The Public Schools and the Question

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equal to four right angles by taking three chairs and walking once round them.]

Professor Lodge, resuming, said a reduction in the number of propositions could be made if a proposition and its converse were lumped into one, and a further simplification would be made if constructions were put in a different part of the book from theorems.

With regard to incommensurables, he considered they were not fitted for elementary subjects nor for practical work.

Rapidity of computation became interesting to boys if they were taught to emulate each other at the proper stage. He wished to call their attention to the fact that the Board of Education would issue, free, as many tables of four-figure logarithms as they liked.

He hoped the committee which was being formed would be able to make valuable suggestions to the B.A. Committee. They would be glad to receive suggestions from any members of the Association; and if these were sent to Mr. Pendlebury, no doubt he would see that they came before the committee.

THE PUBLIC SCHOOLS AND THE QUESTION.

To the Committee appointed by the British Association to Report upon the Teaching of Elementary Mathematics.

Gentlemen,—At the invitation of one of your own body, we venture to address to you some remarks on the problems with which you are dealing, from the point of view of teachers in public schools.

As regards geometry, we are of opinion that the most practical direction for reform is towards a wide extension of accurate drawing and measuring in the geometry lesson. This work is found to be easy and to interest boys; while many teachers believe that it leads to a logical habit of mind more gently and naturally than does the sudden introduction of a rigid deductive system.

It is clear that room must be found for this work by some unloading elsewhere. It may be felt convenient to retain Euclid; but perhaps the amount to be memorised might be curtailed by omitting all propositions except such as may serve for landmarks. We can well dispense with many propositions in the first book. The second book, or whatever part of it we may think essential, should be postponed till it is needed for III. 35. The third book is easy and interesting; but Euclid proves several propositions whose truth is obvious to all but the most stupid and the most intellectual. These propositions should be passed over. The fourth book is a collection of pleasant problems for geometrical drawing; and, in many cases, the proofs are tedious and uninstructive. No one teaches Book V. A serious question to be settled is—how are we to introduce proportion? Euclid's treatment is perhaps perfect. But it is clear that a simple arithmetical or algebraical explanation covers everything but the case of incommensurables. Now this case of incommensurables, though in truth the general case, is tacitly passed over in every other field of elementary work. Much of the theory of similar figures is clear to intuition. The subject provides a multitude of easy exercises in arithmetic and geometrical drawing; we run the risk of making it difficult of access by guarding the approaches with this formidable theory of proportion. We wish to suggest that Euclid's theory of proportion is properly part of higher mathematics, and that it shall not in future form part of a course of elementary geometry. To sum up our position with regard to the teaching of geometry, we are of opinion:

1. That the subject should be made arithmetical and practical by the constant use of instruments for drawing and measuring.
2. That a substantial course of such experimental work should precede any attack upon Euclid's text.
3. That a considerable number of Euclid's propositions should be omitted; and in particular
4. That the second book ought to be treated slightly, and postponed till III. 35 is reached.
5. That Euclid's treatment of proportion is unsuitable for elementary work.

Arithmetic might well be simplified by the abolition of a good many rules which are given in text-books. Elaborate exercises in vulgar fractions are dull and of doubtful utility; the same amount of time given to the use of decimals would be better spent. The contracted methods of multiplying and dividing with decimals are probably taught in most schools; when these rules are understood, there is little left to do but to apply them. Four-figure logarithms should be explained and used as soon as possible; a surprising amount of practice is needed before the pupil uses tables with confidence.

It is generally admitted that we have a duty to perform towards the metric system; this is best discharged by providing all boys with a centimetre scale, and giving them exercise in verifying geometrical propositions by measurement. Perhaps we may look forward to a time when an elementary mathematical course will include at least a term's work of such easy experiments in weighing and measuring as are now carried on in many schools under the name of Physics.

Probably it is right to teach square root as an arithmetical rule. It is unsatisfactory to deal with surds unless they can be evaluated, and the process of working out a square root to five places provides a telling introduction to a discourse on incommensurables; furthermore it is very convenient to be able to assume a knowledge of square root in teaching graphs. The same rule is needed in dealing with mean proportionals in geometry.

Cube root is harder and should be postponed until it can be studied as a particular case of Horner's method of solving equations approximately.

Passing to algebra, we find that a teacher's chief difficulty is the tendency of his pupils to use their symbols in a mechanical and unintelligent way. A boy may be able to solve equations with great readiness without having even a remote idea of the connection between the number he obtains and the equation he started from. And throughout his work he is inclined to regard algebra as a very arbitrary affair, involving the application of a number of fanciful rules to the letters of the alphabet.

If this diagnosis is accepted, we shall be led naturally to certain conclusions. It will follow that elementary work in algebra should be made to a great extent arithmetical. The pupil should be brought back continually to numerical illustrations of his work. The evaluations of complicated expressions in a, b, and c may of course become wearisome; a better way of giving this very necessary practice is by the tracing of easy graphs. Such an exercise as plotting the graph $y=2x-\frac{x^2}{4}$, provides a series of useful arithmetical examples, which have the advantage of being connected together in an interesting way. Subsequently, curve-tracing gives a valuable interpretation of the solutions of equations. Experience shows that this work is found to be easy and attractive.

With the desire of concentrating the attention of the pupil on the meaning rather than the form of his algebraical work, we shall be led to postpone certain branches of the subject to a somewhat later stage than is usual at present. Long division, the rule for H.C.F., literal equations, and the like, will be studied at a period when the meaning of algebra has been sufficiently inculcated by arithmetical work. Then, and not till then, will be the time to attend to questions of algebraic form.

But at no early stage can we afford to forget the danger of relapse into mechanical work. For this reason it is much to be wished that examining bodies would agree to lay less stress upon facility of manipulation in algebra. Such facility can generally be attained by practice, but probably at the price of diminished interest and injurious economy of thought. The educational value of the subject is sacrificed to the perfecting of an instrument which in most cases is not destined for use.

To come to particulars, we think that undue weight is often given to such subjects as algebraic fractions and factors. The only types of factors which crop up continually are those of x^2-a^2, $x^2 \pm 2ax+a^2$, and, generally, the quadratic function of x with numerical coefficients.

In most elementary algebra books there is a chapter on Theory of Quadratic Equations, in which a good deal of attention is paid to symmetric functions of roots of quadratics. No further use is to be made of this till the analytical theory of conics is being studied. Might not the theory of quadratics be deferred till it can be dealt with in connection with that of equations of higher degree?

Indices may be treated very slightly. The interpretation of negative and fractional indices must of course precede any attempt to produce logarithms; but when the extension of meaning is grasped, it is not necessary to spend much more time on the subject of indices; we may push on at once to the use of tables.

It will be seen that our recommendations under the head of Algebra are corollaries of two or three simple guiding thoughts; the object in view being, —to discourage mechanical work; the means suggested,—to postpone the more abstract and formal topics and, broadly speaking, to arithmeticise the whole subject.

The omission of part of what is commonly taught will enable the pupil to study, concurrently with Euclid VI., a certain type of diluted trigonometry which is found to be within the power of every sensible boy. He will be told what is the meaning of sine, cosine, and tangent of an acute angle, and will be set to calculate these functions for a few angles by drawing and measurement. He will then be shown where to find the functions tabulated, and his subsequent work for that term will consist largely in the use of instruments, tables, and common-sense. A considerable choice of problems is available at once. He may solve right-angled triangles, work sums on "heights and distances," plot the graphs of functions of angles, and make some progress in the general solution of triangles by dividing the triangle into right-angled triangles. Only two trigonometrical identities should be introduced—$\sin^2\theta+\cos^2\theta=1$, and $\dfrac{\sin\theta}{\cos\theta}=\tan\theta$. In short, the work should be arithmetic, and not algebra.

Formal algebra cannot be postponed indefinitely; perhaps now will be the time to return to that neglected science. We might introduce here a revision course of algebra, bringing in literal equations, irrational equations, and simultaneous quadratics, illustrated by graphs, partial fractions, and binomial theorem for positive integral index. Side by side with this it ought to be possible to do some easy work in mechanics. Graphical statics may be made very simple; if it is taken up at this stage, it might be well to begin with an experimental verification of the parallelogram of forces, though some teachers prefer to follow the historical order and start from machines and parallel forces. Dynamics is rather more abstract; a first course ought probably to be confined to the dynamics of rectilinear motion.

It is not necessary to discuss any later developments. The plan we have advocated will have the advantage of bringing the pupil at a comparatively early stage within view of the elements of new subjects. Even if this is effected at the sacrifice of some deftness in handling a, b, and c, one may hope that the gain in interest will be a motive power of sufficient strength to carry the student over the drudgery at a later stage. Some drudgery is inevitable if he is ultimately to make any use of mathematics. But it must be borne in mind that this will not be required of the great majority of boys at a public school.—We beg to remain, Gentlemen, yours faithfully,

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REVIEWS AND NOTICES.

Choice and Chance, with 1000 Exercises. By Prebendary W. A. WHITWORTH. Fifth Edition, much enlarged. Pp. viii., 342. 6s. 1901. (Deighton, Bell.)

This interesting volume "contains some 45 pages more than its predecessor." "The most important addition in the body of the work is the very far-reaching theorem which enables us to write down at sight the value of such functions as a^3, $a^3\beta^4$, $a\beta\gamma$... when a, β, γ... are the parts into which a given magnitude is divided at random." The exercises at the end are increased to 1000. Prebendary Whitworth's work is too well known to require further comment.

Algebra for Junior Students. By TELFORD VARLEY. Pp. viii., 152. 1s. 1901. (Allman.)

This little book covers "all examinations intended for younger students." It is thoughtfully written and better than most of its class. It is a pity that detached coefficients are not taught as early as possible, and that the solution of quadratics by splitting into factors is not mentioned. When the author states in his preface that the contents may be mastered in two years, we despair of mathematical teaching in this country and heartily sympathise with Professor Perry. There is no reason why the contents should not be mastered in two terms at most.

Elementary Algebra. By ROBERT GRAHAM. Third Edition. Pp. viii., 312 (34). 6s. 1901. (Longmans, Green.)

This edition contains some 28 pages more than in the first edition. The best chapters in this book are those dealing with imaginary quantities, imaginary factors, and "more difficult equations." The author does not teach the use of detached coefficients, but he is "advanced" enough to use determinants in certain equations of three unknowns, though we cannot see why he did not introduce them to the student who is tackling equations with two unknowns. The examples are very carefully graduated.

Woolwich Mathematical Papers for Admission into the Royal Military Academy for the years 1891-1900. Edited by E. J. BROOKSMITH. 6s. 1901. (Macmillan.)

This volume will be found serviceable to Army Classes. It contains the questions in Pure and Applied Mathematics for the years mentioned, together with the answers. Mr. Brooksmith is an Instructor at the Royal Military Academy.