

ART. XIX.—*Periodic action of Water* ; by LOUIS NICKERSON.

IN reading, some weeks ago, the article by Prof. Loomis, on the vibrations of water flowing over a dam, I was somewhat surprised at the idea of deriving the peculiar motion from a foreign source, as a column of air; surprised, because, however much the air might effect, by reaction, after the action had commenced, the perturbations of a liquid, in whatever state of motion it may exist, have always been so connected with periodic action as to have given use to the name of its most common attribute "the wave," as the characteristic title of nearly all periodic action. Without an attempt to discuss the question with the distinguished gentleman engaged, I shall endeavor to point out the manner in which the vibrations may be considered simply as the result of a wave peculiarly circumstanced.

I was sitting one day upon the bank of a large river in the West. Before me was a strong ripple, supposed by the people around to have been caused by the lodgment of snags upon the bottom. The sound from it was much louder than the roar of the stream—then in a state of freshet, and itself uproarious. But there was a cadence in it, an easily distinguished division into regular periods, which induced an inclination to pause and watch. My position was just upon the middle of an arc, which a late "caving in" of the bank had indented, each point of the arc running past the average bank of the river toward the center. The one down the river being in the greatest projection gave to the arc the appearance of a crescent, quartering into the bank and with its back down-stream-ward. Now a portion of the main current of the river, striking this lower horn, rounded inward as though to make a whirl within the crescent, but, thrown off, shot over toward the center and up stream. Above, another partial current, cut off from the main stream by a shoal or otherwise, came down along shore, and passing the upper horn bore directly down upon the face of the first eddy. Now they attempt to bear each other back, as though striving for the mastery of the crescent. For an instant there is an equilibrium. Both currents at the place of meeting rise rampantly into waves; both seem to receive reinforcements. They might be supposed to be equally matched, but the upper current receives the most water. For another instant they stand poised and opposed, and then the upper rushes, broken, but conquering, down over the surface of the other. Carrying off, however, not only its superabundance, but dragging along a little more water, so that the lower current quickly regains its ascendancy, driving the upper back to be again checked and again overpowered, as before.

Afterward, I watched this place for hours at a time, unfortunately without timing, but yet with so distinct and definite a feeling of the regularity of the periods, that it was easy to estimate in the mind the exact period when, equilibrium having been attained, the lower water would start suddenly back and the accumulated waves from above rush over it, always dragging a sufficient extra quantity of water from the upper current to give for a time the ascendancy to the lower. The space thus fought over was three or four feet.

But mill-dams are certainly not in just this situation. The permanent weir opposes no such active resistance as the elastic and moving weir just described. Its characteristic is passive resistance. Must we therefore look to it for continuous action? I take it to be thus:

1st. That a certain quantity of water arrives at the pool, and is all passed over the weir in the end, but in periods.

2d. That the quantity passed between these points, is, at the lesser points of velocity, of greater transverse section. This is obvious.

3d. That the decrement of velocity, and corresponding increment of section, is greater at a point nearly under the point of greatest depression of the curve of amplitude.

4th. That outside of this stream the pool is made up of water in a state of slow motion, at rest, or even in some cases of reaction, whirling or flowing backward.

Now we may examine the curve of hydraulic amplitude and its changes. For this we take the formulæ of permanent motion; which, though not exact, is sufficiently characteristic, as derived from Weisbach,

$$a - a'_1 = \left( \frac{\sin \alpha - z \frac{P}{ab} \cdot \frac{v^2}{2g}}{1 - \frac{2}{a} \cdot \frac{v^2}{2g}} \right) l,$$

when

$$\left. \begin{array}{l} \sin \alpha = \text{slope of original stream} \\ P = \text{whetted perimeter,} \\ al = \text{transverse section,} \\ v = \text{velocity,} \end{array} \right\} \begin{array}{l} l = \text{distance between } a \text{ and } a'_1. \\ a = \text{depth of dam or known point.} \\ a'_1 = \text{required depth.} \\ z = \text{coefficient of resistance,} \\ g = 32.2, \text{ or } a \text{ gravitation.} \end{array}$$

The form of this curve is represented in the works of almost all hydraulic authors, and its equation shows it to be asymptotic to the original surface. It is easily seen above that when  $\sin \alpha$  of the original surface becomes equal to  $z \frac{P}{ab} \cdot \frac{v^2}{2g}$  and therefore equal to  $\sin \alpha'$  of a transverse section of the pool;  $a - a'_1 = 0$ ,

or the pool is simply a continuation of the stream, and if  $\frac{v^2}{2g}$  becomes equal to  $\frac{a}{2}$ , or the height due to velocity becomes equal to one-half the depth, both of the original stream, then  $a - a_1 = \infty$ , a case which we shall examine more hereafter. We know that when the pool is first filled and the water is just on the point of flowing over the dam, the surface is horizontal, and we call it the hydrostatic amplitude. It is moreover true, that when the flowage commences and the hydraulic amplitude obtains, there is a stream of water passing through the pool, various in its velocities, and with a sheath of water, differently circumstanced, around it, which it in some way affects. We have also the admitted law, (Weisbach, vol. 1, art. 307; D'Aubinson, art. 54), that when any stream of liquid is in motion in any direction, its pressure in all other directions is equal to its hydrostatic pressure, less the pressure in the direction of its motion, and generally, that when a liquid in motion is made to pass through a liquid of less velocity, a part of the latter is dragged along by its greater lateral pressure, and passes off with the stream. In 1797 the engineer Venturi applied this principle successfully to the drainage of public lands. (Ewbank's Hydraulics, p. 478.)

In a pool fed at one point and yielding up the water at another, such as a mill-pond, this state of affairs practically obtains: that there is first a stream of water running some distance into the remou, and another passing out; in long dams only near the ends, perhaps, but in those of ordinary size throughout the whole length. This stream comes in contact with and passes through walls of partly quiescent water, not only that of the dead angles, but also of superimposition. For, says the engineer D'Aubinson, "moreover, the water of flowage seem only to be superimposed above the current, and not to participate wholly in its motion. The engineers who took the levels upon the Weser have observed, at a distance of 3884 feet from the dam, that the velocity of the surface was nearly insensible, whilst that at the bottom was quite strong."

We have now the fact that a stream is running through water much more nearly at rest, and that owing to the difference of pressure, some of the slower water must be dragged along in the course of the faster, in quantity and force varying as the difference of lateral pressure. We must remember that as in the end the weir can only pass over the same amount of water as it has received from the upper end, i. e., the water of the current, there must, then, be a periodic lull until the deficiency caused by this dragging action has been replaced.

It might readily appear that, as the velocity may become less from the interior of the stream to the outside, this might occur

and yet a continuous action be kept up. But a little thought will show that as normally the stream passes out exactly the quantity received through its own continuation outside the pool, and as the water which I have shown to be dragged out is an extra quantity, there must be a pause after its exit until the surface, which has been lowered a very small increment by its departure, attains its own proper regimen.

You will notice that this reasoning requires that there should be a rise and fall of the surface of the curve of amplitude. I have seen no such rise myself, nor been able to obtain a confirmation of its occurrence from others. But these causes of its existence appear to me too plain to be disputed. It must by necessity be extremely small,—too small perhaps for observation.

For the formula  $h = \frac{v^2}{2g}$  when applied to a 4 foot velocity gives us  $h = 0.25$ , and a velocity of 4.3 yields for  $h$  only  $h = 0.263$ . By which we see that a rise which would only add a quantity  $= 0.013$  to the depth would increase the velocity full  $\frac{3}{8}$ . Then for the ordinate of the parabola of theoretic fall, we should have for the *vis viva* of the horizontal component a quantity varying as the square of velocity, or, for velocities varying from 4 ft. to 4.3 ft., a mechanical effect varying as 16 to 18.5 against the resistance of the atmosphere. That the atmosphere both outside of the falling water and the column inclosed between it and the dam, would assist by its elastic reaction in the vibrated distance, there can be no manner of doubt. But a cursory thought strikes me as I write, that were the action of the air truly isochronous with these vibrations, then the vibrations would have a constant tendency to increase. Partly it assists, partly deadens.

Again let us recur to our formula. Although made by Weisbach only to measure the curve of amplitude, and for the ordinary case of remou, it still contains the very elements which we need, and its changes may at least mark corresponding changes in the law which we discuss. In

$$a_1 - a_0 = \left( \frac{\sin \alpha - z \frac{P}{ab} \cdot \frac{v^2}{2g}}{1 - \frac{2}{a} \cdot \frac{v^2}{2g}} \right) l$$

we see that  $\sin \alpha$  is the measure of the slope of the original current,  $z \frac{P}{ab} \cdot \frac{v^2}{2g}$  of the resistances of the whetted perimeter of the pool, whilst the denominator marks the changes which occur in the condition of the stream. To use it for our present purpose we must find the value of  $a$ , for some finite point on the axis of the stream, and then placing  $l =$  to an infinitesimal distance

from that point, the difference  $a_1 - a_0$  should show the fluctuation of height due to a periodic change in the discharge. Now when  $\sin \alpha = z \frac{P}{ab} \cdot \frac{v^2}{2g}$ , it is also by the law of the formula  $= \sin \alpha'$  of that transverse section of which the second member shows the resistance; therefore

$$\sin \alpha = \sin \alpha',$$

therefore the velocity of the pool = the velocity of the stream, and the surface line of the pool is a parallel line with the bed; for  $a_1 - a_0 = 0$ , therefore it is circumstanced as in the original stream. There is no backwater, no difference of pressure and no vibration.

When  $\sin \alpha = 0$

the surface becomes level, for there is no velocity, no flowage, therefore no resistance,  $z \frac{P}{ab} \cdot \frac{v^2}{2g} = 0$ ; again  $a_1 - a_0 = 0$ ; and there is no action, periodic or otherwise.

So we see that there are two points at which the vibrations cease: namely, when the water is sufficiently high to flow over the dam without much remou, as with a stream undammed, and with its surface a line nearly corresponding with the surface of the original stream; and again when the water is so low as to make the difference between the hydrostatic and hydraulic pressures very small. Of course these limits are much circumscribed by the inertia of a large body of water which has constantly a tendency to absorb and soften these vibrations. The most violent palpitation should then occur when

$$\sin \alpha = \left( z \frac{P}{ab} \cdot \frac{v^2}{2g} \right) m,$$

$m$  being a new quantity to be found by a knowledge of the stream.

Again, if we put  $\frac{2 \cdot v^2}{a \cdot 2g} = 1$ , or  $\frac{v^2}{2g} = \frac{a}{2}$ , or when the height due the velocity of the original stream becomes equal to one-half the depth of the same, we have

$$a_1 - a_0 = \infty.$$

Certainly nature admits no such differences as this. Yet, the fact beautifully follows from this, that when the height due the velocity of the original stream is equal to one half the depth of the same, the back water no longer retains the concave form, but, tending to rise infinitely, is checked by the action of gravity, falls backward upon the original stream, and tends to form a convex wave, with a nearly horizontal surface at a height above the bed of about one and one-half times the hydrostatic height. After this height becomes greater than half the depth, the wave

is actually formed, and the water of amplitude flows to and rebounds from the foot of the stream. Bidone discovered this law, and Belanger has applied to it a formula.

The general formula which relates to this action we may gather from what proceeds. If  $h$  be the height of the remou, just before greatest action,  $h-h_2$  = height at the beginning of the lull that succeeds. The velocities are then

$$v = v_1 - v_{11}$$

And the times  $= t = \sqrt{\frac{s_1 - s_2}{g}}$ ,  $s_1$  and  $s_2$ , being spaces due to the

two velocities combined with the time,  $t$ , of the vibration. Or the time of one vibration is equal to the time in which water falls through the height upon the dam, minus a space due to the velocity of different heights of remou, combined with the time observed. In other words, it is the time required for the quick water to draw a certain portion of nearly quiescent water into its own mean velocity—the quantity of quiescent water so drawn, and consequently the time, depending upon the regime of the remou. To this must be added the time of recuperation.

The action when  $\alpha - \alpha_1 = \infty$  would be the formation of a remou, similar to the one described as seen by the writer of this in the first part of this paper, just at the point where the stream runs into the pool, which would cause one set of vibrations there, and perhaps another set similar to those on ordinary dams at the weir.

For a practical solution of this question, to prevent the vibrations, I am only prepared to recapitulate the foregoing examination into the nature of the action.

1st, We may consider that the vibrations become practically small when the inertia of the pool becomes sufficient to absorb them; therefore they are small when the quantity of water running over the dam is small in relation to the pool, and  $\sin \alpha = 0$  or nearly.

2d, The same would be true if there were no backwater, or the stream retained its mean velocity unretarded, obtaining when the  $\sin \alpha = \sin \alpha'$ , as before, or when the surface becomes parallel with the bed; and again, should the pool be so filled as to make the bed become parallel with the surface.<sup>1</sup>

<sup>1</sup> After the horizontal line (or line of hydrostatic amplitude) which bounds the surface of a pool just on the point of running over its dam is found, and the water begins to flow over, the longitudinal outline of the surface changes from its straight and horizontal form, and becomes a curve, which joins the water of flowage at some distance from the dam, and proceeding backward, after the law of the equation given in this paper, becomes asymptotic to the surface of the original stream. It is called the hydraulic amplitude. This curve marks by its changes every alteration or irregularity in the flowage, either of quantity or velocity or resistance, and having been carefully subjected to mathematical laws, answered as a touchstone to the deductions of this paper. And the law developed, not being dependent upon the form or nature of the weir, is a law for the periodic action of all fluids in a state of motion past an obstacle.