HISTORICAL NOTES IN THE MATHEMATICAL TEXTBOOKS.

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On comparing the mathematical textbooks which appeared during the last two or three decades with those published earlier one cannot fail to notice a rapid increase in the number of historical notes. The main object of these notes does not usually seem to be to convey important historical information but to stimulate the interest of the student in the subjects treated. In fact, some authors seem to be more interested in providing this stimulus than in the accuracy of the historical information conveyed by such notes.

As an illustration of somewhat trivial historical notes which appear in many of our elementary geometries we may refer to the statement "There is no royal road to geometry," attributed to Euclid by Proclus. Euclid may have made this statement but it is likely that it was made before his day by others. In fact, an equivalent remark is attributed to an earlier Greek geometer, Menaechmus, who is said to have been asked by Alexander to teach him geometry *concisely*, and to have replied: "O king, through the country there are royal roads and roads for common citizens, but in geometry there is one road for all."

It would be desirable to know why this particular statement has become so popular as a historical note in the elementary geometries. The idea which it conveys is not peculiar to mathematics since there is no royal road to abstract knowledge in general, and geometry is intrinsically the least abstract among the broad mathematical subjects, but its elements are commonly studied in the most abstract manner. It appears not unlikely that the popularity of the statement mentioned above is largely due to the desire of the teacher to have some artificial stimulus to counteract the unnecessarily abstract approach to the subject of geometry and hence this popularity may be regarded as an implicit condemnation of the usual method of approach.

The fact that the historical notes in our textbook should lay a solid foundation for the study of the history of mathematics, provided this can be done without diminishing their stimulating value as regards the particular subjects treated, is in accord with general principles of education. This fact naturally raises the question, What are the most important elements of such a foundation? While it would be difficult to secure general agree-

ment as regards the twenty-five most important elements of the history of mathematics, and their arrangement in order of importance, it seems likely that there would not be much difference of opinion as regards the relative importance of a few of these elements.

In particular, probably nearly all mathematicians would agree that Euclid's *Elements* occupy the foremost place among the records upon which the history of elementary mathematics is based, and some of them might agree with the following statement relating to this work: "Since ancient times the *Elements* of Euclid have been accepted without protest as the foundation of mathematical science and have represented both the concept and the extent of elementary mathematics."[1]

In view of the preeminent position of Euclid's *Elements* in the history of elementary mathematics it is clear that the numerous references to this work in the elementary textbooks are justifiable. Unfortunately some of these references are misleading in various particulars. For instance, some writers refer to these *Elements* as containing *all* that was known about geometry in the days of Euclid.[2] To see that this is far from the truth it is only necessary to recall that the only curved line treated in them is the circle, while earlier Greek geometers had studied other curves. In particular, conic sections had been studied earlier and Euclid himself is believed to have written a work on this subject.

Another incorrect inference to which some of these historical notes give rise is that the *Elements* of Euclid were confined to geometry. In fact, some authors refer to them under the title *Elements of Geometry*. On the contrary, a considerable portion of this work is devoted to theoretical arithmetic, or the theory of numbers, and the entire work seems to have been intended to be an exposition of the elements of *pure* mathematics. As formal Algebra had not then been developed the *Elements* of Euclid fail to present this side of elementary mathematics in its modern form and hence they fail to exhibit clearly some of the most general methods in the field of elementary mathematics.

It was noted above that there would probably be little, if any, difference of opinion as regards the selection of the most important document relating to the history of elementary mathematics.

[1] Vom Altertum an haben die Elemente Euclids unbestritten als Grundlage der mathematischen Wissenschaft gegolten und sowohl Begriff als Umfang der Elementar mathematik bezeichnet. *Encyklopadie der Mathematischen Wissenschaften*, vol. 3, p. 773.

[2] As a recent illustration we may cite the statement on page 2 of *Plane Geometry* by Slaught and Lennes, 1918.

The selection of the document which might be regarded as next to Euclid's *Elements* among the fundamental records relating to this history would probably give rise to a greater diversity of opinions, although the work of Ahmes might reasonably be expected to receive the strongest support. Possibly the *arithmetica* of Diophantus should be regarded third in order of importance among the original sources for the history of elementary mathematics since it is preeminent among Greek works along algebraic lines.

Fortunately two of these three fundamental sources for the history of elementary mathematics are easily accessible to those who read only the English language. The *Elements* of Euclid are provided with a large number of historical and explanatory notes in the three volumes which appeared in 1908 under the title *The Thirteen Books of Euclid's Elements* by T. L. Heath, and two years later there appeared a second edition of *Diophantus of Alexandria, a study in the history of Greek Algebra*, by the same author, which includes a translation of the *arithmetica*.

On the other hand, the work of Ahmes has not yet been made so easily accessible to English readers. The articles by F. L. Griffith which appeared in the *Proceedings of the Society of Biblical Archaeology*, volumes 13 and 16, (1891 and 1894) are very useful, and R. C. Archibald published a helpful note on this work in the *American Mathematical Monthly*, volume 25 (1918), page 36. Hence those who use only the English language can readily secure a fair notion also of this important document bearing on the history of elementary mathematics.

The main objects of the present article are to emphasize the need of greater seriousness as regards the historical notes in our textbooks on elementary mathematics and to point out how improvements may easily be made. Some authors seem to regard these notes as a kind of playground for the imagination, providing even pictures which are supposed to represent ancient mathematicians about whose personal appearance we possess absolutely no reliable information. The trouble about such supposed historical data is that they hinder more than help the student who may later desire to make progress along the line of mathematical history, and it is questionable whether they are intrinsically more interesting than realities in the early history of our subject. At any rate, the latter may become the starting points of scholarly attainments and of increasing intellectual pleasure, while the former necessarily cannot lead further.