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LVIII. An analysis of relationships

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fourth combination-tone, are recorded by König in the neighbourhood of the interval $c : d''$

54. Beats of the mistuned consonance $1 : 7$ are recorded by König. These might be produced by a sixth combination-tone (difference-tone of form $6p - q$) of the primaries n and $7n \pm m$.

55. Beats of the mistuned consonance $1 : 8$ are recorded. These might be produced by a seventh combination-tone (difference-tone of form $7p - q$) of the primaries $8n \pm m$.

56. As far as my own experience goes, however, I have no direct and palpable evidence of beats of mistuned consonances higher than $1 : 4$, or of the existence of combination-tones higher than the third ($3p - q$) in recognizable intensity. Up to this point the phenomena are quite clear; and there is no possible doubt as to their nature.

But in considering these limited results it must be remembered, (1) that I have restricted myself to notes of very moderate intensity, so that the phenomena might correspond as nearly as possible to those which are presented to our ears in practice, and (2) that, although I was unable to get rid entirely of the presence of upper partials in all cases, yet the phenomena were subjected to a careful and prolonged analysis by listening under varied conditions, until the effect of the upper partials could be separated out and eliminated with certainty. And we have at all events no security that these upper partials did not give rise to many of König's results; indeed it is almost certain that they must have entered into those results.

Note.—The present paper was written before the appearance of König's paper in Wiedemann's *Annalen* in the present year. The discussion of that paper, though necessary for a complete view of the subject, must be reserved till after the conclusion of the present paper.

[To be continued.]

LVIII. *An Analysis of Relationships.*

By A. MACFARLANE, M.A., D.Sc., F.R.S.E.*

IN this article I propose to describe some results of several papers on an Algebra of Relationship, which I have recently contributed to the Royal Society of Edinburgh†. The Logic of Relatives‡ has been worked at by De Morgan, Leslie Ellis, Harley, and C. S. Peirce§; and the last-named

* Communicated by the Author.

† Proc. Roy. Soc. Edinb. May 1879, Dec. 1880, and March 1881.

‡ Since writing this article I have had the opportunity of reading two interesting and suggestive papers on the Logic of Relatives, by Mr. J. J. Murphy.

§ For references see Jevons's 'Principles of Science,' p. 23.

philosopher has recently published* the first part of a memoir containing a summary of his investigations. I have not taken up the subject of relation in general, but have restricted myself, in the first place, to a well-defined and important part, namely the relations of men due to consanguinity and affinity.

The particular class of objects considered in the investigation is in its widest extent the natural class *mankind*, by which term I mean the entire number of men who have existed, exist, or will exist. The universal properties of the symbols are based on the universal properties of the class. For particular investigations we may limit in any manner the collection of men considered—for example, to the inhabitants of Christendom, or the subjects of Queen Victoria, or the citizens of a given town, or the members of a given household.

Let U denote any person in the collection of men considered; then U_A is an appropriate mathematical expression for the person whose name is A , and U_B for the person whose name is B . For the sake of shortness U_A may be written A . Also ΣU is an appropriate mathematical expression for all the persons in the collection.

Let cA be used to denote any child of A ; then $\frac{1}{c}A$ denotes either parent of A . To express a certain child of A , two children of A , three children of A , &c., we require $1cA$, $2cA$, $3cA$, &c. The completeness of a number may be indicated by a dot over the number, as $\dot{3}cA$, the three children of A . When the value of the complete number is not expressed, the expression takes the form ΣcA , all the children of A . Subscript numbers, as in c_1A , c_2A , &c., are the appropriate mathematical symbols for expressing the eldest child of A , the second child of A , &c.

Since cA denotes any child of A , ccA will denote any child of any child of A , hence any grandchild of A . Similarly, $c\frac{1}{c}A$ will denote any child of either parent of A —that is, any brother or sister of A , or A himself (or herself). Also $\frac{1}{c}cA$ will denote either parent of any child of A , hence any consort of A or A himself (or herself); and $\frac{1}{c}\frac{1}{c}A$ will denote either parent of either parent of A —that is, any grandparent of A . The expression $\frac{1}{c}A$ may be denoted by $c^{-1}A$; then the above are denoted by c^2A , $c^{1-1}A$, $c^{-1+1}A$, $c^{-2}A$ respectively. The ex-

* American Journal of Mathematics, vol. iii. p. 15.

pressions c^{1-1} and c^{-1+1} may or may not reduce to c^0 , which means *self*.

Relationships expressed in terms of c and c^{-1} take no account of distinctions due to difference of sex; they may therefore be called *general relationships*, in contrast to the specific relationships into which they are broken up by the introduction of distinctions of sex. Let the order of any general relationship be defined as the number of times c enters into its expression, whether directly or inversely. The relationships of the $(n+1)$ th order are derived from those of the n th order by prefixing c and c^{-1} before each of the latter. In the case of the first order c and c^{-1} are applied to 1. Hence, in order to obtain all the relationships of the n th order, we have merely to expand $(c+c^{-1})^n$, but in such a manner that the order of the factors in each term is preserved. It is evident from the mode of development, that the number of general relationships in the n th order is 2^n . Appended is the first portion of the development.

General Relationships.

Order.	Expression.	Meaning, if reducible.	Meaning, if irreducible.
0	c^0	Self.
I.	c^1	Child.
	c^{-1}	Parent.
II.	c^2	Grandchild.
	c^{1-1} .	Child of parent.	Brother or sister.
	c^{-1+1} .	Parent of child.	Consort.
	c^{-2}	Grandparent.
III.	c^3	Greatgrandchild.
	c^{2-1} .	Grandchild of parent.	Nephew or niece.
	c^{1-1+1} .	Child of parent of child.	Step-child.
	c^{1-2} .	Child of grandparent.	Uncle or aunt.
	c^{-1+2} .	Parent of grandchild.	Child-in-law.
	c^{-1+1-1} .	Parent of child of parent.	Step-parent.
	c^{-2+1} .	Grandparent of child.	Parent-in-law.
	c^{-3}	Greatgrandparent.
IV.	c^4	Greatgreatgrandchild.
	c^{3-1} .	Greatgrandchild of parent.	Grandnephew or grandniece.
	c^{2-1+1} .	Grandchild of parent of child.	Child of step-child.
	c^{2-2} .	Grandchild of grandparent.	First cousin.
	c^{1-1+2} .	Child of parent of grandchild.	Child of child-in-law.
	$c^{1-1+1-1}$.	Child of parent of child of parent.	Step-brother or step-sister.
	c^{1-2+1} .	Child of grandparent of child.	Brother-in-law or sister-in-law.
	c^{1-3} .	Child of greatgrandparent.	Granduncle or grandaunt.
	c^{-1+3} .	Parent of greatgrandchild.	Consort of grandchild.
	c^{-1+2-1} .	Parent of grandchild of parent.	Consort of brother or sister.
	$c^{-1+1-1+1}$.	Parent of child of parent of child.	Consort of consort.
	c^{-1+1-2} .	Parent of child of grandparent.	Step-grandparent.
	c^{-2+2} .	Grandparent of grandchild.	Parent of child-in-law.
	c^{-2+1-1} .	Grandparent of child of parent.	Parent-in-law of parent.
	c^{-3+1} .	Greatgrandparent of child.	Grandparent of consort.
	c^{-4}	Greatgreatgrandparent.

When the expression for the relationship contains a change of sign in the index, it may in certain cases be equivalent to a relationship of a lower order; but when the index does not contain a change of sign, the relationship cannot be equivalent to one of a lower order. The third column contains the general meaning, and the fourth column the particular meaning when the relationship is supposed irreducible. The first reducible relationships are c^{1-1} and c^{-1+1} , which may reduce to 1. The relationship c^{2-1} may reduce to c ; c^{2-2} may reduce to c^{1-1} , which we have seen may further reduce to 1. Consider the relationship c^{m-n} ; the reducible forms are $c^{(m-1)-(n-1)}$, $c^{(m-2)-(n-2)}$, and so on, until one of the numbers is reduced to 0. Hence a relationship of an odd order can reduce only to one of an odd order, and a relationship of an even order only to one of an even order.

It is not difficult to conceive how this table of relationships (fully developed) may be useful to legislators in a case where an exact and comprehensive view of relationships is required, as, for example, in making a consistent and logical alteration of the laws of marriage. How many are the ways in which questions of comparative nearness of relationship arise, and how important to have a simple and ready means of settling them! If this table does not classify relationships according to their nearness, it at least provides the means for such a classification. The principles which have to be settled are—Supposing the distance of c to be measured as 1, what ought to be the value assigned to c^{1-1} , and what the value to c^{-1+1} ? The former of these makes a relationship collateral, and the latter makes a relationship one of affinity. According to the method of reckoning degrees adopted by the Greek Church (which is very elaborate) c^{1-1} is reckoned 2, and c^{-1+1} is reckoned 0.

These general relationships are broken up into more specific relationships by the introduction of symbols to denote sex. Let a subscript m denote male, a subscript f denote female. Then

$m c$ denotes son.	$m c^{-1}$ denotes father.
$f c$ „ daughter.	$f c^{-1}$ „ mother.
c_m „ child of man.	c_m^{-1} „ parent of man.
c_f „ child of woman.	c_f^{-1} „ parent of woman.
$m c_m$ „ son of man.	$m c_m^{-1}$ „ father of man.
$m c_f$ „ son of woman.	$m c_f^{-1}$ „ father of woman.
$f c_m$ „ daughter of man.	$f c_m^{-1}$ „ mother of man.
$f c_f$ „ daughter of woman.	$f c_f^{-1}$ „ mother of woman.

Suppose we have a relationship of the second order, as *cc*. We may write :—

$$\Sigma ccU = \Sigma(m+f)ccU, \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$= \Sigma(m+f)^2 ccU, \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$= \Sigma(m+f)^3 ccU. \quad . \quad . \quad . \quad . \quad . \quad (3)$$

In the case of (1), the terms in $m+f$ may be inserted before the first *c*, or between the two *c*'s, or after the second *c*. In the case of (2), the terms of $(m+f)^2$, which are *mm*, *mf*, *fm*, *ff*, may be inserted either in the first and second places, or in the first and third places, or in the second and third places. In the case of (3) the terms, which are *mmm*, *mmf*, *mfm*, &c., can be inserted in only one manner. Hence the number of forms which *cc* can take (counting *cc* itself as one) is 27.

To find the number of elementary relationships of the *n*th Order.

By an elementary relationship is meant a single relationship, in contrast to a relationship expressing the coexistence of several single relationships. The *n*th order has 2^n general relationships. Consider any one of these. A distinction of sex can be introduced before each *c* or c^{-1} and after the last. Hence the number of different ways in which a distinction of sex can be *r* times introduced is equal to the number of combinations of $n+1$ things *r* together. The number of different relationships obtained by the expansion of a term in which a distinction of sex has been *r* times introduced is 2^r . Hence the number of terms for one general notion is

$$1 + (n+1)2 + \frac{(n+1)n}{1 \cdot 2} 2^2 + \dots + 2^{n+1};$$

that is, 3^{n+1} . Hence the total number for the *n*th order is $2^n 3^{n+1}$. For *n* being 5, the number is 23,328.

Cor. The number of elementary relationships included in the first *n* orders is $\frac{18}{5} (6^n - 1)$; for *n* being 7, the number is greater than one million.

Laws of Reduction.—I. When the sex-symbols preceding and succeeding c^{1-1} are the same, then c^{1-1} can reduce to 1; and when they are different, it cannot reduce to 1. This depends on the morphological law, that sex in mankind is dioecious.

II. When the sex-symbols preceding and succeeding c^{-1+1} are the same, then c^{-1+1} must reduce to 1; and when they are different, it cannot reduce to 1. This depends on physio-

logical laws, in addition to the morphological law referred to above. Where polyandry prevails *must* becomes *can* in the case of *m*.

The expression *mm* between two relationship symbols is equivalent to *m*, and *ff* to *f*; but *mf* or *fm* imply a contradiction.

A compound relationship due to the coexistence of several elementary relationships, may be denoted by writing the relationships after one another, the order being immaterial—for example, a child of the man A, who is also a child of the woman B, by $c_m A c_f B$. When B is the same as A, a dot may be used to replace the first A; for example,

$$c_m c^{-1} A c_f c^{-1} A$$

(that is, a child of the father of A who is also a child of the mother of A) may be denoted by

$$c_m c^{-1} . c_f c^{-1} A.$$

It is convenient to have a special notation for a compound relationship which consists of a number of specific forms of one general relationship. This may be done by placing a vinculum over the general relationship, and by means of an index expressing the number of times the general relationship occurs. For example, $c^{\overline{1-1}^2}$ denotes full brother or full sister; $c^{\overline{1-1}^1}$ denotes half brother or half sister; and $c^{\overline{1-1}^0}$ is the appropriate expression for a non-brother or non-sister. According to the laws of this country we may have $c^{\overline{2-2}^4}$, $c^{\overline{2-2}^3}$, $c^{\overline{2-2}^2}$, $c^{\overline{2-2}^1}$, $c^{\overline{2-2}^0}$; that is, first cousin in four ways, first cousin in three ways, first cousin in two ways, first cousin in one way, and first cousin in no way. When distinctions of sex are introduced, the relationship of *first cousin* may have any one of 192 significations.

The notation for a compound term enables us to express the universal law that a person cannot be his or her own descendant. It is

$$1 . c^n A = 0$$

whoever A be, and *n* being any number from 1 upwards. The reciprocal aspect of the law is that

$$c^{-n} . 1A = 0;$$

that is, no person can be his or her own ancestor; while the most general statement of the law is

$$\Sigma c^m . c^{-n} A = 0,$$

provided *m* and *n* are not both 0.

In a similar manner it enables us to express the consequences

of the laws of marriage of a given nation. For example,

$$\Sigma c_m \cdot c_f c_m^{2-1} A = 0,$$

A being any British subject. This means that the children of any man A who are also children of any niece or daughter of the man A are none.

The principles of the English Law of Marriage are expressed as follows:—

I. For the direct line,

$$\Sigma c_m \cdot c_f (c^p + c^{-p})_m A = 0,$$

where p may be any number. This means that the children of any man A must not be the children of any female descendant or ancestress of A.

II. For the collateral line,

$$\Sigma c_m \cdot c_f (c^{2-1} + c^{1-1} + c^{1-2})_m A = 0.$$

This expresses that the children of any man A must not be the children of any niece, sister, or aunt of A.

III. For the direct line with one affinity,

$$\Sigma c_m \cdot c_f \{ (c^p + c^{-p}) c^{-1+1} + c^{-1+1} (c^p + c^{-p}) \}_m A = 0.$$

This means that the children of any man A must not be the children of any female descendant or ancestress of any wife of A, nor the children of any wife of any descendant or ancestor of A.

IV. For the collateral line with one affinity,

$$\Sigma c_m \cdot c_f \{ (c^{2-1} + c^{1-1} + c^{1-2}) c^{-1+1} + c^{-1+1} (c^{2-1} + c^{1-1} + c^{1-2}) \}_m A = 0.$$

This means that the children of any man A must not be the children of any niece, sister, or aunt of any wife of A, nor the children of any wife of any nephew, brother, or uncle of A.

As the result of my investigations, I am led to consider the Analysis of Relationships a special branch of the Algebra of Logic, that the processes of that method as described in my work 'Algebra of Logic' apply to this subject, and that we have special laws in addition, such as those mentioned above. In 1879 Dr. Halsted expressed his opinion* that little cosmos had not yet been brought out of the chaos of Relationships; whether I have been successful in reducing a small part more approximately to order must be left to the reader of this article and of the original papers to judge.

I append some illustrations.

(1) *To express the relationship of Queen Victoria to William*

* Journ. of. Spec. Philos. vol. xiii. p. 107.

the Conqueror. For our present purpose, the symbol s may be used to denote m and d to denote c .

$$\text{Queen Victoria} = i d s_4 (s_1)^3 \text{ George I. ;}$$

that is, Queen Victoria is the only daughter of the fourth son of the first son of the first son of George the First.

$$\text{George I.} = s_1 d_4 i d \text{ James I.}$$

$$\text{James I.} = i s \left\{ i d s_1 \right\}_{s_1 i d} d_1 \text{ (Henry VII. + Elizabeth of York).}$$

$$\text{Henry VII.} = s i d s^2 s_3 \text{ Edward III.}$$

$$\text{Elizabeth of York} = d_1 s_1 s \left\{ s s_4 \right\}_{d s_1 i d s_2} \text{ Edward III.}$$

$$\text{Edward III.} = (s_1)^4 s_5 s_1 i d s_4 \text{ William the Conqueror.}$$

$$\therefore \text{Queen Victoria} =$$

$$i d s_4 s_1^4 d_4 i d i s \left\{ i d s_1 \right\}_{s_1 i d} d_1 \left\{ s i d s^2 s_3 \right\}_{d_1 s_1 s \left\{ s s_4 \right\}_{d s_1 i d s_2}} s_1^4 s_5 s_1 i d s_4 \text{ William the Conqueror.}$$

If we wish to express the general nature of this relationship, we have

$$\text{Queen Victoria} = c^{25^4} . c^{27^2} \text{ William I. ;}$$

that is, Queen Victoria is a descendant of the fourth degree and twenty-fifth order, and also a descendant of the second degree and twenty-seventh order of William the First. This example indicates that the notation of this analysis may prove useful in condensing and rendering more exact the information supplied in *Peerages*.

(2) Given that Stephen was a grandson of William I., and Henry II. a greatgrandson of William I., what follows as to the relationship of Henry to Stephen?

$$S = {}_m c^2 {}_m W, \quad H = {}_m c^3 {}_m W,$$

$$\therefore H = {}_m c^3 {}_m c^{-2} {}_m S;$$

which can reduce to $c^{2-1} S$ and $c S$. Hence Henry was either a son of a first cousin of Stephen, or a nephew of Stephen, or a son of Stephen.

(3) "Sister or brother have I none;

But that man's father was my father's son."

Let A denote the speaker and X the person referred to.

Then the first line gives us the equation

$$\Sigma c c^{-1} A = 2A; \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

and the second line gives the equation

$${}_m c^{-1} {}_m X = {}_m c {}_m c^{-1} A. \quad . \quad . \quad . \quad . \quad (2)$$

From (1) it follows that

$$\Sigma c {}_m c^{-1} A = A;$$

therefore substituting in (2), we get

$${}_m c^{-1} {}_m X = A;$$

that is, A was the father of the man X.

The Rule for transposing a relationship from one side of an equation to the other is as follows:—*Change the order of the symbols, and change each symbol into its reciprocal, the reciprocal of m being m, and of f being f.* Applying this rule to the above, we get

$$X = {}_m c {}_m A;$$

that is, X was a son of the man A.

(4) A lady, on being asked about a photograph in her album, replied:—You know that I have no daughters; well, that person's daughter's son was the father of a grandchild of mine.

Let A denote the lady and X the person. Then

$$\Sigma {}_f c {}_f A = 0, \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

and

$${}_m c {}_f c X = {}_m c^{-1} c {}_f A. \quad . \quad . \quad . \quad . \quad . \quad (2)$$

From (1),

$$\Sigma c {}_f A = \Sigma {}_m c {}_f A;$$

and from (2), by the above rule of transposition,

$$X = c^{-1} {}_f c^{-1} {}_m c^{-1} c {}_f A;$$

therefore from (1),

$$X = c^{-1} {}_f c^{-1} {}_m c^{-1} c {}_f A.$$

But ${}_m c^{-1} c {}_m = 1$ always (p. 440); therefore

$$X = c^{-1} {}_f c^{-1} {}_m c {}_f A.$$

Also ${}_f c^{-1} c {}_f = 1$ always; therefore

$$X = c^{-1} {}_f A;$$

that is, X was either the father or the mother of the lady. Thus the answer given to the question was perfectly determinate.

(5) Deduce all the logical consequences of the law which prohibits a man marrying a sister of his deceased wife.

The primary equation is

$$\Sigma 1 \cdot {}_m c^{-1} c_f c c^{-1} c^{-1} c_m A = 0,$$

where A may be any inhabitant of the British Islands. It expresses that a man cannot (lawfully) be the husband of a sister of his wife. The Rule for transforming an equation which is true for any A is as follows:—

To transform a universal equation which has a compound term of the second degree equated to 0, suppose all the symbols brought to one factor in accordance with the rule of transposition given above (p. 444); then removing a symbol from the front gives one derived equation, and removing a symbol from the end gives another derived equation. Transform each of these two in a similar manner, then each of their four resultants, and so on until all the terms have been brought to the other factor. The total of these derived equations is the total number of transformations of the given universal equation.

Applied to the above equation, transposition of the first symbol gives

$$\Sigma c_m \cdot c_f c c^{-1} c^{-1} c_m A = 0;$$

that is, a child of a man cannot be the child of a sister of a wife of the man. Transposition of the last symbol gives

$$\Sigma {}_m c^{-1} \cdot {}_m c^{-1} c_f c c^{-1} c^{-1} A = 0.$$

that is, the father of a person cannot be the husband of a sister of the mother of that person. By continuing the process other 19 forms (different from one another) are obtained, which respectively express that—

- A wife of a man cannot be the sister of a wife of the man.
- A child of the father of a person cannot be the child of a sister of the mother of the person.
- A husband of a woman cannot be the husband of a sister of the woman.
- A parent of a wife of a man cannot be the parent of another wife of the man.
- A step-mother of a person cannot be the sister of the mother of the person.
- A step-child of a woman cannot be the child of a sister of the woman.
- A son-in-law of a person cannot be the husband of another daughter of the person.
- A parent of a step-mother of a person cannot be the parent of the mother of the person.
- A wife of a husband of a woman cannot be the sister of the woman.
- A child of a husband of a daughter of a person cannot be the child of another daughter of the person.
- A sister of a step-mother of a person cannot be the mother of the person.
- A parent of another wife of a husband of a woman cannot be the parent of the woman.

Another wife of a son-in-law of a person cannot be the daughter of the person.

A step-child of a sister of a woman cannot be the child of the woman.

A child of a sister of a step-mother of a person cannot be the person.

A sister of a wife of a husband of a woman cannot be the woman.

A parent of another wife of a son-in-law of a person cannot be the person.

A wife of a husband of a sister of a woman cannot be the woman.

A child of a husband of a sister of the mother of a person cannot be the person.

LIX. *On an Electrochemical Method of Investigating the Field of Electrolytic Action.* By ALFRED TRIBE*.

THE electrochemical method of investigating the field of electrolytic action has for its basis new facts, the nature of which I propose to set forth in the first part of this communication, reserving the second part for the description and discussion of the results which have recently accrued from its application.

The Method.

When a rectangular plate of metal unconnected with the battery is placed lengthwise in an electrolyte undergoing electrolysis, the plate does work identical in kind with that being done by both electrodes. The electropositive ion of the electrolyte separates and distributes itself on a portion of the plate nearer the + electrode, and the electronegative ion on another part of the plate nearer the - electrode. The respective boundaries of these ions are sharply defined, and the intermedial space free from either ion.

When an aqueous solution of copper sulphate is electrolyzed with silver electrodes, copper of course separates on the - electrode; but more or less of the dark-grey or black silver peroxide forms on the + electrode. $\frac{1}{100}$ of a weber in one minute produces, in fact, a sensible separation of copper and a sensible formation of silver peroxide on a silver plate 34 millim. \times 7 millim.

It follows, therefore, that a silver plate placed in a solution of copper sulphate, under the conditions named in the first paragraph, should have copper deposited on that part which may be supposed to receive - electrification, and silver peroxide on that which receives the positive. Such is the case.

The registration of any set of electrifications in this way by the ions of electrolytes need take only a few minutes, the minimum time being determined by the dimensions of the analyzer †, strength of electrolyte, and available current.

* Communicated by the Author, having been read at the Meeting of the French Association, Algiers, April 1881.

† The rectangular silver plate is named, for convenience, the *analyzing plate*, or, in brief, the *analyzer*.