



370. Three Circles Mutually Orthogonal

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in this polynomial is clearly $\frac{1}{m+1}$, and hence when it is arranged in powers of n (instead of $n+\frac{1}{2}$) the coefficient of n^m is the same as in

$$\frac{1}{m+1}(n+\frac{1}{2})^{m+1}, \text{ that is, } \frac{1}{2}.$$

Again,

$$\begin{aligned} n^m &= \phi(n+\frac{1}{2}) - \phi(n-\frac{1}{2}) \\ &= \phi(n+\frac{1}{2}) + (-1)^m \phi(-n+\frac{1}{2}), \end{aligned}$$

and thus when $\phi(n+\frac{1}{2})$ is arranged in powers of n the powers n^{m-2} , n^{m-4} , ... are all wanting, as well as the absolute term.

If we put $S_m(x)$ for $\phi(x+\frac{1}{2}) \div m!$ we have $S_m(x) - S_m(x-1) = \frac{x^m}{m!}$, and thus $S'_m(x) - S'_m(x-1) = \frac{x^{m-1}}{(m-1)!}$ by differentiating.

$$\text{Hence, } S_{m-1}(x) = S'_m(x) - S'_m(0), \quad S_m(x) = xS'_m(0) + \int_0^x S_{m-1}(x)dx.$$

Thus, $S_2(x)$, $S_3(x)$... can be found by the following rule—when m is odd $S_m(x)$ is the integral of $S_{m-1}(x)$ and vanishes with x : when m is even $S_m(x)$ is the integral of $S_{m-1}(x)$, vanishing with x , plus such a multiple of x that the whole may vanish when $x = -1$.

If we begin with $S_1(x) = \frac{1}{2}x^2 + \frac{1}{2}x$, it is not hard to prove by induction that the polynomials S_m , as formed successively by the rule now stated, have the properties that have been mentioned.

$S_{2m+1}(x)$ contains the factor x^2 , and therefore also $(x+1)^2$,

$S_{2m}(x)$ contains the factors $x(x+\frac{1}{2})$, $x+1$.

If we write as usual

$$S_m(x) = \frac{x^{m+1}}{(m+1)!} + \frac{1}{2} \frac{x^m}{m!} + \frac{B_1 x^{m-1}}{2!(m-1)!} - \frac{B_3 x^{m-3}}{4!(m-3)!} + \dots$$

and

$$\Sigma_m(x) = \frac{1}{2^m} S_m(2x) - S_m(x),$$

we have

$$\begin{aligned} \Sigma_m(x) - \Sigma_m(x-1) &= \frac{1}{2^m m!} \{ (2x)^m + (2x-1)^m \} - \frac{x^m}{m!} \\ &= (x - \frac{1}{2})^m / m! \\ &= \{ \phi(x) - \phi(x-1) \} / m!. \end{aligned}$$

Thus,

$$\phi(x) = m! \{ \Sigma_m(x) - \Sigma_m(\frac{1}{2}) \}$$

$$\text{and } \frac{\phi(x)}{m!} = \frac{x^{m+1}}{(m+1)!} - \left(1 - \frac{1}{2}\right) \frac{B_1 x^{m-1}}{2!(m-1)!} + \left(1 - \frac{1}{2^3}\right) \frac{B_3 x^{m-3}}{4!(m-3)!} - \dots,$$

the coefficients following the law thus indicated except in the absolute term, which is so fixed that $\phi(\frac{1}{2}) = 0$.

Hence the coefficients in $\phi(x)$ as well as in $\phi(x \pm \frac{1}{2})$ can be simply expressed by means of Bernoulli's numbers: also the sum of the m^{th} powers of the first n odd numbers is

$$2^m \{ \phi(n) - \phi(0) \}. \quad \text{A. C. DIXON.}$$

370. [K. 11. a.] *Three circles mutually orthogonal.*

The following theorems have been considered hitherto in relation to a single triangle. It will be found that the proofs are much simplified by considering a system of three circles A , B , C mutually orthogonal.

The figure, from its symmetry, gives three examples of each theorem.

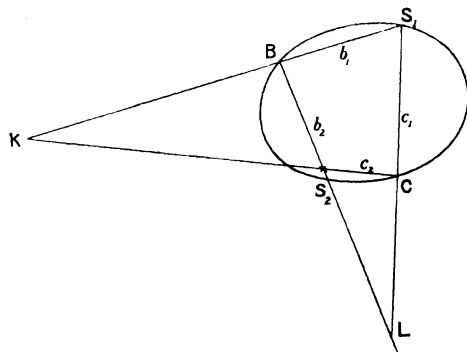
Again, $Y+d=XX'X'=\pi-XX'C=\frac{1}{2}\pi+Y$; $\therefore d=\frac{1}{2}\pi$,
 so that the $\triangle yxx'$, known to be similar to def , has angles $\frac{1}{2}\pi, \frac{1}{4}\pi, \frac{1}{4}\pi$:
 so for the other triangles of $xyx'y'$; another proof that it is a square.

7. Note that A is the orthocentre of BOC ; and the circle A is the polar circle.
 W. GALLATLY.

371. [L¹. 1. d.] *New Proof of Homographic Property of a Conic from Pole and Polar.*

Let S_1, B, S_2, C be four points on a conic.

Then $\left. \begin{matrix} b_2c_1, \\ b_1c_2, \end{matrix} \right\}$ i.e. KL , passes through pole of S_1S_2 , and taking any pt. P on conic, $\left. \begin{matrix} p_1c_2, \\ p_2c_1 \end{matrix} \right\}$ passes through the pole of S_1S_2 , etc.



Call this pole H .

Then

$$\begin{aligned} & H\{p_1b_2, p_1'b_2, p_1''b_2 \dots\} \\ & = H\{p_2b_1, p_2'b_1, p_2''b_1 \dots\}. \end{aligned}$$

\therefore ranges are in perspective,

i.e.

$$\begin{aligned} & S_1\{p_1, p_1', p_1'' \dots\} \\ & = S_2\{p_2, p_2', p_2'' \dots\}. \end{aligned}$$

S. ANDRADE.

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