621. Morley's Theorem

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621. [K[1]. 1. c.] *Morley's Theorem.*

ABC being any triangle with all its angles trisected: if the two trisectors of angle BAC intersect the adjacent trisectors of angles ABC, BCA in R and Q respectively, and if BR, CQ be produced to intersect in L, then

$$RL = QL.$$

For $RL = BL - BR$

$$= BC\left[\frac{\sin\frac{2}{3}C}{\sin\frac{2}{3}(B+C)} - \frac{\sin C\,.\,\sin\frac{1}{3}A}{\sin A\,.\,\sin(60^\circ - \frac{1}{3}C)}\right]$$

$$= BC\left[\frac{\sin\frac{2}{3}C}{\sin\frac{2}{3}(B+C)} - \frac{\sin\frac{1}{3}C\,.\,\sin(60^\circ + \frac{1}{3}C)}{\sin(60^\circ - \frac{1}{3}A)\,.\,\sin(60^\circ + \frac{1}{3}A)}\right]$$

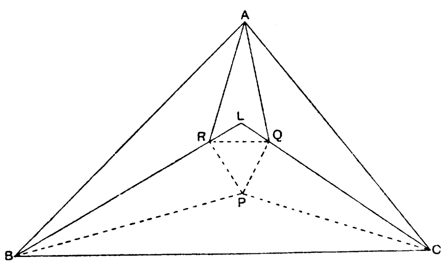
$$= \frac{BC\,.\,\sin\frac{1}{3}C}{\sin\frac{1}{3}(B+C)}\left[\frac{\cos\frac{1}{3}C}{\cos\frac{1}{3}(B+C)} - \frac{\sin(60^\circ + \frac{1}{3}C)}{\sin(60^\circ + \frac{1}{3}(B+C))}\right]$$

$$= \frac{BC\,.\,\sin\frac{1}{3}C\,.\,\sin\frac{1}{3}B}{\sin\frac{2}{3}(B+C)\,.\,\sin(60^\circ + \frac{1}{3}(B+C))}$$

$= QL$, by symmetry.

This equality ($RL = QL$) affords immediate proof of Prof. John Morley's theorem that "the 3 points of intersection of the adjacent trisectors of the angles of any triangle form an Equilateral Triangle."

For, if—in accompanying figure—P be the third point of intersection, obviously, in the triangle BLC, PL will bisect the angle BLC, and the triangles



RLP, QLP will thus be equal in all respects, so that $PR = PQ$; and similarly it can be shown that $RQ = PR$ or PQ. C. H. CHEPMELL.

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622. [K[1]. 1. c.] *Geometrical View of Morley's Theorem.*

Let $\alpha + \beta + \gamma = 60$. Along the circumference of any arbitrarily drawn circle set off an arc ZY whose chord ZY subtends an acute angle of α degrees at the circumference.

This arc may be briefly designated by α.

Draw FE, WU, QP parallel chords to ZY, so that the intercepted arcs YE, EU, UP are designated by α, $\beta + \gamma - \alpha$, α respectively; and divide the arc $PQ(\beta + \gamma)$ in A so that $AP = \beta$, $AQ = \gamma$.

Then the angles WUY, UWZ standing on arcs of $\alpha + \beta + \gamma$ are each 60; and if UY, WZ are produced to meet in X, then XYZ and XUW are equilateral triangles.

Moreover, since arc $WQAU = 2\alpha + \beta + \gamma = WFZYE$, therefore

$$WE = WU = UF.$$

Produce AF, UZ to meet in B; and AE, WY to meet in C. Then the angle $ABZ = AZU - FAZ = (\alpha + \beta) - \alpha = \beta$. Similarly $ACY = \gamma$.