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621. Morley's Theorem

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Source: *The Mathematical Gazette*, Vol. 11, No. 158 (May, 1922), p. 85

Published by: Mathematical Association

Stable URL: <http://www.jstor.org/stable/3602404>

Accessed: 08-11-2015 10:42 UTC

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621. [K<sup>1</sup>. 1. c.] *Morley's Theorem.*

*ABC* being any triangle with all its angles trisected: if the two trisectors of angle *BAC* intersect the adjacent trisectors of angles *ABC*, *BCA* in *R* and *Q* respectively, and if *BR*, *CQ* be produced to intersect in *L*, then

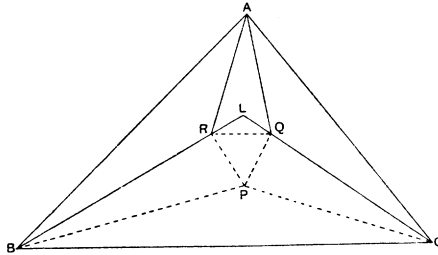
$$RL = QL.$$

For  $RL = BL - BR$

$$\begin{aligned} &= BC \left[ \frac{\sin \frac{2}{3}C}{\sin \frac{2}{3}(B+C)} - \frac{\sin C \cdot \sin \frac{1}{3}A}{\sin A \cdot \sin(60^\circ - \frac{1}{3}C)} \right] \\ &= BC \left[ \frac{\sin \frac{2}{3}C}{\sin \frac{2}{3}(B+C)} - \frac{\sin \frac{1}{3}C \cdot \sin(60^\circ + \frac{1}{3}C)}{\sin(60^\circ - \frac{1}{3}A) \cdot \sin(60^\circ + \frac{1}{3}A)} \right] \\ &= \frac{BC \cdot \sin \frac{1}{3}C}{\sin \frac{1}{3}(B+C)} \left[ \frac{\cos \frac{1}{3}C}{\cos \frac{1}{3}(B+C)} - \frac{\sin(60^\circ + \frac{1}{3}C)}{\sin(60^\circ + \frac{1}{3}(B+C))} \right] \\ &= \frac{BC \cdot \sin \frac{1}{3}C \cdot \sin \frac{1}{3}B}{\sin \frac{2}{3}(B+C) \cdot \sin(60^\circ + \frac{1}{3}(B+C))} \\ &= QL, \text{ by symmetry.} \end{aligned}$$

This equality ( $RL = QL$ ) affords immediate proof of Prof. John Morley's theorem that "the 3 points of intersection of the adjacent trisectors of the angles of any triangle form an Equilateral Triangle."

For, if—in accompanying figure—*P* be the third point of intersection, obviously, in the triangle *BLC*, *PL* will bisect the angle *BLC*, and the triangles



*RLP*, *QLP* will thus be equal in all respects, so that  $PR = PQ$ ; and similarly it can be shown that  $RQ = PR$  or  $PQ$ . C. H. CHEPMELL.

85 Wilbury Crescent, Hove, Sussex, 13th February, 1922.

622. [K<sup>1</sup>. 1. c.] *Geometrical View of Morley's Theorem.*

Let  $\alpha + \beta + \gamma = 60$ . Along the circumference of any arbitrarily drawn circle set off an arc *ZY* whose chord *ZY* subtends an acute angle of  $\alpha$  degrees at the circumference.

This arc may be briefly designated by  $\alpha$ .

Draw *FE*, *WU*, *QP* parallel chords to *ZY*, so that the intercepted arcs *YE*, *EU*, *UP* are designated by  $\alpha$ ,  $\beta + \gamma - \alpha$ ,  $\alpha$  respectively; and divide the arc *PQ* ( $\beta + \gamma$ ) in *A* so that  $AP = \beta$ ,  $AQ = \gamma$ .

Then the angles *WUY*, *UWZ* standing on arcs of  $\alpha + \beta + \gamma$  are each  $60$ ; and if *UY*, *WZ* are produced to meet in *X*, then *XYZ* and *XUW* are equilateral triangles.

Moreover, since arc *WQAU* =  $2\alpha + \beta + \gamma = WFZY, therefore$

$$WE = WU = UF.$$

Produce *AF*, *UZ* to meet in *B*; and *AE*, *WY* to meet in *C*. Then the angle  $ABZ = AZU - FAZ = (\alpha + \beta) - \alpha = \beta$ . Similarly  $ACY = \gamma$ .