EXPERIMENTS IN IMPACT EXCITATION.

BY GEORGE W. NASMYTH.

III. THE FREQUENCY OF THE LEPEL OSCILLATIONS.

§ 14. *Importance of the Variation in Frequency.*

THE frequency of the oscillations produced by the short-arc generators is of considerable practical importance on account of the effect of variations in the frequency upon the sharpness of tuning which can be obtained. The frequency of the oscillations produced by the singing arc is not determined by the inductance and capacity of the shunt circuit alone. With the Duddell arc the variations in the pitch caused by a change in the arc current or arc length are very prominent. In the Poulsen arc the variations in frequency due to changes in the arc are so marked that the advantages of sharp tuning which undamped oscillations ought to give cannot be obtained in practice. On account of the general similarity of the characteristics of the Lepel arc to those of the Poulsen arc, it might be inferred that the frequency of the oscillations produced by the Lepel arc would vary in the same way as the frequency of the singing arc. In view of the importance of the question, however, it was thought desirable that the problem be made the subject of an experimental investigation.

§ 15. *The Frequency of the Duddell and Poulsen Oscillations.*

From theoretical considerations the author has derived[1] a formula for the frequency of the singing arc agreeing closely with all the experimental data which have been published on the Duddel and Poulsen arcs. In its general form this formula is

$$n = \frac{1}{2\pi} \sqrt{\frac{1}{LC} - \frac{(R + dV/dA)^2}{4L^2}}, \qquad (1)$$

[1] Nasmyth, "The Frequency of the Singing Arc," PHYS. REV., *27*, No. 2, p. 117, August, 1908.

where n is the frequency, L is the inductance and C the capacity and R the resistance of the oscillatory circuit, and dV/dA is the slope of the volts-amperes characteristic curve. If we take the value for the slope as given by the author's experiments at high frequencies,

$$\frac{dV}{dA} = -\frac{c + ld}{A}, \tag{2}$$

where l is the arc length, A the arc current, and c and d are constants depending upon the electrodes and the atmosphere in which the arc is formed, the formula for the frequency becomes

$$n = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{\{R - (c + ld)/A\}^2}{4L^2}}. \tag{3}$$

If further, we neglect c and R, which are usually small in comparison with the term ld, we get the approximate formula,

$$n = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{l^2 d^2}{4L^2 A^2}}. \tag{4}$$

This formula shows that if the inductance is increased while the product LC is kept constant, the frequency will increase because the second term under the radical decreases. This effect was first observed experimentally by A. Banti.[1] If the arc current is increased the second term under the radical will decrease and the frequency should increase. Moreover, if the square of the frequency is plotted against the reciprocal of the square of the arc current a straight line with a slope $= - l^2 d^2 / 4L^2$ should result, and the author has shown that this is the case both in his own experiments and in those of L. W. Austin.[2] Austin's data plotted in this way shows that the formula holds not only for the fundamental but for the harmonics which are present in the singing arc oscillations as well. The author has shown that the frequency decreases with increasing arc length and that if the square of the frequency is plotted against the square of the arc length a straight line results. Finally, the formula (3) shows that if the resistance of the oscillation circuit is

[1] A. Banti, Ellettricista, *12*, p. 1, January, 1903. See also L'Elettricita, March 8, 1908, p. 145.

[2] W. Austin, Bulletin, Bureau of Standards, *3*, p. 325, May, 1907.

increased the second term under the radical will approach zero, and the frequency should increase. This prediction has been verified by K. Vollmer,[1] who found that a resistance of 3.3 ohms added to the oscillatory circuit raised the frequency from 350,000 to 359,000 when the arc length was 0.3 mm. and the arc current 1.3 amperes, and similar, though smaller, increases of frequency on the addition of resistance at larger currents and arc lengths. It may be considered, therefore, that the law of the variation of the frequency of the singing arc is fairly well known, and may serve as a guide in the investigation of the frequency of the short-arc, metal-electrode generators.

§ 16. *The Frequency of the Short Arc Oscillations.*

The object of the first experiments on the frequency of the Lepel arc oscillations made by the author was to determine the dependence of the frequency on the arc current. A loosely coupled secondary with variable air condensers and a hot wire ammeter was used to determine the frequency, the connections being the same as in Fig. 1 in the section on the characteristics of short arcs.

Five Leyden jars having a total capacity of 0.0094 mfd. were used in the primary. Each of the three variable air condensers in the secondary had a maximum capacity of 0.00055 mfd., and differences of capacity of 0.000003 mfd. could be determined accurately by means of circular scales mounted on each of the air condensers and calibrated with considerable care by comparison with standard forms of capacity. The resistance of the primary circuit as measured with a Wheatstone bridge, was less than 0.10 ohm, and since stranded wires were used throughout, it is probable that the high frequency resistance did not greatly exceed this value.

The inductance of the primary circuit consisted of a solenoid of 15 turns, 20.55 cm. in length, and 12.85 cm. mean diameter. The stranded conductor was 0.65 cm. in diameter, and consisted of 36 strands each 0.6 mm. in diameter. The coefficient of self induction of this solenoid, as computed by Russell's formula,

$$L = (\pi dn)^2 l[1 - 0.424(d/l) + 0.125(d/l)^2 - 0.0156(d/l)^4],$$

[1] K. Vollmer, Jahrbuch der Drahtlosen Telegraphie und Telephonie, *3*, p. 143 (table 11), December, 1909.

where L is the inductance, d the diameter and l the length, in cm., and n is the number of turns per cm., was 14,100 cm.   The value given by comparison with an Ayrton Perry standard at a frequency of 1,000 cycles was 14,400 cm.   The small difference is probably due chiefly to the connecting wires at the end of the solenoid.   Since these wires were used in the actual connections, the larger value given by direct measurement is probably the more correct.   All the remaining connections in the primary were made so as to give a minimum of inductance, twisted pairs of wires being used wherever possible, and it is probable that the self inductance of the entire primary circuit was within one or two per cent. of 14,400 cm.

The inductance in the secondary circuit was a solenoid of 75 turns, 27.38 cm. long, and 8.87 cm. mean diameter.   The computed coefficient of self induction was 140,200, and the measured value slightly greater than this, giving a mean of about 141,000 cm.

The primary and secondary circuits were coupled by arranging these two solenoids coaxially, with a distance of 7.2 cm. between their adjacent ends.   In response to an inquiry concerning the best formula to use for the coefficient of mutual induction of the two solenoids, Dr. E. B. Rosa, of the Bureau of Standards, very kindly furnished a complete calculation of the coefficient by two formulas. According to an adaptation of the general formula given by Gray the mutual inductance was found to be 1,089.8 cm., while the formula of quadratures by Rayleigh gave 1,089.3 cm.   The coefficient of mutual induction may be considered to be very closely $M = 1,090$ cm. therefore.   Accordingly, the coupling coefficient is $k = M/\sqrt{L_1 L_2}$ $= 1090/\sqrt{14,400 \times 141,000} = 0.0242$ or 2.42 per cent.

§ 17. *Variation with Arc Current.*

With the arc current maintained at a constant value, it was found that resonance could be obtained at a number of different frequencies by varying the capacity in the loosely coupled secondary circuit.   It very soon became evident that the frequencies at which resonance could be obtained varied with the arc current.   The results of the first experiments on the frequency are given later, in Fig. 17.   As some of the resonance frequencies were close together, considerable difficulty was experienced in trying to follow the varia-

tions of a single one as the current was changed, and it was finally decided to make a complete exploration of the field. The results of the more exhaustive experiments at arc lengths of 0.10 and 0.15 mm. are given in Tables IV. and V., and graphically in Fig. 14.

TABLE IV.

Change of Frequency with Arc Current. (Fig. 14.)

Capacity. in M.F.	Fre- quency.	Curve 1. Funda- mental.		Curve 2a.		Curve 2.		Curve 2b.		Curve 3a.		Curve 3b.	
		D.C.	H.W.	D.C.	H.W.	D.C.	H.W.	D.C.	W.	D.C.	H.W.	D.C.	H.W.
.000545	565,000			2.81	.39	1.85	.39	1.42	.33	1.10	.28	.80	.25
.000697	500,000			2.10	.31	1.54	.44	1.22	.44			Curve 3.	
.000822	460,000			1.75	.36	1.40	.34	1.10	.33	.78	.41	.60	.41
.001091	400,000			1.75	.31	1.20	.40	.96	.41				
.001365	357,000			1.44	.30	1.06	.29	.87	.27				
.001635	327,000	2.59	.53	1.19	.23							Curve 1a.	
.001880	305,000	2.09	.29			.98	.14	.82	.12			2.58	.3
.002153	284,000	1.78	.22			.90	.12						
.002425	268,000	1.67	.15			.86	.10					Curve 1a.	
.00270	254,000	1.47	.15			.80	.07					1.85	.08
00297	242,000	1.30	.07										
.00325	232,000	1.23	.07										

D.C. indicates current through the arc, measured by a d.c. ammeter.

H.W. indicates intensity of oscillations, measured by a hot wire ammeter.

Arc. length, 0.10 millimeters, two sheets W. S. & B. Paragon paper.

Computed frequency from formula $n = \frac{1}{2\pi}\sqrt{\frac{1}{L_1 C_1}} = 430,000.$

Primary capacity 0.0094 mfd.   Inductance 14,400 cm.   (October 29, 1909).

It is evident that the frequency of the oscillations produced by the Lepel arc increases with the arc current, as in the singing arc. From the figure, the curves seem to occur in groups of three, and from the tables it is seen that the middle curves in each group, marked respectively 1, 2 and 3, represent the largest amounts of energy for a given current, the companion curves above and below representing comparatively weak oscillations. Curve 1 is apparently the fundamental, and at large currents of three amperes or more, this is the only resonance frequency that appears. It ap-

TABLE V.

Change of Frequency with Arc Current. (*Fig. 14.*)

Capacity in M.F.	Fre-quency.	Curve 1. Funda-mental.		Curve 2a.		Curve 2.		Curve 2b.		Curve 3a.		Curve 3.	
		D.C.	H.W.	D.C.	H.W.	D.C.	H.W.	D.C.	H.W.	D.C.	H.W.	D.C.	H.W.
.000545	565,000	(Fig. 16)		2.70	.44	1.85	.33	1.42	.34	1.15	.33	.95	.33
.000697	500,000			2.36	.33	1.57	.42	1.26	.44	.85	.41	.70	.41
.000822	460,000			2.10	.44	1.45	.47	1.11	.48	.80	.50	.60	.44
.001091	400,000			1.76	.46	1.23	.41	.96	.44	.70	.42	Curve 3b.	
.001242	375,000			1.16	.42	1.18	.42					500,000	
.001365	357,000	3.47	.51	1.52	.38	1.06	.31					.60	.42
.001503	340,000	3.03	.50	1.43	.29								
.001635	327,000	2.58	.47	1.35	.23			.78	.17				
.001880	305,000	2.08	.30			1.05	.14			Curve 1a.		Curve 1b.	
.002153	284,000					.95	.14			2.43	.12	1.46	.21
.002425	268,000	1.76	.09			.89	.13			2.16	.13	1.34	.14
.00297	242,000	1.46	.07							1.85	.09		
.00352	223,000									1.63	.08		

D.C is current through the arc, measured by a d.c. ammeter.

H.W. is intensity of oscillations, measured by hot wire ammeter.

Arc length 0.15 millimeter, three sheets W. S. & B. Paragon paper.

Computed frequency from formula $n = \dfrac{1}{2\pi}\sqrt{\dfrac{1}{L_1 C_1}} = 430{,}000$.

Primary capacity = 0.0094 mfd.   Inductance 14,400 cm.   (November 11, 1909.)

proaches asymptotically the frequency given by the Thomson formula $n = 1/2\pi\sqrt{L_1 C_1}$ as the arc current is increased. The curve 2 is evidently the first harmonic; at 2 amperes, for example, when curve 1 indicates a frequency of 300,000 for the fundamental, the frequency for curve 2 is very closely twice as great, 600,000. The range is not quite large enough to make sure that curve 3 is the next harmonic, but this is probably the case. At one ampere, for example, curve 2 indicates a frequency of about 300,000 and curve 3 about 600,000.

To avoid the necessity for the computation of a large number of frequencies corresponding to a varying capacity in the secondary, the capacity was held constant at the different values and the arc current varied by means of the resistance in the main circuit until

resonance was obtained.   At higher frequencies than about 565,000 the oscillations in the secondary became so weak, on account of the small capacity, that accurate settings could not be obtained.   The observations for the curves at 0.10 mm. were made before the
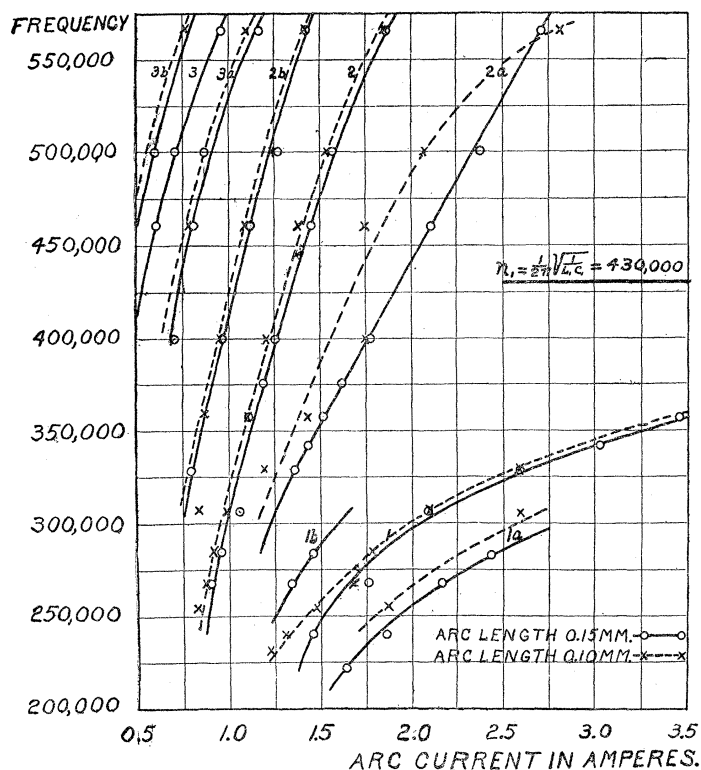

Fig. 14.
Change of frequency with arc current.

experiments at the longer arc length were undertaken, and the curves 1b and 3, which had been overlooked in the first observations were discovered in the more accurate experiments at 0.15 mm.

Influence of the Oscillatory Current.—The interpretation of the apparent discrepancies in the two sets of curves is interesting.   The reason for the flatness of the curve 2a at 0.15 mm. in the region from about 1.5 to 2.5 amperes is clear from Fig. 7 in the section on the characteristics.   Fig. 7 shows that the region of maximum voltage

at 0.15 mm. is between 1.5 and 2.5 amperes, and Fig. 8 shows that the region of minimum oscillations corresponds with the region of maximum voltage. The currents plotted in Fig. 14 were measured by a d.c. ammeter in the main circuit, whereas the actual current which determines the resistance and characteristics of the arc includes not only this current but the oscillatory current as well. The effective current traversing the arc is accordingly less, relatively in the region where the full line curve 2a is flat, and if total currents through the arc were plotted instead of the d.c. current, the flat part of the curve would bend over into its proper place in the region of smaller currents. The apparent shift of the curve 1a, and of most of the points in curve 1, in the same region, is explained in the same way. At 0.10 mm. the region of high voltages and low oscillation currents moves to higher currents as shown in Fig. 7, and the effect of this is apparent in the falling off of the dotted curve 2a at 2.81 amperes d.c. No readings at higher currents than 2.59 amperes were obtained at 0.10 mm. and the dotted curve has been drawn as if it lay above the full line curve all the way out, but the two curves probably cross at larger currents, on account of the influence of the minimum in the oscillatory current. At small currents no irregularities are observed, as no maxima of voltages or minima of oscillations occur in this region.

If the effective currents through the arc were plotted instead of the readings of the d.c. ammeter, the curves at low currents would be affected more than the others, since the oscillatory currents are large even when the d.c. ammeter readings are small. For the same values of effective current, the frequencies indicated by all the curves would be less, but the frequencies at small d.c. readings would be reduced more than the others, and the curves would be steeper.

Influence of Mutual Inductance.—Although the coupling between the primary and the secondary resonance circuit was very loose, the coefficient being only 0.0242 or 2.42 per cent., it was thought possible that the two companion curves above and below the curves 1, 2 and 3 might be accounted for as the two coupled circuit waves which appear even in impact excitation. It is difficult to see how indications of the two coupled waves could be obtained without the

aid of a loosely coupled tertiary circuit, and moreover the largest effect of the loose coupling employed turns out to be far too small to account for the phenomena. At resonance the frequencies[1] of the two waves of the coupled circuit should be

$$T = \sqrt{T_0^2 \pm \theta^2}; \quad (\theta^2 = 4\pi^2 M \sqrt{C_1 C_2})$$

where T_0 is the period of each circuit before coupling, M is the coefficient of mutual induction between the two circuits, and C_1 and C_2 are the capacities in the primary and secondary circuits respectively. Using the actual capacity, 0.0094 mfd. in the primary and the theoretical capacity from the Thomson formula, 0.00092 mfd. in the secondary, we get for the two frequencies 425,000 and 435,000 respectively. This gives a difference of only 1.16 per cent. instead of the difference of more than 10 per cent. required to account for the observations. Using the actual values of the secondary capacity, for a frequency of 350,000 the coupled circuit frequencies would be 347,800 and 352,200, giving a still smaller difference of less than three fourths of one per cent. The phenomenon seems to be therefore a characteristic of the generator itself rather than of the coupling. The presence of the large number of harmonics at which resonance can be obtained with the Lepel arc marks another point of resemblance between it and the singing arc. Austin[2] found three frequencies at which resonance could be obtained, all of which increased with the arc current. In the author's experiments on the frequency of the singing arc formed between a copper and a carbon electrode in illuminating gas six such frequencies were found by means of a loosely coupled resonance circuit, with indications of others at capacities too small to measure accurately.

§ 18. *Variation with Arc Length.*

The full line and dotted line curves for arc lengths of 0.15 and 0.10 mm. given in Fig. 14 show that the frequency decreases with increasing arc length, as in the case of the singing arc and in agreement with the formula (3) or (4). On attempting to follow the changes in the curve 1 as the arc length was increased apparent

[1] See "The Principles of Electric Wave Telegraphy," J. A. Fleming.
[2] Austin, *loc. cit.* See also Phys. Rev., 30, p. 134, August, 1908.

inconsistencies were again met with, the curves falling below each other for a time, then rising to higher frequencies as the arc length was increased beyond a critical value and finally falling again. The attempt to trace the course of the whole curve 1 was finally abandoned, and the variation of the frequency of the fundamental with

TABLE VI.

Fig. 15. Variation of Frequency with Arc Length.

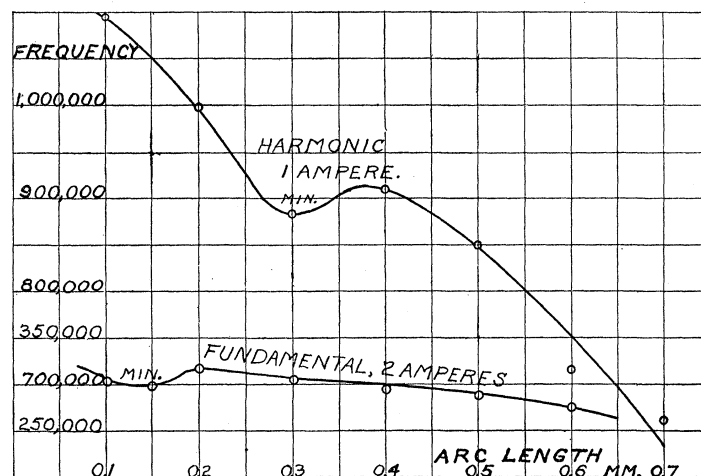| Sheets Paper.	2	3	4	6	8	10	12	14
Arc length, mm.	0.10	0.15	0.20	0.30	0.40	0.50	0.60	0.70
Arc length squared	0.01	0.023	0.04	0.09	0.16	0.25	0.36	0.49
Fundamental at 2.0 amperes. $C_1 = 0.0094$ mfd.								
Frequency $\div 10^3$	303	299	315	305	295	288	275	—
(Frequency)$^2 \div 10^{10}$	9.18	8.94	9.92	9.40	8.70	8.29	7.56	—
Harmonic at 1.0 ampere. $C_1 = 0.00564$ mfd.								
Frequency $\div 10^3$	1,095	—	999	885	910	850	715	662
(Frequency)$^2 \div 10^{10}$	119.9	—	99.8	78.4	82.8	72.3	51.1	43.8

Fig. 15.
Variation of Frequency with Arc Length.

arc length at a constant current of 2 amperes was investigated. The results obtained are given in Table VI. and graphically in Fig. 15. With a primary capacity of 0.00564 mfd. and the same primary

inductance of 14,400 cm., the variation of frequency of one of the harmonics at 1.0 ampere was also observed and the corresponding curve, which is of the same general form, has been plotted. The values of the squares of the arc lengths and frequencies are also given for use later in § 19.

Influence of the Oscillatory Current.—In both the curves of Fig. 15 the frequency decreases with increasing arc length except for regions of strongly marked minima in each case. On referring to
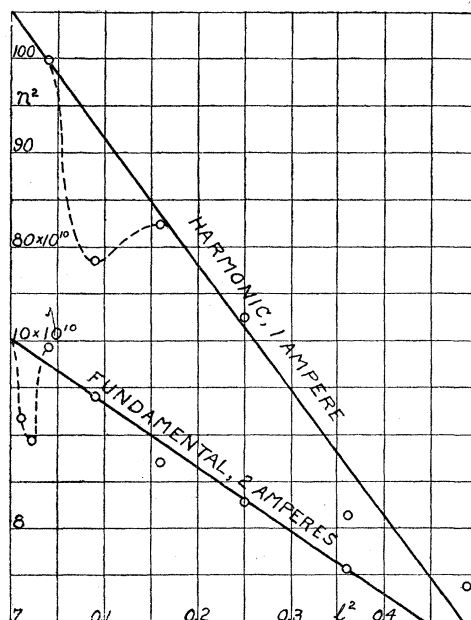

Fig. 16.

Squares of Frequency and Arc Length.

Physical Equation of Straight Lines, $n^2 = -\dfrac{d^2}{16\pi^2 L^2 A^2} \cdot l^2 + \dfrac{1}{4\pi^2 LC}.$

Fig. 7 again, it is found that these minima correspond to regions of voltage maxima and oscillatory current minima, so that the reason for the apparent discrepancies are traced as before, to a decrease in the effective current through the arc. Thus, the minimum in the oscillations and in the frequency for the 2 amperes d.c. curve both come at 0.15 mm. For the harmonic a smaller capacity and a

smaller current were used than for the curves in Fig. 7, and the minimum should be shifted into the region of longer arc lengths or smaller capacities, as it is. If this allowance is made, we may say that the minima in the oscillations and the frequency both come at about 0.3 mm. A second minimum, less pronounced, seems to be indicated at about 0.6 mm. in the curve for the frequency, and it is possible that the corresponding voltage maximum lies just outside the range of Fig. 7.

§ 19. *Proof of the Formula for the Frequency.*

It is apparent from the above curves that the frequency of the Lepel arc agrees qualitatively with the formula which has been found to hold true for the singing arc. With the exception of the

TABLE VII.

Figs. 17 and 18.　D.C. Arc.　Frequency and Arc Current.

Curve 1.

Current=I.	I/I^2.	Frequency $=n$.	n^2	
3.47	0.083	357,000	12.74×10^{10}	Copper electrodes.
3.03	0.109	340,000	11.56	Arc length $=0.15$ mm.
2.58	0.150	327,000	10.69	$L_1 = 14.400$ cm.
2.08	0.230	305,000	9.30	$C_1 = 0.0094$ mfd.
1.76	0.320	268,000	7.18	$N_0 = 430,000$.
1.46	0.460	242,000	5.86	

Curve 2.

Current=I.	I/I^2.	Frequency $=n$.	n^2	
1.18	0.720	322,000	10.36×10^{10}	
1.29	0.600	332,000	11.02	
1.38	0.530	335,000	11.22	
1.43	0.490	337,000	11.36	
1.48	0.456	342,000	11.70	
1.52	0.430	347,000	12.04	
1.55	0.417	352,000	12.39	Copper$+$, brass$-$, electrodes
1.595	0.393	357,000	12.76	Arc length $=0.35$ mm.
1.66	0.362	362,000	13.10	$L_1 = 14,400$ cm.
1.70	0.346	369,000	13.60	$C_1 = 0.00564$ mfd.
1.79	0.312	366,000	13.40	$M = 1089$ cm.
1.83	0.299	370,000	13.70	
1.87	0.286	379,000	14.30	
1.19	0.705	329,000	10.82	
1.08	0.858	315,000	9.91	
.98	1.040	298,000	8.90	
.875	1.310	275,000	7.58	

Curve 3.

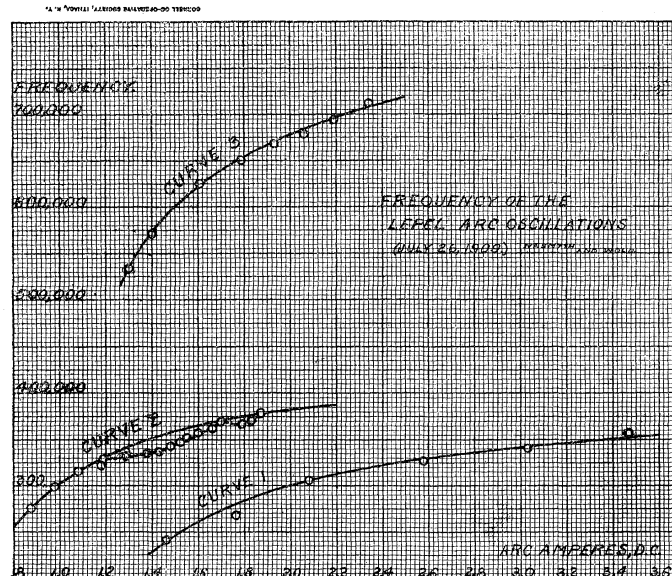|				
2.34	0.183	711,000	50.7×10^{10}	
2.19	0.209	694,000	48.1	
2.06	0.235	678,000	46.0	Copper+, brass—, electrodes.
1.93	0.269	668,000	44.6	Arc length 0.15 mm.
1.79	0.312	650,000	42.3	$L_1 = 14,400$ cm.
1.61	0.389	625,000	39.1	$\quad\quad \left\{ 0.00376 \right.$
1.40	0.510	571,000	32.6	$C_1 = \left\{ 0.00564 \right.$
1.30	0.590	534,000	28.5	

Fig. 17.

Frequency and arc current.

irregularities due to the oscillation minima the curves are of the same form as the corresponding singing arc curves. A more rigorous test of the formula is obtained by plotting the square of the frequency against the square of the arc length, as in Fig. 16. Both these curves prove to be straight lines outside the regions of the above mentioned irregularities due to a decrease in the intensity of the oscillatory current through the arc. If the effective current through the arc A, as measured by a hot wire ammeter, had been maintained constant instead of the direct current I, the agreement

of the curves with the straight line requirement of the formula would have been as satisfactory as in the case of the Poulsen arc.

Additional data for the variation of frequency with the arc current are given in Table VII., and the curves corresponding are plotted in Fig. 17. The data were obtained with the same connections and coupling as for the curves in Fig. 14. The effect of the minima in the oscillations is plainly visible in curve 2, and in the corresponding straight line in Fig. 18. The data for curve 2 are
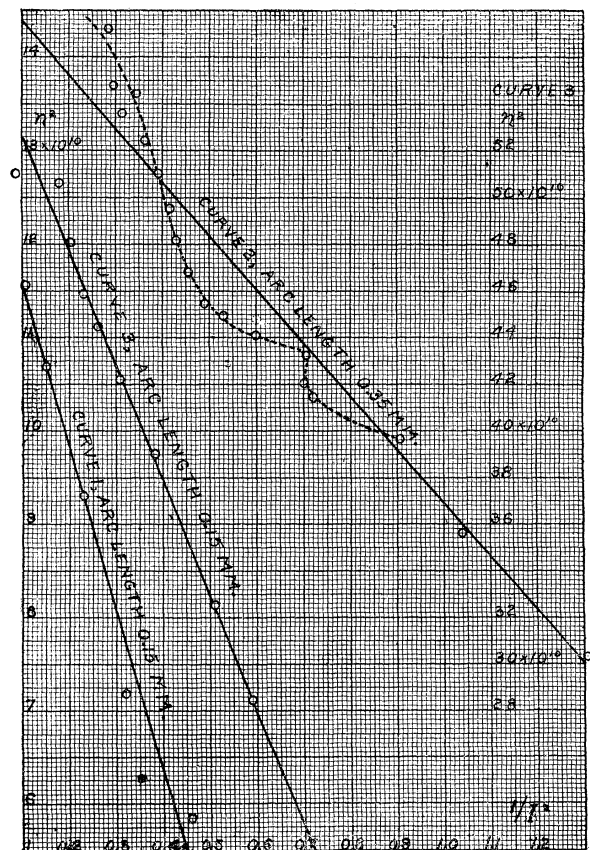

Fig. 18.

Squares of frequency and current reciprocals.

Physical equation of straight lines, $n^2 = -\dfrac{l^2 d^2}{16\pi^2 L^2} \cdot \dfrac{1}{A^2} + \dfrac{1}{4\pi^2 LC}$.

given in Table VII. in the same order in which they were obtained, and it is apparent that the points taken near the end of the rim form a much smoother curve than the earlier ones. It is possible that in this case there was some irregularity in the arc which caused the minima in the oscillations, in addition to the causes which have been traced in the preceding sections on the intensity of the oscillations.

With the exception of the discrepancy caused by using the arc current as measured by a d.c. ammeter instead of the effective current in curve 2, the agreement with the straight line requirement of the formula is again well within the limits of errors of observation.

§ 20. *Related Experiments on Poulsen Oscillations.*

The experiments of the author have shown that the Lepel arc is closely related to the singing arc, especially in its frequency characteristics. Since it appears that the formula derived by the author for the singing arc holds also for the short arc between metal electrodes, experiments on the singing arc which afford additional tests for this formula are of interest to investigators working with the new impact generators.

Some excellent experimental data recently obtained by K. Vollmer[1] working under the direction of Professor Max Wien at Danzig, afford material for a severe quantitative test of the formula as applied to Poulsen oscillations. Vollmer used flat discs of copper and carbon as his electrodes, and formed the arc in an atmosphere of hydrogen. His experiments cover a wide range of frequencies, from 156,000 to 1,000,000 per second corresponding to wave-lengths of from 1,915 to 300 meters. The inductances used range from 33,300 cm. to 1,358,000 cm. and the capacities from 0.00076 mfd. to 0.00585 mfd.

Vollmer measured his arc lengths directly, and did not take account of the variations in the length as indicated by the changes in voltage. Nevertheless the two curves[2] which he plotted to verify the author's formula for the frequency give a good straight line between n^2 and l^2 as formula (4) requires when l is large enough so

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[1] Vollmer, Jahrbuch der Drahtlosen Telegraphie und Telegraphonie, *3*, pp. 117–173.
[2] Vollmer, *loc. cit.*, p. 146.

Variation of Frequency with Arc Length; Poulsen Arc.

Table 2V.

$n_0 = 156,600$; $L_1 = 1,358,000$ cm.; $C_1 = 0.76 \times 10^{-3}$ mf.; $\lambda_0 = 1915$ meters.

(I_g =generator or arc current as measured by a d.c. ammeter; I_c =oscillatory current.)

$I_g = 2.7$ amp. $I_c = 2.0$ amp.

l	l^2	n^2	
1.2 mm.	1.44	2.433×10^{10}	$m = 11 \times 10^6$
1.8	3.24	2.4313	$\sqrt{m} = 3310$
2.4	5.76	2.4298	$d = 154$
2.7	7.29	2.4267	
3.3	10.89	2.4250	$d\sqrt{C_1} = 4.25$
3.6	12.96	2.4212	

Table 3V.

$n_0 = 156,600$; $L_1 = 532,000$ cm.; $C_1 = 1.94 \times 10^{-3}$ mf; $I_g = 2.7$ amperes; $I_c = 2.01$ amp.

l	l^2	n^2	
0.3 mm.	0.09	2.4414×10^{10}	
0.6	0.36	2.4388	
0.9	0.81	2.4363	$m = 27 \times 10^6$
1.2	1.44	2.4348	$\sqrt{m} = 5200$
1.8	3.24	2.4280	$d = 94$
2.1	4.41	2.4267	
2.4	5.76	2.4230	$d\sqrt{C_1} = 4.14$
2.7	7.29	2.4200	
3.0	9.00	2.4154	
3.3	10.89	2.4154	

Table 4V.

$n_0 = 156,600$; $L_1 = 176,500$ cm.; $C_1 = 5.85 \times 10^{-3}$ mf.; $I_g = 2.7$ amp.; $I_c = 2.1$ amp.

l	l^2	n^2	
0.3 mm.	0.09	2.434×10^{10}	$m = 92 \times 10^6$
0.6	0.36	2.421	$\sqrt{m} = 9,600 \ (9,600)$
0.9	0.81	2.419	$d = 57.5$
1.2	1.44	2.416	$d\sqrt{C_1} = 4.40$
1.5	2.25	2.396	
1.8	3.24	2.3815	
2.1	4.41	2.374	
2.4	5.76	2.357	
3.0	9.00	2.343	
3.6	12.96	2.326	
3.9	15.21	2.282	

Table 5V.

$n_0 = 375{,}000;$ $L_1 = 237{,}000$ *cm.;* $C_1 = 0.76 \times 10^{-3}$ *mf.;* $\lambda_0 = 800$ *meters;* $I_g = 2.7$ *amp.;* $I_c^r = 2.15$ *amp.*

l	l^2	n^2	
1.1	1.21	13.684×10^{10}	$m = 5 \times 10^8$
1.2	1.44	13.65	$\sqrt{m} = 2.24 \times 10^4$
1.35	1.82	13.624	$d = 180$
1.5	2.25	13.583	$d\sqrt{C_1} = 4.96$
1.8	3.24	13.55	

Table 6V.

$n_0 = 375{,}000;$ $L_1 = 92{,}900$ *cm.;* $C_1 = 1.94 \times 10^{-3}$ *mf.;* $I_g = 2.7$ *amp.;* $I_c = 2.15$ *amp.*

l	l^2	n^2	
0.3 mm.	0.09	13.86×10^{10}	
0.45	0.203	13.80	$m = 10 \times 10^8$
0.6	0.36	13.73	$\sqrt{m} = 3.16 \times 10^4$
0.9	0.81	13.68	$d = 100$
1.2	1.44	13.63	
1.5	2.25	13.51	$d\sqrt{C_1} = 4.40$
1.8	3.24	13.46	
2.1	4.41	13.35	
2.4	5.78	13.13	

Table 7V.

$n_0 = 375{,}000;$ $L_1 = 30{,}800$ *cm.;* $C_1 = 5.85 \times 10^{-3}$ *mf.;* $I_g = 2.7$ *amp.;* $I_c = 2.25$ *amp.*

l	l^2	n^2	
0.15 mm.	0.0225	13.8×10^{10}	
0.3	0.09	13.4	$m = 33 \times 10^8$
0.6	0.36	13.1	$\sqrt{m} = 5.75 \times 10^4$
0.75	0.5625	12.8	$d = 60$
0.9	0.81	12.6	
1.2	1.44	12.32	$d\sqrt{C_1} = 4.59$
1.5	2.25	12.1	
1.8	3.24	11.7	
2.1	4.41	11.3	
2.4	5.76	11.0	

that the constant c may be neglected. The Thomson frequencies chosen for the two tests were 156,600 with a capacity of 0.00585 mfd. and an inductance of 176,500 cm.; and 375,000, with a capacity of 0.00076 mfd. and an inductance of 237,000 cm. Although the

TABLE 8V.

$n_0 = 1,000,000$; $L_1 = 33,300$ $cm.$; $C_1 = 0.76 \times 10^{-3}$ $mf.$; $\lambda_0 = 300$ $meters$; $I_g = 2.7$ $amp.$; $I_c = 1.7$ $amp.$

l	l^2	n^2	
1.2 mm.	1.44	92.2×10^{10}	$m = 248 \times 10^8$
1.4	1.96	91.0	$\sqrt{m} = 15.7 \times 10^4$
1.5	2.25	89.7	$d = 177$
1.7	2.89	88.7	$d\sqrt{C_1} = 4.89$

TABLE 9V.

$n_0 = 1,000,000$; $I_1 = 13,060$ $cm.$; $C_1 = 1.94 \times 10^{-3}$ $mf.$; $I_g = 2.7$ $amp.$; $I_c = 2.1$ $amp.$

l	l^2	n^2	
0.3 mm.	0.09	91.4×10^{10}	$m = 436 \ \times 10^8$
0.45	0.2025	90.2	$\sqrt{m} = \ 20.9 \times 10^4$
0.9	0.81	86.4	$d = 93$
1.2	1.44	84.2	
1.5	2.25	81.2	$d\sqrt{C_1} = 4.10$
1.7	2.89	78.8	

Recapitulation.

No.	L_1	C_1	I_c	$\sqrt{m}$	d	$d\sqrt{C_1}$
156,600	1,358,000	0.76×10^{-3}	2.0	0.33×10^4	154	4.25
156,600	532,000	1.94	2.01	0.52	94	4.14
156,600	176,500	5.85	2.1	0.96	57.5	4.40
375,000	237,000	0.76	2.15	2.24×10^4	180	4.96
375,000	92,900	1.94	2.15	3.16	100	4.40
375,000	30,800	5.75	2.25	5.75	60	4.59
1,000,000	33,300	0.76	1.7	$15.7 \ \times 10^4$	177	4.89
1,000,000	13,060	1.94	2.1	20.9	93	4.10

$I_g = 2.70$ amperes. Mean 4.47

curves given are straight lines, as required by the formula, the value of the constant d as found from the slope is greater for the large capacity and low frequency curve than for the other. The author found that the product of the constant d and the square root of capacity gives a constant for these cases. Testing Vollmer's other remaining data in the same way, it was found that the product of $d\sqrt{C_1}$ gave a constant within the limits of the errors of observation, with a mean value of 4.47, in all his experiments. Vollmer's data

are not in convenient form for testing the formula, as he gives wave-lengths instead of frequencies. The data are therefore given in the form for direct verification of the results in tables 2V. to 9V., corresponding to Vollmer's table numbers, 2 to 9.

It was predicted in the derivation of the formula[1] that the constant d would change with the Thomson frequency on account of the skin effect of resistance in the arc vapor. It would also change with the capacity in parallel with the arc on account of the effect of capacity on the arc voltage-current characteristic. It appears from the above results, however, that if we use for d the value $d = k/\sqrt{C_1}$, where $k = 4.47 \pm 0.4$, the formula may be extended from a single Thomson frequency (constant value of L_1 and C_1) for which it was at first derived, to include all of Vollmer's experiments from 156,600 to 1,000,000 frequency range and with all values of capacity and inductance used in the practice of radio-telegraphy and radio-telephony.

Using the above value of d, the approximate formula (4) becomes

$$n = \frac{1}{2\pi} \sqrt{\frac{1}{LC} - \frac{l^2 k^2}{4L^2 C A^2}}, \qquad (5)$$

where n is the frequency, L and C the inductance and capacity, A the arc current, l the arc length, and k is a constant for given arc electrodes and atmosphere. The agreement of the generalized formula (5) with the results of experiments may be shown graphically if we plot $n^2 L^2 C$ against l^2. The equation of the straight line in the form $y = mx + b$ is

$$n^2 L^2 C = -\frac{k^2}{16\pi^2 A^2} l^2 + \frac{L}{4\pi}.$$

For a constant current A all the lines should have the same slope, $- k^2/16\pi^2 A^2$. The results from Vollmer's data are given in convenient form in Table X., and the curves at constant slope are plotted in Fig. 17.

The agreement is seen to be well within the limits of the errors of observation in Vollmer's work.

Vollmer measured his arc currents with a d.c. ammeter, whereas

[1] Phys. Rev., *27*, p. 127, formula 13.

the author's experiments on the variation of the frequency of the Lepel arc with current and arc length have demonstrated that the effective current through the arc, including the oscillations, should be used as the arc current A in the formula. The oscillatory current in general increases with the capacity, so that the correction on this account which should be made in Vollmer's data is in the right direction. Whether the Thomson frequency and the oscillatory current corrections together are sufficient to account for the constancy of the product $d\sqrt{C_1}$ must be determined by further experiments in which the arc current is measured by a hot wire ammeter or some equivalent instrument. The relation is at any rate interesting, and enables us to extend the formula from a single frequency as determined by the capacity and inductance, to the general case.

Vollmer's data show also, as mentioned above, that the frequency increases with the resistance in the oscillation circuit. This is in agreement with the prediction of formula (3) and contradicts all

TABLE X.

(Figure 19.)   Frequency and Arc Length; Poulsen Arc.   Values of $n^2 L^2 C$.

| j^2	Table 2V.	Table 3V.	Table 4V.	Table 5V.	Table 6V.	Table 7V.	Table 8V.	Table 9V.
.0225						.766		
.090		13.403	4.437		2.176	.744		.303
.203					2.167			.299
.360		13.389	4.413		2.156	.727		
.5625						.7104		
.81		13.375	4.410		2.148	.699		.286
1.21				5.841				
1.44	34.11	13.367	4.404	5.829	2.140	.684	.777	.279
1.82				5.817				
1.96							.767	
2.25			4.368	5.801	2.121	.672	.756	.269
2.89							.748	.261
3.24	34.08	13.330	4.341	5.786	2.113	.6494		
4.41		13.323	4.328		2.096	.627		
5.76	34.06	13.302	4.297		2.061	.611		
7.29	34.01	13.286						
9.00		13.261	4.271					
10.89	33.99	13.261						
12.96	33.94		4.240					
15.21			4.160					
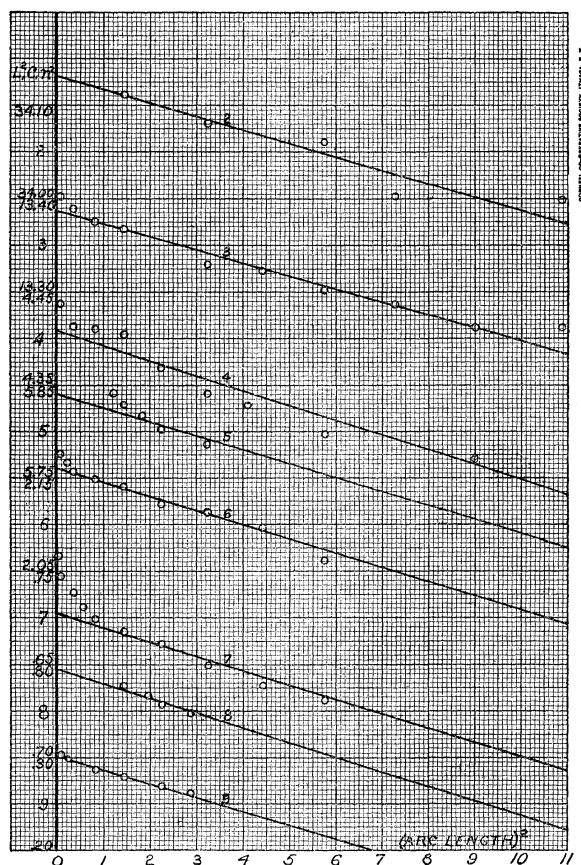
Fig. 19.

Frequency arc length curves with constant slope.

previous assumptions of investigators, based for the most part on reasoning by analogy from the effect of friction in mechanical pendulums and other systems. There is reason to suspect that an increase of frequency with resistance will be found also in the ordinary spark gap excitation as well as in the short spark generators.

§ 21. *Impact Excitation and Forced Undamped Oscillations.*

The resonance curve for the antenna or secondary of the two closely coupled circuits is of interest on account of its bearing upon
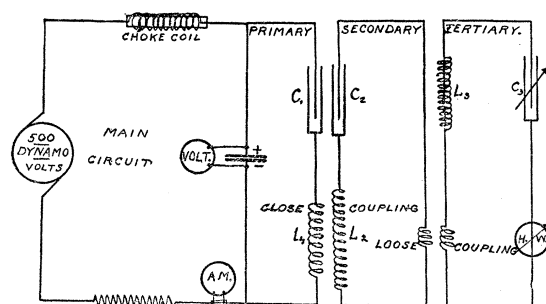
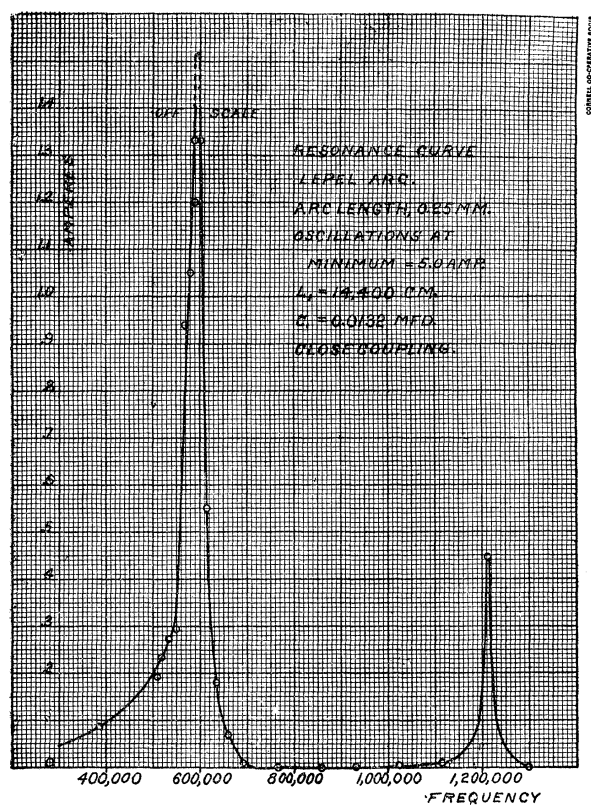Fig. 20.

Diagram of connections for resonance curves.

Fig. 21.

Resonance curve for Lepel arc.

the question of impact excitation.   It may be determined by means
of a tertiary circuit in addition to the connections used thus far
in the experiments and shown in Fig. 1, loosely coupled to the

Table XI.

Resonance Curve for Lepel Arc.

Frequency.	Hot Wire Ammeter Reading.	
385,000	.26	Primary capacity, 7 ja s = 0.0132 mfd.
385,000	.40	Primary inductance, 14,400 cm.
399,000	.40	Arc length, 0.15 mm. Volts, 80.
415,000	.46	Arc current, 2.15 amperes d.c.
420,000	.48	Oscillations at minimum, 4.6 amperes.
440,000	.46	Secondary capacity, 2 jars in series = 0.00094 mfd.
450,000	.43	Secondary inductance, about 180,000 cm.
470,000	.48	
482,000	.52	
492,000	.625	
502,000	.725	Secondary and primary closely coupled. Secondary
506,000	.83	solenoid of 141,000 cm. inductance placed inside
519,000	.94	primary solenoid of 14,100 cm. inductance with
520,000	1.166	lower ends flush. See § 16.
525,000	1.37	
Maximum, off scale.		
567,000	1.37	Tertiary capacity variable.
579,000	.92	Tertiary inductance 160,000.
594,000	.79	Tertiary and secondary loosely coupled.
596,000	.68	
615,000	.40	
660,000	.26	For the resonance curve of Figure 21 the primary and
702,000	.185	secondary capacities and inductance were the same,
765,000	.15	but the arc length was 0.25 mm., and the arc current
980,000	.28	3.03 amperes (volts 66) giving a minimum of 5.0
1,060,000	·.51	amperes in the oscillations.
Maximum.		
First harmonic.		
1,000,000	.29	
1,150,000	.20	
1,220,000	.10	

secondary.   The complete diagram of connections for determining
the form of the wave in the antenna or secondary is given in Fig. 20.
The primary and secondary circuits are maintained constant, and

the readings of the hot wire ammeter are read as the capacity in the loosely coupled tertiary circuit is varied.

With the arc current arranged so as to give a minimum in the intensity of the oscillations at an arc length of 0.25 mm. the resonance curve shown in Fig. 21 was obtained.

The primary and secondary circuits were not tuned to the same frequency, the lowering of the frequency of the oscillations by the arc itself increasing the so-called "untuning." The arc current was 3.03 amperes d.c., and the voltage 66. The remaining constants are given in connection with Table XI., which gives the data for a similar resonance curve (not plotted) at an arc length of 0.15 mm. and with the oscillations again at a minimum.

Both the resonance curves show but a single wave, the smaller maximum being evidently the harmonic of double frequency. So far they are in agreement with the theory of impact excitation, the steep curves and sharp peak being especially characteristic. On the other hand, the frequency of the oscillations should remain constant, as long as the secondary circuit is not changed, whereas the frequency of the oscillations changes with the constants of the primary circuit, according to the above figure and table. Further experiments gave the following increase of frequency of the oscillations in the secondary circuit with increasing primary arc current.

TABLE XII.

Arc Current Amperes, D.C.	Arc Volts, D.C.	Oscillation Amperes.	Resonance Frequency.
1.30	90	3.25	500,000
1.50	88	3.40	507,000
1.75	84	3.45	519,000
2.00	90	4.27	528,000
2.00	80	3.68	545,000
2.00	70	3.68	552,000
3.00	66	4.84	563,000

The conclusion is that in these experiments at least, the Lepel arc did not give free impact excitation, but *forced oscillations* which the form of the resonance curve shows to be almost *undamped*

at arc currents which give a minimum in the intensity of the oscillations.

Summary.

The Lepel generator resembles the older Duddell and Poulsen arc generators in having a number of harmonics, besides the fundamental, to which a loosely coupled resonance circuit may be tuned.

The following laws hold for both fundamental and harmonics, as in the case of the older generators:

1. The frequency of the oscillations increases with increasing arc current at constant arc length.

2. The frequency of the oscillations increases with decreasing arc length at constant arc current.

The variation in the frequency is least with a large primary inductance.

It is predicted, though not yet experimentally verified, that a small increase in the resistance of the oscillatory circuit will increase the frequency of the Lepel arc oscillations as in the case of the Duddell and Poulsen arcs.

The formula derived by the author in 1908 for the frequency of the singing arc, which includes all the known facts and is in agreement with the results of all the experiments on the frequency of the Duddell and Poulsen arc oscillations, holds also for the Lepel arc oscillations. The formula is

$$n = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{\{R-(c+ld)/A\}^2}{4L^2}},$$

where n is the frequency, L, C and R the inductance, capacity and resistance of the condensor circuit, l the arc length and A the arc current, and c and d are constants depending upon the electrodes and gas in which the arc is formed. Usually R and c are small compared with the other terms, so that the formula may be written in an approximate form,

$$n = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{l^2d^2}{4L^2A^2}}.$$

Recent extensive experiments on the Poulsen arc have shown that if the value of $d = k/\sqrt{C}$ where d is the arc constant and C

the primary capacity, k has the same value for a given arc for all the values of inductance, capacity and Thomson frequencies used in practice.   In its more general form the formula thus becomes

$$n = \frac{1}{2\pi} \sqrt{\frac{1}{LC} - \frac{l^2 k^2}{4L^2 A^2 C}}.$$

The effect of capacity on the oscillation intensity and the form of the voltage current characteristics makes it very probable that the more generalized formula holds for the Lepel arc also, but sufficient data for a rigorous test are not yet available.

With a closely coupled secondary circuit and the arc current arranged to give minimum oscillations, a very sharp resonance curve with only a single wave is obtained by means of a loosely coupled tertiary circuit, as required by the theory of impact excitation. The maximum shifts with change of arc current, however, leading to the conclusion that the sharp resonance curve of the Lepel arc is due, not to free oscillations, but to *forced undamped* oscillations.

The author wishes to express his appreciation of the sympathetic aid and many valuable suggestions which he has received in the course of his researches from Dean Ernest Merritt, of the Graduate School of Cornell University.