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THE INVERTING PRISM

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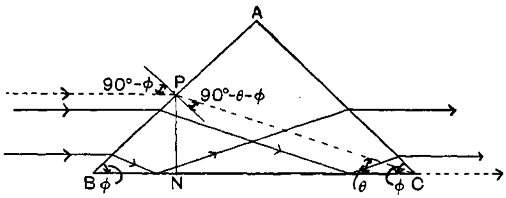
In many instances a prism is used in an instrument to invert an image by a single reflection without alteration of the direction of the light. A very common form of this is the ordinary prism, with angles 45°, 45°, and 90° which is used so that light enters parallel to the hypotenuse face and after refraction at one of the short sides is reflected in the hypotenuse and subsequently emerges parallel to its original direction. It is quite clear that exactly the same effect could have been obtained with any prism in which the sides AB and AC are equal and that there is no necessity to limit the shape to one in which the angles at B and C are 45°.

The extreme ray which can get through the prism in this way is shown in pecked lines and what we may call the numerical aperture of the prism (i.e. the



ratio $BC : PN$) varies with the base angles and the refractive index of the glass. As in certain circumstances as large an aperture as is possible may be required, the following note discusses the relationship of base angle with refractive index in order to give maximum aperture. If the base angles B and C are each ϕ and the angle PCB is θ the refraction equation becomes

$$\cos\phi = n\cos(\phi + \theta).$$

We also obtain $BC/PN = \cot\phi + \cot\theta$.

Hence, to fulfil the required conditions $\cot\phi + \cot\theta$ must be a minimum with the condition

$$\cot\phi\,\mathrm{cosec}\,\theta = n(\cot\phi\cot\theta - 1).$$

Writing $\cos\theta = x$, we have

$$\cot\phi = n\sqrt{1 - x^2}/(nx - 1),$$

and

$$\frac{n\sqrt{1-x^2}}{nx-1} + \frac{x}{\sqrt{1-x^2}}$$

is then to be a minimum.

This leads to the following equation for the determination of x

$$x^3 - 2nx^2 + x + n - 1/n = 0.$$

This is a cubic equation which gives as solutions

(i) a value between $-1/n$ and $-1/2n$,
(ii) ,, ,, $1/n$ and 1,
(iii) ,, ,, n and $2n$.

It is clear that the only possible practical value is the second root for which $x = (7n + 6)/(10n + 3)$ is closely approximate.

The values of θ and ϕ for the values of $n = 1{\cdot}5$, $1{\cdot}55$, $1{\cdot}60$, $1{\cdot}65$, $1{\cdot}70$ are tabulated below:

n	θ	ϕ	Aperture ratio base/height	Aperture ratio with 45° prism	Aperture ratio with 30° prism
1·50	23° 36′	31° 57′	3·89	4·29	3·90
1·55	24 25	32 43	3·76	4·11	3·78
1·60	25 9½	33 23	3·65	3·94	3·68
1·65	25 51	33 59	3·55	3·81	3·59
1·70	26 29	34 32	3·46	3·69	3·51

These tabulated values show that no very large improvement is possible but that a prism of angles 30°, 30°, 120° will, in all cases, be better than one of the ordinary 45° pattern.