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Professor H. Hennessy F.R.S.

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where s is the heat-capacity in electric units (water = 4.49). For a certain allowable gain in temperature per unit time and a given mass, that material is the best which has the greatest specific heat. A metal of low atomic weight would therefore have the preference. If the volume of the substance be taken as fixed, we must make the product of density and specific heat as great as possible. This product is highest for iron and steel (.90), then follows copper (.80), platinum (.72), and gold (.62). There would be an appreciable advantage in substituting a steel wire for a platinum one, if small differences of temperature are to be detected by a change of resistance, especially as the temperature-coefficient of electric conductivity is considerably higher than that of the other metals. There are, however, obvious disadvantages connected with the use of steel which in most cases would counterbalance its greater sensibility.

XVIII. Ronayne's Cubes.

By Professor H. HENNESSY, F.R.S.*

A FEW years since I was presented by Mr. S. Yeates with a pair of equal cubes, one of which could pass obliquely through the other. These objects had been for a long time in his father's possession, but there was no record as to where or by whom they were made. At first I paid little attention to the cubes, as I looked upon them merely as mathematical curiosities. Not long since, on perusing Gibson's 'History of Cork,' I came on a passage extracted from Smith's 'History of Cork,' published in 1750, in which a pair of interpenetrating cubes is distinctly referred to as the invention of Mr. Phillip Ronayne, of the Great Island near Cove (now Queenstown). The cubes were said to be constructed after Mr. Ronayne's design by Mr. Daniel Voster, who was at this time teacher of mathematics in Cork, and editor of the well-known book on Arithmetic, compiled during the early part of the last century by his father, Elias Voster†.

* Communicated by the Author.

† Passage in Smith's 'History of Cork':—

"Not far from the castle of Belvelly is Ronayne's Grove formerly called Hodnets Wood; [now Marino] a good house and handsome improvements of Philip Ronayne, Esquire. From the gardens one has a charming view of the river and shipping up to Cork, as also the town of Passage on the opposite shore. This gentleman has distinguished himself by several essays in the most sublime parts of mathematics; among others by a treatise on Algebra, which has passed through several editions, and is much read and esteemed by all the philomaths of the present time. He has invented a cube which is perforated in such a manner

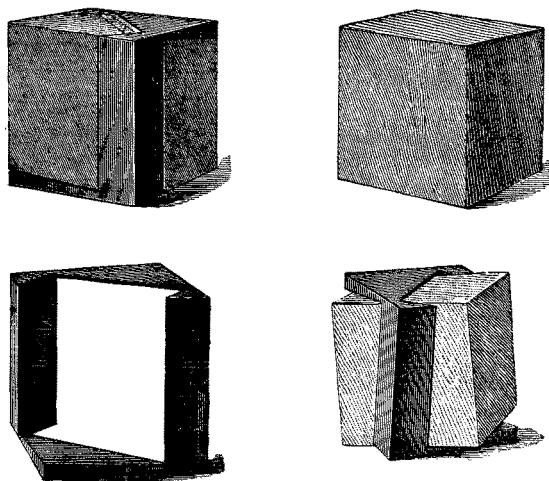
I was led by this passage in Smith's 'History' to more closely examine the cubes, together with the box in which they are contained. The box is mahogany, and it is fastened by hasps of an old-fashioned pattern; it is itself manifestly in an old style of work. One of the cubes is made of hard wood, the other consists of a brass shell or frame, in which two pieces of hard wood fill up the vacant space so as to constitute a complete cube. The brass work was not a casting, but the result of combining pieces of hammered or sheet-brass suitably cut out which were welded or soldered together, and the whole was manifestly finished by the use of file and hammer. The only mark on the brass is the number 1, placed on a part of the shell as a guide for inserting one of the supplementary wooden pieces. This figure is undoubtedly in the old style, and Mr. Yeates is distinctly of opinion that the brass work is old-fashioned, as he satisfied himself, by comparing it with old brass instruments, that it could be very well assigned a date in the early part of the eighteenth century. It was most probably the work of an amateur, or done under the direction of a mathematician for a special object; for if it had been made by an instrument-maker for the purpose of sale a number of similar brass cubical shells could more easily have been made by casting.

I have never seen any demonstration of the structure of the cubical shell, but it soon appeared manifest that it is based on the properties of a square. Lay off on the diagonal of one of the faces of the cube from its middle point two parts, each equal to half the side of the cube. Draw lines at the extremities of these perpendicular to the diagonal, and two isosceles right-angled triangles will be formed which constitute the bases of two equal triangular prisms, between which a cube equal to that from which the prisms were cut can slide. If the cube slides parallel to the faces of the original cube the prisms will be totally unconnected, and the whole problem turns upon their material connexion while allowing the sliding cube to pass between them.

The annexed figures show the cubes separately, and also

that a second cube of the same dimensions may be passed through the same, the possibility of which he has demonstrated, both geometrically and algebraically, and which has been actually put in practice by the ingenious Mr. Daniel Voster of Cork, with whom I saw two such cubes." (Smith's 'History of Cork,' 1st edition, 1750, vol. i. p. 172.) This passage is quoted by Gibson, who says that Daniel Voster was probably father to Elias, the author of the *Arithmetic*. But this is not correct—Daniel was the son of Elias, as I find from the eighth edition published in 1766 that Daniel was editor of the book to which the name of Elias is prefixed as the author.

the penetration of the second cube through the brass which forms the frame of the first.



The prisms must be connected at top and bottom by two similar and equal flanges, the edges of which must lie between the diagonal of the cube and the extreme corner of the flange. If too far from the diagonal, connexion with the prisms would cease ; if too close to the diagonal, the second cube could not pass between the flanges.

The geometrical conception of a cube and the material solid representing such a conception to the sight and touch are different things. It is easy to conceive how one geometrical cube can pass through another of equal dimension by a square opening whose diagonals are perpendicular to the sides of the first cube. The triangular prisms left by this opening would be equal ; but they would touch by mere lines, and could not be represented by a continuous solid material substance. They would necessarily be distinct and separate.

If the second cube were passed through the first parallel to the diagonal of the square on one of its faces, two triangular prisms would be cut off each distant from the other by the side of the cube. The sides of the base of each prism p would be manifestly $p = a \left(1 - \frac{1}{\sqrt{2}} \right)$, and the long sides $a(\sqrt{2} - 1)$.

But these prisms would again be unconnected. They could be connected by triangular flanges having knife-edges, and

equally inclined planes terminating at the corners of the cube. These knife-edges must manifestly be equal to the side of the cube; and as the sliding cube on each flange has its side perpendicular to the flange, the two flanges must have their edges a little distant from the diagonal to which each is parallel. The interval secures junctions of the flange with the two prisms. The thickness t of flange downwards must be also secured in order that the flange holds its place. The inclination of the face of the flange will depend upon these two quantities.

The relations between x the distance of the knife-edge from the corner of the cube, t the thickness of the flange at the points of junction with the prisms, and θ the angle of inclination of the inner face of the flange to the face of the cube can be easily found. As the sliding cube must have one of its sides always perpendicular to the face of the flange, the following equation must subsist:—

$$a = (a - t) \cos \theta + [p\sqrt{2} - (x - \frac{1}{2}a)] \sin \theta.$$

As the least thickness of the flange parallel to the long side of the prism may be represented by

$$t = (x - \frac{1}{2}a) \tan \theta,$$

the above becomes

$$(a - 2t) \cos \theta + p\sqrt{2} \sin \theta = a, \quad . \quad . \quad . \quad (1)$$

or

$$a = a \cos \theta + [p\sqrt{2} - 2(x - \frac{1}{2}a)] \sin \theta,$$

$$a - a \cos \theta = a(\sqrt{2} - 2x) \sin \theta, \quad . \quad . \quad . \quad (2)$$

remembering that $p = a \left(1 - \frac{1}{\sqrt{2}}\right)$.

From (2) the relation between x and θ gives for θ ,

$$\sin \theta = \frac{2a(a\sqrt{2} - 2x)}{3a^2 + 4x^2 - 4ax\sqrt{2}}.$$

From (1) the value of θ which makes t a maximum can be found by the usual methods,

$$2t = a(\sqrt{2} - 1) \tan \theta - a(\sec \theta - 1),$$

$$2 \frac{dt}{d\theta} = \frac{a(\sqrt{2} - 1)}{\cos^2 \theta} - \frac{a \sin \theta}{\cos^2 \theta},$$

$$2 \frac{d^2t}{d\theta^2} = \frac{a \cos^3 \theta - 2(\sqrt{2} - 1) \cos \theta \sin \theta - 2a \sin^2 \theta \cos \theta}{\cos^4 \theta}.$$

When $\frac{dt}{d\theta}=0$, $\sin \theta = \sqrt{2}-1$, this, substituted in the value for $\frac{dt^2}{d\theta^2}$, shows that the latter must be negative; hence $\sin \theta = \sqrt{2}-1$ gives for t a maximum, or $\theta = 24^\circ 28'$ would give the greatest thickness for t . Between this and zero the thickness would give a smaller inclination and also a different value for x . In the model x has been chosen between the two extreme values $\frac{a}{\sqrt{2}}$ and $\frac{a}{2}$, or $x = \frac{a}{4}(\sqrt{2}+1)$.

This value of x would allow greater values of t and θ than in the model, but they have been both determined by making the greatest thickness of the flange at its extreme end equal to $\frac{p}{2\sqrt{2}}$. This gives

$$\begin{aligned}\tan \theta &= \frac{p}{2x\sqrt{2}} \\ &= \frac{p\sqrt{2}}{4x};\end{aligned}$$

with the assumed value of x ,

$$\tan \theta = \frac{\sqrt{2}-1}{\sqrt{2}+1}.$$

Hence $\theta = 9^\circ 45'$ nearly.

The value of x numerically is $a \cdot .60355$ nearly.

$$\begin{aligned}p &= a \cdot 29289, \quad t = \frac{a(\sqrt{2}-1)^2}{4(\sqrt{2}+1)} = \frac{a(3-2\sqrt{2})}{4(1+\sqrt{2})}, \\ t &= \frac{p^2}{8x}.\end{aligned}$$

The side of the cube is very nearly 1.92 inch, and the above results agree with the measured values of x , p , θ , and very nearly with that of t .

It thus follows that a material solid cube can be so constructed as to allow of a cube of the same dimensions passing through it by an aperture cut in the former without separating the remaining portions. As crystals are known to be penetrated by others of similar shape, this problem may possibly illustrate questions connected with the study of isomorphous groups of the cubical type which are frequently known to present the appearance of interpenetration.