Theory of Magneto-Mechanical Systems as Applied to Telephone Receivers and Similar Structures

BY R. L. WEGEL

Research Laboratories of the American Tel. & Tel. Co. and Western Elec. Co., Inc., New York, N. Y.

A sufficiently general theory of magnetic vibratory apparatus is deduced to cover all practical problems in the operation of the various types of telephone receivers as transformers of electrical into acoustical or acoustical into electrical energy. A set of linear equations is derived relating currents in windings, mounted either on a stationary support (pole piece) or on the moving part (diaphragm or armature), and the motion of the moving part. The effect of eddy currents is included by treating each eddy circulation as a circuit. From the equations derived, it is possible by simple algebraic processes to calculate the effect of eddy currents and diaphragm or armature motion in terms of the constants or impedances of the system.

By making certain constants of the equations either infinite or zero the different receiver types are represented including (1) simple receiver without eddy currents, (2) receiver with eddy currents in the pole pieces, (3) receiver with eddy currents in the diaphragm, (4) receiver with eddy currents in both pole pieces and diaphragm, (5) receiver with the driving coil on the diaphragm, and (6) receiver with non-magnetic but conducting diaphragm.

INTRODUCTORY

THE theory of the permanent magnet telephone receiver was first worked out from fundamental considerations of energy, using the Lagrangian method of obtaining the equations of motion, by Poincare, *Eclairage Electrique*, vol. 50, page 221, 1907, et seq. Interesting experimental studies of the ordinary telephone receiver have been published by Kennelly and his associates. The first of these, by Kennelly & Pierce, was "The Impedances of Telephone Receivers as Affected by the Motion of their Diaphragms" (*Proceedings of the American Academy of Arts and Sciences*, 1912, Vol. 48.) The general method used throughout these studies is the inference of the characteristics of the telephone from its impedance under various conditions.

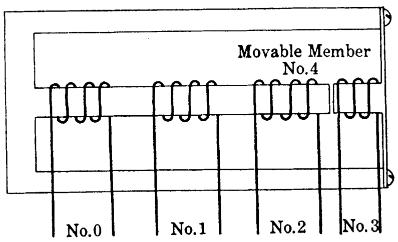


FIG. 1—SCHEMATIC RECEIVER

A number of other articles on the receiver has appeared in American and foreign periodicals during the last few years. They are concerned principally with methods of measurement of various constants.

This paper includes a somewhat more general theory of receiver structures and its application to the technique of measurement. The method used is applicable to any number of mutually dependent or independent electrical circuits magnetically coupled to any number of mechanical members, mutually dependent or independent. A paper by H. W. Nichols, "Theory of Variable Dynamical—Electrical Systems," *Phys. Rev.* August 1917, gives the general considerations of variable systems, of which the telephone receiver is a particular case.

GENERAL THEORY

This section covers the theory of a sufficiently general electromagnetic vibratory mechanism to be applicable to any type of telephone receiver or transmitter operating on the principle of the reaction between relatively movable magnetic fields produced either by magnets or electric currents.

In Fig. 1, four coils are shown which are parts of four circuits. Coils Nos. 0, 1, 2, and 3 are situated so as to be inductively coupled each to each. Circuits Nos. 0, 1, and 2 are relatively fixed. No. 4 is a mechanical member free to move with one degree of freedom by a bending of its supports with respect to the support or core of members Nos. 0, 1, and 2. Coil No. 3 is constrained to move either with the mechanical member No. 4, or with a motion having a definite linear relation to the movement of No. 4. Fig. 1 illustrates schematically a single-pole telephone receiver. The movable member, No. 4, represents the diaphragm. Coil No. 0 carries a direct current to supply the steady magnetizing flux for the pole piece on which coils Nos. 0, 1, and 2 are mounted. Coil No. 1 represents the receiving coil which carries the telephone current. Coil No. 2 represents the eddy current circuit in the pole piece. Coil No. 3 mounted on the diaphragm represents the eddy current circuit in the diaphragm. A movement of the diaphragm, No. 4, alters the self and mutual inductances of all four electrical circuits.

Notation. Let L_0, L_1, L_2, and L_3 represent the self-inductances of circuits of which coils Nos. 0, 1, 2, and 3 are parts, L_{01}, L_{02}, L_{03}, L_{12}, L_{13}, and L_{23}, the mutual inductances between circuits as indicated by their subscripts, and L_4, the vibrating mass of the movable member.

Let R_0, R_1, R_2, and R_3 be the resistances of circuits of which the coils are parts and R_4—the mechanical dissipative resistance to motion of the movable member.

R_4 is assumed independent of the velocity of motion and gives, when multiplied by the square of the velocity of motion, the rate of dissipation of energy mechanically by the movable member. In order to conform with the electrical units adopted this resistance will be so chosen that the power dissipated is measured in watts.

Let C_0, C_1, C_2, and C_3 represent series capacitances or displacement of electricity in the condensers per unit of electromotive force in the corresponding circuits, and C_4 the mechanical "capacitance" or displacement per unit of force acting on the diaphragm. The reciprocal $1/C_4$ is the stiffness of the mechanical member.

Let i_0, i_1, i_2, and i_3 be currents flowing through the corresponding circuits, and $\dot{x}$ the velocity of motion of the movable member.

Let q_0, q_1, q_2, and q_3 be the quantities of electricity displaced as defined by $d\,q/d\,t = i$, and x the displacement of the movable member definable by $d\,x/d\,t = \dot{x}$.

Let E_0, E_1, E_2, and E_3 be the electromotive forces tending to produce displacement of electricity or current in the circuits and F the force tending to produce a displacement of the mechanical member.

f = frequency, $w = 2\,\pi\,f$ and $j = \sqrt{-1}$.

The first step in formulating a theory of this structure is to obtain the electromotive force equations or equations of motion; that is, a relation between the currents in the various circuit branches, velocity of motion of the mechanical member, electromotive and mechanical forces and electrical and "mechanical" impedances.

The reactions due to resistances and capacities in the various circuits are $i_0 R_0$, $i_1 R_1$, $i_2 R_2$, $i_3 R_3$, and q_0/C_0, q_1/C_1, q_2/C_2, q_3/C_3. Similarly the mechanical resistance and stiffness reactions are $\dot{x} R_4$ and x/C_4, respectively.

The reaction in an invariable circuit due to inductance is $L\,d\,i/d\,t$. In this case the inductances are variable, depending on the position, x, of the mechanical member. The motion is assumed to be small enough so that each inductance is a linear function of the mechanical displacement x and of the form $L\,(1 + k\,x_4)$ where L is the mean value of an inductance about which the variation takes place and k is a constant. On this assumption, $k\,x$ is small compared to unity. The inductive reactions are obtained from the kinetic energy, T, of the system by means of a differential formula due to Lagrange for mechanical systems and Maxwell for electrical systems. This formula is:

$$\frac{d}{d\,t}\left(\frac{d\,T}{d\,\dot{i}}\right) - \frac{d\,T}{d\,q}$$

To get the inductive reaction in circuit No. o, i_0 and q_0 are used. The reactions in the other circuits are obtained by using the proper i and q. The mass reaction is found by using $\dot{x}$ and x.

The instantaneous kinetic energy of the whole system is

$$\begin{aligned} T = {} & 1/2\,L_0\,i_0^2 + 1/2\,L_1\,i_1^2 + 1/2\,L_2\,i_2^2 + 1/2\,L_3\,i_3^2 \\ & + 1/2\,L_4\,\dot{x}^2 + L_{01}\,i_0\,i_1 + L_{02}\,i_0\,i_2 + L_{03}\,i_0\,i_3 \\ & + L_{12}\,i_1\,i_2 + L_{13}\,i_1\,i_3 + L_{23}\,i_2\,i_3 \end{aligned} \qquad (1)$$

The force equation of any member, electrical or mechanical, is then given by an expression of the form

$$E = R\,i + \frac{q}{C} + \frac{d}{d\,t}\left(\frac{d\,T}{d\,i}\right) - \frac{d\,T}{d\,q}$$

Applying this formula to each member of the system, the following five equations of motion are obtained:

$$\left.\begin{aligned}
E_0 &= R_0\,i_0 + \frac{1}{C_0}\,q_0 + \frac{d}{d\,t}\,[L_0\,(1 + k_0\,x)\,i_0 \\
&\quad + L_{01}\,(1 + k_{01}\,x)\,i_1 + L_{02}\,(1 + k_{02}\,x)\,i_2 \\
&\quad + L_{03}\,(1 + k_{03}\,x)\,i_3] \\
E_1 &= R_1\,i_1 + \frac{1}{C_1}\,q_1 + \frac{d}{d\,t}\,[L_{01}\,(1 + k_{01}\,x)\,i_0 \\
&\quad + L_1\,(1 + k_1\,x)\,i_1 + L_{12}\,(1 + k_{12}\,x)\,i_2 \\
&\quad + L_{13}\,(1 + k_{13}\,x)\,i_3] \\
E_2 &= R_2\,i_2 + \frac{1}{C_2}\,q_2 + \frac{d}{d\,t}\,[L_{02}\,(1 + k_{02}\,x)\,i_0 \\
&\quad + L_{12}\,(1 + k_{12}\,x)\,i_1 + L_2\,(1 + k_2\,x)\,i_2 \\
&\quad + L_{23}\,(1 + k_{23}\,x)\,i_3] \\
E_3 &= R_3\,i_3 + \frac{1}{C_3}\,q_3 + \frac{d}{d\,t}\,[L_{03}\,(1 + k_{03}\,x)\,i_0 \\
&\quad + L_{13}\,(1 + k_{13}\,x)\,i_1 + L_{23}\,(1 + k_{23}\,x)\,i_2 \\
&\quad + L_3\,(1 + k_3\,x)\,i_3] \\
F &= R_4\,\dot{x} + \frac{1}{C_4}\,x + L_4\,\frac{d\,\dot{x}}{d\,t} - [1/2\,(L_0\,k_0\,i_0 \\
&\quad + L_{01}\,k_{01}\,i_1 + L_{02}\,k_{02}\,i_2 + L_{03}\,k_{03}\,i_3)\,i_0 \\
&\quad + 1/2\,(L_{01}\,k_{01}\,i_0 + L_1\,k_1\,i_1 + L_{12}\,k_{12}\,i_2 \\
&\quad + L_{13}\,k_{13}\,i_3)\,i_1 + 1/2\,(L_{02}\,k_{02}\,i_0 + L_{12}\,k_{12}\,i_1 \\
&\quad + L_2\,k_2\,i_2 + L_{23}\,k_{23}\,i_3)\,i_2 + 1/2\,(L_{03}\,k_{03}\,i_0 \\
&\quad + L_{13}\,k_{13}\,i_1 + L_{23}\,k_{23}\,i_2 + L_3\,k_3\,i_3)\,i_3]
\end{aligned}\right\} \quad (2)$$

Suppose now that the capacitance, C_0, is omitted, and a relatively large direct current flows in circuit No. 0 through a high enough impedance so that the reactions depending on $\dfrac{d\,i_0}{d\,t}$ can be considered negligible. The first equation then becomes $E_0 = i_0\,R_0$ and in this investigation will not be considered. The purpose of this circuit, it will be seen later, is only to supply a large permanent steady magnetic field. In a permanent magnet receiver this is a fictitious winding so disposed as to give the field, in magnitude and distribution, of the permanent magnet.

In the foregoing equations, i_0 occurs in products with other currents. Products of the small i_1, i_2, i_3, and x also occur. The terms containing these products involve reactions which are negligibly small compared to those of the terms containing i_0. The term $1/2\, L_0\, k_0\, i_0^2$ is a constant entering the equation of mechanical force and determines the permanent deflection of the mechanical member from its equilibrium position. This term can be eliminated by measuring the displacement x from the equilibrium position when the large current i_0 is flowing.

If, in addition, we define

$$L_{01}\, k_{01}\, i_0 = b_{14},\; L_{02}\, k_{02}\, i_0 = b_{24},\; L_{03}\, k_{03}\, i_0 = b_{34},$$

and neglect the second order terms as explained above, the above five equations are reduced to four as follows:

$$\left.\begin{aligned}
&L_1\, d\, i_1/d\, t + R_1\, i_1 + (1/C_1)\, q_1 + L_{12}\, d\, i_2/d\, t \\
&\qquad + L_{13}\, d\, i_3/d\, t\; b_{14}\, \dot{x} = E_1 \\
&L_2\, d\, i_2/d\, t + R_2\, i_2 + (1/C_2)\, q_2 + L_{12}\, d\, i_1/d\, t \\
&\qquad + L_{23}\, d\, i_3/dt + b_{24}\, \dot{x} = E_2 \\
&L_3\, d\, i_3/d\, t + R_3\, i_3 + (1/C_3)\, q_3 + L_{13}\, d\, i_1/d\, t \\
&\qquad + L_{23}\, d\, i_2/d\, t + b_{34}\, \dot{x} = E_3 \\
&L_4\, d\, \dot{x}/d\, t + R_4\, \dot{x} + (1/C_4)\, x - b_{14}\, i_1 - b_{24}\, i_2 \\
&\qquad - b_{34} i_3 = F
\end{aligned}\right\} \quad (3)$$

It will be seen from dimensions that the quantities b_{14}, b_{24} and b_{34} are either back electromotive forces per unit velocity of No. 4 or forces on No. 4 per unit of current.

The free vibrations of this system are obtained by placing $E_1 = E_2 = E_3 = F = 0$ and solving for the free periods and the damping in the usual way. The expressions thus obtained, except for the simplest cases, are very complex and will not be of interest in connection with this treatment.

For forced vibrations, each value of E and F may be considered to be of single frequency. If a number of frequencies act simultaneously, they may be considered separately since in any system represented by a set of linear equations such as these the law of superposition holds.

We will therefore consider each value of E to be sinusoidal and to contain e^{jwt} as a factor. The forms of the i's, q's, $\dot{x}$, and x will then be complex and contain the periodic symbol e^{jwt} as a factor. Under these conditions $d/d\, t$ can be replaced by $j\, w$ and $\int d\, t$ can be replaced by $1/j\, w$. When this is done the above equations become

$$\left.\begin{aligned}
&(i\, w\, L_1 + R_1 + 1/j\, w\, C_1)\; i_1 + j\, w\, L_{12}\, i_2 \\
&\qquad + j\, w\, L_{13}\, i_3 + b_{14}\, \dot{x} = E_1 \\
&(j\, w\, L_2 + R_2 + 1/j\, w\, C_2)\; i_2 + j\, w\, L_{12}\, i_1 \\
&\qquad + j\, w\, L_{23}\, i_3 + b_{24}\, \dot{x} = E_2 \\
&(j\, w\, L_3 + R_3 + 1/j\, w\, C_3)\, i_3 + j\, w\, L_{13}\, i_1 \\
&\qquad + j\, w\, L_{23}\, i_2 + b_{34}\, \dot{x} = E_3 \\
&(i\, w\, L_4 + R_4 + 1/j\, w\, C_4)\, \dot{x} - b_{14}\, i_1 - b_{24}\, i_2 \\
&\qquad - b_{34}\, i_3 = F
\end{aligned}\right\} \quad (4)$$

The coefficients of currents i_1, i_2, i_3, and velocity $\dot{x}$ are seen to be impedances. Using the standard notation, Z, for impedance, these equations can be written

$$\left.\begin{aligned}
Z_{11}\, i_1 + Z_{12}\, i_2 + Z_{13}\, i_3 + Z_{14}\, \dot{x} &= E_1 \\
Z_{12}\, i_1 + Z_{22}\, i_2 + Z_{23}\, i_3 + Z_{24}\, \dot{x} &= E_2 \\
Z_{13}\, i_1 + Z_{23}\, i_2 + Z_{33}\, i_3 + Z_{34}\, \dot{x} &= E_3 \\
-Z_{14}\, i_1 - Z_{24}\, i_2 - Z_{34}\, i_3 + Z_{44}\, \dot{x} &= F
\end{aligned}\right\} \quad (5)$$

The significance of these equations will be better understood by pointing out their similarity to those of a familiar electrical analog.

Fig. 2 is the diagram of a four-winding transformer. Coils Nos. 1, 2, and 3 are parts of three circuits which correspond to circuits 1, 2, and 3 of the mechanism of Fig. 1. It will be of interest to compare the effect of circuit No. 4, which is like Nos. 1, 2, and 3, with that of the mechanical member, No. 4.

The same method of deriving the "equations of motion" or impedance equations can be employed for this simple transformer, but the theory of electrical networks is in such common use that this is unnecessary. The equations can be written directly.

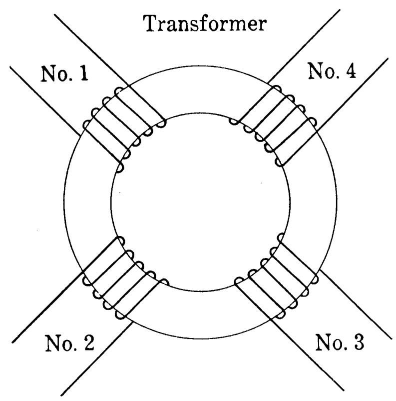


FIG. 2—FOUR-WINDING TRANSFORMER

$$\left.\begin{aligned}
&(j\, w\, L_1 + R_1 + 1/j\, w\, C_1)\; i_1 + j\, w\, L_{12}\, i_2 \\
&\qquad + j\, w\, L_{13}\, i_3 + j\, w\, L_{14}\, i_4 = E_1 \\
&(j\, w\, L_2 + R_2 + 1/j\, w\, C_2)\; i_2 + j\, w\, L_{12}\, i_1 \\
&\qquad + j\, w\, L_{23}\, i_3 + j\, w\, L_{24}\, i_4 = E_2 \\
&(i\, w\, L_3 + R_3 + 1/j\, w\, C_3)\; i_3 + j\, w\, L_{23}\, i_2 \\
&\qquad + j\, w\, L_{13}\, i_1 + j\, w\, L_{34}\, i_4 = E_3 \\
&(j\, w\, L_4 + R_4 + 1/j\, w\, C_4)\; i_4 + j\, w\, L_{14}\, i_1 \\
&\qquad + j\, w\, L_{24}\, i_2 + j\, w\, L_{34}\, i_3 = E_4
\end{aligned}\right\} \quad (6)$$

The coefficients of the currents are impedances and can be written:

$$\left.\begin{aligned}
Z_{11}\, i_1 + Z_{12}\, i_2 + Z_{13}\, i_3 + Z_{14}\, i_4 &= E_1 \\
Z_{12}\, i_1 + Z_{22}\, i_2 + Z_{23}\, i_3 + Z_{24}\, i_4 &= E_2 \\
Z_{13}\, i_1 + Z_{23}\, i_2 + Z_{33}\, i_3 + Z_{34}\, i_4 &= E_3 \\
Z_{14}\, i_1 + Z_{24}\, i_2 + Z_{34}\, i_3 + Z_{44}\, i_4 &= E_4
\end{aligned}\right\} \quad (7)$$

There are two notable differences between this set of equations and that of the "magneto-mechanical" device. The mutual impedances b_{14}, b_{24}, b_{34}, between the coils and the mechanical member are constants, independent of frequency and involve no phase difference between the current and induced force, while in the transformer the corresponding mutual impedances, $j\, w\, L_{14}$, $j\, w\, L_{24}$, and $j\, w\, L_{34}$ are proportional to frequency and involve a phase difference of 90 deg. between the inducing current and induced electromotive force. The mutual impedances, b_{14}, b_{24}, b_{34}, are of the character of resistances, in that they are in phase with the currents but are unlike resistances in that they are non-dissipative, which is shown by the fact that their signs in the electrical equations are opposite to those in the mechanical equation (See equations **4** and **5**). The analogy between the transformer and the receiver is not complete because of the difference indicated by these differences in sign.

The difference may be illustrated in another way, by an application of the "principle of reciprocity" which for a pure electrical network may be stated;

In any invariable electrical network, if any electromotive force, E, is applied in any branch and the current I measured in any other branch their ratio E/I is equal in magnitude and phase to the ratio obtained if the positions of E and the measurement of I be reversed.

This principle can be proved directly from the symmetrical form in magnitude and phase of the equations of a network of which the transformer (Eq. **7**) is an example.

The same reciprocal relation holds between displacement or velocity and mechanical force in an invariable "mechanical network," composed of any combination of elastic and inertia members, with or without damping.

In a "magneto-mechanical network" such as the one under discussion in this paper a similar principle can be formulated. For the sake of brevity a variable dynamical-electrical system in which the only variations are small changes of inductance will be alluded to as a "magneto-mechanical network."

In a "magneto-mechanical network," the ratio of an electromotive force placed in any electrical branch to the resulting displacement (or velocity) of a mechanical member (or branch) is equal in magnitude but opposite in phase (180 deg.) to the ratio of a mechanical force acting on the same mechanical member to the resulting current in the same electrical branch.

This can be proved directly from the symmetry of magnitude and asymmetry in signs in the characteristic equations No. 5, of a "magneto-mechanical network" of which the subject of this paper is an example.

In regard to the transfer of power from one circuit to another, from one mechanical vibratory element to another or from an electrical to a mechanical element or vice versa these systems are alike. For a maximum transfer of power the impedance of an element which is to be fitted to another must be the conjugate of the element to which it is fitted. This follows directly from equations 5 or 7, which are typical though not general.

A number of general expressions will next be derived for the calculation of the various currents and velocity and impedance characteristics. A set of linear equations of this kind is manipulated most easily by the short hand method of determinants.

For convenience, define

$$Z_{21} = Z_{12}, Z_{31} = Z_{13}, Z_{32} = Z_{23}, Z_{41} = -Z_{14}, \quad Z_{42} = -Z_{24}, Z_{43} = -Z_{34}$$

Rewrite equations **5**.

$$\left.\begin{aligned} Z_{11}\, i_1 + Z_{12}\, i_2 + Z_{13}\, i_3 + Z_{14}\, \dot{x} &= E_1 \\ Z_{21}\, i_1 + Z_{22}\, i_2 + Z_{23}\, i_3 + Z_{24}\, \dot{x} &= E_2 \\ Z_{31}\, i_1 + Z_{32}\, i_2 + Z_{33}\, i_3 + Z_{34}\, \dot{x} &= E_3 \\ Z_{41}\, i_1 + Z_{42}\, i_2 + Z_{43}\, i_3 + Z_{44}\, \dot{x} &= E_4 \end{aligned}\right\} \quad (\mathbf{8})$$

Denote the discriminant of this system, or determinant of the coefficients of current by D, and its minors by D with proper subscripts. To illustrate: D_{13} is the first minor of D obtained by suppressing row 1, and column 3 of the coefficients; $D_{13}{}^{24}$ is a second minor determinant with row 2 and column 4 eliminated in addition.

Inspection will show that a relation similar to that between mutual impedances holds for the first minors of this discriminant,

$$D_{21} = D_{12}, D_{31} = D_{13}, D_{32} = D_{23},$$
$$D_{41} = -D_{14}, D_{42} = -D_{24}, D_{43} = -D_{34},$$

In the purely electrical system, as in equations **7**, corresponding minors are identical in sign as well as in magnitude.

The solutions for currents i_1, i_2, i_3 and velocity of motion of No. 4, $\dot{x}$, are directly written.

$$\left.\begin{aligned} i_1 &= (E_1 D_{11} - E_2 D_{21} + E_3 D_{31} - F D_{41}) / D \\ i_2 &= (- E_1 D_{12} + E_2 D_{22} - E_3 D_{32} + F D_{42}) / D \\ i_3 &= (E_1 D_{13} - E_2 D_{23} + E_3 D_{33} - F D_{43}) / D \\ \dot{x} &= (- E_1 D_{14} + E_2 D_{24} - E_3 D_{34} + F D_{44}) / D \end{aligned}\right\} \quad (\mathbf{9})$$

As a rule, of course, only one electromotive force, E, or mechanical force, F, acts at a time so these expressions are greatly simplified by placing the others equal to zero. The expressions for various current ratios, ratios of current or velocity to applied electromotive force or mechanical force are easily written from equations 9. In particular, the ratio of any electromotive force, E, to the resulting current i in the same element is the "driving point" impedance of the system. The ratio $F/\dot{x}$ is a driving point impedance and is given in mechanical units—dynes per centimeter per second. The "driving point" impedances measured in the four circuits (the mechanical member may by analogy be referred to as a mechanical circuit) are:

$$Z_1 = D/D_{11}, Z_2 = D/D_{22}, Z_3 = D/D_{33}, \quad Z_4 = D/D_{44} \quad (\mathbf{10})$$

Since all of the impedances are involved in these determinants, the driving point impedances Z_1, Z_2, Z_3, Z_4 are seen to depend on the reactions of all the circuits and the mechanical member.

The "damped" impedance of a receiver is the impedance of the coil measured with the diaphragm constrained from moving. This may be regarded as making the mechanical impedance infinite or "opening" the "mechanical circuit." The damped impedance of the system measured from circuit No. 1 is obtained from the driving point impedance by suppressing row 4 and column 4. Designating the impedance measured in circuit No. 1 with No. 4 damped by Z_{1d4}:

$$\left.\begin{aligned} Z_{1d4} &= D_{44}/D_{11}^{44} \\ \text{Similarly } Z_{2d4} &= D_{44}/D_{22}^{44} \\ Z_{3d4} &= D_{44}/D_{33}^{44} \end{aligned}\right\} \quad \textbf{(11)}$$

These give the damped inpedances as measured from any of the three electrical circuits. These expressions also hold for the driving point impedance of the transformer, Fig. 2, equations **7**, with circuit No. 4 "opened."

The mechanical impedance of the vibrating member No. 4 under the influence of the reactions of circuits Nos. 2 and 3, but with No. 1 open, which by analogy can be called "damped mechanical impedance" is given similarly by

$$Z_{4d1} = D_{11}/D_{11}^{44}$$

Motional impedance is defined as the difference between the driving point impedance with the mechanical member free to move and with it damped.

$$Z_{1m4} = Z_1 - Z_{1d4}$$

In terms of the determinants of the system

$$Z_{1m4} = -D_{14}\,D_{41}/D_{11}\,D_{11}^{44}$$

This is typical of any electrical network as well and gives the effect on driving point impedance in any circuit No. 1, of any other circuit No. 4. In the case of the magneto-mechanical system $D_{14} = -D_{41}$.

$$\left.\begin{aligned} & Z_{1m4} = (D_{14})^2/D_{11}\,D_{11}^{44} \\ \text{Similarly}\quad & Z_{2m4} = (D_{24})^2/D_{22}\,D_{22}^{44} \\ & Z_{3m4} = (D_{34})^2/D_{33}\,D_{33}^{44} \end{aligned}\right\} \quad \textbf{(12)}$$

These give the motional impedances measured in any of the three electrical circuits.

In any invariable electrical, mechanical, or magneto-mechanical network, the driving point impedance Z_p measured from the p^{th} element, can be expressed as the sum of two terms, the first of which is independent of any other q^{th} circuit and the second of which is due to the reaction of the q^{th} circuit.

$$Z_p = Z_{pdq} + Z_{pmq}$$

Where

$$Z_{pdq} = D_{qq}/D_{pp}^{qq};\ Z_{pmq} = -D_{pq}\,D_{qp}/D_{pp}\,D_{pp}^{qq}$$

The first term may be called "damped" impedance and the second "motional" impedance.

The force per unit current in circuit No. 1 when the reactions of Nos. 2 and 3 are eliminated by "opening" is Z_{14}. A current in No. 1, however, induces currents in Nos. 2 and 3 which in turn act on No. 4 through their force factors Z_{24} and Z_{34} respectively. The total force acting on No. 4 is

$$Z_{14}\,i_1 + Z_{24}\,i_2 + Z_{34}\,i_3$$

A motion of the mechanical member causes modifying reactions on these currents producing a different force on this member than when constrained. By solving for the force resulting from unit current in circuit No. 1 after constraining No. 4 by suppressing row 4 and column 4 of the discriminant, it is found to be

$$\left.\begin{aligned} & B_{1d4} = D_{14}/D_{11}^{44} \\ \text{Similarly}\quad & B_{2d4} = D_{24}/D_{22}^{44} \\ & B_{3d4} = D_{34}/D_{33}^{44} \end{aligned}\right\} \quad \textbf{(13)}$$

These give the force per unit current in magnitude and phase with an electromotive force acting in circuits Nos. 1, 2 or 3 respectively as modified by induced currents in the other two circuits.

The electromotive force in circuit No. 1 per unit velocity of No. 4 is given by

$$\left.\begin{aligned} & B_{4d1} = D_{41}/D_{44}^{11} = -B_{1d4} \\ \text{Similarly}\quad & B_{4d2} = D_{42}/D_{44}^{22} = -B_{2d4} \\ & B_{4d3} = D_{43}/D_{44}^{33} = -B_{3d4} \end{aligned}\right\} \quad \textbf{(14)}$$

This property holds generally for a "magneto-mechanical" network, and like that given above in contrast with the principle of reciprocity for invariable electrical and mechanical systems, is evident directly from the characteristic determinant by the fact that $D_{14} = -D_{41}$, $D_{24} = -D_{42}$, $D_{34} = -D_{43}$.

"The available force on a mechanical member per unit current in a circuit is equal in magnitude but opposite in sign to the available electromotive force in the same circuit per unit velocity of the same mechanical member."

In general, for an invariable electrical or mechanical network, or for a magneto-mechanical network,

$$B_{pdq} = D_{pq}/D_{pp}^{qq}$$

Motional impedance and the "damped" force factor B_{1d4} are therefore related by:

$$\left.\begin{aligned} & Z_{1m4} = \frac{(B_{1d4})^2}{Z_{4d1}} \\ \text{Similarly}\quad & Z_{2m4} = \frac{(B_{2d4})^2}{Z_{4d2}} \\ & Z_{3m4} = \frac{(B_{3d4})^2}{Z_{4d3}} \end{aligned}\right\} \quad \textbf{(15)}$$

Equations **15** give the most useful form for motional impedance.

The most useful substitutes for the general equations, No. 8, of motion are two in number, involving the current in the circuit in which the driving electromotive force acts and the velocity of the mechanical member on which a mechanical force may act.

$$\left.\begin{aligned} Z_{1d4}\,i_1 + B_{1d4}\,\dot{x} &= E_1 \\ B_{4d1}\,i_1 + Z_{4d1}\,\dot{x} &= F \end{aligned}\right\} \quad \textbf{(16)}$$

These correspond to equations No. 8 with currents i_2 and i_3 eliminated. They hold for invariable electrical and mechanical networks as well. Z_{1d4} is the apparent electrical impedance of the receiving coil and is independent of diaphragm motion. B_{1d4} is electromotive force generated in the coil per unit velocity, $\dot{x}$ of the diaphragm. B_{4d1} is force on the diaphramg per unit current in the coil and is equal to B_{1d4} in magnitude but opposite in sign. Z_{4d1} is the apparent mechanical impedance of the diaphragm. The reaction of the diaphragm in response to an applied force is modified, both in magnitude and phase, by eddy currents.

Applications of the General Theory to Particular Receiver Structures

The application of the general theory to particular types of receivers consists in first showing the modification of the general structure, (Fig. 1) to which each corresponds, and modifying the characteristic equations (No. 5) accordingly. For example, if the simple receiver without eddy currents is to be studied the effects of circuits 2 and 3 are eliminated. In the equations the mutuals Z_{12}, Z_{13}, Z_{23}, Z_{24}, Z_{34}, or currents i_2, i_3, are placed equal to zero. The various types and discussions of their characteristics will be taken up separately.

A. Simple Receiver. The simple receiver is defined as one having no eddy currents in the core or diaphragm. This sort of receiver is never actually met with in practise but may be approximated by the use of high resistivity material or laminated cores and diaphragms.

The diaphragm of the receiver is not a mechanical element with one degree of freedom but may be so regarded without involving serious error. While vibrating, all parts of the diaphragm do not move with the same amplitude. The integrated product of surface density of the diaphragm and square of the velocity (r. m. s.) over the surface, gives the kinetic energy of vibration. The apparent vibrating mass, L_4 of the diaphragm is found by dividing the kinetic energy by the square of the velocity (r. m. s.) at the center. The shape of the curve of bending caused by the electromagnetic force acting about the center of the diaphragm varies with frequency of excitation, and hence, the apparent mass, L_4 must be considered as varying to a certain extent with the frequency. Similarly, the apparent elasticitance, C_4 is determined by the integrated potential energy of flexure which also varies with the frequency. The variation in L_4 and C_4 to be expected between the frequencies of zero and 3000 cycles is probably a matter of a 10 or 20 per cent gradual change for an ordinary receiver diaphragm. Similarly a variation in R_4 occurs in the receiver.

Attention must be called to the fact that even though L_4, C_4, and R_4 vary with frequency, they do not vary appreciably with amplitude of vibration at a single frequency for the small motions of a diaphragm. As a consequence, the law of superposition holds and the equations which have been deduced are valid. In complete theory, L_4, C_4, and R_4 should be replaced by their proper functions of frequency but since these changes are gradual they will not be included in this study of fundamental principles of the device.

The equations for the simple receiver are

$$\left.\begin{aligned} Z_{11}\, i_1 + Z_{14}\, \dot{x} &= E_1 \\ -\, Z_{14}\, i_1 + Z_{44}\, \dot{x} &= F \end{aligned}\right\} \quad (17)$$

From these equations we find

$$i_1 = \frac{E_1\, Z_{44} - F\, Z_{14}}{Z_{11}\, Z_{44} + Z_{14}^2}$$

$$\dot{x} = \frac{E_1\, Z_{14} + F\, Z_{11}}{Z_{11}\, Z_{44} + Z_{14}^2}$$

These equations give either current or velocity re-

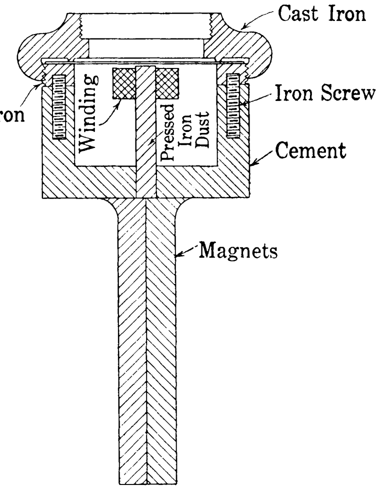


Fig. 3—Cross-Section of Experimental Receiver

sulting from an electromotive force acting in the winding as in a telephone receiver, or from a force acting on the diaphragm as in the case of a magnetic telephone transmitter, or a combination of both.

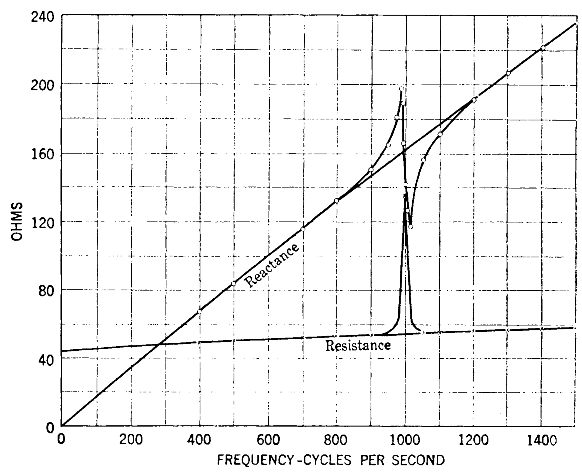


Fig. 4—Resistance and Reactance Curves of Experimental Receiver

The electrical impedance of the receiver is the ratio of the electromotive force acting to the resulting current in the coil and is given by

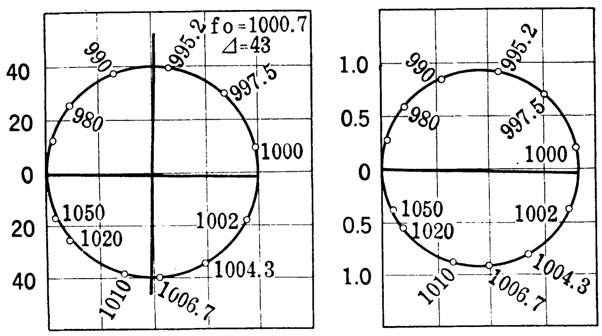
$$Z_1 = Z_{11} + Z_{14}^2 / Z_{44}$$

The damped impedance is simply the impedance of the coil Z_{11}, and the motional impedance is $Z_{14}^2\ Z_{44}$. The "vector" locus of this motional impedance is a circle with its diameter coinciding with the real axis and tangent to the positive side of the imaginary axis.

Fig. 3 shows a cross-section of an experimental simple receiver. The pole piece was made of finely divided iron pressed together with shellac and energized by a pair of bar magnets. The cup was made of cement to reduce any possible eddy current effects and to minimize differential temperature expansions. The cap and seating surfaces for the diaphragm were cast iron. The diaphragm was a standard disk 0.009 in. thick. The coil had a resistance of 48 ohms and an inductance of 0.026 henry.

Fig. 4 shows plots of measured resistance and reactance of this receiver as functions of frequency. The eddy current effect is small as is shown by the relatively small increase in damped resistance, R, with frequency.

Fig. 5 shows the circular "vector" plots of measured motional impedance and computed velocity of motion of the center of the diaphragm at various frequencies in the region of resonance. The resonant frequency is seen to be 1000.7 cycles.



Motional Impedance Vector Plot Velocity Vector Plot

FIG. 5

The effective vibrating mass, L_4, of the diaphragm, is 0.68 gram as calculated from the observed change in natural frequency caused by adding a small known weight to the center of the diaphragm. The logarithmic decrement per second as determined from the motional impedance circle (Fig. 5) is $\Delta = 48$. The mechanical resistance, R_4, of the diaphragm obtained from the relation $\Delta = R_4/2L_4$ is 58.5 dynes per centimeter per second. At resonance of the diaphragm, $Z_{1m4} = b_{14}^2/R_4$. From this relation the force factor b_{14} was computed and found to be 2.18×10^6 dynes per absolute electromagnetic unit of current. The apparent elasticity $1/C_4$ of the diaphragm is 26.7×10^6 dynes per centimeter.

The mechanical impedance is the ratio of force F to velocity of motion, $\dot{x}$.

$$Z_4 = Z_{44} + Z_{14}^2/Z_{11}$$

Z_{44} is analogous to damped impedance because it is the impedance of the diaphragm when the electrical circuit is "damped" or "opened." Z_{14}^2/Z_{11} is analogous to motional impedance because it is the effect on the diaphragm of a "motion" of electricity in the coil. The "vector" diagram of this motional mechanical impedance is a semicircle and gives the damping effect of the circuit on the diaphragm at any frequency. If a condenser is placed in the circuit of the receiver coil the locus of Z_{14}^2/Z_{11} ($= Z_{4m1}$) is a circle exactly as in the case of Z_{1m4} but is given in mechanical units (mechanical ohms) instead of in electrical ohms.

The ratio of power dissipated in diaphragm motion to the total input is easily calculated. This ratio should not be confounded with the efficiency of the instrument as a telephone receiver because a large part of the power is dissipated in diaphragm friction as its supports and in air vibrations which do not reach the ear.

The mechanically dissipated power is *mod* $\dot{x}^2 R_4$, ("*mod*" is an abbreviation for modulus or "absolute" value). The input power is *mod* $i_1^2 R$ where R is total apparent resistance. The ratio e, at any frequency is

$$\begin{aligned} e &= mod\ (\dot{x}^2 R_4/i_1^2 R) \\ &= mod\ (Z_{14}^2 R_4/Z_{44}^2 R) \\ &= mod\ (Z_{14m4}^2/Z_{1M4} R) \end{aligned}$$

The symbol Z_{1M4} is used to denote the maximum motional impedance. At resonance, or the frequency of maximum response this reduces to

$$e_0 = mod\ Z_{1M4}/R$$

The theory of this receiver is the same as that of the vibration galvanometer.

B. Receiver with Eddy Currents in the Core. This receiver corresponds closely to the commercial instrument in which the effect of eddy currents in the diaphragm is usually small.

The theory of a receiver with eddy currents in the diaphragm only, as seen from the characteristic equations is identical with that to be considered here. A little consideration will make this apparent physically. When the diaphragm is damped an eddy current in the core obviously has the same qualitative effect as one in the diaphragm for they are coaxial. Similarly when the receiving coil, No. 1, is opened and the diaphragm moves toward the pole piece, an increase in magnetic flux in both circuits results. The damping effect on diaphragm motion is qualitatively the same due to either circuit.

We are regarding the eddy current in the core as a circuit. This is as well justified as regarding the diaphragm as a member of one degree of freedom. The principal body of eddy current flows around the core surface in a layer the thickness of which decreases roughly as the square root of the frequency. In an infinite cylindrical core inside of a solenoid the current distribution is a Bessel's function of the radial distance from the axis. Regarded as a circuit, the core eddy current consists of an elongated coil of one turn short-circuited on itself. The resistance to flow of eddy current is determined by the resistivity of the iron and the thickness of the layer. The resistance of the eddy current circuit, therefore, increases roughly as the square root of the frequency. Its inductance is fairly constant since the flux linkage per unit current in this layer is about constant. At very low frequencies eddy currents are not confined to the surface, and the effective resistance and inductance of the eddy current circuit are, therefore, very small. The inductance approaches a constant value, as frequency increases, of the order of magnitude of the inductance of the receiving coil divided by the square of the number of turns.

In a common type of receiver having 700 turns per spool and 0.020-henry inductance, the inductance of the eddy current circuit would be of the order of $0.020/(700)^2$ or about 0.041×10^{-6} henry per pole. The eddy current circuit force factor, b_{24}, would be given, in order of magnitude, by that of the receiving winding divided by the number of turns, $5 \times 10^6/700$ or 0.71×10^4 dynes per absolute electromagnetic unit of current. As in the case of the diaphragm, the constants of the eddy current circuit vary with frequency, and to some extent with the character of the diaphragm motion, but do not vary appreciably with amplitude of current for the small values considered here.

The equations in this case are

$$\left.\begin{aligned} Z_{11}\, i_1 + Z_{12}\, i_2 + Z_{14}\, \dot{x} &= E_1 \\ Z_{12}\, i_1 + Z_{22}\, i_2 + Z_{24}\, \dot{x} &= 0 \\ -Z_{14}\, i_1 - Z_{24}\, i_2 + Z_{44}\, \dot{x} &= F \end{aligned}\right\} \quad (\mathbf{18})$$

By eliminating i_2 these equations can be written

$$\left.\begin{aligned} (Z_{11} - Z_{12}^2/Z_{22})\, i_1 + (Z_{14} - Z_{24}\, Z_{12}/Z_{22})\, \dot{x} &= E_1 \\ -(Z_{14} - Z_{24}\, Z_{12}/Z_{22})\, i_1 + Z_{44} + (Z_{24}^2/Z_{22})\, \dot{x} &= F \end{aligned}\right\} \quad (\mathbf{19})$$

In terms of quantities already defined, these equations are identical with

$$\left.\begin{aligned} Z_{1d4}\, i_1 + B_{1d4}\, \dot{x} &= E \\ -B_{1d4}\, i_1 + Z_{4d1}\, \dot{x} &= F \end{aligned}\right\} \quad (\mathbf{20})$$

The form of these equations is similar to that of equations (**17**) for the simple receiver. Z_{1d4} corresponds to Z_{11}, B_{1d4} to Z_{14}, and Z_{4d1} to Z_{44}. The quantity Z_{1d4} or impedance of the receiver with the diaphragm constrained has been generally defined (equations **10**). Z_{4d1} is the apparent mechanical impedance of the diaphragm when the effect of the receiving coil is eliminated by "opening."

This shows how the receiver with eddy currents in the core can be treated by the same method used for a simple receiver by defining the coil, diaphragm and mutual impedances as proper functions of frequency. The motional impedance is given by

$$Z_{1m4} = B_{1d4}^2/Z_{4d1}$$

We will now compare the characteristics of these new parameters with those of the simple receiver. In the simple receiver the force factor, Z_{14}, is a constant. In this case it (B_{1d4}) varies with frequency.

$$B_{1d4} = b_{14} - b_{24}\,\frac{j\,w\,L_{12}}{R_2 + j\,w\,L_2} \quad (\mathbf{21})$$

Fig. 6 shows a schematic "vector" plot of the force factor, B_{1d4}, as dependent on frequency.

The locus is a semicircle below the real axis. At low frequencies the force factor is greatest approaching b_{14}, which is in phase with the current at zero frequency. At high frequency the force factor approaches the smaller value

$$b_{14} - b_{24}\, L_{12}/L_2$$

which is also in phase with current.

The motional impedance is given by

$$Z_{1m4} = (B_{1d4})^2/(Z_{44} + Z_{24}^2 / Z_{22}) \quad (\mathbf{22})$$

The locus of Z_{1m4} in a "vector" plot is not a circle. For circuit and diaphragm constants such as occur in most telephone receivers it is very nearly so, however. The character of the apparent mechanical impedance of the diaphragm Z_{4d1} as influenced by eddy currents can be shown best by its expansion. (Represent the modulus or absolute value of an impedance by z.)

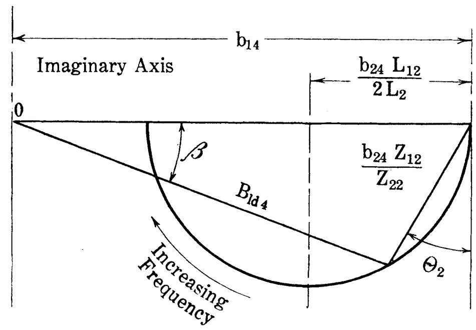


FIG. 6—SCHEMATIC "VECTOR" PLOT OF FORCE FACTOR

$$Z_{4d1} = \left(R_4 + R_2\,\frac{b_{24}^2}{z_2^2} \right) + j\,w\left(L_4 - L_2\,\frac{b_{24}^2}{z_2^2} \right) + \frac{1}{j\,w\,C_4} \quad (\mathbf{23})$$

The first term is the apparent mechanical resistance of the diaphragm. It is increased over the pure mechanical dissipative resistance R_4 by the addition of an eddy current component $R_2\, b_{24}^2/z_2^2$. This component decreases with frequency due to increase of z_2 with frequency.

The second term shows how the apparent mass of the diaphragm may be regarded as being decreased by the influence of eddy currents. The effect is to modify the mechanical reactance in such a way as to increase the frequency of maximum response. The decrease in apparent mass becomes less as the frequency of excitation is increased due to the variation of z_2.

The "vector" plot of the reciprocal of Z_{4d1} is not strictly a circle because of the variation of its real component with frequency. Variation of z_2 with frequency through the range of resonance of a receiver diaphragm is small enough to make the deviation from circular form inappreciable. The variation of the reactive terms has no influence on this circular form of motional impedance. They determine only the distribution of points about the circumference of the circle and therefore lead to errors in the calculation of logarithmic decrement per second by the standard method from resonance circles or curves. The errors are only serious when the resonant region is large.

The variation in force factor B_{1d4} with frequency due to the term $b_{24} Z_{12}/Z_{22}$, representing the eddy current reaction, also produces a small deviation from circular form in motional impedance. This term shows how the eddy current produces a depression in the principal diameter of the motional impedance circle. This depression angle 2β, Fig. 6, is twice the angle of B_{1d4}. Since β is a function of frequency the angle of depression 2β, of motional impedance, is a function of the frequency to which the diaphragm is tuned, being small at low frequencies, higher at intermediate frequencies, and small at very high frequencies.

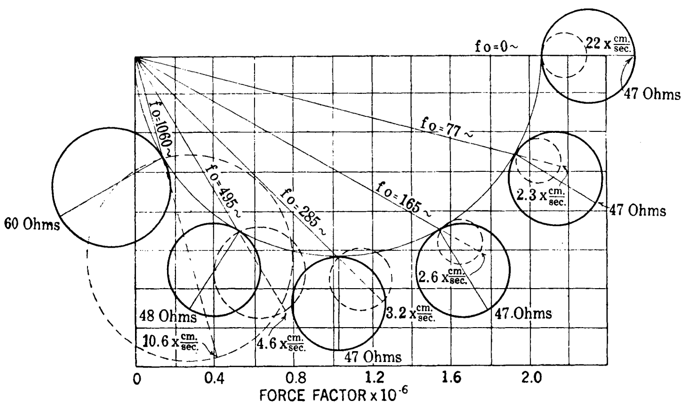


FIG. 7—FORCE FACTOR, MOTIONAL IMPEDANCE AND VELOCITY AT DIFFERENT FREQUENCIES

Velocity given per milliampere—dotted circles.

To illustrate the variation of force factor and motional impedance in this type of receiver, a simple case shown in the diagram (Fig. 7) was computed.

The constants of the experimental receiver were used with the exceptions that no mechanical resistance R_4 was assumed in the diaphragm and in place of eddy current in the core, a second winding was assumed of the same constants as the receiving coil completely coupled to it and short-circuited. In this case the force factor semicircle has a diameter of b_{14}. Motional impedance circles with the diaphragm tuned to six different frequencies are shown in heavy lines at the ends of their corresponding force factor "vectors." The frequencies of tuning are 0, 77, 165, 285, 495, and 1060 cycles. The circles increase in diameter with frequency of tuning of the diaphragm. This is because the eddy current component of damping resistance decreases at a faster rate than B_{1d4}. The dotted circles are corresponding "vector" diagrams of root mean square velocity of the diaphragm per milliampere of current in the winding. These increase in diameter indefinitely with frequency. In practise, these circles may either increase or decrease with frequency of tuning depending upon the relative magnitudes and variations of R_4, B_{1d4}, and eddy damping reaction $R_2\, b_{24}^2/z_2^2$.

The force factor semicircle can be determined experimentally for any receiver. The value of B_{1d4} is found in phase and magnitude at resonance from the motional impedance circle, or resonance curve, by a method such as that given under "Simple Receiver." By reference to Fig. 6, it will be seen that the additional measurement of the angle θ_2 determines the semicircle. The angle θ_2 is the phase angle of the eddy current circuit at the frequency of diaphragm resonance and is found from the damped impedance by the following reasoning.

$$Z_{1d4} = (R_1 + R_2\, z_{12}^2/z_2^2) + j\, w\, (L_1 - L_2\, z_{12}^2/z^2) \qquad \textbf{(24)}$$

At low frequencies the apparent resistance and inductance approach the values R_1 and L_1. These may be determined by extrapolation from measurements of damped impedance at low frequencies. The differences between the damped resistance and inductance at resonance and these values give the last terms in these two parentheses. The ratio of the differences give, $\tan\theta_2 = w\, L_2/R_2$. Having found the semicircle, the force factor B_{1d4} can be found at any frequency by simply measuring θ_2 in this way from the damped impedance, drawing a line from "a" (Fig. 6) at this angle. The line connecting its intersection with the semicircle and the origin is the force factor at the frequency in question.

A variation in inductance L_2 of the eddy circuit will cause a deviation from the circular form of the force factor locus through the speech range of the frequencies and above it. The eddy circuit inductance is usually very nearly constant in commercial receivers. The variation in the resistance R_2 of the eddy circuit alters the simple theoretical distribution of points about this locus. R_2 increases indefinitely with frequency. This tends to retard the progress of the force factor around this locus as frequency increases. In one type of receiver investigated with a single cylindrical core, the eddy current resistance approaches linear increase with frequency such that the eddy current phase angle θ_2 approaches a constant value of about 70 deg. instead of 90 deg. at high frequencies.

To illustrate the application of this theory, an analysis of a particularly efficient standard bipolar head receiver was made. It is assumed that the coils and eddy currents in the two poles are approximately the same. This allows of treating two eddy circuits as one. Fig. 8 shows curves of measured resistance and reactance plotted as functions of frequency from 0 to 1500 cycles. The distortion of these curves from a smooth variation in the neighborhood of 1000 cycles is due to a large vibration of the diaphragm in the region of its natural frequency. In a small region between 1000 and 1050 cycles, the vibration of the diaphragm is in such a phase and large enough actually to give the receiver as a whole an effective capacity reactance.

The smooth curves are drawn asymptotically and represent to a fair accuracy the "damped" values which would be obtained by restraining diaphragm motion. The circular "vector" plot of motional impedance is given in Fig. 9. The frequency of the maximum response of the diaphragm is 1005 cycles, at which the

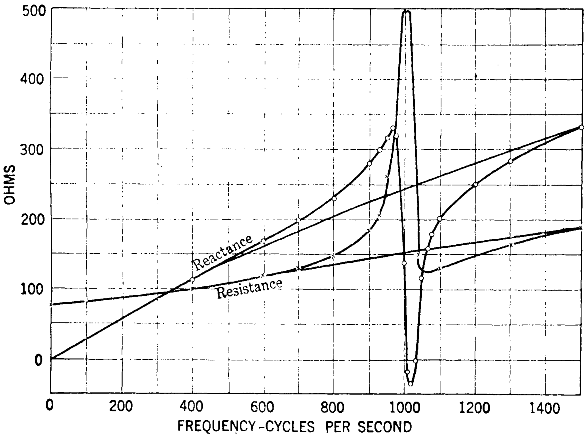


FIG. 8—STANDARD BIPOLAR HEAD RECEIVER—RESISTANCE AND REACTANCES CURVES

motional impedance is $332 - j\,180$ or 380 ohms at an angle 2β of $-$ 28.5 deg. The logarithmic decrement per second, of the diaphragm is 138. This is obtained from the circle by multiplying the difference between the frequencies at opposite ends of the diameter perpendicular to the principal diameter of the motional impedance circle by π.

The apparent mass of the diaphragm $(L_4 - L_2\, b_{24}^2/z_2^2)$ is 0.400 gram. This is found by a calculation from the measured change in frequency of maximum response by the addition of a small known mass at the center of the diaphragm. The resistance to motion of the

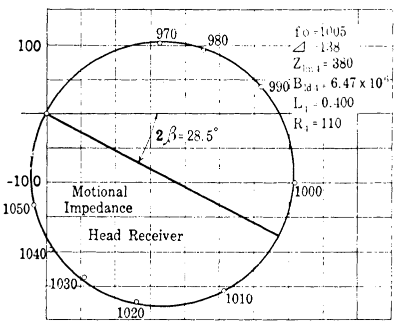


FIG. 9—CIRCULAR "VECTOR" PLOT OF MOTIONAL IMPEDANCE

diaphragm $(R_4 + R_2\, b_{24}^2/z_2^2)$ is 110 dynes per centimeter per second and is found from the relation

$$\Delta = (R_4 + R_2\, b_{24}^2/z_2^2)/2(L_4 - L_2\, b_{24}^2/z_2^2)$$

The motional impedance is given by

$$Z_{1m4} = B_{1d4}^2/(Z_{44} + Z_{24}^2/Z_{22})$$

(see equation 22), which owing to the cancelling of the mass and elastic reactances of Z_{44} becomes at the frequency of maximum response

$$Z_{1m4} = B_{1d4}^2/(R_4 - R_2\, b_{24}^2/z_2^2)$$

From this, the damped force factor B_{1d4} was computed at the frequency of maximum response to be $(6.28 - j\,1.59) \times 10^6$ or 6.47×10^6 dynes per absolute electromagnetic unit of current, at an angle of $-$ 14.3 deg. This is plotted "vectorially" in Fig. 10, the reference phase being that of i_1, the current in the receiving winding. The semicircle given in this figure is the locus of B_{1d4} as frequency is varied. The force factor at zero frequency, b_{14}, is 8.44×10^6 and is in phase with the received current. The semicircular locus was determined by the method given in the discussion of equation **21**. The force factors at 500 and 1500 cycles are also given.

The part of the resistance due to eddy current $(R_2\, b_{24}^2/z_2^2)$ is computed to be 42 dynes per centimeter per second which is about 38 per cent of the total. The eddy current component of resistance is computed from the force factor diagram, Fig. 10, and damped

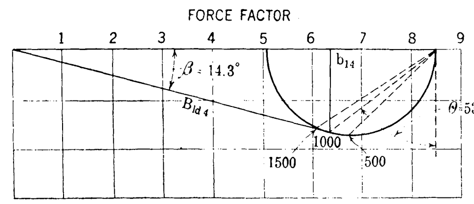


FIG. 10—HEAD RECEIVER-PLOT OF FORCE FACTOR B_{1d4}

impedance, Fig. 8. A little calculation will show that this is equal to the product of the real and imaginary components of the "vector," $- b_{24}\, Z_{12}/Z_{22}$ of the experimentally determined force factor diagram (of Fig. 6) divided by $w\, L_2\, z_{12}^2/z_2^2$ obtained from damped reactance (see discussion of equation **24**).

The apparent mass of the diaphragm is 0.400 gram. This is less than the purely mechanical apparent mass by 0.009 gram or about 2 per cent which gives an increase in maximum response frequency of about 10 cycles. The eddy current component of apparent mass can be seen to be equal to the product of the real and imaginary parts of the "vector" $- b_{24}\, Z_{12}/Z_{22}$ in the force factor diagram divided by the product of the radial frequency w, and the added damped resistance $R_2\, z_{12}^2/z_2^2$ due to eddy currents (see equation **24**).

C. Simple Induction-Type Receiver. In principle, an ordinary telephone receiver using a non-magnetic conducting diaphragm such as aluminum or copper is an inefficient induction receiver. The most familiar example of a device working on this principle is the Fessenden submarine oscillator. Motion of the diaphragm causes no change of inductance of any circuit except the eddy current circuit on the diaphragm which moves with it. This circuit is No. 3 in Fig. 1. As a consequence, $b_{14} = b_{24} = 0$. b_{34} is therefore the only acting force factor.

Excluding all eddy currents other than that in the diaphragm, the characteristic equations are

$$\left.\begin{aligned} Z_{11}\,i_1 + Z_{13}\,i_3 \qquad\quad &= E_1 \\ Z_{13}\,i_1 + Z_{33}\,i_3 + Z_{34}\,\dot{x} &= O \\ -\,Z_{34}\,i_3 + Z_{44}\,\dot{x} &= F \end{aligned}\right\} \quad (25)$$

Eliminating i_3

$$\left.\begin{aligned} (Z_{11} - Z_{13}^2/Z_{33})\,i_1 + (-\,Z_{34}\,Z_{13}/Z_{33})\,\dot{x} &= E_1, \\ -\,(-\,Z_{34}\,Z_{13}/Z_{33})\,i_1 + (Z_{44} + Z_{34}^2/Z_{33})\,\dot{x} &= F \end{aligned}\right\} \quad (26)$$

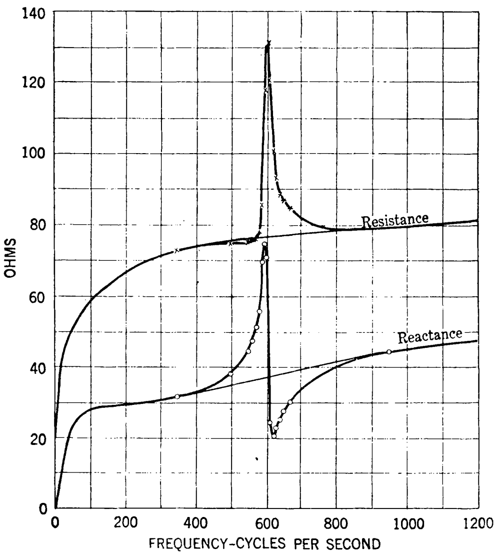


Fig. 11—Induction-Type Receiver—Resistance and Reactance Curves

Following the reasoning given under "*B. Receiver with Eddy Currents in the Core.*" These equations are identical with

$$\begin{aligned} Z_{1d4}\,i_1 + B_{1d4}\,\dot{x} &= E_1 \\ -\,B_{1d4}\,i_1 + Z_{4d1}\,\dot{x} &= F \end{aligned} \qquad (27)$$

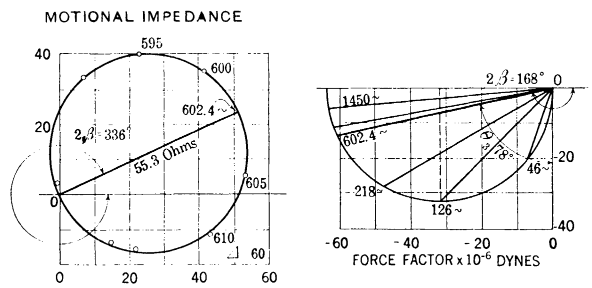


Fig. 12—Induction-Type Receiver—Motional Impedance and Force Factor Diagrams

These are the equations of the induction-type receiver reduced to the form of the simple receiver. As before the motional impedance is given by

$$Z_{1m4} = B_{1d4}^2/Z_{4d1}$$

The forms of Z_{1d4} and Z_{4d1} are identical with the former case and the same discussion holds here. The force factor B_{1d4} and the motional impedance Z_{1m4} are somewhat different.

Fig. 11 shows the curves of measured resistance and reactance as a function of frequency for an experimental induction-type receiver. The smooth asymptotic curves represent damped values.

Fig. 12 shows the motional impedance and force factor diagrams of this receiver at different frequencies.

The force factor for an induction-type receiver is given by

$$B_{1d4} = -\,b_{34}\,\frac{j\,w\,L_{13}}{R_3 + j\,w\,L_3} \qquad (28)$$

By comparison with equation **21**, it will be seen that this has the same semicircular form as the force factor

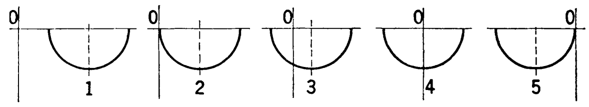


Fig. 13—Possible Force Factor Semicircles

for a receiver with eddy currents in the core, but is displaced on the real axis to a point of tangency on the negative side of the imaginary axis. This is due to the absence of the direct force factor, b_{14}, between the receiving winding and the diaphragm. In order for this type of receiver to be efficient, the semicircle must be large, and the phase angle, θ_3, of circuit No. 3 must be large. This means that R_3 must be small.

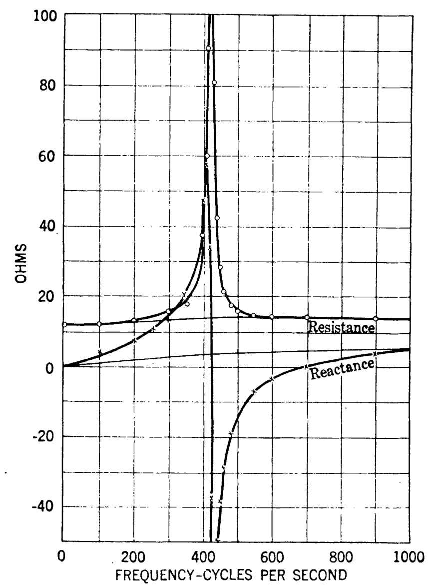


Fig. 14—Electrodynamic Receiver—Resistance and Reactance Curves

The force factor is determined experimentally by the same method as that used in the previous case "B".

The angle, 2β, of maximum motional impedance is 336 deg. and that of the force factor at this frequency is $\beta = 168$ deg. at the frequency, 602.4 cycles, of resonance. The phase angle of circuit No. 3 is $\theta_3 = 78$ deg. at resonance. The theory of this apparatus when used to receive mechanical energy is obtained from the equations by allowing the vibrating force F to act instead of the electromotive force E_1.

It is interesting to note that the theory of this system is the same as that for the simple receiver (without eddy currents) but working through a transformer.

The force factor semicircles of cases "B" and "C" are in a sense closely related.

$$B_{1d4} = Z_{14} - Z_{24} Z_{12}/Z_{22}$$

This is the equation of the force factor for case "B". By changing the relative values of the first and second terms, the semicircle representing it is moved along the real axis from any position on the positive side (Z_{14} larger), down through a point of tangency on the positive side of the imaginary axis as in Fig. 6 to tangency on the negative side which corresponds to the induction-type receiver in which case $Z_{14} = 0$. Fig. 13 illustrates a series of possible force factor circles with one circuit in addition to the receiver circuit, No. 1.

Diagram 1. $b_{14} > b_{24} L_{12}/L_2$

This is the case in which the eddy current is least. If the semicircle shrinks to a point we have a case of a simple receiver.

Diagram 2. $b_{14} = b_{24} L_{12}/L_2$

For this condition it is necessary that circuits Nos. 1 and 2 have 100 per cent inductive coupling. Fig. 7 illustrates this case.

Diagram 3. $b_{24} L_{12}/L_2 > b_{14} > b_{24} L_{12}/2\, L_2$

This is the first case in which the phase of the force factor can be greater than 90 degrees.

Diagram 4. $b_{24} L_{12}/2\, L_2 = b_{14}$

The magnitude of the force factor here is independent of frequency but its phase increases from zero to $-$ 180 deg.

Diagram 5. $b_{14} = 0$

This is the case of the induction-type receiver, or simple receiver, working through a transformer.

D. Electrodynamic Receiver. The electrodynamic receiver operates by placing the driving coil on a movable member or diaphragm which is non-magnetic. Referring to Fig. 1, this case corresponds in principle to using coil No. 3 as the receiving coil. The theory of this case is identical with that for the vibration galvanometer which corresponds to a simple receiver if no eddy currents are present.

We will consider the case where an eddy circuit No. 2 is present in the stationary magnetic poles supplying the polarizing flux. The characteristic equations are

$$\left.\begin{aligned} Z_{22}\, i_2 + Z_{23}\, i_2 \qquad\quad &= 0 \\ Z_{23}\, i_2 + Z_{33}\, i_3 + Z_{34}\, \dot{x} &= E_3 \\ - Z_{34}\, i_3 + Z_{44}\, \dot{x} &= F \end{aligned}\right\} \quad \textbf{(29)}$$

This is different from the induction-type receiver, case "C," only in that the driving electromotive force acts in circuit No. 3 instead of the stationary circuit. In this case we are interested in the impedance measured from circuit No. 3.

Eliminating i_2

$$\left.\begin{aligned} (Z_{33} - Z_{23}^2/Z_{22})\, i_3 + Z_{34}\, \dot{x} &= E_3 \\ - Z_{34}\, i_3 + Z_{44}\, \dot{x} &= F \end{aligned}\right\} \quad \textbf{(30)}$$

As before the coefficient of i_3 in the first equation is the damped impedance Z_{3d4} of the receiver. The force factor is independent of frequency and uninfluenced by the eddy current. The motion of the diaphragm is independent of eddy current which adds no electromagnetic damping.

$$Z_{3m4} = Z_{34}^2/Z_{44}$$

Motional impedance as in the case of the simple receiver is represented by a circle with its principal diameter coinciding with the real axis as in the case of the simple receiver. Eddy currents act simply as an inductively coupled external circuit.

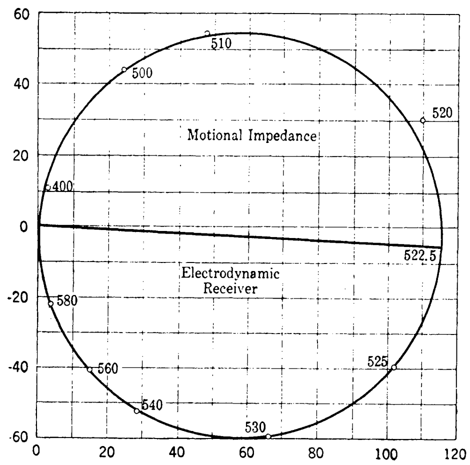


FIG. 15—VECTOR PLOT OF MOTIONAL IMPEDANCE

Fig. 14 shows plots of measured resistance and reactance of such a receiver. Fig. 15 is a vector plot of motional impedance. The natural frequency is 522.5 cycles. The depression angle of maximum motional impedance is small due to a small effect of eddy current in the coil supports on the diaphragm.

CONCLUSION

The object of this paper is to reduce the theory of typical magneto-mechanical vibratory structures to the simplicity of the electrical network theory, which amounts essentially to a manipulation of complex algebra.

The method is illustrated by experimental data on representatives of the leading types of magnetic receivers and transmitters. It is equally applicable to types having any number of circuits or mechanical vibratory systems including electromagnetic galvanometers and meters.