

A STUDY OF SOME OF THE ELASTIC PROPERTIES
OF A PLATINUM-IRIDIUM WIRE.

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I. INTRODUCTORY.

THE remarkable elastic properties of a certain platinum-iridium wire containing 40 per cent. of iridium were first announced by Guthe.¹ In his experiments the wire was used as the suspension of a torsion pendulum. Although the amplitudes of vibration were less than 50 degrees, a marked increase both in the period and in the logarithmic decrement accompanied the increase in amplitude. Cylinders of equal mass but of different moments of inertia were suspended from the wire, set in vibration, and timed, but as a result of these experiments no change was observed in the logarithmic decrement-amplitude curves. This absence of any effect led to the supposition that the damping was proportional to the amplitude, and independent of the velocity.

The study of the elastic properties of such wires was continued by Guthe and the writer² during the latter part of 1908 and the early part of 1909. In that work, wires containing different percentages of iridium were used. The apparatus was so constructed that much larger amplitudes could be obtained than were obtainable before. As a result of the use of the larger amplitudes, additional information concerning the relation of period and of logarithmic decrement to the amplitude was obtained. It was found that although both the period and the logarithmic decrement varied directly with the amplitude for small values of the amplitude, they did not continue to do so for larger values of the amplitude. The periods at these large amplitudes tended to become constant, and at the same time the logarithmic decrements reached a maximum,

¹K. E. Guthe, *Proc. Iowa Acad. Sci.*, 15, p. 147, 1908; *Abs. in Phys. Rev.*, 26, p. 201, 1908.

²K. E. Guthe and L. P. Sieg, *Phys. Rev.*, 30, p. 610, 1910.

and then diminished. The peculiarities in the elastic behavior of these wires were found to increase as the percentage of iridium was increased. It was found that when the wires were tested by a static method they followed Hooke's¹ law, and that the torsional moment calculated from this static experiment was larger than the largest torsional moment found by the kinetic method.

In the present paper, the observations have been confined to the 40 per cent. platinum-iridium wire used by Guthe in his first experiments. It developed during the preparations of the second paper² that if any further experiments with the wire were to have value, a method of getting a definite period-amplitude relation must be found. The search for the determination of such a method has been the principal problem attacked in the present work, although several other points of interest have been developed. The writer believes that the value of these experiments, aside from their own interest, will be to suggest problems connected with ordinary wires, and methods of attacking these problems.

II. APPARATUS AND METHOD OF TIMING.

The apparatus was the same as that described in the former paper,² with the complete circular scale which was used in the last set of experiments therein described. The amplitudes of the vibrations were read by focusing the image of a wire, illuminated by an arc light, on the circular scale graduated directly in double degrees. The diameter of the scale used was 149.2 cm., thus giving a space of 2.6 cm. to a degree, and so enabling one to read with accuracy to tenths and to estimate in the smaller amplitudes to one one-hundredth of a degree. The double mirror was the same one used in the previous experiments.² It was possible, by the use of the complete circle for a scale and the double mirror, to read all the amplitudes with the exception of a very few when the mirror was end on. This equipment was a decided improvement over that used in the preceding experiments, for in those some of the important amplitudes were obtained only by interpolation. In all cases where

¹Historically speaking, this is not Hooke's Law, but an extension of it discovered experimentally by Coulomb. Cf. Kelvin, *Phys. Papers*, Vol. III., p. 54.

²Guthe and Sieg, *loc. cit.*

the word "amplitude" is used, the word refers to what is ordinarily thought of as double the amplitude in angular harmonic motion, *i. e.*, the angular displacement from one extremity of the vibration to the other. In the few cases where the term is used in its strict sense, the phrase "amplitude from the center" is employed. The method of fastening the wire to the supports, and of attaching the various moments of inertia was the same as that described in the former paper. To the upper rod holding the wire was attached a torsion head, graduated into degrees, and having adjustable stops. This device made it possible to set the wire into vibration, and to hold it at practically constant amplitude. The method of timing was, with only one or two changes, essentially the same as that explained in the former paper. In all cases, unless otherwise stated, the periods are in reality half-periods. One important change, which in the light of some recently performed experiments proved of considerable value, was to tap the key at the end of the swing instead of at the middle. As this is contrary to usual procedure, it needs perhaps some justification. Briefly, if the amplitudes were measured at the ends of the swings and the periods were measured at the center, it would be necessary to plot the amplitudes and to obtain the amplitudes corresponding to the different periods by interpolation. While this refinement would not be necessary in an ordinary wire, it was in this case a decided necessity. However, the timing at the ends of the swings made it unnecessary to plot the amplitudes in order to get corrected values, for in each case tried the points observed fell on a smooth curve. Of course at the end of each experiment, when the amplitudes were down to about forty degrees, it was necessary to transfer observations to the center. yet as this resulted in a break of only a quarter period this error was of vanishingly small significance. From this point down the periods were taken in one of two ways: either by the method of coincidences or by the method of middle elongations. To average twenty readings on the tapes was found to give practically as smooth a curve as to average forty readings and the curve so obtained more closely agreed with the facts. To make perfectly clear how the periods were obtained, the method will be briefly described. A drum geared to a motor running at nearly constant speed wound

the tape from the roll, drawing it under a pen attached to the armature of an electromagnet. A Bond and Son's siderial, break-circuit chronometer in circuit with the electromagnet gave double-second marks on the tape. Connected with the recorder by a separate circuit was a tap key, which was closed, as explained above, at the ends of the vibrations. In this way there was recorded on the same tape the times at which these positions were reached and the double-seconds. When the amplitude had decreased to a value of from 15 to 20 degrees, the tape was abandoned, and the periods thereafter were obtained as explained above. Upon the tape record, the chronometer times of these turning points were recorded in hours, minutes, seconds, and hundredths of a second. Supposing that t is this recorded time for the fifth vibration, and that t' is the time for the twenty-fifth vibration, then $t' - t$ is the time elapsed between these two vibrations. This difference divided by 20 is used as the period of vibration of the mean swing; in this case, the fifteenth. In this way the errors arising from individual observations are diminished; and while this method is not an exact one, numerous tests have shown that it gives essentially correct values. A sample of one of the curves connecting period and vibration number given in Fig. 1. The figure shows how well the observed values fit a smooth curve, and how in most cases it was unnecessary to correct the period readings.

III. RELATION BETWEEN AMPLITUDE AND PERIOD.

In the former experiments it became quite evident that the variation of period of vibration with the amplitude was, except under very special circumstances, not a definite thing. It seemed highly necessary, therefore, to determine if possible some definite treatment of the wire which for any initial amplitude would lead to a fixed relation between the subsequent amplitudes and periods. The necessity of determining some definite method of treatment is shown by the widely different relations existing between these two quantities, when they are observed under different circumstances. In the former experiments, with the exception of a few observations taken at the end of the work, this peculiar variation of amplitude and period was overlooked on account of the fact that preceding

any set of observations the wire was nearly always annealed, and was always twisted through a large amplitude. The following experiments seem at first sight to indicate that the elastic behavior of the wire is of an extremely unconstant and variable nature, but this conclusion is contradicted by the more usual behavior of the wire under static conditions.

1. *Increasing Initial Amplitudes.*—In the first place a set of experiments was carried out, showing the variation of the period-ampli-

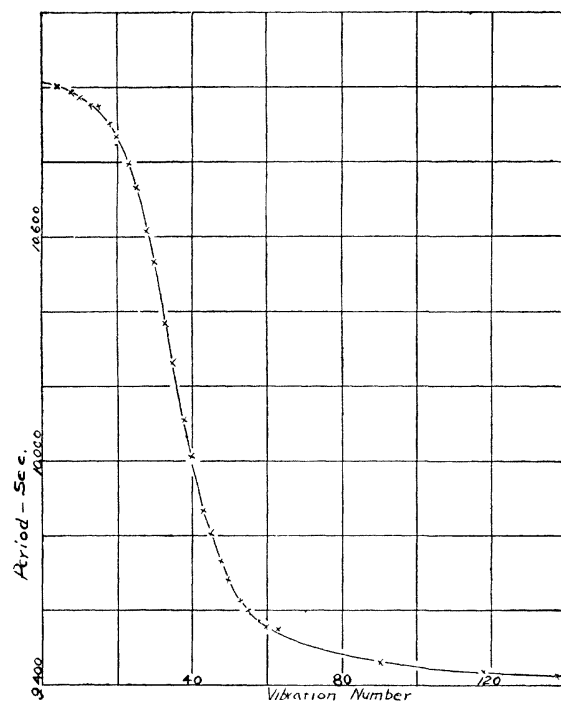


Fig. 1.

tude curve with different initial amplitudes. The wire was annealed at dull red heat by a current of 2.5 amperes, for about 30 seconds. Care was taken not to allow the wire to vibrate through more than fifteen or twenty degrees after this treatment. The wire was then carefully twisted through a certain small amplitude and timed by the method described above. After a rest of a day the wire was

twisted through a larger amplitude and timed as before. This was continued with constantly larger initial amplitudes until at the sixth observation the amplitude had reached a value of 788 degrees. This was an amplitude from the zero position of 394 degrees, and a twist per centimeter of length of 9.8 degrees. The results of this set of experiments are given in Table I., and graphically shown in

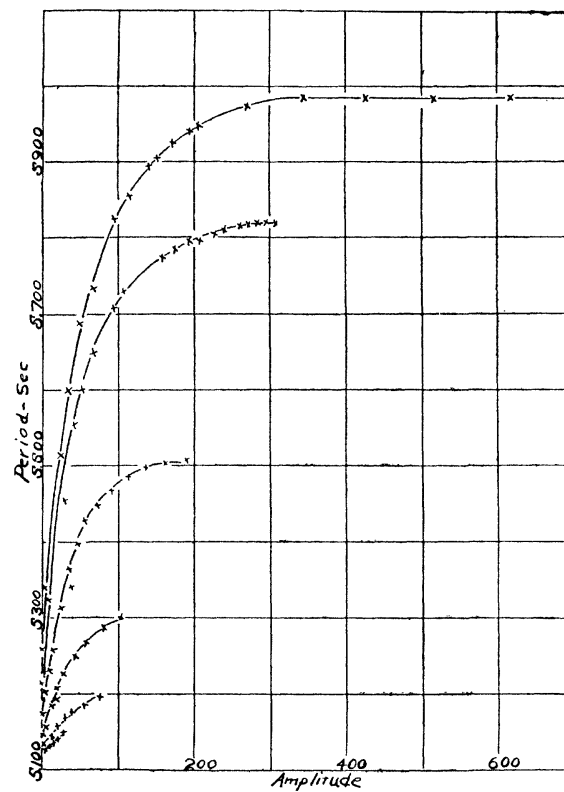


Fig. 2.

Fig. 2. This particular experiment was recorded in the former paper, but as the experiment had been rather hastily done, it was for the sake of certainty repeated here.

It will be seen that for small initial amplitudes the wire is in its behavior not far different from ordinary wires, *i. e.*, its change in period with amplitude is small, but it will be seen also that as the

TABLE I.
Length 40.2 cm. Moment of Inertia 980 g. cm².

November 30.			December 1.			December 2.		
No.	Amp.	Period.	No.	Amp.	Period.	No.	Amp.	Period.
0	28.1	5.149	0	74.5	5.198	0	102.0	5.300
10	26.6	.147	15	62.2	.192	10	83.0	.287
20	25.1	.145	25	55.0	.186	30	52.8	.262
30	23.6	.144	35	49.0	.183	40	42.6	.250
40	22.4	.142	50	41.3	.178	50	34.2	.238
50	21.2	.141	70	33.3	.171	60	28.3	.228
70	19.2	.139	90	27.3	.165	70	23.8	.218
90	17.4	.138	120	21.3	.158	80	19.8	.208
110	16.2	.136	150	17.0	.153	90	16.8	.200
130	15.0	.135	180	13.5	.149	110	12.1	.188
150	14.0	.135	200	12.0	.147	130	10.0	.180
170	13.2	.134	210	11.3	.145	140	8.9	.175
190	12.4	.133	230	10.3	.143	160	7.3	.167
210	11.8	.133	240	9.8	.143	180	6.2	.161
230	—	.132	250	9.4	.142	200	5.3	.157
300	9.2	.131	300	7.6	.140	230	4.4	.155
350	8.2	.130	350	6.3	.138	250	3.9	.154
400	7.4	.129	400	5.4	.135	350	2.6	.150
450	6.9	.129	450	4.7	.133	400	2.3	.148
973	3.31	.128	874	2.35	.133	450	2.1	.147
December 3.			December 4.			December 5.		
No.	Amp.	Period.	No.	Amp.	Period.	No.	Amp.	Period.
0	190.0	5.508	0	308	5.820	0	738	5.986
10	135.0	.498	5	249	.813	5	619	.986
20	91.0	.468	10	195	.795	10	517	.986
30	58.2	.427	15	150	.771	15	426	.986
40	36.3	.364	20	109	.730	20	344	.985
50	24.8	.312	25	78.0	.676	25	270	.972
60	17.3	.274	30	53.0	.601	30	206	.946
70	13.3	.249	35	37.0	.520	35	150	.905
90	8.6	.226	40	27.0	.435	40	104	.847
110	6.0	.213	45	20.0	.390	50	46	.673
130	4.6	.201	50	16.0	.365	60	22	.515
150	3.7	.193	60	10.0	.326	65	17	.465
160	3.4	.191	70	7.0	.300	70	13	.425
170	3.1	.189	80	6.0	.271	80	9	.383
180	2.8	.187	90	4.8	.250	90	6.1	.362
200	2.5	.184	100	4.1	.234	100	5.0	.340
220	2.2	.182	120	3.08	.227	120	3.7	.300
240	2.1	.180	140	2.4	.222	140	2.75	.280
250	2.0	.178	160	2.09	.216	160	2.1	.265
300	1.6	.172	170	1.98	.214	210	1.4	.259

initial amplitude increases, the periods corresponding to a given amplitude increase in a remarkable manner. Not only does the period increase, but so also does the internal friction, as is evident when we examine the total number of vibrations necessary to bring the wire down to a certain small amplitude, say five degrees.¹ This number will be noted in Table II.

TABLE II.

Date.	θ°	Vibrations to 5° .	t	θ^2	$\theta^2 - 25$	$\frac{\theta^2 - 25}{t} \times 10^2$
Nov. 30	14.1	625	3,270	199	174	5
Dec. 1	37.3	425	2,190	1,391	1,366	.62
" 2	51.0	210	1,093	2,601	2,576	236
" 3	95.0	125	664	9,025	9,000	1,355
" 4	154	88	484	23,716	23,691	4,895
" 5	369	100	567	136,161	136,136	24,010

In the column headed θ° is the first obtained reading for the maximum amplitude from the rest point. In column headed t is the number of seconds which elapsed while the pendulum fell from θ° to 5° . The numbers in column $\theta^2 - 25$ give a measure of the loss in energy during this time, and the numbers in the last column give the relative values for the rate of dissipation of this energy. Relative values can be obtained by representing the loss in energy in falling from a twist θ° to, say, an angle of 5° , by the expression $C(\theta^2 - 5^2)$, where C is a constant, and by dividing this by the number of seconds required for the wire to come down to five degrees. If it were desirable the actual energy dissipation in ergs per second might be determined from the knowledge of the torsional moment of the wire. However as the torsional moment requires for its determination a knowledge of n , the coefficient of simple rigidity, and as n will be shown later to have different values, depending on whether it is measured by the kinetic or by the static method, the exact determination of the torsional moment is perhaps a difficult

¹Kelvin [Math. and Phys. Papers, Vol. III., p. 25] observed in the case of an aluminum wire that the vibration "subsided much more rapidly from amplitude 20 to amplitude 10, when the initial amplitude was 40, than when it was 20." In the former case the number of vibrations from 20 to 10 was 96, and in the latter case the number from 20 to 10 was 112.

matter, and would largely rest on the definition for n . Still further it must be admitted that, in this calculation, it has been assumed that the torque possessed at the angle θ in the actual experiment is the same as the static torque for the same angle θ . So with this assumption, the relative values for the rate of energy loss are calculated. It is of course evident that this method gives only the average values. Somewhat similar values would be found for ordinary wires, and these values are of chief interest when compared with the values obtained under different circumstances with the same wire (cf. Table Vb). It will be noticed, under the column "vibrations to 5° ," that down to a certain point the number decreases notwithstanding the fact that the amplitude continually increases, but that at a certain point the number of vibrations commences to increase. This is comprehensible when we examine the period-amplitude curve for the largest initial amplitude and note that after a certain amplitude has been reached, the period approaches saturation, as it were. That is to say, after a certain starting amplitude has been reached, the further increases in amplitude tend to give period-amplitude curves that more nearly approach each other when traversing common ground; and whatever it is that has caused this enormous increase in internal friction, has apparently reached such a state that no large changes are possible. So from this point on, we should expect that for larger amplitudes more vibrations would be necessary to bring the amplitude down to 5° . This point is also evident from similar reasoning, when we examine the curves and values for the logarithmic decrement. These latter have been calculated for this set of experiments, and although the logarithmic decrement here loses its original significance, it is still very instructive in the information it gives as to the damping. The following values for the decrement (Tables III. and IV.) were calculated from another set of observations, similar to these in every respect, except that the wire was annealed between each two experiments. The final results in the two cases are practically identical. To save space only one complete table is given, and for the other six experiments, are recorded merely the finally calculated decrements and the corresponding amplitudes. In all cases common logarithms are used. Owing to the large initial

decrease in the amplitudes, the decrement is taken over a very small range of amplitude, this range being gradually increased. To get the amplitude corresponding to a given value of the decrement, the value is taken from the original tables corresponding to the mean vibration number used in the interval. The results are graphically shown in Fig. 3, where curves are drawn corresponding to the gradually increasing initial amplitudes.

TABLE III.

Length 40.2 cm. Moment of Inertia 980 g. cm².

<i>m-n</i>	<i>A_m</i>	<i>A_n</i>	Log <i>A_m</i>	Log <i>A_n</i>	Log (<i>A_m/A_n</i>)	Log. Dec.	Mean Amp.
4	639	526	2.8055	2.7210	.0845	.0211	582
4	582	476	.7649	.6776	.0873	.0218	526
4	526	427	.7210	.6304	.0906	.0227	476
4	476	381	.6776	.5809	.0967	.0242	427
4	427	337	.6304	.5276	.1028	.0257	381
4	381	295	.5809	.4698	.1111	.0278	337
4	295	220	.4698	.3424	.1274	.0319	256
4	220	158	.3424	.1987	.1437	.0359	188
4	158	110	.1987	.0414	.1573	.0393	132
4	110	75	.0414	1.8751	.1663	.0416	90.7
4	75	53	1.8751	.7243	.1508	.0377	62.3
8	62.3	33.8	.7945	.5289	.2656	.0332	44.4
11	53.2	26.3	.7259	.4200	.3059	.0278	38.4
14	44.4	21.9	.6474	.3404	.3070	.0219	33.8
17	38.4	19.3	.5843	.2856	.2987	.0176	26.3
20	33.8	16.9	.5289	.2279	.3010	.0151	21.9
20	16.9	12.3	.2279	.0899	.1380	.0069	14.2
20	12.3	10.1	.0899	.0043	.0856	.0043	11.1
40	10.1	8.0	.0043	0.9031	.1012	.0025	9.0
145	8.0	4.54	0.9031	.6571	.2460	.0017	5.82
159	4.54	3.21	.6571	.5065	.1506	.0009	3.80

2. *Decreasing Initial Amplitudes.*—The next set of experiments was similar to that mentioned in the preceding section, except that the wire was started at large amplitude at the first, and then successively started at lower amplitudes. It was thought that this would settle the question as to whether the period depends merely upon the amplitude, or whether, in a more complicated fashion, it depends to a large extent upon the preceding history of the wire. The latter was found to be true, and the results for the amplitude-

TABLE IV.

Mean Amp.	Log. Dec.	Mean Amp.	Log. Dec.	Mean Amp.	Log. Dec.
18.2	.00072	46.1	.00208	59.3	.00403
17.8	98	43.8	218	54.1	393
17.4	100	41.7	219	49.4	386
17.0	102	39.7	208	45.4	354
16.6	105	37.7	219	41.9	342
16.3	80	35.9	205	39.0	313
16.0	81	33.8	216	36.3	287
15.7	84	32.6	213	34.0	268
15.3	85	31.1	195	28.5	233
15.0	86	27.8	195	24.0	198
14.7	88	23.4	176	20.3	165
13.3	80	20.4	212	13.0	109
11.5	66	18.1	150	6.72	060
7.95	72	14.4	174	3.88	044
6.08	53	10.7	119		
5.00	45	7.83	106		
4.26	50	5.57	088		
3.72	41				
3.27	41				
115	.00944	194	.0196	400	.0193
103	950	177	203	366	199
92.4	942	161	202	333	209
82.9	953	147	207	302	216
74.2	958	133	220	273	227
66.5	944	120	226	245	240
59.7	944	96.7	239	194	262
53.5	930	77.7	240	150	304
43.3	919	62.2	237	112	331
35.2	887	50.1	228	82	366
28.9	824	41.0	205	59.2	334
24.1	756	34.3	186	45.9	313
20.5	656	27.5	150	32.6	296
17.7	590	21.7	112	25.6	245
13.7	502	14.7	0646	21.0	186
11.2	386	12.4	0324	14.0	118
7.2	132	9.8	0387	9.45	069
5.33	095	7.6	0172	6.76	0458
3.64	119	6.2	0181	4.59	0262
		4.19	0081	3.31	0156

period relations are shown in Table Va, Fig. 4, and for the amplitude-logarithmic decrement relations in Table VI., Fig. 5. In Table Vb is a calculation of the relative values for the rate of dissipation of

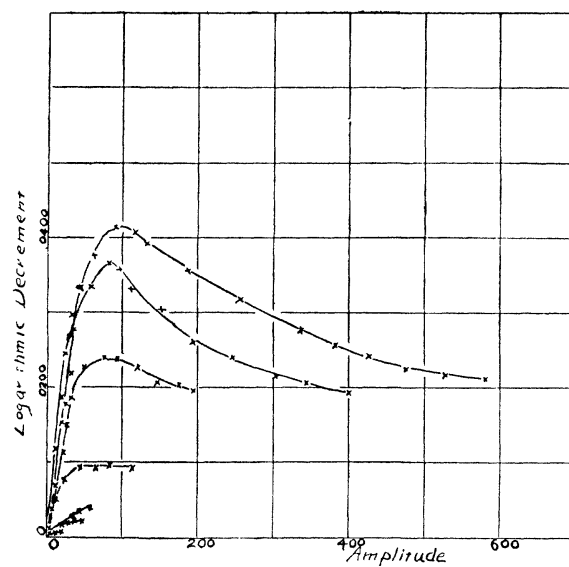


Fig. 3.

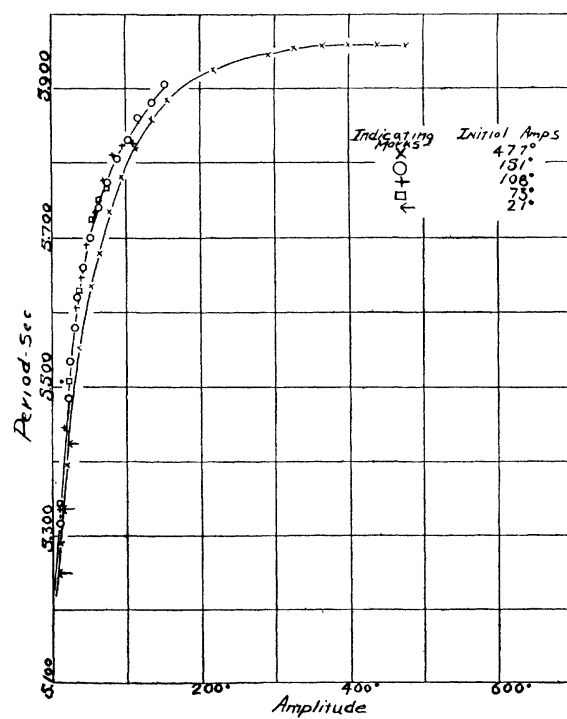


Fig. 4.

TABLE V (a).

Length 40.2 cm. Moment of Inertia 980 g. cm².

December 7.			December 9.			December 10.		
No.	Amp.	Period.	No.	Amp.	Period.	No.	Amp.	Period.
0	477	5.958	0	151	5.905	0	108	5.825
2	437	.958	4	117	.860	4	81	.810
4	398	.958	10	74	.773	10	48.3	.690
6	361	.957	16	43	.660	14	33.3	.604
8	326	.953	20	30	.580	16	27.9	.562
15	219	.925	24	21	.485	20	19.9	.485
24	113	.820	28	16.2	.415	26	13.3	.387
30	64	.678	35	11.4	.345	30	10.9	.350
34	43	.595	45	8.1	.297	36	8.7	.314
40	26	.475	55	6.4	.267	40	7.8	.297
55	13	.330	65	5.4	.253	50	6.1	.263
65	10	.290	75	5.0	.247	55	5.5	.253
80	7	.255	85	4.4	.244	60	5.1	.245
85	6.7	.248	95	4.0	.240	80	4.2	.240
100	5.7	.236	100	3.8	.239	100	3.4	.236
101	5.55	.236	121	3.23	.231	136	2.52	.231
179	3.06	.231	174	2.47	.225	190	2.01	.222
237	2.39	.227	232	2.02	.224	244	1.67	.222
301	1.92	.219	299	1.64	.222	303	1.41	.222
370	1.57	.216	371	1.38	.222	362	1.22	.222
December 11.			December 12.					
No.	Amp.	Period.	No.	Amp.	Period.			
0	73.3	5.765	0	20.7	5.425			
10	30.0	.580	6	14.0	.375			
12	21.4	.509	10	11.4	.348			
20	13.9	.403	14	9.6	.320			
26	10.2	.342	16	8.9	.308			
30	8.7	.315	20	7.8	.292			
36	7.2	.289	24	6.9	.280			
40	6.4	.277	26	6.6	.275			
50	5.2	.256	30	6.1	.265			
55	4.8	.250	35	5.4	.257			
60	4.4	.245	40	5.0	.250			
70	3.9	.241	45	4.6	.245			
80	3.4	.239	50	4.3	.240			
90	3.2	.236	60	3.8	.235			
100	3.0	.235	70	3.4	.238			
115	2.8	.232	80	3.2	.232			
125	2.7	.230	90	3.0	.230			
135	2.6	.228	100	2.8	.228			
142	2.62	.225	157	2.01	.222			
238	1.49	.222	220	1.66	.222			

TABLE V (*b*).

Date.	θ°	Vibrations to 5° .	t	θ^2	$\theta^2 - 25$	$\frac{\theta^2 - 25}{t} \times 10^2$
Dec. 7	239	114	620	57121	57096	9209
" 9	75.5	75	407	5700	5675	1394
" 10	54	63	335	2916	2891	863
" 11	36.7	52	286	1347	1322	462
" 12	10.4	40	212	108	83	39

energy (obtained as described in the preceding section), together with a column for the number of vibrations down to 5 degrees. A comparison of the two tables [Tables II. and V*b*] will clearly bring out the essential differences between these two sets of experiments.

Since the points in Fig. 4 lie so closely together, only through those of the two largest initial amplitudes are curves drawn. However the loci corresponding to the different initial amplitudes are indicated on Fig. 4, and the general relations of their paths can be

TABLE VI.

Dec. 7.		Dec. 9.		Dec. 10.		Dec. 11.		Dec. 12.	
Amp.	Log.Dec.	Amp.	Log.Dec.	Amp.	Log.Dec.	Amp.	Log.Dec.	Amp.	Log.Dec.
437	.0197	143	.0260	94	.0312	62.3	.0367	17.9	.0293
398	208	126	295	69	363	43.4	400	14.0	246
361	214	110	298	48.3	398	30.0	351	11.4	209
326	227	95	346	33.3	397	19.9	371	9.6	169
271	247	81	352	23.3	367	13.2	251	8.1	144
219	274	62	383	17.1	307	10.2	181	6.8	107
175	309	52	397	13.3	242	8.7	148	5.4	0836
133	350	43	430	10.0	175	7.4	136	4.6	0655
94	377	35	391	8.7	135	6.4	109	3.6	0428
78	417	21	335	7.1	110	5.0	0749	3.0	0290
43	394	16.2	241	6.1	0933	3.9	0560	2.4	0253
26	302	11.4	177	5.1	0540	3.1	0241	1.84	0132
19	222	8.1	125	2.52	0291	2.75	0158		
13	1505	6.4	0881	1.84	0149	2.6	0077		
10	1054	5.2	0494	1.54	0125	2.09	0026		
8	0870	4.7	0464	1.32	0106				
6.5	0589	3.8	0373						
6.1	0511	3.6	0357						
2.16	0149	2.9	0220						
1.75	0127	1.51	0104						

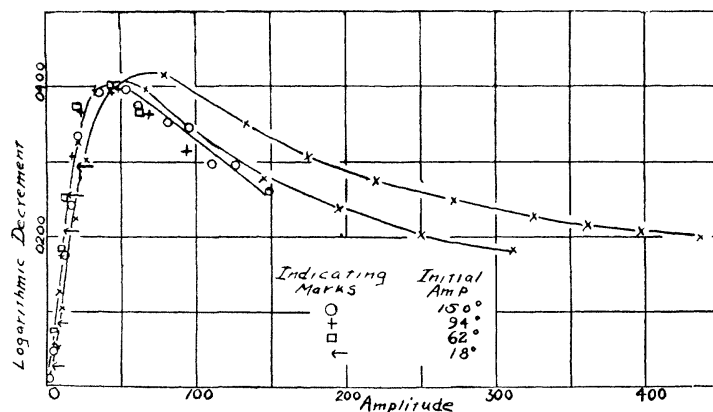


Fig. 5.

seen. It appears at first sight from this figure that in this case the wire had been put in a certain condition as a result of the first large distortion, and that this condition had remained in the wire. There seems to have been a slight tendency for the wire in the lowest starting amplitude to regain the original values it had in the experiments which were the reverse of this one. To satisfy myself concerning this point one last experiment was made which quite clearly brought out the tendency toward self-restoration in the wire. It seemed possible that in coming by *gradual* steps from the large amplitudes down to the smaller ones, the tendency for the wire to restore itself had been masked, and that a better procedure would have been to treat the wire as follows: first, anneal to red heat with a current of 2.5 amperes; second, vibrate through the maximum amplitude, 360 degrees from the center; allow to die down naturally without timing; and third, vibrate at a small amplitude, say 40 degrees, and time in the usual way. For the next experiment, repeat steps one and two, and then for the third step vibrate through about 70 degrees, and so on with larger amplitudes for this last step. These experiments were performed, and the results are shown in Table VII., Fig. 6. Curve (a) represents the period-amplitude relation in the lower values for the large original twist (second step above; in this one observation the step was timed), while curves (b), (c) and (d) represent the values for these quantities when the

TABLE VII.

Length 35.5 cm. Moment of Inertia 3,611 g. cm².

March 24—III.			March 24—IV.			March 24—V.		
No.	Amp.	Period.	No.	Amp.	Period.	No.	Amp.	Period.
0	43.59	9.820	0	72.42	10.050	0	143.9	10.450
2	37.90	770	2	60.33	9.970	2	120.4	370
4	33.57	712	4	50.83	883	4	100.1	290
6	30.10	670	6	43.50	815	6	83.25	220
8	27.37	630	8	37.89	763	8	69.67	150
10	24.99	592	10	33.48	710	10	58.15	054
12	23.08	560	12	30.10	668	12	49.08	9.990
14	21.51	542	14	27.23	630	14	41.78	912
16	20.21	528	16	24.96	595	16	36.09	848
18	19.01	514	18	23.16	565	18	31.64	789
20	18.06	503	20	21.62	541	20	28.11	732
22	17.16	493	22	20.35	520	22	25.32	688
24	16.40	486	24	19.16	505	24	23.16	650
26	15.69	480	26	18.16	494	26	21.26	617
28	15.05	473	28	17.31	484	28	19.80	593
30	14.52	468	30	16.52	474	30	18.41	572
32	14.03	460	32	15.82	468	32	17.31	556
34	13.56	453	34	15.20	460	34	16.38	542
36	13.14	449	36	14.68	455	36	15.48	531
38	12.78	443	38	14.17	450	38	14.73	521
40	12.40	440	40	13.71	445	40	14.10	511
71	9.13	413	90	8.45	403	76	8.54	448
100	7.37	404	123	6.87	394	102	6.79	434
127	6.35	398	154	5.94	388	130	5.64	427

distortions following the large one of 320° from the center were respectively 21.8° , 36.2° and 72° from the center, or in our present nomenclature double amplitudes of 43.6° , 72.4° and 143.9° respectively. Curve (*d*) is decidedly below (*a*), while (*b*) and (*c*) are below (*d*). The curves (*b*) and (*c*) seem to fall almost exactly together as far as they cover common ground. It would seem from this fact that there is a certain limiting amplitude below which it takes the wire a longer time to recover. In order to make certain of this last point, further experiments will be necessary. The rather complicated behavior of the wire is then brought out by these experiments; for a given amplitude we find that the period depends upon whether the wire has been freshly annealed or not,

upon whether it has approached the amplitude from a larger one or from a smaller one, and finally upon how long the wire has rested since its last distortion. There seems good reason to believe that with a long rest the wire might in great part regain its original condition. During the present investigation, however, there has been no opportunity to substantiate this point. From the results

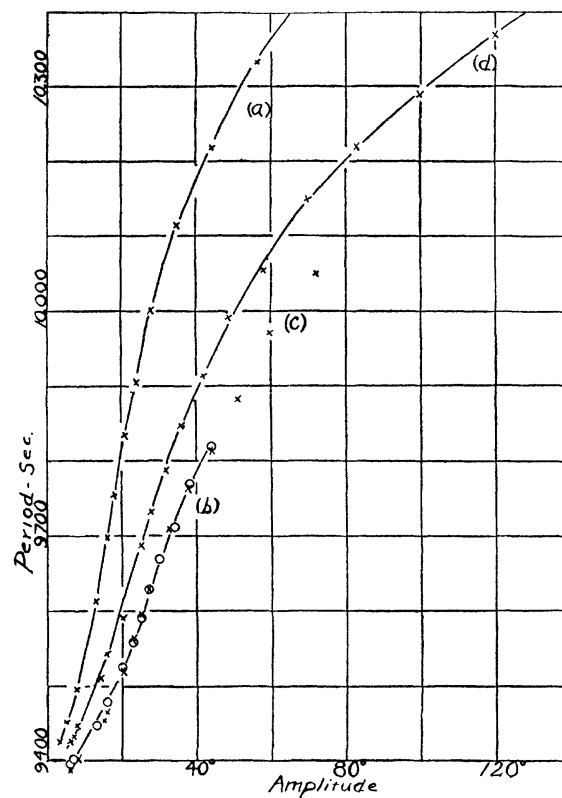


Fig. 6.

of the preceding observations, it is evident that experiments of other kinds upon this wire (for example the variation of the period with the moment of inertia of the suspended mass, the temperature coefficient, heat treatment, etc.) would be of little or no value unless a definite relation under varying conditions could be obtained between the period and the amplitude. It is true, that after anneal-

ing the wire with a current sufficient to bring it to a red heat a fairly definite relation between period and amplitude can with proper care be obtained. The annealing appears to destroy the previous history of the wire as far as elastic after effects are concerned. This method has been used in many of the experiments. However one must be extremely careful in the treatment of the wire after annealing it, as a single accidental vibration of even thirty or forty degrees is sufficient to change its condition most evidently.

3. *Attempts to Put the Wire in a Definite State.*—Since the method of annealing led to somewhat uncertain results, it seemed desirable to find another method of putting the wire into a definite state. To find the proper conditions, many experiments were attempted before what now seems to be the proper solution was found. Of these experiments only a few of the significant ones will be given.

In the first place an attempt was made to obtain the period at a given amplitude by keeping the pendulum vibrating at a fixed amplitude by means of the torsion head. It will be remembered that the wire dies down so rapidly that the period for a given amplitude can be obtained only by means of the tape and the chronograph, and even in this way only a single value is obtained, so quickly does the amplitude fall off. This ability to gain only a single value for the period might lead to the suspicion that the possible error in timing would vitiate any attempt at refined results. So the continued vibration at constant amplitude served two purposes: viz., to see if in timing by ordinary methods at constant amplitude there was any change in the period as time went on; and also to see if the periods obtained by finally letting the vibrations die down in the usual way (timing them with the chronograph and the tape) would join those obtained at constant amplitude without any break. The first point—the change in period resulting from continued vibration at constant amplitude—is brought out in Table VIII. In this table it appears from the experiment of November 2, that the period gradually increased from 5.875 sec. to 5.886 sec., while the amplitude was held within 2 or 3 degrees of 269 degrees, and it appears also, from the experiment of November 5, that the period increased from 5.900 sec. to 5.908 sec., while the amplitude remained at 299 degrees.

TABLE VIII.

Length 40.2 cm. Moment of Inertia 980 g. cm². Amplitude 269°.

Coincidences.			Interval (sec.).	No. of Vibrations.	Period.
3h	20m	32s			
	22	06	94	16	5.875
	27	00	388	66	.879
	28	34	482	82	.878
	32	06	694	118	.881
	35	50	918	156	.885
	39	34	1142	194	.886
	46	26	1554	264	.886
<i>Amplitude 299°.</i>					
5h	3m	28s			
	5	26	118	20	5.900
	9	34	366	62	.903
	11	44	496	84	.905
	13	54	626	106	.906
	18	26	898	152	.908
	20	36	1028	174	.908
	22	46	1158	196	.908

Further information on this point is shown in Table IX., based on a set of experiments taken in such close succession that the wire was kept in almost continuous vibration. To be exact, the wire was vibrated 23 minutes, rested 22 minutes, vibrated 28 minutes, rested 14 minutes, and finally vibrated 24 minutes. During the periods of rest the wire was not wholly quiet, for it was during these intervals that the stops on the torsion head were adjusted for the new amplitudes, and many trials were necessary before the desired amplitudes could be reached and maintained. So it can be stated correctly that the vibration continued practically two hours. This may seem a trivial point, but later experiments have shown its importance.

In this table are recorded data for experiments at three different amplitudes. During this group of experiments the temperature was constant within about 1° C.; it might be added here that the same care as to constancy of temperature was observed in each set of experiments. The three experiments in this group showed some interesting results. It will be noticed that for the amplitude of

TABLE IX.

Length 40.2 cm. Moment of Inertia 980 g. cm². Amplitude 825°.

Coincidences.			Interval (sec.).	No. of Vibrations.	Period.
10h	53m	56s			
	59	10	314	52	6.032
11	3	24	568	94	.048
	7	2	786	130	.055
	10	28	992	164	.059
	13	42	1,186	196	.063
<i>Amplitude 748°</i>					
11h	38m	44s			
	41	34	170	28	6.071
	44	24	170	28	.071
	49	40	316	52	.077
	54	44	304	50	.080
	59	48	304	50	.080
12	4	40	292	48	.083
<i>Amplitude 579°</i>					
12h	21m	6s			
	24	8	182	30	6.067
	27	22	194	32	.063
	33	14	352	58	.069
	33	16	182	30	.067
	39	6	170	28	.071
	41	56	170	28	.071
	44	58	182	30	.067

825 degrees the period increases from 6.032 sec. to 6.063 sec., that for the amplitude of 748 degrees the period increases from 6.071 sec. to 6.083 sec., and that for the amplitude of 579 degrees the period is practically constant and smaller than in the other two experiments. The fact that, in the third experiment the period is lower, is not unreasonable as all of the preceding experiments have indicated shorter periods at the smaller amplitudes; and though accompanying the fall in amplitude from 825° to 748° there is an actual rise in the period, still this rise could not well continue indefinitely. Ultimately at some amplitude the rise in period would cease, and the period would fall off. This rise in the period due to continued vibrations at a given amplitude, or even at a somewhat smaller amplitude, will be further discussed in a following section.

However the important point now is that the period seems to have reached, in this third experiment, practically a constant value for the given amplitude. There are, of course, variations due very probably to errors of observation, but admitting even the maximum variations in the periods, the percentage changes in the three cases are respectively, 0.52 per cent., 0.20 per cent. and 0.15 per cent. Obviously, the deduction is that a fair constancy of period for a given amplitude can be obtained only after the wire has been vibrated for some time.

The second purpose of the above experiment was, it may be remembered, to test whether or not the periods obtained when the torsion head was stopped and the wire allowed to die down in the usual way, follow without any break the periods obtained by coincidences. A comparison of Table VIII. and Table X. will make this point clear. In Table VIII. (lower half) are recorded the periods,

TABLE X.

Length 40.2 cm. Moment of Inertia 980 g. cm².

No.	Amp.	Period.	No.	Amp.	Period.	No.	Amp.	Period.
0	292	5.898	25	116	5.850	70	7	5.441
2	277	898	30	92	822	75	6	415
4	260	897	35	67	786	80	5	403
6	244	895	40	48	730	85	5	395
8	230	894	45	35	672	98	3.59	376
10	214	892	50	24	615	119	2.27	333
12	199	890	55	17	560	149	1.67	333
15	177	885	60	12	515	179	1.16	333
20	147	870	65	9	473			

obtained by coincidences, where the vibrations had been held at constant amplitude. The subsequent periods of this same torsion pendulum, as it died down naturally, are given in Table X. It is evident that these latter periods join the former periods without any serious break.

In most of the experiments performed, the amplitudes were not taken below one degree, and very commonly not below two degrees. This was usually because a small side swing had developed as a result of the disturbances incident on setting the pendulum into the

large vibrations. While these small side swings did not appreciably vitiate the results for amplitudes down to five or six degrees, they made it impossible to get accurate readings much below this point. Many of the curves for amplitude and period seemed to indicate that at low values for the amplitude there was a tendency towards constancy in period. This point was tested by several carefully performed experiments at small amplitudes. The damping was so small here that the periods could easily be obtained by the method of coincidences. The results are given in Table XI., and in Fig. 7.

TABLE XI.

Length 40.2 cm. Moment of Inertia 980 g. cm².

Amp.	Period.	Amp.	Period.
4.1	5.333	.91	.292
2.96	.321	.75	.290
2.3	.313	.68	.290
1.8	.308	.58	.288
1.3	.288	.50	.287
1.19	.296	.41	.286
1.07	.295		

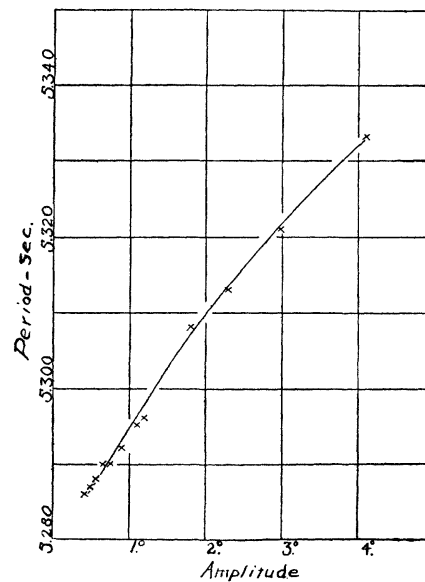


Fig. 7.

From these results it is evident that the relation between the period and the amplitude is, for these small values, a linear one, and that there is no evidence of constancy of period, at least not above 0.4 degree. Below this amplitude, it is very difficult to time with any accuracy.

4. *Effect of a Slow Torsion of the Wire through the Maximum Range of Amplitude.*—The next query that arose was whether the peculiar state of the wire caused by vibrating it through a large amplitude after annealing, was due merely to the amplitude of the torsional strain, or to some internal phenomena accompanying the actual vibration. This point was tested in the following simple experiment. The weight attached to the wire was firmly clamped. Then, after the wire had been annealed to red heat, the torsion head was slowly turned through 360 degrees. After being held in this position 30 seconds, it was twisted back to zero, and then on to the other side to the same amplitude. It was held there 30 seconds, and finally returned to the zero point. The weight was then carefully released, and the wire allowed to rest 24 hours, at the end of which time a regular experiment was made with an initial amplitude of 35 degrees. It was necessary to take this small amplitude, for otherwise the effect sought would have been lost in the complications that arise from the larger amplitudes. The results for period and amplitude are given in Fig. 8, curve (c), where for comparison two other curves covering about the same range of amplitudes are given; curve (a) illustrating an experiment where the wire had been freshly annealed, and curve (b) illustrating an experiment where the wire had been vibrated for some time at larger amplitudes. The results are very striking, and show clearly what effect the mere torsional strain has on the state of the wire. Here clearly this single twist does not cause as great an effect as do long continued vibrations, but still the single twist does cause a decided change. In the same figure (Fig. 8) are platted from the same experiments the variation of the logarithmic decrement with the amplitude, the curves corresponding to (a), (b) and (c) being marked (a'), (b') and (c'). From these logarithmic decrements it is seen that the larger the frictional losses, the larger the variation in the period.

5. *Successive Observations without Preliminary Vibrations.*—It was

thought barely possible that, although the period for a given amplitude does depend upon the previous state of the wire, yet three observations taken in succession with no more vibration than necessary might yield results that would be essentially alike. While this solution of the problem would not be satisfactory, it still would serve to give useful comparative results on a short series of experiments. As might have been expected, perhaps, the desired constancy was not obtained: this will be evident from an inspection

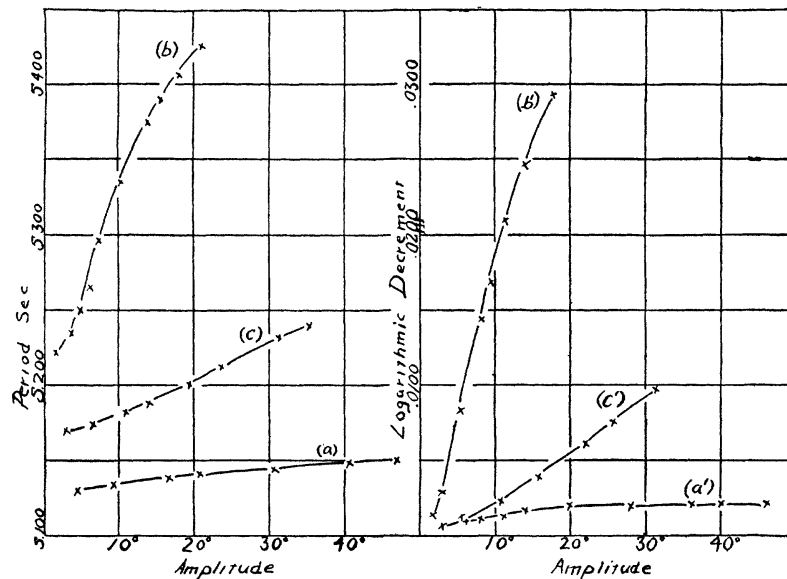


Fig. 8.

of Fig. 9 which represents the results of three successive experiments. The wire was annealed to red heat and set into vibration to its full amplitude by one single twist, then allowed to die down and timed. The results are given in curve (a). When this experiment had been completed, the wire was again set into vibration and timed. This was done a third time. These last two experiments are represented by the curves (b) and (c), respectively. The fact that the pendulum was put into vibration by means of one twist is mentioned because often before this it had been set into vibration by means of the

torsion head, by which the pendulum was twisted back and forth, "working it up" as it were.

This completed the first group of three experiments. As had been expected the results were unsatisfactory, yet it was thought best to repeat the three experiments varying, however, the first one. Even by this varied method, small chance seemed to remain of getting results of any value. The wire was annealed and then,

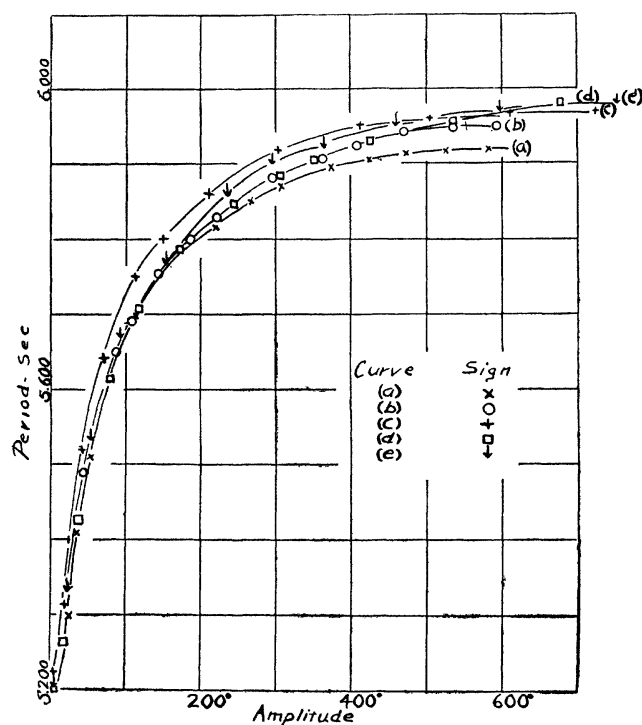


Fig. 9.

the weight being clamped, the torsion head was twisted as described above to the right 360 degrees, back to zero, then to the left, etc., and after that two observations were taken. It was thought that the variation in the period-amplitude curves as indicated in curves (a), (b) and (c), Fig. 9, was largely due to the first experiment, following the annealing. When the table of results (which for the sake of space is not given) was examined it was found that in the

two experiments the values of the periods for both the maximum and the minimum amplitudes were practically the same. In the first experiment (curve *(d)*, Fig. 9) after the wire had been annealed and twisted, the range of periods was from 5.980 sec. to 5.200 sec., where the maximum amplitude was 677 degrees; in the second (curve *(e)*, Fig. 9) the range in period was the same, while the maximum amplitude was 751 degrees. However when all the values were platted, the two curves were found to be decidedly different in form, as is evident from an inspection of curves *(d)* and *(e)*. This general tendency of the curves to show, after repeated vibrations, a sharper curvature in the region where they bend over toward the amplitude axis, has been frequently observed.

6. *Effect of Long-continued Vibration on the Logarithmic Decrement-amplitude Curves.*—Along with this variation of the period-amplitude curves following continued vibrations, it is interesting to notice

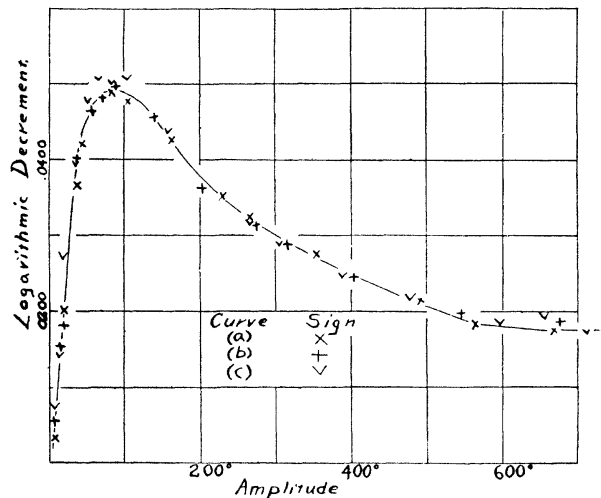


Fig. 10.

the corresponding variations of the logarithmic decrement-amplitude curves. To give but one illustration of this point, there are platted in Fig. 10, three curves, based upon three experiments performed on the same day, for the variation of the logarithmic decrement with the amplitude. The conditions governing these experiments

were as follows: The wire, being first annealed to red heat, was turned through 360° and timed. Curve (*a*) represents the variation of the logarithmic decrement. The wire was next vibrated by means of the torsion head through a total amplitude of 720° for 30 minutes and was then allowed to die down and was timed as usual. Curve (*b*) represents the variation of the logarithmic decrement. Lastly the wire was again vibrated as explained above, for 30 minutes. The values of the resulting decrement as the wire died down are given in curve (*c*). It is quite evident that the repeated vibrations had increased the internal friction of the wire, as the decrement, to a slight extent, progressively increased.¹ This increase of the decrement as well as of the period, following repeated vibrations, was first announced by Lord Kelvin² who experimented particularly with copper wires. J. O. Thompson³ later showed that as long as the amplitude and temperature remain constant there is no change in the period or in the logarithmic decrement. Still later this same phenomenon was found by Bouasse⁴ to hold for repeated vibrations on a hard drawn copper wire. He expressed the belief that this change did not come through a change in the elasticity, but through a change in the cohesion of the molecules of the wire.

7. *Long-continued Vibrations.*—The last step, and the one that seems to have given a solution of the problem of this paper, was to vibrate the wire for much longer periods than hitherto, before taking observations on the relation between period and amplitude. Had the information recorded in the earlier sections of this paper been previously known to the writer, this method of treatment would have been arrived at much sooner. The following figure (Fig. 11) represents the results of the variation of period and amplitude for ten experiments to be described, but to save confusion in the curves, with the exception of the results from two experiments, only the values of the periods for amplitudes above 300 degrees are given.

¹H. G. Tammen [Carl Rep., 18, p. 350, 1882] found with copper wire an increase of the log. dec. for a given amplitude with the duration of the vibration. He found also a decrease in the log. dec. with increase in the amplitude.

²Kelvin, Math. and Phys. Papers, III., p. 26.

³J. O. Thompson, PHYS. REV., 8, p. 141, 1899.

⁴H. Bouasse, Ann. Chim. Phys. (8), 2, pp. 5-77, 1904.

period had, due to the week's rest, fallen back slightly; the results are shown in (*d*). The pendulum was now vibrated an additional 30 minutes, and then was given a second vibration of 30 minutes. The results of these two vibrations are given in curves (*e*) and (*f*) respectively. It began to seem as though the pursuit of an upper limit for the period would be hopeless unless stronger measures were taken, so after a three days' rest the pendulum was kept in continuous vibration for two and one half hours.¹ The results of this experiment are given completely in (*g*). After this observation the wire was at once twisted to the maximum amplitude. The observation for the succeeding periods is recorded in curve (*h*), which agrees with (*g*) throughout the entire range. Curve (*i*), the record for an experiment performed two days later, shows the usual tendency of the period to fall off after an interval of rest. An objection to the above experiments is that following (*g*) the pendulum was not vibrated a longer time before (*h*) was taken, for it could be argued that there might still be a further rise in the period. The objection may be well founded; but what was obtained was really what was sought, viz., to get a period-amplitude curve that could be, within the limits of error in timing, exactly duplicated. The almost exact duplication of curve (*g*) by curve (*h*) seems then to show that the wire, at least for the time being, had reached a definite and final condition. Further experiments on this point are planned for the near future.

IV. EFFECT OF VARIATION OF MOMENT OF INERTIA ON THE PERIOD-AMPLITUDE CURVES.

In the former paper,² it was found that the nature of the period-amplitude curves was not appreciably affected by a variation in the moment of inertia of the suspended weight. Since in the earlier investigation no especial precautions were taken to eliminate the past history of the wire, there was a possibility that the effect of variation of moment of inertia was so small that it might have been masked by the irregularities resulting from the past treatment

¹H. Streintz (Carl Rep., 16, p. 476, 1880) found that a definite elastic condition for ordinary wires was not reached even after they had been vibrated for three and one-half months.

²Guthe and Sieg, *loc. cit.*

of the wire. This the present investigation attempts to guard against. In all the preceding experiments mentioned in this paper the moment of inertia of the suspended weight was always the same in any one set of observations, so that its effect was a constant one. In the present observations four different cylinders, agreeing in mass (mass = 229 gr.) within 0.5 gram were used. The values of their moments of inertia, including that of the central supporting rod and the double mirror, were respectively: *A*, 3,611 c.g.s.; *B*, 980 c.g.s.; *C*, 474 c.g.s.; and *D*, 367 c.g.s. In each case the wire was annealed to red heat with a current of 2.5 amperes, and no preliminary vibrations were taken. With proper care this method was found in any given case to give consistent results. The curves, in order to be compared, had to be platted on different scales, since of course the periods varied with the different cylinders used. What amounted to the same thing as changing the scales used was to make the four results agree at some one amplitude, and then to use the factor thus determined to multiply all the values of the period in any one set. Some care has to be used here in choosing the proper amplitude at which to make comparisons. At first it was attempted to make all the periods agree at an amplitude of five degrees, but this led to serious variations, and the explanation was quite natural. At an amplitude as small as five degrees, or even in that neighborhood, the variation of the period with the amplitude was extremely large, and a slight error in properly choosing the true period at this amplitude made a large error in the corrected periods, especially in the cases of the cylinders of smallest moment of inertia. On the other hand it was undesirable to match values of the period at the largest amplitudes, because in the method of timing used these values were least accurate. In short, the amplitude of 500 degrees was taken as the point of reference because this amplitude was in a part of the curve far enough from the beginning to warrant accurate results, and still in a part of the curve where the variation of period with amplitude was small. A period of ten seconds was taken in each case for this amplitude, and the factors properly chosen to make all four curves agree at this point. The results are given in Table XII., and are graphically shown in Fig. 12, where the curves corresponding to the different cylinders

TABLE XII.

Length 36.1 cm. Moment of Inertia: A 3,611, B 980, C 474
and D 367 g. cm².

Cylinder A.			Cylinder B.		
Amp.	Period.	Corrected Period.	Amp.	Period.	Corrected Period.
735	11.141	10.006	741	5.720	10.017
515	.132	.000	500	.710	.000
500	.126	.000	485	.709	9.999
341	.096	9.968	375	.694	.972
310	.084	.956	261	.653	.901
281	.070	.944	242	.644	.885
252	.051	.927	210	.620	.843
226	.030	.907	165	.580	.773
200	.002	.882	140	.547	.716
165	10.955	.832	129	.525	.676
154	.931	.819	98.3	.466	.573
131	.890	.782	75.4	.415	.484
111	.810	.710	53.2	.334	.342
83.4	.689	.602	35.6	.240	.214
76.	.638	.556	25.6	.170	.055
50.2	.487	.420	16.1	.082	8.900
41.	.391	.412	9.37	.028	.806
20.2	.114	.162	7.2	.012	.782
5.29	9.721	8.805	5.0	4.998	.754
2.44	.644	.735	3.71	.989	.738
Cylinder C.			Cylinder D.		
826	3.964	10.025	890	3.488	10.032
675	.961	.018	730	.484	.020
540	.958	.010	591	.480	.009
500	.954	.000	500	.477	.000
312	.926	9.929	356	.461	9.954
272	.913	.896	312	.451	.925
199	.888	.823	237	.429	.862
166	.862	.767	201	.411	.810
137	.837	.704	168	.398	.773
111	.809	.633	138	.375	.707
90.4	.773	.542	112	.350	.635
73.1	.733	.441	91	.322	.554
53.8	.685	.320	74	.291	.465
44.1	.650	.231	60.5	.260	.376
33.9	.603	.112	49.5	.233	.298
22.3	.529	8.925	37.6	.194	.186
8.2	.450	.725	17.1	.078	8.852
7.0	.438	.695	10.8	.042	.749
5.0	.430	.674	5.00	.013	.666
2.50	.415	.637	2.20	2.998	.623

are clearly indicated on the figure. It is evident that although the curves are in general much alike, there still is a change that progresses with the change in the moment of inertia of the vibrating system; the larger the moment of inertia, the more tendency for the curves to remain parallel to the amplitude axis and the more abrupt the final bend toward the period axis. The curves are so

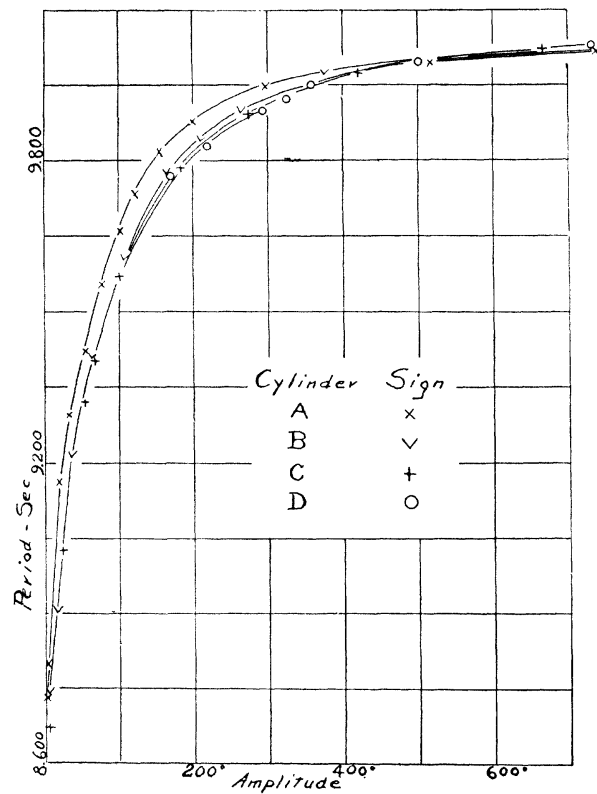


Fig. 12.

nearly alike that it is certain that this whole effect of variation of period with amplitude is largely a function of the amplitude only, but it is equally certain that the velocity of the vibration is a determining factor, even though a small one.

V. DETERMINATION OF THE COEFFICIENT OF RIGIDITY.

One obvious explanation of the remarkable variation in the the period-amplitude curves, would be to assume that with increased amplitude Hooke's law falls off as the period increases. This point was tested in the experiments recorded in the former paper,¹ by balancing different torques of the platinum-iridium wire against the torques of a steel wire. Assuming that the steel wire followed Hooke's law perfectly, then the wire in the above-mentioned investigation likewise followed the law up to the widest range of amplitudes used. To verify the above point, and to obtain a value for the coefficient of rigidity, measured statically, it was thought well to repeat this experiment by a more absolute method and one independent of any other wire. The method was to balance the torque of the wire by the couple caused by the bending of two delicate glass fibers. The fibers were drawn, and the end of each bent into a sharp hook. Then the two fibers were supported exactly as they were to be used in the experiment, calibrated by suspending small milligram weights from the hooks at the ends, and the resulting deflections read with a traveling microscope. In the final experiments on the wire, a microscope was used for each fiber. In calibrating the fibers and in the actual experiments with the wire, the deflections of the fibers were always small enough to follow Hooke's law almost perfectly. It was impossible to draw two fibers of exactly the same rigidity, but in all cases the two torques constituting the moment were made so nearly alike that the wire was undeviated from the cross wire of a reading telescope. Table XIII gives a résumé of the trials made. In the first column, under the heading θ° , is the torsional strain in degrees from the position of rest. In the second and third columns, under f_1 and f_2 respectively, are the forces exerted on the two fibers, expressed in milligram weights. In the fourth column, under the head of torque (c.g.s.), is the torque in dyne-cms. obtained by multiplying the sum of f_1 and f_2 by 980 and by the radius of the cylinder to which the glass fibers were attached by silk threads. In the last column under the heading torque/ θ° is the torque per degree of torsional

¹Guthe and Sieg, *loc. cit.*

TABLE XIII.

θ°	f_1 (mg. wt.)	f_2 (mg. wt.)	Torque (c. g. s.)	Torque/ θ° .
36.90	41.19	70.64	192.7	5.568
37.24	49.97	70.36	207.4	.582
38.17	43.60	80.00	213.1	.239
40.18	42.56	79.55	210.5	.223
143.6	290.7	159.5	776.1	.404
233.4	240.5	448.9	1,188.	.092
308.6	477.6	469.8	1,633.	.292
313.0	390.8	591.2	1,693.	.408
349.8	701.8	428.4	1,948.	.570
372.8	629.2	497.1	1,941.	.207
380.6	540.3	577.8	1,927.	.064
382.0	630.5	553.9	2,042.	.345
384.5	659.7	529.1	2,049.	.330
463.1	737.1	733.5	2,535.	.474
525.4	870.5	698.6	2,705.	.148
530.7	727.4	861.8	2,740.	.162
710.0	994.9	1,224.0	3,825.	.388
				Mean 5.323

$$\frac{\text{mean torque}}{\text{radian}} = 305 \text{ c.g.s.}$$

strain. From these data it is evident that this method is not productive of as accurate results as the method involving the use of the iron wire, and in itself this method would not establish the truth of Hooke's law. However among the values for the torque per unit angle there seems to be no tendency either to increase or to decrease at the larger values of the amplitudes. Using the mean value for the torque per radian as 305 c.g.s., and substituting in the formula $n = 2LT/\pi r^4$, the value $7.63 \cdot 10^{11}$ dynes/cm². is obtained for n , the coefficient of simple rigidity. This is about equal to the value of n for wrought iron. Even granting an error as large as 5 per cent. the value thus obtained for n is decidedly different from any value obtained for a small amplitude by the method of torsional vibrations. The results in Table XIV. giving the values of n for the four different moments of inertia used, illustrate this wide variation. In each case two values are given, one (n_1) for an amplitude of 500 degrees and one (n_2) for an amplitude of 5 degrees. Two interesting facts are to be noticed: first, the coeffi-

TABLE XIV.

Radius of wire .00975 cm.

Cyl- inder.	Moment of Inertia.	Length.	T_1 (mean solar) for Amp. 500°.	T_2 (mean solar) for Amp. 5°.	n_1	n_2
A	3611 g.cm ² .	35.5 cm.	22.020	19.380	7.35×10^{11}	9.50×10^{11}
B	980	36.1	11.390	9.969	7.58	9.90
C	474	36.1	7.887	6.842	7.65	10.17
D	367	36.1	6.935	6.010	7.66	10.20

cient of rigidity thus determined increases as the moment of inertia decreases; and second, the value of n for 500 degrees is smaller than the value obtained for smaller amplitudes and is much more nearly the value obtained statically, than is the value obtained at the smaller amplitudes. This is turning the usual order of things topsy turvy. But it is only at large amplitudes that the wire approaches anything like a constant period, and with a constant period the wire tends to act more like an ordinary wire. It is well known that even for ordinary wires the value of n obtained in these two different ways is different,¹ and it would be interesting to see if these two values would more nearly agree at the larger amplitudes.

As has been stated in the paragraph above, from Table XIV. it appears that there are apparently two different tendencies toward increase in the value of n . The first tendency toward increase, following a higher velocity of vibration due to the use of smaller moments of inertia, seems to indicate that possibly there is in every case a dragging of the instantaneous center toward the direction of vibration, but that the closeness with which this center follows the displacement is less in the case of the rapidly moving cylinders than in the case of the slowly moving ones. If we postulate that the torsional restoring couple on the wire is proportional to the angular distance of the wire from this instantaneous center and not from the original zero, then in the rapidly moving cylinders this center is at any time during the outward motion, farther away from the angular displacement than it is in the case of the slowly moving

¹H. Tomlinson (Proc. Roy. Soc. Lond., 43, pp. 88-108, 1887) found that for iron wire the kinetic determination of n was larger than the static by 2 per cent. A like but smaller change was found to hold for Pt., Ag. and Al.

cylinders. So the restoring couple would be greater and the apparent elasticity of the wire greater. If, on the other hand, the observations be confined to any one cylinder, it is to be noted (Table XIV.) that the value of n increases with the smaller amplitudes. This exemplifies the second tendency mentioned above, and would seem to indicate that for the smaller amplitudes there is little or no yielding of the instantaneous rest point from the average rest point, and that therefore the restoring couple is correspondingly larger. This observation should be repeated with more widely varying moments of inertia.

VI. HEAT TREATMENT.

The following experiments are to be regarded merely as preliminary observations to indicate problems for future work on wires of this type. In the first place the question arose as to the effect of the annealing current, and as to what current, and hence what temperature, was necessary to restore the wire to the perfectly annealed condition. The first part of this present paper indicated that when the wire had been annealed, then vibrated through the usual amplitude of 360 degrees from the center, and then finally vibrated through a smaller angle, the change in period with amplitude was much larger for this small amplitude than it would have been if the preliminary large vibration had not occurred. The procedure in this present set of experiments was the following: (1) The wire was annealed to a bright red heat; (2) with a single twist, the suspended weight was turned through a displacement of 360 degrees from the center, and allowed to die down without timing; (3) a certain measured current was sent through the wire, care being taken not to allow any vibration during this process; (4) the pendulum was set into vibration through an amplitude of about 50 degrees, and timed in the usual way as the vibrations died down. No larger vibration than 50° was used, because a much larger value would annul the restoring effect of the current. This procedure constituted one experiment. In the next experiment, the first two steps were the same as those given above; the third step was the heating of the wire by a slightly larger current; the fourth step was the same as above. This was repeated with currents in

the third step ranging in value from zero to an intensity sufficient to raise the wire to red heat. In Fig. 13, in curves (*a*) to (*j*) inclusive, are given the results of this set of experiments, while in Table XV. are given for the corresponding curves the currents sent through the wire, and the resulting temperatures.

TABLE XV.

No.	Curve.	Current (Amperes).	Temperature (deg. Cent.).
1	<i>a</i>	0.00	22
2	<i>b</i>	0.50	72
3	<i>c</i>	1.02	205
4	<i>d</i>	1.21	262
5	<i>e</i>	1.29	290
6	<i>f</i>	1.38	318
7	<i>g</i>	1.47	406
8	<i>h</i>	2.03	648
9	<i>i</i>	2.50	851
10	<i>j</i>	2.70	930

In Fig. 14 are given the values of the logarithmic decrement for the preceding set of observations. It will be noticed in both figures that between curves (*d*) and (*e*), there has been a rapid change in both decrement and period accompanying a small increase in the temperature of the wire. Reference to Table XV. will show that the temperature corresponding to curve (*d*) was about 262 degrees, and that the change in temperature between that curve and (*e*) was only 28 degrees. It seems that a certain critical point had been reached in the neighborhood of these values. This is a fact that will receive further consideration. The temperatures were determined by an indirect method, but still by a method sufficiently accurate for preliminary experiments. The method used will be merely outlined. To equalize the effects of radiation, the room was kept at the same temperature that existed during the foregoing experiments. Various currents, covering more than the range used in the preceding experiments, were sent through the wire, and the fall of potential over the wire was measured by a potentiometer method in which the accuracy of the readings depended ultimately upon resistance readings. The current and the fall of potential

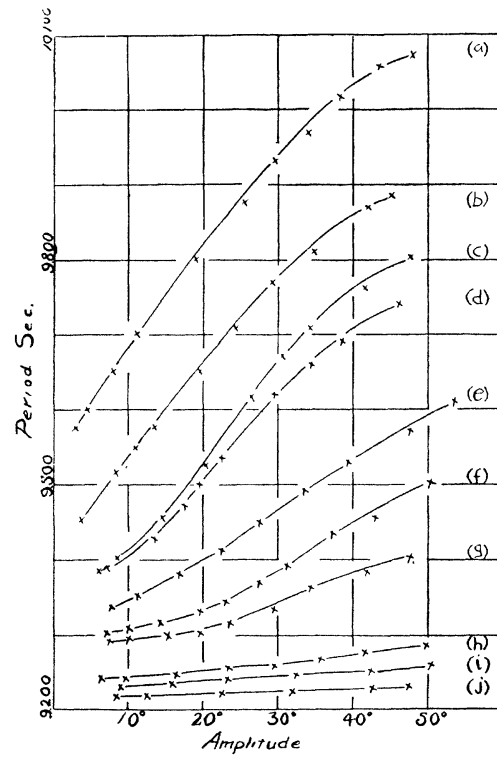


Fig. 13.

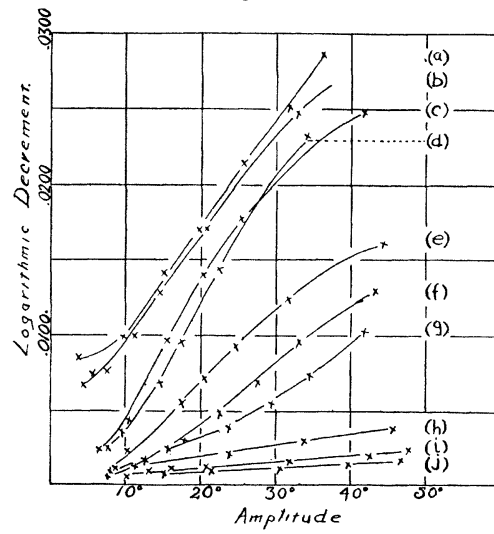


Fig. 14.

being known, the resistance could easily be calculated. This gave a graph of current and resistance. Then the wire was immersed in a bath of paraffin and the resistance for temperatures up to 280°C . determined. For values of the temperature above this point, the wire was removed from the paraffin, and calibrated by noting the melting points of crystals, observed under the microscope. The most satisfactory crystals used were sodium sulphate and potassium sulphate. The melting point of the former was taken at 880°C ., and of the latter at 1066°C . In this way a graph for temperature and resistance was obtained which, within the limits of error, was a straight line. With these two curves, one for resistance and current, and the other for resistance and temperature, it was of course simple to construct a graph of temperature and current.

It was thought desirable to determine the variations in the period-amplitude curves which the wire showed at different temperatures, and for this purpose an electric furnace was constructed. It was realized that to get any accurate results the entire vibrating system should be enclosed in the furnace, with a window provided for reading the periods and the amplitudes. However as time was lacking, as only preliminary results were aimed at, and as such a furnace as that described above was difficult to construct on short notice, a temporary heater was used which enclosed only the wire. This unfortunately proved quite inadequate, for on account of the necessary clearance at the top and the bottom, unavoidable convection currents were set up. While the results were probably not far in error at the lower temperatures, at the higher ranges there was such a wide variation in the temperatures at the different parts of the furnace that the results obtained were worthless. There was another source of error in these experiments, which was not realized at the time they were performed, and that was in connection with point mentioned in the earlier part of the paper. During the set of experiments with the furnace, the wire was not annealed after being set in position in the heater, and as such a procedure would lead to a gradually increased period, and as an increase in temperature would lead undoubtedly to the same general change in the period, the results obtained, even with a perfect furnace would be of no particular value. The proper procedure, granting a satisfac-

tory furnace, would be to anneal the wire to exactly the same temperature before each experiment, and then after the desired temperature had been reached to set the wire in vibration, always in exactly the same way. This method ought not only to give the temperature coefficient of the period at different amplitudes, but also ought to indicate whether or not there is any change with the temperature in the shape of the principal curves discussed in this paper. It is hoped that such an experiment can be tried in the near future.

VII. SUMMARY.

1. In a 40 per cent. platinum-iridium wire, the torsional period as a function of the amplitude depends to a marked degree upon the previous treatment of the wire. Various preliminary treatments and their effects are summarized below.

(a) If after annealing the wire at red heat observations for the relation of period to amplitude are taken, it is found, where the initial amplitudes in the several experiments are gradually increased, that there is an increasing range in the variation of the period, and that for any given amplitude there is an increase in the successive values for the periods. In this same set of observations, it was found that up to a certain amplitude the number of vibrations occurring while the pendulum dies down to an amplitude of 5 degrees steadily diminishes, notwithstanding the fact that the amplitudes continually increase. The values of the logarithmic decrements as functions of the amplitudes, increase with the increasing amplitudes, and at a certain amplitude for a given experiment this function reaches a maximum.

(b) If the order of the above experiments is reversed, *i. e.*, if the original amplitude is gradually diminished, the curves for period and amplitude as long as they pass over common ground are much more nearly alike than they were in (a) above. There is also in this set (b) a marked agreement among the logarithmic decrement curves. Finally, the number of vibrations occurring while the pendulum dies down to 5 degrees, is less the smaller the initial amplitude.

(c) If the general method outlined above is varied by following the large initial amplitude immediately with a very small amplitude,

a noticeable difference in the results is obtained. In this last experiment with the small amplitude, the period-amplitude curve is decidedly below the curve obtained in the experiment with the large amplitude, and further, the curves obtained by this gradual increase of the smaller amplitude tend to assume forms intermediate between those described in sections (a) and (b) above. This indicates a tendency for the wire to restore itself to the annealed condition after a short period of rest. This tendency was, to a certain extent, masked in the observations recorded under section (b).

2. A method of preliminary treatment of the wire was sought, which would put the wire invariably into such a state that similar successive experiments would each time give the same relation between period and amplitude. The facts observed as results of these attempts are summarized as follows:

(a) By keeping the wire in continuous vibration at nearly constant amplitude with the aid of the torsion head it was found that the periods increased at first, but that after sufficient vibration they approached constant values.

(b) In observations of the period, when the wire was allowed to die down naturally it was found that by preceding any observation for a given amplitude by a long continued vibration of the wire at that amplitude, the period was increased. Likewise, as a result of these preliminary vibrations, the form of the period-amplitude curve was changed, the period maintaining high values to lower amplitudes, and then more rapidly falling off with the further decrease in the amplitude. The increase in the period for a given amplitude, and the change in the form of the period-amplitude curves following the preliminary vibrations did not go on indefinitely, but in the case observed, reached after a vibration of two and one-half hours, practically constant values.

3. With constant load, the variation of the moment of inertia had a small but noticeable effect on the form of the period-amplitude curves; the larger the moment of inertia, the stronger was the tendency for the period to keep at high values while the amplitudes were decreasing, and then to fall off very rapidly as the amplitudes reached the lowest values.

4. The coefficient of rigidity of the wire was determined by a

static method, and when compared to the values determined by the kinetic method was found to agree with the latter more nearly when the values determined kinetically were for larger amplitudes. For the small amplitudes the values determined by the kinetic method were larger than those determined by the static method. These observations are not in agreement with the results obtained by Guthe and Sieg, who employed another method, and the point should be investigated further. For the coefficient of rigidity for a given amplitude, the values obtained by the kinetic method were found to increase with decrease in the moment of inertia.

5. Some preliminary experiments were tried in which different currents were passed through the wire, after the wire had been annealed to red heat and vibrated through a large amplitude. It was found that the greater this current through the wire, the greater was the restoring action of the resulting increase in temperature. When the temperature of the wire was raised by this current to about 275° C., there seemed to have been a marked acceleration of this restoring power.

In conclusion, the writer wishes to express his appreciation of the help and suggestions received from Professor K. E. Guthe, who suggested the problem, and from Professor G. W. Stewart, both of whom throughout the work have shown a most helpful interest.

PHYSICAL LABORATORY,
STATE UNIVERSITY OF IOWA,
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