

LIGHT EMISSION FROM A MOVING SOURCE IN CONNECTION WITH THE RELATIVITY THEORY.

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SYNOPSIS.

Light Emission from a Moving Source; a Generalized Explanation of the Michelson-Morley Experiment.—The paper is based upon unpublished work left by the late *Nikolas Pashsky* of Kieff. It is pointed out that the experiment does not prove the constancy of the velocity of light but merely that if light is sent out in all directions from the center of a spherical mirror, all rays after reflection will return simultaneously to the center no matter what the velocity of the system through space. This result requires merely that with respect to axes moving with the source, the velocity of light in a direction making an angle φ with the direction of motion is given by $\sigma_{\varphi} = \sigma / \{1 + \sqrt{[\psi \cos \varphi / (\sigma + \psi)]}\}$, when σ and ψ are arbitrary functions of the velocity of translation v , with the only conditions that for $v = 0$, $\psi = 0$ and $\sigma = c$. There are therefore an *infinite number of possible hypotheses* which will explain the experiment, *in addition to the relativity theory*, which assumes that $\sigma_{\varphi} = \text{constant}$. The only way to decide the matter would be to measure the velocity of light by a method in which the ray of light would not return to its starting point.

THE following paper is based upon the unpublished works of *Nikolas Pashsky*, late assistant in physics at the University of Kieff.¹ His manuscripts were entrusted to me by his widow to prepare for publication, but I was forced to leave them in Kieff.

Wishing to delay no longer in presenting his ideas to the scientific world I am presenting my own explanation of some of his ideas.

The present paper represents only a very small portion of the large mass of work which he had nearly ready for publication.

1. The classical theory of the electromagnetic field, among other fundamental statements, on which it is based, contains the following postulate:

(a) The velocity of propagation of any electromagnetic disturbance in free space is independent of the motion of the source, and is, relative to a system at rest, equal to $c = 3 \times 10^{10}$ cm./sec.

Einstein, denying *the reality of absolute rest and absolute motion* and requiring that the velocity of light should have the same value c , relative to any chosen system, came to the familiar conclusions, which all together form the relativity theory.

The principal experimental fact, on which are based his speculations, is the Michelson experiment, to which however the relativity theory

¹ Died June 26, 1918, aged 32 years.

must ascribe more than it gives in reality. According to this theory the velocity of light in a moving system is in all directions equal to c , *i.e.*, all the points of a sphere, from the center of which at the moment t an electromagnetic disturbance is sent out, will receive the impulse simultaneously, while the Michelson experiment shows only that all those disturbances, reflected by the points of the sphere, will *come back to the center simultaneously*, and this is not the same.

H. Lorentz, wishing to give an explanation for this true result of the Michelson experiment, made the contraction hypothesis. Einstein, going beyond the limits of the experiment and requiring the simultaneity of the arrival of impulses on the sphere, was *forced* to make the assumption as to the change of time-units in the moving system.

The purpose of this article is to show that without going further than the experiment leads us, and consequently abandoning the principle of relativity, we can, without making the contraction assumption, find an infinite number of hypotheses satisfying the Michelson experiment and that all the theories based on those hypotheses are equivalent one to another and to a *certain extent* equivalent to the relativity theory. But for this purpose *we must abandon the postulate I of the constancy of velocity of light*.

Such an attempt was made by W. Ritz, who made the assumption that the velocity of light emitted by a moving source is equal, relative to a system at rest, to $c + v$ and $c - v$ respectively in the direction of movement of the source and opposite it, where v is the velocity of translation. We shall now make a more general assumption.

(b) The velocity of light emitted by a moving source is equal, relative to a system of rest, to $c + f_1(v)$ in the direction of motion and $c + f_2(v)$ in the opposite direction, where $f_1(v)$ and $f_2(v)$ are arbitrary functions of v , with the only limiting condition that

$$f_1(0) = f_2(0) = 0. \quad (1)$$

Relative to axes moving with the source the velocities will be respectively

$$\begin{aligned} c + v + f_1(v) &= U_1(v) = \sigma + F_1(v), \\ c - v + f_2(v) &= U_2(v) = \sigma + F_2(v), \end{aligned} \quad (2)$$

where σ is a function of v with the condition $\sigma(0) = c$; and $F_1(v)$ and $F_2(v)$ are other arbitrary functions of v with the same limiting conditions

$$F_1(0) = F_2(0) = 0.$$

Here it will be of use to define what we understand by the words "velocity of light."

All the methods employed up to the present to measure this velocity are based on the principle of Fizeaux or Foucault in both of which the

light travels along a path of a known length, and comes back again in the opposite direction. The time necessary to traverse this path is measured, and the velocity is obtained from the equation

$$T = \frac{2l}{\sigma},$$

where l is the length of the path. It is obvious that this does not give us in any case the "true" velocity of light, for the velocities in opposite directions may be different. The velocity obtained in this way we shall denote as "measured velocity" of light, in distinction to the "true velocity" which could be obtained by a method using the propagation of light in one direction. But in all our physical experiments we know only the "measured velocity," a fact which is of importance. Now we shall make the assumption that σ is the "measured velocity" of light in our moving system. But, as our hypothesis must satisfy the conditions of the Michelson experiment, we must require that the time necessary for light to pass from a moving point A to another point B , situated at the distance l from A and moving with it, and return to A , must be independent of whether the system is moving in the direction AB or BA and equal to σ .

Hence

$$\frac{l}{\sigma + F_1(v)} + \frac{l}{\sigma + F_2(v)} = \frac{2l}{\sigma}, \quad (3)$$

or, after some transformations,

$$-\frac{\sigma}{2} [F_1(v) + F_2(v)] = F_1(v)F_2(v). \quad (4)$$

If we put

$$F_1(v) + F_2(v) = 2\psi(v), \quad (5)$$

where $\psi(v)$ is an arbitrary function with the same limiting condition

$$\psi(0) = 0,$$

we have

$$\begin{aligned} -\sigma\psi(v) &= F_1(v)F_2(v), \\ 2\psi(v) &= F_1(v) + F_2(v); \end{aligned} \quad (6)$$

we see that $F_1(v)$ and $F_2(v)$ are roots of the quadratic equation

$$-F^2 + 2\psi(v)F + \sigma\psi(v) = 0.$$

Hence

$$\begin{aligned} F_1(v) &= \psi(v) + \sqrt{\psi^2(v) + \sigma\psi(v)}, \\ F_2(v) &= \psi(v) - \sqrt{\psi^2(v) + \sigma\psi(v)}, \end{aligned} \quad (7)$$

and for the velocities relative to a moving system we find respectively

$$\sigma + \psi(v) + \sqrt{\psi^2(v) + \sigma\psi(v)}$$

and

$$\sigma + \psi(v) - \sqrt{\psi^2(v) + \sigma\psi(v)}. \quad (8)$$

But from (3) we see that we can change the rôles of the functions $F_1(v)$ and $F_2(v)$ and so finally we get

$$\begin{aligned} \sigma + \psi(v) &\pm \sqrt{\psi^2(v) + \sigma\psi(v)}, \\ \sigma + \psi(v) &\mp \sqrt{\psi^2(v) + \sigma\psi(v)}. \end{aligned} \quad (9)$$

So far we have neglected the velocity of light in a direction making a definite angle with the direction of translation. We may show that we obtain a result consistent with the Michelson experiment if we assume that the hodograph of velocities relative to moving axes is an ellipsoid of revolution along the major axis, this axis being parallel to the direction of motion, and the source being situated in one of the foci of the ellipsoid. The velocities along the axis of revolution are given by (9).

Then we have for the major semi-axis

$$a = \sigma + \psi(v), \quad (10)$$

the linear excentricity is

$$e = \sqrt{\psi^2(v) + \sigma\psi(v)}, \quad (10a)$$

the parameter which gives the velocity in a direction perpendicular to the direction of motion is

$$p = \sigma. \quad (10b)$$

The numerical excentricity is

$$\frac{e}{a} = \sqrt{\frac{\psi(v)}{\sigma + \psi(v)}}. \quad (10c)$$

Hence the velocity of light in a direction making an angle φ with the line of motion is

$$\sigma_\varphi = \frac{\sigma}{1 \pm \sqrt{\frac{\psi(v)}{\sigma + \psi(v)}} \cos \varphi}. \quad (11)$$

The positive sign in the denominator corresponds to the assumption that the source lies in the front focus; and the negative that it lies in the rear one.

It is easy to show that our assumption explains the Michelson experiment, if we make the assumption that every point of the reflecting sphere behaves like an independent source, emitting light in accordance with (11).

Really the time necessary for the light to pass from the center to a point of the sphere, situated in a direction defined by the angle φ is, if l = radius of the sphere,

$$T_1 = \frac{l}{\sigma_\phi} = \frac{l \left(1 \pm \sqrt{\frac{\psi(v)}{\sigma + \psi(v)}} \cos \varphi \right)}{\sigma}.$$

The time necessary to return by the same path from the sphere to the center is

$$T_2 = \frac{l}{\sigma_{\pi+\phi}} = \frac{l \left(1 \mp \sqrt{\frac{\psi(v)}{\sigma + \psi(v)}} \cos \varphi \right)}{\sigma}.$$

The change of sign before the radical is due to the fact that

$$\cos(\pi + \varphi) = -\cos \varphi.$$

The entire time necessary for the light to go and return is

$$T = T_1 + T_2 = \frac{2l}{\sigma}, \quad (12)$$

i.e., independent of the direction of propagation.

Formula (12) shows also that if we measure the velocity of light by ordinary methods, sending the beams twice by the same path and dividing the double length of the path by the time that is necessary for the light to go and return, we will always obtain the value σ . If σ is independent of the velocity of translation of the source, and $\sigma = \text{const.} = c$, then we will never detect in this manner the absolute movement of the system.

If σ is a function of v , this would be possible.

As $\psi(v)$ is a quite arbitrary function, only limited by the condition

$$\psi(0) = 0,$$

we have thus found an infinite number of solutions of the "Michelson problem," all equivalent one to another.

2. To show now the mutual relation between our theory and the relativity theory, we may proceed as follows:

Let us consider a sphere of radius l , moving with a uniform velocity v .

Let the center of the sphere be the origin of moving coördinate axes and let the x -axis lie in the direction of motion. Let us consider now the phenomena of light propagation from the point of view of the classical theory.

The velocity of light referred to the moving axes will be, as easily shown, respectively $c - v$ and $c + v$ in the direction of motion, and $\sqrt{c^2 - v^2}$ in a direction perpendicular to this. The time necessary for the light to travel from the center of the sphere and back will be in the latter case

$$\frac{2l}{\sqrt{c^2 - v^2}}, \quad (13)$$

and in the former

$$\frac{l}{c+v} + \frac{l}{c-v} = \frac{2lc}{c^2 - v^2}, \quad (14)$$

i.e., if this time is greater, and if we desire to maintain the simultaneity of return of light to the center, we *must* assume that l is contracted along the line of motion in the ratio

$$\frac{\sqrt{c^2 - v^2}}{c} = \sqrt{1 - \frac{v^2}{c^2}} \quad (\text{assumption 1}),$$

and then it is easily shown that the time of return of light to the center will be independent of the direction of propagation.

This would account for the Michelson experiment and, as Lorentz showed, for all the other known experimental phenomena.

But Einstein, as remarked above, went further and postulated, that the "measured velocity" of light in a moving system should be c ; as in our case it is equal to

$$\sqrt{c^2 - v^2},$$

there remains nothing but to make the assumption that in the moving system time runs slower in a ratio

$$\frac{\sqrt{c^2 - v^2}}{c} = \sqrt{1 - \frac{v^2}{c^2}} \quad (\text{assumption 2}),$$

for then the time required for light to go and return will be not

$$\frac{2l'}{\sqrt{c^2 - v^2}} \quad \text{but} \quad \frac{2l'}{c}.$$

But even if we make this assumption we will find that the absolute movement of the system could be detected, for, though the time of return of the light to the center and its measured velocity are independent of the direction of propagation, the time of arrival of light at the sphere and the "true velocity" will depend upon this direction, and to remove this difficulty, we must assume that a watch whose x -coördinate is equal to x' , shows the time $t - (u/c^2)x'$, if t is the time at the center (assumption 3).

In this way we obtain the complete set of assumptions which form the basis of relativity theory.

Let us now return to our hypothesis. Formula (11) without any contraction hypothesis satisfies the conditions of the Michelson experiment and shows that the measured velocity of light is independent of the direction of propagation. Thus our hypothesis is equivalent to the contraction one and furnishes an equally good explanation of all the

known experimental facts, our infinite set of assumptions giving the same result as the contraction hypothesis. All the experiments made up to the present had to do with the propagation of light in a closed path, this being true also for experiments of electromagnetic character, where we always have to do with "mutual actions" of two charges. No experiment has been made that shows the necessity of assumptions 2 and 3. They are beyond the limits of experiment. And in the same way, as between the modified Lorentz theory and the relativity theory of Einstein the only decisive experiment would be to measure the velocity of light by a method involving its propagation in one direction only; our assumptions are *to this extent* equivalent to the relativity theory.

If we should postulate the simultaneity of arrival of the light impulses at the surface of the sphere in our theory, we should be obliged to assume that the watch on the surface of the sphere shows a time

$$t' = t \mp \frac{k}{\sigma} x, \quad (15)$$

where x is its moving coördinate and

$$k = \sqrt{\frac{\psi(v)}{\sigma + \psi(v)}}. \quad (16)$$

In reality the time necessary for the light to pass from the center to a point of the sphere is equal to

$$\frac{l(1 \pm k \cos \varphi)}{\sigma},$$

and if we introduce t' , it becomes

$$\frac{l(1 \pm k \cos \varphi)}{\sigma} \mp \frac{k}{\sigma} l \cos \varphi = \frac{l}{\sigma},$$

i.e., the velocity of light measured by a "one direction method" will be σ .

Of course, if we desire that even from a source at rest the velocity of light in the moving system should be σ , we must change in an appropriate manner the unit of time in our moving system.

But, without this assumption, since all the apparatus with which we make our measurements has the same velocity of translation as the earth, we shall find (applying the principle used in the deduction of 12) for the "measured velocity" of light even from extra-terrestrial sources the value σ .

If by astronomical methods, given by de Sitter and others, we obtain a constant value of the velocity of light, even from extra-terrestrial sources,¹ it does not disprove our theory; it shows only that the value of $\psi(v)$ for a given v is below a certain limit, for example

¹ By "one-direction method."

$$\psi(v) < \frac{v^2}{c^2}, \text{ etc.}$$

It will be the problem of experimental physics to find out the precise form of $(\psi(v))$.

By applying the Huygens principle and taking instead of spherical elementary waves, ellipsoidal ones, all the optics of moving systems can be built up. This was done by the late N. Pashsky. Those results I hope to publish later.

In conclusion I wish to express my sincerest thanks to Dr. P. H. Dike for the revision of the English translation of the Russian original manuscript, and for many critical remarks.

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