

ANALYSIS OF COMPLEX SOUND WAVES.

BY C. W. HEWLETT.

THE object of this work was the analysis of the complex sound waves sent out by violins. The work is a continuation of that begun in this laboratory by Dr. P. H. Edwards (PHYS. REV., 32, Jan., 1911), who studied quantitatively the analysis of the sound waves sent out from several instruments including the violin, 'cello, flute, cornet and clarinet.

The same method was used by both Dr. Edwards and myself. A set of resonators each provided with a Rayleigh disc was used to analyze the waves. The resonators were cylindrical and were composed of two pieces of telescoping brass tubing, one piece being closed at one end with a solid disc of brass, and the other closed at one end with a disc of brass having a circular opening at its center. This opening was the mouth of the resonator. This arrangement enabled the resonators to be tuned over quite a wide range. A set of thirty-nine resonators was designed with frequencies ranging from 128 to 2,552 vibrations per second. They were constructed by Mr. Frank Smith, one of the mechanics in this laboratory. A drawing showing the design of the resonators, and a table giving their dimensions is given farther on. The resonators were mounted side by side on the four shelves of a wooden case. This was a framework whose front sashes were covered with very thin celluloid, transparent to both light and sound; the other sashes being covered loosely with light tissue paper.

The room in which the experiments were carried out was approximately four meters each way. Reflections from the walls, floor and ceiling distort the sound waves to such an extent that absolute measurements of the intensity of a source would be almost impossible if some means were not devised to eliminate these disturbances. With this end in view the walls and ceiling were hung with a preparation of flax called Linofelt. This material is a fluffy, fibrous substance sewed between two strips of heavy paper. This was first suspended by wires so as to completely hide the ceiling and walls and then the strips were sewed together. The surface of the paper on the walls exposed to the room was then stripped off, leaving the soft hairy surface exposed. On account of the lack of mechanical strength of the material the paper was not stripped from the ceiling. The floor was also covered with Linofelt and then

with a layer of burlap. The reflections were greatly decreased, although not entirely destroyed.

The resonator system for measuring the intensity of sound was as follows: A thin mica disc whose diameter was about half that of the opening in the resonator was suspended by means of a fine quartz fiber close in the opening. A small glass rod attached to the lower side of the disc terminated in a small glass dumbbell which dipped into a cup of kerosene oil. The object of the dumbbell and oil was to damp the motions of the disc. In the center of the mica disc was fastened a small chip of platinized concave mirror. The disc was turned so that its plane made an angle of 45° approximately with the mouth of the resonator.

When the resonator responds to a tone of its frequency the disc experiences a couple whose magnitude depends upon the angle between the plane of the disc and that of the mouth. The disc will therefore turn till the couple due to the fiber is equal and opposite to that caused by the vibrating air. The formula given by Lord Rayleigh for this couple (Theory of Sound, Vol. II.) is

$$M = \frac{4}{3}\rho a^3 W^2 \sin 2\theta,$$

where a is the radius of the disc, W is the mean velocity of the vibrating particles of air, ρ is the density of the air, and θ is the inclination of the disc to the plane of the mouth. Now the couple may be calculated from a knowledge of the torsional moment of the fiber and the deflection, so that the mean kinetic energy per cubic centimeter of the air in the mouth of the resonator, $\frac{1}{2}\rho W^2$, may be calculated. Helmholtz has shown that the intensity of the waves which would exist at a given point were the resonator not there is to be obtained from that in the mouth of the resonator by multiplying by the factor $2R^3/\pi^2 S$, where R is the radius of the mouth and S is the volume of the resonator.

If δ is the deflection in degrees of the disc and if K is the torsional moment of its suspending fiber, then the moment acting on the disc due to the fiber is $2\pi\delta K/360$. When equilibrium has been established, the moment due to the vibrating air and that due to the fiber must be equal and opposite. Equating the two we have

$$\frac{2\pi\delta}{360} K = \frac{4}{3}\rho a^3 W^2 \sin 2\theta,$$

$$\frac{2\delta}{\sin 2\theta} = \frac{480}{\pi} \cdot \frac{a^3}{K} \cdot \rho W^2,$$

$$\text{Kinetic energy per cm}^3 = \frac{1}{2}\rho W^2 = \frac{\pi}{960} \cdot \frac{K}{a^3} \cdot \frac{2\delta}{\sin 2\theta}.$$

The kinetic energy per cubic centimeter in the air is the quantity we wish to determine when analyzing a complex wave. From the above formula it is seen that the kinetic energy per cubic centimeter in the air in the mouth of the resonator may be determined from a knowledge of δ , θ , and the ratio K/a^3 . By using Helmholtz's formula for the multiplying power of a resonator the kinetic energy per cubic centimeter which would exist at that point if the resonator were not there may be calculated. It is thus seen that it is not necessary to know the torsional moment of the fiber and the radius of the disc separately. The method by which K/a^3 was determined will be discussed shortly.

A large translucent paper scale with vertical rulings 1 cm. apart was mounted in front of the case of resonators. The faces of the resonators were adjusted parallel to this screen. Four Nernst glowers, one for each shelf of the resonators, were provided with hoods each containing a small aperture. These were mounted at the sides of the case, the apertures facing small celluloid windows. The rays of light entering these windows were therefore perpendicular to the axis of the resonator.

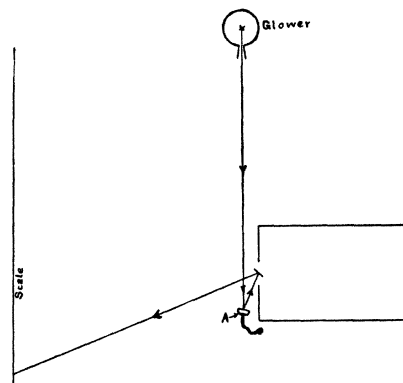


Fig. 1.

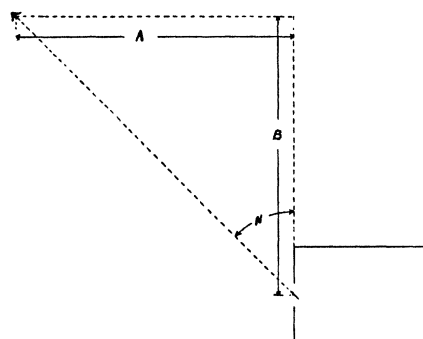


Fig. 2.

The disc was turned so that the chip of the mirror on its face was hidden from the glower and an auxiliary mirror *A* mounted on a lead wire served to illuminate its surface. By this means the image of the glower on the scale could be placed where desired without interfering with the disc or glower. The surface of the mica disc was blackened with soot to avoid reflections from its surface. The deflections on the scale were reduced to degrees by a graphical method. The angle between the disc and the face of the mouth of the resonator was found by placing the eye in the plane of the disc and then measuring the tangent of the angle. The zero of the image was noted at the same time.

$$\tan N = \frac{A}{B}.$$

This angle, of course, was the zero angle, but knowing the deflection, half its value could be subtracted from the zero angle thus giving the θ in Rayleigh's formula.

Edwards showed that, if the disc was very close in the opening of the resonator, the deflection produced by a sound of a given intensity was independent of the distance of the disc from the mouth. In the present work this experiment was repeated and found to agree with the work of Edwards when the size of the disc was small compared to that of the opening; but, if the disc was larger than half the opening, the deflection increased very rapidly with approach to the opening when this distance was small. It was found that with a disc whose diameter was half that of the opening the deflection was practically constant within a distance of about .1 of the diameter of the opening. All the suspensions hung within this distance, and those in front of the larger resonators were much closer.

The torsional moments K of three of the fibers were determined by measuring the periods of vibration in vacuum, first when loaded with the usual system of disc and glass damper, second with a small auxiliary load whose moment of inertia was known. The diameter of the disc was also measured and K/a^3 calculated for each of these three suspensions. K/a^3 for each of the suspensions used in the experiments was then determined by comparison with these standards. To make this comparison a suspension and one of the standard suspensions whose disc was nearest the size of the one which it was wished to compare were suspended successively in front of the same resonator at the same distance from its face. The resonator was excited by an organ pipe or another resonator blown by air kept at a constant pressure. We have from the equation previously given,

$$\frac{K}{a^3} = C \frac{\sin 2\theta}{2\delta}$$

where

$$C = \frac{480}{\pi} \rho W^2.$$

Using the standard suspension, C may be determined. The suspension for which K/a^3 was required was then substituted for the standard suspension and the deflection in degrees of the disc determined. θ the angle between the disc and the mouth of the resonator was also determined. Substituting these quantities in the previous equation K/a^3 was

calculated. K/a^3 was measured for all the suspensions at least three times, and the suspensions were compared among each other in order to get some idea as to the accuracy attainable. The results which are given further on are the averages of all the determinations and are probably correct to within 10 per cent. The comparison resonator was mounted at the center of curvature of a circular paper scale free to rotate about a perpendicular axis through its center of curvature. This scale was divided into degrees, so that the deflections were read directly in degrees. In this case the zero angles were determined by measuring the angle between the image when the disc lay flat against a piece of glass clapped over the opening of the resonator, and when hanging free.

A word should be said in regard to the way in which the resonators were tuned. A violin was first tuned by means of an "A" tuning fork, and then the resonators were tuned to the partials of the violin. The resonator was first tuned approximately by ear. It was then placed in its position back of its disc and its pitch found by playing the violin and sliding the finger along the string till a maximum deflection was obtained. The resonator was then sharpened or flattened as required by sliding the telescoping tubes, and the process repeated until the resonator was so exactly in tune that no difference could be observed between its pitch and the corresponding partial. The pitches of some of the partials of the different strings so nearly coincided that it was found possible in some cases to use the same resonator for a partial of two or three strings.

This work has been confined almost exclusively to the analysis of the quality of the tone from the open strings of violins, although when the apparatus was designed it was intended to include some other instruments. In making the tests the performer took his position in front of the case of resonators so as to hold the violin about the same distance from all the resonators. This distance was about 240 cm. The observer's position was in front of the scale but slightly to one side so as not to obstruct the waves on their way to the resonators. In taking a record, the performer drew the bow to and fro over the open string as evenly as possible several times in succession. On account of the varying intensity from one end of the stroke to the other, and the changes in quality as the position of the bow changed the positions of the spots of light on the scale were quite variable during this interval. However after some practice it was found possible to select a short interval during which the quality of the tone seemed the best and to locate the image with some accuracy. Thus during an interval of from five to ten drawings of the bow to and fro it was usually possible to form a pretty fair estimate of the

deflection for any one partial. This same process was repeated for each partial. The image giving the largest deflection was observed during the whole procedure, and it was ascertained that it kept on the average the same position throughout the test. Should its average position change owing to change of pitch or intensity the performer was immediately notified. It might be mentioned here, that invariably it was found comparatively easy to produce a tone of fairly constant intensity throughout the stroke on the best violins, while with violins of poor quality this was very difficult to do.

Constants for Fibers Used.

Numbers correspond to number of resonator with which the fibers were used.

No.	$K/a^3 \times 10^2$.	No.	$K/a^3 \times 10^2$.	No.	$K/a^3 \times 10^2$.	No.	$K/a^3 \times 10^2$.
1	3.4	11	21.1	21	1.93	31	4.34
2	12.3	12	19.7	22	135.0	32	8.0
3	2.0	13	69.0	23	2.23	33	3.88
4	24.8	14	18.9	24	62.5	34	21.30
5	6.9	15	36.5	25	80.4	35	2.97
6	6.5	16	3.91	26	4.59	36	4.52
7	4.8	17	125.0	27	7.61	37	1.08
8	15.0	18	21.4	28	61.30	38	8.74
9	6.4	19	46.3	29	1.61	39	3.00
10	274.0	20	36.4	30	.60		

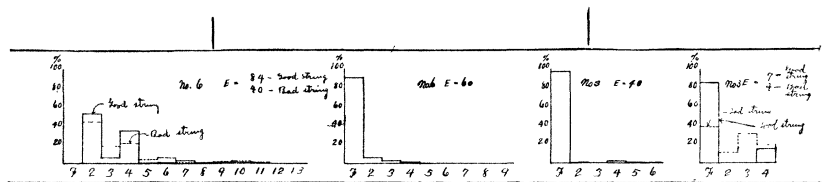
K represents the torsional moment of the fiber and a the radius of the disc to which it is attached.

Number of Resonator System.	G-string.	Partials.		
		D-string.	A-string.	E-string.
5	1			
8		1		
11	2			
12			1	
15	3	2		
16				1
19	4			
21		3	2	
23	5			
26	6	4		
28			3	2
29	7			
30		5		
31	8			
33	9	6	4	
35	10			3
36		7		
37	11		5	
38	12	8		
39	13	9	6	4

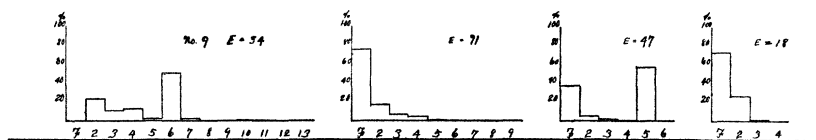
Records were made of the tone quality of twenty-nine violins and one viola. The records of a number of these instruments were made but once, while anywhere from two to six records of some of the others were made. A few of these records illustrating the points brought out in this work are reproduced at the end of this paper. It will be observed that each record is distinguished by a number. These numbers indicate approximately the order in which the records were made. The abscissæ represent the fundamental and successive partials of the string being played. The ordinate at any partial gives the energy in that partial expressed as a percentage of the total energy given out by the string. The energy per cubic centimeter, E , given out by each string is also given. To reduce to ergs per cubic centimeter multiply by 10^{-7} . The distance of the resonators from the position occupied by the violin when the record was being made was about 240 centimeters, so that the output of the violin may be calculated if one neglects the distortion of the waves due to reflections at the walls, and if it be assumed that the violin sends

out waves equally in all directions. The four curves in each set correspond to the open G, D, A, and E strings respectively, reading from left to right, unless otherwise specified.

There are differences in the quality of tone produced by different performers on the same instrument, sometimes even by the same performer at different times. However, fairly good agreement was usually obtained between the records made by the same performer on the same instrument under the same conditions. Some idea as to the degree in which the experiments could be repeated may be obtained by comparing the following two sets of records, (16, 18, 20), (24, 31, 37). The last group were all made by different performers playing with quite different degrees of loudness. The following groups of records are representative of some of the best instruments studied: (56, 57), 19, (16, 18, 20). Record 17 and the group (24, 31, 37) were made from two violins of exceedingly poor tone quality. The violins studied were roughly classified under three heads, the criterion being the general opinion of musical people in regard to the quality of the instrument. About one sixth of the instruments belonged to the class of the "Best Violins," about one sixth to the class of the "Poor Violins," and the remaining two thirds to the class of the "Ordinary Violins."



RECORDS 3 AND 6. These are the records of two violins bought in the white and finished and graduated by Mr. Della Torre. They have been treated with Mr. Della Torre's varnish and oil preparation. Both violins were made as much alike as possible. Record 3 is of the violin designated as "A," while record 6 is of the violin designated "W." Mr. Della Torre played for both records. It was found that both violins were beginning to "open up," *i. e.*, the glue holding the ribs, back and belly together had turned loose in one place. This may account for the fact that the tone was no better, as Dr. Edwards recorded the violins as having very good tone quality.

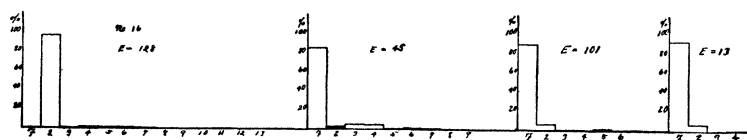


RECORD 9. This is a trade violin bought from Eisenbrandt, of Baltimore, by Mr. Della Torre for \$5.00. It is designated by Mr. Della Torre as No. 2. This record, played by Mr. Della Torre, shows the violin as having a very poor weak tone. The sound post was directly under the bridge, and the base bar quite stiff.

DISCUSSION OF RECORDS.

G-string.—The intensity of the fundamental of the G-string on all the violins is negligible in comparison to the second partial (first overtone). On all the best instruments the intensity of the second partial lies between 90 and 100 per cent. of the total intensity of the string. The third partial has an intensity of less than 10 per cent. of the total intensity of the string. After the third, the intensity of the partials drop off very rapidly, the fourth having less than 2 per cent. of the total intensity, there being no measurable energy beyond the sixth partial.

In the case of the poor violins, the second partial rarely has more than 60 per cent. of the total intensity and very often less than 20 to 30 per cent. The third partial is usually the strongest and attains at times more than 70 per cent. of the total intensity. In one case it was observed that the sixth partial contained 50 per cent. of the total intensity (see record 9). After the third partial the intensity is distributed among the partials as far out as the thirteenth, dying out very slowly in some cases. Nothing more general can be said in regard to the ordinary violins than that the second partial usually contains from 60 to 90 per cent. of the total intensity, the third and fourth partials being often quite prominent. Sometimes the third partial is the most prominent, but in this case it is probable that the merits of one of its other strings saves it.

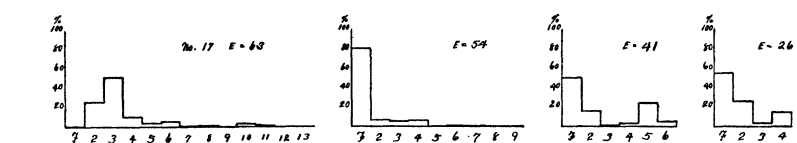


RECORD 16. This violin was made by Luther Heigis in 1911 and is numbered 106. The instrument has a fine tone quality, especially on the G string. Mr. Della Torre played for this record just before applying to it his oil preparation. The intensity of the playing was medium.

D-string.—On all the best violins the intensity of the fundamental on the D-string lies between 80 and 100 per cent. of the total intensity. The remaining 20 per cent. is distributed irregularly between the second, third, and fourth partials, the fifth and sixth receiving some in some cases, but there is nothing beyond the sixth or seventh. On the poor violins the intensity of the fundamental may rise as high as 70 per cent. but usually is as low as 60 per cent. or lower. In one case it was found to be as low as 20 per cent., the second partial being 74 per cent. A peculiarity of these poor violins is the way in which the intensities of the successive partials fall off, "step" fashion (see records 24, 31, 37, 9). On the ordinary violins the fundamental of the D-string may have

intensities as low as 50 per cent. and as high as the best violins. Usually the intensity of the fundamental lies above 70 per cent. and very often the record appears very much the same as do those on the best violins.

A-string.—On the best instruments the intensity of the fundamental lies between 91 and 100 per cent. of total intensity. The intensity of the second partial is less than 8 per cent. while the third and fourth partials are absent. The fifth partial appears quite frequently in all classes of violins. In all the best violins it has an intensity of less than 6 per cent., and there is no energy in the sixth or higher partials. The intensity of the fundamental of the A-string on the poor violins may be anywhere from 30 per cent. to 90 per cent. or more of the total intensity.

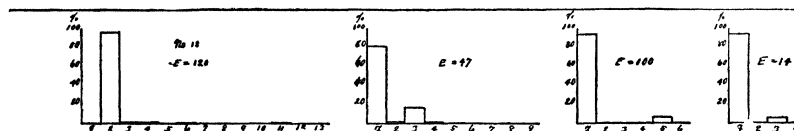


RECORD 17. This violin is owned by Mr. Grubmeyer, of Baltimore. It is a very poor "trade" violin, and has a tone which is very weak in intensity and poor in quality. Mr. Della Torre played for the record.

The intensity of the fundamental usually lies between 50 per cent. and 70 per cent. of the total intensity. The balance of the energy is irregularly distributed among the second, third, fourth, fifth and sixth overtones, the fifth overtone sometimes getting a large share (see record 9). The intensity of the fundamental on the A-string of the ordinary violins may lie between 70 per cent. and 95 per cent. of the total intensity, but most often lies between 80 per cent. and 95 per cent. Some of them resemble the poor instruments in that the energy is scattered among the partials, while most of them approach the records given by the best instruments.

E-string.—In the best violins the fundamental has an intensity of from 70 per cent. to 95 per cent. the total intensity. The balance of the energy appears quite irregularly distributed among the second, third, and fourth partials. It is hardly possible to make a distinction between the ordinary violins as a whole and the poor violins. In both classes the intensity of the fundamental is frequently between 70 per cent. and 95 per cent. Often however it falls below this, and occasionally the second, third, or fourth partial appears the most prominent.

All of the best violins have certain strings which are considered better than others. I have selected these best strings and plotted them on a single sheet which I have labelled "Ideal violin tone." The curves for all the best strings on the best violins fall within the shaded portions of these curves.



RECORD 18. Same violin as in Record 16. Mr. Della Torre played for this record two days later than for record 16, having applied the oil preparation in the meantime. The "A" string seemed to give a brighter tone than in the previous test. The intensity of the playing was moderate.

The effects of certain changes in the adjustment of the violin were also studied. It is seen that bad strings throw the energy out into the higher partials (see records 3 and 6).

It was thought that strings stopped by holding them down to the finger board with the fingers might give a tone in which the overtones were less prominent than with the open string. This was tried by playing the notes corresponding to the open D, A, and E strings on the G, D and A strings respectively. A comparison of records 20 and 57 will show, however, that while this may be true in some cases, it is not necessarily always true.

The effect of hard and soft bowing was tried at various times. Record 55 shows the results of forcing a very fine violin. Contrary to expectation the overtones are fewer and less intense than with the same instrument played lightly (see record 43). Record 20 shows the result of soft bowing on a good violin. Records 16 and 18 are for the same instrument when bowed with medium intensity. Again it does not appear that on the whole the number and relative intensities of the overtones are decreased. Record 31 represents a violin bowed with medium intensity, and 37 represents the same violin bowed softer.

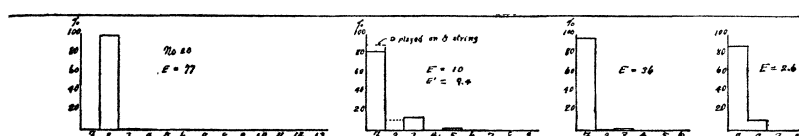


RECORD 19. This violin is owned by Professor Aperda, of Baltimore, the age and make of the instrument are unknown. The instrument has a very fine tone quality, and an exceptionally powerful "A" string. Mr. Della Torre played for the record.

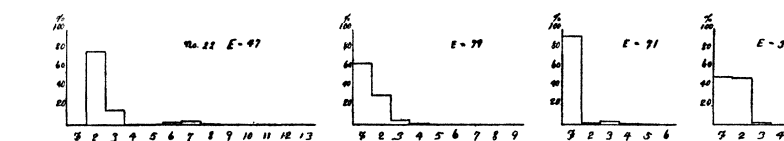
A violin was found in which the sound post was directly under the bridge. A record was made (9), and then the sound post was moved to its proper position back of the bridge. Another record (22) was then made. The total intensity of the sound was increased about 36 per cent. by this procedure, and a wonderful change produced in the relative intensities of the overtones, shifting the energy into the overtones of lower

frequency. The base bar of this same violin was found to be very thick. Two strips were accordingly removed, and a record made after each removal. Finally, the base bar was about half as thick as previously, the intensity about 50 per cent. greater than before the base bar was split and the overtones fewer and the energy more concentrated into the lower overtones (see record 54).

The reflections from the walls is one of the main difficulties in the work, and I would suggest that any one wishing to continue it devise some scheme to do away with walls or put them at a sufficiently great distance.



RECORD 20. Same violin as in records 16 and 18. Mr. Della Torre played for this record on the same date as for record 18. The playing was soft. The D played on the G string seems to be a purer tone than when played on the open strings.



RECORD 22. Same violin as in record 9. The sound post had just been moved to its proper position. It will be seen that the total intensity of the sound was 36 per cent. higher than the previous one. Mr. Della Torre played for the record.

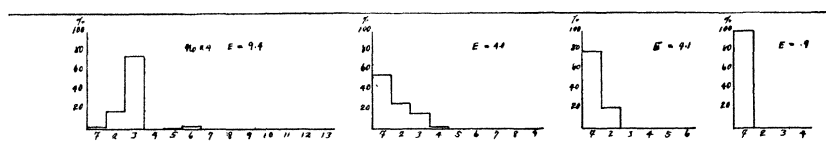
It has been suggested to do the work in a very large tent. I think that the work should be carried on in some locality where the evenings are calm. A collapsible covering should be built around the apparatus in a large field. When one wishes to work the covering could be removed.

SUMMARY.

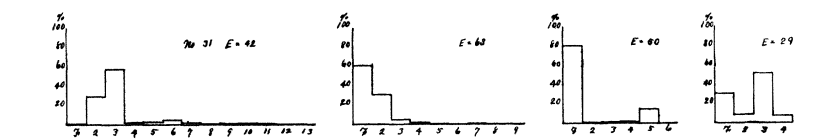
1. Trustworthy records of the tone quality of several violins have been obtained.
2. A set of curves has been drawn defining tentatively an ideal violin tone, the data being taken from the best violins obtainable.
3. Soft bowing does not necessarily produce a purer tone than medium bowing, and hard bowing does not necessarily produce a tone less pure than obtained with medium bowing.
4. The method furnishes a powerful means of studying the effects produced by different kinds of strings and bridges, and by various changes produced in the adjustment of the sound post, base bar and bridge.

In conclusion I wish to thank Professor Ames, Dr. Anderson, Dr. Pfund

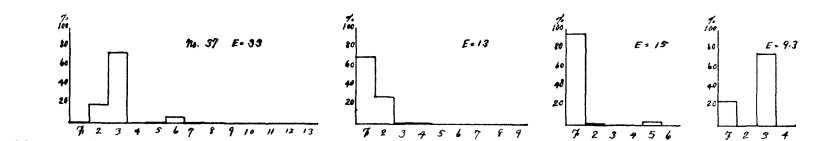
and others in the laboratory for their assistance and advice, and for the interest they have shown in my work. I wish also to thank numerous friends in the city and members of the Peabody Conservatory of Music for the loan of violins and for personal assistance in the work. In conducting this investigation I have had the great benefit of the advice and assistance of Mr. Frank Della Torre, of Baltimore. He placed at my disposal his entire collection of violins, which is a very fine one; he took apart and altered one of these violins for me, a kind of work in which he is exceptionally skillful; and he played various violins for me while I made the observations.



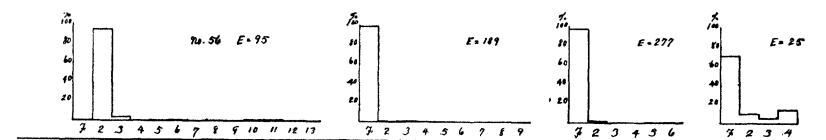
RECORD 24. This violin is owned by Mr. Wear, of Baltimore. It is a comparatively new and cheap instrument, with a weak, scratchy tone quality. The sound post was in front of the bridge. The writer played for this record, and having to make observations at the same time, he was located unsymmetrically with respect to the resonators.



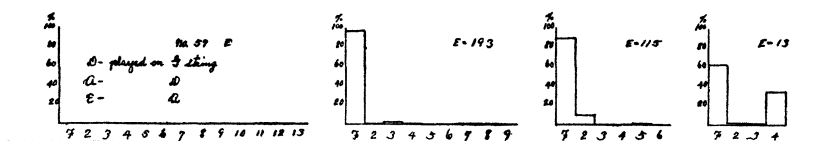
RECORD 31. Same violin as in record 24. Mr. Della Torre played for this record.



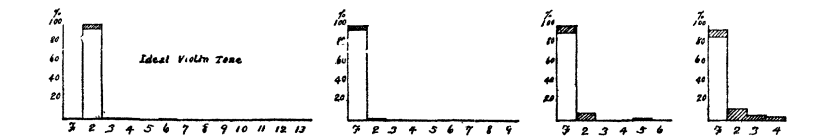
RECORD 37. Mr. Wear played for this record. He played more softly than did Mr. Della Torre in record as above.



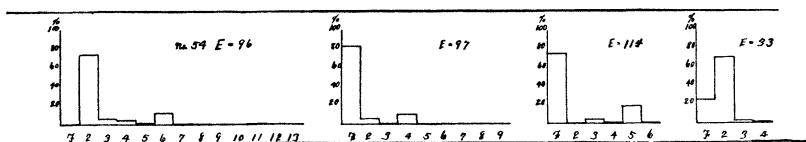
RECORD 56. Professor Van Hulsteyn's violin. This violin is a Regiera and has a very fine tone quality. The "E" string is the poorest string on the instrument. Professor Van Hulsteyn played for the record. The tone was pleasing, and the playing of medium intensity.



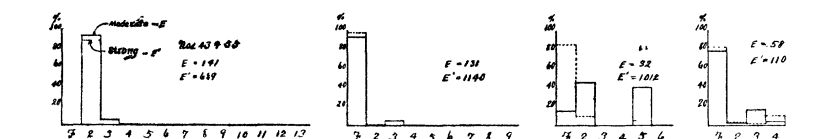
RECORD 57. Professor Van Hulsteyn's violin. The record consists of D played on the G string, A played on the D string and E played on the A string. Professor Van Hulsteyn played for the record, and the playing was of medium intensity.



Ideal violin tone. This set of curves represents tentatively the characteristics of the ideal violin tone. The characteristics of the best strings on the best violins tested fall within the shaded portions of these curves.



RECORD 54. Same violin as in record 9. Two strips had been removed from the base bar, so that the bar was about one half of its original width. The total intensity was about 100 per cent. greater than in record 9. Mr. Della Torre played for the record.



RECORDS 43 AND 55. Grancino violin (1727). This instrument has a powerful tone and a fine tone quality. Record 43 was made with Mr. Della Torre playing. The instrument was played lightly because louder playing sent the spots of light off the scale. Mr. Della Torre also played for record 55. The intensity of the sound was very great. An auxiliary scale was necessary to read the deflections. A bamboo was used. The tone quality of the instrument was very similar to that of Professor Van Hulsteyn's violin.

THE JOHNS HOPKINS UNIVERSITY,
June 4, 1912.