



Review

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where Δ is the area of $ABCD$. This leads to an algebraic expression of every magnitude in the spherical triangle in terms of the homogeneous parameters a, b, c, d .

Now there are three different shapes of cyclic quadrilaterals with these given sides, and three different diagonals e, f, g within the circle, where $ef=ac+bd$, $fg=ab+cd$, $ge=ad+bc$. By forming the ratios $\frac{a \pm b}{e}$, $\frac{a+e}{b}$ and similar ones for all combinations of a, b, c, e, f, g we obtain Delambre's analogies. Similarly $\frac{e-a}{e+a}$ is the type which leads to Napier's, and $\frac{e-c+d}{e+c-d}$ to L'Huilier's formulae. It turns out that these well-known analogies are not a complete set, but by taking all the forms of these ratios they can be made complete.

This treatment makes the book very instructive, and throws much light upon the whole subject.

Theorie der Zahlenreihen und der Reihengleichungen. By Prof. Dr. ANDREAS VOIGT. Pp. 133. Leipzig, 1911.

This book is occupied with the algebra of number-rows which naturally arise in the theory of Finite Differences applied to the summation of Arithmetical Series. Starting with an arbitrary number-row we can deduce any number of such rows from the given one by a given law, an example of which is the scheme of Figurative Numbers in which the r^{th} term of the n^{th} row equals the sum of the first r terms of the $n-1^{\text{th}}$ row, the first row being 1, 0, 0, 0, ... Looking upon such numbers as functions in which n is the independent variable, and confining the investigation to two classes of number-rows whose terms are really binomial coefficients, a remarkable theory can grow up with important bearings on the summation of series and in particular $\sum_{r=1}^n r^k$, and on the integral solution of indeterminate equations. How this is effected cannot be shortly explained, though the germ of the idea lies in the following:

If N is a positive integer, it has a unique expression in the scale of x ;

$$N = a'x^r + b'x^{r-1} + \dots + g'x^2 + h'x + k', \text{ say.}$$

This may be transformed into the unique series

$$a \binom{x}{r} + b \binom{x}{r-1} + \dots + g \binom{x}{2} + h \binom{x}{1} + k,$$

where $\binom{x}{r} \equiv \frac{x!}{r!(x-r)!}$. Thus N has become a linear function of $r+1$ consecutive terms of a number-row, without confining the variable x to any special value.

Perhaps the reason for considering such rows may be most easily illustrated in this way: in the theory of rational numbers the number is rarely looked upon as a single object. Thus to say that a is a multiple of b is really to say that a is a member of the row bx , ($x=0, 1, 2, \dots$).

The author has dealt with the subject very fully, and has made the theory of practical value by providing numerical tables to facilitate the actual solution of equations by this method.

Analytical Geometry. By N. C. RIGGS, M.S., Assistant Professor of Mathematics, Carnegie Technical Schools, Pittsburgh, Pa. Pp. 294. (The Macmillan Company. 1910.)

This book has been written with a view to bring out clearly the fundamental principles and methods of the subject, and to make it a natural introduction to more advanced work. If analytical geometry means simply the analytical discussion of the geometry of a single curve considered as the locus of a moving point depending on one parameter x or t , then the book has served its purpose well. For there is a splendid introduction explaining the use of linear measurements to represent number, and the ideas involved in projection, coordinates and a geometrical locus. This is followed by a practical treatment of the simpler equations which represent curves, algebraic and transcendental, all with a view to its application to the calculus. Prominence is given to the parametric representation of the cartesian coordinates of a point on a curve, which is always instructive and forms a good foundation for more advanced work. Then the

book concludes with an introduction to the calculus and a short account of three dimensional coordinate geometry.

If, however, one takes a broader view of the meaning of analytical geometry the book fails signally in being an introduction to more advanced work. The subject always seems to begin with the meaning of the equation $\alpha - \lambda\beta = 0$, where α and β are short for linear functions of x and y . There are plenty of simple uses to which this idea leads, so that a book need not omit it because of its difficulty. At the same time it helps to form the idea that points are not the only geometrical elements which can be specified by coordinates. The same might be said of the analytical representation of coaxial circles as $S - \lambda S' = 0$. Now this book omits all mention of such systems of equations. It would therefore not be so suggestive an introduction to a beginner with a liking for geometry as some of the older books on the subject.

H. W. TURNBULL.

The Teaching of Arithmetic. By D. E. SMITH, LL. D., Professor of Mathematics in Teachers College, Columbia University. Pp. 120. 75 cents. 1910. (Teachers College, Columbia University.)

Arithmetical Abilities and some Factors determining them. By C. W. STONE, Ph.D. Pp. 101. \$1.00. 1909. (Teachers College, Columbia University.)

If modern American text-books in arithmetic offer any criterion of the teaching of the subject in the States, it would appear that America lags behind this country, and indeed most European countries. For the most part they are characterised by a drab monotony of treatment, by a lack of vitality, and by an entire absence of any correlation with other branches of a mathematical education. Professor Smith's book affords ample confirmation of these views, and yet he leads among the reformers in his country. In his general remarks upon the teaching of the subject he is sound, and what he has to say is worthy of the attention of all who are grappling with this subject. He sees the defects in American teaching, he diagnoses the weakness, but, in our opinion, he fails to find proper remedies. A complete treatise would be needed to treat the question adequately, and in the short space of a review it is not possible to do more than to indicate briefly what we consider to be two or three defects in Prof. Smith's treatment of the subject. In the first place, we find no suggestion of anything in the way of practical work, a prominent feature in modern arithmetical teaching in this country. Tables of measurement, the fundamental operations, the use of decimals, elementary mensuration, and kindred matters, seem to be dealt with entirely in an abstract fashion from the blackboard. In the second place, there seems to be no suggestion of the desirability, by means of the practical work and otherwise, of linking up the arithmetic with geometry, algebra, mensuration, or physics. Thirdly, we believe that the monotonous character of the examples must fail to provide that stimulus which is necessary for intelligent work in the subject. Prof. Smith asks for the rejection of the academic type of example which America has inherited from Italy *via* England, and suggests more practical examples. If they are to be of the types which he gives as specimens, we doubt whether there will be much improvement, for his "practical" examples seem to be confined largely to trade and agriculture. We cannot imagine any English boy being stimulated by such examples as: "Silver is sold by the troy ounce. This is what per cent. of the avoirdupois ounce (7000 gr. = 16 av. oz., 5760 gr. = 12 troy oz.)." Or "When Iowa's annual product amounted to 305,800,000 bu., this was how many times the 440,000 bu. produced by Maine?" Perhaps the American boy thrives better than the English on such fare. It seems as though the American teacher has yet to learn that the phenomena of our daily existence, our simplest acts and experiences, present a rich field for those seeking for quantitative problems which will enliven the arithmetic lesson, quicken the intelligence of the pupil, and lead him to feel that his school arithmetic is intimately associated with the world in which he lives and acts.

Dr. Stone's book affords evidence that American educationists are addressing themselves seriously to the scientific study of pedagogy. It is an account of a series of experiments undertaken in a number of American primary schools with a view to an examination of the factors determining arithmetical ability. The method of conducting the investigation is interesting, and similar work might