

## On the Use of Mutual Inductometers

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XV. *On the Use of Mutual Inductometers.**By* ALBERT CAMPBELL, B.A.\*

(From the National Physical Laboratory.)

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§ 1. *Introductory.*

IN a former paper † I have described arrangements, which we may call mutual inductance bridges, by which self inductances, even of very small amount, can be directly measured ‡. I wish here to discuss further these and other methods in which use is made of variable mutual inductances or, to give them the more convenient name, mutual inductometers.

§ 2. *Modified Mutual Inductance Bridge.*

In the equal arm bridge already discussed there is considerable loss of sensitivity resulting from the insertion of an auxiliary balancing self inductance in one of the arms. By a modification of the arrangement of the bridge, however, the use of the auxiliary coil is rendered unnecessary and the whole apparatus becomes simpler and more efficient. Let us first consider the more general case shown in fig. 1, where the ratio arms are not equal.

Let  $N$  be the self inductance of the coil to be measured,  $r$  a constant inductance rheostat,  $L_1$   $L_2$  self inductances, while the current  $i$  acts inductively on *both* sides of the bridge as shown, the mutual inductances being  $M$  and  $m$ . The coils  $L_1$   $L_2$  may be the upper and lower fixed coils in the

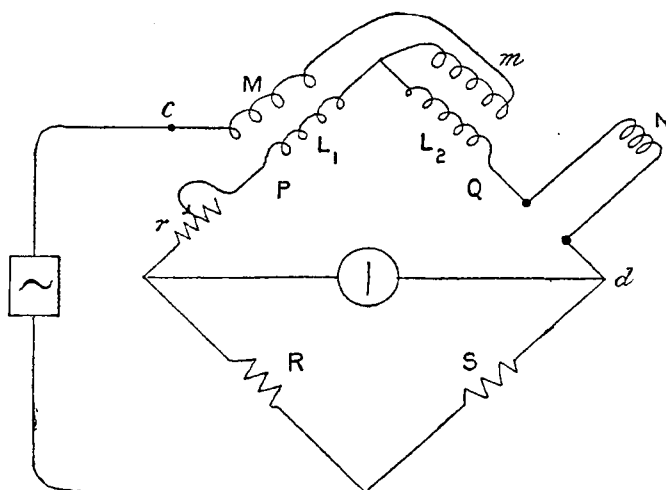
\* Read January 21, 1910.

† Phil. Mag. Jan. 1908, p. 155.

‡ I would mention here that many years ago Dr. Oliver Heaviside investigated a very general case of inductance bridges (Phil. Mag. p. 173, vol. xxiii. 1887). Most of the possible combinations are included in his paper.

inductometer which I have before described (*loc. cit.*), the inducing coils carrying the current  $i$ . Thus  $L_1$  and  $L_2$  will usually have mutual inductance (call it  $y$ ) between themselves.

Fig. 1.



Let  $i_1$  and  $i_2$  be the instantaneous values of the currents (of sine wave form) in the arms  $L_1$  and  $L_2$  respectively, the resistances of these arms being  $P$  and  $Q$ . Let  $\alpha = \omega \sqrt{-1}$ , where  $\omega = 2\pi n$ ,  $n$  being the frequency, and let  $R/S = \sigma$ .

Then, when there is no current through the galvanometer, we have

$$Ri_1 = Si_2 \quad \text{or} \quad \sigma i_1 = i_2$$

and

$$(P + L_1\alpha)i_1 + Mai + yai_2 \\ = (Q + L_2\alpha + N\alpha)i_2 - mai + yai_1.$$

Since  $i = i_1 + i_2$  we have

$$Pi_1 + L_1\alpha i_1 + M\alpha(i_1 + i_2) + yai_2 \\ = Qi_2 + L_2\alpha i_2 + N\alpha i_2 - m\alpha(i_1 + i_2) + yai_1.$$

Separating the real and imaginary parts, we obtain

$$P = \sigma Q$$

$$\text{and} \quad \sigma(L_2 + N) - L_1 = (m + M)(\sigma + 1) - v(\sigma - 1) \quad \dots (1)$$

The most useful case of this is when the ratio arms are

equal, *i.e.*  $\sigma=1$ , and then these equations reduce to

$$P=Q \quad , \quad , \quad , \quad , \quad , \quad , \quad , \quad , \quad (2)$$

and

$$L_2 - L_1 + N = 2(m + M) \dots \dots \dots (3)$$

The best arrangement is to make  $L_2=L_1$  permanently in the inductometer, and then the unknown inductance is given directly by

$$N=2(m+M), \quad , \quad , \quad , \quad , \quad (4).$$

When  $L_1$  and  $L_2$  are the upper and lower fixed coils in the inductometer, the reading of the instrument is  $m + M$  and thus  $N = 2 \times \text{Reading}$ , and thus is read directly. The sensitivity of the bridge is here much greater than when  $L_2$  consists of a separate balancing coil.

### § 3. *Measurement of Effective Resistance.*

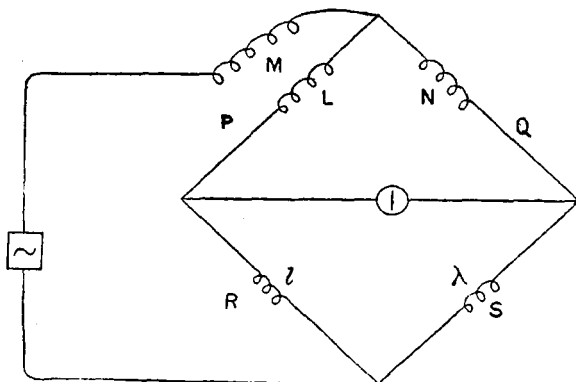
If an alternating current at  $n \sim$  per second having an effective value  $I$  in flowing through a circuit wastes energy in it at the rate of  $R' I^2$  watts (including losses due to eddy currents, magnetic and dielectric hysteresis, &c.), then  $R'$  is called the Effective Resistance of the circuit (for frequency  $n$ ). In general it increases with the frequency, and when telephonic frequencies (500 to 2000  $\sim$  per sec.) are reached, it may often become very much larger than the ohmic resistance. As its value governs the loss of energy, it is of the utmost importance in telephone work to be able to measure it with accuracy. In former papers already cited I have shown how it can be directly measured by a mutual inductance bridge simultaneously with the effective self inductance of the circuit. When the bridge has equal ratio arms which can be interchanged to ensure that they are identical, the method is free from serious error; but if unequal ratio arms have to be employed, large errors may arise from the very small but unavoidable self inductances of these arms. The importance of this in the simple self-inductance bridge was pointed out lately by Giebe \*, and upon his mathematical result he based his ingenious method of measuring very small inductances.

\* *Ann. der Physik*, p. 941 (24), 1907.

I have applied similar treatment to the somewhat more complicated case of the mutual inductance bridge as follows.

In fig. 2 let the resistances of the four arms be  $P$ ,  $Q$ ,  $R$ , and  $S$ , and their self inductances  $L$ ,  $N$ ,  $l$ , and  $\lambda$  respectively.

Fig. 2.



Let  $M$  be the mutual inductance as shown. By procedure similar to that in §2 it is easy to show that

$$PS - QR = \omega^2[(L - M)\lambda - (N + M)l] \quad . \quad . \quad (5)$$

and  $SL - RN = (S + R)M - P\lambda + Ql \quad . \quad . \quad . \quad (6)$

Several cases are important.

Case (1). When  $M = 0$  we have

$$PS - QR = \omega^2(L\lambda - Nl) \quad . \quad . \quad . \quad (7)$$

and  $SL - RN = Ql - P\lambda \quad . \quad . \quad . \quad (8)$

These are Giebe's equations for the ordinary self-inductance bridge.

Case (2). If  $R = S$ ,

then  $(P - Q)R = \omega^2[(L - M)\lambda - (N + M)l]$

and  $(L - N - 2M)R = Ql - P\lambda$ .

If also  $\lambda = l$ ,

then  $(P - Q)R = \omega^2 l (L - N - 2M)$

and  $(Q - P)l = R(L - N - 2M)$ .



and a closer approximation then is

$$X \doteq M[1 + \sigma - \omega^2(\sigma\lambda - l)\lambda/S] \quad . \quad . \quad . \quad (15)$$

Thus the unknown  $T$  and  $X$  are obtained from the change made in  $P$ , the reading of  $M$ , and the corrections due to  $\lambda$  and  $l$ . It will be found that the correction is usually almost negligible in the expression for the inductance  $X$ . On the contrary, for the effective resistance  $T$  the correction may become very important, since  $\omega^2$  is large even for moderate frequencies. For example, let  $R=99$ ,  $S=1$ ,  $P_0-P_1=20$ ,  $M=1$  millihenry,  $l=10$  microhenries,  $\lambda=1$  microhenry, and  $n=1000 \sim$  per sec., giving  $\omega^2 \doteq 40 \times 10^6$ .

Then we have

$$T \doteq P_0 - P_1 + 3.6 \doteq 23.6 \text{ ohms,}$$

while

$$\begin{aligned} X &= 100M - 3.6 \times 10^{-6} \\ &= 0.1 - 3.6 \times 10^{-6} \text{ henry.} \end{aligned}$$

Thus the small inductances  $l$  and  $\lambda$  affect the measurement of the effective resistance  $T$  by as much as 15 per cent., while the self inductance  $X$  is only affected by 3.6 parts in 100,000.

This example shows how much more difficult it usually is to measure effective resistance than self inductance. The difficulty is got over, however, if we can make  $R\lambda = Sl$  (*i. e.*  $\sigma\lambda = l$ ), for then the terms involving  $l$  and  $\lambda$  disappear and we have

$$X = (1 + \sigma)M \quad . \quad . \quad . \quad . \quad (16)$$

and

$$T = P_0 - P_1 \quad . \quad . \quad . \quad . \quad (17)$$

To ensure that  $R\lambda = Sl$ , or in other words that the proportional arms have self inductances in the ratio of their resistances, is not a very easy matter. The following is the best method that I have tried. A coil ( $A$ ) is constructed of highly stranded wire to give moderately high self inductance (say 0.1 henry) with as high effective insulation resistance as possible\*. Its distributed capacity  $k$ , which should be as small as possible, may be measured by connecting it through a thermoammeter to a small variable condenser and adjusting

\* See 'Electrician,' Dec. 10, 1909.

the latter to give resonance with an alternating current of known frequency (2000 to 10,000 ~ per sec.) in a loosely coupled neighbouring coil. If  $K$  be the reading of the variable condenser in mfd., and  $L$  the inductance in henries of the coil (A), then  $n\sqrt{(K+k)L}=159.3$ , and so  $k$  can be found. It is well known that the effective resistance  $T'$  and inductance  $X'$  are given by Dolezalek's formulas,

$$X' \doteq X(1+2\omega^2 Lk) \quad . \quad . \quad . \quad (18)$$

$$\text{and} \quad T' \doteq T(1+\omega^2 Lk), \quad . \quad . \quad . \quad (19)$$

where  $X$  and  $T$  are the values if  $k$  were absent (in this case the values for very low frequency).

The coil (A) is then tested in the bridge, and if the values obtained for  $X'$  and  $T'$  are not those given by equations (18) and (19), a small amount of inductance is added to  $R$  or  $S$  until  $X'$  and  $T'$  are read correctly. When this adjustment has been made once for all, it will be found that any other effective resistance will be measured correctly.

It may be mentioned that a well proportioned 0.1 henry coil of 7-strand wire, each strand being of 0.2 mm. diameter and separately insulated, should show practically no variation in effective inductance and resistance due to skin effect at 1000 ~ per sec.\*; its distributed capacity should be less than 0.0001 mfd. If a coil when tested at various frequencies shows percentage variations in its effective resistance which are double those in its inductance, then these variations can be accounted for by distributed capacity; but if the variation of the effective resistance is at a greater relative rate, then we must look for additional causes such as leakage or skin effect (or errors in the bridge).

When the self inductances of the inductometer coils are high, their distributed capacities may cause errors at the higher frequencies. The effects of the capacities of the coils  $L_1$  and  $L_2$  (fig. 1) are got rid of by the method of preliminary balancing; unfortunately this does not eliminate the effect of the capacity  $k$  of the external inducing coil. If its resistance and self inductance are  $\rho$  and  $x$  respectively, then

\* See M. Wien, *Ann. der Physik*, p. 1 (6), 1904, and Dolezalek, *l. c.* p. 1142 (12) 1903.



for an equal arm bridge, instead of equations (2) and (4) we have

$$Q = P + 2\omega^2 k \rho M - \omega^2 k M^2 \quad . \quad . \quad . \quad (20)$$

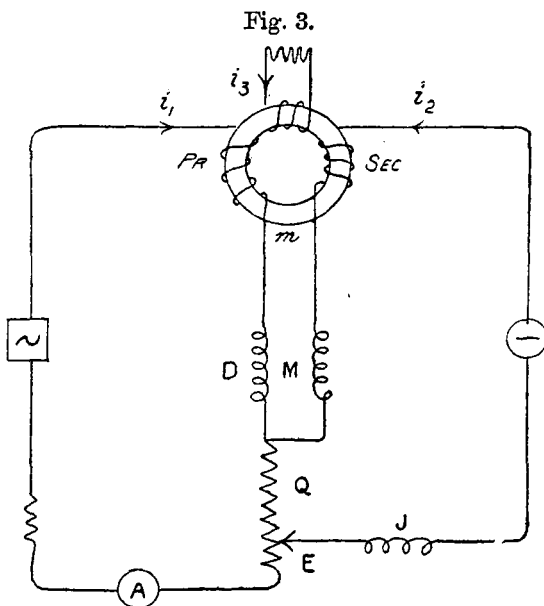
$$\text{and} \quad N = 2M(1 + \omega^2 k x), \quad . \quad . \quad . \quad (21)$$

where  $k$  is in farads.

The error in  $Q$  due to the second term in (20) is usually the most serious. Both errors can be approximately eliminated by connecting a small capacity of suitable amount across the points  $c, d$  (fig. 1).

#### § 4. Null Method in Iron Testing.

The use of a mutual inductometer affords a null method for the magnetic testing of iron; this is analogous to the self inductance method which Max Wien investigated very thoroughly some years ago \*, and there are cases in which it may prove of distinct value. The connexions are shown in fig. 3.



An iron ring is wound with superimposed primary and secondary coils of turns  $N_1$  and  $N_2$  respectively,  $m$  being the mutual inductance between them. These coils are connected,

\* *Ann. der Physik*, p. 859 (66), 1898.

as shown, through the coils of a mutual inductometer D to a source of alternating current and a vibration galvanometer (or tuned telephone), a sliding contact E allowing a part Q to be selected from a resistance connected with the junction of the coils of D. The galvanometer circuit is best made highly inductive by a coil J. Let the hysteresis\* and eddy current loss in the ring be represented by a tertiary closed winding, evenly distributed, of resistance R and self inductance L, having mutual inductances F and G to the primary and secondary coils respectively. If  $N_1/N_2=b$ , then  $G=bF$ . Let  $i_1, i_2, i_3$  be the instantaneous values of the currents in the primary, secondary, and tertiary coils respectively,  $I_1, I_2$ , and  $I_3$  being their effective values. Also let the current  $i_1$  be of sine wave form, which may be attained by electrical tuning or otherwise, the period of the galvanometer being also tuned to that of  $i_1$ . The galvanometer deflexion can now be reduced to zero by adjusting D and E to values M and Q respectively.

Then we have

$$(R + L\alpha)i_3 = F\alpha i_1,$$

and therefore

$$I_3^2(R^2 + L^2\omega^2) = F^2\omega^2 I_1^2. \quad . \quad . \quad (22)$$

Also

$$\begin{aligned} (m - M)\alpha i_1 &= -Q i_1 - G\alpha i_3, \\ &= -Q i_1 + \frac{FG\omega^2 i_1}{R + L\alpha}, \\ &= -Q i_1 + \frac{FG\omega^2 R i_1}{R^2 + L^2\omega^2} - \frac{FGL\omega^2 \alpha i_1}{R^2 + L^2\omega^2}. \end{aligned}$$

Hence

$$m - M = -\frac{FGL\omega^2}{R^2 + L^2\omega^2},$$

and

$$Q = \frac{FGR\omega^2}{R^2 + L^2\omega^2} = \frac{F^2 R \omega^2}{b(R^2 + L^2\omega^2)} = \frac{R I_3^2}{b I_1^2}.$$

Therefore the total iron loss  $R I_3^2$

$$= Q I_1^2 b = Q I_1^2 \cdot N_1/N_2. \quad . \quad . \quad . \quad (23)$$

Also

$$m - M = -QL/R. \quad . \quad . \quad . \quad . \quad (24)$$

\* If this be not considered rigorous enough for the hysteresis loss, the part relating to it can be proved by another method.

I have assumed that the conditions are not sensibly affected by the small harmonic currents of higher frequency which pass through the galvanometer without any considerable effect on its deflexion. The object of the inductance coil J is to check these, and this could be done even more effectively by adding also a condenser of suitable capacity.

Since the magnetizing current  $i_1$  is of sine wave form, the flux density  $\mathcal{B}$  and the total flux  $\Phi$  will in general not be sinoidal. It is usually necessary to reduce the results of iron tests to a standard value of  $\mathcal{B}_{\max}$  with induced secondary voltage of sine wave form. When this is required the  $\mathcal{B}_{\max}$  and the form factor of the secondary voltage are observed simultaneously with the above measurement of power. This is done in the usual way by the help of a synchronous commutator\*. By making tests at two different frequencies the hysteresis and eddy current losses may be separated.

It should be noticed that the work done in the iron by the current  $i_1$  is got by integrating  $i_1 d\mathcal{B}$ ; if  $i_1$  has no harmonics, the harmonics in  $\mathcal{B}$  disappear in the integration. I have tried the method with a ring of stampings from ordinary transformer sheet, and obtained a result in very fair agreement with the ordinary wattmeter method; but further experiments are desirable. I have found the method convenient for testing small samples of iron for telephonic work where the tests have to be made for very low values of  $\mathcal{H}$  (say about 0.01), and in such cases the method is so sensitive that it is easy to test very small rings weighing only a gram or two. When the eddy currents are very small, equation (24) becomes  $m = M$ ; and since

$$m = 4\pi \times 10^{-9} N_1 N_2 \mu s / (\text{circumf. of ring}),$$

where  $s$  = section of ring, we can at once find the permeability  $\mu$  for various values of  $I_1$  (and hence of  $\mathcal{H}$ ). In such cases I have found the values obtained at moderate frequencies in agreement with the results of ballistic tests.

When the eddy currents are large enough to affect the

\* See Lloyd, 'Bulletin Bureau of Standards,' p. 467, vol. iv. 1908 also the Author, Proc. Inst. El. Eng. p. 553, vol. xliii. 1909.

phase of  $\Phi$ , but do not alter its magnitude appreciably, as Mr. T. L. Eckersley has pointed out to me, we can still find  $\mu$ ; in the galvanometer circuit in fig. 3 the vectors  $QI_1$ ,  $\omega MI_1$ , and  $\omega N_2\Phi$  form a right-angled triangle ( $\Phi$  being the effective value), and hence

$$\omega^2 N_2^2 \Phi^2 = (Q^2 + \omega^2 M^2) I_1^2. \quad \dots \quad (25)$$

Thus we can find  $\Phi$  and  $\mu$  for any value of  $I$ , by observing  $Q$ ,  $M$ , and the frequency.

### § 5. *Tests of Current Transformers.*

In current transformers used with ammeters, wattmeters, &c., it is necessary to be able to test accurately the ratio of current transformation ( $I_1/I_2$ ) and the angle of lag ( $\phi$ ) between the secondary and the reversed primary current. The above method may be used for this as shown in fig. 4, where the transformer has any desired load; but part of the load is the known resistance  $S$ . By considering the galvanometer circuit, when a balance is obtained, the vectors  $\omega MI_1$ ,  $QI_1$ , and  $SI_2$  form a right-angled triangle, and hence

$$\tan \phi = \frac{\omega M}{Q} \quad \text{and} \quad I_1^2/I_2^2 = S^2/(Q^2 + M^2\omega^2). \quad \dots \quad (26)$$

$$\text{When } \phi \text{ is small, this} \quad \doteq \frac{S^2}{Q^2} \left(1 - \frac{M^2\omega^2}{Q^2}\right),$$

$$\begin{aligned} \text{or} \quad I_1/I_2 &\doteq \frac{S}{Q} \left(1 - \frac{1}{2} \tan^2 \phi\right) \\ &\doteq \frac{S}{Q} \left(1 - \frac{\phi^2}{2}\right). \end{aligned}$$

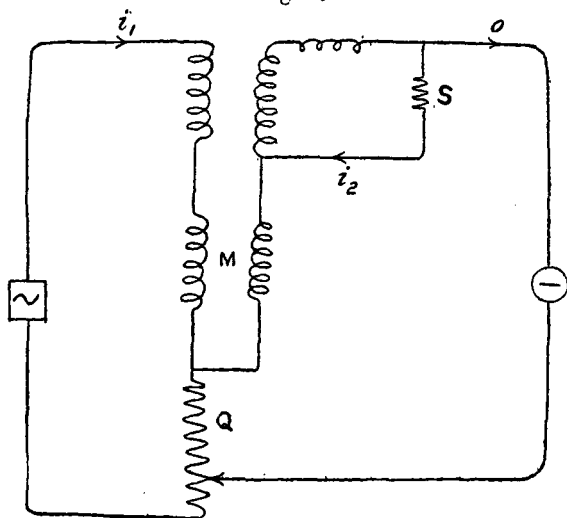
Usually  $\phi$  is so small that we may take

$$I_1/I_2 = S/Q. \quad \dots \quad (27)$$

With regard to the actual working of the method, the inductometer should be a low-reading one whose primary coils can carry the large primary current. The resistance  $Q$  may be a standard low resistance shunted by a slide wire along which the slider runs. Tests by this method on a

commercial transformer gave results in practical agreement with those obtained by Mr. C. C. Paterson by a wattmeter method, the source of current being a sine-wave alternator. This agreement is interesting as showing that the harmonics

Fig. 4.



in the secondary current (which are not taken account of by the vibration galvanometer) have very slight effect on the results. The method should only be used where the primary current can be made approximately sinoidal.

In conclusion, I would express my best thanks to Dr. Glazebrook for his kind interest in the work, and to Mr. T. L. Eckersley for valued and suggestive criticism.

#### ABSTRACT.

In the use of mutual inductometers (or variable inductances) already described by the Author, the use of a balancing coil in one arm of the bridge causes considerable loss of sensitivity. With an equal-arm bridge this difficulty is overcome by putting the two halves of the secondary circuit in adjacent arms of the bridge. The auxiliary balancing coil is thus dispensed with and the usual formula is still applicable.

The Author next discusses the measurement of effective resistance, which is in general much more troublesome than that of self-inductance. As the effective resistance determines the total power spent by a given

alternating current in a conductor, it is a most important quantity in telephonic and other high-frequency work. When it is measured by an ordinary self-inductance bridge, Giebe has shown that large errors may be introduced by the small residual inductances of the ratio arms. The Author works out the analogous formulas for mutual inductance bridges, which indicate that the inductances of the ratio arms must be accurately proportional to their resistances, if errors are to be avoided.

He next describes a null method in iron testing analogous to Max Wien's self-inductance method. The ring to be tested is wound with primary and secondary coils. The magnetizing current  $I_1$  is passed through the primary coil, the primary circuit of a mutual inductometer, and a slide-wire resistance. The detecting instrument, a vibration galvanometer or a tuned telephone, is put across a circuit consisting of the secondaries of the ring and the inductometer in opposition and a part  $Q$  of the slide-wire resistance. By adjusting  $Q$  and the reading  $M$  of the inductometer a balance is obtained, in which case the power lost in the ring (due to hysteresis and eddy currents) is equal to  $QI_1^2 \times N_1/N_2$ , where  $N_1$  and  $N_2$  are the numbers of turns in the windings of the ring. In certain cases the permeability can also be directly found. The method is immediately applicable to the testing of current transformers. If the instrument usually in the secondary circuit of the transformer be replaced by a suitable low resistance  $S$ , then when a balance is obtained

$$\tan \phi = 2\pi nM/Q \quad \text{and} \quad I_1/I_2 = S/Q,$$

where  $\phi$  is the angle of lag between the primary and the reversed secondary current, and  $n$  is the frequency. The primary current should have a sine wave form. The method gives directly the two quantities wanted in practice, but, owing to considerations of wave form, the results must be interpreted with caution.

#### DISCUSSION.

Mr. C. C. PATERSON congratulated the Author upon his null method of iron testing and pointed out some of its advantages.

Mr. W. DUDELL remarked with regard to the self-inductance of the ratio arms that the time constants of the two should be the same, and suggested the use of woven resistances.

Mr. CAMPBELL, in reply to Mr. Duddell, stated that when the ratio arms of the bridge could be of high resistance the employment of woven resistances could ensure that their time constants were equal. The main difficulty, however, arose with ratio arms of relatively low resistance (e.g. 10 : 1000 ohms); for such values woven resistances did not appear quite applicable.

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