

PROCEEDINGS
OF THE
AMERICAN PHYSICAL SOCIETY.

MINUTES OF THE NINETY-SIXTH MEETING.

THE ninety-sixth meeting of the American Physical Society was held in Fayerweather Hall of Columbia University, New York, on February 28 and March 1, 1919.

A regular meeting of the Council was held on February 28, five members being present, and President Ames presiding. Professor Theodore Lyman, of Harvard University, was elected vice-president of the society for the unexpired term caused by the death of Professor Wallace C. Sabine. The following were recommended to the National Research Council as representatives of the American Physical Society in the Division of Physical Sciences: H. A. Bumstead, Wm. Duane, Irving Langmuir, Earnest Merritt, R. A. Millikan, and E. B. Wilson. The following were appointed representatives of the American Physical Society in the International Astronomical Union: J. S. Ames, Henry Crew, and Theodore Lyman.

Elections to membership were made as follows: to *regular membership*, J. A. Fleming, Carl Hering, Paul W. Merril, Henry Norris Russell; to *associate membership*, Carl Darnell, J. P. Delaney, Carl F. Eyring, Joseph Foster, Clifford G. Formd, John G. Homan, Horton W. Hall, G. A. Johnstone, Edwin W. Kelly, C. N. Moore, A. M. Mahon, B. A. Orndorff, K. G. Szlupas, Paul B. Taylor, Phillips Thomas; *transferred from associate to regular membership*, S. J. Mauchly, Peter I. Wold.

Sessions for the reading of papers were held in the morning and in the afternoon of Saturday, March 1, 1919, at which twenty-seven papers were presented, six being read by title. The program was as follows:

Water Vapor Pressure in Units of Force. ALEXANDER MCADIE. (Read by title.)

The Thermal Expansion of Living Tree Trunks. C. C. TROWBRIDGE (deceased) and MABEL WEIL.

Pyrometer Absorption Glasses. PAUL D. FOOTE, C. O. FAIRCHILD and F. L. MOHLER. (Read by title.)

The Function of Phase Difference in the Binaural Location of Pure Tones. R. V. L. HARTLEY.

Temperature Coefficient of Resistance of Molybdenum. E. C. BLOM.

Measurement of Small Alternating Currents by Means of a Rayleigh Resonator. F. R. WATSON. (Read by title.)

An Addition to the Theory of the Quadrant Electrometer. A. H. COMPTON and K. T. COMPTON.

Permanent-Contact Crystal Detectors. LOUISE S. McDOWELL.

Are the Frequencies in the Series of X-rays the Highest Frequencies Characteristic of Chemical Elements? WILLIAM DUANE and TAKEO SHIMIZU.

The Spectral Photoelectric Sensitivity of Silver Sulphide. W. W. COBLENTZ and H. KAHLER. (Read by title.)

Some Photoelectric Lecture Experiments. JACOB KUNZ. (Read by title.)

The Crystal Structure of Carborundum. A. W. HULL.

A Possible Relation between the Resonance Potential and Specific Inductive Capacity of a Metallic Vapor and its Properties in the Solid State. K. T. COMPTON.

Collisions of Slow Moving Electrons with Nitrogen Molecules. S. KARRER. (Read by title.)

The Ionizing Potentials of Argon, Neon and Helium. H. C. RENTSCHLER.

Note on the Use of the Hot Cathode Vacuum Tube for the Measurement of Voltage. LYNDE P. WHEELER.

An Interferential Method for Measuring the Expansion of very small Samples. C. G. PETERS and IRWIN G. PRIEST.

On the Measurement of the Detection Coefficient of Thermionic Vacuum Tubes. H. J. VAN DER BIJL.

The Properties of the Electron as Derived from the Chemical Properties of the Elements. IRVING LANGMUIR.

The Law of Absorption of High Frequency Radiation. ARTHUR H. COMPTON.

Some Physical Properties of Dental Materials. C. G. PETERS and W. H. SOUDER.

The Origin of the General Radiation Spectrum of X-Rays. DAVID L. WEBSTER.

Note on Bomb Trajectories. EDWIN B. WILSON.

On the Relation Between the *K* Series and the *L* Series of X-Rays. WILLIAM DUANE and TAKEO SHIMIZU.

A New Formula for the Spectral Distribution of Energy from a Complete Radiator. IRWIN G. PRIEST.

Transmission of a Quartz Grating Replica. R. S. WETZEL and WALTER SCHWANHAUSER.

On the Measurement of the True Barometric Pressure in a Rapidly Moving Current of Air. J. G. COFFIN.

DAYTON C. MILLER,
Secretary.

THERMIONIC AMPLIFIER.¹

BY H. J. VAN DER BIJL.

THE current voltage characteristic of the vacuum-tube amplifier manufactured by the Western Electric Company can be represented by the equation

$$I = \alpha(\gamma E_B + E_C + \epsilon)^2, \quad (1)$$

where I is the current in the filament-plate circuit, E_B the potential difference between filament and plate, E_C the potential difference between filament and grid, and ϵ a small quantity which depends on the contact potential difference between filament and grid, the potential drop in the filament, etc.

The equation is obtained as follows: Suppose the grid and filament be electrically connected and a potential difference E_B be applied between them and the plate, then the field at a point near the filament is the same as if a potential difference $E_B + \epsilon$ be applied between the filament and a plane coincident with the grid. If a potential difference E_C be also applied between the filament and the grid the effective voltage is the sum, namely $\gamma E_B + E_C + \epsilon$. The exponent 2 has been found empirically.

With the help of this equation the amplification equations of the tube can be obtained. It shows that the relation between plate current and grid voltage is of the same form as the relation between plate current and plate voltage. If an alternating voltage $e \sin pt$ be applied between grid and filament the current is

$$I = \alpha(\gamma E_B + E_C + \epsilon + e \sin pt)^2. \quad (2)$$

The admittance of the tube K is given by

$$\frac{1}{2\kappa} \int_0^{2\pi} \frac{\partial I}{\partial E_B} dt = 2\alpha\gamma (\gamma E_B + E_C + \epsilon).$$

The impedance is $1/K$, and this can be expressed as

$$R_0 = \frac{E_B + \mu_0 (E_C + \epsilon)}{2I}, \quad (3)$$

where $\mu_0 = 1/\gamma$ and is the maximum voltage amplification obtainable from the tube.

If a resistance R be placed in the filament plate circuit, the voltage E_B between filament and plate does not remain constant, but is given by

$$E_B = E - RI, \quad (4)$$

where E is the constant voltage of the battery in the plate circuit. The characteristic equation then becomes

$$I = \alpha [\gamma(E - RI) + E_C + \epsilon + e \sin pt]^2. \quad (5)$$

It is seen from this that the external resistance has the effect of straightening out the characteristic. For a perfect amplifier the characteristic must be

¹ Abstract of a paper presented at the New York meeting of the American Physical Society, April 27, 1918.

straight, otherwise distortion is produced, because in virtue of the curvature of the characteristic an increase in the input voltage (filament grid voltage) produces a greater increase in current than the current decrease produced by an equivalent decrease in the input voltage. It has been found that if the external resistance is equal to or greater than the impedance of the tube itself, the characteristic is sufficiently straight to reduce distortion to a negligible quantity.

Considering now the case in which the circuit contains an external resistance R , it is seen that a variation in the current in R due to a variation in the grid voltage causes the plate voltage also to be variable, that is,

$$I = f(E_B, E_C)$$

and hence

$$\frac{dI}{dE_C} = \frac{\partial I}{\partial E_B} \frac{dE_B}{dE_C} + \frac{\partial I}{\partial E_C}. \quad (6)$$

Referring to equations (1) and (4) this gives

$$\frac{dI}{dE_C} = 2\alpha (\gamma E_B + E_C + \epsilon) \left(\gamma \frac{d(E - RI)}{dE_C} \right),$$

i. e.,

$$\frac{dI}{dE_C} = \frac{2\alpha (\gamma E_B + E_C + \epsilon)}{1 + 2d\gamma R(\gamma E_B + E_C + \epsilon)}.$$

Multiplying by R and putting $\gamma = 1/\mu_0$ we obtain

$$R \frac{dI}{dE_C} = \frac{\mu_0 K}{R + \frac{E_B + \mu_0(E_C + \epsilon)}{2I}}. \quad (7)$$

Now RdI is the voltage change set up in the resistance R and dE_C is the change in the input voltage, so that equation (7) gives the voltage amplification μ which with the help of (3) can be written

$$\mu = \frac{\mu_0 R}{R + R_0}. \quad (8)$$

From this it readily follows that if R_i be the input impedance of the tube, the power amplification is given by

$$\eta = \frac{\mu^2 R R_i}{(R + R_0)^2} \quad (9)$$

and the current amplification by

$$\xi = \frac{\mu_0 R_i}{R + R_0}. \quad (10)$$

These equations have been tested and found to hold with sufficient accuracy to serve for the computation of circuits involving thermionic amplifiers.

It must be remarked that the characteristic equation used here is not sufficiently accurate to apply in the case in which the tube is used as a radio-detector. The detecting action of the tube depends on the second derivative of the characteristic, so that when using the tube as a detector a more accurate equation for the characteristic is used.

RESEARCH LABORATORY,
WESTERN ELECTRIC CO.,
NEW YORK.

WATER VAPOR PRESSURE IN UNITS OF FORCE.¹

BY ALEXANDER MCADIE.

THERE are many instructors in physics and chemistry who are not at the present time, for good and sufficient reasons, fully conversant with the units used by aërographers.

Probably no one feature of the war, not even the use of gas, nor the success of the submersible, stands out so sharply as the development of the airship. Whether we be advocates of the plane or the blimp, representing in general heavier-than-air and lighter-than-air craft, we all recognize that a new and vastly important field of engineering has been opened up in aëronautics, and a new and promising field of investigation for the physicist in studies of the structure of the atmosphere, more briefly called aërography.

With the first use of the sounding balloon it became apparent that the Fahrenheit scale would have to be scrapped and that the Centigrade scale was likewise unserviceable. The reasons are self-evident. And so the temperatures in all upper air work are given on the Absolute scale. The form used however is not the most convenient, and we have been using with success at Blue Hill for several years a new form which has been called the Kelvin kilograde and which has been described elsewhere. The zero is the absolute zero and the melting point of ice under standard pressure is marked 1000. No degree signs are used. The values used later are in these units.

In upper air work it also became evident that inches and millimeters as units for the expression of atmospheric pressure were, like the temperature units, out-of-date; and so the aërographic services of many countries, especially Great Britain, France and Italy have been expressing pressure in units of force. This usage is spreading rapidly.

So far as I can ascertain credit for the suggestion and first use of an absolute unit to express pressure must be given to Professor Theodore Richards. In the classic paper on "New Method of Determining Compressibility" (Carnegie Inst., 1903), the *bar* is clearly defined as the pressure of a dyne per square centimeter. The pressure of an atmosphere is expressed as one million bars or megabar. For convenience this is written 1,000 kilobars. The pressure of the atmosphere at a height of 1 km. is approximately 900 kb., at an elevation of 10 km. the pressure is about 300 kb., and at the top of the atmosphere or as high as we may hope to explore for some time to come, say 50 km., the pressure is as low as 5 kb.

The use of these units of force has been extended at Blue Hill to measurements of water vapor pressure. There appears to be no difficulty in using the new units and the formulæ below.

European aërographic services use the following formulæ when the cover of the wet bulb is not frozen, and it may be noted here that in high level observations the temperature is generally so low that the wet bulb is covered with ice. No satisfactory solution has yet been reached.

¹ Abstract of a paper presented at the New York meeting of the American Physical Society, March 1, 1919.

With wind velocity 0 to 0.5 m/s (meter per second)

$$p_s = p_e - .0012 P(t - f_1) (1 + f'/610)$$

with wind velocity 1 to 1.5 m/s

$$p_s = p_e - .0008 P(t - t_1) (1 + t'/610)$$

with wind velocity exceeding 2.5 m/s

$$p_s = p_e - .000656 P(t - t_1) (1 + t'/610)$$

When the water is frozen, the numerical coefficients .001060, .000706 .000579, are used, and 610 in the last factor changed to 689.

We prefer to express the relation in the following form

$$p_s = p_e - PK(t - t')$$

in which p_s is the pressure of the water vapor at the temperature of the dew-point or saturation, p_e the pressure at the temperature of evaporation or wet-bulb, P the pressure of the atmosphere, K a constant which like most constants varies somewhat (with ventilation of the bulb and probably other conditions). All pressures are given in the same unit, kilobar, and all temperatures in same unit, Kelvin kilograde. The value of K within ordinary limits is .00179. For quick determination of vapor pressure, absolute humidity and relative humidity, the value of PK may be taken as .18.

The wind effect is an uncertain quantity. All the tables of relative humidity so laboriously compiled by various meteorological services are invalidated by the omission of statement of temperature and wind values. Reference is made to papers by the author: "New Units in Aërology," PHYSICAL REVIEW, N.S., Vol. VI., No. 6, December, 1915; "Comparative Accuracy of Whirled Psychrometer," etc., PHYS. REV., February, 1918.

THE THERMAL EXPANSION OF LIVING TREE TRUNKS.¹

BY C. C. TROWBRIDGE (DECEASED) AND MABEL WEIL.

THE transverse coefficient of expansion of living tree trunks has been found to be extremely large below 0° C. and very much smaller, *i. e.*, of about the order of the coefficient of expansion of dead wood, above that temperature. In the longitudinal direction a small *expansion* with *rise* of temperature was noted above the freezing point while an equally small *expansion* with *fall* of temperature was measured below that point. When, however, the temperature fell to about -18° C., a slight *contraction* with *fall* of temperature was observed. This behavior indicates the presence of two processes superposed, one a thermal coefficient of expansion, the other a physiological or cellular expansion. A lag of the transverse expansions and contractions behind temperature changes, also points to the same conclusion.

¹ Abstract of a paper presented at the New York meeting of the American Physical Society, March 1, 1919.

PYROMETER ABSORPTION GLASSES.¹

BY PAUL D. FOOTE, C. O. FAIRCHILD AND F. L. MOHLER.

THE proper methods of using absorption glasses and the choice of a suitable absorption glass for extending the high temperature scale are questions of considerable importance. The calibration of an absorption glass is given by the formula

$$1/\theta - 1/S = A,$$

where θ is the absolute temperature of a black body, S the absolute observed temperature with the absorption glass in place and A is in general a function of θ .

A glass is described for which A is a constant. Various methods of determining A are given. The use of two absorption glasses together is discussed and several theorems are developed in regard to effective wave-lengths of colored glasses, effective wave-length when sighted on non-black radiators, etc. A synopsis of the paper will appear in the Journal of the Optical Society and the complete paper will be published in the Bulletin of the Bureau of Standards.

THE MEASUREMENT OF SMALL ALTERNATING CURRENTS BY MEANS OF A RAYLEIGH RESONATOR.¹

BY F. R. WATSON.

THE resonator consisted of a brass cylinder closed at one end by a telephone receiver and open at the other end through a small tube in which a Rayleigh disc was suspended by a quartz fiber. Alternating currents sent through the telephone receiver set up vibrations of the air in the cylinder that caused the disc to rotate, the amount of rotation being noted by the deflection of a beam of light reflected from the disc. A 110-volt, 60-cycle alternating current gave readable deflections for currents of the magnitude of 1×10^{-5} amperes. Later experiments indicated a greater sensitivity.

Various modifications of the apparatus were tested. A double resonator was made of two resonators similar to the one described connected by a tube with the Rayleigh disc in the center of the connecting tube. This arrangement allowed the use of two telephones and, being closed from the outside air, eliminated almost entirely the effect of extraneous sounds. An ordinary Bell telephone arrangement of a transmitter, dry cell, induction coil and receiver gave deflections for speech and musical sounds. Sensitiveness was increased by tuning the resonators and the electrical circuit to the source of sound. The frequencies of the currents ranged from 60 to 1,200 per second.

UNIVERSITY OF ILLINOIS,
December 1, 1918.

¹ Abstract of a paper presented at the New York meeting of the American Physical Society, March 1, 1919.

AN ADDITION TO THE THEORY OF THE QUADRANT ELECTROMETER.¹

BY A. H. COMPTON AND K. T. COMPTON.

THE ordinary theory of the quadrant electrometer is extended to take account of a tilt of the needle about its long axis, a vertical displacement of one pair of quadrants with respect to the other by an amount δ and a vertical displacement of the needle from its position of symmetry by an amount p . Under these conditions, if the slope of the tilted needle is s , the torque due to electrical forces is

$$T = K_1 V(v_2 - v_1) + V^2 \left(K_3 \frac{Rs\delta}{h^2} \theta + K_4 + K_5 \frac{\delta p}{h^2} \right), \quad (1)$$

neglecting terms of second and higher order of small quantities. Here V is the potential of the needle, v_1 and v_2 are the potentials of the two pairs of quadrants, h is the height of the quadrants, R is the radius of the needle and θ is the deflection.

The terms

$$\left(K_4 + K_5 \frac{\delta p}{h^2} \right) V^2$$

cause the deflection usually observed when the needle is charged. By suitably adjusting δ or p these terms may be made to cancel. Then the remaining terms are balanced against the torque $K_2\theta$ from the suspension, giving

$$K_1 V(v_2 - v_1) = \left(K_2 - K_3 \frac{Rs\delta}{h^2} V^2 \right) \theta; \quad (2)$$

whence the sensitiveness is

$$S = \frac{\theta}{v_2 - v_1} = \frac{K_1 V}{K_2 - K_3 \frac{Rs\delta}{h^2} V^2}. \quad (3)$$

By suitably adjusting s , δ and V , the magnitude and sign of the second term in the denominator may be varied within wide limits permitting a range of sensitiveness, theoretically from zero to infinity, to be obtained with any given suspension.

A model of an electrometer designed to take advantage of these considerations is demonstrated.

PERMANENT—CONTACT CRYSTAL DETECTORS.¹

BY LOUISE S. McDOWELL.

THE following investigation of possible methods of securing a permanent-contact crystal detector was initiated by a request from the Signal Corps to the Bureau of Standards to investigate a method proposed by Second Lieutenant Henri Lauer. His proposal to secure a permanent multiple-contact crystal detector by alternating-current electrolysis was based upon the hypothesis that, when immersed in an electrolyte, the rectifying action of the crystal would permit current to flow in only one direction through the sensitive areas and that

¹ Abstract of a paper presented at the New York meeting of the American Physical Society, March 1, 1919.

upon these areas alone there should be a deposit of the metal since the flow of current in both directions through the insensitive areas would prevent deposition. To the deposits upon the sensitive areas permanent contact could then be made by direct-current electrolysis or otherwise.

Audibility tests made upon two or more detectors in parallel and upon single crystals with contacts in parallel to two or more sensitive areas indicated that no gain in loudness of signal was to be expected from the use of multiple contacts. Crystals were further divided into three classes according to the direction of flow of the rectified current. Class 1, in which the direction of the current was usually from point to crystal, included molybdenite, zincite, chalcopyrites, iron pyrites, and bornite. Class 2, in which the direction of current flow varied from one sensitive point to another on the same crystal, included carborundum, approximately fifty per cent. of the specimens of silicon tested and one or two specimens of galena and molybdenite. Class 3, in which the direction of the current was usually from crystal to point included galena and fifty per cent. of the silicon tested.

Alternating-current electrolysis, with currents of 60 cycles and of radio frequency, was tried with crystals of class 1. A few adherent deposits of copper were obtained, but results indicated that these were due to accidental causes, not to rectification by the crystal since (1) only a few of the sensitive spots were coated, (2) there were many insensitive deposits, (3) lack of perfect adhesion between deposit and crystal increased the sensitiveness when contact was made to the deposit.

With galena, of class 3, an attempt was made to secure a metallic deposit upon the sensitive spots by cathode sputtering, using an alternating discharge and making the crystal one electrode. Results indicated that under these conditions no rectification occurs of a nature to permit deposition upon the sensitive areas of the crystal.

Further attempts to obtain a fixed, sensitive contact by the use of direct-current electrolysis, by soldering, and by welding proved unsuccessful with the more sensitive crystals such as galena and silicon although partially successful with molybdenite and carborundum. The results point to the necessity of a high resistance at the contact and suggest that with the more sensitive crystals the only satisfactory permanent-contact detector will be one with a mechanically maintained, high resistance contact.

BUREAU OF STANDARDS,
WASHINGTON, D. C.

ARE THE FREQUENCIES IN THE *K* SERIES OF X-RAYS THE HIGHEST FREQUENCIES CHARACTERISTIC OF CHEMICAL ELEMENTS?¹

BY WILLIAM DUANE AND TAKEO SHIMIZU.

A NUMBER of investigators have found experimental evidence, which they interpreted as indicating the existence of radiation or absorption

¹ Abstract of a paper presented at the New York meeting of the American Physical Society, March 1, 1919.

characteristic of a chemical element, and of higher frequency than those of its *K* series of X-rays. On the other hand curves have been published showing the emission of X-rays by chemical elements such as rhodium and molybdenum, which include rays in the general X-radiation up to frequencies 50 per cent. and 75 per cent. greater than those of the *K* radiation and these curves furnish no evidence of the existence of characteristic rays of higher frequency than the *K* absorption frequency.

In this paper we wish to report to the society the results of experiments on the X-rays emitted by an aluminium target. The wave-lengths of the X-rays that we have examined lie between $.1820 \times 10^{-8}$ cm. and 1.259×10^{-8} cm. This is the portion of the X-ray spectrum usually obtained from an ordinary X-ray tube excited by a difference of potential of 71,200 volts. Such a difference of potential does not produce X-rays shorter than $.1820 \times 10^{-8}$ cm. and rays longer than about 1.26×10^{-8} cm. are almost completely absorbed by the glass walls of the tube. In order to investigate such long waves, specially constructed tubes must be employed.

As the shortest wave-length in the *K* series of aluminium is 7.98×10^{-8} cm., the range of frequencies in our experiments extends from 6.34 to 43.8 times the highest frequency in this *K* series.

In making the measurements we used the apparatus described in the PHYSICAL REVIEW for December, 1917, p. 624. This consisted of an X-ray spectrometer with a calcite crystal, the beams of X-ray being defined by two slits in lead blocks, which lay between the X-ray tube and the spectrometer. The current through the tube came from a high tension transformer connected to a system of kenotrons and condensers, which produced approximately a constant voltage. During part of the work the difference of potential amounted to 53,200 volts and during the rest of it, to 71,200 volts. The X-ray tube (kindly furnished us by Dr. W. D. Collidge) was similar to the ordinary Coolidge tube, except that an aluminium target replaced the usual tungsten target.

The curves obtained by plating the ionization currents against the grazing angles of incidence are, within the errors of measurement, smooth curves without such prominences as indicate characteristic rays, except at the following points:

(a) Near the angles corresponding to $\lambda = .622 \times 10^{-8}$ cm. and $\lambda = .709 \times 10^{-8}$ appears a pair of well defined prominences, indicating characteristic radiation amounting to 2 per cent. and 5 per cent. of the general X-radiation. These are undoubtedly due to the characteristic *K* radiation of molybdenum, the wave-lengths of the β and α lines of which are $.641 \times 10^{-8}$ cm. and $.716 \times 10^{-8}$ cm. respectively, as given by Wooton.¹

(b) There are also prominences near the angles corresponding to $\lambda = .975 \times 10^{-8}$ cm. $\lambda = 1.18 \times 10^{-8}$ cm. These lie very close to the β and α lines in the *L* series of lead. According to Siegbahn and Friman's tables the wave-lengths of the strong β and α lines in the *L* series of lead are $\lambda = .981 \times 10^{-8}$ cm., and $\lambda = 1.17 \times 10^{-8}$ cm.

¹ PHYSICAL REVIEW, January, 1918.

Part of the Coolidge cathode in the X-ray tube consists of metallic molybdenum so that the presence of the molybdenum lines may be accounted for either as coming from a thin layer of metallic molybdenum deposited on the target during the construction of the tube, or else as the result of secondary radiation from the molybdenum in the cathode producing tertiary radiation from the target.

The lead lines undoubtedly are due to secondary radiation from the lead blocks containing the slits through which the X-rays passed before reaching the spectrometer.

If the voltage applied to an X-ray tube lies considerably above that required to produce the characteristic rays of its target (as is the case here with molybdenum and lead), the characteristic radiation is many times more intense than the general radiation in its neighborhood. Further molybdenum and lead are far more efficient radiators than aluminium is (their atomic numbers being much higher, 47 and 82 respectively instead of 13 for aluminium).

In addition to these prominences there appears a sharp break in the ionization curve at an angle corresponding to $\lambda = .3727 \times 10^{-8}$ cm. which is the characteristic ionization wave-length of iodine. This break is due to the fact that methyl-iodide was used as the gas in the ionization chamber. To make sure that no radiation characteristic of aluminium occurs near this wave-length, we removed the methyl-iodide, and refilled the ionization chamber with ethyl-bromide. Under these conditions no prominence nor break appears in the curve near $\lambda = .3727 \times 10^{-8}$ cm.

We conclude, therefore, that aluminium possesses no characteristic radiation within the range of wave-lengths examined.

THE SPECTRAL PHOTO-ELECTRIC SENSITIVITY OF SILVER SULPHIDE.¹

BY W. W. COBLENTZ AND H. KAHLER.

BOTH the natural mineral acanthite and a laboratory preparation by Mr. G. W. Vinal, were examined. The latter material which was hammered into a thin plate, was found insensitive photo-electrically, at room temperature, but at -157° C. a sharp maximum of photo-electrical sensitivity was observed for radiations of wave-length $\lambda = 1.05 \mu$.

At room temperatures the natural crystalline mineral differs from other photoelectrically sensitive substances studied in that the photo-electric response becomes fatigued and after an exposure of 2 to 3 seconds to light, the positive resistance change (galvanometer deflection) begins to be effective. For unlimited exposure, the galvanometer deflection returns to about one fourth of its

¹ Abstract of a paper presented at the New York meeting of the American Physical Society, March 1, 1919.

maximum deflection. On removing the light stimulus, the galvanometer gives a negative deflection, which in the course of a few minutes returns to the original zero scale reading. In other words, the change in resistance of the crystal when exposed to radiation, is first negative then positive, the resultant change being negative and roughly one fourth the original change. At low temperatures, -158° C., this polarization phenomenon disappears, and the response to radiation is the same as that of other substances, *e. g.*, selenium and molybdenite.

At room temperature, acanthite has a maximum of photo-electrical sensitivity for radiations of wave-length $\lambda = 1.35 \mu$ with a region of high sensitivity at 0.8 to 1μ .

At -158° C. the sensitivity curve is quite symmetrical with a maximum at 1.2μ . Mechanical working of the crystal (hammering it into a thin plate) has a marked effect upon this maximum which, in one sample, occurred at 1.13μ to 1.23μ , depending upon the amount of hammering the crystal had undergone.

BUREAU OF STANDARDS,
WASHINGTON, D. C.,
February 5, 1919.

THE CRYSTAL STRUCTURE OF CARBORUNDUM.¹

By A. W. HULL.

IN the December number of the Journal of the American Chemical Society Burdick and Owen have described experiments on the crystal structure of carborundum. They conclude that the structure is like that of diamond, with half of the carbon atoms replaced by silicon, with this exception, that the carbon atoms are not in the centers of the tetrahedra formed by the silicon atoms, but are displaced toward the apices of the tetrahedra, corresponding to a lack of symmetry in the carbon atom for which we have no other evidence. Their conclusions are based entirely upon the relative intensities of the X-ray reflections, and their calculations are made in exactly the same manner as those of the Braggs on calcite and similar crystals. It is assumed that the scattering electrons of each atom are all concentrated at the center of the atom, and that some undefined cause produces a "normal" fall of intensity in successive orders represented by the numbers 100, 20, 7 and 4. The variations from this normal fall of intensity are the basis of the conclusions drawn regarding the positions of the atoms.

Inasmuch as the assumptions on which this analysis is based are certainly not valid, it seemed worth while to investigate what variations in intensity might be expected from a *perfect* diamond lattice, composed of atoms having

¹ Abstract of a paper presented at the New York meeting of the American Physical Society, March 1, 1919.

stationary electrons in cubical arrangement, such as have been found to account for the intensities of the reflection observed in diamond and crystal silicon respectively. The only assumptions made are that the electrons are fairly stationary in the positions specified, and that they scatter X-rays independently according to classical electromagnetic theory.

In order to make the conclusions as definite and free from arbitrary assumptions as possible, the *C* atom is assumed to have *exactly the same size and shape* as in diamond, namely, two electrons very near the nucleus, and four electrons at the opposite (tetrahedral) corners of a cube at distances of .77 aengstroms from the nucleus; and the silicon atom to be *exactly as in crystal silicon*, except that its four outer electrons are drawn in about seven per cent. It has two electrons very near the nucleus, eight at the corners of a cube at distances of .15 aengstroms from the nucleus, and the remaining four at opposite (tetrahedral) corners of a larger cube at distances of 1.12 aengstroms from the center. The atoms are assumed to be all similarly oriented, with the axes of their cubes parallel to the crystallographic axis of the carborundum crystal (assumed cubic). The calculations are made in a manner exactly analogous to that of the Braggs, applying Darwin's formula to *single electrons*.

The following table gives the intensities calculated in this way for the different reflections observed by Burdick and Owen.

The agreement with the experimental values is as good as the data warrants, and is fully as good as in Burdick and Owen's calculations. When more exact experimental data is available it will probably be found that the arrangement of electrons in exact cubes as assumed above is not correct, but that these cubes are distorted, at least slightly, by the forces of attraction holding the atoms together.

The facts which it is desired to emphasize here are (1), that the assumption of simple cubical atoms, that is, atoms composed of a nucleus surrounded by stationary electrons in cubical arrangement, accounts completely for the observed variation in the intensity of the reflections of different orders in the case of "normal" planes. (2) that in the case of carborundum this "shape factor" is capable of explaining completely the observed *variations* from the "normal" intensity, without assuming any lack of symmetry in the arrangement of the atoms.

The structure of carborundum may therefore be considered, as far as our present information goes, to be a lattice *exactly* like that of diamond, in which half the carbon atoms, that is, those belonging to one of the two face-centered cubic lattices, are replaced by silicon. Each carbon atom is surrounded by two electrons very near its center, plus 8 equidistant electrons, four of its own and one each from the four surrounding silicon atoms along the four "chemical bonds" joining these silicon atoms to the carbon. These eight electrons form an exact cube of side .89 Å. around the carbon atom. Each silicon atom has two electrons very near its center, surrounded by eight electrons in a cube of side .17 aengstroms, surrounded in turn by the four above mentioned electrons

Reflecting Plane.	Order of Reflection.	Intensity of Lines.			
		Observed.		Calculated.	
		Crystal I.	Crystal II.	Hull.	Burdick and Owen.
100	1	100	100	100	100
	2	126	470	47	72
	3	57	196	130	17
110	1	100	100	100	100
	2	61	38	77	20
	3	22	7	29.2	7
111	1	100	100	100	100
	2	35	55	18	31
	3	19	38	16	19
	4	13	23	3.2	4
	5	3	—	8	2
	6	1	—	3.5	2
$11\bar{1}$	1	100	100	100	100
	2	35	37	18	7
	3	44	57	16	3
	4	18	14	3.2	5
$2\bar{1}1$	1	100	100	100	100
	2	23	27	38	20
$11\bar{3}$	1	100	100	100	100
	2	12	12	18	14
$31\bar{1}$	1	100	100	100	100
	2	21	21	18	31
	3	—	8	10	19
210	1	100	100	100	100
	2	10	11	14	13

in tetrahedral arrangement, on the lines joining it to the four nearest carbon atoms. The distance of these outer four electrons from the center of the silicon atom is 1.12 aengstroms. This is slightly less than their distance, 1.20 aengstroms, from the centers of silicon atoms in crystal silicon. In other words, the atoms of carbon and silicon are about 4 per cent. closer to each other in carborundum than would be obtained by packing together carbon and silicon atoms of the exact size and shape as found in diamond and crystal silicon. This closer approach should correspond to the chemical stability of carborundum as compared with diamond and silicon. For simplicity, it has been assumed in the above calculations that the carbon atom maintained its

exact size, the shrinkage being attributed entirely to the silicon electrons as being the more mobile.

February 14, 1919.

A POSSIBLE RELATION BETWEEN THE RESONANCE POTENTIAL AND SPECIFIC
INDUCTIVE CAPACITY OF A METALLIC VAPOR AND ITS PROPERTIES IN THE
SOLID STATE.¹

BY K. T. COMPTON.

RECENT discoveries have shown that the long wave-length limit of photoelectric sensitiveness of a metal is greater in the solid than in the vapor state. In other words, the energy required to liberate an electron from an atom of the vapor is greater than that required to liberate an electron from a metal. However the work required to extract an electron from the metal is strikingly similar to that required to produce the "single line" spectrum of the vapor. A table is given showing the degree to which these quantities are identical, from which it appears that in the case of a majority of the metals the discrepancies fall within the limits of experimental error. Theoretical considerations indicate that exact coincidence would not be expected even if the relation suggested were true.

If this relation is true, it is suggestive of interesting speculations regarding the condition of the electrons in a metal. It would appear that the atoms retain practically the same structure as in the vapor state as far as the equilibrium orbits lying within that orbit involved in the production of the single lined spectrum are concerned, but that an electron outside this orbit becomes a "free electron" in that it is not subject to appreciable attractions from the parent atom. Thus the work required to extract an electron from a metal is very nearly that required to withdraw it from the "single line" orbit to the one next outside. These considerations are highly speculative at present, though it would not be surprising if an attempt to apply the Bohr theory of the atom to solids would yield fruitful results.

Attention is also called to the fact the argument used by the writer to derive the ionizing potential of a gas in terms of the specific inductive capacity of the gas² applies equally well to the resonance potential, applying to whichever has the lower value. Recent published and unpublished data on ionizing and resonance potentials and absorption spectra are shown to support the validity of the argument as a first approximation.

PRINCETON UNIVERSITY.

¹ Abstract of a paper presented at the New York meeting of the American Physical Society, March 1, 1919.

² *PHYS. REV.*, 8, p. 412, 1916.

THE LAW OF ABSORPTION OF HIGH FREQUENCY RADIATION.¹

BY ARTHUR H. COMPTON.

OWENS' law for the absorption of X-rays, that

$$\mu = k_1 \lambda^3,$$

where μ is the mass-absorption coefficient, λ is the wave-length and k a constant, has been modified by Barkla to account for the scattering which becomes important at very high frequencies. In his expression,

$$\mu = k_2 \lambda^3 + \sigma_0$$

the mass-scattering coefficient σ_0 was originally considered as a constant, having the value 0.2, calculated by J. J. Thomson on the usual electron theory. Barkla and White have recently shown, however, that this quantity must diminish at short wave-lengths, but were unable to suggest any reason for the decrease.

If the electron is taken as a ring of electricity with a radius comparable with the wave-length of very hard X-rays and V-rays, the mass-scattering coefficient should vary according to the expression recently proposed by the writer,

$$\sigma = \sigma_0 \left(1 - 29.61 \frac{a^2}{\lambda^2} + 524.2 \frac{a^4}{\lambda^4} - 5396 \frac{a^6}{\lambda^6} + \dots \right), \quad (1)$$

where σ_0 is the mass scattering coefficient as calculated by Thomson, and a is the radius of the electronic ring. At the same time the true or fluorescent absorption will depend upon the magnitude of the action of the incident radiation upon the electron. This will diminish for short wave-lengths, since in this case different parts of a wave may work against each other in trying to pry an electron loose from its fixed position. The energy absorbed in displacing an electron may be shown to be proportional to $\sqrt{\sigma}$. The mass absorption coefficient may therefore be represented by

$$\mu = k_3 \sqrt{\sigma} \lambda^3 + \sigma, \quad (2)$$

where σ is the function of a/λ represented by equation (1).

This expression is compared with the experimental values for the absorption coefficient of aluminium as given by Hull and Rice, Williams, and Bragg and Pierce, and is found to be satisfactory, especially for the short wave-lengths where the formulas of Owen and Barkla fail.

The value of *the radius of the electron* necessary to give best agreement with the experimental values is 1.85×10^{-10} cm. Unless there is some consistent error in the measurements of Hull and Rice, this value is determined within a probable error of about 5 per cent.

¹ Abstract of a paper presented at the New York meeting of the American Physical Society, March 1, 1919.

COLLISIONS OF SLOW MOVING ELECTRONS WITH NITROGEN MOLECULES.¹

BY S. KARRER.

THE collisions of slow-moving electrons with nitrogen molecules has been studied by projecting electrons having known velocities into a retarding electric field between two parallel plate electrodes. The variation of the current between the electrodes with the distance between them indicates that the collisions are inelastic, if the velocity of the electrons is below the resonance velocity and the number of collisions per second is not too great. The nature of the collisions seems to be somewhat influenced by the rapidity with which they follow each other.

No traces of ionization of nitrogen were observed with electron velocities as high as ten volts, a result in accord with the recent report of Davis and Goucher.

A plane equipotential source of electrons was used.

PHYSICAL LABORATORY,
UNIVERSITY OF ILLINOIS.

THE IONIZING POTENTIALS OF ARGON, NEON AND HELIUM.¹

BY H. C. RENTSCHLER.

THE object of the investigation was to measure the ionizing potentials of these gases and to determine whether they show the two types of inelastic impact between an electron and an atom of the gas, as found in the case of the metallic vapors. The first type of inelastic impact occurs when the colliding electron produces a displacement of an electron of the atom without detaching the electron from the atom known as the "resonance potential." The second type of inelastic impact occurs when the colliding electron produces ionization.

The test for "resonance potential" was made by the method first used by Tate in his work on mercury vapor.

The ionizing potentials were measured by Lenard's method, and also by measuring the current between a hot tungsten filament and a surrounding cylinder as a function of the potential between the cylinder and filament.

Argon was found to have a resonance potential of about 12 volts and an ionizing potential of about 17 volts.

Neon shows no resonance potential. The ionizing potential was found to be about 21 volts instead of 16 as determined by Franck and Hertz. This difference is probably due to the greater purity of the neon used in the present experiment.

Helium shows no "resonance potential." The ionizing potential was found to be about 27 volts instead of 20 volts as found by Franck and Hertz, Powlow, and Bazzoni. This value is in close agreement with Bohr's theory and also with the value calculated by the quantum theory from the limit of the helium spectrum measured by Richardson and Bazzoni.

¹ Abstract of a paper presented at the New York meeting of the American Physical Society, March 1, 1919.

Experiments are described showing the probable cause for the difference between the present and former values.

RESEARCH LABORATORY,
WESTINGHOUSE LAMP COMPANY.

NOTE ON THE USE OF THE HOT CATHODE VACUUM TUBE FOR THE MEASUREMENT OF VOLTAGE.¹

BY LYNDE P. WHEELER.

1. *Methods Employed.*—Two general schemes of connection have been tried. In one the voltage to be measured is applied between the grid and the filament of a three element tube and the change thus produced in the plate current observed. In the other, the voltage is introduced between the plate and filament of a two element tube and the resulting change in plate current measured. A third method (for higher voltages) in which the voltage is applied between the plate and filament of a three element tube and the change in grid current measured, is still under investigation.

2. *Results Obtained.*—Using the first of the above methods it has been found possible to devise a compact portable apparatus which serves satisfactorily for the measurement of low voltages (up to 50 volts) in A.C. circuits of any frequency. Such voltmeters are in every day use in the high frequency laboratory of the Sloane Laboratory at Yale University. By the second method somewhat higher voltages can be measured, and it is hoped by means of the third to extend the range still further. Only the results obtained by the first method are discussed in this note. These instruments have been found to hold their calibration well, are not too complicated in their manipulation to be put into the hands of electrical engineering seniors, and if properly used their indications can be made independent of wave form as well as frequency.

3. *Details of Operation.*—Only the smaller sizes of the high vacuum tube used for sending and receiving in radio have been tried. The best plate and grid potentials to use vary with the characteristic of the particular tube. They are determined (a) by the condition that the grid current must be kept very small in order not to draw appreciable energy from the circuit in which the voltage is being measured; (b) by the sensibility of the instrument used to measure the plate current, (c) by whether it is desired to make the scale of the instrument uniform when calibrated in volts. The first condition necessitates in general a negative, zero or at most a small positive grid potential. A considerable range in the choice of plate potential in no way affects the usefulness of the contrivance (provided the plate current does not surpass that at the upper bend of the characteristic) and enables the second and third conditions to be met. Any device for flattening out the characteristic which does not introduce inductance in the plate circuit will aid in meeting the third condition.

¹ Abstract of a paper presented at the New York meeting of the American Physical Society, March 1, 1919.

If care is taken to keep the variations of plate current within the limits of the straight portion of the characteristic of the tube, the calibrated scale is linear. Also since under these conditions there is no distortion, the impressed voltage wave form will be faithfully reproduced.

The calibration must be carried out on a low frequency A.C. circuit. For measurements up to five volts, using tubes such as the V T-I or V T-II, it has been found satisfactory to work with the grid at zero and the plate at one and a half volts in connection with a microammeter having a full scale reading of 120. With these values of the grid and plate potentials the scale is not quite uniform, nor are the readings independent of wave form. The compensating advantages are simplicity and that owing to the rectifying action of the tube when so used no thermocouple is needed.

4. *Limitations.*—These are of two kinds. The first is that imposed by the necessity of using two auxiliary batteries. This restricts the portability of the contrivance and of course it is essential that each battery should yield a strictly constant current. The second lies in the variability of the tubes commercially available. Each tube requires individual adjustment, although once a good tube has been selected and the working potentials properly adjusted its indications seem to be reliable for the best part of the life of the filament. Until the manufacture of tubes has been better standardized however, this variability is a distinct handicap to the use of these methods of measuring voltage.

AN INTERFERENTIAL METHOD FOR MEASURING THE EXPANSION OF VERY SMALL SAMPLES.¹

BY C. G. PETERS AND IRWIN G. PRIEST.

THIS paper describes an interferential method for measuring accurately the thermal expansion of samples even much smaller than can be used in the Fizeau interferential method, which requires *three* similar pins, usually about ten millimeters long or a homogeneous ring about forty millimeters in diameter and usually ten millimeters long in the direction of the measured expansion. Data are presented relating to *single* samples about five millimeters in cross section and ten millimeters in the direction of the measured expansion. Even smaller samples may be used if necessary.

The essential novelty of the method lies in the fact that measurements are made on the change in width instead of the displacement of interference fringes. For a fuller explanation of this, the complete paper must be consulted.

The new method yields results of the same accuracy as the Fizeau method while it has the following advantages over the latter:

1. Only one small pin is needed for a sample.
2. Contact errors are decreased and the error due to creeping of the cover plates is eliminated.

¹ Abstract of a paper presented at the New York meeting of the American Physical Society, March 1, 1919.

3. The computations are simplified by having the standard and sample nearly equal in length.
4. The trouble of counting the fringes while the temperature is changing is eliminated entirely.
5. The correction for change of refractive index with the density of the air is eliminated without evacuating the container.
6. The method is especially advantageous in determining small differential expansions, for the difference in expansion of small samples may be obtained directly by carrying them together through the same temperature cycle.

The complete paper will be published as a Scientific Paper of the Bureau of Standards.

BUREAU OF STANDARDS,
WASHINGTON, D. C.,
February 14, 1919.

THE PROPERTIES OF THE ELECTRON AS DERIVED FROM THE CHEMICAL PROPERTIES OF THE ELEMENTS.¹

BY IRVING LANGMUIR.

IN a paper soon to appear in the *Journal of the American Chemical Society* a theory of the arrangement of electrons in atoms and molecules will be given. This theory explains the periodic properties of all the elements including those of the eighth group and the rare earths. It also meets with success in explaining the magnetic properties of the elements. It leads to a simple theory of chemical valence applying equally well to polar and to non polar substances. In the case of organic compounds the results are identical with those of the ordinary valence theory, but with oxygen, nitrogen, chlorine, sulphur and phosphorus compounds the new theory applies as well as to organic compounds, while the ordinary valence theory fails completely. The so-called physical properties such as boiling points, freezing points, electric conductivity, etc., are accounted for as easily as the chemical properties. The arrangement of electrons even in molecules of nitrogen, carbon monoxide, nitric oxide and cyanogen is worked out and explains the properties of these remarkable substances.

The theory is an extension of Lewis' theory of the "cubical atom."²

The very satisfactory agreement between this theory and the enormous number of experimental data of organic and inorganic chemistry gives great importance to the postulates underlying the theory for these represent the conditions which must apparently be fulfilled if the properties of matter are to be explained. Thus far the physicist has studied the electron from a rather narrow field of observation. The chemical data now gives much additional information about the electron, some of which seems inconsistent with conclusions resulting from a study of the problem by physical methods.

¹ Abstract of a paper presented at the New York meeting of the American Physical Society, March 1, 1919.

² *Jour. Amer. Chem. Soc.* 38, 762, 1916.

The theory is based on the following postulates, given here in rather incomplete form.

1. The electrons in atoms are either stationary or rotate, revolve or oscillate about certain definite positions in the atom. In the most stable atoms, namely those of the inert gases, the electrons are arranged in pairs symmetrically placed with respect to a plane called the equatorial plane. No electrons lie in this plane. There is an axis of symmetry (polar axis), perpendicular to this plane, through which four secondary planes of symmetry pass forming angles of 45° with each other. The atoms as a whole thus have the symmetry of a tetragonal crystal.

2. The electrons in any given atom are distributed through a series of concentric (nearly) spherical shells, all of equal thickness. Thus the mean radii of the shells form an arithmetic series 1, 2, 3, 4, and the effective areas are in the ratios $1:2^2:3^2:4^2$.

3. Each shell is divided into cellular spaces or cells occupying equal areas in their respective shells and distributed over the surface, of the shells according to the symmetry required by postulate 1. The first shell thus contains 2 cells, the second 8, the third 18, and the fourth 32.

4. Each of the cells in the first shell can contain only one electron, but each other cell can contain either one or two electrons. All the inner shells must have their full quotas of electrons before the outside shell can contain any. No cell in the outside layer can contain two electrons until all the other cells in this layer contain at least one.

The inert gases are those in which all the cells in the outside shell have equal numbers of electrons. Thus according to the first four postulates helium has two electrons, neon has ten, argon 18, krypton 36, xenon 54, and niton 86. All atoms with an atomic number greater than that of helium (2) have as their first shell a pair of electrons close to the nucleus. The line connecting the two electrons establishes the polar axis for the atom. Neon has in its second shell eight electrons, four in each hemisphere (*i. e.*, above and below the equatorial plane), arranged symmetrically about the polar axis. The eight electrons are thus nearly at the corners of a cube. In argon there are eight more electrons in the second shell. In all elements of higher atomic number the second shell is like that in argon. Krypton has in its third shell nine electrons in each hemisphere symmetrically placed with respect to the polar axis and to the four electrons in the second shell. The ninth electron in each hemisphere goes into the polar axis. Xenon is like krypton except that it has twice as many electrons in its third shell. Niton has sixteen electrons in each hemisphere of its fourth shell. These are easily placed symmetrically with respect to the polar axis and the eight underlying electrons.

5. Two electrons in the same cell do not repel nor attract one another with strong forces. This probably means that there is a magnetic attraction (Parson's Magneton Theory) which nearly counteracts the electrostatic repulsion.

6. When the number of electrons in the outside layer is small, the arrange-

ment of the electrons is determined by the (magnetic?) attraction of the underlying electrons. But when the number of electrons increases, especially when the layer is nearly complete, the electrostatic repulsion of the underlying electrons and of those in the outside shell becomes predominant.

7. This postulate refers to the properties of atoms and need not be considered here.

8. The stable and symmetrical arrangements of electrons corresponding to the inert gases are characterized by strong internal and weak external fields of force. The smaller the atomic number, the weaker the external field.

9. The most stable arrangement of electrons is that of the pair in the helium atom. A stable pair may also be held by: (a) a single hydrogen nucleus; (b) two hydrogen nuclei; (c) a hydrogen nucleus and the kernel of another atom; (d) two atomic kernels (very rare).

10. The next most stable arrangement of electrons is the *octet*, that is, a group of eight electrons like that in the second shell of the neon atom. Any atom with atomic number less than eighteen, and which has more than three electrons in its outside layer tends to take up enough electrons to complete its *octet*.

11. Two octets may hold one, two, or sometimes three pairs of electrons in common. One octet may share one, two, three or four pairs of its electrons with one, two, three or four other octets. One or more pairs of electrons in an octet may be shared by the corresponding number of hydrogen nuclei. No electron can be shared by more than two octets.

The results obtained by the use of these postulates are so striking that one may safely reason that the results establish the fundamental correctness of the postulates.

How can these results be reconciled with Bohr's theory and with our usual conception of the electron? It is too early to answer. Bohr's stationary states and the cellular structure postulated above have many points of similarity. It seems that the electron must be regarded as a complex structure which undergoes a series of discontinuous changes while it is being bound by the nucleus or kernel of an atom. There seems to be strong evidence that an electron can exert magnetic attractions on other electrons in the atom even when not revolving about the nucleus of the atom.

RESEARCH LABORATORY,
GENERAL ELECTRIC CO.,
SCHENECTADY, N. Y.

SOME PHYSICAL PROPERTIES OF DENTAL MATERIALS.¹

BY C. G. PETERS AND W. H. SOUDER.

IN carrying out an investigation of dental amalgams, the following properties are being considered: thermal expansivity, dimensional changes, and crushing strengths.

¹ Abstract of a paper presented at the New York meeting of the American Physical Society, March 1, 1919.

Thermal Expansion Coefficients.—In determining the thermal expansion coefficients, a differential interference method, was adopted as the most accurate and convenient.

The thermal expansion of different sections of teeth, immersed in a solution, determined over the range 20 to 70 degrees centigrade were approximately 6.5×10^{-6} per degree centigrade for the root and 10×10^{-6} for the enamel.

The coefficient of a specimen of synthetic porcelain tested under similar conditions was about 8×10^{-6} .

Similar tests on amalgams showed coefficients from 24 to 28×10^{-6} . The critical point near 75° centigrade, discovered by other observers while making different tests, was confirmed.

This differential expansion between tooth structure and amalgam, amounting to .2 micron per centimeter per degree centigrade, is thought to be worthy of serious consideration, by dental clinicians.

Dimensional Changes During Amalgamation.—Samples of different amalgams were packed in a mold of definite shape such that these would fit the interferometer and their changes measured exactly as in the previous case. These samples were placed in position in a few minutes after amalgamation and observations taken over a period of one to three days.

Curves plotted to show change in length with time show an initial contraction which lasts about 20 minutes and is thought to be due to the specimens returning to the equilibrium temperature. This contraction is followed by an expansion of from 4 to 8 microns per centimeter at the end of 3 hours. Additional changes with time are relatively small.

Flow Under Continuous Pressure.—Subjecting specimens of amalgam to a constant pressure of about 3,000 pounds per square inch (beginning 48 hours after amalgamation) caused a flow in all cases. In most instances this was less than 2 per cent. at the end of five days. In some cases it caused a change of 10 per cent. or caused a rupture of the material.

It is thought that the rate of application of pressure modifies the crushing strength quite materially.

THE ORIGIN OF THE GENERAL RADIATION SPECTRUM OF X-RAYS.¹

BY DAVID L. WEBSTER.

AT a meeting of this society, four years ago, Duane and Hunt² announced the experimental proof that the spectrum of X-rays contained no frequencies above the value whose quantum is the energy of a cathode ray; and I pointed out³ the fact that this law meant that the old pulse theory could not hold, but that the general radiation, just as much as the characteristic, must consist of long trains of continuous waves. Since then the evidence for the

¹ Abstract of a paper presented at the New York meeting of the American Physical Society, March 1, 1919.

² W. Duane and F. H. Hunt, *PHYS. REV.*, 6, 166, 1915.

³ D. L. Webster, *PHYS. REV.*, 6, 56, 1915.

wave train theory has been strengthened by a later analysis showing that for an infinitely thin anticathode, not only the derivatives of the intensity, but the intensity itself, would be discontinuous at the limit of the spectrum.

The question suggested by this is, What is the nature of the oscillators producing these trains of waves? Are they cathode rays or are they something fixed in the atom? If they are cathode rays, we may expect them to show the Doppler effect, and with it also, according to the electromagnetic theory, we may expect an unsymmetrical distribution of intensity of X-rays as though the X-rays were thrown forward by the cathode rays. Such a combination of effects was indeed found by Kaye,¹ in 1909, using very thin targets of Al, Cu, Au and Pt. In every case he found more rays thrown forward than back, and in most cases the forward rays were more penetrating, showing the higher frequency that one would expect from the Doppler effect. With thick targets of carbon, the same sort of asymmetry of intensity was found by Stark,² and Kirschbaum,³ but they could not be sure of any difference in frequency. And in thick targets of Pb investigated by Ham,⁴ silver by Miller⁵ and Pt by Kirschbaum,⁶ no asymmetry could be detected. From this we should infer that the asymmetry was a property of light elements or thin layers of heavier ones, but not of thick layers of heavy ones. Before drawing any conclusions we should therefore try to see why the effect is limited in this way.

One obvious reason for expecting less asymmetry from heavy elements is the presence of characteristic rays, which, for various reasons, are likely to be uniformly distributed. But this is not enough. Another significant fact is that none of the experiments on the limit of the spectrum have shown any Doppler effect there, although with the high speeds of cathode rays such an effect might be expected even at angles not much different from a right angle from the cathode rays. This indicates that the highest frequencies emitted are emitted by electrons that have already lost all their velocity before emitting, although they were necessarily the fastest ones present at all when they struck the atoms that caused the emission. If so, the electrons that emit while in motion are therefore those that emit lower frequencies than the highest they are capable of, or in other words the lower frequencies will only show asymmetry when produced by electrons whose energy is initially much greater than the quanta for those frequencies. With aluminum, which gives no errors from characteristic rays, exactly this effect was found by Kaye, since the softer components of a hard beam showed more asymmetry than the harder ones. This explains at once why thick targets show less asymmetry than thin ones. They contain cathode rays of low velocity, and a large proportion of their low frequency X-rays are due to these cathode rays. And it also explains why

¹ G. W. C. Kaye, *Proc. Camb. Phil. Soc.*, 15, 269, 1909.

² J. Stark, *Phys. Zeitschr.*, 10, 902, 1909.

³ H. Kirschbaum, *Ann. der Phys.*, 46, 85, 1915.

⁴ W. R. Ham, *Phys. Rev.*, 30, 96, 1910.

⁵ F. C. Miller, *Franklin Inst. Journal*, 171, 457, 1910.

⁶ Loc. cit.

heavy elements show less asymmetry than light ones if we may assume that their atomic electrons are bound more firmly and are less easy for the cathode rays to dislodge and move in their own direction, and their more powerful nuclei will cause greater scattering of the moving electrons.

This brings the experiments into line with each other, but leaves us with two very startling conclusions. First, that electrons can radiate continuous wave trains while in rapid forward motion; and second, that when an electron radiates all its energy, it stops first and radiates after stopping. If the electron is in motion, and moves the diameter of an atom while radiating a wave train, where does it get the elastic force under which it vibrates? How can it carry its elastic restoring force and whole vibrating mechanism with it for such a distance? And if an electron does radiate all its energy, as at the high frequency limit of the spectrum, and must stop first, before radiating, where is its energy after its forward motion has stopped but before it radiates? These questions seem very hard to answer on the hypothesis that the only properties of the electron are those of a small uniformly charged sphere.

But they are easily answered by the theory of heat radiation that I developed some time ago on the basis of a modified form of the Parson magneton.¹ For in this theory the electron is a thin conducting ring and the emitting oscillation is that of the electricity on the ring, and this oscillation can occur without the help of elastic forces from outside, as it must when the electron is moving. The magneton also provides for the disposal of the energy between the stoppage of the cathode ray and the radiation of the X-rays, transforming it into energy of the oscillation. And it explains why, if all the energy of translation of the cathode ray has gone into the electric oscillation it will be radiated from a stationary center, but if only part of it goes into the oscillation, the translational motion will continue during the oscillation and cause the observed asymmetry.

MASSACHUSETTS INSTITUTE OF TECHNOLOGY,
CAMBRIDGE, MASS.,
March 1, 1919.

NOTE ON BOMB TRAJECTORIES.²

By EDWIN B. WILSON.

THE trajectory in vacuo would be a parabola of which the equation is $y = gx^2/2u_0$, if u_0 be the initial velocity (supposed horizontal) of the bomb. It is possible to make a simple approximation to the path when the resistance of the air is taken into account.

Namely:

$$y = \frac{gx^2}{2u_0^2} + \frac{g^2x^3}{3u_0^2U^2},$$

¹ A. L. Parson, Smithsonian Miscellaneous Collections, 65, 11, 1, 1915. D. L. Webster, Proc. Amer. Acad., 50, 131, 1915.

² Abstract of a paper presented at the New York meeting of the American Physical Society, March 1, 1919.

where U is the limiting velocity of the bomb and where it must be assumed that the length of the path of the trajectory is not over one or two thousand feet. The equation may be solved for x in terms of y , or rather in terms of the square root of y , with the result

$$x = \sqrt{\frac{2y}{g}} u_0 - \frac{2}{3} \frac{u_0^2 y}{U^2}.$$

The first term in this expression represents the horizontal travel of the bomb in vacuo during a drop of amount y and the second term represents the distance that the bomb is retarded relative to the parabolic path by the action of the air supposed to resist according to the ordinary law of the square of the velocity. The approximation is good only when $\sqrt{2yg/3} \cdot u_0/U^2$ is small. If the limiting velocity of the bomb is 900 feet per second and the initial velocity is 100 feet per second, the correction remains small for very considerable lengths of drop y —in fact, for lengths much greater than appear valid for approximations previously introduced in the work.

ON THE RELATION BETWEEN THE K SERIES AND THE L SERIES OF X-RAYS.

BY WILLIAM DUANE AND TAKEO SHIMIZU.

SOME years ago Kossel called attention to the fact that the difference between the frequencies of the β and α lines in the K series of X-rays characteristic of a chemical element very nearly equals the frequency of one of the strong lines in the L series of that element. Since then the relation between the two series of X-rays has been the subject of considerable discussion, especially with regard to its bearing on the modern theories of the structure of the atom and of the mechanism that produces characteristic line spectra. Kossel's equation cannot represent a general relation between the lines in the K series and those in the L series, however, for the L series contains many more lines than the K series contains.

In the researches described in this paper we have endeavored to measure the absorption and emission frequencies characteristic of tungsten in both the K and the L series, using the same calcite X-ray spectrometer throughout the work.

In the experiments on emission lines the X-rays came from the tungsten target of an ordinary Coolidge tube, but in most of the experiments on absorption we replaced this tube with one containing a molybdenum target, and inserted a thin layer of potassium tungstate between the tube and the spectrometer.

In each experiment the current exciting the X-ray tube came from a high tension transformer with a system of condensers and kenotrons attached to it for producing approximately a constant difference of potential.

In order to avoid errors due to the penetration of the X-rays into the crystal

¹ Abstract of a paper presented at the New York meeting of the American Physical Society, March 1, 1919.

we placed the two narrow slits that define the X-ray beam between the X-ray tube and the spectrometer.

The curves shown in the lantern slides represent the ionization currents as functions of the readings of one of the verniers attached to the crystal table.

To avoid making an accurate determination of the zero point directly readings were taken on both sides of the zero line of the spectrometer.

The curves on the first slide represent the *K* series of lines in both the first and the second order spectra. To produce the *K* series in tungsten requires a difference of potential greater than 69,200 volts applied to the X-ray tube, and in order that the peaks on the curves that represent characteristic lines should be well marked we employed a difference of potential of 100,000 volts.

The curves show the following characteristics:

(a) At the angles corresponding to $\lambda = .1785 \times 10^{-8}$ cm. (calculated from the grazing angle of incidence θ by the formula

$$n\lambda = 6.056 \times 10^{-8} \times \sin \theta)$$

appear sharp drops in the curves. This, then, is the characteristic absorption wave-length of tungsten in the *K* series.

(b) At the angles corresponding to $\lambda = .2087 \times 10^{-8}$ cm. and $\lambda = .2134 \times 10^{-8}$ cm. occur two sharp peaks completely separated from each other. These are the wave lengths of the two emission lines α_1 and α_2 in the *K* series. The peak corresponding to α_2 does not appear to be quite symmetrical. It is slightly broader on the long-wave length side than on the short, suggesting that there may be a weak line α_3 very close to α_2 , with a wave-length of about $.214 \times 10^{-8}$ cm. There is also a well defined peak at the angle corresponding to $\lambda = .1842 \times 10^{-8}$ cm. (the β line in the *K* series), and a slight elevation close to the absorption drop (a γ line in the *K* series).

In the neighborhood of the peaks readings were taken 15'' of arc apart; and, since the double grazing angle is about 4° , an error of 15'' in estimating the point of a peak would produce an error of 1/10 per cent. in the value of λ . If the same error were made in corresponding peaks on the two sides, an error 1/5 per cent. would result.

The curves representing experiments with a molybdenum target and a tungsten absorbing screen show marked breaks at $\lambda = .1785 \times 10^{-8}$ cm., $\lambda = 1.230 \times 10^{-8}$ cm. (strong) and $\lambda = 1.08 \times 10^{-8}$ cm. (medium) and a less well defined break at $\lambda = 1.025 \times 10^{-8}$ cm. The first is the characteristic *K* absorption wave-length, and the other three are the characteristic *L* absorption wave-lengths of tungsten.

The following table contains the frequencies of vibration calculated from the wave-lengths obtained from the curves.

Emission and Absorption Frequencies of Tungsten $\times 10^{10}$. (Probable error ± 0.003 .)

K Absorption Frequencies.

First order tungsten target	1.680
First order molybdenum target	1.679
Second order molybdenum target	1.681
Average ν	= 1.680

*L Absorption Frequencies.*Strong $\nu_1 = .2438$ Medium $\nu_2 = .2773$ Weak $\nu_3 = .2922$ *K Emission Frequencies.*

	β Medium	α_1 Strong	α_2 Medium	α_3 Weak
1st order.....	1.628	1.435	1.405	1.40?
2d order.....		1.437	1.405	1.40?
Average.....		1.436	1.405	1.40?

Relation between K Series and L Series.

Absorption	Emission
$\nu - \nu_1 = 1.436$	$\nu_{\alpha_1} = 1.436$
$\nu - \nu_2 = 1.403$	$\nu_{\alpha_2} = 1.405$
$\nu - \nu_3 = 1.388$	$\nu_{\alpha_3} = 1.40?$

It appears from the last five lines in the table that *the difference between the K absorption frequency and one of the L absorption frequencies equals one of the α emission frequencies in the K series* to within the limits of error of the measurements. Further, if we subtract the strong L absorption frequency ν_1 from the K absorption frequency ν we get the frequency of the strong α_1 line, if we subtract the medium L absorption frequency ν_2 from ν we get the frequency of the medium K emission line α_2 and, if we use the weak L absorption frequency we get the third α line in the K series, α_3 .

The curve representing experiments with a tungsten target shows about 12 lines in the L series between $\lambda = 1.2 \times 10^{-8}$ cm. and $\lambda = 1.50 \times 10^{-8}$ cm. Six of these lines can be accounted for by an extension of the above law, that an emission frequency is the difference between two absorption frequencies, if we assume the existence of two M absorption wave-lengths between $\lambda = 6 \times 10^{-8}$ cm. and $\lambda = 7.5 \times 10^{-8}$ cm. Such M absorption wave-lengths would account, also, for the presence of the β line in the K emission series.

If the above law proves to be of general applicability, characteristic absorption frequencies similar to those of X-rays should be sought for in other parts of the spectrum (the visible and ultra-violet, etc.). Characteristic X-ray absorption frequencies are quite different from the absorption lines that obey Kirchhoff's law.

It is unnecessary to lay further stress upon the importance of this absorption-emission relation in speculations on the structure of matter and on the mechanism of the absorption and emission of radiation.

TEMPERATURE COEFFICIENT OF RESISTANCE OF MOLYBDENUM.¹

BY E. C. BLOM.

THESE experiments were conducted to determine the temperature coefficient of resistance of molybdenum over a greater range of temperature than had previously been recorded.

¹ Abstract of a paper presented at the New York meeting of the American Physical Society, March 1, 1919.

Some 10 mil (0.254 mm.) drawn molybdenum wire was made into lamps. The filament length of one group of these was 150 mm. while that of the other was 120 mm. With everything else the same, this gave a filament length difference of 30 mm., which was used for all resistance calculations.

For the temperature from 0° C. to 30° C. the lamps were immersed in a water-bath until constant conditions were reached. The resistance was then measured with a Wheatstone Bridge. The temperature was measured with a calibrated thermometer. Over the range 800° C. to 2000° C. the temperature

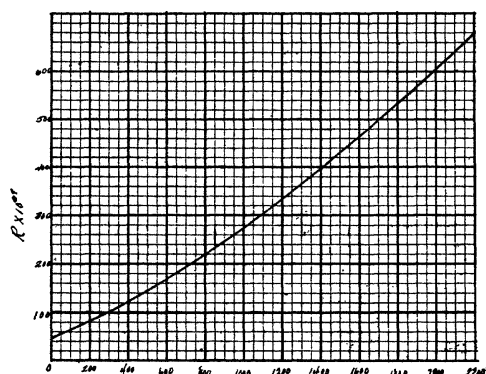


Fig. 1.

was measured with both brightness and color match optical pyrometers. Direct current was used to light the lamps and the resistance was determined by measuring volts and amperes with laboratory standard instruments. For the interval 30° C. to 300° C., the bulbs were removed, and the filaments immersed in a paraffine bath where again the temperature was measured with a calibrated thermometer and the resistance measured with the bridge. The interval 300° C. to 800° C. was not covered but the results up to 300° C. and beyond 800° C. ran so evenly that it seems plausible to assume that the curve which fits those temperatures already covered will fit the others also.

The accompanying curve gives a summary of the results obtained. The following empirical equation was found to yield the correct values:

$$R \cdot 10^7 = 44 + 0.177t + 0.000053t^2,$$

where t is in °C and R in ohms per cm.³

PHYSICAL LABORATORY,
WESTINGHOUSE LAMP CO.,
BLOOMFIELD, N. J.,
February 11, 1919.

SOME PHOTOELECTRIC LECTURE EXPERIMENTS

BY JACOB KUNZ.

IN a short article on the photoelectric relay in the *Electrical World*, Vol. 66, p. 934, October, 1915, the writer has indicated some simple experiments with the photoelectric cell and the three electrode photoelectric relay. Three more experiments will here be indicated by which some properties of the photoelectric cell can be demonstrated.

The tube *A* represents a photoelectric cell containing potassium sensitized with hydrogen and argon. The argon is used at such a pressure that the

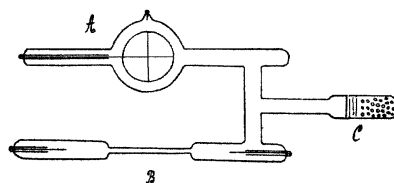


Fig. 1.

maximum sensitiveness of the cell for 130 volts, for instance, is surpassed, *B* represents a spectroscopic tube and *C* a bulb containing charcoal. Now for a given tube the photoelectric current depends essentially on three independent variables: the intensity of light, the potential difference, and the gas pressure. Let us start with a certain light intensity, for instance, with an incandescent lamp of 16 candlepower at a distance of two meters, with a certain pressure and a potential difference of 80 volts. We shall find in the galvanometer a certain deflection which will increase with increasing potential difference, first slowly, later very rapidly until the gas begins to glow. This point should not be reached with the ordinary cells, because the sensitiveness of potassium cells as a rule decreases when the glow occurs.

In a second experiment let us keep the potential difference, say 110 volts. and the gas pressure constant, and increase the intensity of light. The current is proportional to the illumination. Many photoelectric cells have been constructed in which this proportionality does not hold. The current may either increase more slowly or more quickly than the illumination. If the current approaches a saturation value with increasing illumination, it is apparently due to volume charges; and if it increases more rapidly than the illumination it is due to the fact that the gas approaches the glowing stage, which depends on the pressure, potential difference and illumination, like the photoelectric current itself.

In a third experiment let us keep the illumination and the potential difference constant and vary the pressure by putting the charcoal into liquid air. As the argon is absorbed the photoelectric current will at first increase, reach a maximum, then decrease and become very small when all the argon is absorbed.

¹ Abstract of a paper presented at the New York meeting of the American Physical Society, March 1, 1919.

The absorption of the argon can be seen in the changes of the light in the tube *B*. When the liquid air is withdrawn, argon is emitted, the photoelectric current passes through the same maximum as before and falls off to the initial small value (Stoletow's law).

The point at which the gas of the photoelectric cell becomes luminous depends on the potential difference, the intensity of illumination and the gas pressure. If for a certain pressure and potential difference we approach the source of light, suddenly the gas begins to glow, and of course continues to glow when the light is withdrawn. This beautiful experiment is interesting insofar as it shows that an increase of the number of electrons emitted from the metal leads to such an increase in ionization that luminous discharge occurs in which the current rises enormously over the value attained without glow. The photoelectric cell becomes a fairly good source of cold light. In such experiments the sensitive layer changes considerably and it is better to use two different tubes for this set of experiments and for the previous set, which does not lead to glow discharge.

If we place a rotating sector disc between the source of light and the photoelectric current, amplify the current by means of an audion as shown by E. Pike in a recent article in the *PHYSICAL REVIEW*, and use a telephone instead of a galvanometer in the secondary of the audion, then we can hear the source of light, even a very weak source, quite distinctly. This principle may be used in an instrument by which a blind man may hear the letters of a printed page.

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URBANA, ILLINOIS.
January 11, 1919.

ON THE MEASUREMENT OF THE DETECTION COEFFICIENT OF THERMIONIC VACUUM TUBES.¹

BY H. J. VAN DER BIJL.

THE relation between current and voltage in thermionic tubes of the audion type, can be represented by

$$(1) \quad I = f\left(\frac{1}{\mu} \Sigma E_B + \Sigma E_c\right),$$

where ΣE_B and ΣE_c are the potentials of the anode and grid with respect to the filament. The function f is non-linear and hence the device can be used as a radio detector. If, for example, a varying e.m.f. e , be impressed between filament and grid, the varying current i in the circuit connecting anode and filament can be represented by a simple convergent power series:

$$(2) \quad i = a_1 e + a_2 e^2 + a_3 e^3 + \dots$$

In order to properly design tubes it is necessary to know the relation between

¹ Abstract of a paper presented at the New York meeting of the American Physical Society, March 1, 1919.

the coefficients $a_1, a_2 \dots$ and the structural parameters of the tube. This requires a knowledge of the form of the function f . When using the tube as an amplifier f can be given a simple quadratic form.¹ This is, however, not sufficiently accurate for purposes of radio detection which is determined by the second derivative of the characteristic, *i. e.*, by the coefficient a_2 of the above series. The problem we want to consider here is the direct determination of a_2 , which may be called the detection coefficient. The difficulty of this deter-

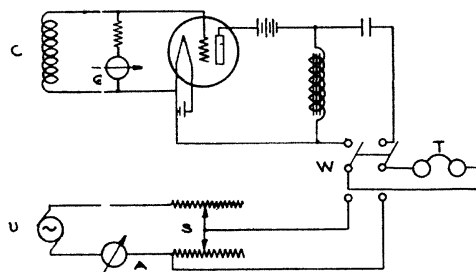


Fig. 1.

mination lies in the necessity for measuring extremely small alternating currents,—ranging from 10^{-3} to 10^{-6} amp. The use of hot wire meters is therefore entirely out of the question. Hence the method shown in the accompanying diagram was used.

The input voltage impressed on the tube at C ranged from a few hundredths to a few tenths of a volt, and was measured with the Duddell galvanometer G . It was of the form

$$(3) \quad e = A \sin pt (1 + \sin qt)$$

i. e., it was a voltage of high frequency $p/2\pi$ completely modulated by a voltage of frequency $q/2\pi$ lying within the audible range. If (3) be inserted in the second term of (2) the resulting current will be seen to have an audible component of frequency $q/2\pi$ which can be heard in the telephone receiver T . In order to measure this current the generator U is used to give a current of the same frequency as that in the receiver and which is strong enough to be measured with a thermo-couple and microammeter A . In fact, the same generator supplied the current used to modulate the high frequency input. This current is then attenuated by means of the shunt S until the intensity of the note in the receiver is the same for both positions of the switch W . Knowing the value of the shunt S the current i in the receiver can be computed from i_1 as measured by A . This shunt is so designed that for all its adjustments the impedance into which the generator U works is constant, so that i_1 remains constant.

For convenience in representing the results we can put

¹ H. J. Van der Bijl, *PHYS. REV.*, 12, p. 171, 1918.

$$(4) \quad d = \log \frac{i_1}{i},$$

where d can be previously determined for any combination of shunt and series resistances of the shunt S . Since the audible current i in the output of the detector is given by

$$(5) \quad i = a_2 e^2$$

we get

$$(6) \quad d = -2 \log e + (\log i_1 - \log a_2).$$

The accuracy with which this relation holds is shown in the diagram, the circles and crosses indicating measurements made by two different observers.

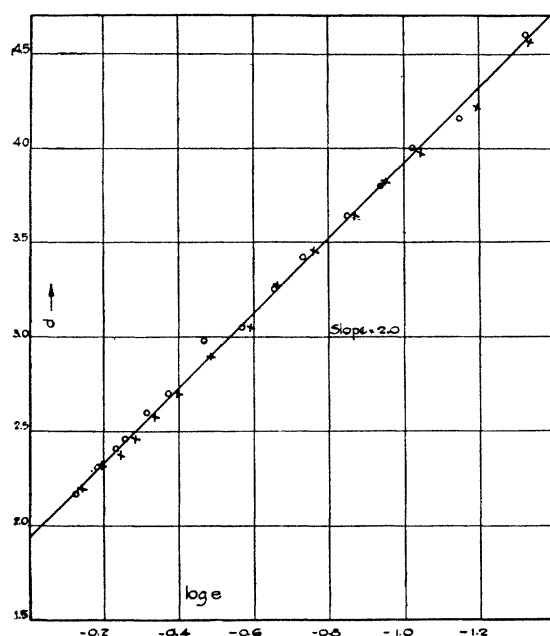


Fig. 2.

Since i_1 is large enough to be measured with the thermo-couple and microammeter A the detection coefficient a_2 can be obtained from the intercept of the line.

The detector used in these experiments was a standard Western Electric tube of the type that was designed specially for aeroplane radio telephone service. Its detecting efficiency had not changed to any noticeable extent over a period of more than seven months, during which time it was in frequent use. In these experiments the operating voltages were not adjusted to give the best efficiency.

From these results the detection coefficient was found to be 36×10^{-6} amp./ $(\text{volt})^2$, the resistance of the receiver being 6,400 ohms. This corresponds to a power dissipation in the receiver of 8.3×10^{-6} watt/ $(\text{volt})^4$. An idea of this

amount of power in the receiver can be obtained by noting that the minimum power dissipation necessary in this receiver to give a sound that just barely enables one to distinguish signals is about 3×10^{-12} watt.

RESEARCH LABORATORY OF THE WESTERN ELECTRIC CO., INC.,
NEW YORK CITY.
February, 1919.

A NEW FORMULA FOR THE SPECTRAL DISTRIBUTION OF ENERGY FROM A COMPLETE RADIATOR.¹

BY IRWIN G. PRIEST.

THE radiant energy from a complete radiator ("black body") may be expressed as a function of temperature and wavelength by the following empiric formula which does not appear to have been noticed heretofore:

$$E_{\lambda} = D_1 T^5 e^{-D_2 [A^{-1/3} - (\lambda T)^{-1}]^2} \quad (1)$$

where

E_{λ} \equiv energy of wave-length λ in the same sense as in the Planck equation.
 D_1 and D_2 are empiric constants.

T \equiv absolute temperature in degrees Kelvin.

A \equiv Wien displacement constant in micron degrees. ($A \equiv \lambda_m T$, where
 $\lambda_m \equiv$ wave-length of maximum energy.)

λ \equiv wave-length in microns.

From equation 1, we have for relative energy at any temperature

$$E_{\lambda r} \equiv \frac{E_{\lambda}}{E_m} = e^{-D_2 [A^{-1/3} - (\lambda T)^{-1}]^2} \quad (2)$$

where $E_m \equiv$ maximum energy, of wave-length λ_m .

This formula was originally obtained as an empiric approximation to the Planck formula by graphic transformations of the latter. It was later found, for $D_2 = 4481$ and $A = 2940$, that this formula represents some of the best of Coblentz' experimental distribution data *better* than the Planck formula can represent them for any value of A and the corresponding value of C_2 ($C_2 = 4.965A$). This conclusion may be verified by reference to the accompanying table.

The proposed equation includes all of the following previously recognized radiation laws, each of which may be derived from it:

1. The Wien displacement law.

The special case of Wien's displacement law ($\lambda_{\max} T = A$) may be inferred from inspection of the equation (1 or 2) for condition of maximum E_{λ} , viz: $A^{-1/3} - (\lambda T)^{-1/3} = \text{zero}$.

Further, the following equation may be derived from equation 2:

¹ Abstract of a paper presented at the New York meeting of the American Physical Society, March 1, 1919.

Priest: A New Formula for Spectral Distribution, etc.

λT ↓	Values of E_λ as a Function of λT					Residuals			
	Computed by Formulas					Exp. Data	Planck for A=2890 minus	Proposed Equation for A=2890 minus	Planck for A=2940 minus
	For A=2890		For A=2940			Coblentz B.S. Sci. Pap. 284, Fig. 2 T=1598.5 K	Exp. Data	Exp. Data	Exp. Data
	Wien $C_1=14450$	Planck $C_2=14350$	Proposed Equation Priest Dec 28	Planck $C_2=14587$	Proposed Equation Priest Dec 28				Proposed Equation for A=2940 minus
Column No.	(I)	(II)	(III)	(IV)	(V)	(VI)	(II)-(VI)	(III)-(VI)	(IV)-(VI)
500	0.27×10^{-6}	0.32×10^{-6}	0.87×10^{-6}	Note	0.72×10^{-6}				
800	$.13 \times 10^{-2}$	$.14 \times 10^{-2}$	$.18 \times 10^{-2}$		$.16 \times 10^{-2}$				
942		$.94 \times 10^{-2}$	$.11 \times 10^{-1}$	0.78×10^{-2}	0.9×10^{-2}	0.15×10^{-1}	-0.006	-0.004	-0.007
1000	$.16 \times 10^{-1}$	$.17 \times 10^{-1}$	$.19 \times 10^{-1}$						-0.005
1102		$.39 \times 10^{-1}$	$.42 \times 10^{-1}$	$.34 \times 10^{-1}$	$.3 \times 10^{-1}$	$.35 \times 10^{-1}$	+0.004	+0.007	+0.001
1200		$.80 \times 10^{-1}$	$.82 \times 10^{-1}$	$.71 \times 10^{-1}$	$.7 \times 10^{-1}$	$.58 \times 10^{-1}$	+0.022	+0.024	+0.013
1300		$.1381$	$.1420$	$.1254$	$.1330$	$.110$	+0.029	+0.032	+0.015
1400		$.2525$	$.2558$	$.2334$	$.2406$	$.233$	+0.019	+0.023	+0.000
1500	2.582	.2646	.2678	.3904	.3973	.395	+0.020	+0.022	-0.005
1600		.4151	.4166	.4877	.4929	.495	+0.019	+0.020	-0.001
1700		.5135	.5150	.5827	.5863	.581	+0.027	+0.029	+0.002
1800		.6079	.6096						
2000	6.810	.6871	.6859	.6959	.6963	.700	+0.018	+0.018	-0.004
2200		.7924	.7926	.7700	.7722	.797	-0.005	-0.004	-0.027
2400		.9238	.9240	.9111	.9034	.929	-0.005	-0.005	-0.018
2700		.9964	.9965	.9858	.9848	.994	-0.004	-0.004	-0.008
3000			.9665	.9675	.9769	.9753	+0.003	+0.005	+0.014
3200									
3500			.9180	.9167	.9322	.9296	-0.022	-0.023	-0.008
3600			.8916	.8900	.9078	.9053	-0.019	-0.021	-0.003
3800			.8368	.8332	.8553	.8509	-0.027	-0.031	-0.009
4000	7.887		.7967	.7933					
4071		.7788	.7730	.7976	.7929	.771	+0.008	+0.002	+0.027
5000	5.323	.5515		.4117	.3999	.397	-0.002	-0.016	+0.015
5843	3.465	.3712	.3566						
6000									
6003		.3712	.3570	.3868	.3751	.374	-0.003	-0.017	+0.013
6434		.3433	.2979	.3274	.3146	.313	+0.000	-0.015	+0.014
7089		.2439	.2288	.2558	.2437	.244	-0.000	-0.015	+0.012
7408		.2165	.2021	.2275	.2155	.218	-0.002	-0.016	+0.010
7695		.1949	.1807	.2047	.1926	.197	-0.002	-0.016	+0.008
7983		.1756	.1615	.1847	.1731	.179	-0.003	-0.017	+0.006
8000	1.501	.1746	.1607						
8238		.1606	.1466	.1690	.1574	.160	+0.001	-0.013	+0.009
8493		.1468	.1340	.1546	.1437	.147	-0.000	-0.013	+0.008
8655		.1340	.1212	.1412	.1305	.132	+0.002	-0.011	+0.009
9930		.9200	.8100	.9700	.8600	.860	+0.006	-0.005	+0.011
10000	.7190								
20000	$.45 \times 10^{-2}$	$.86 \times 10^{-2}$	$.63 \times 10^{-2}$	Note	$.77 \times 10^{-2}$				
30000	$.76 \times 10^{-2}$	$.19 \times 10^{-2}$	$.15 \times 10^{-2}$	Agreement of					
40000	$.20 \times 10^{-3}$	$.65 \times 10^{-3}$	$.54 \times 10^{-3}$	of	$.63 \times 10^{-3}$				
50000	$.72 \times 10^{-4}$	$.28 \times 10^{-4}$	$.25 \times 10^{-4}$	Col. II, III, V, or Low values in I					
60000	$.30 \times 10^{-4}$	$.14 \times 10^{-4}$	$.13 \times 10^{-4}$		$.15 \times 10^{-4}$				
70000	$.15 \times 10^{-4}$	$.74 \times 10^{-5}$	$.74 \times 10^{-5}$		$.60 \times 10^{-4}$				
80000	$.76 \times 10^{-5}$	$.45 \times 10^{-5}$	$.51 \times 10^{-5}$						
90000	$.43 \times 10^{-5}$	$.28 \times 10^{-5}$	$.35 \times 10^{-5}$						
100000	$.28 \times 10^{-5}$	$.18 \times 10^{-5}$	$.23 \times 10^{-5}$						
Notes to Column Headings III, V, VI.						for Arith.	+0.015	+0.016	+0.009
* Proposed Equation $E_\lambda = e^{-.4481[A^2/(\lambda T)^2]}$						for $\lambda < \lambda_m$ Algeb.	+0.012	+0.013	-0.004
* Omitting points affected by atmospheric absorption.						for Arith.	+0.006	+0.014	+0.011
						for $\lambda > \lambda_m$ Algeb.	-0.004	-0.013	+0.008
						for all values of λ	+0.010	+0.015	+0.010
							+0.003	-0.002	+0.003

Notes: (I)-(VI) are in units of 10^{-10} Watts/cm²/cm²/steradian. (VII)-(VIII) are in units of 10^{-10} Watts/cm²/cm²/steradian. (IX)-(X) are in units of 10^{-10} Watts/cm²/cm²/steradian. (XI)-(XII) are in units of 10^{-10} Watts/cm²/cm²/steradian. (XIII)-(XIV) are in units of 10^{-10} Watts/cm²/cm²/steradian. (XV)-(XVI) are in units of 10^{-10} Watts/cm²/cm²/steradian. (XVII)-(XVIII) are in units of 10^{-10} Watts/cm²/cm²/steradian. (XIX)-(XX) are in units of 10^{-10} Watts/cm²/cm²/steradian. (XXI)-(XXII) are in units of 10^{-10} Watts/cm²/cm²/steradian. (XXIII)-(XXIV) are in units of 10^{-10} Watts/cm²/cm²/steradian. (XXV)-(XXVI) are in units of 10^{-10} Watts/cm²/cm²/steradian. (XXVII)-(XXVIII) are in units of 10^{-10} Watts/cm²/cm²/steradian. (XXIX)-(XXX) are in units of 10^{-10} Watts/cm²/cm²/steradian. (XXXI)-(XXXII) are in units of 10^{-10} Watts/cm²/cm²/steradian. (XXXIII)-(XXXIV) are in units of 10^{-10} Watts/cm²/cm²/steradian. 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$$\lambda T = \left[\frac{1}{A^{-1} \pm \sqrt{\frac{\log_e E_{\lambda r}}{-D_2}}} \right]^3 \quad (3)$$

which is in accord with the general case of Wien's law.

2. The law that the maximum energy is proportional to the fifth power of the absolute temperature.

This may be seen by inspection of the equation for the condition

$$\lambda = \lambda_m = \frac{A}{T}.$$

3. The empiric Stefan law that total energy (integral with respect to wave-length) is proportional to the fourth power of absolute temperature.

This follows from the fact that the definite integral between any wave-length limits for equal values of $E_{\lambda r} > 0.33 \times 10^{-9}$ is proportional to the fourth power of temperature.

From the experimental point of view, therefore, the proposed equation accords with the fourth power law just as well as the Planck equation does.

However, the theoretical Stefan-Boltzmann law that the definite integral with respect to wave-length between *zero* and *infinity* is proportional to the fourth power of absolute temperature can *not* be derived from this equation, because E_{λ}/E_m has a finite although very small value (0.33×10^{-9}) for $\lambda T = \text{infinity}$.

Another interesting property of the proposed equation is the fact that it has precisely the form of the well known equation of the "probability curve." If $E_{\lambda r}$ (equation 2) be plotted as ordinate against $\sqrt[3]{1/\lambda}$ or $\sqrt[3]{300/\lambda}$ (cube root of frequency) as abscissa, the resulting curve is strictly symmetrical about the maximum ordinate. It is possible that this is the mathematical essence of what may be sometime recognized as a theoretical law of radiation.

If the general validity of this equation as representative of experimental data be admitted, the law of symmetry just mentioned provides a very simple method of determining the wave-length of maximum energy from an isothermal curve.

Graphically, the intersection of the axis of symmetry with the abscissa scale gives $\sqrt[3]{1/\lambda_m}$ from which λ_m may be found directly. Or, by algebraic solution,

$$\lambda_m = \frac{8}{\left(\sqrt[3]{\frac{1}{\lambda_1}} + \sqrt[3]{\frac{1}{\lambda_2}} \right)^3}$$

where λ_1 and λ_2 are any wave-lengths for equal relative energy.

The departures of the proposed formula from the corresponding formulas of Wien and Planck for various values of λT may be seen in the accompanying table. It will be noticed:

1. Throughout the region best known by experimental investigation, it does not depart much from the Planck formula, although it approximates closer to the experimental data.

2. The curve of the proposed equation intersects the Planck curve at a point in the "Reststrahlung" region ($\lambda T =$ about 60,000).

For numerical evaluation this formula is more convenient than the Planck formula.

The author has no theoretical basis for proposing this equation; but recalling that the Planck equation was first an *empiric* relation to which theory was later *forced* to conform, he has thought that it might be worthy of notice and perhaps of some theoretical consideration by others.

P.S. April 18, 1919.—A second abstract giving a revised value of the constant D^2 on the basis of isochromatic data, will appear in the Proceedings of the Physical Society, Washington Meeting, April 25–26, 1919.

NATIONAL BUREAU OF STANDARDS,
February 25, 1919.

ON THE MEASUREMENT OF THE TRUE BAROMETRIC PRESSURE IN A RAPIDLY MOVING CURRENT OF AIR.¹

BY J. G. COFFIN.

THE accuracy of the measurement of barometric pressure in rapidly moving airplanes is open to serious question.

If a body of any shape is moving rapidly through the air, there are regions on the surface of it which receive an aerodynamic pressure greater than the normal static or barometric pressure and there are also regions under smaller pressure than normal.

For a sphere, for example, the pressure variation is shown in Fig. 1, where radial distances outside the circle represent pressures in excess of normal and radial distances inside represent pressures less than normal. The radius, therefore, represents true barometric pressure, but not necessarily to scale.

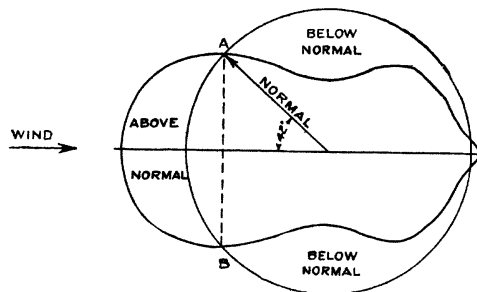


Fig 1

It is seen that there is an excess pressure on the nose where the air strikes the sphere directly and that at points on a circle of which AB is the trace there exists exactly normal pressure. At points aft of AB the pressure is below

¹ Abstract of a paper presented at the New York meeting of the American Physical Society, March 1, 1919.

normal and on the sides reaches values as much or more under normal than the values over normal at the nose. Any form of barometric instrument must necessarily be open to the air whose pressure it is to measure, hence the position of the opening or openings is of importance.

For example, at 60 miles per hour the total possible error may be as much as corresponds to an altitude of 240 feet and at 120 miles per hour, a usual speed at the present time, it can be nearly 1,000 feet!

This difficulty cannot be met by placing the barometer in any other receptacle or in particular, inside the fusilage; as the outside container must also be open to the air and as before the instrument will measure the total pressure obtaining at this opening.

In order to overcome the difficulty Dr. Zahm proposed, some years ago, to surround the barometer with a streamline body with a small opening at a point corresponding to a point on the circle *AB* on the sphere; that is at the point of zero aërodynamic pressure.

As is seen from Fig. 1, the rate change of aërodynamic pressure due to slight displacements along the surface is a maximum and any slight change in the air flow would cause large variations in the registered pressure. There could also be a large constant error due to initial uncertainty in the knowledge of the actual direction of air flow where the body is attached.

It occurred to the writer that these objections could be overcome by placing the barograph inside of a small sphere which is fully exposed to the airstream and connecting its hermetically tight case to a small opening in the sphere.¹ If this sphere be now rotated by clockwork or other means at a uniform angular speed the opening will be carried around in a horizontal great circle and the

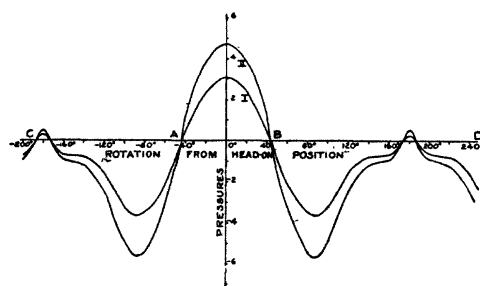


Fig II

barograph will register all the pressures, of course, passing through normal pressure two (or more) times per revolution.

The trace registered will be similar to Fig. 2.

From experiments made in the Curtiss Wind Tunnels at Garden City it is found that the zero points *A* and *B* remain fixed at all speeds experimented with. On the other hand, the change in speed produces an enlargement of the

¹ The barograph can, of course, also be connected to this opening by means of an air tight tube and be placed in any convenient location for observation.

pressure scale and the pressures follow exactly the same law as if registered with a Pitot tube of standard design.

It is a simple matter to calibrate such an instrument and draw the line CD , representing true barometric pressure, so that this troublesome problem is completely solved.

As a by-product of this idea we obtain the following advantages: If a barograph of this type is placed on an airplane and the trace examined on its return from a flight, this trace will show the true barometric pressure at every instant during the flight, the amplitude of the variations in the trace, will show at every instant the speed of the airplane and as equal periods are marked off on the graph by the rotations of the sphere, the slope of the graph will show the rate of climb at every instant so that this single trace contains a complete record of the performance of the airplane.

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THE FUNCTION OF PHASE DIFFERENCE IN THE BINAURAL LOCATION OF PURE TONES.¹

BY R. V. L. HARTLEY.

THE location of a source emitting a pure tone is a binaural process depending upon differences in the intensity and phase of the sounds at the two ears. Considerable experimental work has been done in which the intensities were kept equal and only the phases varied. A good review of this has been given by Stewart.¹

If the observed location effects are the result of the listener's subconsciously perceiving the phase difference and associating it with past experiences in which he has encountered the same phase difference when the position of the source was known; it should be possible to trace a correlation between these experimental sound images and the positions of sources which would cause the phase differences used in producing the images.

Using a method due to Stokes,² Stewart³ and, more recently, T. C. Fry of the Western Electric Co. have made calculations, from which, on the assumption that the head is a rigid sphere with the ears on a horizontal great circle, it is possible to plot the phase difference, P , against the lateral angular displacement of the source, for various distances and frequencies of the source. We have then to compare the direction corresponding to a particular phase difference, as given by the curves, with the direction of the image obtained experimentally with that phase difference.

¹ Abstract of a paper presented at the New York meeting of the American Physical Society, March 1, 1919.

² G. W. Stewart, *PHYS. REV.*, June, 1917, p. 502.

³ Lord Rayleigh, *Theory of Sound*, Ch. 17.

⁴ G. W. Stewart, *PHYS. REV.*, 1914, 4, p. 252.

The agreement revealed by this comparison is striking. The curves account for the rotation of the image, with continuously varying P , that is observed at frequencies below about 600 cycles, including its motion toward the ear, followed by a range of more or less uncertainty, just before P reaches 180° . They also indicate that the value of P for maximum lateral displacement increases with the frequency as Bowlker¹ has observed. This value reaches 180° for about 600 cycles. Above that frequency, the curves indicate the appearance of a second image on the opposite side. At first this is present only for values of P near 180° , but with increasing frequency, it extends to smaller values, finally covering the whole cycle. A third image then appears near $P = 0$, and so on for still higher frequencies. This accounts for the generally observed falling off in the definiteness of the image with increasing frequency above about 600 cycles. Bowlker however was able to distinguish as many as three simultaneous images, and his description of them shows an agreement with those deduced from the theoretical curves which is remarkable considering the assumptions involved.

This general correlation constitutes strong evidence in favor of the theory of direct perception of phase in sound location.

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WESTERN ELECTRIC COMPANY, INC.

¹ T. J. Bowlker, *Phil. Mag.*, (6), 15, p. 318, 1908.