

*On the Homogeneous Equation of a Plane Section of a Geometrical Surface.* By J. J. WALKER.

[Read May 8th, 1884.]

1. The formation of the equation to a plane section of a geometrical surface (given by an equation in rectangular Cartesian coordinates) as referred to rectangular axes in that plane, appears to have been first given by Leroy, §§ 85 to 88. Mr. Whitworth, in his "Trilinear Coordinates," 1866, p. 161, formed the trilinear equation; the consideration of the general question arising out of the particular case of the determination of the equation of the section of a right cone.

Two years later, in the *Nouvelles Annales*, June, 1868, M. Housel showed how to form the Cartesian equation to a plane section of a surface, referred to axes with any angles of ordination, the axes of the plane section being two of the three lines in which its plane meets the coordinate planes.

The object of the present paper is to complete the treatment of the question, by forming the trilinear equation to the section, whatever the angles of ordination of the Cartesian axes to which the surface is referred, and then to deal with the case in which the equation to the surface is expressed in a homogeneous form, either by means of tetrahedral or quadriplanar coordinates.

2. Suppose the surface referred to Cartesian axes having any angles of ordination, and

$$lx + my + nz = p$$

to be any plane intersecting the axes  $OX$ ,  $OY$ ,  $OZ$  in  $A$ ,  $B$ ,  $C$ ; then

$$OA = p/l, \quad OB = p/m, \quad OC = p/n,$$

and if  $p$  be the perpendicular from  $O$  on the plane, then  $l$ ,  $m$ ,  $n$  will be the cosines of the angles which  $p$  makes with the axes; and

$$\begin{aligned} BC^2 &= OB^2 + OC^2 - 2OB \cdot OC \cos YOZ \\ &= p^2 m^{-2} + p^2 n^{-2} - 2p^2 m^{-1} n^{-1} \cos YOZ, \end{aligned}$$

$$mnBC/p = (m^2 + n^2 - 2mn \cos YOZ)^{\frac{1}{2}}.$$

Now, if  $x'$ ,  $x_0$  are the perpendiculars from any point in the plane  $ABC$ , say  $P$ , and from  $A$  respectively on  $BC$ ,

$$x : x' = OA \text{ or } p/l : x_0,$$

or

$$\begin{aligned}x : x' \cdot BC &= p^3 : lp x_0 \cdot BC \\ &= mn : p \sin O,\end{aligned}$$

since  $px_0 \cdot BC = 6 \text{ vol. } OABC = OA \cdot OB \cdot OC \sin O = p^3 \sin O / lmn$ ;

hence  $x = mn x' \cdot BC / p \sin O = (m^3 + n^3 - 2mn \cos YOZ)^{\frac{1}{3}} / \sin O$ .

Similarly  $y = y' (n^3 + l^3 - 2nl \sin ZOZ)^{\frac{1}{3}} / \sin O$ ,

$$z = z' (l^3 + m^3 - 2lm \sin XOY)^{\frac{1}{3}} / \sin O.$$

3. If  $F(xyz) = 0$  be the equation of any geometrical surface referred to the axes  $OX, OY, OZ$ , the substitution of the above values for  $x, y, z$  will plainly give the trilinear equation of the section made by the plane  $lx + \dots = p$ , the result being made homogeneous by means

of the factor  $xx_0^{-1} + yy_0^{-1} + zz_0^{-1} = 1$ .

It remains to express  $x_0, y_0, z_0$  in terms of  $l, m, n, p$ , and the angles of ordination.

It has just been shown that

$$\begin{aligned}x_0 &= p^3 \sin O / lmn \cdot BC \\ &= p \sin O / l (m^3 + n^3 - 2mn \sin YOZ)^{\frac{1}{3}},\end{aligned}$$

or  $x_0^{-1} = (m^3 + n^3 - 2mn \sin YOZ)^{\frac{1}{3}} l / p \sin O$ .

Similarly  $y_0^{-1} = (n^3 + l^3 - 2nl \sin ZOZ)^{\frac{1}{3}} m / p \sin O$ ,

$$z_0^{-1} = (l^3 + m^3 - 2lm \sin XOY)^{\frac{1}{3}} n / p \sin O.$$

The unit factor is, therefore,

$$\begin{aligned}(m^3 + n^3 - 2mn \sin YOZ)^{\frac{1}{3}} lx' + (n^3 + l^3 - 2nl \sin ZOZ)^{\frac{1}{3}} ny' \\ + (l^3 + m^3 - 2lm \sin XOY)^{\frac{1}{3}} nz'\end{aligned}$$

divided by  $p \sin O$  or

$$p (1 - \cos^3 YOZ - \cos^3 ZOZ - \cos^3 XOY + 2 \cos YOZ \cos ZOZ \cos XOY)^{\frac{1}{3}}.$$

4. Briefly the result may be stated thus:—

If the equation to the surface be

$$F(xyz) \equiv u_n + u_{n-1} + \dots + u_0 = 0,$$

and the plane

$$v_1 \equiv ax + by + cz = 1,$$

then the equation of the section of  $F$  made by  $v_1$  will be

$$u_n + v_1 u_{n-1} + v_1^2 u_{n-2} + \dots + v_1^n u_0 = 0,$$

where the  $x, y, z$  are certain multiples of the perpendiculars  $(x'y'z')$  from any point of the section on the lines in which the plane cuts the coordinate planes  $YOZ, ZOZ, XOY$ ; viz.,

$$x = x' (b^3 + c^3 - 2bc \cos YOZ)^{\frac{1}{3}} \div,$$

$$y = y' (c^3 + a^3 - 2ca \cos ZOZ)^{\frac{1}{3}} \div,$$

$$z = z' (a^3 + b^3 - 2ab \cos XOY)^{\frac{1}{3}} \div,$$

$x, y, z$  being connected by the relation,

$$ax + by + cz = 1,$$

and the divisor being the square root of

$$\Sigma a^3 \sin^2 YOZ + 2\Sigma bc (\cos ZOX \cos XOY - \cos YOZ),$$

since  $p = \sin O \div$  that same divisor.

If the axes are rectangular, then

$$x = x' (b^2 + c^2)^{\frac{1}{2}} \div (a^2 + b^2 + c^2)^{\frac{1}{2}},$$

$$y = y' (c^2 + a^2)^{\frac{1}{2}} \div (a^2 + b^2 + c^2)^{\frac{1}{2}},$$

$$z = z' (a^2 + b^2)^{\frac{1}{2}} \div (a^2 + b^2 + c^2)^{\frac{1}{2}}.$$

5. In employing a trilinear equation, it is important to have ascertained the values of the sines and cosines of the triangle of reference. In the present case,

$$\begin{aligned} \sin A &= 2\Delta / CA \cdot AB = 6Y/p \cdot CA \cdot AB \\ &= OA \cdot OB \cdot OC \sin O / p \cdot CA \cdot AB \\ &= l \sin O / (n^2 + l^2 - 2nl \cos ZOX)^{\frac{1}{2}} (l^2 + m^2 - 2lm \cos XOY)^{\frac{1}{2}} \\ &= a \{ \Sigma a^3 \sin^2 YOZ - 2\Sigma bc (\cos YOZ - \cos ZOX \cos XOY) \}^{\frac{1}{2}} \div \\ &\quad (c^2 + a^2 - 2ca \cos ZOX)^{\frac{1}{2}} (a^2 + b^2 - 2ab \cos XOY)^{\frac{1}{2}}, \end{aligned}$$

according as the cutting plane is defined by

$$lx + \dots = p \text{ or } ax + by + cz = 1;$$

$$\begin{aligned} \cos A &= (l^2 + mn \cos YOZ - nl \cos ZOX - lm \cos XOY) \div \\ \text{or} \quad &= (a^2 + 2bc \cos YOZ - ca \cos ZOX - ab \cos XOY) \div, \end{aligned}$$

the divisors being the same as for  $\sin A$ .

If the plane of the section should pass through the origin, the process above followed becomes nugatory. In that case, to obtain a homogeneous equation, the equations of the surface and plane may first be transferred to any convenient point as a new origin, and the process then be carried out. Or M. Housel's plan may be adopted by using, as Cartesian axes, the lines in which the plane of section cuts two of the coordinate planes.

6. Suppose now the surface to be defined by an equation,

$$F(a\beta\gamma\delta) = 0,$$

among tetrahedral coordinates, the four corners of the fundamental tetrahedron being  $A, B, C, D$ ; and let

$$ka + l\beta + m\gamma + n\delta = 0 \dots\dots\dots(1)$$

be a plane cutting the edges  $AB, AC, AD$  in the points  $B', C', D'$ .

The coordinates of these points will be

$$\left. \begin{aligned} \alpha_1 &= \frac{l}{l-k}, & \beta_1 &= \frac{k}{k-l}, & \gamma_1 &= 0, & \delta_1 &= 0 \\ \alpha_2 &= \frac{m}{m-k}, & \beta_2 &= 0, & \gamma_2 &= \frac{k}{k-m}, & \delta_2 &= 0 \\ \alpha_3 &= \frac{n}{n-k}, & \beta_3 &= 0, & \gamma_3 &= 0, & \delta_3 &= \frac{k}{k-n} \end{aligned} \right\} \dots\dots(2).$$

If  $P$  be any point lying in the plane (1), and  $\lambda, \mu, \nu$  its areal co-ordinates relative to the triangle  $B'C'D'$ , then

$$\begin{aligned} \lambda &= PC'D' : A'C'D' \\ &= \text{vol. } PAO'D' : \text{vol. } A'AC'D' \\ &= \text{vol. } PACD : \text{vol. } A'ACD \\ &= \beta : \beta_1, \end{aligned}$$

$$\left. \begin{aligned} \beta &= \lambda\beta_1 = \frac{k\lambda}{k-l} \\ \text{Similarly,} \quad \gamma &= \frac{k\mu}{k-m} \\ \delta &= \frac{k\nu}{k-n} \end{aligned} \right\} \dots\dots\dots(3),$$

and, since the  $\alpha$  of any point on the plane is equal to

$$\begin{aligned} &-(l\beta + m\gamma + n\delta) \div k, \\ \alpha &= \frac{l\lambda}{l-k} + \frac{m\mu}{m-k} + \frac{n\nu}{n-k}; \end{aligned}$$

$$\text{or, say,} \quad \alpha = l'\lambda + m'\mu + n'\nu \dots\dots\dots(4).$$

These substitutions for the tetrahedral coordinates in the equation to the surface will therefore give the equation of the section by the plane (1) in areal coordinates, the triangle of reference being that formed by the three lines in which the plane cuts the three faces of the tetrahedron meeting in the vertex  $A$ .

7. Symmetric substitutions may be obtained in lieu of the above by taking as a new triangle of reference,  $LMN$ , that formed by the three diagonals of the quadrilateral made up of the four lines in which the plane  $k\alpha + \dots$  cuts the four faces of the tetrahedron  $ABCD$ ; viz., the lines

$$\lambda = 0, \quad \mu = 0, \quad \nu = 0, \quad l'\lambda + m'\mu + n'\nu = 0 \dots\dots\dots(4),$$

the diagonals of which are

$$m'\mu + n'\nu = 0, \quad n'\nu + l'\lambda = 0, \quad l'\lambda + m'\mu = 0.$$

Let  $\xi, \eta, \zeta$  be the areal coordinates of  $P$  relative to this new triangle; viz.,  $\xi = PMN : LMN$ ,  $\eta = PNL : LMN$ ,  $\zeta = PLM : LMN$ .

Now the coordinates of  $L, M, N$ , relative to  $B'C'D'$ , are

$$\lambda' = -l'^{-1} \div, \quad \mu' = m'^{-1} \div, \quad \nu' = n'^{-1} \div,$$

the divisor being  $-l'^{-1} + m'^{-1} + n'^{-1}$ ;

$$\lambda'' = l'^{-1} \div, \quad \mu'' = -m'^{-1} \div, \quad \nu'' = n'^{-1} \div,$$

the divisor being  $l'^{-1} - m'^{-1} + n'^{-1}$ ;

$$\lambda''' = l'^{-1} \div, \quad \mu''' = m'^{-1} \div, \quad \nu''' = -n'^{-1} \div,$$

the divisor being  $l'^{-1} + m'^{-1} - n'^{-1}$ ;

and if  $\Delta$  represents the area of the triangle  $B'C'D'$ ,

$$\begin{aligned} \text{area } PMN &= \Delta \begin{vmatrix} \lambda, & \mu, & \nu \\ l'^{-1}, & -m'^{-1}, & n'^{-1} \\ l'^{-1}, & m'^{-1}, & -n'^{-1} \end{vmatrix} \\ &\div (l'^{-1} - m'^{-1} + n'^{-1})(l'^{-1} + m'^{-1} - n'^{-1}) \\ &= 2\Delta (m'\mu + n'\nu) \div l'm'n' (\dots) (\dots). \end{aligned}$$

Or,  $\text{area } PMN = 2\Delta (m'\mu + n'\nu)(-l'^{-1} + m'^{-1} + n'^{-1}) \div,$

$$\text{area } PNL = 2\Delta (n'\nu + l'\lambda)(l'^{-1} - m'^{-1} + n'^{-1}) \div,$$

$$\text{area } PLM = 2\Delta (l'\lambda + m'\mu)(l'^{-1} + m'^{-1} - n'^{-1}) \div,$$

$$\text{area } LMN = 4\Delta \div,$$

the common divisor being

$$l'm'n'(-l'^{-1} + m'^{-1} + n'^{-1})(l'^{-1} - m'^{-1} + n'^{-1})(l'^{-1} + m'^{-1} - n'^{-1});$$

and the value of the fourth area being obtained from that of the first by writing  $\mu', \nu'$  for  $\mu, \nu$ ; or, analogously from either of the other two of the first three.

Hence the equations

$$2\xi = 2(-l'^{-1} + m'^{-1} + n'^{-1})\xi = m'\mu + n'\nu,$$

$$2\eta = 2(l'^{-1} + m'^{-1} - n'^{-1})\eta = n'\nu + l'\lambda,$$

$$2\zeta = 2(l'^{-1} + m'^{-1} - n'^{-1})\zeta = l'\lambda + m'\mu,$$

giving

$$l'\lambda = -\xi + \eta + \zeta,$$

$$m'\mu = \xi - \eta + \zeta,$$

$$n'\nu = \xi + \eta - \zeta,$$

from which, observing (3) that  $l\beta \div k = -l'\lambda$ ,  $m\gamma \div k = -m'\mu$ ,  $n\delta \div k = n'\nu$ ,  $-(l\beta + m\gamma + n\delta) \div k = k\alpha \div k$ ,

$$k\alpha = k(\xi + \eta + \zeta),$$

$$l\beta = k(\xi - \eta - \zeta),$$

$$m\gamma = k(-\xi + \eta - \zeta),$$

$$n\delta = k(-\xi - \eta + \zeta).$$

Finally, substituting for  $l^{-1}$ ,  $m'^{-1}$ ,  $n'^{-1}$ , their values  $1 - kl^{-1}$ ,  $1 - km^{-1}$ ,  $1 - kn^{-1}$ , ..

$$-l^{-1} + m'^{-1} + n'^{-1} = k(k^{-1} + l^{-1} - m^{-1} - n^{-1}),$$

$$l^{-1} - m'^{-1} + n'^{-1} = k(k^{-1} - l^{-1} + m^{-1} - n^{-1}) \dots,$$

the substitutions may be written

$$\alpha = k^{-1}(\xi' + \eta' + \zeta'),$$

$$\beta = l^{-1}(\xi' - \eta' - \zeta'),$$

$$\gamma = m^{-1}(-\xi' + \eta' - \zeta'),$$

$$\delta = n^{-1}(-\xi' - \eta' + \zeta'),$$

where now

$$\xi' = \xi \div (k^{-1} + l^{-1} - m^{-1} - n^{-1}),$$

$$\eta' = \eta \div (k^{-1} - l^{-1} + m^{-1} - n^{-1}),$$

$$\zeta' = \zeta \div (k^{-1} - l^{-1} - m^{-1} + n^{-1}).$$

8. To express the parts of the triangle of reference  $LMN$  in terms of  $k \dots n$  and the edges of the fundamental tetrahedron  $ABCD$ ; first,

$$\begin{aligned} MN^2 &= (\mu'' - \mu''')(\nu''' - \nu'') C'D^2 + (\nu'' - \nu''')(\lambda''' - \lambda'') D'B^2 \\ &\quad + (\lambda'' - \lambda''')(\mu''' - \mu'') B'C^2, \\ &= 2l'^{-2} \{ m'^{-1} n'^{-1} C'D^2 + n'^{-1} (-m'^{-1} + n'^{-1}) D'B^2 \\ &\quad + m'^{-1} (m'^{-1} - n'^{-1}) B'C^2 \} \div \\ &\quad (l'^{-1} - m'^{-1} + n'^{-1})^2 (l'^{-1} + m'^{-1} - n'^{-1})^2 \\ &= 2(l-k)^2 \{ mn(m-k)(n-k) C'D^2 + (n-k)(n-m) km D'B^2 \\ &\quad + (m-k)(m-n) kn B'C^2 \} \div \\ &\quad k^2 l^2 m^2 n^2 (k^{-1} - l^{-1} + m^{-1} - n^{-1})^2 (k^{-1} - l^{-1} - m^{-1} + n^{-1})^2. \end{aligned}$$

Representing the edges of the tetrahedron by

$$\begin{aligned} CD &= a, \quad DB = b, \quad BC = c, \quad AB = a', \quad AC = b', \quad AD = c', \\ C'D^2 &= \gamma_2 \delta_3 a^2 + (a_3 - a_2) \gamma_2 b^2 + (a_2 - a_3) \delta_3 c^2 \\ &= k^2 (m-k)^{-1} (n-k)^{-1} a^2 + k^2 (n-n)(m-k)^{-2} (n-k)^{-1} b^2 \\ &\quad + k^2 (n-m)(n-k)^{-2} (m-k)^{-1} c^2; \\ D'B^2 &= k^2 (n-k)^{-1} (l-k)^{-1} b^2 + k^2 (n-l)(n-k)^{-2} (l-k)^{-1} c^2 \\ &\quad + k^2 (l-n)(l-k)^{-2} (n-k)^{-1} a^2; \\ B'C^2 &= k^2 (l-k)^{-1} (m-k)^{-1} c^2 + k^2 (l-m)(l-k)^{-2} (m-k)^{-1} a^2 \\ &\quad + k^2 (m-l)(m-k)^{-2} (l-k)^{-1} b^2. \end{aligned}$$

Substituting these values in the expression above for  $MN^2$ ,

$$\begin{aligned} MN^2 &= 2 \{ mn(l-k)^2 a^2 - kl(m-n)^2 a^2 \\ &\quad + (l-k)(n-m)(kmb^2 - knc^2 - nlb^2 + lmc^2) \} \div \\ &\quad k^2 l^2 m^2 n^2 (k^{-1} - l^{-1} + m^{-1} - n^{-1})^2 (k^{-1} - l^{-1} - m^{-1} + n^{-1})^2. \end{aligned}$$

The terms within the { } have a remarkable significance; for it

may readily be verified that they form the expression for the square of the line  $(a_1)$ , joining the points in which the plane  $ka + \dots$  cuts the opposite edges  $AB, CD$  of the tetrahedron  $ABCD$ , multiplied by the factor

$$(k-l)^2 (m-n)^2;$$

thus  $MN = \sqrt{2} (k-l)(m-n) a_1 \div klmn (k^{-1} - \dots)(k^{-1} - \dots);$

similarly  $NL = \sqrt{2} (k-m)(n-l) b_1$

$$\div klmn (k^{-1} - l^{-1} - m^{-1} + n^{-1})(k^{-1} - l^{-1} - m^{-1} - n^{-1}),$$

$$LM = \sqrt{2} (k-n)(l-m) c_1$$

$$\div klmn (k^{-1} + l^{-1} - m^{-1} - n^{-1})(k^{-1} - l^{-1} + m^{-1} - n^{-1}),$$

$b_1, c_1$  being the lengths of the connectors of the points in which the plane  $ka + \dots$  meets  $AC, BD$  and  $AD, BC$  respectively.

The area of the triangle  $LMN$  has been expressed in terms of that of the triangle  $B'C'D'$ . If  $V$  represents the volume of the tetrahedron  $ABCD$ ,  $V'$  that of the tetrahedron  $AB'C'D'$ , and  $p$  the length of the perpendicular from  $A$  on the plane  $ka + \dots$ ; then

$$p \cdot B'C'D' = 3V' = 3AB' \cdot AC' \cdot AD' \cdot V \div AB \cdot AC \cdot AD.$$

$$\text{Now } p = 3V \cdot k \div (k^3 A^2 + \dots - 2mn CD \cos \widehat{CD})^{\frac{1}{2}},$$

where  $A \dots D$  stand for the areas of the faces opposite the corners  $A \dots D$ ; and

$$AB' : AB = k : k-l, \quad AC' : AC = k : k-m, \quad AD' : AD = k : k-n,$$

so that the area

$$B'C'D' = k^3 (k^2 A^2 + \dots - 2mn CD \cos \widehat{CD})^{\frac{1}{2}} \div (k-l)(k-m)(k-n),$$

and it was shown that

$$\begin{aligned} LMN &= 4B'C'D' \div l'm'n' (-l'^{-1} + \dots)(l'^{-1} + \dots)(l'^{-1} + m'^{-1} - n'^{-1}) \\ &= 4(l-k)(m-k)(n-k) B'C'D' \div k^3 lmn (k^{-1} + l^{-1} - m^{-1} - n^{-1})(\dots)(\dots). \end{aligned}$$

Hence, finally,

$$\begin{aligned} LMN &= 4(k^3 A^2 + \dots - 2mn CD \cos \widehat{CD})^{\frac{1}{2}} \div \\ &klmn (k^{-1} + l^{-1} - m^{-1} - n^{-1})(k^{-1} - l^{-1} + m^{-1} - n^{-1})(k^{-1} - l^{-1} - m^{-1} + n^{-1}). \end{aligned}$$

9. If the original equations of the surface and plane had been given in quadriplanar coordinates, then everywhere, in what precedes,  $ka_0, l\beta_0, m\gamma_0, n\delta_0$  should be written for  $k \dots \delta$  respectively,  $a_0 \dots \delta_0$  being the perpendiculars from the four vertices of the tetrahedron of reference on the opposite faces; viz.,

$$a_0 = 3V/A, \quad \beta_0 = 3V/B, \quad \gamma_0 = 3V/C, \quad \delta_0 = 3V/D.$$

And to obtain the symmetrical trilinear equation of the section for  $\xi, \eta, \zeta$ , in the values of  $a \dots \delta$ , must, of course, be written  $\xi/x_0 \dots, x_0 y_0 z_0$ , being the perpendiculars from  $L \dots$  on the opposite

sides  $MN$  ... of the triangle  $LMN$ ; viz.,

$$\begin{aligned}
 x_0 &= 2LMN / MN = 24V (k^2 + \dots - 2mn \cos \widehat{OD})^{\frac{1}{2}} \\
 &\quad \times [(ka_0)^{-1} + (l\beta_0)^{-1} - (m\gamma_0)^{-1} - (n\delta_0)^{-1}] \div \sqrt{2} (ka_0 - l\beta_0)(m\gamma_0 - n\delta_0) a_1; \\
 y_0 &= 24V (k^2 + \dots)^{\frac{1}{2}} [(ka_0)^{-1} - (l\beta_0)^{-1} + (m\gamma_0)^{-1} - (n\delta_0)^{-1}] \\
 &\quad \div \sqrt{2} (ka_0 - m\gamma_0)(n\delta_0 - l\beta_0) b_1, \\
 z_0 &= 24V (k^2 + \dots)^{\frac{1}{2}} [(ka_0)^{-1} - (l\beta_0)^{-1} - (m\gamma_0)^{-1} + (n\delta_0)^{-1}] \\
 &\quad \div \sqrt{2} (ka_0 - n\delta_0)(l\beta_0 - m\gamma_0) c_1.
 \end{aligned}$$

*Motion of a Network of Particles with some Analogies to Conjugate Functions.* By Dr. E. J. ROUTH.

[Read May 8th, 1884.]

*Summary.*—In this paper we propose to discuss briefly the motion of certain networks with material particles placed at the intersection of the threads. We begin with a network in which the openings between the threads are rectangular. Expressing the equations of motion by a single equation of differences and writing down the solution, we find that there are at least two distinct kinds of motion. Either of these might be produced by a forced agitation at a boundary. The distinction between the two types of motion depends on the period of the forced oscillation. If this period exceed a certain limit, waves will travel over the network. If the period fall short of this limit, the distant parts of the network will be permanently at rest. There are also other differences between the two kinds of motion. When the network reduces to a membrane, the second kind of motion is found not to exist.

When the network is equally stretched in all directions, the wave motions will travel quickest when the front is along the diagonals of the openings, and slowest when the fronts are along the threads.

Passing next to the case of a network in which the openings form quadrilaterals, we find that there is a class of such networks which move exactly as if the openings were rectangular. A criterion is given to distinguish such networks and also a method of constructing them. Thus the periods of oscillation of any rectangular network being known, we can immediately state the periods of oscillation of the corresponding quadrilateral network.

By noticing the meaning of this transformation when the network is so fine as to become a membrane, we find it is analogous to that of conjugate functions. Thus the transformation here effected for the