



176. The Fundamental Exponential Limit

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for every p , $|s_n - s_{n+p}| < \epsilon'$. In these two inequalities s and m may be chosen at will. Give them the values s_n and n , which are definite. Moreover, p may be chosen at will. Give it the value q , which is made definite by the definite choices of s and m . We thus get two inequalities,

$$|s_n - s_{n+q}| \nless \epsilon' \text{ and } |s_n - s_{n+q}| < \epsilon',$$

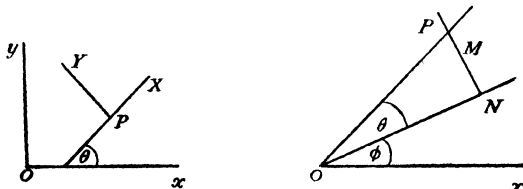
which are inconsistent. We have then supposed an impossibility.

E. B. ELLIOTT.

175. [K. 20. a.]. *The Addition Formulae for Cosine and Sine.*

If PX makes an angle θ with Ox , then the projection of PX on Ox is $PX \cos \theta$. Hence if Oy , PY are obtained from Ox , PX by a turn through a right angle in the positive direction, it follows that the projection of PY on Ox is $PY \cos(\theta + \frac{\pi}{2})$, and that the projection of PX on Oy is $PX \cos(\theta - \frac{\pi}{2})$.

Now let ON make an angle ϕ with Ox and OP an angle θ with ON . Let NM be the direction obtained from ON by a turn through a right angle in



the positive direction. Let NM cut OP in P and take OP as the unit of length. Then the projection of OP on Ox is $\cos(\theta + \phi)$. But this projection is also the sum of the projections on Ox of ON , NP , that is of $\cos \theta$, $\cos(\theta - \frac{\pi}{2})$. Now the projecting factors are $\cos \phi$, $\cos(\phi + \frac{\pi}{2})$. Hence we have

$$\begin{aligned} \cos(\theta + \phi) &= \cos \theta \cos \phi + \cos(\theta - \frac{\pi}{2}) \cos(\phi + \frac{\pi}{2}) \\ &= \cos \theta \cos \phi - \sin \theta \sin \phi. \end{aligned}$$

The proof applies for angles of any size or sign.

Replacing ϕ by $\phi - \frac{\pi}{2}$, we get the addition formula for sine, and then, in the two addition formulae, replacing ϕ by $-\phi$, we get the formulae for the sine and cosine of $\theta - \phi$.

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176. [D. 6. b.]. *The Fundamental Exponential Limit.*

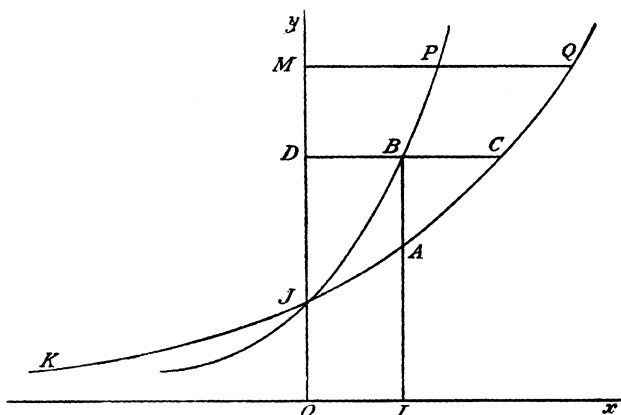
Let the curve KJQ be the graph of $y = a^x$ and let $OI = OJ = 1$, so that $IA = a$. In IA or IA produced take any point B where $IB = b$ and through B draw DBC parallel to Ox cutting Oy in D and the graph in C . Then if $DC = c$, we have $b = a^c$, and therefore $b^x = a^{cx}$. Hence if through any point P on the graph JBP of $y = b^x$ MPQ is drawn parallel to Ox cutting Oy in M and the graph of $y = a^x$ in Q , then $MQ = c \cdot MP$.

Thus from the graph of any one exponential a^x that of any other exponential b^x can be deduced by cutting all the ordinates to y in the proper ratio. Conversely, whatever ratio is used, the graph of some exponential is obtained. By properly choosing the ratio we can therefore make the derived graph have any slope we please at J . There must then be some value of b which gives unit slope at J . Denote this value of b by e , then from the definition of a tangent it follows that

$$\lim_{x \rightarrow 0} (e^x - 1)/x = 1.$$

The differential coefficients of e^x , x^x , $\log x$ are at once found, and if

$$(-1)^n y_n = 1 - \frac{x}{1} + \frac{x^2}{2} - \dots + (-1)^n \frac{x^n}{n} - e^{-x},$$



we deduce that $dy_n/dx = y_{n-1}$, and hence that y_1, y_2, \dots, y_n are all positive if x is positive, so that e^{-x} lies between s_n and s_{n+1} where

$$s_n = 1 - \frac{x}{1} + \frac{x^2}{2} - \dots + (-1)^n \frac{x^n}{n}.$$

Hence as close an approximation as we please can be found for e^{-1} , and therefore also for e . E. J. NANSON.

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