, connected on

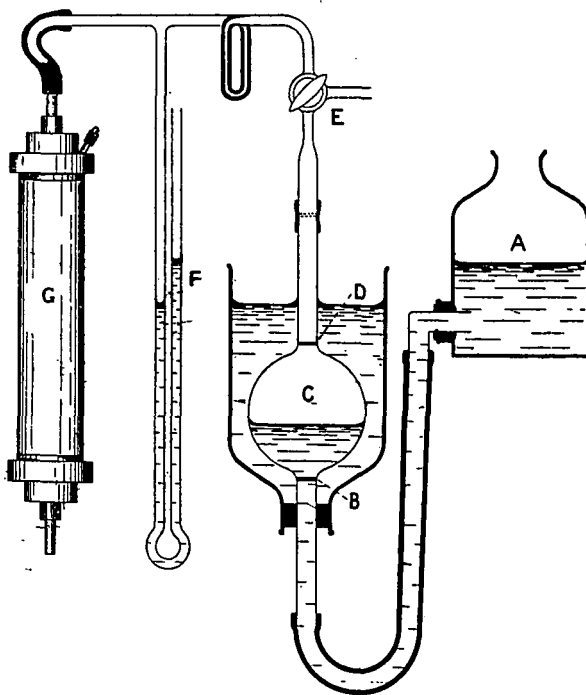


FIG. 2. Apparatus used for the measurement of the permeability to air of a glass tube filled with beads.

the one side to an adjustable water reservoir *A*, and on the other to the tube *G* containing the glass beads (or the capillary tube) whose permeability to air was to be measured.

To carry out an experiment the reservoir *A* was adjusted so as to give the required pressure and, by means of the three-way tap *E*, air was either forced or aspirated through until the water level was several centimetres either below *B* or above *D* according as the measurements were to be made under "head" or "tail" of pressure. The tap *E* was then turned so as to make communication between *C* and *G*, and the water level rapidly rose as the air in *C* was being forced through *G*. At the moment of passing *B* the stopwatch was started and the first reading of the pressure at *F* obtained by means of a reading telescope; further readings were made at regular time intervals and the watch stopped as the water level passed *D*.

The mean pressure *h* was obtained by averaging these readings, half weight only being given to the initial and final pressures (*h*₀ and *h*_{*f*}).

Whilst the temperature coefficient for the viscosity of air is only small, yet it is necessary to waterjacket the bulb *C* to prevent *changes* of temperature, and therefore of effective volume, *during* the progress of an experiment. For a similar reason *C* and *G* must be maintained at the *same* temperature; consequently, when the capillary tube was being experimented on, it was enclosed in a Liebig's condenser through which the water that had circulated round *C* was passed.

TABLE IV.

Time readings	<i>h</i> (cm.)	
	(a) using "head" of pressure	(b) using "tail" of pressure
Initial	9.75	10.7
15"	7.4	8.1
30"	6.7	7.45
45"	6.1	6.8
1' 0"	5.6	6.2
1' 15"	5.15	5.7
1' 30"	4.75	5.25
1' 45"	4.35	4.75
2' 0"	3.95	4.3
2' 15"	3.55	3.85
2' 30"	3.25	3.35
2' 45"	2.85	2.8
3' 0"	2.4	—
3' 15"	2.0	—
Final	1.05	1.9
Mean value of <i>h</i>	4.53	5.41

Temp. = 9.7° C.

(a) Using "head" of pressure ;
 $t = 3.425$ mins.
 $h = 4.53$ cm.

(b) Using "tail" of pressure ;
 $t = 2.855$ mins.
 $h = 5.41$ cm.

A typical set of readings are given in Table IV for a pair of experiments in which a "head" and "tail" of pressure respectively were used.

§ 12. A number of experiments were made with this capillary tube under widely different conditions of pressure in order to decide whether the formulae given by Meyer and others would hold good for our apparatus and eliminate the various errors due to loss of kinetic energy etc.

TABLE V.

Measurements of the Permeability (to air) of a glass capillary tube.

t (secs.)	h (cms.)	$P_{\text{obs.}}$	h_0	h_f	$P_{\text{corr.}}$	Temp.	$\eta \times 10^6$	$\eta P_{\text{corr.}}$
(i) Using "head" of pressure								
54.8	+ 17.84	11.64	23.4	14.5	12.10	8.8°	177.2	.002144
67.0	14.47	11.74	19.9	11.0	12.13	8.8°	177.2	2148
83.1	11.57	11.85	17.0	8.15	12.19	10.0°	177.8	2168
100.6	9.44	11.96	14.8	5.95	12.26	9.0°	177.2	2172
125.7	7.54	12.00	12.95	4.05	12.27	9.0°	177.2	2174
171.9	5.47	12.09	10.9	2.0	12.31	9.4°	177.4	2184
215.5	4.35	12.22	9.85	1.05	12.36	9.4°	177.4	2192
250.4	3.67	12.38	9.25	0.5	12.38	9.8°	177.7	2236
(ii) Using "tail" of pressure								
50.3	- 19.28	11.83	24.8	16.0	12.06	10.5°	178.0	.002146
53.5	17.89	11.87	23.35	14.5	12.10	9.7°	177.7	2152
58.9	16.24	11.88	21.6	12.75	12.12	9.9°	177.7	2152
67.1	14.18	11.93	19.5	10.7	12.13	10.1°	177.8	2154
69.3	13.61	12.03	18.95	10.1	12.23	9.6°	177.6	2176
80.3	11.83	11.95	17.0	8.2	12.13	10.1°	177.8	2154
96.3	9.86	11.97	15.0	6.1	12.15	9.6°	177.6	2156
117.6	8.015	12.06	13.05	4.3	12.22	9.6°	177.6	2170
140.0	6.71	12.10	11.8	3.05	12.26	9.7°	177.7	2178
171.3	5.41	12.28	10.7	1.9	12.43	9.7°	177.7	2208
205.4	4.53	12.23	9.75	1.05	12.36	9.7°	177.7	2196

§ 13. Both of these series of experiments (in Table V) exhibit a slight regular progression, and seem to indicate that the various corrections leave some residual effect unbalanced. It was found difficult to eliminate the error due to differing surface tension forces acting on the water in the two limbs of the manometer; but the zero error, sometimes 0.1 or 0.2 mm., was always read and allowed for.

The true value of ηP should apparently be obtained by working at an infinitely small pressure or by extrapolation of the values obtained

at higher pressures, but owing to the shape of the measuring bulb the lowest mean pressure that could be worked with, in the manner described above, was about 4 cms. The device was therefore adopted of inserting a "resistance in the circuit," between E and G , in the shape of a suitable piece of capillary tubing. This allowed of mean pressures of less than 1 cm. being utilized without loss of accuracy although initial and final readings of the pressure in the bulb (h_0 and h_f) as well as at the manometer (h_1 and h_2) had to be taken in order to apply the usual corrections indicated in § 10.

TABLE VI.

Measurements of the Permeability (to air) of a glass capillary tube, using a "resistance" capillary in the circuit.

t (secs.)	h (cms.)	$P_{\text{obs.}}$	h_0	h_f	h_1	h_2	Temp.	$\eta P_{\text{corr.}}$
216.9	-4.299	12.23	23.5	14.7	5.37	3.41	10.0°	.002166
216.7	-4.276	12.28	23.5	14.7	5.37	3.41	8.5°	2164
240.9	+3.960	11.90	22.1	13.3	5.16	3.09	10.0°	2174
240.9	+3.960	11.92	22.1	13.3	5.16	3.09	9.0°	2170
603.7	+1.579	12.15	12.0	13.1	2.82	0.73	9.8°	2194
607.5	-1.512	12.35	11.5	2.6	2.69	0.64	9.4°	2198
990.7	-0.940	12.20	9.3	0.4	2.20	0.13	10.2°	2195
1053.5	-0.887	12.17	9.5	0.6	2.18	0.10	10.2°	2192

§ 14. All the values of $\eta P_{\text{corr.}}$ obtained are shown in Fig. 3 and it will be seen that the "gang" is independent of experimental error and that the results for both "head" and "tail" pressures, with and without

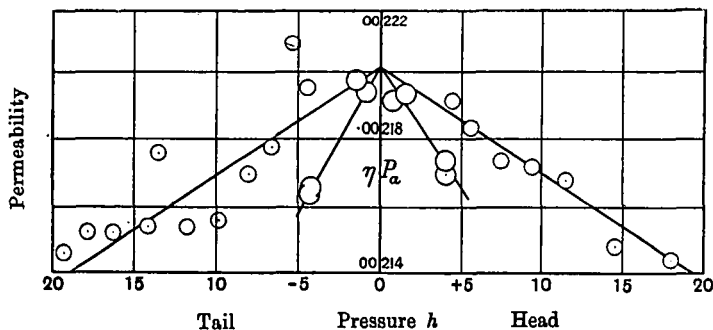


FIG. 3. Permeability of capillary tube to air under varying pressures.

the capillary in series, all extrapolate almost exactly to $\eta P = 0.002202$. This is 2.2 per cent. greater than 0.002153 the value calculated from

the measured dimensions of the capillary and taking $\eta_0 = 0.0001730$ [Meyer and Breitenbach]; but it must be borne in mind that the absolute value for the viscosity of air obtained by different observers varies by at least this amount.

Consequently we may fairly claim for this apparatus an accuracy quite sufficient for our needs.

§ 15. *Permeability to water.* The water was supplied from a large reservoir and after passing through the capillary was, in the preliminary experiments, allowed to drip from the curved exit tube into the receiving vessel—the head of pressure being taken as the difference in level between the orifice of this exit tube and the water in the reservoir.

The capillarity effects at the outlet are considerable however, and were overcome by reading the pressure from two side tubes connected to each end of the tube whose permeability is being measured. The nozzle of the outlet tube had also to be immersed in liquid and placed in contact with the wall of the burette that was used as a receiving vessel, otherwise a fluctuation of pressure was observed as each drop detached itself from the orifice.

The results obtained in a series of experiments at different pressures are given in Table VII. The decrease of ηP_w at low pressures is to be accounted for by the loss due to evaporation of the effluent water, and the percentage error due to this cause will evidently be proportional to the reciprocal of the rate of flow or, more conveniently, driving pressure.

TABLE VII.

Measurement of the Permeability (to water) of a glass capillary tube.

h (cms.)	$\eta_w P_w$	$\frac{1}{h}$
19.39	0.002158	.052
11.21	2148	.089
10.41	2147	.096
10.20	2143	.098
4.119	2107	.243
3.025	2087	.331
2.048	2028	.489
1.980	2030	.506
1.098	1907	.911
0.56	1740	1.787

The results obtained for $\eta_w P_w$ are therefore plotted (in Fig. 4) against $1/h$ and the graph is, as was expected, a straight line extrapolating to $\eta P = 0.002173$.

The kinetic energy correction, even for the highest rates of flow, is almost insignificant and only raises this extrapolated value of ηP to 0.002183.

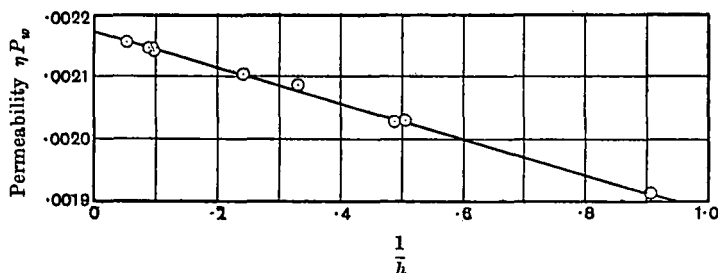


FIG. 4. Permeability of capillary tube to water under varying pressures.

§ 16. All the results obtained with this capillary tube may be summarized thus:

$$\begin{aligned} \eta P \text{ (calculated from the measurements of the tube)} &= 0.002153, \\ \eta_a P_a \text{ (from observed permeability to air)} &= 0.002202, \\ \eta_w P_w \text{ (from observed permeability to water)} &= 0.002183. \end{aligned}$$

The accuracy shown here is abundantly sufficient for the purpose in view.

Construction and Testing of Apparatus for holding Beads.

§ 17. Many measurements were made of the permeability of each of the various grades of beads when contained in narrow tubes, such as had been previously used for soils, but it was found that the error in the measurement of the pore-space in these narrow tubes was too great for the results to be of value in our present enquiry.

§ 18. A special containing vessel, shown in Figs. 2 and 5, was therefore designed. It consisted of a glass tube with brass cone-shaped end pieces which were carefully ground to fit the glass tube and were capable of exact measurement before being cemented in place. The inner surfaces of these brass ends were turned into hollow cones whose apices were connected with the exterior by means of short pieces of brass tubing into which corks and glass tubes were inserted.

The beads were retained in place by a disc of wire gauze (shown in the figure by a line of dots) and were fed in through a small hole drilled diagonally through the brass ends. This arrangement allowed

of the accurate determination of volume and other dimensions and proved thoroughly satisfactory.

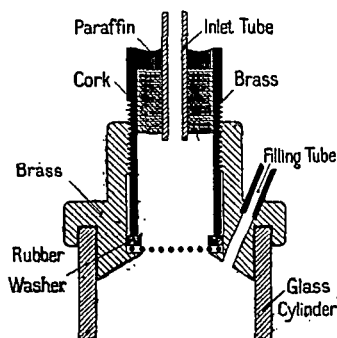


FIG. 5.

A knowledge of the volume is necessary to determine the pore-space S , and $\frac{l}{A}$ is also required since it enters into the formula

$$P = \frac{v}{th} \cdot \frac{l}{A}.$$

The value of l/A was separately determined for the body of the tube and the conical ends.

For a truncated cone

$$\begin{aligned} \frac{l}{A} &= \frac{1}{\pi \tan \alpha} \left(\frac{1}{r} - \frac{1}{R} \right) \\ &= \frac{h}{\pi r R}, \end{aligned}$$

where α is the angle of the cone, r and R the radii of its top and base respectively and h its length.

§ 19. The measurements of the cones and tubes, made with specially accurate calipers, were as follows for the two vessels that were employed.

Vessel I was only used in the earlier experiments as the glass cylinder was repeatedly breaking. Nearly all the recorded experiments were made with Vessel II which was carefully made to fit a glass tube of better quality and very uniform bore.

§ 20. Bead sample D (diameter = 0.319 mm.; density = 2.750) was first experimented with. Vessel I was loosely filled with about 200 grams of the beads and its permeability to air measured by means of the apparatus described in § 11; then, by gentle packing, an extra one or two grams of beads were introduced and the permeability again measured. This

process was repeated until, even on continued "dumping" and rotating, the tube would hold no more beads.

TABLE VIII.

Dimensions of Vessels in which Permeabilities of Beads were measured.

	Vessel I		Vessel II	
Area of cross-section of glass cylinder...	7.09 cm. ²		3.567 cm. ²	
Length of glass cylinder.....	12.13 cm.		23.60 cm.	
Length of cones = h	0.394	0.362	0.328	0.342
Radius of base of cones = R	1.473	1.470	1.052	1.048
Radius of apex of cones = r	0.720	0.720	0.500	0.482
Volume of cones	1.55 cm. ³	1.42 cm. ³	0.65 cm. ³	0.66 cm. ³
Value of l/A for cones	0.119	0.109	0.198	0.215
Total volume of vessel	88.97 cm. ³		85.50 cm. ³	
l/A for whole vessel	1.943		7.03	

After measuring the permeability to air for this minimum pore-space, the tube was connected with a water-pump and exhausted. Air-free distilled water was allowed to enter and remain at rest for some considerable time to dissolve any air that had escaped removal by pumping.

The permeability to water was then measured by observing the rate of flow through the column under a measured pressure of water.

TABLE IX.

Preliminary measurements of the Permeability of Bead Sample D in an unlined tube.

S	$k = \frac{1-S}{B^2}$	$\eta_a P_a \times 10^3$	$\eta_w P_w \times 10^3$	$\frac{\eta_a P_a}{d^3} \cdot k$
0.379	26.06	0.780		19.97
0.3695	26.39	.746		19.34
0.366	27.28	.7075		18.95
0.364	27.79	.682	0.545	18.62

§ 21. The disagreement between $\eta_a P_a$ and $\eta_w P_w$ in the last line of Table IX is too great to be accounted for by ordinary experimental

errors and was eventually found to be due to imperfect removal of the air from between the interstices of the beads. In subsequent experiments special precautions were taken to ensure the complete removal of the air and the disagreement practically vanished.

More difficult of explanation however were the values of $\frac{\eta_a P_a}{d^3} \cdot k$ given in the last column; these should, from Slichter's calculation (see § 3), be equal to 10.2. The high permeability observed might, it was thought, be due to the channels, of a larger area than those between the beads themselves, existing all round the inner surface of the tube (Fig. 6). The error due to this cause will depend on the relative diameters of the beads and the containing tube. Where the diameter of a bead is one per cent. of that of the tube a simple calculation shows that the permeability will be increased by about twenty-eight per cent., if the beads are packed in the closest possible manner; for ordinary packing the error will be diminished to about eight per cent.

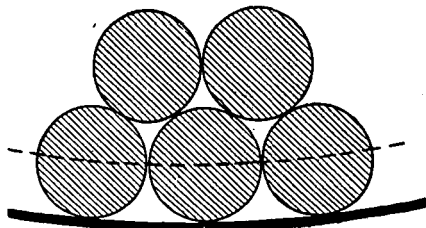


FIG. 6.

But the want of concordance with Slichter's formula cannot be accounted for in this way.

§ 22. If the wall of the tube could be covered with hemispheres fitting closely together, this correction would be practically eliminated; this condition was attained by the following indirect device.

The inner surface was lined with a layer of cement (Chatterton's compound) approximately calculated to be sufficient to embed the beads to half their diameter. In order to obtain an even coating the cement was dissolved in chloroform and poured into the vessel and the chloroform completely evaporated by blowing air through the tube while the latter was continuously rotated in a horizontal position. The tube was then filled with beads and gently warmed to melt the cement into which a layer of beads sank and was firmly held on cooling.

The cement and half of the embedded beads were considered as belonging to the wall and the other half of the glass beads to the main

body whose permeability and pore-space were to be measured. In calculating the latter, half the weight of these embedded beads was added to the weight of the loose beads with which the vessel was filled.

The "effective" volume and area of the tube were obtained from appropriate weighings of the attached cement and beads. In one case the following measurements were obtained:

Unlined tube: original volume = 89.0 c.c.; $\frac{l}{A} = 1.943$.

The inner surface was then lined with cement (cones excepted). 5.80 grams of beads—diameter = 0.319 mm., density = 2.75—attached themselves and occupied a volume of 2.10 c.c.

<i>Lined tube:</i> volume (determined by water content)	= 85.15 c.c.
half-volume of beads in lining	= 1.05 c.c.
"effective" volume	86.20 c.c.
volume of cones (from previous measurement)	= 2.97 c.c.
" cylinder (by difference)	= 83.23 c.c.
length "	= 12.04 cm.
therefore area of lined part of cylinder	= 6.910 sq. cm.
and $\frac{l}{A}$ for " " "	= 1.744
but $\frac{l}{A}$ for cones (from previous measurement)	= .228
therefore "effective" $\frac{l}{A}$ for whole vessel	= 1.972

§ 23. Table X gives the results of the first experiments carried out with the tube lined as described above; and in Fig. 7 curves are drawn comparing the unlined and lined tubes filled with the same beads.

TABLE X.

Measurements of the Permeability of Bead Sample D in a lined tube.

S	$k = \frac{1-S}{B^2}$	$\eta_a P_a \times 10^3$	$\eta_w P_w \times 10^3$	$\frac{\eta_a P_a}{d^2} \cdot k$
0.3895	22.19	0.901		19.63
0.3835	23.39	.851		19.55
0.3835 (?)	23.39	.811		18.62 (?)
0.373	25.62	.736		18.51
0.3625	28.17	.626	0.611	17.31

Whilst attempting to duplicate these measurements the glass tube broke and it was at this stage that the second vessel was constructed and brought into use. It was also lined with cement and a series of readings obtained with it, for the same size of beads is included in Table XI and in Fig. 7.

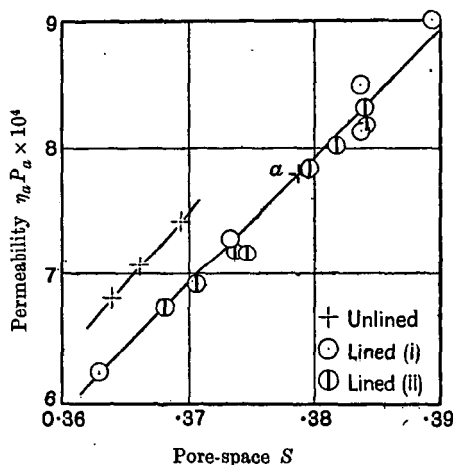


FIG. 7. Comparison of the permeabilities of beads (sample D) in unlined and lined tubes.

It will be seen that while the two lined vessels give results agreeing within about one per cent., the beads in the unlined tube had a permeability some seven per cent. higher. This agrees with the estimate previously given (§ 21).

The highest pore-space which we attempted to work with in the unlined tube gave a point, marked (a), considerably below the true curve; obviously the slight vibration due to unavoidable handling of the apparatus further compacted the beads and the reading must therefore be discarded. With the lined tubes there was naturally much less tendency for this to occur.

Results obtained with Glass Beads.

§ 24. Series of measurements were now made with five sizes of the carefully graded glass beads, the glass tube being in each case lined with some of the same size beads embedded in cement. After the permeability to air was observed for several degrees of compactness, then water was admitted as described and a reading of the permeability to water obtained.

TABLE XI.

Summary of the Permeabilities of Glass Beads, of various grades, as measured in lined tubes.

Diameter of "beads"	Pore-space	Intrinsic Permeability		$\frac{\eta P}{d^2} \times 10^3$	$\frac{\eta P}{d^2} \cdot k$
		to air	to water		
d	S	$\eta_a P_a \times 10^3$	$\eta_w P_w \times 10^3$		
A 0.938 mm.	0.391	6.76		7.68	17.10
	.370	5.21		5.92	15.54
	.3675	5.125		5.82	15.65
	.3635	4.96		5.63	15.69
			4.70	5.33	14.90
B 0.709 mm.	0.400	4.685		9.31	18.90
	.3925	4.19		8.33	18.02
	.388	3.93		7.80	17.52
	.384	3.806		7.56	17.61
	.373	3.264		6.48	16.58
			3.33	6.63	16.93
C 0.497 mm.	0.376	1.776		7.18	17.89
	.373	1.677		6.77	17.33
	.366	1.538		6.22	16.96
	.366	1.564		6.34	17.28
	.361	1.489		6.02	17.18
			1.523	6.15	17.57
D 0.319 mm.	0.3895	0.901		8.85	19.63
	.3835	.851		8.35	18.62
	.373	.736		7.22	18.51
	.3625	.626		6.15	17.31
			0.611	6.00	16.90
	0.384	0.831		8.15	18.98
	.3795	.785		7.71	18.65
	.3745	.719		7.06	17.83
	.3705	.692		6.795	17.77
			0.6645	6.52	17.05
	0.382	0.802		7.875	18.65
	.3735	.7195		7.065	18.00
	.368	.676		6.63	17.73
			0.655	6.43	17.40
E 0.250 mm.	0.3905	0.516		8.25	18.30
	.384	.470		7.525	17.52
	.379	.4385		7.02	17.05
	.373	.4135		6.62	16.94
	.370	.4035		6.45	16.94
	.370	.410		6.56	17.03
	.366	.377		6.03	16.45
			0.373	5.97	16.27

§ 25. The summarized results are given in Table XI and Fig. 10 and a satisfactory basis for discussing the accuracy of Slichter's calculations for spherical particles of uniform size within the range of the diameters available. The values of k used throughout this paper are those taken from his formulae (see Table I).

Results obtained with Sands.

§ 26. It was decided to extend the investigation to ordinary sand grains also of uniform size. The quartz sand utilized was but slightly water-worn and, after being chemically cleaned, three grades were separated by sifting and elutriation by the same apparatus and in the same manner as the glass beads.

The accompanying photographs enable their shape, size and uniformity to be examined and compared. The smaller grades originally contained much ilmenite, the bulk of which however was removed by careful "panning off."

§ 27. The average diameter of each grade was determined in the usual way by counting out several thousand grains and weighing them. Much greater deviations from the mean were found than with the glass spherical beads, possibly due in part to the variations in composition and consequently of specific gravity.

The actual weighings were as follow:—

TABLE XII.
Diameter and Specific Gravity of Sand Grains.

Number weighed	Coarse Sand		Medium Sand	Fine Sand
	1000		500	500
Weighings	0.7364 .7261 .6933 .7405 .7415	0.7251 .7883 .9207 .8226 .9076	0.0170 0.0167	0.0045 .0045 .0045 .0043 .0045
Average weight per thousand ...	0.7802 gram.		0.0337 gram.	0.00892 gram.
Specific Gravity	2.654		2.653	2.648
Average Diameter	0.825 mm.		0.289 mm.	0.186 mm.

§ 28. The same apparatus was used for measuring the permeabilities of the quartz sands to both air and water as had previously been used

for the glass beads. It was considered unnecessary to line the vessel with cement and a layer of the sand on account of the irregular and angular shapes of the individual grains. The results justified this decision.

TABLE XIII.
Summary of Permeabilities of Quartz Sands.

Diameter of Grains	Pore-space	Intrinsic Permeability		$\frac{\eta P}{d^2} \times 10^3$	$\frac{\eta P}{d^2} \cdot k$
		to air	to water		
d	S	$\eta_a P_a \times 10^3$	$\eta_w P_w \times 10^3$		
a 0.825 mm.	0.386	3.186		4.68	10.71
	.378	3.018		4.43	10.84
	.3695	2.706		3.975	10.48
	.361	2.545		3.74	10.68
	.3485	2.144		3.155	10.12
	.347	2.070		3.04	9.89
			2.026	2.97	9.68
b 0.289 mm.	0.412	0.447		5.35	9.37
	.393	.3640		4.36	9.40
	.385	.3265		3.91	9.03
	.381	.3065		3.67	8.76
	.378	.2955		3.535	8.66
	0.408	0.4525		5.42	10.33
	.395	.383		4.585	10.08
	0.403	0.4005		4.795	9.51
	.385	.3385		4.055	9.36
	0.401	0.3885		4.65	9.37
	.373	.294		3.525	9.02
			0.294	3.525	9.02
c 0.186 mm.	0.435	0.2525		7.30	11.20
	.4045	.1820		5.265	10.33
	.3945	.1650		4.78	10.17
	.377	.1320		3.82	9.44
			0.1290	3.74	9.22

Discussion of results.

§ 29. The justifiability of Slichter's assumptions and calculations may be judged by the agreement of these results with the formula

$$\eta P = 10.2 \frac{d^2}{k}.$$

I.e. the expression $\frac{\eta P k}{d^2}$ should be equal to 10.2 for all sizes of particles and for each arrangement of pore-space.

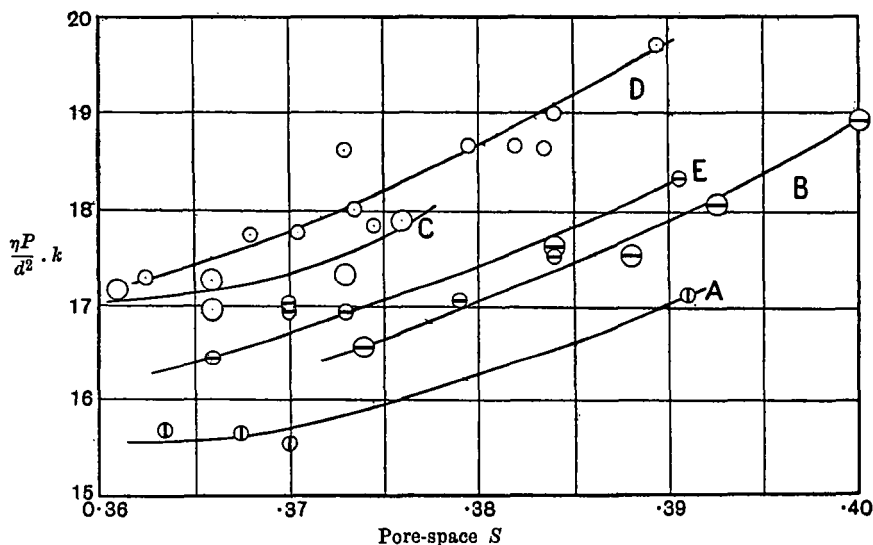


FIG. 10. Summary of results with glass beads (see Table XI).

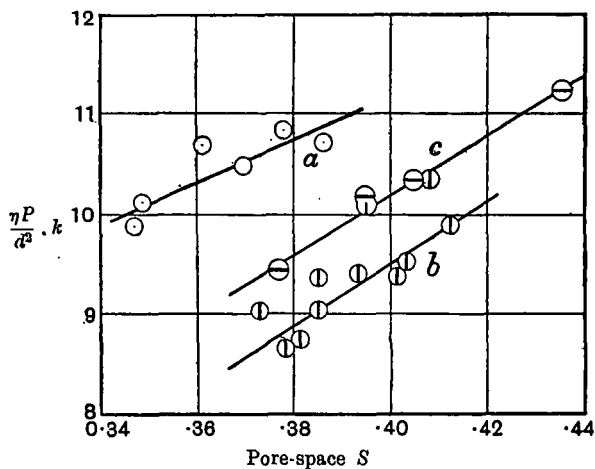


FIG. 11. Summary of results with sands (see Table XIII).

§ 30. It will be observed that for the glass beads however no such agreement is to be found, for the value obtained varies from 15.5 to 19, and increases with increase of pore-space. In other words, the permeabilities both to air and water are from 50 to 85 per cent. greater than Slichter's calculated values.

The explanation of this is almost certainly to be found in his method of considering each soil capillary as if it were a double triangular-shaped pore with a *partition down the centre* instead of as an *undivided* more or less rhomboidal pore at its narrowest part. It is obvious that this assumption must considerably undervalue the permeability of the pore,

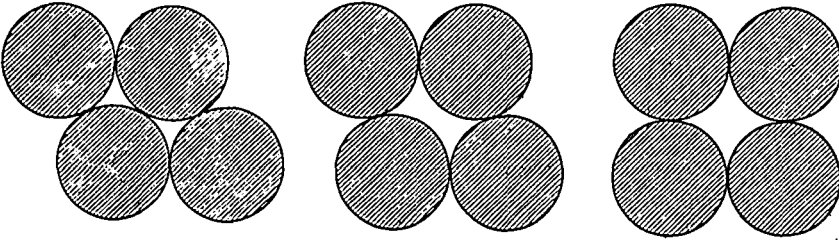


FIG. 12.

but it would be difficult to make anything like an exact estimate of the deficiency. Any such correction should vanish when the angle of packing is sixty degrees, for, as the pore-space approaches the minimum possible, so does the pore actually become divided into two and Slichter's method of discussion would then hold good. [See Fig. 12.]

An examination of the graph (Fig. 10) bears this out, for if extrapolated to the minimum pore-space the value of $\frac{\eta P k}{d^2}$ evidently will in general approximate to the calculated figure—10·2.

§ 31. On the other hand, the experiments with ordinary sands, though showing a rather large percentage error, give an average result of 9·45 for the permeability to air ($\eta_a P_a$) and 9·31 for the permeability to water ($\eta_w P_w$). This is, considering the difficulties of accurate measurement, a satisfactory concordance.

The obvious explanation (of this less perfect material agreeing more perfectly with the theoretical formula) is that the angular shapes of the particles do practically have the effect of dividing the pore into two triangular passages as assumed in the formula.

Conclusion.

§ 32. As the particles in ordinary soils are not perfect spheres but more or less angular in shape, *the experiments described in this paper show that the formula $\eta P = 10\cdot2 \frac{d^3}{k}$ holds quantitatively for variations of the pore-space and of the diameters of the soil particles.* This will be so

whether the permeating fluid be *air or water, provided that the actual sizes of the soil particles are unaffected by the presence of water*¹.

With this factor taken into account it is therefore legitimate to consider a soil as statistically composed of a bundle of capillary tubes when discussing the movements of air and water through it.

In conclusion we have to acknowledge our indebtedness to Professor T. R. Lyle for valuable advice and suggestions, to Mr H. J. Grayson for assistance in preparing the micro-photographs and to the Victorian Government for financial assistance towards the expenses of this research.

¹ It was pointed out in Part I. of these researches (*loc. cit.* §§ 9, 32) that the behaviour with water is a most important property of the soil; for whereas with clean sands the ratio $\frac{\eta_a P_a}{\eta_w P_w}$ will be but slightly greater than unity, the amount of colloidal matter present will cause a corresponding increase in its magnitude.