

mary winding Y-connected, and on the 75 per cent machine with 75 per cent secondary winding and full primary winding delta-connected. The last or running voltages would of course be obtained in the usual way.

Where three-wire converters with a high unbalance capacity are installed, it may be possible to obtain the desired reduced voltage by having arrangements made so that either the positive or negative circuit breaker of the converter may be left open and the converter neutral connected to the bus on which the breaker is left open. This will give voltages about one-half as large as given in Table I for primary starting.

If the stations are operated with several busses at different voltages, it is of course necessary to take this into account when starting up the station. It might be found advisable to have the arrangements made for closing the d-c. circuit breakers on several

of the converters at the same time by means of special control connections, so that the first machines which are connected to the system may be connected simultaneously, thus avoiding the possibility of having one or two trip off on overload.

The exact procedure to be followed in adapting the substations in any particular system to this method of reestablishing the Edison service can only be arrived at after making a thorough analysis of the problem.

While it is thought that the method of reenergizing outlined in detail will be found to be satisfactory, it is not the intention to give the impression that it is in general better than the other methods mentioned, as it is thoroughly realized that under certain conditions the method discussed would not be applicable at all. Which one of the methods of reestablishing service is made use of should be determined by making a comprehensive study of the situation.

## Tooth Frequency Losses in Rotating Machines

BY THOMAS SPOONER

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*Data are presented showing the hysteresis and eddy current losses due to minor displaced hysteresis loops superimposed on major loops of various amplitudes.*

*In all cases the amplitude of the minor loops is made proportional to the displacement which approximates the conditions which occur in machines. These data are applied to the case of the induction motor having a sine wave field distribution and a sample case worked out showing that the hysteresis losses in the teeth due to the tooth pulsations may be of the same order of magnitude as the losses due to the fundamental frequency. The eddy current losses in thin sheets due to the high-frequency pulsations are in general negligible but may be quite appreciable in the pole faces where the sheets are thicker. There are of course other losses due to tooth pulsations such as eddy current and circulating losses in the copper, etc., but these are beyond the scope of the present investigation.*

### INTRODUCTION

THE calculation of core losses in rotating machines is based very largely on empirical data derived from tests of complete machines. That this procedure is followed is due to two chief causes.

1. The large magnitude and uncertainty of the illegitimate losses due to imperfections in manufacture such as bending strains in the sheet, burrs, filing of slots, etc.

2. Lack of sufficient fundamental data and design formulas.

The illegitimate losses are so large and variable at times that the designer feels that any refinement in calculations is a waste of time. This feeling on the part of the designer is largely responsible for the lack of design data since the designer has not demanded it.

When small changes only are made from standard designs the empirical data are fairly satisfactory, but when radically new designs are worked out, it frequently happens that the calculated losses are far from the true values. When a machine fails to meet the

calculated losses, the designer should know whether the fault is due to his design or to poor shop practise. Of the various factors at present not subject to calculation which go to make up iron losses in rotating machines, those due to high-frequency tooth pulsations seemed to be the most important. In some cases it seems probable that these tooth pulsations may be responsible for more than 50 per cent of the total iron losses.

A few years ago a paper<sup>1</sup> was presented before the A. I. E. E. giving some data on displaced hysteresis loops. These data were rather meager and not very directly applicable to rotating machines since the authors considered only the displacement factor or the increase in hysteresis loss of a displaced minor loop occurring at the tip of a major loop over that of a symmetrical loop of the same amplitude. Referring to Fig. 1, the displacement factor is the ratio of the area of loop No. 1 to that of a small symmetrical loop at the center having the same  $B$  amplitude.

Recently the author has undertaken a further investigation along the same line in an effort to obtain results which are more nearly applicable to the conditions which exist in rotating machines.

1. *The Effect of Displaced Magnetic Pulsations on the Hysteresis Loss of Sheet Steel*. L. W. Chubb and Thomas Spooner, *TRANSACTIONS A. I. E. E.*, 1915, Vol. XXXIV, page 2671.

## EXPERIMENTAL PROCEDURE

**Test Samples.** The experimental data were obtained on ring punchings from three kinds of material. The samples were 4 7/16 in. (11.27 cm.) outside diameter and 3 7/16 in. (8.73 cm.) inside diameter. These samples were mill annealed but not reannealed after punching. They weighed about one pound apiece and had the following characteristics:

Sample	Gage Inches	Silicon Content
A	0.028	2.25%
B	0.012	1.0 %
C	0.017	1.0 %

These materials are all standard electrical sheet. The silicon contents are only approximate.

**Test Methods.** These samples were wound with suitable primary and secondary windings and tested

TABLE I

Loop	$B_{\max} = 10,000$		$B_{\max} = 15,000$		$B_{\max} = 17,000$	
	Lower tip	Upper tip	Lower tip	Upper tip	Lower tip	Upper tip
1	8000	10000	12000	15000	13600	17000
2	4800	6000	8000	10000	9600	12000
3	2400	3000	4000	5000	4800	6000
4	-3000	-2400	-5000	-4000	-6000	-4800
5	-6000	-4800	-10000	-5000	-12000	-9600

ballistically, using a modification of a ring testing apparatus<sup>2</sup> previously described. The main hysteresis loops were measured by determining each point in-

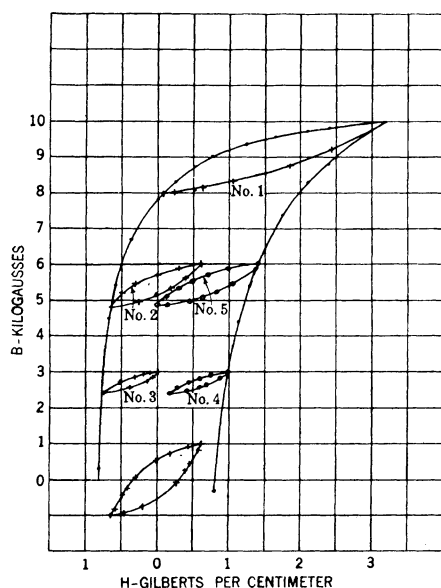


FIG. 1—SAMPLE C

$B_{\max}$  equals 10 kilogausses. Amplitude of displaced loops equals 20 per cent of displacement.

dependently with reference to the tip value. The displaced loops were, for convenience, measured by means of the step by step method. The samples were placed in oil to avoid heating. Two primary and two secondary coils were used one set of a few turns and

2. "Rapid Testing of Magnetic Materials," T. Spooner, *Electrical World*, Vol. 74, July 5, 1919.

the other of many, thus making it possible to obtain high sensitivity for both low and high main loops. For testing high induction major loops, a sensitive ammeter was automatically switched into circuit when reading low values.

The procedure for determining a displaced minor loop was as follows:

The sample was brought into a cyclic condition for the major loop by reversing the magnetizing current

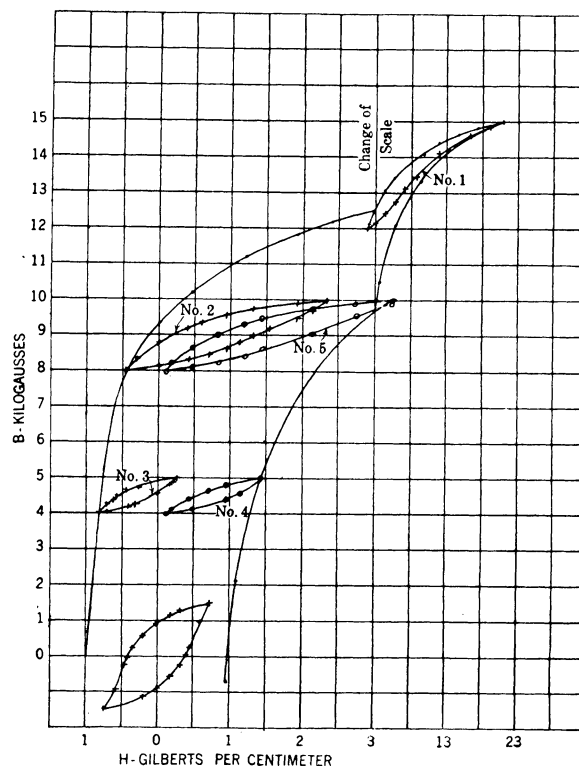


FIG. 2—SAMPLE C

$B_{\max}$  equals 15 kilogausses. Amplitude of displaced loops equals 20 per cent of displacement.

a number of times. Then starting from the tip of the major loop, the induction was reduced to the lower tip of the minor loop in one step. From this point, step by step, we ascended to the upper tip of the displaced loop and then descended to the induction of the lower tip.

In all cases the amplitude of each displaced loop was 20 per cent of the displacement of the tip farthest from the zero line of induction. For each sample three maximum inductions for major loops were taken, namely,  $B = 10, 15$  and  $17$  kilogausses. Table I gives the upper and lower tip values for all loops.

## TEST RESULTS

A typical set of hysteresis loops for sample C are given by Figs. 1, 2 and 3 for maximum inductions of 10, 15 and 17 kilogausses respectively. The inductions were calculated from the net section as determined by the weight of the samples and an assumed specific gravity of 7.7. The losses for the major loops expressed in ergs per cu. cm. per cycle are given in Table II.

The losses for the displaced loops are given by Table III.

These losses expressed in per cent of the major loop losses are given by Figs. 4, 5 and 6.

It will be noted that with the different major loops there happened to be associated a number of minor

of the *C* sample, assuming a sine-wave field form. On this assumption, the reference tip of each minor loop will be proportional to the cosine of the angle of rotation in electrical degrees, assuming that *B*<sub>max</sub> occurs at an angle of 0 deg. Suppose now we assume 18 teeth per pole. This will correspond to a minor

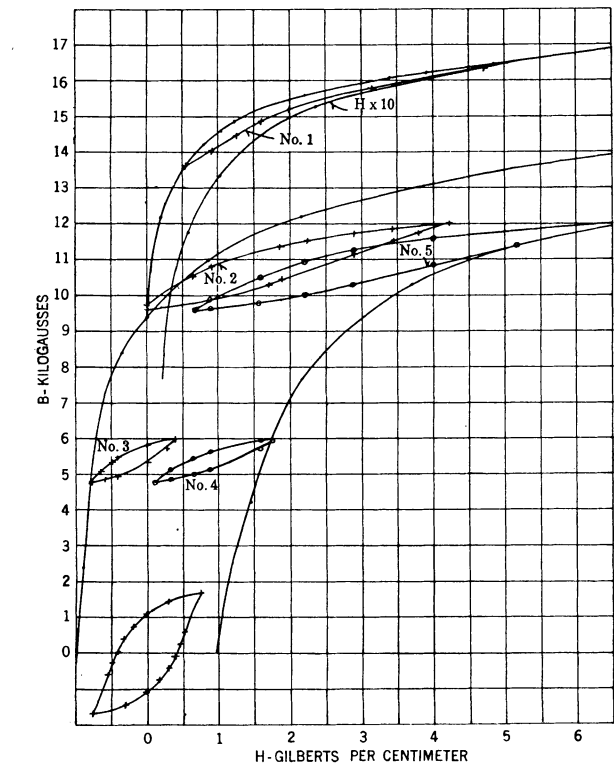


FIG. 3—SAMPLE C  
*B*<sub>max</sub> equals 17 kilogausses. Amplitude of displaced loops equals 20 per cent of displacement.

loops having the same amplitude and displacement. The losses for these loops have been collected in Table IV for convenient comparison.

For comparison with previous work the displacement factors have been calculated for the No. 1 loops. Results are given in Table V.

TABLE II			
<i>B</i> max	Sample A	Sample B	Sample C
10000	3110	2960	2780
15000	6000	7700	6252
17000	7930	10400	9010

Fig. 7 shows the effect of repeated reversals of magnetizing force on the hysteresis loss between the same maximum and minimum values of *H*. This particular curve is for a No. 4 displaced loop superimposed on a major loop of *B* = 17 kilogausses for the *B* ring and shows the difference between the first and fourth cycles for the same *H* amplitude.

CALCULATED RESULTS

*Hysteresis Loss.* In order to apply these results to specific conditions such as the loss in the teeth of an induction motor, calculations were made from the data

Table III			
Losses (Ergs) <i>B</i> max = 10			
Loop	Ring No. A	Ring No. B	Ring No. C
1	147	148	162
2	33	37	37
3	8.6	10.5	10.5
4	8.0	9.8	9.8
5	46	49	47
6	147	148	162
<i>B</i> max = 15			
1	407	650	550
2	152	141	140
3	26	29	26
4	24	36	29.5
5	190	179	148
6	407	650	550
<i>B</i> max = 17			
1	790	980	890
2	222	237	218
3	39	47	41
4	46.5	52.3	48.5
5	237	340	292
6	790	980	890

loop at each 10 electrical degrees. Now, if we multiply the cosine of the angle by *B*<sub>max</sub>, this will give us the reference tip for each displaced loop corresponding to the abscissa of Fig. 6. Next we take the corresponding percentage losses from this curve for each 10 deg. over a range of 180 deg., add the percentages, multiply by

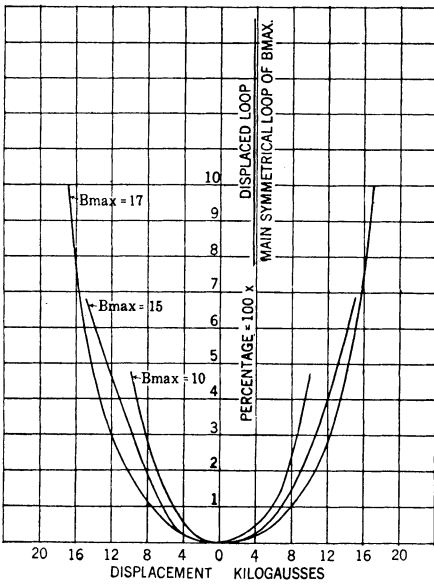


FIG. 4

two and we have the percentage hysteresis losses due to the displaced loops for a complete cycle. If this be done for various numbers of teeth it will be found that the losses are directly proportional to the number of teeth. We have then:

$$P = K T \tag{1}$$

where  $P$  = the loss of the minor loops in per cent of the loss of the major loop,

$K$  = a constant

$T$  = teeth per pair of poles.

If the percentage loss as calculated above be divided by the number of teeth per pair of poles the results will be  $K$  for the given  $B_m$  and for an assumed pulsating induction of 20 per cent. In order to obtain  $K$  for other

area of hysteresis loop as determined ballistically. In case of the induction motor,  $T$  must be reduced in proportion to the slip, namely,

$$T = T_1 \times \frac{\text{Rotor Speed}}{\text{Syn. Speed}}$$

where  $T_1$  is the actual number of teeth.

This applies both to the rotor and stator. If the number of teeth in the rotor and stator are different the pulsating losses in the rotor teeth are based on the number of teeth in the stator, and the losses in the stator on the number of teeth in the rotor.

The same formula may be used for calculating the losses with the inductions expressed in lines per square inch, and the losses in watts per pound or per cubic inch. In this case the three values of  $B_m$  for the curve of Fig. 8 expressed in lines per square inch will be 64.5, 86.8 and 98.3 corresponding to 10, 15 and 17 kilogausses respectively. The only other change needed is to express  $C$  in term of watts per pound or watts per cubic inch as desired.

**Eddy Losses.** A formula for the corresponding eddy current losses in the iron may be calculated as follows;

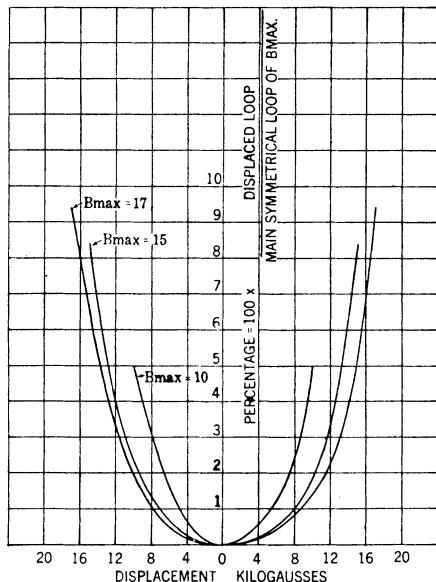


FIG. 5

pulsating amplitudes we assume that the hysteresis loss for the displaced loops is proportional to 1.6 power of the amplitude of the displaced loop. That this assumption is not far from the truth is shown by previous investigation<sup>3</sup> in which the exponent varied from about 1.6 to 1.5 for the larger displacements. Table VI gives these constants.

$$P = K T$$

$P$  = percentage loss

$K$  = constant

$T$  = teeth per pair of poles

$K_{10/30} = K$  for  $B_m = 10$  and ampl. of 30 per cent.

These results are plotted in Fig. 8. The total hysteresis loss due to the displaced loops may be calculated from the following formula:

$$W_{hd} = C K T f \quad (2)$$

where  $W_{hd}$  = watts per kilogram for a given  $B_m$ .

$C$  = hysteresis loss in watts per kilogram per cycle for a given  $B_m$ , due to the major hysteresis loops.

$K$  = a constant

$T$  = teeth per pair of poles.

$f$  = fundamental cycles per second.

$C$  may be determined by the ordinary Epstein test for the particular material considered. The eddy current losses must of course be deducted from the Epstein test value and the result divided by the frequency. Or  $C$  may be calculated directly from the

3. Ibid.

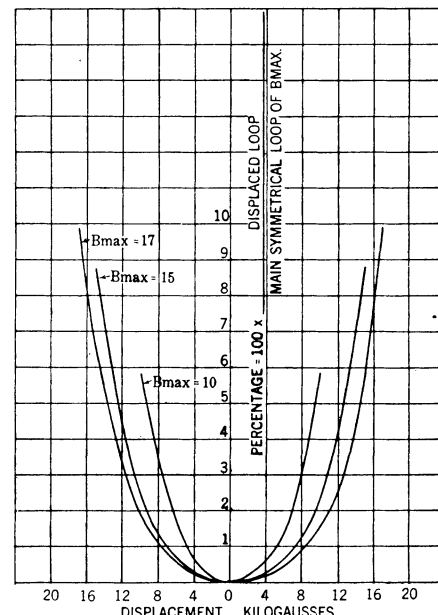


FIG. 6—SAMPLE C—1 PER CENT SILICON—0.0172-IN. GAGE  
Displacement is based on tip farthest from  $O B$ . Amplitude of displaced loop is 20 per cent of displacement.

assuming as before a sine-wave field form. Let us assume an eddy current loss of 1.0 watt per kilogram at  $B_m = 10$  kilogausses and 60 cycles. This is a fair value for 17-mil 1 per cent silicon steel. Assuming that the eddy loss is proportional to the square of the frequency and the square of the induction we get,

$$W_e = 2.78 \times 10^{-6} \times B^2 \times f^2 \quad (3)$$

where  $W_e$  is in watts per kilogram

$B$  is in kilogausses

$f$  is in cycles per second

Let us assume 36 teeth per pair of poles (one tooth every 10 electrical degrees) a fundamental frequency

of 60 cycles, a  $B_m$  of 10 kilogausses and an amplitude of 20 per cent for the displaced loop. The harmonic frequency would then be  $60 \times 36$  or 2160 cycles per second and the amplitude of the maximum displaced loop would then be 2 kilogausses corresponding to a  $B$  of one kilogauss. Applying these data to formula (3) and dividing by the harmonic frequency, we get

TABLE IV  
Comparison of displaced loops having same amplitude.

Sample	Amplitude	$B_{max}$	Loop No.	$W_h$
A	2	10	1	147
"	"	15	2	152
"	"	"	5	190
B	"	10	1	148
"	"	15	2	141
"	"	"	5	179
C	"	10	1	162
"	"	15	2	140
"	"	"	5	148
A	1.2	10	2	33
"	"	"	5	46
"	"	17	3	39
"	"	"	4	46.5
B	"	10	2	37
"	"	"	5	49
"	"	17	3	47
"	"	"	4	52.3
C	"	10	2	37
"	"	"	5	47
"	"	17	3	41
"	"	"	4	48.5

the eddy loss for the maximum loop at 0 deg. This procedure is repeated for every 10 deg. the induction being numerically equal to the cosine of the angle. The eddy losses are then summed up for the complete cycle. The result for the eddy losses due to the displaced loops for the particular conditions specified in watts per kilogram is 1.08. Assuming that the

TABLE V  
Displacement factors for rings A, B and C for 20 per cent loops.

Ring	$B = 10$	$B = 15$	$B = 17$
A	2.45	2.35	5.16
B	1.81	4.08	4.87
C	2.24	3.90	5.17

Amplitude of minor symmetrical and displaced loops are as follows:

$B^*$	Ampl.	Mean Displ.
10	2	9
15	3	13.5
17	3.4	15.3

\* $B$  is the upper tip of the displaced loop.

eddy losses vary as the square law we may write the formula directly.

$$W_{ed} = K (T \times t \times f \times P \times B_m)^2 \quad (4)$$

$W_{ed}$  is expressed in watts per kilogram

$T$  = teeth per pair of poles (with suitable corrections for slip as explained above.

$t$  = thickness of laminations in inches

$f$  = fundamental frequency in cycles per second

$P$  = per cent amplitude of displaced loop

$B_m$  = maximum induction in kilogausses per net section

By supplying 1.08 as calculated above in (4)

$K = 1.96 \times 10^{-9}$  which is an average constant for 17-mil 1 per cent silicon steel.

Of course, if the losses are to be expressed in units such as watts per cubic inch or the induction, thickness, etc., in other units the constant may be readily con-

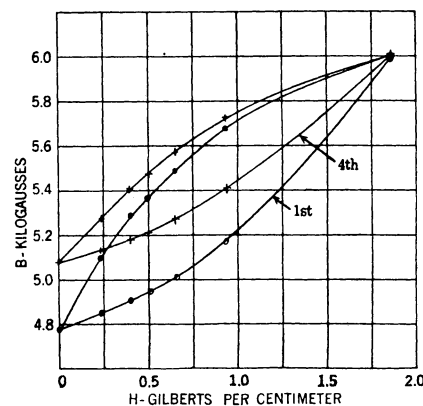


FIG. 7—SAMPLE B

Showing effect of repeated reversals of magnetizing force between the same  $H$  limits.

$W_h$  equals 52.3 (1st) ergs per cu. cm. per cycle.

$W_h$  equals 32.3 (4th) ergs per cu. cm. per cycle.

$$32.3 \left( \frac{1.2}{0.92} \right)^{1.6} \text{ equals } 49.5 \text{ (cf. 52.3)}$$

verted to take care of these units. It must of course be changed for other materials, and will be approximately inversely proportional to the resistivity of the material. The assumed losses were for sheet having a

TABLE VI

$B_m = 10$	$B_m = 15$	$B_m = 17$
K 10/30 = 4.96	K 15/30 = 7.04	K 17/30 = 7.56
K 10/25 = 3.69	K 15/25 = 5.25	K 17/25 = 5.63
K 10/20 = 2.59	K 15/20 = 3.68	K 17/20 = 3.95
K 10/15 = 1.63	K 15/15 = 2.33	K 17/15 = 2.49
K 10/10 = 0.85	K 15/10 = 1.22	K 17/10 = 1.30
K 10/5 = 0.28	K 15/5 = 0.40	K 17/5 = 0.43

resistivity of about 23 microhms per centimeter cube. The resistivity for other silicon alloys may be calculated by the following formula:

$$R = a + k S \quad (5)$$

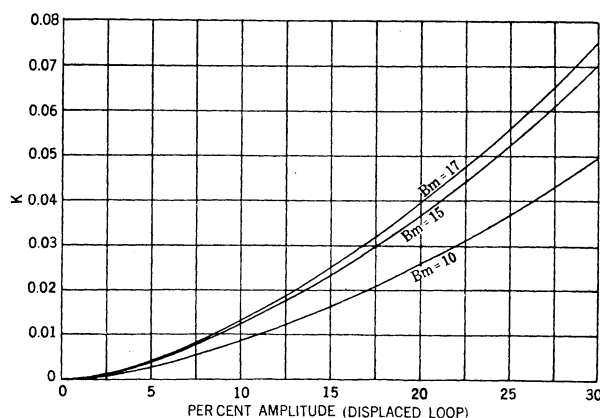


FIG. 8—FACTOR  $K$  FOR FORMULA (2) CALCULATED FROM DATA OF C SAMPLE

where  $R$  = The resistance in microhms per cm.<sup>3</sup>

$a$  = 12

$k$  = 11

$S$  = silicon content in per cent.

As an example of the use of the above mentioned formulas and curves calculations will be made for the following conditions:

Material	= 1 per cent silicon steel
Thickness	= 17 mils
$f$	= 60 cycles
$B_m$	= 17 kilogausses
$T$	= 40 teeth per pair of poles
$P$	= 20 per cent (amplitude of displaced loops)

Let us also assume that the iron has a total loss due to the fundamental frequency of 12 watts per kilogram at  $B_m = 17$  kilogausses which is a reasonable figure.

The eddy losses will then be from (3).

$$W_e = 2.78 \times 10^{-6} \times 17^2 \times 60 = 2.90 \text{ watts per kilogram.}$$

The hysteresis losses is  $12 - 2.9 = 9.1$  at 60 cycles and 0.152 per cycle.

The hysteresis loss due to the displaced loops is now from (2)

$$W_{hd} = 0.152 \times 0.0398 \times 40 \times 60 = 14.5.$$

The eddy losses due to the tooth pulsation are as follows from (4)

$$W_{ed} = 1.96 \times 10^{-9} (40 \times 0.017 \times 60 \times 20 \times 17)^2 = 0.38. \text{ Therefore } W \text{ (total)} = 12 + 14.5 + 0.38 = 26.9 \text{ watts per kilogram.}$$

#### DISCUSSION OF RESULTS

The displaced hysteresis loops as determined by the method used do not altogether represent the conditions which exist in machines due primarily to the fact that these loops were obtained independently by starting each time from the tip of the major loop and going directly to the lower tip of the minor loops. In machines the minor loops follow one another with increasing or decreasing amplitude with a considerable number of minor loops superimposed on each major loop. It might be thought that this would have a demagnetizing effect tending to decrease the area of the loops. As a matter of fact there is such a tendency but it is small. Fig. 7 shows a fourth loop compared to a first loop, the former having a considerably smaller area than the latter. This is due chiefly to the fact that the fourth loop under the conditions of test has a smaller  $B$  amplitude although the  $H$  amplitude is the same. When the area of the fourth loop was multiplied by the 1.6 power of the difference in  $B$  amplitudes, the result nearly equals the areas of the No. 1 loop. In machines the conditions would correspond more nearly to definite  $B$  limits than definite  $H$  limits. In the A. I. E. E. paper referred to above, Fig. 24, page 2689, gives results on a first and tenth displaced loop taken between definite  $B$  limits of 7 and 9 kilogausses. The area of the first loop is 2.58 and of the tenth 2.32. We believe therefore that these different conditions will not introduce large errors. It is possible also that the minor loops may have an appreciable effect on the area of the major loops. If such an effect exists, it would be to decrease the losses due to the fundamental frequency.

It will be noted from Table IV and from Figs. 1 2 and 3 that the area of the displaced loops in general increases not only with the  $B$  displacement but with the  $H$  displacement also. In other words the No. 4 loop is usually larger than the No. 3 loop and the No. 5 loop larger than the No. 2. It will be noted that the percentage curves Figs. 4, 5 and 6 are nearly alike although the three samples are quite different in gage, chemical composition and losses. Therefore no large errors would probably be introduced by using these curves or the  $K$  curves of Fig. 8 for any class of electrical sheet having a silicon contents of not much over 2 per cent.

In order to calculate losses at high inductions, it is probably not feasible to obtain loops accurately at much higher values ballistically. We believe, however, that the percentage losses will not increase very much beyond those given for  $B_m = 17$ . Let us refer to Fig. 3 showing the data for  $B_m = 17$ . At higher values of  $B$  the ascending and descending branches apparently nearly coincide. It is possible that a displaced loop at very high  $B$  would have little or no area. This should certainly be true if the loop limits were entirely above the saturation point. At very high  $B_m$  values then the percentage curves might even decrease. There seems to be a tendency in this direction, at least for the  $B$  and  $C$  samples. We believe therefore that no serious errors would be introduced by using the  $K$  curves for  $B = 17$  kilogausses (98.3 lines per square inch) for all higher inductions.

For other kinds of material the only other factor necessary to consider for approximate results is the specific normal hysteresis loss per cycle at the given  $B_m$  or  $C$  of formula (2).

Of course, when the designer comes to apply these results to a specific problem there will be a number of uncertain factors such as amplitude, percentage of tooth pulsation at various parts of the tooth, flux density of the teeth, etc. Moreover, these results have been put into usable form only for a sine wave field form and would have to be recalculated in a different way for other field forms. In addition to tooth pulsation losses, it is believed that the results may be used for the calculation of pole face losses as well by proper manipulation.

Finally these data should be applicable where the whole flux pulsates due to high-frequency reluctance variations in the air gap.

#### FUTURE WORK

In order to apply these data, a more accurate knowledge of the magnitude, distribution and depth of penetration of these tooth pulsation seems essential. An estimate of the magnitude of these pulsations is made very difficult due to the damping action of the eddy and circulating currents in the iron and copper.

In conclusion I wish to acknowledge the assistance of Mr. G. H. Keulegan who did most of the experimental work.