

## THE THIRD LAW OF MOTION.

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"To every action, there is an equal and opposite reaction." This short statement looks very unpretentious yet it is so subtle and full of meaning. As a student in high school and university, the writer failed to get a realizing sense of the meaning of this law. As a teacher first thrusting himself on the unsuspecting pupils, he always felt a heavy load lifted when this point in the book was passed.

To the illustrations that are usually given to exemplify this law, the average student readily assents. He can easily see that if one pushes against the wall with a certain force, the wall must push back with a reaction that is equal and opposite. It is perfectly evident to him that if one stands on the floor and thereby exerts a force on the floor, the floor must necessarily exert an equal and opposite reaction. Most students will agree that both boys pulling on opposite ends of a rope must pull with equal force. "Action is equal to reaction" runs glibly off the tongue. The texts give numerous examples of the above character with the result that both teacher and student congratulate one another on their appreciation and understanding of this fundamental proposition.

On the other hand, illustrative examples of the third law applied to problems in kinetics are not so numerous. In no case, where such examples are given, are the discussion and explanation lucid, accurate or satisfactory. The problem of a horse pulling a wagon is a typical problem of this type. When the horse pulls the wagon, then the wagon must pull back with an equal force. If the wagon pulls back equally, then one asks how is it possible for the wagon to move?

As a member of a group of a dozen or more graduate students, the third law applied to such problems frequently came up for discussion. It was interesting to note that no one of the group ever talked with thorough understanding or convincing argument. Rather it appeared that all were trying to get light on the subject. Several teachers of physics in good positions have been questioned on the above horse and wagon problem with the result that the person sometimes admitted that he didn't see how the third law could be true. One teacher said that he had told the pupils that no doubt the horse pulled ahead just a little bit more than the wagon pulled back.

No attempt to elaborate on the third law could be made without incorporating a discussion of the first two laws. Apply the first law to the wagon. The wagon is at rest, it tends to stay at rest, it is "lazy" and does not want to move. Indeed it is more than "lazy," it actually opposes any force that tries to move it, it "strikes back." If you do compel it to move, you therefore bring out this reaction. It resists, it pulls back since according to the first law it tends to stay at rest. In other words, it reacts because some force has made it move. However, once started, it likewise reacts against being given further acceleration, either negative or positive. If then, to repeat, it persists in a state of rest or uniform motion in a straight line, it will necessarily react with a push or pull when made to change that state.

The second law tells us how to measure this "striking back," this reaction mentioned or rather implied in the first law. If a body is made to move with an acceleration of  $\frac{dv}{dt}$ , the second law states that the body reacts against being given this acceleration by an amount proportional to its mass and its acceleration. As is usually stated, the force brought out by the fact that the body is made to change its velocity is proportional to time rate of change in momentum.

$$f \propto m \frac{dv}{dt}$$

If we choose our units properly we have

$$f = m \frac{dv}{dt} = ma$$

If, according to the first two laws, the body reacts with a force of  $ma$ , while it is moving, the third law completes the argument by saying that the "outside" force mentioned in the first two laws is equal to the one that the body exerts because it is moving. To reverse the order; the outside force acts and if motion results, immediately the reactive force comes into existence and is equal to the applied force. When we write  $f = ma$ , we think of  $f$  as the applied force and with no misgivings write it equal to  $ma$ , the reactive force.

By the very manner in which the unit of force, the dyne, is defined, we tacitly write down the third law. The force which is applied gives to a certain mass a certain acceleration. Necessarily then, the mass reacts with a certain force and we are

enabled to define the applied force by its being equal to the reactive force. Every time we write  $f = ma$ , we thereby say that the wagon pulls back just as hard as the horse pulls ahead.

When a horse pulls a wagon, the wagon pulls back as hard as the horse pulls ahead. If this is true how can the wagon move? The answer is, that it is not a question of how the wagon can move. The burden of proof is placed on the wrong phase of the problem. The wagon pulls back because it is moving. If it were not moving, that is of course with an acceleration, it would not pull back.

Suppose a man takes hold of an object which weighs 10 lbs. This body has the appearance of an object much heavier, so the man thinks he must lift considerably more than this, say 50 lbs. He exerts a force of 50 lbs. and what is the result? There is to be an equal reaction. The only way the 10-lb. object can resist with a force of 50 lbs. is to move. By its being given an accelerated motion, it resists to the amount of 40 lbs. and its own weight makes up the equal reaction of 50 lbs. A 50-lb. object can resist with a force of 50 lbs. Likewise a 10-lb. object can resist with a force of 50 lbs. too. The first object remains at rest, while the second must move. In the first case, the 50-lb. reaction comes from the pull of the earth, in the second case, the 50-lb. reaction exists because the 10-lb. mass moves with accelerated motion.

To the student of electricity, Kirchoff's second law really involves the same question as brought out by the third law of motion. Kirchoff's Law states that in any closed circuit the sum of the voltage is equal to zero.

Ohm's Law states that  $E = RI$ . Here we have Kirchoff's Law in its simplest form.  $E - RI = 0$ . The impressed voltage,  $E$ , is opposed by an equal voltage,  $RI$ . Here then are two equal and opposite voltages. One asks, how is it possible for any current to pass when the above is true? Here again the answer is that by the very fact that a current flows, the counter voltage is immediately produced.

Consider a circuit having resistance and self inductance. In this case we write

$$E = RI + L \frac{di}{dt}$$

At every instant of time while the current is increasing in value, that is all the time that there is a  $\frac{di}{dt}$ , the applied voltage

is opposed equally by the sum of two others. One asks, how can any current flow at all; further, how can the current actually increase when the above is true? The answer is similar to the case in mechanics; by the very fact that the current is increasing or decreasing either, the counter voltage of self induction is brought into existence. This counter voltage due to inductance in a circuit, while the current is increasing, is analogous to the counter force an object exerts when the velocity changes.

We have a similar case in the operation of the electric motor. The applied voltage is opposed equally in this case by the sum of the so-called armature drop and the back voltage of the motor itself. This back voltage is made to exist by the very fact that a current passes through the armature coils. Hence the concept given in Kirchoff's Law is readily explained in this case.

When a permanent magnet is thrust into a close circuited coil, it is a well known phenomena that a current flows in this coil. Lenz's Law tells us that the current flows in such a direction as to oppose the action producing it. Assume that a magnet is being thrust into a coil with uniform velocity and a constant current is therefore flowing and a uniform field of intensity,  $H$ , exists within the coil. The motion of the magnet whose pole strength is  $m$  will necessarily be opposed by a force,  $Hm$ . Disregarding the purely mechanical forces involved which have been considered before, the opposing force  $Hm$  is exactly equal to the force which is exerted by the experimenter. If the moving magnet is repelled with a force exactly equal and opposite to that which the experimenter exerts, how is it possible that the magnet can be inserted at all? Here it is more easily understood than in the other cases, the force  $Hm$  exists because of the fact that the magnet is moving. Let the experimenter push harder and as a result the magnet moves faster; immediately a greater current is produced, a stronger field results and a force  $Hm$  opposes him equally and opposite as before.

The idea discussed in the previous pages also finds application in other phases of physics, including the transformer, differential equations and other more advanced branches of physical science.

If anyone will get as much straightened out in their understanding of the third law by reading this paper as the writer did in preparing it, then its preparation will have been worth while.