



212. The Exponential Function

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Source: *The Mathematical Gazette*, Vol. 3, No. 60 (Dec., 1906), pp. 403-404

Published by: The Mathematical Association

Stable URL: <http://www.jstor.org/stable/3602516>

Accessed: 27-06-2016 02:38 UTC

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REVIEW.

Text-Book of Light. 4th Edition. Univ. Tut. Press.

This book has been thoroughly revised and very considerably enlarged. The experimental side of the subject receives attention, and the clear practical directions for performing experiments which are given greatly enhance the value of the work. A chapter is devoted to the experimental determination of the constants of Lenses, etc. Dispersion is fully treated in a simple manner; the chapter on it includes a good discussion of ordinary spectra, some remarks on Absorption Spectra, Colour, Rainbows, etc., and forms a notable section in the book. The chapter on Velocity of Light is improved by an account of the Corpuscular and Wave Theories. The number of problems given has been increased; and the student who works through them will have ample exercise in this important branch of optics. The get-up of the work is in the main highly satisfactory; the figures are well and clearly executed. Here and there one finds an overcrowded page; this however detracts but little from the general excellence of the volume. The book, as is stated in the preface, is a very full treatment of those parts of geometrical optics which do not require advanced Mathematics, and it is, indeed, difficult to imagine a better.

T. R. DINGAD DAVIES.

MATHEMATICAL NOTES.

212. [D. 6. b.] *The Exponential Function.*

How to best introduce the learner to the exponential function is a difficulty which, it seems to me, needs to be discussed.

I take it that hitherto the first appearance of this function has been in Algebra. After the binomial theorem for positive integral indices, came a short discussion of convergence, and then, in rapid succession, the general binomial series, the exponential series, and the logarithmic series.

In this order the extension of the binomial theorem seems natural to the learner, and as the exponential theorem leads rapidly to that by which logarithms can be calculated, its introduction probably appears natural also.

In future, however, we are to cater for army candidates and the would-be engineer, as well as for those about to read for the mathematical tripos: one result of this is that the exponential function will first appear in the differential calculus, and will form the most serious difficulty in the earlier part of that subject.

In attacking this difficulty it seems to me that, first, the importance of a function y such that $\frac{dy}{dx} = ky$ should be impressed on the learner by examples such as: increase of population; compound interest; diminution of light in passing through several consecutive panes of glass; increase of resistance when a rope is coiled round a post; cooling of a hot body; etc.

After this several methods may be adopted.

(a) Start fresh, and try to differentiate $y = a^x$;

here $dy = a^x(a^x - 1)$.

If we assume $a^x - 1$ to be a multiple of dx , say $A dx$, then A will be a function of a ; let e be the value of a that makes $A = 1$; the e is defined by $d \cdot e^x = e^x \cdot dx$. If now we assume that e^x can be expressed as a series of powers of x , we deduce $e^x = 1 + x + \frac{x^2}{2} + \dots \equiv \exp(x)$, say.

Here by a series of assumptions we have proved, or rather avoided the necessity of proof of the 'exponential theorem' that $\{\exp 1\}^x = \exp x$.

(β) Again, we may lay stress on compound interest when the interest-period is small, and consider $L \left(1 + \frac{x}{n}\right)^n$.

By using the binomial theorem, and by burking the question of approaching the limit except near the beginning of the series, we can easily obtain a sort of proof that $\lim_{n \rightarrow \infty} \left(1 + \frac{x}{n}\right)^n = \exp x$, and $\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n = \exp 1$.

Here again we have avoided the proof of the exponential theorem.

(γ) On the other hand, it seems more straightforward to proceed directly from $\frac{dy}{dx} = ky$. If we assume for y a power series in terms of x , we obtain $y = A \exp(kx)$, where A is a constant; and in particular if $k=1$ and A be dropped, $y = \exp(x)$.

Unfortunately by this method we have not proved that $\exp x = (\exp 1)^x$.

Personally I should like to adopt (γ). Having seen the importance of the solution of $\frac{dy}{dx} = ky$, and having determined its form, $y = \exp x$, I should like to discuss the multiplication $\exp x_1 \cdot \exp x_2$, and prove this product to equal $\exp(x_1 + x_2)$, in fact to prove the exponential theorem in the most satisfactory way.

To give a rigorous proof by this method is certainly easier than to make (β) rigorous, but it would be interesting to collect opinions on the merits of (α), and to see whether a satisfactory proof of the exponential theorem can be given by that method.

C. O. TUCKEY.

213. [K. 2. c.] *The nine-point circle.*

It is well known that if O is the orthocentre of a $\triangle ABC$, then A is the orthocentre of $\triangle OBC$; and so on.

The symmetry that thus exists between the four points, leads to many interesting properties, especially in connection with the nine-point circle. In stating these properties, however, the symmetry is not often mentioned, so that the following account of well-known theorems may perhaps interest some teachers.

Let O be the orthocentre of $\triangle ABC$, then $OABC$ may be regarded as a tetrahedron in one plane, with 6 edges.

The 4 points form 3 \triangle s with a common pedal $\triangle XYZ$ (X on BC , etc.).

The 6 mid-points of the edges (D, E, F for BC , etc., P, Q, R for AO , etc.), form 3 rectangles (such as $PQDE$) whose sides are parallel to and half of the edges.

Each pair of rectangles has a common diagonal, and the 3 diagonals of the rectangles meet at a point (N say) which is the centre of a \odot round DEF , PQR and passing through XYZ (since $\angle DXP$ is right, etc.).

This circle has radius half the circum-radius for either ABC , OBC , etc. since in each case it is the circum-circle of a similar \triangle of half the linear dimensions.

The point N is the c.g. of equal particles at A, O, B, C (which shows that PD, QE, RF meet in a point) and therefore divides in ratio 3 to 1 the joins of O to c.g. of $\triangle ABC$, A to c.g. of $\triangle OBC$, etc.

The point N also bisects the join of O to the circum-centre of $\triangle ABC$ (being centre of \odot through DX, FZ) and the join of A to circum-centre of OBC , etc.

C. O. TUCKEY.

214. [V.] *An illustration by dissection.*

The following is an illustration by dissection of the theorem: "If a parallelogram $ABCD$ and a triangle ABE be on the same base AB and between the same parallels AB, DCE , the parallelogram shall be double of the triangle," for the case when the apex of the triangle falls outside the parallelogram and CE is not greater than AB .