the deepest interest. But I venture to think that when any expedition succeeds in reaching Mount Everest it will not be either from the east or north. My own impression is that far the simplest and shortest way is through Nepal. I am quite well aware that if political authorities were asked as to whether it was possible to go through Nepal we should hear at once that it was frankly impossible. I believe myself that this is the way in which finally Mount Everest will be reached.

I have only to ask you now to join in a vote of thanks to Major Noel for his very interesting and attractive lecture, and I hope at some time not very remote we shall be able to hear more about the proposed expedition to Mount Everest.

THE ADMIRALTY TIDE TABLES AND NORTH SEA TIDAL PREDICTIONS

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THE Admiralty Tide Tables were first published for the year 1833, and then contained the times of high water only for the four principal ports in the United Kingdom. By the year 1912 the information had increased to predictions for 54 ports, 26 in the British Islands and 28 in foreign countries and the Colonies: times and heights of both high and low water were predicted at 28 ports, 7 of which were in the British Islands, the times and heights of high water only being given at the remaining 26 ports. The Hydrographic Department does not undertake the calculation of all predictions given in the Admiralty Tide Tables, the present general rule being that predictions for ports in the British Islands and foreign countries are calculated in the Department, those for colonial ports being supplied by the authorities concerned.

The aim of the Tide Tables is to supply, by means of tidal predictions for standard ports and tidal differences for secondary ports on the standard ports, information which will enable seamen to make use of any port in the world independently of information obtained locally.

This aim was far from adequately fulfilled in the tables for 1912, the greater portion of the world being entirely unrepresented in the standard and secondary ports.

Although there had, between the years 1833 and 1912, been a great increase in information, there had been no corresponding increase in accuracy, so far at least as the ports predicted in the Hydrographic Department were concerned, the method of calculation being the same in the latter year as in the former; predictions for colonial ports, however, supplied by the colonial authorities, were in 1912 mostly calculated by the harmonic method. The method used in the Hydrographic Department was that introduced by the late Sir John Lubbock in about the year 1830 and described in the *Philosophic Transactions* for 1836; in this method as used in the Hydrographic Department tables showing the lunitidal intervals and heights of high and low water for all times of the moon's transit, with the declination and parallax of the moon and sun at their mean values, were calculated from tidal observations obtained at each port to be predicted, these intervals and heights being corrected for the actual declination and parallax of the moon and sun by means of tables calculated in accordance with Bernouilli's theory. The accuracy of these tables of corrections was tested by Sir John Lubbock by means of nineteen years' observations of the time and height of high water obtained at the London Docks, observation and theory being found to be in close agreement.

Predictions by this method fulfilled all requirements at the time of its introduction; the increase in size, speed and draft of vessels however necessitated increased accuracy long before the year 1912. The main defect in Lubbock's method is the entire omission of the diurnal tide; in about the year 1870, therefore, an attempt was made to remedy this by calculating a diurnal correction; very little success was attained. Spasmodic attempts at increased accuracy were continued, but by the year 1912 it had become evident that no real increase in accuracy was possible, and that nothing less than a radical change in method would meet requirements.

The only alternative method then available was the harmonic, which was therefore investigated with a view to its adoption. This method had met with undoubted success at ports in the tropics; there were, however, grave reasons for doubting its accuracy at ports in European waters. It had been tried by the Germans and rejected, the reasons for rejection being, as stated in their published Tide Tables, "the serious discrepancies which exist between predictions and observations"; and predictions for ports in the United Kingdom, calculated by this method, published in the Tide Tables of the United States Coast and Geodetic Survey, were even less accurate than those by Lubbock's method in the Admiralty Tide Tables. The U.S. predictions were, however, mostly calculated from insufficient observations, and but little importance could be attached to their lack of accuracy; the deliberate German statement, on the other hand, appeared to be of great importance.

Extended investigation, whilst proving that a very high degree of accuracy was reached at ports out of Europe, brought to light certain minor but irremediable defects, only one of which need be mentioned. To the seaman the most important part of tidal predictions is the time of high water; for actual depth of water at any place at any moment accurate knowledge of the times and heights of both high and low water is essential; but as, in Admiralty publications, the turn of the tidal stream is always referred to the time of high water, this time is doubly important. It was almost invariably found that, in harmonic predictions, the heights reached a considerably higher degree of accuracy than the times, the probable reason being that, as the times depend on the amplitudes of the component waves, small errors in these amplitudes, due possibly to the short length of the series of observations usually analyzed, cause considerable errors in the actual moment of high or low water without appreciably affecting the heights.

A point which has since been noticed, but which may be mentioned here, is the fact that harmonic predictions obtained by different authorities by means of different machines, but using the same set of constants, may differ materially. The predictions for London and for Panama, published in the Tide Tables of the United States Coast and Geodetic Survey and in the Admiralty Tide Tables, are calculated from the same sets of constants, yet differences in time of as much as fifteen minutes exist, showing that harmonic predictions are affected by either personal or instrumental errors, or both.

The decision reached at the conclusion of the investigation was to make a commencement with harmonic predictions, but at the same time to attempt to evolve a new method which, whilst retaining the advantage, the separate and independent prediction of times and heights, of Lubbock's method, would give the desired degree of accuracy. In consequence of this decision Mr. Edward Roberts was employed to calculate harmonic predictions for certain ports, investigation of a new method being continued in the Hydrographic Department.

Predictions for Dover, London, Immingham, Stromness, Oban, Liverpool, and Avonmouth, calculated by the harmonic method, are now included in the Admiralty Tide Tables, and these predictions reach a high degree of accuracy. Success has also been attained in the investigation of a new method of prediction, and predictions for 19 ports, 15 in the British Islands, 3 in Germany, and 1 in the Netherlands, based on this method, which has been called the Equation method, will be included in the Tide Tables for 1920 now in course of preparation. In the tables for 1919 predictions for 18 ports depend on this method.

In addition to the increase in accuracy, due to the change in methods of prediction, there has been a considerable increase in the quantity of information given between the Tide Tables for 1912 and those for 1920. The tables for 1912 included, as mentioned above, predicted times and heights of high and low waters at 28 standard ports and times and heights of high water at 26 standard ports. Differences in time and height of high water on standard ports for some 220 secondary ports were also given. The tables for 1920 will include predicted times and heights of high and low water at 62 standard ports, and differences, in most cases of time and height of both high and low water, on standard ports, for some 2000 secondary ports.

Though improvements of considerable magnitude have thus been made in the Tide Tables, much work yet remains, neither standard nor secondary ports in the United States or in the greater part of South America, Africa, and Eastern Asia being given. Work has been delayed by war requirements, also by investigation of methods of prediction; but as war requirements are reaching their end, and as investigations are for the time being completed, improvements should now proceed with increasing rapidity.

Before describing the equation method of tidal predictions, some information as to the degree of accuracy attained in the prediction of the tides at ports in the British Islands may be of interest. The proper judging of the accuracy of tidal predictions is not a simple matter. The tides are affected by meteorological conditions, and, unless corrections for these conditions are applied, there is no means of telling whether differences between predictions and observations are due to errors in prediction or to meteorological causes. Tables of corrections for meteorological conditions have not been calculated for any port in the British Islands; as the predictions include tidal changes due to seasonal variations in wind, rainfall, and barometric height, the calculation of such tables involves the comparison of a very considerable series of observations and predictions, separate tables being required for each month, or, at any rate, for each season. No time has been available for this work.

Tidal predictions calculated from the short series of observations usually employed are not exact; to obtain, therefore, a true measure of the relative accuracy of different methods, predictions by each method should be calculated for the same port from the same series of observations, and the same days' predictions should be compared in each case. This again requires much time, which has not been available

There is no standard method of ascertaining the accuracy of predictions; the method usually employed is to count all predictions which differ from observations by not more than ten minutes in time or 1/10 of the range of the tide in height as correct: all others as incorrect. This method is not satisfactory, for predictions correct within 5 minutes are of more value than those within 10 minutes, and so long as the error does not exceed 15 minutes the predictions are of some value. The height allowance of 1/10 of the range is also unsatisfactory; this may, at St. Helier or Avonmouth, for instance, exceed 4 feet, and a predicted height 4 feet in error is of no value anywhere.

The following method of comparison is now used in the Hydrographic Department: summer tides only are compared, thus eliminating, as far as possible, meteorological disturbances; a maximum of 3 marks is given for each predicted time which differs by not more than 5 minutes from the observed time, and for each height which differs by not more than 0·5 foot from the observed height; 2 marks are given for times which differ by more than 5, but not more than 10 minutes, from the observed time, and for heights which differ by more than 0·5 foot and not more than 1 foot from the observed height; where differences are greater than 10 minutes or 1 foot, but not more than 15 minutes or 1·5 feet, 1 mark is given; for differences of more than 15 minutes or 1·5 feet no marks are given. The total obtained is then finally given in the form of a percentage of the possible total.

For the comparison of the relative accuracy of the harmonic and equation methods the following ports have been chosen:

1. Liverpool, predicted by the harmonic method from eight years' observations, 1857–1860, 1867–1870, and 1902.
2. Immingham, predicted by the harmonic method from one year's observations, 1912.
3. River Tees entrance, predicted by the equation method from one year's observations, 1897.
4. River Tyne entrance, predicted by the equation method from one year's observations, 1915–16.

The comparisons in each case covered three months, May to July 1918, except at Immingham, where comparisons were made for May to July 1917.

Results are given for comparisons both by the Hydrographic Department method and by that usually employed, and are:

Liverpool.	90 per cent. correct within 10 minutes or 1/10 range.
	81·5 per cent. by Hydrographic Department method.
Immingham.	91 per cent. correct within 10 minutes or 1/10 range.
	82·8 per cent. by Hydrographic Department method.
River Tees.	95 per cent. correct within 10 minutes or 1/10 range.
	84·4 per cent. by Hydrographic Department method.
River Tyne.	98 per cent. correct within 10 minutes or 1/10 range.
	90·4 per cent. by Hydrographic Department method.

A high degree of accuracy is thus reached in all cases, results by the equation method being slightly the better; it must, however, be admitted that the equation method decreases in accuracy as the diurnal tide increases and cannot be used at all at places where single day tides occur, and is therefore valueless in many parts of the world. It should perhaps also be stated that, in the comparisons given, the predicted tides for Liverpool were compared with the times and heights of high and low water as supplied by the Mersey Docks and Harbour Board; those for Immingham, the River Tees, and the River Tyne were compared with the original automatic tide-gauge records.

The equation method of prediction has been of considerable value during the past four years. Prior to 1914, in the course of the ordinary exchange of Hydrographic information, tidal predictions were exchanged with the German Hydrographic Department, predictions for German ports included in the Admiralty Tide Tables being supplied by the German Hydrographer in exchange for predictions for British ports included in the German tables. In 1914 this source of supply ceased but accurate predictions for German ports were of almost vital importance. The ports which it was desired to predict were Wilhelmshaven, Cuxhaven, and Heligoland, the material available being:

Wilhelmshaven: Harmonic constants obtained from German publications; these were, in all probability, the constants at one time used for the German predictions which were said to be unsatisfactory.

Cuxhaven: Incomplete observations of the times and heights of high and low water for the years 1841, 1842, and 1843.

Heligoland: Incomplete hourly ordinates for the year 1880, obtained from the records of an automatic gauge which the German Government had been allowed to establish while the island was still in British occupation, together with a few observations obtained by Admiralty surveyors in the summer of 1855.

These could hardly be regarded as satisfactory data on which to base important predictions, but inquiry led to the obtaining, in addition, of harmonic constants for Heligoland, calculated by the U.S. Coast and Geodetic Survey, derived from four complete years' hourly ordinates, 1882, 1883, 1894, and 1895.

As the only available information for Wilhelmshaven and Heligoland consisted of harmonic constants, it was decided to use the harmonic method for predicting the tides at all three ports, and Mr. Roberts was convinced of his ability to produce accurate predictions from the material available. The predictions, when calculated, were found to differ very materially from those calculated in Germany for the use of the German fleet; time differences of an hour or more were found at all ports; and the height differences were also very considerable, the differences both in time and in height exhibiting great similarity in all cases. As a check on accuracy, harmonic predictions for Heligoland for the summer of 1855 were then calculated and compared with the Admiralty surveyors' observations of that year. Differences were again large and similar to those existing between the harmonic and German predictions.

The exact method by which the German predictions are calculated is not known, but it is known that it is not harmonic and that the only port for which direct predictions are calculated is Wilhelmshaven, where they depend on seven years' observations, 1903–1909, predictions for other ports being calculated from the Wilhelmshaven predictions by means of variable differences obtained from several years' observations; as observations were not available, except for Cuxhaven, it was decided to calculate predictions for Wilhelmshaven by the equation method, using the German predictions as basis, and to calculate Cuxhaven and Heligoland by means of variable differences on Wilhelmshaven obtained from the German predictions.

Assuming the German predictions to be accurate, complete success was obtained, the Admiralty figures agreeing almost exactly with those calculated in Germany; but as a final test predictions for Heligoland were calculated for the summer of 1855, for comparing with the Admiralty surveyors' observations. Results showed a considerable improvement on the harmonic predictions, but the expected degree of accuracy was not attained. By the Admiralty method of marking, the accuracy of the harmonic and equation methods of prediction, respectively, from comparison with the 1855 observations was—harmonic 59·7 per cent., equation 70·4 per cent. By the same method of marking, comparing with the German predictions for Heligoland, accuracy was—harmonic 62·1 per cent., equation 96·8 per cent.

It is evident from these figures that the German predictions and the 1855 observations are not in exact agreement, but, as the observations on which the predictions depend were obtained at the south end of the island whereas the 1855 tidal station was near the north point, very little importance can be attached to the differences which exist; the German predictions are, in fact, known to reach a reasonable degree of accuracy.

Differences between the German and harmonic predictions have been partially analyzed, and it has been found that, though there are errors of other periods which cannot be easily placed, the principal error in the time of high water has a period of half a synodic month, and is therefore in the phase inequality. This error varies with approximate regularity from about zero to about +40 minutes, and the predicted times of high water could be materially improved by applying a periodical correction of this nature; but as the corrections required to the times of high and low water differ, and the heights require separate corrections, and as the predictions after correction do not attain the accuracy of those by the equation method, harmonic predictions for German ports have been discontinued.

Tidal predictions for the Admiralty Tide Tables are prepared some years in advance, and those for German ports in the Tables for 1916, 1917 and 1918 were calculated by the harmonic method; special predictions for those years were issued to H.M.'s ships and it was not considered necessary to correct those published in the Tide Tables, but the German Admiralty have considered that correction should be made, the following being a translation of a portion of Berlin Notice to Mariners, No. 1319/18:

"English Tide Tables.—When using English Tide Tables it should be noted that the predictions for Wilhelmshaven are calculated by means of harmonic constants. The use of harmonic constants for Wilhelmshaven gives values which are inadmissible owing to their inaccuracy. The errors amount to as much as 45 minutes."

Harmonic predictions for German ports can thus not be regarded otherwise than as a failure; the ascertaining of the true reason for failure is a matter of considerable importance, for unexpected failure in one case is liable to lead to doubts as to accuracy in other cases. The great majority of the ports hitherto predicted by the harmonic method have deep-water approaches, very few being situated on rivers or shallow estuaries. Exceptions to this are the ports on the River Hugli included in the Tide Tables published by the Government of India, Liverpool, Avonmouth, Immingham, and possibly also some United States ports. The predictions for the Indian Tide Tables are calculated at the National

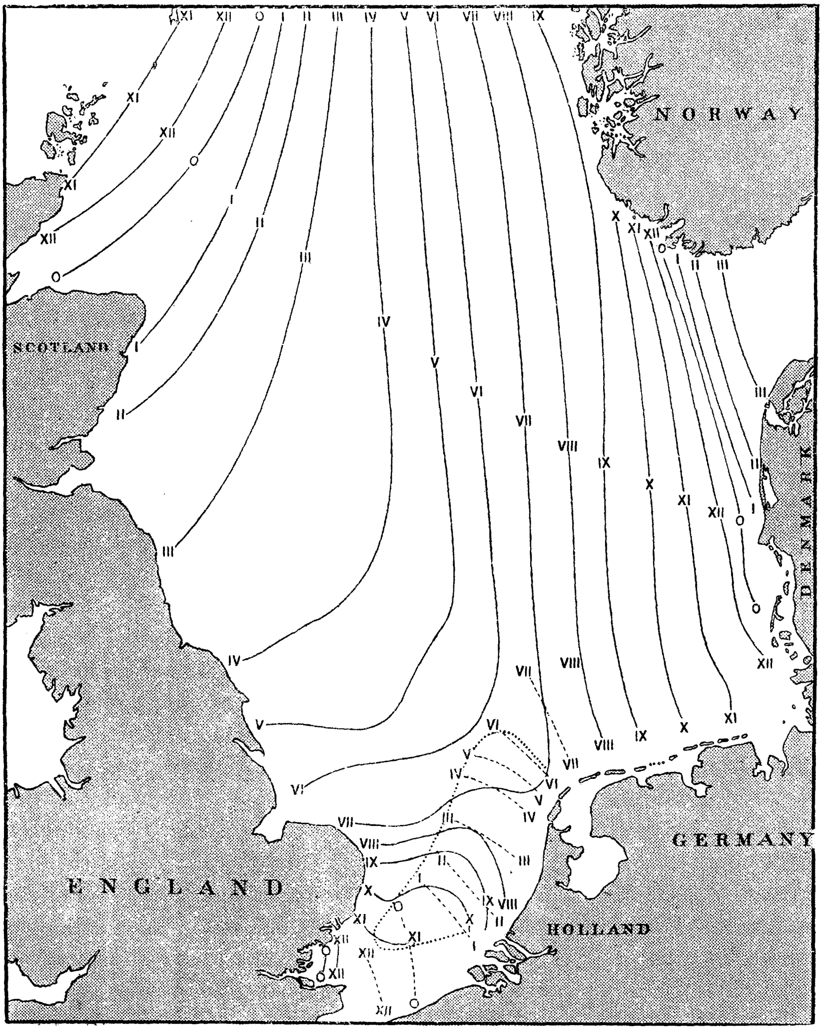
Physical Laboratory, Teddington, those for the English ports by Mr. Roberts; and in both cases non-harmonic corrections to the low-water predictions have been found necessary. Mr. Roberts, I believe, applies such corrections at Avonmouth only, though the Liverpool predictions would probably be improved by their use, for at this port the errors are considerably greater in the low than in the high water predictions. Partial failure of harmonic predictions, owing, in all probability, to the extreme alteration in the shape of the tide wave due to shallow water and other local circumstances, is thus not entirely unknown.

At the German ports tidal conditions are even more involved; not only is the shape of the wave changed by shallow water, but the tides on the German North Sea coast are due to the combination of two separate and distinct waves.

The co-tidal lines shown on the chart of the North Sea have been taken from an Admiralty chart, and are founded on observations obtained by Admiralty surveyors; the observations are incomplete, especially in the northern portions, and a considerable amount of filling in was required in compiling the chart; the main features of the progress of the North Sea tide waves were, however, distinctly shown by the observations. The advance of the northern wave is shown by continuous lines, that of the southern wave by pecked lines; the Roman figures give the lunitidal interval on full and change days.

As the chart shows, the North Sea is affected by two tide waves, one entering through the Straits of Dover, causing high water on the coasts of Belgium and the Netherlands, the other entering between Scotland and Norway, causing high water on the coasts of Scotland, England, Norway, and Denmark. Between the coasts of Norfolk and Suffolk and the Netherlands these waves meet, and an area accordingly exists in this locality in which double tides occur, the approximate limit of double tides being within the area surrounded by a dotted line. Double tides occur at places on the coast of the Netherlands, double low waters at the Hook of Holland and double high waters at the Helder; and double high waters are not unknown in the entrance to the River Thames, but generally speaking the double tides do not reach the shore. Near the centre of the area of double tides the times of high water due to the two waves are about six hours apart, and consequently there is no regular tide at all; it is interesting to note that the existence of this area was originally suspected by Whewell, and confirmed by observations by Captain Hewitt, R.N., in the year 1840. Unfortunately the original observations, which were obtained by measuring the depth of water at a moored boat, though undoubtedly still in existence, cannot at present be found, and exact particulars can therefore not be given. The region of no tide was placed by Captain Hewitt some distance to the northward of that resulting from modern observations, but as the observations now available are insufficient for settling exactly the boundaries either of the region of no tides or that of double tides, no importance can be attached to this discrepancy. It is even possible, in fact probable, that the boundaries are not fixed and differ at different periods of a lunation and at different seasons.

Off the north coast of the Netherlands, on a line situated in about



Co-tidal lines in the North Sea

6° E. and running nearly north and south, the waves combine, and the tides of the bay formed by the coasts of Germany and Slesvig are due to the combined wave.

That the ratio between the solar and the lunar tides varies in different regions is a well-known fact; the variations are probably greater in the

North Sea than in any other area of similar size. At Ostende, in the southern tide wave, the ratio of the solar to the lunar wave is 1 : 4·6; at Thurso, in the northern wave, the ratio is 1 : 2·5. Assuming that these ratios for the southern and northern waves remain constant, and that the lunar components of the waves are of equal magnitude, then, at springs the ratio, in the combined wave, of northern to southern would be 1·2 : 1, the corresponding ratio at neaps being 0·8 : 1, the combined wave being thus predominantly northern at springs and southern at neaps. It is probable that this change in composition of the wave accounts for that part of the difference between harmonic prediction and observation at German ports which has a period of semi-lunation, and the error could possibly be eliminated by the inclusion of a special constant. This would, however, though increasing accuracy, not be sufficient, and, if the theory of different composition of the wave at springs and at neaps is correct, additional special constants would be required for each of the more important harmonic constants.

Though this theory may be correct as to the spring and neap variation it is not correct as to amount, for, if the ratios given for the northern and southern waves were correct, the resulting average ratio on the German coast of the solar to the lunar tide would be about 1 : 3·5, whereas it is at Heligoland 1 : 5·5 and at Wilhelmshaven 1 : 7·8. These values have been calculated from the phase inequality of height; at Wilhelmshaven the ratio of the solar to the lunar tide, as calculated from the phase inequality of time, is 1 : 2·7 only.

In the harmonic method of tidal prediction the analysis includes the whole tide wave; in the equation, as in Lubbock's, method only the lunitidal intervals and heights of high and low water are dealt with, and prediction consists in the application to the mean lunitidal intervals and heights of corrections for phase inequality, and for changes in the parallax and declination of the moon and sun.

Lubbock's method depends on Bernouilli's theory, and therefore, at any rate when first introduced, assumed a constant ratio between solar tides, lunar tides, and changes due to changes in declination and parallax; the corrections made to the lunitidal intervals for phase-inequality, parallax, and declination were therefore the same at all places, but the corresponding height corrections varied with the mean range of the tide. Corrections calculated by theory were tested by means of corresponding corrections calculated from nineteen years' observed times and heights of high water at London; the phase correction was found to differ materially in observation and theory; the parallax and declination corrections were, on the other hand, in close agreement. In predictions by this method, therefore, the mean lunitidal intervals and heights and the phase inequality corrections were obtained from observations at each port predicted; the parallax and declination corrections, on the other hand, were calculated by theory, and were the same for all ports.

It is unfortunately a fact that the London corrections, as calculated by Sir John Lubbock, agree very closely with Bernouilli's theory, and that diurnal inequality is practically non-existent in the London high-waters, for these facts led to the assumption that the tides at all ports could be predicted by this method, and that the omission of the diurnal tide was of no importance. This method was therefore used for predicting tides not only in the British Islands, but also at Georgetown, British Guiana, and, though predictions were never printed, at Hong Kong.

Shortly after the publication of Lubbock's tables in the *Philosophical Transactions* for 1836, Whewell calculated corresponding tables from tidal observations at Devonport. These tables differed materially from Lubbock's, but, though they were used for predicting the Devonport tides, it appears to have been assumed either that these tides were exceptional, or that, as the time corrections were calculated from only three, and the height corrections from only five, years' observations, the tables were inaccurate. Whewell also calculated a simple table of diurnal corrections to the heights at Devonport. The publication of these Devonport tables appears to have caused some uneasiness in the Hydrographic Department, for the calculation of corrections from seven years' observations at Leith was commenced; this work was never completed, and Lubbock's tables continued in use for all ports in the United Kingdom predicted in the Admiralty Tide Tables, with the exception of Devonport.

That all was not well was known, and from time to time attempts at correction were made. The only attempts, however, which reached any practical results were the calculation, by Captain W. N. Goalen, R.N., on a method of his own, of tables for predicting the Harwich tides, and of a table of diurnal corrections to the heights at Portsmouth. Both of these were adopted, and added materially to the accuracy of the predictions for these ports.

In Lubbock's method it is assumed that the ages of the phase, declinational, and anomalistic inequalities are the same. This is not correct, and, though alterations in method could no doubt be made, and accurate tables of all necessary corrections calculated, the period of observations required would, owing to the necessity for calculating corrections for both increasing and decreasing declination and parallax, be long, probably at least ten years. There are not many ports for which so long a series of observations is available, and were all necessary observations obtained, the labour of dealing with them would be enormous; it was therefore decided to attempt the calculation of corrections from a comparatively short series of observations and by a new method.

The tide wave, and changes or inequalities, to be considered in a method of prediction where times and heights are separately considered, are:

(*a*) The mean lunar semidiurnal wave.

(*b*) The phase-inequality, or effect of the mean solar semidiurnal wave on the mean lunar semidiurnal wave. Period, half a synodic month.

(*c*) The lunar tropical diurnal inequality, or diurnal effect on the phase-inequality of the changes in the moon's declination. Period, one tropical month.

(*d*) The lunar tropical semidiurnal inequality, or semidiurnal effect on the phase-inequality of changes in the moon's declination. Period, half a tropical month.

(*e*) The lunar anomalistic semidiurnal inequality, or effect on the phase-inequality of changes in the moon's parallax. Period, one anomalistic month.

(*f*) The solar tropical diurnal inequality, or diurnal effect on the phase-inequality of changes in the sun's declination. Period, one solar year.

(*g*) The solar tropical semidiurnal inequality, or semidiurnal effect on the phase-inequality of changes in the sun's declination. Period, half a solar year.

(*h*) The solar anomalistic semidiurnal inequality, or effect on the phase-inequality of changes in the sun's parallax. Period, one solar year.

(*k*) The annual variation in height, due in all probability to seasonal meteorological changes. Period, one solar year.

The mean lunar tide and phase-inequality are easily found, by a system of means, from a series of tidal observations extending over a year or more; if the mean tide for any time of moon's transit be deducted from an observed tide of the same series and time of transit, the remainder will be the "total correction" or sum of inequalities (*c*) to (*k*) for the time of moon's transit, date, parallax, and declination of the moon and sun.

The solar inequalities (*f*) to (*h*) are known to be small, and the finding of each separately complicates the analysis; all but (*g*) have a period of a solar year, and, as this is also the period of (*k*), their inclusion with (*k*) in a single solar inequality will involve but little error and will simplify calculations. This inequality will vary with the date.

The attempt to divide the total correction into inequalities is also complicated by the fact that it contains an unknown correction due to temporary meteorological conditions, and by the minutes in the time of moon's transit and the change in the moon's parallax and declination in the interval between the upper and lower transits. In order, as far as possible, to overcome these difficulties, the whole series of observations, which should extend over fourteen synodic months, is plotted on section paper, the diagram showing date; time of moon's transit, upper in black, lower in red; lunitidal intervals and heights of high and low water, tides following upper transit in black, lower transit in red; moon's declination, north in black, south in red; and moon's parallax. Mean curves of intervals and heights following the upper and lower transits of the moon are then drawn through the points plotted, thus eliminating to some extent the effect of temporary meteorological disturbances, and from these curves the intervals and heights corresponding to exact hours of moon's transit may be obtained, the tides following the upper and lower transits appearing simultaneous.

From the diagram twelve tables are compiled, for moon's transits 0^h or 12^h, 1^h or 13^h, 2^h or 14^h . . . 11^h or 23^h, each table giving date, time of moon's transit, moon's declination and parallax, and the lunitidal intervals and heights of high and low water. In the tables the colours of the diagram are followed, and the moon's declination and parallax are further distinguished as "increasing" and "decreasing."

The colours black and red are used in order that the lunar tides may be separated into "like" and "unlike." Like tides occur after moon's upper transit with north declination (both black) and lower transit with south declination (both red); unlike tides occur after moon's upper transit with south declination (black and red), or lower transit with north declination (red and black). The colours of transit and declination thus distinguish tides into like and unlike; the colours of the tides, however, distinguish those following the moon's upper transit (black) from those following the lower transit (red).

Each table consists of twenty-eight dates, half a synodic month apart, with one lunar declination and parallax, two transits, and two tides for each date.

The high and low water lunitidal intervals and heights in each table are now meaned, red and black separately, the mean giving, for each table, a value for mean lunar tide $\pm$ phase-inequality for the tides following the upper and lower transits, the difference between each mean and the quantities meaned giving total correction according to time of transit, upper or lower, date, declination and parallax, like or unlike tide.

On any date a, time of moon's upper transit T^h, lower transit $T + 12^h$, declination Δ° increasing, parallax (measured from the mean parallax) P' increasing: if x and x' are the total corrections to tides following upper and lower transit, D and d the diurnal and semidiurnal corrections for moon's declination, p the correction for moon's parallax, s the solar correction,

$$(1)\ x = (d + D) + p + s \text{ (Like tides).}$$
$$(2)\ x' = (d - D) + p + s \text{ (Unlike tides).}$$

At the next date, half a synodic month from a, assuming the synodic, tropical, and anomalistic months to be of equal length and each to be a very small proportion of a solar year, the moon's declination will be $-\Delta^\circ$ increasing and parallax $-P'$ increasing, and if x'' and x''' are the total corrections following the moon's upper and lower transits,

$$(3)\ x'' = (d - D) - p + s \text{ (Unlike tides)},$$
$$(4)\ x''' = (d + D) - p + s \text{ (Like tides)}.$$

After the lapse of $6\frac{1}{2}$ synodic months from α the moon's declination and parallax will be nearly as at α, and if X and X′ are the total corrections following the moon's upper and lower transits, assuming that $6\frac{1}{2}$ synodic = 6 solar months,

$$(5)\ X = (d + D) + p - s \text{ (Like tides)}.$$
$$(6)\ X' = (d - D) + p - s \text{ (Unlike tides)}.$$

Seven synodic months from α, X″ and X‴ being the total corrections following the moon's upper and lower transits,

$$(7)\ X'' = (d - D) - p - s \text{ (Unlike tides)}.$$
$$(8)\ X''' = (d + D) - p - s \text{ (Unlike tides)}.$$

From these eight equations, on which the equation method of predicting tides depend, and at date α and $\alpha + 6\frac{1}{2}$ synodic months,

$$p = \tfrac{1}{16}\{(x - x''') + (x' - x'') + (X - X''') + (X' - X'')\}$$
$$(d + D) = \tfrac{1}{4}\{(x + x''') + (X + X''')\},\ (d - D) = \tfrac{1}{4}\{(x' + x'') + (X' + X'')\}$$
$$s = \tfrac{1}{8}\{(x + x''') - (X + X''') + (x' + x'') - (X' + X'')\}$$

As the synodic, tropical and anomalistic months are not exactly equal, the corrections for each date should be calculated for half a synodic month back as well as half a synodic month on; the corrections $(d + D)$ and $(d - D)$ accepted as corrections for the mean declination at dates α and $\alpha + 6\frac{1}{2}$ synodic months, p should be accepted as the correction

TABLE I.

As obtained from Diagram.

(All Intervals +2 days.)

Date.	Moon's			High water.				Low water.			
	Transit.	Dec.	H.P,	Interval.	Corr.	Height.	Corr.	Intetval.	Corr.	Height.	Corr.
	H. M.	°	′	H. M.	M.	F.	F.	H. M.	M.	F.	F.
16 Sept. 1915	18 00	*27*	59·2 d.	5 18	+ 7	10·6	− 0·7	12 04	+ 27	2·0	− 2·0
	6 00			*5 35*	+ 20	*10·7*	− 0·5	*11 55*	+ 17	*3·2*	− 0·9
1 Oct.	6 00	26 d.	54·5 i.	5 30	+ 19	9·1	− 2·2	11 58	+ 21	4·6	+ 0·6
	18 00			*5 15*	0	*9·6*	− 1·6	*12 07*	+ 29	*3·7*	− 0·4
16 Oct.	18 00	*24* d.	58·8 d.	5 07	− 4	10·7	− 0·6	11 58	+ 21	1·9	− 2·1
	6 00			*5 33*	+ 18	*10·7*	− 0·5	*11 52*	+ 14	*3·8*	− 0·3
25 Mar. 1916	6 00	*26* d.	59·2 i.	4 50	− 21	12·5	+ 1·2	11 30	− 7	3·9	− 0·1
	18 00			*5 10*	− 5	*12·2*	+ 1·0	*11 40*	+ 2	*5·5*	+ 1·4
10 April	18 00	24 d.	54·2 i.	5 34	+ 23	11·1	− 0·2	11 44	+ 7	6·0	+ 2·0
	6 00			*5 20*	+ 5	*11·1*	− 0·1	*11 41*	+ 3	*4·9*	+ 0·8
24 April	6 00	*21* d.	59·2 d.	5 00	− 11	11·7	+ 0·4	11 41	+ 4	2·0	− 2·0
	18 00			*5 33*	+ 18	*11·3*	+ 0·1	*11 38*	0	*3·9*	− 0·2

Means (as derived from complete table) { 5 11 — 11·3 — 11 37 — 4·0
{ *5 15* — *11·2* — *11 38* — *4·1*

Note: figures printed in italics are to be considered RED.

Y

for the mean parallax at α and $\alpha + 6\frac{1}{2}$ synodic months, s as the solar correction at α, and at $\alpha + 6\frac{1}{2}$ synodic months.

As an example the tides necessary for calculating the corrections for one lunar declination, one parallax and two dates have been taken from the tidal diagram by means of which the tables for predicting tides at the River Tyne entrance were calculated and the corrections worked out.

TABLE II.

Total corrections.

Date.	Moon's			Like tides.				Moon's transit.	Unlike tides.			
				High water.		Low water.			Low water.		Low water.	
	Transit.	Dec.	H.P.	Int.	Height.	Int.	Height.		Int.	Height.	Int.	Ht,
	H.	°	′	M.	F.	M.	F.	H.	M.	F.	M.	F.
16 Sept. 1915	6	27	59·2 d.	÷ 20	− 0·5	+ 17	− 0·9	18	+ 7	− 0·7	+ 27	−2·0
1 Oct. ,,	6	26 d.	54·5 i.	+ 19	− 2·2	+ 21	+ 0·6	18	0	− 1·6	+ 29	−0·4
16 Oct. ,,	6	24 d.	58·8 d.	+ 18	− 0·5	+ 14	− 0·3	18	− 4	− 0·6	+ 21	−2·1
25 Mar. 1916	18	26 d.	59·2 i.	− 5	÷ 1·0	+ 2	+ 1·4	6	− 21	+ 1·2	− 7	−0·1
10 April ,,	18	24 d.	54·2 i.	+ 23	− 0·2	+ 7	+ 2·0	6	+ 5	− 0·1	+ 3	+0·8
24 April ,,	18	21 d.	59·2 d.	+ 18	+ 0·1	0	− 0·2	6	− 11	+ 0·4	+ 4	−2·0

TABLE III.

Half sum of lines from Table II.

Date.	Moon's		Like tides.				Moon's transit.	Unlike tides.			
			High water.		Low water.			High water.		Low water.	
	Transit.	Dec.	Int.	Height.	Int.	Height.		Int.	Height.	Int.	Height.
	H.	°	M.	F.	M.	F.	H.	M.	F.	M.	F.
1 Oct. 1915	6	26 d.	+ 19·5	− 1·35	+ 19·0	− 0·15	18	+ 3·5	− 1·15	+ 28·0	− 1·20
1 Oct. 1915	6	26 d.	+ 18·5	− 1·35	+ 17·5	+ 0·15	18	− 2·0	− 1·10	+ 25·0	− 1·25
10 April 1916	18	24 d.	+ 9·0	+ 0·40	+ 4·5	+ 1·70	6	− 8·0	+ 0·55	− 2·0	+ 0·35
10 April 1916	18	24 d.	+ 20·5	− 0·05	+ 3·5	+ 0·90	6	− 3·0	+ 0·15	+ 3·5	− 0·60
1 Oct. 1915	6	26 d.	+ 19·0	− 1·35	+ 18·2	0·00	18	+ 0·7	− 1·12	+ 26·5	− 1·22
10 April 1916	18	24 d.	+ 14·7	+ 0·17	+ 4·0	+ 1·30	6	− 5·5	+ 0·35	+ 0·7	− 0·12

TABLE IV.

Correction for Moon's Declination.

(Half sum of last lines of Table III.)

Moon's Dec.	Like tides.				Unlike tides.			
	High water.		Low water.		High water.		Low water.	
	Int.	Height.	Int.	Height.	Int.	Height.	Int.	Height.
°	M.	F.	M.	F.	M.	F.	M.	F.
25 d.	+ 16·8	− 0·59	+ 11·1	+ 0·65	− 2·4	− 0·38	+ 13·6	− 0·67

TABLE V.

Solar Correction.

(Half difference of last lines of Table III.)

Date.	Mean tides. High water. Int.	Mean tides. High water. Height.	Mean tides. Low water. Int.	Mean tides. Low water. Height.
	M.	F.	M.	F.
1 October	+ 2·1	− 0·76	+ 7·1	− 0·65
1 October	+ 3·1	− 0·73	+ 12·9	− 0·55
10 April	− 2·6	+ 0·75	− 10·0	+ 0·60
1 October	+ 2·6	− 0·75	+ 10·0	+ 0·60

TABLE VI.

Correction for Moon's Parallax.

(Half difference of lines from Table II.)

Moon's H.P.	Mean tides. High water. Int.	Mean tides. High water. Height.	Mean tides. Low water. Int.	Mean tides. Low water. Height.
′	M.	F.	M.	F.
54·5 i.	− 0·5	− 0·85	+ 2·0	+ 0·75
54·5 i.	− 3·5	− 0·45	+ 1·0	+ 0·80
54·5 i.	+ 0·5	− 0·85	+ 3·5	+ 0·45
54·5 i.	+ 2·0	− 0·50	+ 4·0	+ 0·85
54·2 i.	+ 14·0	− 0·60	+ 2·5	+ 0·30
54·2 i.	+ 13·0	− 0·65	+ 5·0	+ 0·45
54·2 i.	+ 2·5	− 0·15	+ 3·5	+ 1·10
54·2 i.	+ 8·0	− 0·25	− 0·5	+ 1·40
54·35 i.	+ 4·5	− 0·54	+ 2·6	+ 0·76

From the complete diagram and 12 complete tables, 12 values are obtained for lunar tide ± phase-inequality for the tide following the moon's upper transit, 12 values for the tide following the lower transit; values for $(d + D)$ and $(d - D)$ for 13 declinations, values of p for 13 parallaxes and values of s for 26 dates in each of the 12 tables. These values are plotted on squared paper and curves drawn, the complete tables of corrections compiled from the curves giving

I. Mean lunar tide ± phase-inequality for all times of moon's upper and lower transits.

II. Corrections for moon's declination for like and unlike tides for all declinations and times of moon's transit.

III. Corrections for moon's parallax for all parallaxes and times of moon's transit.

IV. Solar corrections for all dates and times of moon's transit.

In predicting tides by means of these tables the algebraic sum of the quantities obtained by entering with

I. Moon's transit, upper or lower.

II. Moon's transit and declination, like or unlike tide.

III. Moon's transit and parallax.

IV. Moon's transit and date

gives the lunitidal intervals and heights of high and low water, the lunitidal intervals added to the times of transit giving the times.

A complete example requires the complete tables, the following, working back to Table I, however, shows the method, though the corrections for moon's declination and parallax are not exact.

Required the lunitidal intervals and heights on October 1, moon's upper transit 6^h, lower transit 18^h, declination 26° decreasing and parallax 54·5 increasing.

Upper transit 6^h. (Like tides.)

	H.	M.	F.	H.	M.	F.
Mean intervals and heights ...	5	11	11·3	11	37	4·0
Correction for dec. 26° d. ...		+ 17	− 0·6		+ 11	+ 0·6
,, ,, H.P. 54·5 i. ...		+ 4	− 0·5		+ 3	+ 0·8
Solar correction 1 Oct. ...		+ 3	− 0·7		+ 10	− 0·6
	5	35	9·5	12	01	4·8

Lower transit 18^h. (Unlike tides.)

	H.	M.	F.	H.	M.	F.
Mean intervals and heights ...	5	15	11·2	11	38	4·1
Correction for dec. 26° d. ...		− 2	− 0·4		+ 14	− 0·7
,, ,, H.P. 54·5 i. ...		+ 4	− 0·5		+ 3	+ 0·8
Solar correction 1 Oct. ...		+ 3	− 0·7		+ 10	0·6
	5	20	9·6	12	05	3·6

In the years 1915–16, for which the River Tyne Entrance prediction tables are calculated, the moon's maximum declination was 27°; the maximum declination is not constant, varying from 18° to 29°, and this variation affects both the mean lunar tide and the correction for moon's declination.

Lubbock's tables for London afford a means of ascertaining the change to be expected in the mean lunar tide, for when maximum declination is 29° mean is about 20°, when maximum is 18 mean is about 13; and, as Lubbock's tables are calculated for mean declination 15°, the correction then being zero, correction at 20° will give the London correction to the mean lunar tide in a year when this is the value of mean declination. The correction for London being thus ascertained, the height corrections for other ports are found from the formula:

Correction required : London correction :: range : London range

the time corrections being the same as at London.

Except in cases where the prediction tables are calculated for years of maximum or minimum lunar declination, and the tides are required for years of minimum or maximum, the corrections, especially to the times, thus obtained are so small as to be hardly worth applying.

The correction to the declination correction is calculated, by means of two factors, on the assumption that the correction for declination varies directly with the declination.

$$\text{Factor I} = \frac{\text{maximum declination for which tables were calculated}}{\text{maximum declination of the year}}$$

$$\text{Factor II} = \frac{\text{maximum declination of the year}}{\text{maximum declination for which tables were calculated}}$$

Each declination for which the correction is required is multiplied by Factor I; the correction for the result multiplied by Factor II gives the correction to be applied. These corrections have been found satisfactory in practice, there being little, if any, falling off in accuracy from year to year.

Predictions calculated by the equation method are not exact, for some small inequalities are entirely omitted, and others are not found exactly. This method should not be used when any inequality is large in comparison with the mean lunar tide, for inaccuracies increase with the inequalities, and with large rapidly changing corrections interpolation becomes difficult. The object of the investigations resulting in this method was the prediction of tides in European waters, without the use of a machine, and by a method which could be used by the unscientific staff of the tidal branch of the Hydrographic Department, and this object appears to have been attained. With practice, the calculation of the prediction tables from observations extending over fourteen lunations is about 120 hours' work, and the prediction of tides for one port for one year about twenty-four hours' work.

The tidal requirements of seamen are not completely fulfilled by predictions alone; lack of space and financial considerations will not permit the calculation and publication of the tides for more than a very limited number of ports, or even of the prediction of the complete wave at the few ports for which the tides are calculated; simple, but so far as possible accurate, methods of extending the predicted tides to secondary ports, and of finding the height at other than the predicted times, are therefore required. The present methods of extending predictions by means of differences, and of finding the height by means of diagrams or tables based on harmonic formulæ, are far from satisfactory, and investigation should lead to improvement.

Standard ports are now chosen with very little regard to anything but their importance as ports; the choice should depend on their suitability as standards. The accuracy of tidal predictions for a standard port is of more than local importance, for the predictions are used for finding times and heights at other ports, and it will be understood that, if approximate differences are applied to approximate predictions, and an approximate method of finding the height then used, the result may be so far in error as to be not only valueless but dangerous. It is therefore of the utmost importance that the starting-point of all navigational tidal problems, the predicted times and heights of high and low water at the standard port, should be accurate.

Tide tables for the use of seamen should thus contain:

1. Accurate tidal predictions for a number of well-chosen standard ports.
2. A method of extending the predictions to secondary ports.
3. A method of finding the height of the tide at times between high and low water.

2 and 3 should be accurate if possible, but must be simple.

It is possible that the solution of the problems of extending the prediction and finding the height at other than predicted times lies in the division of the world into "tide districts," or regions in which tidal characteristics are similar, and the prediction of the average tide of each district instead of the tides of any one port. Standard ports would no doubt lose in accuracy were this course followed, but the general gain in accuracy would be very great, and the tides of important ports might well be calculated in addition to the average tides of the district; there can be no doubt that the tides of secondary ports could be more accurately obtained by means of differences on the average tide of their district, than by means of differences on a standard port situated perhaps some miles up a river.

Discussion after the above paper:

ADMIRAL J. F. PARRY (Hydrographer of the Navy): Commander Warburg during his lecture stated that one of the essential requirements of tidal investigation was to obtain some simple formula which could be easily used by the unscientific staff of the Hydrographic Department; I do not think that Commander Warburg should apply that description to himself, as personally I consider him a true scientist on his own subject of tides. The greater part of the lecture that we have heard this afternoon has left me in much the same condition, I think, as it has left a good many of you, that is, in the main part, as being well over my head; but I specially wish to add that Commander Warburg and I have had dealings with each other for a considerable number of years, and I have the very highest respect for him and for all that he has carried out respecting tidal investigation work; I have been, of course, specially concerned with the actual working of his methods, and I know not only from my own observations and personal testing of his methods, but also from reports received from officers on board ships in a great variety of places, and especially in the North Sea, that his new method of prediction has given the most satisfactory results one could possibly hope for. It must be pointed out that in a great many cases the matter of tides is literally a question of life or death to a ship; the general public are very apt to look upon tides as a matter of interest which only concerns scientific people and does not really affect the practical man to any serious extent, but at all times knowledge of tides is most essential to the seaman: to take one example, namely