Let ρ be the radius of curvature of the parabola in which the particle commences to move at P, and let a be the radius of the circle.

$$\therefore \frac{mv^2}{a}=mg\cos\theta,$$

$$\frac{mv^2}{\rho}=mg\cos\theta;$$

$$\therefore \rho=a.$$

$\therefore$ the circle is the circle of curvature to the parabola at P.

Hence if the particle meets the circle again at Q, PQ is equally inclined to the vertical with PT the tangent at P. P. SCOONES.

233. [D. 6. b.] The following modification of Stolz and Gmeiner's method (which Mr. Hardy numbers (iv)) was suggested by Mr. M'Cleland, one of my pupils, and appears to me simple and satisfactory.

$$\begin{array}{ll}\text{Let} & \left.\begin{array}{l}\exp(ix)=u+iv,\\ \exp(-ix)=u-iv,\end{array}\right\} x \text{ being real.}\\ \text{then} & \end{array}$$

Then $$u^2+v^2=\exp(ix).\exp(-ix)=1.$$

$\therefore$ $\exp ix$ has modulus unity, *i.e.* is represented by a point on the circle radius unity, centre origin;

$$\therefore \exp(ix)=\cos\phi+i\sin\phi, \text{ where } \phi \text{ is a real angle.}$$

Differentiating, $\exp(ix).idx=(-\sin\phi+i\cos\phi)d\phi$;

$$\therefore \frac{idx}{d\phi}=\frac{-\sin\phi+i\cos\phi}{\cos\phi+i\sin\phi}=i; \quad \therefore \frac{dx}{d\phi}=1;$$

$$\therefore x=\phi+\text{constant}=\phi+c, \text{ say}$$

(thus to each value of ϕ, a value of x corresponds, as well as conversely);

$$\therefore \cos\phi+i\sin\phi=\exp(i\phi+ic) \text{ for all real values of } \phi;$$

$$\therefore \cos(-c)+i\sin(-c)=\exp(0)=1;$$

$$\therefore c=2k\pi \text{ where } k \text{ is an integer};$$

$$\therefore \cos\phi+i\sin\phi=\exp i(\phi+2k\pi).$$

This proof has the advantage of avoiding the initial assumption in Mr. Godfrey's way of writing the proof (viz. that y can be found so that $\cos x+i\sin x=\exp y$). C. O. TUCKEY.

234. [L[1]. 17. e.] *On a Certain Double Envelope.*

How often in Conics must poor despised Geometry come to the rescue of Analysis—to test, interpret, and illumine results of themselves stale, flat, and unprofitable!

A striking illustration of the superiority of geometrical treatment is to be found in the consideration of the following attractive question due to Mr. C. E. Youngman, of Cheltenham College:

"Prove that similar ellipses having a common diameter envelope a pair of circles." [*Educational Times*, October 1906, No. 16087.]

(I.) The analyst sets to work somewhat in this fashion:

Assume $x^2-d^2+2\lambda xy+\beta y^2=0$ as the equation to the ellipse, where λ, β are variable with the condition $\sqrt{(\lambda^2-\beta)}/(1+\beta)=\text{const.}$ Though never in any actual difficulty he is certainly completely in the dark from the outset until the end. Finally perseverance is rewarded by some such result for the envelope as $(x^2+y^2-d^2)^2=\kappa^2y^2$. Then with a sigh of relief he feels he has achieved great things; and that the question, as far as he is concerned, is completely disposed of.