The Principles of Dynamics (Continued)

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Source: *The Mathematical Gazette*, Vol. 4, No. 61 (Jan., 1907), pp. 1-5
Published by: The Mathematical Association
Stable URL: http://www.jstor.org/stable/3604262
Accessed: 06-06-2016 09:00 UTC

THE

MATHEMATICAL GAZETTE.

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LONDON:
GEORGE BELL & SONS, PORTUGAL STREET, LINCOLN'S INN,
AND BOMBAY.

Vol. IV. | January, 1907. | No. 61.

THE PRINCIPLES OF DYNAMICS.

(Continued from p. 394, Vol. III.)

IX. The expression "The Kinetic Energy of a body."

(1) We have, above, assigned an intelligible meaning to this expression *if the body be considered as a member of a system.* But, taken absolutely, it has no meaning.

(2) If our shot, now considered separately, meet directly a target of mass M' and there be inelastic collision, and if the relative velocity be U, then the velocity v' of the shot relatively to the *c.m.* of this new system will be $\frac{M'+m}{M'} \times U$, and the velocity V' of the target relatively to this *c.m.* will be $\frac{m}{M'+m} \times U$.

The total internal energy of this system as measured by the heat given out will be $[\frac{1}{2}m(v')^2 + \frac{1}{2}M'(V')^2]$, and of this the shot may with some reason be said to contribute the first term, and the target the second, in virtue of its K.E. [see VII., (3) and (4)].

(3) So that we cannot in general attach any meaning even to the more guarded expression "the K.E. of a body relatively to another body." Only if the target had relatively infinite mass could the shot be said to have a definite "kinetic energy relatively to the target." More correctly we should say that the internal kinetic energy of the *system* of shot and target may now be expressed in terms of the mass of the shot and the relative velocity of shot with respect to the target, viz. as $\frac{1}{2}mU^2$. For, with M' infinite, we may take v' as equal to U, and may neglect $\frac{1}{2}M'.(V')^2$ in comparison with $\frac{1}{2}m(v')^2$; so that the K.E.

$$[\tfrac{1}{2}m(v')^2 + \tfrac{1}{2}M'.(V')^2]$$

reduces, as we have just said, to $\frac{1}{2}mU^2$.

X. The Kinetic Energy of a System. Conservation of Energy.

(1) If we refer the motions of any system to any origin O and "fixed" axes, and if v' gives the velocity of any particle in it relatively to O, and v its velocity relatively to the *c.m.* G of the system, and V the velocity of this *c.m.* G relatively to the origin O, then we know that, by the properties of the centre of mass,

$$\Sigma[\tfrac{1}{2}m(v')^2]=\tfrac{1}{2}[\Sigma(m)]\,.\,V^2+\Sigma[\tfrac{1}{2}mv^2]. \quad\text{(i)}$$

When we speak of "the K.E. of the solar system" (*e.g.*) we usually refer to its centre of mass G, and so get the simpler expression

$$\Sigma(\tfrac{1}{2}mv^2). \quad\text{(ii)}$$

(2) This last expression, the second term of the more general expression given in (i), is the term affected by internal reactions within the system; while if O were the *c.m.* of a still larger system, the expression given in (i) would represent the contribution of K.E. made by the solar system towards the K.E. of this larger system; the term $\frac{1}{2}[\Sigma(m)]\,.\,V^2$ being unaffected by reactions within the solar system (which is now a sub-system).

(3) Hence it seems best to define "the K.E. of a system," when the system is considered by itself, by the term $\Sigma(\frac{1}{2}mv^2)$, where velocities are referred to the *c.m.* of the system.

(4) *Conservation of Energy.* This law refers to a self-contained system, or one that includes all the bodies between which reactions occur. Part of its energy at any moment will be the visible dynamical kinetic energy referred to above. [In the above we have not intended to include the energy of molecular vibration].

Referred to any origin whatever, this part of the energy of the system takes the form given in (1) (i) above.

(5) When V is constant, the term $\frac{1}{2}[\Sigma(m)]\,.\,V^2$ is constant, it is true; but it has nothing to do with the energy of the system considered by itself.

(6) While if the *c.m.* G of the system have an acceleration relatively to the origin O chosen, this term will in general change its value as time goes on, since V is now not constant.

(7) Hence in applying the law of conservation of energy to any self-contained system we may consider it best to refer all velocities, etc., to the *c.m.* of the system (and to "fixed" axes), rather than to any other origin.

XI. Cases in which there is no need to refer explicitly to "the system."

(1) As explained in V., our first ideas as to the laws of dynamics are derived from experiments, made on the earth's surface, in which we start with the ideas of *force, mass, velocity, direction,* etc., already existing in the mind of what may be

termed "the natural man." But laws based on ideas of velocity, acceleration, and displacement as referred to the earth's surface and to landmarks on it do not suffice when we (*e.g.*) deal with the behaviour of the gyroscope, or of Foucault's pendulum, and still less when we deal with the solar system. So we proceeded to give them a wider basis (see VI.). In a vast number of cases, however, we can, quite consistently with these more general principles, revert to the simpler conceptions of V.; and we can in these cases, without sensible error and with much gain as regards simplicity of treatment, desert *the system* and *stresses,* and consider *single bodies* and *forces.*

(2) If (*e.g.*) we are considering the action between the earth and a stone when the latter falls from the top of a house, the laws of VI. tell us that we can without sensible error refer to the *c.m.* of the earth instead of to the common *c.m.* of stone and earth which practically coincides with the former.

(3) If further we disregard the diurnal rotation, we do not even at the equator make an error of more than about 0·35 per cent. in judging of the pull exerted by the earth on the stone from the observed approach of the latter to the former.

(4) Or, more generally, when dealing with cases in which one mass is relatively infinite, and the rotation relatively to the distant stars (or to "fixed" axes) so small that the force required to deflect the path of the smaller mass is but a small fraction of the whole force with which it is urged towards the large mass, *we may refer velocities, etc., to the surface of the large mass and to landmarks on it.* It is for this reason that our first experiments can give us a very good idea of the more general laws.

(5) *Work and kinetic energy.* So again, referring once more to the simple case of VI., (3) and to IX., (3), we see that in the consideration of work and kinetic energy we may in such cases, without sensible error, *refer the displacement and velocity of the smaller body to the surface of the larger body, and to landmarks on it* when considering the work and kinetic energy done in and appearing in any reaction between the two bodies; *and may consider the kinetic energy of the smaller body only.*

(6) So in all similar cases—as (*e.g.*) the action of steam in causing the piston and other movable parts of an engine to move relatively to the framework of an engine and to the earth to which the framework is attached—we may refer to the earth as fixed.

(7) In the less simple case of a locomotive again, we reckon the work done by the steam by considering its pressure and change of volume, and can measure the corresponding kinetic energy that appears by referring the motion of the whole train and of the smaller parts in it to the earth (regarded as fixed); the

reaction in this case also taking place between the earth, of relatively infinite mass, and a group of bodies of masses that are very small relatively to the earth.

XII. Conclusion.

In conclusion it may not be amiss to re-state briefly some of the points insisted on in the above.

(1) *Origin and axes.* In most text-books no clue is given as to the origin and axes to which displacements, velocities, and accelerations are referred. There is presented to the student a body and force acting on it; and the acceleration of, or work done on, the body is discussed.

But we can conceive of any number of origins in space, having any velocities *or accelerations* relatively to the body; which are we to choose? And what axes are we to regard as fixed? If we begin by taking origin and axes fixed relatively to the earth, it should be clearly stated that we do so, as is done in V., (1). If we take origin and axes for which the laws of dynamics hold good with accuracy, as in VI., (1), this again should be stated clearly.

(2) *Force and Mass.* It should be made clear whether we start with independent measures of force and mass and search by experiment for the laws of dynamics—[the course taken in this paper]—or whether we define force and mass dynamically, and then try by experiment whether measures so obtained agree with what may be called statical measures. It appears to the writer that the student is, as a rule, left in doubt as to whether Newton's laws are definitions or true experimental laws; it is hardly made clear where experiment "comes in."

(3) *Energy.* There is no doubt but that, with specification of origin and axes, a perfectly definite meaning can be assigned to the expression "The Energy of a system."

But, too frequently, the "kinetic energy of a body," or "the work done on a body," is spoken of, without any reference to a system; and, as has been pointed out in this paper, this leads to lack of significance or even to error.

XIII. Some suggestions as to the teaching of dynamics.

(1) Care must of course be taken not to confuse the beginner by presenting to him too many ideas at once. He must learn bit by bit; but he should be given to understand that the first bit of truth is not the whole truth, that it is an approximation to it.

(2) *Kinematics.* In the usual preliminary discussion of kinematics, the essentially *relative* nature of all motion as we know it can be insisted on at once; and axes and origin should be specified. In dealing with the movements on a chess-board, we may take the corner and edges of the board as origin and axes, however the board itself may be moved about.

For movements about in a railway carriage we may refer to the carriage as fixed. For movements of the train, or of ships at sea, to the surface of the earth as fixed. For movements of the moon round the earth we may refer to the centre of the earth as origin, and to lines fixed relatively to the distant stars as axes. And so on. The conception of absolute velocity should be discouraged by continual recurrence to this idea of the relativity of motion.

(3) *Kinetics.* In beginning to discuss forces, masses, and accelerations, etc., it would be better to state plainly that we are going at first to take the surface of the earth as "at rest," in the sense that we are going to refer to an origin and to axes fixed on it; and that, in the problems at first discussed, the choice of such origin and axes is justified by experiment. The writer is of opinion that the line of approach to the laws should be that indicated in V.

The discussion of "circular motion" (stones whirled at the end of a string, etc.) would naturally lead up to the consideration of what is meant in dynamics by "change in direction of motion"; and the dynamical definition of "fixed axes" as "axes fixed with respect to distant stars" should be given as one based on experiment, [see III., (4) and (5)]; it should be understood that the expression "absolute rotation" can have for us no more than a dynamical meaning.

Later on, the consideration of "the system" would be needed for a proper understanding of the principles of *Conservation of Momentum* and of *Conservation of Energy*; and working back from this, the considerations presented in XI. would show the student why, and to what extent, we were justified in referring to origin and axes fixed on the earth in the first discussion of the laws of dynamics and in the earlier (terrestrial) problems considered. In fact a re-consideration of all the earlier problems from the point of view of "the system of reacting bodies, a suitable origin [see VI., (1), (ii)], and fixed axes," would probably add much to the student's grasp of the whole subject.

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THE INTEGRAL CALCULUS THEOREM.

THE following presentation of the fundamental theorem of The Integral Calculus has been evolved as the result of attempts to put § 31 of Whittaker's "Analysis" before a University class. It is to be regarded as the first application of the Infinite Sequence Theory which has been the subject of articles in recent numbers