

A la fin, M. Russell n'a pas l'air très satisfait de ce que je dis de la probabilité. Je n'en suis pas très satisfait non plus et je serais heureux si M. Russell avait quelque chose de plus satisfaisant à proposer.

POINCARÉ.

M. Poincaré's reply to my review in *MIND*, July, 1905, calls for a few words of explanation.

On the subject of mathematical induction I await his forthcoming article in the *Revue de Métaphysique et de Morale*. But I should like to clear up a misunderstanding as to the sense in which, as I hold, mathematical induction does not proceed from the particular to the general. (Mathematical induction, by the way, does not define *integers*, but *finite integers*.) The principle may be stated as follows: "A number n is said to obey mathematical induction if it possesses every property which (1) belongs to 0, and (2) belongs to $m + 1$ whenever it belongs to m ". Here the principle itself is doubly general, since (a) it makes a statement about all properties, (b) it makes a statement about all numbers. The statement about all numbers occurs in (2) above. And when we have taken a particular property, and thus ceased to concern ourselves with the general principle of induction, we still have a general statement about all numbers. Let us take an instance: Suppose we wish to prove that if n obeys mathematical induction, then n is not equal to $n + 1$. We prove (a) that 0 is not equal to 1, (b) that if m is not equal to $m + 1$, then $m + 1$ is not equal to $m + 2$. Here (b) is a statement about *all* numbers. It is only from (a) and (b) together that we reach the desired conclusion. The generality of (b) is not the kind of generality that M. Poincaré supposes me to mean when he suggests that I wish to adduce the principle of mathematical induction itself as a necessary premise in all its applications.

As regards geometry, I do not think it is necessary to my point to decide what is meant by perception. My point is that relations of *order*, as opposed to metrical relations, are in some sense given in experience, and that this appears to show that spatial relations are to some extent empirically determined.

I regret that my remark about "Abracadabra" appeared to be a mere epigram. I meant to suggest that what it is convenient to suppose must have some meaning, and I did not suppose that I was "profiting by an ambiguity," which I should be most unwilling to do consciously.

B. RUSSELL.

MR. MACCOLL'S VIEWS ON LOGICAL EXISTENCE.

Mr. MacColl in the last number of *MIND* replied to my note in the previous number. He has put the matter so clearly that no doubt whatever can be entertained as to the position he occupies, but I still think that there are certain important considerations that prevent the general adoption of his view. Mr. Russell has well expounded from one standpoint the doctrine commonly held. The following criticisms may throw some further light upon the subject.

In my view it is not permissible to consider the Universe of Discourse as made up of two *universes*. The Universe of Discourse in Symbolic Logic means all the things that we are talking about. Now such a universe may be divided into two compartments, but each of these does not form a universe by itself. This is not a matter of mere words, but is one of principle. Within our Universe of Discourse there is not a universe of unrealities: all the members of the Universe of Dis-

course are on an equal footing as regards logical reality. The premisses decide what compartments are to be erased, i.e., what members are to be declared non-existent. When the Universe of Discourse is divided into two compartments, the fact that a subdivision of one compartment is destroyed does not place that subdivision in the other compartment. The number of divisions in the Universe of Discourse is limited by the number of terms mentioned in the premisses: it is not allowable, after having arrived at our compartments in this way, to go over them and state that some of them have existence—in Mr. Russell's (a) sense—and some have not, for to do such a thing would be to *assume* the presence of a further premiss, giving this additional information. Since, then, in the same argument there cannot be two universes, it is correct, in describing 0, to speak of a class with no members rather than to speak of a universe of unrealities.

When Mr. Russell said that some meanings of existence lie 'wholly outside Symbolic Logic,' he did not mean, we may be quite sure, that the logician cannot manipulate arguments dealing with the various kinds of existence. What was meant was that Symbolic Logic, in occupying itself 'with any question whatever on which it can throw any light'—questions of existence among others—does not adopt any special meaning of existence that may be found in Metaphysics.

Mr. MacColl believes that his fundamental division into realities and unrealities supplies a method of getting rid of certain paradoxes that ordinary symbolists have to encounter; but from what I have said above it will be seen that this advantage is realised only by means of a procedure that is based on unjustifiable assumptions. He says that, whereas ordinary symbolists are led to state that 'Every round square (a null-class) is a triangle,' he can say 'No round square is a triangle.' Such a universal negative can be reached only by labelling some of the compartments real, and some unreal, and to do this two premisses are assumed, *viz.*, 'No round squares are real,' and 'All triangles are real.'

My division of the Universe of Discourse, when there are two terms 0 and 'existent,' into four compartments was only intended to show that the second and the third compartment do not constitute all the existing things in the universe. It is quite true, as Mr. MacColl suggests, that, with a premiss as to the meaning of 0, these four compartments might be decreased in number. But, when the reduction commenced, we should not proceed on the same lines, because he would say that the compartment '0-existent' is to find a place in his universe of unrealities, whereas I should say that the compartment in question has no occupants.

In the concluding paragraphs of his reply to my note Mr. MacColl, in discussing the implications $AB : A$ and $\neg A : A$, introduces doctrines which cannot be discussed here, but which I have criticised in another place.¹ I can only just state that with Mr. Johnson I think (1) it is not correct to speak of propositions as 'always true,' and (2) the term 'certainly' refers to a relation in which the thinker stands to an implication, and, where such term occurs, this relation must be definitely stated before the logician has material upon which to work. And, in contradistinction to the way in which Mr. MacColl speaks at the close of his reply to Mr. Russell, I should say that it is inadvisable to think of several symbolic systems, each equal to dealing with a certain class of problems. A generalised logic should embrace all that is correct in all the so-called systems, and should be able to deal with every problem of a logical character.

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¹ In my paper, "Some Controverted Points in Symbolic Logic," *Proc. of the Arist. Soc.*, N.S., vol. v.).