Physics Made Simple

Part I & II

Q & A and Solutions to Problems for Class XI

(as per NCERT Syllabus)

BY

Mrs. P. SHUBHRAJYOTSNA AITHAL M.Sc.(Chem), M.Sc. (Mat. Sc.), M.A. (Eng.) M.Phil. (Chem), (Ph.D.)

&

Dr. P. S. AITHAL M.Sc. (Phys), Ph.D.(Phys), M.I.T.(I.T.), M.Sc. (E-Bus.), M.Tech.(I.T.), Ph.D. (Bus. Mngt.)

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UNITS & WEIGHTAGE

One Paper **Three Hours** **Max Marks: 70**

	Class XI	Weightage
Unit I	Physical World & Measurement	03
Unit II	Kinematics	10
Unit III	Laws of Motion	10
Unit IV	Work, Energy & Power	06
Unit V	Motion of System of particles & Rigid Body	06
Unit VI	Gravitation	05
Unit VII	Properties of Bulk Matter	10
Unit VIII	Thermodynamics	05
Unit IX	Behaviour of Perfect Gas & Kinetic Theory of gases	05
Unit X	Oscillations & Waves	10
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Centre for Science Popularization
Srinivas Global Education & Research
Srinivas Group of Institutions, Mangalore – 575 001
www.srinivasgroup.com

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ABOUT THE CONTRIBUTORS

Mrs. Shubhrajyotsna Aithal is belonging to Mangalore, India, born on 19/11/1970. She has M.Sc. in Material Science from Mangalore University, India, M.Sc. in Chemistry from Kuvempu University, India, and M.Phil. in Chemistry, Vinayaka University, India. Presently she is doing her Ph.D. in the field of Characterization of nonlinear optical materials in Rayalaseema University, India. She has 10 years teaching experience in teaching Chemistry for undergraduate students. Presently she is working as Senior Lecturer in Chemistry at Srinivas College, Pandeshwar, Mangalore, Karnataka State, India. Her research interests are in nonlinear absorption, nonlinear refraction, optical limiting and generation of Phase Conjugated signal in dye doped polymers.
Mrs. Aithal has published 06 papers in refereed journals in the field of characterization of nonlinear optical materials.

Dr. P.S. Aithal is belonging to Udupi, India, born on 04/04/1966. He has M.Sc. in Physics from Mangalore University, India, M.Sc. in E-Business from Manipal University, India, M.Tech. in Information Technology from Karnataka University, India, Ph.D. in Physics from Mangalore University, India, and Ph.D. in Management from Manipal University, India. His major field of study are characterization of nonlinear optical materials, optical solitons, e-commerce and mobile business. He has two years post doctoral research experience at Physical Research Laboratory, Ahmedabad, India and one year post doctoral research experience at CREOL, University of Central Florida, USA, in the field of Characterization of nonlinear optical materials. He has about 22 years teaching experience both at UG and PG level in Electronics, Computer Science and Business management. Currently he is working as PRINCIPAL at Srinivas Institute of Management Studies, Mangalore, India. He has published about 55 research papers in peer reviewed journals and two text books on physics and Electronics for Engineering students. He has research interest in Nonlinear optical absorption, Optical Phase Conjugation, Photorefractive materials, e-business, m-business, ideal business, and nanotechnology business Opportunities. Dr. Aithal is member of World Productivity Council, U.K., member of Strategic Management Foruum, India, member of Photonics Society of India, CUSAT, Cochin, senior member of IEDRC.org, Singapore.

Unit I
Physical World (2H – 2M (1Q – 2M))
(2 Hours Q = 3M) (1M – 1Q, 2M – 1Q)

Syllabus : Physics - scope and excitement; nature of physical laws; Physics, technology and society.

I. Questions based on the Syllabus :

1.1 What is Physics ?
Physics is a branch of natural science, deals with study of matter, properties of various components of matter and interactions among them. Physics is the study of nature and natural phenomena.

1.2 Physics - scope and excitement ?
Basically, in physics, there are two domains of interest : macroscopic and microscopic. The macroscopic domain includes phenomena at the laboratory, terrestrial and astronomical scales. The microscopic domain includes atomic, molecular and nuclear phenomena.

(1) Classical Physics deals mainly with **macroscopic phenomena** and includes subjects like Mechanics, Electrodynamics, Optics and Thermodynamics.
(a) Mechanics founded on Newton's laws of motion and the law of gravitation is concerned with the motion (or equilibrium) of particles, rigid and deformable bodies, and general systems of particles. The propulsion of a rocket by a jet of ejecting gases, propagation of water waves or sound waves in air, the equilibrium of a bent rod under a load, etc., are problems of mechanics.
(b) Electrodynamics deals with electric and magnetic phenomena associated with charged and magnetic bodies. Its basic laws were given by Coulomb, Oersted Ampere and Faraday, and encapsulated by Maxwell in his famous set of equations. The motion of a current-carrying conductor in a magnetic field, the response of a circuit to an ac voltage (signal), the working of an antenna, the propagation of radio waves in the ionosphere, etc., are problems of electrodynamics.
(c) Optics deals with the phenomena involving light. The working of telescopes and microscopes, colours exhibited by thin films, etc., are topics in optics.
(d) Thermodynamics, in contrast to mechanics, does not deal with the motion of bodies as a whole. Rather, it deals with systems in macroscopic equilibrium and is concerned with changes in internal energy, temperature, entropy, etc., of the system through external work and transfer of heat. The efficiency of heat engines and refrigerators, the direction of a physical or chemical process, etc., are problems of interest in thermodynamics.

(2) The microscopic domain of physics deals with the constitution and structure of matter at the minute scales of atoms and nuclei (and even lower scales of length) and their interaction with different probes such as electrons, photons and other elementary particles. **Classical physics** is inadequate to handle this domain and **Quantum Theory** is currently accepted as the proper framework for explaining microscopic phenomena.

1.3 Nature of physical laws :

The physical quantities that remain unchanged in a process are called conserved quantities. Some of the general conservation laws in nature include the laws of conservation of mass, energy, linear momentum, angular momentum, charge, parity, etc. Some conservation laws are true for one fundamental force but not for the other. Conservation laws have a deep connection with symmetries of nature. Symmetries of space and time, and other types of symmetries play a central role in modern theories of fundamental forces in nature.

1.4 Physics, technology and society :

Technology	Scientific principle(s)
Steam engine	Laws of thermodynamics
Nuclear reactor	Controlled nuclear fission
Radio and Television	Generation, propagation and detection of electromagnetic waves
Computers	Digital logic
Lasers	Light amplification by stimulated emission of radiation
Production of ultra high magnetic fields	Superconductivity
Rocket propulsion	Newton's laws of motion
Electric generator	Faraday's laws of electromagnetic induction
Hydroelectric power	Conversion of gravitational potential energy into electrical energy
Aeroplane	Bernoulli's principle in fluid dynamics
Particle accelerators	Motion of charged particles in electromagnetic fields
Sonar	Reflection of ultrasonic waves
Optical fibres	Total internal reflection of light
Non-reflecting coatings	Thin film optical interference
Electron microscope	Wave nature of electrons
Photocell	Photoelectric effect
Fusion test reactor (Tokamak)	Magnetic confinement of plasma
Giant Metrewave Radio Telescope (GMRT)	Detection of cosmic radio waves
Bose-Einstein condensate	Trapping and cooling of atoms by laser beams and magnetic fields.

1.5 Fundamental Forces in Nature & Unification of them :

The four fundamental forces in nature are : Gravitational force, Electromagnetic force, Strong nuclear force, and Weak nuclear force.

(1) **The gravitational force** is the force of mutual attraction between any two objects by virtue of their masses. It is a universal force. Every object experiences this force due to every other object in the universe. All objects on the earth, for example, experience the force of gravity due to the earth. It plays a key role in the large-scale phenomena of the universe, such as formation and evolution of stars, galaxies and galactic clusters.

(2) **Electromagnetic force** is the force between charged particles. In the simpler case when charges are at rest, the force is given by Coulomb's law : attractive for unlike charges and repulsive for like charges. Charges in motion produce magnetic effects and a magnetic field gives rise to a force on a moving charge. Electric and magnetic effects are, in general, inseparable – hence the name electromagnetic force. Like the gravitational force, electromagnetic force acts over large distances and does not need any intervening medium. It is enormously strong compared to gravity. The electric force between two protons, for example, is 10^{36} times the gravitational force between them, for any fixed distance. Gravity is always attractive, while electromagnetic force can be attractive or repulsive.

(3) **The strong nuclear force** binds protons and neutrons in a nucleus. It is evident that without some attractive force, a nucleus will be unstable due to the electric repulsion between its protons. This attractive force cannot be gravitational since force of gravity is negligible compared to the electric force. A new basic force must, therefore, be invoked. The strong nuclear force is the strongest of all fundamental forces, about 100 times the electromagnetic force in strength. It is charge-independent and acts equally between a proton and a proton, a neutron and a neutron, and a proton and a neutron. Its range is, however, extremely small, of about nuclear dimensions (10^{-15m}). It is responsible for the stability of nuclei. The electron, it must be noted, does not experience this force. Protons and neutrons are built out of still more elementary constituents called quarks.

(4) **The weak nuclear force** appears only in certain nuclear processes such as the β-decay of a nucleus. In β-decay, the nucleus emits an electron and an uncharged particle called neutrino. The weak nuclear force is not as weak as the gravitational force, but much weaker than the strong nuclear and electromagnetic forces. The range of weak nuclear force is exceedingly small, of the order of 10^{-16} m.

Unification of different forces/domains in nature is a basic quest in physics. Newton unified terrestrial and celestial domains under a common law of gravitation. Maxwell unified electromagnetism and optics with the discovery that light is an electromagnetic wave. Einstein attempted to unify gravity and electromagnetism but could not succeed in this venture. The electromagnetic and the weak nuclear force have now been unified and are seen as aspects of a single 'electro-weak' force. Attempts have been (and are being) made to unify the electro-weak and the strong force and even to unify the gravitational force with the rest of the fundamental forces.

Table 1.1 : Fundamental forces of nature

Name	Relative strength	Range	Operates among
Gravitational force	10^{-39}	Infinite	All objects in the universe
Weak nuclear force	10^{-13}	Very short, Sub-nuclear size ($\sim 10^{-16}$m)	Some elementary particles, particularly electron and neutrino
Electromagnetic force	10^{-2}	Infinite	Charged particles
Strong nuclear force	1	Short, nuclear size ($\sim 10^{-15}$m)	Nucleons, heavier elementary particles

1.6 Nature of physical laws :

The physical quantities that remain unchanged in a process are called conserved quantities. Some of the general conservation laws in nature include the laws of conservation of mass, energy, linear momentum, angular momentum, charge, parity, etc. are considered to be fundamental laws in physics. Some conservation laws are true for one fundamental force but not for the other. Conservation laws have a deep connection with ***symmetries of nature***. Symmetries of space and time, and other types of symmetries play a central role in modern theories of fundamental forces in nature.

(1) Law of Conservation of Mass : The total mass of the system is conserved.

(2) Law of Conservation of Energy : Energy can neither be created nor destroyed, however, it may change from one form to another.

(3) Law of Conservation of Linear Momentum : If no external force acts on the system, the linear momentum of the system remains constant.

(4) Law of Conservation of Angular momentum : The total angular momentum of the system remains constant when no external torque acts on the system.

(5) Law of Conservation of Charge : The total electrical charge of an isolated system is conserved.

I. VERY SHORT ANSWER QUESTIONS (1 MARK)

1. Name the strongest force in nature. What is its range?

The strong nuclear force is the strongest of all fundamental forces, about 100 times the electromagnetic force in strength. It is charge-independent and acts equally between a proton and a proton, a neutron and a neutron, and a proton and a neutron. Its range is, however, extremely small, of about nuclear dimensions (10^{-15}m).

2. Give two discoveries of Physics used in your daily life.

Radio, Television, Telephone, Computer, Laser, Electricity, Motors,

3. What is the relation between light year and par sec.

1 parsec = 3.26 light year

4. Give the order of magnitude of the following :

(i) size of atom

(ii) size of our galaxy.

Size of atom = 10^{-10} m (b) size of galaxy = 10^{22} m
Order = – 10, Order of magnitude = 22.

5. How many kg make 1 unified atomic mass unit?
1u = 1.66×10^{-27} kg

6. Name same physical quantities that have same dimension.
Work, energy and torque.

7. Name the physical quantities that have dimensional formula $[ML^{-1}T^{-2}]$
Stress, pressure, modulus of elasticity.

8. Give two examples of dimension less variables.
Strain, refractive index.

9. State the number of significant figures in
(i) $0.007m^2$ (ii) 2.64×1024 kg (iii) 0.2370 g cm^{-3} (iv) 0.2300m (v) 86400 (vi) 86400 m
(i) 1, (ii) 3, (iii) 4, (iv) 4, (v) 3, (vi) 5 since it comes from a measurement the last two zeros become significant.

10. Given relative error in the measurement of length is 0.02, what is the percentage error?
2%

11. If a physical quantity is represented by $X = M^a L^b T^{-c}$ and the percentage errors in the measurements of M, L and T are α, β and γ. What will be the percentage error in X.
% error in measurement of X = aα + bβ + cγ

12. A boy recalls the relation for relativistic mass(m) in terms of rest mass (m_0), velocity of particle *v*, but forgets to put the constant *c* (velocity of light). He writes m = $\frac{m_0}{(1-V^2)^{1/2}}$ correct the equation by putting the missing '*c*'.
Since quantities of similar nature can only be added or subtracted, v^2 cannot be subtracted from 1 but v^2/c^2 can be subtracted from 1.
Therefore, m = $\frac{m_0}{(1-V^2)^{1/2}}$

13. Name the technique used in locating. (a) an under water obstacle (b) position of an aeroplane in space.
(a) SONAR → Sound Navigation and Ranging.
(b) RADAR → Radio Detection and Ranging.

14. Deduce dimensional formulae of— (i) Boltzmann's constant (ii) mechanical equivalent of heat.
(i) Boltzmann Constant :

$$k = \frac{\text{Heat}}{\text{Temperature}} \Rightarrow [k] = \frac{ML^2T^{-2}}{K} = M^1L^2T^{-2}K^{-1}$$

(ii)

$$[J] = \left[\frac{Work}{Heat}\right] = \frac{M^1L^2T^{-2}}{M^1L^2T^{-2}} = \left[m^0L^0T^0\right]$$

15. Give examples of dimensional constants and dimensionless constants.
Dimensional Constants : Gravitational constant, plank's constant.
Dimensionless Constants : π, e.

16. Who proposed elliptical orbit of the plants around sun?
Kepler

17. Mention one of the demerits of Newtonian mechanics .
It fails to explain some basic features of atomic phenomenon.

18. What is the result of α- ray scattering experiment conducted by Rutherford ?
It established nuclear model of the atom.

19. Who introduced the concept of anti particle theoretically for the first time ?
Paul Dirac

20. Who confirmed the existence of positron experimentally?
Anderson

21. Mention the phenomenon included in (i) macroscopic domain (ii) microscopic domain
Macroscopic domain includes phenomena at the laboratory, terrestrial and astronomical scales. Microscopic domain includes atomic, molecular and nuclear phenomenon.

22. What are the important topics converted under classical physics?
Mechanics, electrodynamics, optics, thermodynamics.

23. A feather and a stone are dropped from same height. Do they reach the ground at the same time (i) in presence of atmosphere? (ii) in absence of atmosphere?
(i) No (ii) Yes

24. Does the value of acceleration due to gravity depends on the mass of the falling body?
No

25. Discoveries in physics generates new technology . Give an evidence to this.
Wireless communication technology follows the discovery of the basic laws of electricity and magnetism.

26. Who discovered the phenomenon of neutron induced fission of uranium ?
Hahn and Meitner.

27. Mention the important contribution by Archimedes in fluid mechanics.
Principle of buoyancy.

28. Who proposed wave theory of light?

Huygens.

29. Who proposed the universal law of gravity?
Newton.

30. Who proposed the laws of electromagnetic induction?
Faraday.

31. Light is an electromagnetic wave – who proposed this?
Maxwell

32. Who discovered X – ray?
Roentgen

33. Who discovered electrons?
J J Thomson

34. Who discovered radium and polonium?
Marie Curie

35. Who was awarded a Nobel prize for the explanation of photo electric effect?
Einstein

36. Who won the Nobel prize in the field of inelastic scattering of light by molecules?
C V Raman

37. Who is the founder of controlled nuclear fission reaction? (nuclear power reactor)
Fermi

38. Who discovered neutron?
Chadwick.

39. Mention the fundamental forces in nature.
(i) Gravitational force
(ii) Weak nuclear force
(iii) Electromagnetic force
(iv) Strong nuclear force

40. Name the (i) strongest (ii) weakest fundamental force in nature
(i) Stronger nuclear force
(ii) Gravitational force

41. Does the laws of nature change with time?
No

42. Name the elementary particle emitted during β- decay along with the electron.
Anti Neutrino.

43. Does the law of gravitation valid on the surface of moon?
Yes

45. The value of acceleration due to gravity is different for different planets. Is it the violation of the law of gravitation?
No

46. Name the principle behind the uplift of aircraft (aeroplane)
Bernoulli’s principle

47. What is the principle behind the working of electric generator.
Faraday’s law of electromagnetic induction.

EXERCISES

1.1 Some of the most profound statements on the nature of science have come from Albert Einstein, one of the greatest scientists of all time. What do you think did Einstein mean when he said : "The most incomprehensible thing about the world is that it is comprehensible"?
Ans : The whole of physical world is complex in nature. The biological world has its own complexities. Moreover, vastly different orders of magnitudes are involved in space, time and mass. Inspite of all this, all the physical phenomena can be expressed in terms of a few basic laws. When viewed in this context, Einstein's statement becomes very meaningful.

1.2 "Every great physical theory starts as a heresy and ends as a dogma". Give some examples from the history of science of the validity of this incisive remark.
Ans : Dogma is an established opinion which is questioned by only a few. On the other hand, anything against the established belief, known as heresay, does cause a few ripples in the mind of the intelligent. Thomas Young's wave theory of light started as a heresay and finally ended as dogma when Einstein and others replaced it by quantum theory of light.

1.3 "Politics is the art of the possible". Similarly, "Science is the art of the soluble". Explain this beautiful aphorism on the nature and practice of science.
Ans : Science is a systematic study of observations in nature. A scientist patiently analyses these observations and comes out with certain laws. As an illustration, Tycho Brahe worked for twenty long years to make observations on planetary motions. It is from this huge reservoir of observations that J. Kepler formulated his three famous laws of planetary motion. Thus, science is the art of the soluble just as politics is the art of the possible.

1.4 Though India now has a large base in science and technology, which is fast expanding, it is still a long way from realising its potential of becoming a world leader in science.
Name some important factors, which in your view have hindered the advancement of science in India.

Ans. One of the main factors which has hindered India's march towards becoming a world leader in science and technology is that the young scientists and technologists are denied the academic freedom which is so very necessary for making advances in science and technology. The management of science education in our country is bureaucratic. In addition to this, there is practically no co-ordination between the researchers and the industrialists. The industrialists are the actual consumers of new research and technology. The industrialist of this country has little confidence in the ability of the Indian scientists. He prefers to import technology from advanced countries. The indigenous technology does not find favour with the Indian scientist. Another important factor that has hindered India's progress in science and technology is 'brain drain'. There has been a large scale migration of scientists and technologists.

1.5 No physicist has ever "seen" an electron. Yet, all physicists believe in the existence of electrons. An intelligent but superstitious man advances this analogy to argue that 'ghosts' exist even though no one has 'seen' one. How will you refute his argument ?

Ans. Many phenomena which depend upon the existence of atom have been predicted and actually observed in everyday life. There is no phenomenon which can be explained on the basis that ghosts exist though they are not seen. So, obviously, the comparison between two situations does not make any sense.

1.6 The shells of crabs found around a particular coastal location in Japan seem mostly to resemble the legendary face of a Samurai. Given below are two explanations of this observed fact. Which of these strikes you as a scientific explanation ?
(a) A tragic sea accident several centuries ago drowned a young Samurai. As a tribute to his bravery, nature through its inscrutable ways immortalised his face by imprinting it on the crab shells in that area.
(b) After the sea tragedy, fishermen in that area, in a gesture of honour to their dead hero, let free any crab shell caught by them which accidentally had a shape resembling the face of a Samurai. Consequently, the particular shape of the crab shell survived longer and therefore in course of time the shape was genetically propagated. This is an example of evolution by artificial selection.
Ans : (b) is the scientific explanation.

1.7 The industrial revolution in England and Western Europe more than two centuries ago was triggered by some key scientific and technological advances. What were these advances ?

Ans. Prior to 1750 AD when Industrial revolution happened, simple tools and machines were used. But industrial revolution brought new machinery. Some of the outstanding contributions of the industrial revolution were (*i*) steam engine (*ii*) blast furnace which converts low grade iron into steel (*iii*) cotton gin which separates the seeds from cotton three hundred times faster than by hand (*iv*) power loom etc.

1.8 It is often said that the world is witnessing now a second industrial revolution, which will transform the society as radically as did the first. List some key contemporary areas of science and technology, which are responsible for this revolution.

Ans. The key areas which will transform radically the present are (*i*) super fast computers (*ii*) biotechnology (*iii*) development of superconducting materials at room temperature etc.

1.9 Write in about 1000 words a fiction piece based on your speculation on the science and technology of the twenty-second century.

Ans. Let us imagine a spaceship moving towards a distant star, 500 light years away. Suppose this is propelled by current fed into the electric motor consisting of superconducting wires. In space, suppose there is a particular region which has such a high temperature that destroys the superconducting property of the electric wires of the motor. At this stage, another spaceship filled with matter and anti-matter comes to the rescue of the first ship. And the first ship continues its onward journey.

1.10 Attempt to formulate your 'moral' views on the practice of science. Imagine yourself stumbling upon a discovery, which has great academic interest but is certain to have nothing but dangerous consequences for the human society. How, if at all, will you resolve your dilemma ?

Ans. A scientist aims at truth. A scientific discovery reveals a truth of nature. So, any discovery, good or bad for mankind, must be made public. A discovery which appears dangerous today may become useful to the mankind sometimes later. In order to prevent misuse of scientific technology, we must build up a strong public opinion. Scientists should in fact take up two roles—to discover truth and to prevent its misuse.

1.11 Science, like any knowledge, can be put to good or bad use, depending on the user. Given below are some of the applications of science. Formulate your views on whether the particular application is good, bad or something that cannot be so clearly categorised :
(a) Mass vaccination against small pox to curb and finally eradicate this disease from the population. (This has already been successfully done in India). Good

(b) Television for eradication of illiteracy and for mass communication of news and ideas. Good
(c) Prenatal sex determination - Bad
(d) Computers for increase in work efficiency – Good
(e) Putting artificial satellites into orbits around the Earth - Good
(f) Development of nuclear weapons – Bad
(g) Development of new and powerful techniques of chemical and biological warfare. - Bad
(h) Purification of water for drinking – Good
(i) Plastic surgery – Good
(j) Cloning – Can not say.

1.12 India has had a long and unbroken tradition of great scholarship — in mathematics, astronomy, linguistics, logic and ethics. Yet, in parallel with this, several superstitious and obscurantistic attitudes and practices flourished in our society and unfortunately continue even today — among many educated people too. How will you use your knowledge of science to develop strategies to counter these attitudes ?
I will check each and every superstitious and obscurantistic attitudes and practices with scientific base and spirit.

1.13 Though the law gives women equal status in India, many people hold unscientific views on a woman's innate nature, capacity and intelligence, and in practice give them a secondary status and role. Demolish this view using scientific arguments, and by quoting examples of great women in science and other spheres; and persuade yourself and others that, given equal opportunity, women are on par with men.

Ans. **The nutrition content of pre-natal and post-natal diet contributes a lot towards the development of human mind. If equal opportunities are afforded to both men and women, then the female mind will be as efficient as male mind.**

1.14 "It is more important to have beauty in the equations of physics than to have them agree with experiments". The great British physicist P. A. M. Dirac held this view. Criticize this statement. Look out for some equations and results in this book which strike you as beautiful.

Ans. There is no contradiction in the given statement. A basic equation of Physics which agrees with experiment must necessarily be both simple and beautiful. The equation $E = mc^2$ is one example of a beautiful equation.

Multiple Choice Questions :

1. Sky appears to be blue in clear atmosphere due to lights ______
(a) Scattering
(b) Polarization
(c) Diffraction
(d) Dispersion [AIIMS 1999] Ans : (a)

2. The number of electrons for one coulomb of charge are ______
(a) 6.23×10^{24}
(b) 6.23×10^{22}
(c) 6.23×10^{18}
(d) 6.23×10^{20} [AIIMS 1999] Ans : (c)

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CH 2

Units & Measurement

(04 Hours, 03 Marks (1Q-1M, 1Q-2M))

Syllabus : Need for measurement: Units of measurement; systems of units; SI units, fundamental and derived units. Length, mass and time measurements; accuracy and precision of measuring instruments; errors in measurement; significant figures. Dimensions of physical quantities, dimensional analysis and its applications.

2.1 Need for measurement:

Measurement is needed to represent the magnitude of a physical quantity. By measurement, we can find out how many times a standard amount of that quantity is present in the quantity being measured.

2.2 Units of measurement:

Measurement of any physical quantity involves comparison with a certain basic, arbitrarily chosen, internationally accepted reference standard called unit. The number so obtained is known as the magnitude of the physical quantity.
The unit of a physical quantity is a definite and convenient amount of that quantity.
Physical quantity = numerical measure x unit
Types of Units : (1) Fundamental units, (2) Derived units, (3) Supplementary units and (4) Practical units.

2.3 Systems of units; SI units :

The units for the fundamental or base quantities are called **fundamental or base units**. The units of all other physical quantities can be expressed as combinations of the base units. Such units obtained for the derived quantities are called **derived units**. A complete set of these units, both the base units and derived units, is known as the **system of units**.
The followings are four main systems of units in Physics.

(1) MKS (Metre, Kilogram, Second) system
(2) CGS (Centimetre, Gram, Second) system
(3) FPS (Foot, Pound, Second) system
(4) SI (International System of units)

• In CGS system, base units for length, mass and time were centimetre, gram and second respectively.
• In FPS system, base units for length, mass and time were foot, pound and second respectively.
• In MKS system, base units for length, mass and time were metre, kilogram and second respectively.

The extended MKS system called the International System of Units (SI) based on seven base units is at present internationally accepted unit system and is widely used throughout the world. The SI units are used in all physical measurements, for both the base quantities and the derived quantities obtained from them. Certain derived units are expressed by means of SI units with special names (such as joule, newton, watt, etc).

The SI units have well defined and internationally accepted unit symbols (such as m for metre, kg for kilogram, s for second, A for ampere, N for newton etc.).
Of unit is applicable to the whole of Physics, i.e.,
Out of four systems of units, the first three are applicable only to one branch of Physics called Mechanics and not for the whole of Physics, where as SI system of unit is applicable to the whole of Physics, i.e., the units of all physical quantities can be obtained in SI system of units.

Coherent System : A system of unit is said to be coherent if the units of all physical quantities can be obtained by multiplying or dividing the fundamental units of the system without involving any numerical value.

2.4 Fundamental and derived units.

Certain physical quantities have been chosen as fundamental or base quantities (such as length, mass, time, electric current, thermodynamic temperature, amount of substance, and luminous intensity). Each base quantity is defined in terms of a certain basic, arbitrarily chosen but properly standardised reference standard called unit (such as metre, kilogram, second, ampere, kelvin, mole and candela). The units for the fundamental or base quantities are called fundamental or base units.

Certain derived units are expressed by means of SI units with special names (such as joule, newton, watt, etc).

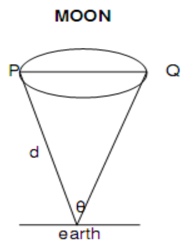
2.5 Length, mass and time measurements;

(1) Measurement of Length :

A metre scale is used for lengths from 10^{-3} m to 10^{2} m. A vernier callipers is used for lengths to an accuracy of 10^{-4} m. A screw gauge and a spherometer can be used to measure lengths as less as to 10^{-5} m. To measure lengths beyond these ranges, we make use of some special indirect methods.
Large distances such as the distance of a planet or a star from the earth cannot be measured directly with a metre scale. An important method in such cases is the **parallax method.**

Let 'D' be the diameter of the moon, when moon is observed from a place 'E' on earth, let 'θ' be the angle made by two diametrically opposite ends P and Q of the moon called parallax angle. If 'd' is the distance of moon from earth , then

$\theta = \frac{PQ}{d} = \frac{D}{d}$ and hence $d = \frac{D}{\theta}$



Using this relation the size of moon can be determined.

Example 2.1 Calculate the angle of (a) 1° (degree) (b) 1′ (minute of arc or arcmin) and (c) 1″ (second of arc or arc second) in radians. Use 3600=2π rad, 1^0 = 60′ and 1′ = 60 ″

Answer (a) We have $360^0 = 2\pi$ rad, $1^0 = (\pi/180)$ rad $= 1.745\times10^{-2}$ rad
(b) $1^0 = 60' = 1.745\times10^{-2}$ rad, $1' = 2.908\times10^{-4}$ rad ; 2.91×10^{-4} rad
(c) $1' = 60'' = 2.908\times10^{-4}$ rad, $1'' = 4.847\times10^{-6}$ rad ; 4.85×10^{-6} rad

Example 2.3 The moon is observed from two diametrically opposite points A and B on Earth. The angle θ subtended at the moon by the two directions of observation is 1^{O} 54′. Given the diameter of the Earth to be about 1.276 × 10^7 m, compute the distance of the moon from the Earth.

Answer : We have θ = 1° 54′ = 114′ = (114 x 60) ″ × (4.85 x 10^{-6}) rad = 3.32 × 10^{-2} rad, since 1″ =4.85×10^{-6} rad., Also b = AB =1.276 ×10^7 m. Hence from Eq. (2.1), we have the earth-moon distance, D = b /θ

$$= \frac{1.276\times10^{7}}{3.32\times10^{-2}}$$

= 3.84 × 10^8 m

(2) Measurement of Mass :

The SI unit of mass is kilogram (kg). The prototypes of the International standard kilogram supplied by the International Bureau of Weights and Measures (BIPM) is available in India at the National Physical Laboratory (NPL), New Delhi.

While dealing with atoms and molecules, the kilogram is an inconvenient unit. In this case, a unit of mass, called the unified atomic mass unit (u), which has been established for expressing the mass of atoms as 1 unified atomic mass unit = 1u = (1/12) of the mass of an atom of carbon-12 isotope ($^{12}_{6}C$) including the mass of electrons = 1.66 × 10^{-27} kg.

Mass of commonly available objects can be determined by a **common balance** like the one used in a grocery shop. Large masses in the universe like planets, stars, etc., based on Newton's law of gravitation can be measured by using **gravitational method**. For measurement of small masses of atomic/sub-atomic particles etc., we make use of **mass spectrograph**.

(3) Measurement of Time :

The unit of time – one second is based on atomic standard of time, which is based on the periodic vibrations produced in a cesium atom called cesium atomic clock.

In the cesium atomic clock, the second is taken as the time needed for 9,192,631,770 vibrations of the radiation corresponding to the transition between the two hyperfine levels of the ground state of cesium-133 atom.

2.6 Accuracy and precision of measuring instruments :

Measurement is the foundation of all experimental science and technology. The result of every measurement by any measuring instrument contains some uncertainty. This uncertainty is called error. Every calculated quantity which is based on measured values, also has an error.

The accuracy of a measurement is a measure of how close the measured value is to the true value of the quantity. Precision tells us to what resolution or limit the quantity is measured.

The accuracy in measurement may depend on several factors, including the limit or the resolution of the measuring instrument.

Problems with *accuracy* are due to errors. The *precision* describes the limitation of the measuring instrument.

2.7 Errors in measurement :

Difference between measured value and true value of a quantity represents error of measurement. Thus every measurement is approximate due to errors in measurement. In general, the errors in measurement can be broadly classified as (a) systematic errors and (b) random errors.

(1) Systematic errors :

The systematic errors are those errors that tend to be in one direction, either positive or negative. Some of the sources of systematic errors are :

(a) **Instrumental** errors that arise from the errors due to imperfect design or calibration of the measuring instrument, zero error in the instrument, etc.

(b) **Imperfection in experimental technique or procedure** To determine the temperature of a human body, a thermometer placed under the armpit will always give a temperature lower than the actual value of the body temperature.

(c) **Personal errors** that arise due to an individual's bias, lack of proper setting of the apparatus or individual's carelessness in taking observations without observing proper precautions, etc.

(2) Random errors :

The random errors are those errors, which occur irregularly and hence are random with respect to sign and size. These can arise due to random and unpredictable fluctuations in experimental conditions (e.g. unpredictable fluctuations in temperature, voltage supply, mechanical vibrations of experimental set-ups, etc), personal (unbiased) errors by the observer taking readings, etc.

(3) Least count error :

The smallest value that can be measured by the measuring instrument is called its least count. All the readings or measured values are good only up to this value. The least count error is the error associated with the resolution of the instrument. Least count error belongs to the category of random errors but within a limited size; it occurs with both systematic and random errors. Using instruments of higher precision, improving experimental techniques, etc., we can reduce the least count error.

Absolute Error, Relative Error and Percentage Error :

The magnitude of the difference between the true value of the quantity and the individual measurement value is called the **absolute error** of the measurement.

The relative error is the ratio of the mean absolute error $\Delta\alpha_{mean}$ to the mean value α_{mean} of the quantity measured. Relative error = $\Delta\alpha_{mean}/\alpha_{mean}$

When the relative error is expressed in per cent, it is called the percentage error ($\delta\alpha$).

$$\delta a = (\Delta a_{mean}/a_{mean}) \times 100\%$$

Combination of Errors :

(a) Error of a sum or a difference :

When two quantities are added or subtracted, the absolute error in the final result is the sum of the absolute errors in the individual quantities.

(b) Error of a product or a quotient :

When two quantities are multiplied or divided, the relative error in the result is the sum of the relative errors in the multipliers.

(c) Error in case of a measured quantity raised to a power :
The relative error in a physical quantity raised to the power k is the k times the relative error in the individual quantity.

Formulae in Errors :

(i) Mean of n measurements

$$a_{mean} = \frac{a_1 + a_2 + a_3 + ...a_n}{n} \qquad = \frac{1}{n}\sum_{i=1}^{n} a_i$$

(ii) Mean absolute error

$[|\Delta a_1| + |\Delta a_2| + |\Delta a_3|$

$$\Delta a_{mean} = \frac{[|\Delta a_1| + |\Delta a_2| + |\Delta a_3 + ... + |\Delta a_n|]}{n} \qquad = \frac{\sum_{l=1}^{n}|\Delta a_1|}{n}$$

(iii) Relative error =

$$= \frac{\Delta a_{mean}}{a_{mean}}$$

(iv) Percentage error =

$$= \frac{\Delta a_{mean}}{a_{mean}} \times 100\%$$

The errors are communicated in different mathematical operations as detailed below :
if $\pm\,\Delta a$, $\pm\,\Delta b$ and $\pm\,\Delta x$ are absolute errors in a, b and x respectively, then

(i) for $x = (a + b)$, $\quad \Delta x = \pm\,[\Delta a + \Delta b]$

(ii) for $x = (a - b)$, $\quad \Delta x = \pm\,[\Delta a + \Delta b]$

(iii) for $x = a \times b$, $\quad \dfrac{\Delta x}{x} = \pm\left[\dfrac{\Delta a}{a} + \dfrac{\Delta b}{b}\right]$

(iv) for $x = a/b$, $\quad \dfrac{\Delta x}{x} = \pm\left[\dfrac{\Delta a}{a} + \dfrac{\Delta b}{b}\right]$

(v) for $x = \dfrac{a^n b^m}{c^p}$, $\quad \dfrac{\Delta x}{x} = \pm\left[n\dfrac{\Delta a}{a} + m\dfrac{\Delta b}{b} + p\dfrac{\Delta c}{c}\right]$

2.8 Significant Figures :

Significant Figures : In the measured value of a physical quantity, the digits about the correctness of which we are sure plus the last digit which is doubtful, are called the significant figures. For counting significant figures rules are as :

1. All the non-zero digits are significant. In 2.738 the number of significant figures is 4.

2. All the zeros between two non-zero digits are significant, no matter where the decimal point is, if at all. As examples 209 and 3.002 have 3 and 4 significant figures respectively.
3. If the measurement number is less than 1, the zero(s) on the right of decimal point and to the left of the first non-zero digit are nonsignificant. In 0.00807, first three underlined zeros are non-significant and the number of significant figures is only 3.
4. The terminal or trailing zero(s) in a number without a decimal point are not significant. Thus, 12.3 m = 1230 cm = 12300 mm has only 3 significant figures.
5. The trailing zero(s) in number with a decimal point are significant. Thus, 3.800 kg has 4 significant figures.
6. A choice of change of units does not change the number of significant digits or figures in a measurement.

In any mathematical operation involving addition, subtraction, decimal places in the result will correspond to lowest number of decimal places in any of the numbers involved.
In a mathematical operation like multiplication and division, number of significant figures in the product or in the quotient will correspond to the smallest number of significant figures in any of the numbers involved.

2.9 Dimensions of physical quantities :

The dimensions of a physical quantity are the powers to which the fundamental (base) quantities are raised to represent that quantity.

Dimensionless Physical Quantities : Angle, solid angle, relative density, specific gravity, strain, Poisson's ratio, Reynold's number, all trigonometric ratios, refractive index, mechanical efficiency, relative permittivity, dielectric constant, relative permeability, electric susceptibility, magnetic susceptibility.

The **three main uses** of dimensional analysis are :
(i) **Conversion of one system of units into another for which we use**

$$n_2 = n_1 \left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c$$

where M1, L1, T1 are fundamental units on one system; M2, L2, T2 are fundamental units on the other system; *a, b, c* are the dimensions of the quantity in mass, length and time; *n*1 is numerical value of the quantity in one system and *n*2 is its numerical value in the other system.
(ii) Checking the dimensional correctness of a given physical relation.
(iii) Derivation of formulae.

Principle of Homogeneity of Dimensions : According to this principle, a correct dimensional equation must be homogeneous, *i.e.*, dimensions of all the terms in a physical expression must be same LHS = RHS

2.10 Dimensional analysis and its applications.

Dimensional analysis guide the description of physical behaviour is of basic importance as only those physical quantities can be added or subtracted which have the same dimensions. A thorough understanding of dimensional analysis helps us in deducing certain relations among different physical quantities and checking the derivation, accuracy and dimensional consistency or homogeneity of various mathematical expressions. When magnitudes of two or more physical quantities are multiplied, their units should be treated in the same manner as ordinary algebraic symbols. We can cancel identical units in the numerator and denominator. The same is true for dimensions of a physical quantity. Similarly, physical quantities represented by symbols on both sides of a mathematical equation must have the same dimensions. Some of the applications are :

1. Checking the Dimensional Consistency of Equations : The magnitudes of physical quantities may be added together or subtracted from one another only if they have the same dimensions. In other words, we can add or subtract similar physical quantities. Thus, velocity cannot be added to force, or an electric current cannot be subtracted from the thermodynamic temperature. This simple principle called the principle of homogeneity of dimensions in an equation is extremely useful in checking the correctness of an equation. if an equation fails this consistency test, it is proved wrong, but if it passes, it is not proved right. Thus, a dimensionally correct equation need not be actually an exact (correct) equation, but a dimensionally wrong (incorrect) or inconsistent equation must be wrong.

2. Deducing Relation among the Physical Quantities : The method of dimensions can sometimes be used to deduce relation among the physical quantities.

I. VERY SHORT ANSWER QUESTIONS (1 MARK)

1. What is a physical quantity?
Any quantity which can be measured is called a physical quantity.

2. What is a unit?
A certain basic, arbitrarily chosen, internationally accepted standard of reference for making measurements of a physical quantity is called a unit.

3. Which are fundamental or basic units?
The unit of fundamental quantities is called fundamental units

4. What are derived units?
The units of derived quantities that can be expressed as combination of basic units are called derived units.

5. What is meant by SI system?
SI system means the international system of units, containing seven basic units.

6. Name the system of units accepted internationally.
SI system,

7. Give the basic units of length in CGS / MKS / FPS / SI system.
Centimeter / meter / foot / meter respectively.

8. Given the base units of mass in CGS / MKS / FPS / SI system.
gram / kilogram / pound / kilogram respectively.

9. Name the unit of time in all systems.
seconds

10. How many base units are there in SI system?
There are seven basic units in SI system.

11. Name the SI unit of current / temperature / amount of a substance / luminous intensity.
ampere / Kelvin / mole / candela.

12. Name the SI unit of angle in a plane.
radian

13. Name the SI unit of solid angle.
steradian

14. Express the relation for angle in a plane.

$$\text{angle } (d\theta) = \frac{length\ of\ arc\ (ds)}{radius\ (r)}$$

15. Express the relation for solid angle.

$$\text{A: solid angle } (d\Omega) = \frac{area\ (dA) of\ spherical\ surface}{square\ of\ radius\ (r^2)}$$

16. Mention some direct method of measuring length
Using metallic scale, vernier scale or screw gauge.

17. Name the method of measuring long distances.
Parallax method

18. What is parallax?
The change in position of an object with respect to a point when viewed with left and right eye.

19. What is basis?
The distance between the two points of observation is called the basis.

20. What is meant by parallax angle?
The angle between two directions of observation of a point (object) is called parallax angle.

21. Which are the shorter units of length? Express them in meters. fermi and angstrom
1 fermi = 10^{-15}m and 1 angstrom = 10^{-10}m

22. Name the larger units of length.
Astronomical unit (AU) and light year (ly), parsec (pc)

23. What is meant by light year?

The distance travelled by light in one year of time.

24. Define unified atomic unit.
One unified atomic mass unit is equal to 1/12th of the mass of an atom of carbon – 12 isotope including the mass of electron.

25. Which is the instrument used to measure small masses like atom?
: Mass spectrograph.

26. What is the basis of working of cesium clock or atomic clock?
Periodic vibration of cesium atom.

27. Name the type of clock which gives accurate time.
Cesium atom clock.

28. What is error?
The uncertainty in measurement is called error.

29. What is meant by accuracy?
The accuracy is the measure of how close the measured value is to the true value of the quantity.

30. What is meant by precision?
Precision means, to what resolution or limit of the instrument, the quantity is measured. It is given by least count

31. What are the types of error?
Systematic and random error.

32. Why does a measurement give approximate value ?
It is due to error.

33. What is meant by systematic error?
The systematic errors are that tend to be in one direction and affects each measurement by same amount.

34. What are random errors?
The random errors are those errors which occur irregularly due to random and unpredictable fluctuations in experimental conditions.

35. What is least count?
The smallest value that can be measured by an instrument is called the least count.

36. What is least count error?
It is the error associated with the resolution of the instrument.

37. What is absolute error?

The magnitude of the difference between the individual measurement and the true value of the physical quantity is called absolute error. It is always positive.

38. How would you determine the true value of a quantity measured several times ?
By taking arithmetic mean

39. What is relative error?
The relative error is the ratio of the mean absolute error to the mean value of the quantity measured.

40. Write the relation for relative error.

$$\text{relative error} = \frac{\Delta a \text{ mean}}{a \text{ mean}}$$

41. What is percentage error?
The relative error expressed in percentage is called percentage error.

42. Write the expression for percentage error.

$$\delta a \text{ (\% error)} = \frac{\Delta a \text{ mean}}{a \text{ mean}} \times 100$$

43. Define astronomical unit (AU)
The average distance between earth and sun is known as astronomical unit.

44. Define parsec.
One parsec is the distance at which an arc of length equal to one AU subtends an angle of one second at a point.

45. Express parsec in terms of light years.
1 parsec = 3.26 light year.

46. What are significant figures?
The reliable digits plus the first uncertain digit are known as significant figures.

47. Does the number of significant figures depend on the choice of unit?
No

48. State the number of significant figures in the following
a) 0.006 m^2 (b) 2.65 x 10^3 kg (c) 0.2309 m^{-3} (d) 6.320 J (e) 0.006032 m^2

(a) 1 (b) 3 (c) 3 (d) 4 (e) 4

49. Round off the following result to three significant figures
(a) 2.746 (b) 2.744 (3) 2.745 (4) 2.735
(a) 2.75 (b) 2.744 (c) 2.74 (d) 2.74

50. Define dimension of a physical quantity.

The dimensions of a physical quantity are the powers to which the base quantities are raised to represent that quantity.

51. Write the dimensions of work.

Work = $[M^1 L^2 T^{-2}]$. So dimensions of work are one in mass, two in length and -2 in time

52. Define dimensional formula of physical quantity.

The expression which shows how and which of the base quantities represent the dimensions of a physical quantity is called the dimensional formula.

53. Write the dimensional formula of volume.

$[V] = [M^0 L^3 T^0]$

54. Define dimensional equation of a physical quantity.

An equation obtained by equating a physical quantity with its dimensional formula is called the dimensional equation of the physical quantity.

55. Name a physical quantity which has no units and no dimensions

strain.

56. Name a physical quantity which has units but no dimensions.

Angle has unit i.e. radian but it is dimensionless.

57. Name a physical quantity which has neither unit nor dimension.

Relative density (specific gravity)

58. Can a physical quantity have dimension but no unit?

No

59. State the principle of homogeneity of dimensions.

It states that the dimensions of all the terms on either side of an equation must be the same

Fill up the blanks

1. The range of weak nuclear force is exceedingly small, of the order of 10^{-16} m.

SHORT ANSWER QUESTIONS (2 MARKS)

1. What is a physical standard? What characteristics should it have?

2. Define the term unit. Distinguish between fundamental and derived units.

3. Describe the principle and use of SONAR and RADAR.

1. Mention the base quantities in SI system

Length, mass, time, electric current, thermodynamic temperature, amount of substance and luminous intensity

2. Name any two derive SI unit with the name of scientist.

newton, joule, watt etc (any two)

3. What are sources of systematic error?

(i) Instrumental error (ii) imperfection in experimental procedure
(iii) personal error

4. Explain the method of reducing systematic error.

It can be minimized by improving experimental techniques, selecting better instruments and removing personal bias.

5. Give any two methods of reducing least count error.

Least count error can be reduced by using instruments of higher precision, improving experimental techniques and taking mean of all observations.

6. The distance 'D' of the sun from the earth is 1.496×10^{11} m. if sun's angular diameter is 9.31×10^{-3} rad as measured from earth, find the diameter of the sun.

Sun's diameter d = *D = $(9.31 \times 10^{-3}) \times 1.496 \times 10^{11}$ = 1.39×10^{9} m

7. State the rule to find the absolute error when two quantities are added or subtracted. Write the expression.

When two quantities are added or subtracted the absolute error in the final result is the sum of the absolute error in the individual quantities.

$$\Delta Z = \Delta A + \Delta B$$

8. State the rule to find the relative error when two quantities are multiplied or divided. Write an expression.

When two quantities are multiplied or divided , the relative error in the result is the sum of the relative error in the multipliers

$$\frac{\Delta Z}{Z} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$$

9. Find the relative error in Z, if Z = $(A^3 \ B^{1/2}) / (C \ D^{3/2})$

The relative error in Z is

$$\frac{\Delta Z}{Z} = \frac{3\Delta A}{A} + \frac{1}{2}\frac{\Delta B}{B} + \frac{\Delta C}{C} + \frac{3}{2}\frac{\Delta D}{D}$$

10. Explain scientific notation method of finding the number of significant figures.

In this notation, every number is expressed as a x 10^{b}, where 'a' is a number between 1 and 10, and 'b' is any power of 10. The number of digits in the decimal number gives significant figures.

11. Write the dimension of universal gravitational constant ?

$$F = \frac{G\, m_1 m_2}{d^2}$$

$$\therefore G = \frac{F\, d^2}{m_1 m_2}$$

writing the dimensions, $[G] = \frac{[M\, L\, T^{-2}]\,[L^2]}{[M][M]}$

$$= [M^{-1}\, L^3\, T^{-2}]$$

∴ dimensions are -1 in mass, 3 in length and -2 in time.

12. Mention two pairs of physical quantities which have the same dimensions.
(i) work and energy
(ii) pressure and stress.

13. Mention the physical quantities whose dimensions are (i) $[M^1\, L^{-1}\, T^{-2}]$ (ii) $[M^1\, L^2\, T^{-3}]$
(i) pressure (ii) power

14. What are the dimensions of a and b in the relation $F = a\sqrt{x} + bt^2$ where F is force, x is distance and t is time.
By principle of homogeneity, dimensions of every term on RHS should be same as that on LHS i.e. force

$$\therefore [a] = \frac{[F]}{[\sqrt{x}]} = \frac{[M\, L\, T^{-2}]}{[L^{1/2}]} = [M\, L^{1/2}\, T^{-2}]$$

$$[b] = \frac{[F]}{[t^2]} = \frac{[M\, L\, T^{-2}]}{[T^2]} = [M\, L\, T^{-4}]$$

15. Mention any two constants which have dimensions.
Gravitational constant, Plank's constant.

16. Mention any two applications of dimensional analysis.
1) To check the correctness of a physical relation
2) to derive relationship between different physical quantities.

17. Mention any two limitations of dimensional analysis.
1) dimensionless constants in a relation cannot be determined by this method
2) it cannot derive the exact relationship between physical quantities in any equation.

4. State the principle of homogeneity. Test the dimensional homogeneity of equations—

(i) $S = ut + \frac{1}{2}at^2$

(ii) $S_n = u + \frac{a}{2}(2n - 1)$

Ans :

(i) Dimension of L.H.S. = $[s] = [M^oL^1T^o]$

Dimension of R.H.S = $[ut] + [at^2] = [M^oL^1T^{-1}] + [M^oL^1T^{-2}.T^2]$

$= M^oL^1T^o$

∴ The equation to dimensionally homogeneous.

(ii) S_n = Distance travelled in n^{th} sec that is $(S_n - S_{nm})$

∴ $S_n = u \times l + \frac{a}{2}(2n - 1) \times 1$

Hence this is dimensionally incorrect.

5. In van der Wall's gas equation □$(p + \frac{a}{v^2})$ □ (v – b) = RT. Determine the dimensions of *a* and *b*.

Since dimensionally similar quantities can only be added

∴ $[P] = \left[\frac{a}{v^2}\right] \Rightarrow [a] = [Pv^2] = M^1\,L^5\,T^{-2}$

$[b] = [v] = L^3$

6. Using dimensions convert (a) 1 newton into dynes (b) 1 erg into joules.

7. Magnitude of force experienced by an object moving with speed *v* is given by F = *kv*^{2}. Find dimensions of *k*.

$$[k] = \frac{[F]}{[v^2]} = \frac{M^1L^1T^{-2}}{[LT^{-1}]^2} = \frac{M^1L^1T^{-2}}{M^0L^2T^{-2}} = [M^1L^{-1}]$$

8. A book with printing error contains four different formulae for displacement. Choose the correct formula/formulae

(a) $y = a\sin\frac{2\pi}{T}t$ (b) $y = a\sin vt$

(c) $y = \frac{a}{T}\sin\left(\frac{t}{a}\right)$ (d) $y = \frac{a}{T}\left(\sin\frac{2\pi}{T}t + \cos\frac{2\pi}{T}t\right)$

The argument of sine and cosine function must be dimension less so (a) is the probable correct formula. Since

(a) $y = a\sin\left(\frac{2\pi}{T}t\right) \quad \because \left[\frac{2\pi t}{T}\right] = \left[T^0\right]$ is dimensionless.

(b) $y = a\sin\mu t \quad \because [\mu t]=[L]$ is dimensional so this equation is incorrect.

(c) $y = \frac{a}{t}\sin\left(\frac{t}{a}\right) \quad \left[\frac{t}{a}\right]$ is dimensional so this is incorrect.

(d) $y = \frac{a}{t}\left(\sin\frac{2\pi}{T}t + \cos\frac{2\pi t}{T}\right)$: Though $\frac{2\pi t}{T}$ is dimensionless $\frac{a}{T}$ does not have dimensions of displacement so this is also incorrect

9. Give limitations of dimensional analysis.

Limitation of dimensional analysis :-

1. The value of proportionality constant cannot be obtained
2. Equation containing sine and cosine, exponents, logx etc cannot be analysed.
3. If fails to derive the exact form of physical relation which depends on more than three fundamental quantities
4. It does not tell whether a quantity is scaler or vector.

10. For determination of '*g*' using simple pendulum, measurements of length and time period are required. Error in the measurement of which quantity will have larger effect on the value of '*g*' thus obtained. What is done to minimize this error?

SHORT ANSWER QUESTIONS (3 MARKS)

1. Give the name of six Indian Scientists and their discoveries.

2. Name the discoveries made by the following scientists :
(a) Faraday (b) Chadwick
(c) Hubble (d) Maxwell
(e) Newton (f) Bohr.

3. Name the scientific principle on which the following technology is based.
(i) Steam engine (ii) Laser (iii) Aeroplane (iv) Rocket propulsion (v) Radio and T.V. (vi) Production of Ultra high magnetic field.

4. Describe a method for measuring the molecular size of Oleic acid.

5. What types of phenomena can be used as a time standard. What are the advantages of defining second in terms of period of radiation from cesium –133 atom.

6. Deduce the dimensional formula for the following quantities
(i) Gravitational constant (ii) Yung's modules
(iii) Coefficient of viscosity.

7. Define the following units :
(i) Light year (ii) Parsec
(iii) Astronomical unit (Au)

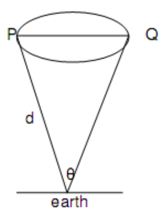
LONG ANSWER QUESTIONS (5 MARKS)

1. Name the four basic forces in nature. Write a brief note of each. Hence compare their strengths and ranges.

2. Distinguish between the terms precision and accuracy of a measurement.

3. Explain
(i) absolute error (ii) mean absolute error (iii) relative error (iv) percentage error (v) random error

1. Explain parallax method of determining the size of moon.



Let 'D' be the diameter of the moon, when moon is observed from a place 'E' on earth, let 'θ' be the angle made by two diametrically opposite ends P and Q of the moon called parallax angle. If 'd' is the distance of moon from earth, then $\theta = PQ/d = D/d$
Therefore, $D = d/\theta$.
Using this relation the size of moon can be determined.

2. The period of oscillation of a simple pendulum is $T = 2\pi\sqrt{\frac{L}{g}}$, the measured value of L is 20.0 cm known to 1 mm accuracy and time for 100 oscillations of the pendulum is found to be 90 s using a watch of 1s resolution. Find accuracy in % error.

We have $g = 4\pi^2 \frac{L}{T^2}$

Here, $T = \frac{t}{n}$ and $\Delta T = \frac{\Delta t}{n}$.

$\therefore \frac{\Delta T}{T} = \frac{\Delta t}{t}$

Here errors are least count errors

$$\frac{\Delta g}{g} = \left(\frac{\Delta L}{L}\right) + 2\left(\frac{\Delta T}{T}\right)$$

$$= \frac{0.1}{20.0} + 2\left(\frac{1}{90}\right)$$

$$= 0.027$$

Thus percentage error in g is 100 x $\frac{\Delta g}{g}$ = 100 x 0.027 = 2.7%

3. Check the correctness of following equation by dimensional analysis. $x = x_0 + v_0 t + \frac{1}{2}at^2$

For LHS , dimension of x = [L]

For RHS, dimension of x_0 = [L]

dimension of v_0t = $[LT^{-1}]$ [T] = [L]

dimension of $\frac{1}{2}at^2$ = $[LT^{-2}]$ $[T^2]$ = [L] here ½ is a constant having no dimension.

∴ [L] = [L] + [L] + [L] i.e. the dimensions of each term on both sides of the equation are the same. Thus the equation is dimensionally correct.

4. Check the correctness of following equation by dimensional analysis. F x = ½ mv^2 – ½ mv_0^2

Where F – force, x - distance, v0 – initial velocity, v – final velocity

Consider Fx = ½ mv^2 – ½ mv_0^2

For LHS, dimensions of Fx = $[M L T^{-2}]$ [L] = $[M L^2 T^{-2}]$

For RHS, dimensions of ½ mv^2 = [M] $[L T^{-1}]^2$ = $[M L^2 T^{-2}]$

Similarly , dimensions of ½ mv_0^2 = [M] $[L T^{-1}]^2$ = $[M L^2 T^{-2}]$

Where ½ is a constant, has no dimension.

; $[M L^2 T^{-2}] = [M L^2 T^{-2}] - [M L^2 T^{-2}]$

Since dimensions of each term on both sides of the equation are the same. Thus the equation is dimensionally correct.

5. The period of oscillation of a simple pendulum depends on its length (l), mass of the bob (m) and acceleration due to gravity (g). Derive the expression for its time period using method of dimensions.

We can write T α $l^a m^b g^c$ T = K $l^a m^b g^c$ ---------- (1)

Where K is dimensionless constant.

Writing the dimensions of all terms, we get $[T] = [L]^a [M]^b [L T^{-2}]^c$

$[L^0 M^0 T^1] = [L^{a+c} M^b T^{-2c}]$

Equating the powers of L, M and T on both sides

We get, a + c = 0 ; b = 0 ; and -2c = 1

Thus, a = ½ ; b = 0 ; c = - ½

Substituting in equation (1) we get $T = K l^{1/2} m^0 g^{-1/2}$ or $T = K\sqrt{\frac{l}{g}}$

6. The centripetal force (F) acting on a particle moving uniformly in a circle depends upon its mass (m), velocity (v) and radius of circle (r). Derive the expression for centripetal force using method of dimensions.

We can write,

F α $m^a v^b r^c$; F = K $m^a v^b r^c$ ----- (1)

where K is a constant having no dimension. Writing the dimensions of all terms in the equation,

we get, $[M L T^{-2}] = [M]^a [L T^{-1}]^b [L]^c$; $[M L T^{-2}] = [M^a L^{b+c} T^{-b}]$

Equating the powers of M, L and T on both sides

We get, a = 1, b+c = 1 and –b = -2

i.e. a = 1, b = 2 , c = -1 putting these values in equation (1)

$F = K\, m^1 v^2 r^{-1}$ or $F = K \frac{mv^2}{r}$

7. Check the correctness of the relation, $S_n = u + \frac{a}{2}(2n - 1)$ where 'u' is the initial velocity, 'a' is the acceleration and 'S_n' is the distance travelled by the body in the n^{th} second.

$$S_n = u + \frac{a}{2}(2n - 1)$$

Writing the dimensions on the either side, we have

$$LHS = S_n = \frac{distance}{time} = \frac{L}{T} = [M^0 L^1 T^{-1}]$$

$$RHS = u + \frac{a}{2}(2n - 1) = L\,T^{-1} + L\,T^{-2}\,(T)$$

$$= L\,T^{-1} + LT^{-1}$$

$= [M^0 L^1 T^{-1}]$

Thus dimension of all terms are same. LHS = RHS dimensionally. So the relation is correct.

NUMERICALS

1. Determine the number of light years in one metre.

$1 \text{ ly} = 9.46 \times 10^{15} \text{ m}$

$$1 \text{ m} = \frac{1}{9.46 \times 10^{15}} = 1.057 \times 10^{-16} \text{ly}$$

2. The sides of a rectangle are (10.5 ± 0.2) cm and (5.2 ± 0.1) cm. Calculate its perimeter with error limits.

$P = 2(l + b)$

$= 2(10.5 + 5.2) + 2(0.2 + 0.1)$

$= (31.4 + 0.6)$ cm

3. The mass of a box measured by a grocer's balance is 2.3 kg. Two gold pieces 20.15 g and 20.17 g are added to the box.

(i) What is the total mass of the box?

(ii) The difference in masses of the pieces to correct significant figures.

(i) Mass of box : 2.3 kg

Mass of gold pieces = 20.15 + 20.17 = 40.32 g = 0.04032 kg.

Total mass = 2.3 + 0.04032 = 2.34032 kg

In correct significant figure mass = 2.3 kg (as least decimal)

(ii) Difference in mass of gold pieces = 0.02 g

In correct significant figure (2 significant fig. minimum decimal) will be 0.02 g.

4. 5.74 g of a substance occupies 1.2 cm3. Express its density to correct significant figures.

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}} = \frac{5.74}{1.2} = 4.783 \text{ g/cm}^3$$

Here least significant figure is 2, so density = 4.8 g/cm^3

5. If displacement of a body $s = (200 \pm 5)\ m$ and time taken by it $t = (20 + 0.2)\ s$, then find the percentage error in the calculation of velocity.

Percentage error in measurement of displacement $= \frac{5}{200} \times 100$

Percentage error in measurement of time $= \frac{0.2}{20} \times 100$

$\therefore$ Maximum permissible error = 2.5 + 1 = 3.5%

6. If the error in measurement of mass of a body be 3% and in the measurement of velocity be 2%. What will be maximum possible error in calculation of kinetic energy.

$$\text{K.E.} = \frac{1}{2}mv^2$$

$$\therefore \quad \frac{\Delta k}{k} = \frac{\Delta m}{m} + \frac{2\Delta v}{v} \Rightarrow \frac{\Delta k}{k} \times 100 = \frac{\Delta m}{m} \times 100 + 2\left(\frac{\Delta v}{v}\right) \times 100$$

$\therefore$ Percentage error in K.E. = 3% + 2 × 2% = 7%

7. The length of a rod as measured in an experiment was found to be 2.48m, 2.46m, 2.49m, 2.50m and 2.48m. Find the average length, absolute error and percentage error. Express the result with error limit.

Average length

$$= \frac{2.48 + 2.46 + 2.49 + 2.50 + 2.48}{5} = \frac{12.41}{5} = 2.48\ \text{m}$$

Mean absolute error

$$\frac{0.00 + 0.02 + 0.01 + 0.02 + 0.00}{5} = \frac{0.05}{5} = 0.013\text{n}$$

$$\text{Percentage error} = \frac{0.01}{2.48} \times 100\% = 0.04 \times 100\%$$

$$= 0.40\%$$

Correct length = (2.48 ± 0.01)m

Correct length = (2.48 M ± 0.40%)

8. A physical quantity is measured as a = (2.1 ± 0.5) units. Calculate the percentage error in (i) Q2 (2) 2Q.

$P = Q^2$

$$\frac{\Delta p}{p} = \frac{2\Delta Q}{Q} \qquad \left(\frac{0.5}{2.1}\right) = \frac{1.0}{2.1} = 0.476$$

$$\frac{\Delta p}{p} \times 100\% = 47.6\% = 48\%$$

R = 2Q

$$\frac{\Delta R}{R} = \frac{\Delta Q}{Q} \Rightarrow \frac{0.5}{2.1} = 0.238$$

$$\frac{\Delta R}{R} \times 100\% = 24\%$$

9. When the planet Jupiter is at a distance of 824.7 million km from the earth, its angular diameter is measured to be 35.72′′ of arc. Calculate diameter of Jupiter.

Q = 35.72”

$1” = 4.85 \times 10^{-6}$ radian $\Rightarrow = 35.72 \times 4.85 \times 10^{-6}$

$d = DQ = 824.7 \times 10^{6} \times 35.72 \times 4.85 \times 10^{-6}$

$= 1.4287 \times 10^{5}$ km

10. A lesser light beamed at the moon takes 2.56 and to return after reflection at the moon’s surface. What will be the radius of lunar orbit.

t = 2.56 s

$\therefore$ t’ = time taken by laser beam to go to the man $= \frac{t}{2}$

distance between earth and moon = $d = c \times \frac{t}{2}$

$$= 3 \times 10^{8} \times \frac{2.56}{2}$$

$$= 3.84 \times 10^{8} \text{ m}$$

11. Convert

(i) 3m . S^{-2} to km h^{-2}

(ii) G = 6.67 × 10^{-11} N m^2 kg–2 to cm3 g^{-1} S^{-2}

(i) $3 \text{ m s}^{-2} = \left(\frac{3}{1000}\text{km}\right)\left(\frac{1}{60 \times 60}\text{hr}\right)^{-2}$

$$= \frac{3 \times (60 \times 60)^2}{1000} = 3.9 \times 10^{4} \text{km h}^{-2}$$

(ii) $G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$

$= 6.67 \times 10^{-11} (\text{kg m s}^{-2})(\text{m}^2 \text{ kg}^{-2})$

$= 6.67 \times 10^{-11} \text{ kg}^{-1} \text{ m}^3 \text{ s}^{-2}$

$= 6.67 \times 10^{-11} (1000 \text{ g})^{-1} (100 \text{ cm})^3 (\text{s}^{-2})$

$= 6.67 \times 10^{-11} \alpha \frac{1}{1000} \times 100 \times 100 \times 100$

$= 6.67 \times 10^{-8} \text{g}^{-1} \text{cm}^3 \text{s}^{-2}$

12. A calorie is a unit of heat or energy and it equals 4.2 J where IJ = 1kg m2S–2. Suppose we employ a system of units in which unit of mass is □□kg, unit of length is □□m, unit of time is □□s. What will be magnitude of calorie in terms of this new system.

$$n_2 = n_1 \left[\frac{m_1}{m_2}\right]^a \left(\frac{L_1}{L_2}\right)^b \left(\frac{T_1}{T_2}\right)^c$$

$$= 4.2 \left(\frac{kg}{\alpha\, kg}\right)^1 \left(\frac{m}{\beta\, m}\right)^2 \left(\frac{s}{r\, s}\right)^{-2}$$

$$n_2 = 4.2\ \alpha^{-1}\ \beta^{-2}\ \rho^{+2}$$

13. The escape velocity *v* of a body depends on–

(i) the acceleration due to gravity 'g' of the planet,

(ii) the radius R of the planet. Establish dimensionally the relation for the escape velocity.

$v\ \alpha\ g^a\ R^b$ Þ $\mu = k\ g^a\ R^b$ K → dimensionless proportionality constant

$[v] = [g]^a\ [R]^b$

$[M^oL^1T^{-1}] = [M^0\ L^1T^{-2}]^a\ [m^0\ L^1\ T^0]^b$

equating powers

$1 = a + b$

$$-1 = -2a \Rightarrow a = \frac{1}{2}$$

$$b = 1 - a = 1 - \frac{1}{2} = \frac{1}{2}$$

$$\therefore v = k\sqrt{gR}$$

14. The frequency of vibration of a string depends of on, (i) tension in the string (ii) mass per unit length of string, (iii) vibrating length of the string. Establish dimensionally the relation for frequency.

$n\ \alpha\ l^a\ T^b\ m^c\ [l] = M^oL^1T^0$

$[T] = M^1\ L^1\ T^{-2}$ (force)

$[M] = M^1\ L^{-1}\ T^0$

$[M^0\ L^0\ T^{-1}] = [M^0\ L^1\ T^0]^a\ [M^1\ L^1\ T^{-2}]^b\ [\ M^1\ L^{-1}\ T^0]^c$

$b + c = 0$

$a + b - c = 0$

$-2b = -1\ b = 1/2$

$$c = -\frac{1}{2} \qquad a = 1 \qquad n\ \alpha\ \frac{1}{l}\sqrt{\frac{T}{M}}$$

15. One mole of an ideal gas at STP occupies 22.4 L. What is the ratio of molar volume to atomic volume of a mole of hydrogen? Why is the ratio so large. Take radius of hydrogen molecule to be 1°A.

$1\ A^0 = 10^{-10}m$

Atomic volume of 1 mole of hydrogen = Avagadios number × volume of hydrogen molecule

$$= 6.023 \times 10^{23} \times \frac{4}{3} \times \pi \times \left(10^{-10}\right)^3$$

$= 25.2 \times 10^{-7}\ m^3$

Molar volume $= 22.4\ L = 22.4 \times 10^{-3}\ m^3$

$$\frac{\text{Molar volume}}{\text{Atomic volume}} = \frac{22.4 \times 10^{-3}}{25.2 \times 10^{-7}} = 0.89 \times 10^4 \approx 10^4$$

This ratio is large because actual size of gas molecule is negligible in comparison to the inter molecular separation.

TEXTBOOK PROBLEMS

2.1 Fill in the blanks

(a) The volume of a cube of side 1 cm is equal to __10^{-6}_ m^3

(b) The surface area of a solid cylinder of radius 2.0 cm and height 10.0 cm is equal to ___________ $(mm)^2$

(c) A vehicle moving with a speed of 18 km h^{-1} covers ________ m in 1s

(d) The relative density of lead is 11.3. Its density is _______ g cm^{-3} or ______ kg m^{-3}.

Ans : 1 cm = 1/100 m

Volume of the cube = 1 cm^3 = 1cm × 1cm × 1 cm = 1/100 m × 1/100 m × 1/100 m = 10^{-6} m.

(b) The total surface area of a cylinder of radius r and height h is

S = 2πr (r + h).

Given that,

r = 2 cm = 2 × 1 cm = 2 × 10 mm = 20 mm

h = 10 cm = 10 × 10 mm = 100 mm

$\therefore S = 2\times3.14\times20\times(20+100)$ = 15072 = 1.5 × 104 mm2

(c) Using the conversion,

1 km/h = $\frac{5}{18}$ m/s

$$18 \text{ km/h} = 18\times\frac{5}{18} = 5 \text{ m/s}$$

Therefore, distance can be obtained using the relation:

Distance = Speed × Time = 5 × 1 = 5 m

Hence, the vehicle covers 5 m in 1 s.

(d) Relative density of a substance is given by the relation,

Relative density = $\frac{\text{Density of substance}}{\text{Density of water}}$

Density of water = 1 g/cm3

$$\text{Density of lead} = \text{Relative density of lead}\times\text{Density of water}$$
$$= 11.3\times1 = 11.3 \text{ g/cm}^3$$

Again, 1g = $\frac{1}{1000}$ kg

1 cm3 = 10−6 m3

1 g/cm3 = $\frac{10^{-3}}{10^{-6}}$ kg/m^3 = 10^3 kg/m^3

□ 11.3 g/cm3 = 11.3 × 103 kg/m3

2.2 Fill in the blanks by suitable conversion of units

(a) 1 kg m^2 s^{-2} = ________ g cm^2 s^{-2}

(b) 1 m = __________ ly

(c) 3.0 m s^{-2} = ________ km h^{-2}

(d) G = 6.67 × 10^{-11} N m^2 (kg)$^{-2}$ = _______ (cm)^3 s^{-2} g^{-1}.

Answer:

(a) 1 kg = 103 g

1 m2 = 104 cm2

1 kg m2 s−2 = 1 kg × 1 m2 × 1 s−2

=103 g × 104 cm2 × 1 s−2 = 107 g cm2 s−2

(b) Light year is the total distance travelled by light in one year.

1 ly = Speed of light × One year

= (3 × 108 m/s) × (365 × 24 × 60 × 60 s)

= 9.46 × 1015 m

$\therefore 1 \text{ m} = \frac{1}{9.46 \times 10^{15}} = 1.057 \times 10^{-16}$ ly

(c) 1 m = 10−3 km

Again, 1 s = $\frac{1}{3600}$ h

1 s−1 = 3600 h−1

1 s−2 = (3600)2 h−2

□3 m s−2 = (3 × 10−3 km) × ((3600)2 h−2) = 3.88 × 10−4 km h−2

(d) 1 N = 1 kg m s−2

1 kg = 10−3 g−1

1 m3 = 106 cm3

□ 6.67 × 10−11 N m2 kg−2 = 6.67 × 10−11 × (1 kg m s−2) (1 m2) (1 s−2)

= 6.67 × 10−11 × (1 kg × 1 m3 × 1 s−2)

= 6.67 × 10−11 × (10−3 g−1) × (106 cm3) × (1 s−2)

= 6.67 × 10−8 cm3 s−2 g−1

2.3 A calorie is a unit of heat or energy and it equals about 4.2 J where 1J = 1 kg m^2 s^{-2}. Suppose we employ a system of units in which the unit of mass equals α kg, the unit of length equals β m, the unit of time is γ s. Show that a calorie has a magnitude 4.2 α^{-1} β^{-2} γ^2 in terms of the new units.

Answer:

Given that,

1 calorie = 4.2 (1 kg) (1 m2) (1 s−2)

New unit of mass = α kg

Hence, in terms of the new unit, 1 kg = $\frac{1}{\alpha} = \alpha^{-1}$

In terms of the new unit of length,

$$1\ \text{m} = \frac{1}{\beta} = \beta^{-1} \text{ or } 1\ \text{m}^2 = \beta^{-2}$$

And, in terms of the new unit of time,

$$1\ \text{s} = \frac{1}{\gamma} = \gamma^{-1}$$

$$1\ \text{s}^2 = \gamma^{-2}$$

$$1\ \text{s}^{-2} = \gamma^2$$

□1 calorie = 4.2 (1 α−1) (1 β−2) (1 γ2) = 4.2 α−1 β−2 γ2

2.4 Explain this statement clearly :
"To call a dimensional quantity 'large' or 'small' is meaningless without specifying a standard for comparison". In view of this, reframe the following statements wherever necessary :
(a) atoms are very small objects
(b) a jet plane moves with great speed
(c) the mass of Jupiter is very large
(d) the air inside this room contains a large number of molecules
(e) a proton is much more massive than an electron
(f) the speed of sound is much smaller than the speed of light

Answer:

The given statement is true because a dimensionless quantity may be large or comparison to some standard reference. For example, the coefficient of frictio dimensionless. The coefficient of sliding friction is greater than the coefficient friction, but less than static friction.

(a) An atom is a very small object in comparison to a soccer ball.

(b) A jet plane moves with a speed greater than that of a bicycle.

(c) Mass of Jupiter is very large as compared to the mass of a cricket ball.

(d) The air inside this room contains a large number of molecules as compare present in a geometry box.

(e) A proton is more massive than an electron.

(f) Speed of sound is less than the speed of light.

2.5 A new unit of length is chosen such that the speed of light in vacuum is unity. What is the distance between the Sun and the Earth in terms of the new unit if light takes 8 min and 20 s to cover this distance ?

Answer:

Distance between the Sun and the Earth:

= Speed of light × Time taken by light to cover the distance

Given that in the new unit, speed of light = 1 unit

Time taken, t = 8 min 20 s = 500 s

□Distance between the Sun and the Earth = 1 × 500 = 500 units

2.6 Which of the following is the most precise device for measuring length :
(a) a vernier callipers with 20 divisions on the sliding scale
(b) a screw gauge of pitch 1 mm and 100 divisions on the circular scale
(c) an optical instrument that can measure length to within a wavelength of light ?

Answer:

A device with minimum count is the most suitable to measure length.

(a) Least count of vernier callipers

= 1 standard division (SD) − 1 vernier division (VD)

$$= 1 - \frac{9}{10} = \frac{1}{10} = 0.01 \text{ cm}$$

(b) Least count of screw gauge = $\frac{\text{Pitch}}{\text{Number of divisions}}$

$$= \frac{1}{1000} = 0.001 \text{ cm}$$

(c) Least count of an optical device = Wavelength of light □ 10−5 cm

= 0.00001 cm

Hence, it can be inferred that an optical instrument is the most suitable device

measure length.

2.7 A student measures the thickness of a human hair by looking at it through a microscope of magnification 100. He makes 20 observations and finds that the average width of the hair in the field of view of the microscope is 3.5 mm. What is the estimate on the thickness of hair ?

Answer:

Magnification of the microscope = 100

Average width of the hair in the field of view of the microscope = 3.5 mm

□Actual thickness of the hair is $\frac{3.5}{100}$ = 0.035 mm.

2.8 Answer the following :

(a) You are given a thread and a metre scale. How will you estimate the diameter of the thread ?

(b) A screw gauge has a pitch of 1.0 mm and 200 divisions on the circular scale. Do you think it is possible to increase the accuracy of the screw gauge arbitrarily by increasing the number of divisions on the circular scale ?

(c) The mean diameter of a thin brass rod is to be measured by vernier callipers. Why is a set of 100 measurements of the diameter expected to yield a more reliable estimate than a set of 5 measurements only ?

Answer:

(a) Wrap the thread on a uniform smooth rod in such a way that the coils thus

are very close to each other. Measure the length of the thread using a metre s

diameter of the thread is given by the relation,

$$\text{Diameter} = \frac{\text{Length of thread}}{\text{Number of turns}}$$

(b) It is not possible to increase the accuracy of a screw gauge by increasing t

of divisions of the circular scale. Increasing the number divisions of the circula

increase its accuracy to a certain extent only.

(c) A set of 100 measurements is more reliable than a set of 5 measurements

random errors involved in the former are very less as compared to the latter.

2.9 The photograph of a house occupies an area of 1.75 cm2 on a 35 mm slide. The slide is projected on to a screen, and the area of the house on the screen is 1.55 m2. What is the linear magnification of the projector-screen arrangement.

Answer:

Area of the house on the slide = 1.75 cm2

Area of the image of the house formed on the screen = 1.55 m2

= 1.55 × 104 cm2

Arial magnification, ma = $\frac{\text{Area of image}}{\text{Area of object}} = \frac{1.55}{1.75} \times 10^4$

□Linear magnifications, ml = $\sqrt{m_a}$

$= \sqrt{\frac{1.55}{1.75} \times 10^4} = 94.11$

2.10 State the number of significant figures in the following :
(a) 0.007 m^2
(b) 2.64 × 10^{24} kg
(c) 0.2370 g cm^{-3}
(d) 6.320 J
(e) 6.032 N m^{-2}
(f) 0.0006032 m^2

Answer:

(a) Answer: 1

The given quantity is 0.007 m2.

If the number is less than one, then all zeros on the right of the decimal point the first non-zero) are insignificant. This means that here, two zeros after the are not significant. Hence, only 7 is a significant figure in this quantity.

(b) Answer: 3

The given quantity is 2.64 × 1024 kg.

Here, the power of 10 is irrelevant for the determination of significant figures. digits i.e., 2, 6 and 4 are significant figures.

(c) Answer: 4

The given quantity is 0.2370 g cm−3.

For a number with decimals, the trailing zeroes are significant. Hence, besides and 7, 0 that appears after the decimal point is also a significant figure.

(d) Answer: 4

The given quantity is 6.320 J.

For a number with decimals, the trailing zeroes are significant. Hence, all four appearing in the given quantity are significant figures.

(e) Answer: 4

The given quantity is 6.032 Nm−2.

All zeroes between two non-zero digits are always significant.

(f) Answer: 4

The given quantity is 0.0006032 m2.

If the number is less than one, then the zeroes on the right of the decimal poi to the first non-zero) are insignificant. Hence, all three zeroes appearing befor significant figures. All zeros between two non-zero digits are always significan the remaining four digits are significant figures.

2.11 The length, breadth and thickness of a rectangular sheet of metal are 4.234 m, 1.005 m, and 2.01 cm respectively. Give the area and volume of the sheet to correct significant figures.

Answer:

Length of sheet, l = 4.234 m

Breadth of sheet, b = 1.005 m

Thickness of sheet, h = 2.01 cm = 0.0201 m

The given table lists the respective significant figures:

Quantity	Number	Significant Figure
l	4.234	4
b	1.005	4
h	2.01	3

Hence, area and volume both must have least significant figures i.e., 3.

Surface area of the sheet = 2 (l × b + b × h + h × l)

= 2(4.234 × 1.005 + 1.005 × 0.0201 + 0.0201 × 4.234)

= 2 (4.25517 + 0.02620 + 0.08510)

= 2 × 4.360

= 8.72 m2

Volume of the sheet = l × b × h

= 4.234 × 1.005 × 0.0201

= 0.0855 m3

This number has only 3 significant figures i.e., 8, 5, and 5.

2.12 The mass of a box measured by a grocer's balance is 2.300 kg. Two gold pieces of masses 20.15 g and 20.17 g are added to the box. What is (a) the total mass of the box, (b) the difference in the masses of the pieces to correct significant figures ?

Answer:

Mass of grocer's box = 2.300 kg

Mass of gold piece I = 20.15g = 0.02015 kg

Mass of gold piece II = 20.17 g = 0.02017 kg

(a) Total mass of the box = 2.3 + 0.02015 + 0.02017 = 2.34032 kg

In addition, the final result should retain as many decimal places as there are number with the least decimal places. Hence, the total mass of the box is 2.3

(b) Difference in masses = 20.17 − 20.15 = 0.02 g

In subtraction, the final result should retain as many decimal places as there a number with the least decimal places.

2.13 A physical quantity P is related to four observables a, b, c and d as follows :

$$P = a^3b^2/\left(\sqrt{c}\,d\right)$$

The percentage errors of measurement in a, b, c and d are 1%, 3%, 4% and 2%, respectively. What is the percentage error in the quantity P ? If the value of P calculated using the above relation turns out to be 3.763, to what value should you round off the result ?

Answer :

$$P = \frac{a^3b^2}{\left(\sqrt{c}d\right)}$$

$$\frac{\Delta P}{P} = \frac{3\Delta a}{a} + \frac{2\Delta b}{b} + \frac{1}{2}\frac{\Delta c}{c} + \frac{\Delta d}{d}$$

$$\left(\frac{\Delta P}{P} \times 100\right)\% = \left(3 \times \frac{\Delta a}{a} \times 100 + 2 \times \frac{\Delta b}{b} \times 100 + \frac{1}{2} \times \frac{\Delta c}{c} \times 100 + \frac{\Delta d}{d} \times 100\right)\%$$

$$= 3 \times 1 + 2 \times 3 + \frac{1}{2} \times 4 + 2$$

$$= 3 + 6 + 2 + 2 = 13\ \%$$

Percentage error in P = 13 %

Value of P is given as 3.763.

By rounding off the given value to the first decimal place, we get P = 3.8.

2.14 A book with many printing errors contains four different formulas for the displacement y of a particle undergoing a certain periodic motion :

(a) y = a sin 2π t/T

(b) y = a sin vt

(c) y = (a/T) sin t/a

(d) y = (a √2) (sin 2πt /T + cos 2πt/T)

(a = maximum displacement of the particle, v = speed of the particle. T = time-period of motion). Rule out the wrong formulas on dimensional grounds.

Answer:

(a) Answer: Correct

$$y = a \sin \frac{2\pi t}{T}$$

Dimension of y = M0 L1 T0

Dimension of a = M0 L1 T0

Dimension of $\sin \frac{2\pi t}{T}$ = M0 L0 T0

∵ Dimension of L.H.S = Dimension of R.H.S

Hence, the given formula is dimensionally correct.

(b) Answer: Incorrect

y = a sin vt

Dimension of y = M0 L1 T0

Dimension of a = M0 L1 T0

Dimension of vt = M0 L1 T−1 × M0 L0 T1 = M0 L1 T0

But the argument of the trigonometric function must be dimensionless, which

the given case. Hence, the given formula is dimensionally incorrect.

(c) Answer: Incorrect

$$y = \left(\frac{a}{T}\right) \sin\left(\frac{t}{a}\right)$$

Dimension of y = M0L1T0

Dimension of $\frac{a}{T}$ = M0L1T−1

Dimension of $\frac{t}{a}$ = M0 L−1 T1

But the argument of the trigonometric function must be dimensionless, which

the given case. Hence, the formula is dimensionally incorrect.

(d) Answer: Correct

$$y = \left(a\sqrt{2}\right)\left(\sin 2\pi \frac{t}{T} + \cos 2\pi \frac{t}{T}\right)$$

Dimension of y = M0 L1 T0

Dimension of a = M0 L1 T0

Dimension of $\frac{t}{T}$ = M0 L0 T0

Since the argument of the trigonometric function must be dimensionless (whic

the given case), the dimensions of y and a are the same. Hence, the given for

dimensionally correct.

2.15 A famous relation in physics relates 'moving mass' m to the 'rest mass' mo of a particle in terms of its speed v and the speed of light, c. (This relation first arose as a consequence of special relativity due to Albert Einstein). A boy recalls the relation almost correctly but forgets where to put the constant c. He writes :

$$m = \frac{m_0}{\left(1 - v^2\right)^{1/2}}.$$

Guess where to put the missing c.

Answer:

Given the relation,

$$m = \frac{m_0}{\left(1 - v^2\right)^{\frac{1}{2}}}$$

Dimension of m = M1 L0 T0

Dimension of m_0 = M1 L0 T0

Dimension of v = M0 L1 T−1

Dimension of v2 = M0 L2 T−2

Dimension of c = M0 L1 T−1

The given formula will be dimensionally correct only when the dimension of L.I

same as that of R.H.S. This is only possible when the factor, $\left(1 - v^2\right)^{\frac{1}{2}}$ is dimen

i.e., (1 − v2) is dimensionless. This is only possible if v2 is divided by c2. Henc

correct relation is

$$m = \frac{m_0}{\left(1 - \frac{v^2}{c^2}\right)^{\frac{1}{2}}}.$$

Answer:

Radius of hydrogen atom, r = 0.5 Å = 0.5 × 10−10 m

Volume of hydrogen atom = $\frac{4}{3}\pi r^3$

$=\frac{4}{3}\times\frac{22}{7}\times\left(0.5\times10^{-10}\right)^3$

$=0.524\times10^{-30}\ \mathrm{m}^3$

1 mole of hydrogen contains 6.023 × 1023 hydrogen atoms.

□ Volume of 1 mole of hydrogen atoms = 6.023 × 1023 × 0.524 × 10−30

= 3.16 × 10−7 m3

2.16 The unit of length convenient on the atomic scale is known as an angstrom and is denoted by Å: 1 Å = 10^{-10} m. The size of a hydrogen atom is about 0.5 Å. What is the total atomic volume in m3 of a mole of hydrogen atoms ?

Answer:

Radius of hydrogen atom, r = 0.5 Å = 0.5 × 10−10 m

Volume of hydrogen atom = $\frac{4}{3}\pi r^3$

$=\frac{4}{3}\times\frac{22}{7}\times\left(0.5\times10^{-10}\right)^3$

$=0.524\times10^{-30}\ \mathrm{m}^3$

1 mole of hydrogen contains 6.023 × 1023 hydrogen atoms.

□ Volume of 1 mole of hydrogen atoms = 6.023 × 1023 × 0.524 × 10−30

= 3.16 × 10−7 m3

2.17 One mole of an ideal gas at standard temperature and pressure occupies 22.4 L (molar volume). What is the ratio of molar volume to the atomic volume of a mole of hydrogen ? (Take the size of hydrogen molecule to be about 1 Å). Why is this ratio so large ?

Answer:

Radius of hydrogen atom, r = 0.5 Å = 0.5 × 10−10 m

Volume of hydrogen atom = $\frac{4}{3}\pi r^3$

$=\frac{4}{3}\times\frac{22}{7}\times\left(0.5\times10^{-10}\right)^3$

$=0.524\times10^{-30}\ \mathrm{m}^3$

Now, 1 mole of hydrogen contains 6.023 × 1023 hydrogen atoms.

□ Volume of 1 mole of hydrogen atoms, Va = 6.023 × 1023 × 0.524 × 10−3(

= 3.16 × 10−7 m3

Molar volume of 1 mole of hydrogen atoms at STP,

Vm = 22.4 L = 22.4 × 10−3 m3

$$\therefore \frac{V_m}{V_a} = \frac{22.4\times10^{-3}}{3.16\times10^{-7}} = 7.08\times10^{4}$$

Hence, the molar volume is 7.08 × 104 times higher than the atomic volume.

reason, the inter-atomic separation in hydrogen gas is much larger than the s

hydrogen atom.

2.18 Explain this common observation clearly : If you look out of the window of a fast moving train, the nearby trees, houses etc. seem to move rapidly in a direction opposite to the train's motion, but the distant objects (hill tops, the Moon, the stars etc.) seem to be stationary. (In fact, since you are aware that you are moving, these distant objects seem to move with you).

Answer:

Line of sight is defined as an imaginary line joining an object and an observer'

When we observe nearby stationary objects such as trees, houses, etc. while s

moving train, they appear to move rapidly in the opposite direction because th

sight changes very rapidly.

On the other hand, distant objects such as trees, stars, etc. appear stationary

the large distance. As a result, the line of sight does not change its direction r

2.19 The principle of 'parallax' in section 2.3.1 is used in the determination of distances of very distant stars. The baseline AB is the line joining the Earth's two locations six months apart in its orbit around the Sun. That is, the baseline is about the diameter of the Earth's orbit ≈ 3 × 10^{11}m. However, even the nearest stars are so distant that with such a long baseline, they show parallax only of the order of 1" (second) of arc or so. A parsec is a convenient unit of length on the astronomical scale. It is the distance of an object that will show a parallax of 1" (second) of arc from opposite ends of a baseline equal to the distance from the Earth to the Sun. How much is a parsec in terms of metres ?

Answer:

Diameter of Earth's orbit = 3 × 1011 m

Radius of Earth's orbit, r = 1.5 × 1011 m

Let the distance parallax angle be $1''$ = 4.847 × 10−6 rad.

Let the distance of the star be D.

Parsec is defined as the distance at which the average radius of the Earth's orl

subtends an angle of $1''$.

☐ We have $\theta = \frac{r}{D}$

$$D = \frac{r}{\theta} = \frac{1.5\times10^{11}}{4.847\times10^{-6}}$$
$$= 0.309\times10^{-6} \approx 3.09\times10^{16} \text{ m}$$

Hence, 1 parsec ≈ 3.09 × 1016 m.

2.20 The nearest star to our solar system is 4.29 light years away. How much is this distance in terms of parsecs? How much parallax would this star (named Alpha Centauri) show when viewed from two locations of the Earth six months apart in its orbit around the Sun ?

Answer:

Distance of the star from the solar system = 4.29 ly

1 light year is the distance travelled by light in one year.

1 light year = Speed of light × 1 year

= 3 × 108 × 365 × 24 × 60 × 60 = 94608 × 1011 m

☐4.29 ly = 405868.32 × 1011 m

∵ 1 parsec = 3.08 × 1016 m

☐4.29 ly = $\frac{405868.32\times10^{11}}{3.08\times10^{16}}$ = 1.32 parsec

Using the relation,

$$\theta = \frac{d}{D}$$

where,

Diameter of Earth's orbit, $d = 3 \times 10^{11}$ m

Distance of the star from the Earth, $D = 405868.32 \times 10^{11}$ m

$$\therefore \theta = \frac{3\times10^{11}}{405868.32 \times 10^{11}} = 7.39\times10^{-6} \text{ rad}$$

But, 1 sec = 4.85 × 10−6 rad

☐ $$7.39\times10^{-6} \text{ rad} = \frac{7.39\times10^{-6}}{4.85\times10^{-6}} = 1.52''$$

2.21 Precise measurements of physical quantities are a need of science. For example, to ascertain the speed of an aircraft, one must have an accurate method to find its positions at closely separated instants of time. This was the actual motivation behind the discovery of radar in World War II. Think of different examples in modern science where precise

measurements of length, time, mass etc. are needed. Also, wherever you can, give a quantitative idea of the precision needed.

Answer:

It is indeed very true that precise measurements of physical quantities are ess

the development of science. For example, ultra-shot laser pulses (time interva

s) are used to measure time intervals in several physical and chemical process

X-ray spectroscopy is used to determine the inter-atomic separation or inter-p

spacing.

The development of mass spectrometer makes it possible to measure the mas

precisely.

2.22 Just as precise measurements are necessary in science, it is equally important to be able to make rough estimates of quantities using rudimentary ideas and common observations. Think of ways by which you can estimate the following (where an estimate is difficult to obtain, try to get an upper bound on the quantity) :

(a) the total mass of rain-bearing clouds over India during the Monsoon

(b) the mass of an elephant

(c) the wind speed during a storm

(d) the number of strands of hair on your head

(e) the number of air molecules in your classroom

Answer:

(a) During monsoons, a metrologist records about 215 cm of rainfall in India i

height of water column, h = 215 cm = 2.15 m

Area of country, A = 3.3 × 1012 m2

Hence, volume of rain water, V = A × h = 7.09 × 1012 m3

Density of water, ρ = 1 × 103 kg m−3

Hence, mass of rain water = ρ × V = 7.09 × 1015 kg

Hence, the total mass of rain-bearing clouds over India is approximately 7.09

kg.

(b) Consider a ship of known base area floating in the sea. Measure its depth i

d1).

Volume of water displaced by the ship, Vb = A d1

Now, move an elephant on the ship and measure the depth of the ship (d2) in

Volume of water displaced by the ship with the elephant on board, Vbe= Ad2

Volume of water displaced by the elephant = Ad2 − Ad1

Density of water = D

Mass of elephant = AD (d2 − d1)

(c) Wind speed during a storm can be measured by an anemometer. As wind l

rotates. The rotation made by the anemometer in one second gives the value

speed.

(d) Area of the head surface carrying hair = A

With the help of a screw gauge, the diameter and hence, the radius of a hair c

determined. Let it be r.

□Area of one hair = πr2

Number of strands of hair $\approx \frac{\text{Total surface area}}{\text{Area of one hair}\pi} = \frac{A}{r^2}$

(e) Let the volume of the room be V.

One mole of air at NTP occupies 22.4 l i.e., 22.4 × 10−3 m3 volume.

Number of molecules in one mole = 6.023 × 1023

□Number of molecules in room of volume V

$= \frac{6.023\times10^{23}}{22.4\times10^{-3}}\times V$ = 134.915 × 1026 V

= 1.35 × 1028 V

2.23 The Sun is a hot plasma (ionized matter) with its inner core at a temperature exceeding 107 K, and its outer surface at a temperature of about 6000 K. At these high temperatures, no substance remains in a solid or liquid phase. In what range do you expect the mass density of the Sun to be, in the range of densities of solids and liquids or gases ? Check if your guess is correct from the following data : mass of the Sun = 2.0 × 10^{30} kg, radius of the Sun = 7.0 × 108 m.

Answer:

Mass of the Sun, M = 2.0 × 1030 kg

Radius of the Sun, R = 7.0 × 108 m

Volume of the Sun, V = $\frac{4}{3}\pi R^3$

$= \frac{4}{3}\times\frac{22}{7}\times\left(7.0\times10^8\right)^3$

$= \frac{88}{21}\times343\times10^{24} = 1437.3\times10^{24}\ \text{m}^3$

Density of the Sun = $\frac{\text{Mass}}{\text{Volume}} = \frac{2.0\times10^{30}}{1437.3\times10^{24}} \sim 1.4\times10^3\ \text{kg/m}^3$

The density of the Sun is in the density range of solids and liquids. This high d
attributed to the intense gravitational attraction of the inner layers on the out
the Sun.

2.24 When the planet Jupiter is at a distance of 824.7 million kilometers from the Earth, its angular diameter is measured to be 35.72" of arc. Calculate the diameter of Jupiter.

Answer:

Distance of Jupiter from the Earth, D = 824.7 × 106 km = 824.7 × 109 m

Angular diameter = $35.72'' = 35.72 \times 4.874 \times 10^{-6}$ rad

Diameter of Jupiter = d

Using the relation,

$$\theta = \frac{d}{D}$$

$$d = \theta D = 824.7 \times 10^{9} \times 35.72 \times 4.872 \times 10^{-6}$$
$$= 143520.76 \times 10^{3} = 1.435 \times 10^{5} \text{ km}$$

2.25

A man walking briskly in rain with speed v must slant his umbrella forward ma
angle θ with the vertical. A student derives the following relation between θ ar
= v and checks that the relation has a correct limit: as v →0, θ → 0, as expec
are assuming there is no strong wind and that the rain falls vertically for a sta
man). Do you think this relation can be correct? If not, guess the correct relat

Answer:

Incorrect; on dimensional ground

The relation is $\tan\theta = v$.

Dimension of R.H.S = M0 L1 T−1

Dimension of L.H.S = M0 L0 T0

(∵ The trigonometric function is considered to be a dimensionless quantity)

Dimension of R.H.S is not equal to the dimension of L.H.S. Hence, the given re
not correct dimensionally.

To make the given relation correct, the R.H.S should also be dimensionless. O
achieve this is by dividing the R.H.S by the speed of rainfall v'.

Therefore, the relation reduces to

$\tan\theta = \frac{v}{v'}$. This relation is dimensionally correct.

2.26 It is claimed that two cesium clocks, if allowed to run for 100 years, free from any disturbance, may differ by only about 0.02 s. What does this imply for the accuracy of the standard cesium clock in measuring a time-interval of 1 s ?

Answer:

Difference in time of caesium clocks = 0.02 s

Time required for this difference = 100 years

= 100 × 365 × 24 × 60 × 60 = 3.15 × 109 s

In 3.15 × 109 s, the caesium clock shows a time difference of 0.02 s.

In 1s, the clock will show a time difference of $\frac{0.02}{3.15\times10^9}$ s.

Hence, the accuracy of a standard caesium clock in measuring a time interval

$$\frac{3.15\times10^9}{0.02}=157.5\times10^9 \text{ s} \approx 1.5\times10^{11} \text{ s}.$$

2.27 Estimate the average mass density of a sodium atom assuming its size to be about 2.5 Å. (Use the known values of Avogadro's number and the atomic mass of sodium). Compare it with the density of sodium in its crystalline phase : 970 kg m–3. Are the two densities of the same order of magnitude ? If so, why ?

Answer:

Diameter of sodium atom = Size of sodium atom = 2.5 Å

Radius of sodium atom, r = $\frac{1}{2}\times2.5 \text{ Å} = 1.25 \text{ Å}$

= 1.25 × 10−10 m

Volume of sodium atom, V = $\frac{4}{3}\pi r^3$

$$=\frac{4}{3}\times3.14\times\left(1.25\times10^{-10}\right)^3$$

According to the Avogadro hypothesis, one mole of sodium contains 6.023 ×

and has a mass of 23 g or 23 × 10−3 kg.

□ Mass of one atom = $\frac{23\times10^{-3}}{6.023\times10^{23}}$ kg

Density of sodium atom, ρ = $\dfrac{\dfrac{23\times10^{-3}}{6.023\times10^{23}}}{\dfrac{4}{3}\times3.14\times\left(1.25\times10^{-10}\right)^3}=4.67\times10^{-3} \text{ kg m}^{-3}$

It is given that the density of sodium in crystalline phase is 970 kg m−3

Hence, the density of sodium atom and the density of sodium in its crystalline not in the same order. This is because in solid phase, atoms are closely packe the inter-atomic separation is very small in the crystalline phase.

2.28.

The unit of length convenient on the nuclear scale is a fermi : 1 f = 10– 15 m.

sizes obey roughly the following empirical relation : $r = r_0 A^{\frac{1}{3}}$

where r is the radius of the nucleus, A its mass number, and r0 is a constant e about, 1.2 f. Show that the rule implies that nuclear mass density is nearly co different nuclei. Estimate the mass density of sodium nucleus. Compare it with average mass density of a sodium atom obtained in Exercise. 2.27.

Answer:

Radius of nucleus r is given by the relation,

$r = r_0 A^{\frac{1}{3}}$ ... (i)

r_0 = 1.2 f = 1.2 × 10−15 m

Volume of nucleus, V = $\frac{4}{3}\pi r^3$

$$= \frac{4}{3}\pi\left(r_0 A^{\frac{1}{3}}\right)^3 = \frac{4}{3}\pi r_0^3 A \qquad \text{... (i)}$$

Now, the mass of a nuclei M is equal to its mass number i.e.,

M = A amu = A × 1.66 × 10−27 kg

Density of nucleus,

$$\rho = \frac{\text{Mass of nucleus}}{\text{Volume of nucleus}}$$

$$= \frac{A \times 1.66 \times 10^{-27}}{\frac{4}{3}\pi r_0^3 A} = \frac{3 \times 1.66 \times 10^{-27}}{4\pi r_0^3} \text{ kg/m}^3$$

This relation shows that nuclear mass depends only on constant r_0. Hence, th mass densities of all nuclei are nearly the same.

Density of sodium nucleus is given by,

$$\rho_{\text{Sodium}} = \frac{3\times1.66\times10^{-27}}{4\times3.14\times\left(1.2\times10^{-15}\right)^3}$$

$$= \frac{4.98}{21.71}\times10^{18} = 2.29\times10^{17}\ \text{kg m}^{-3}$$

2.29 A LASER is a source of very intense, monochromatic, and unidirectional beam of light. These properties of a laser light can be exploited to measure long distances. The distance of the Moon from the Earth has been already determined very precisely using a laser as a source of light. A laser light beamed at the Moon takes 2.56 s to return after reflection at the Moon's surface. How much is the radius of the lunar orbit around the Earth ?

Answer:

Time taken by the laser beam to return to Earth after reflection from the Moor

Speed of light = 3 × 108 m/s

Time taken by the laser beam to reach Moon = $\frac{1}{2}\times2.56 = 1.28\ \text{s}$

Radius of the lunar orbit = Distance between the Earth and the Moon = 1.28 ›

= 3.84 × 108 m = 3.84 × 105 km

2.30 A SONAR (sound navigation and ranging) uses ultrasonic waves to detect and locate objects under water. In a submarine equipped with a SONAR the time delay between generation of a probe wave and the reception of its echo after reflection from an enemy submarine is found to be 77.0 s. What is the distance of the enemy submarine? (Speed of sound in water = 1450 m s^{-1}).

Answer:

Let the distance between the ship and the enemy submarine be 'S'.

Speed of sound in water = 1450 m/s

Time lag between transmission and reception of Sonar waves = 77 s

In this time lag, sound waves travel a distance which is twice the distance bet

ship and the submarine (2S).

Time taken for the sound to reach the submarine $= \frac{1}{2}\times77 = 38.5\ \text{s}$

☐ Distance between the ship and the submarine (S) = 1450 × 38.5 = 55825 ı

km

2.31 The farthest objects in our Universe discovered by modern astronomers are so distant that light emitted by them takes billions of years to reach the Earth. These objects (known as quasars) have many puzzling features, which have not yet been satisfactorily explained. What is the distance in km of a quasar from which light takes 3.0 billion years to reach us ?

Answer:

Time taken by quasar light to reach Earth = 3 billion years

= 3 × 109 years

= 3 × 109 × 365 × 24 × 60 × 60 s

Speed of light = 3 × 108 m/s

Distance between the Earth and quasar

= (3 × 108) × (3 × 109 × 365 × 24 × 60 × 60)

= 283824 × 1020 m

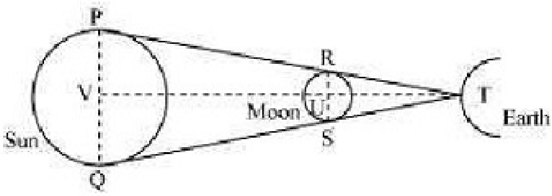
= 2.8 × 1022 km

2.32 It is a well known fact that during a total solar eclipse the disk of the moon almost completely covers the disk of the Sun. From this fact and from the information you can gather from examples 2.3 and 2.4, determine the approximate diameter of the moon.

Answer:

The position of the Sun, Moon, and Earth during a lunar eclipse is shown in th

figure.



Distance of the Moon from the Earth = 3.84 × 108 m

Distance of the Sun from the Earth = 1.496 × 1011 m

Diameter of the Sun = 1.39 × 109 m

It can be observed that ΔTRS and ΔTPQ are similar. Hence, it can be written a

$$\frac{PQ}{RS} = \frac{VT}{UT}$$

$$\frac{1.39\times10^{9}}{RS} = \frac{1.496\times10^{11}}{3.84\times10^{8}}$$

$$RS = \frac{1.39\times3.84}{1.496}\times10^{6} = 3.57\times10^{6}\ \text{m}$$

Hence, the diameter of the Moon is 3.57× 106 m.

2.33.

Question 2.33:

A great physicist of this century (P.A.M. Dirac) loved playing with numerical va
Fundamental constants of nature. This led him to an interesting observation. D
that from the basic constants of atomic physics (c, e, mass of electron, mass o
and the gravitational constant G, he could arrive at a number with the dimens
time. Further, it was a very large number, its magnitude being close to the pre
estimate on the age of the universe (~15 billion years). From the table of func
constants in this book, try to see if you too can construct this number (or any
interesting number you can think of). If its coincidence with the age of the uni
significant, what would this imply for the constancy of fundamental constants?

Answer:

One relation consists of some fundamental constants that give the age of the
by:

$$t=\left(\frac{e^2}{4\pi\varepsilon_0}\right)^2\times\frac{1}{m_p m_e^2 c^3 G}$$

Where,

t = Age of Universe

e = Charge of electrons = 1.6 ×10−19 C

ε_0 = Absolute permittivity

m_p = Mass of protons = 1.67 × 10−27 kg

m_e = Mass of electrons = 9.1 × 10−31 kg

c = Speed of light = 3 × 108 m/s

G = Universal gravitational constant = 6.67 × 1011 Nm2 kg−2

Also, $\frac{1}{4\pi\varepsilon_0}=9\times10^9$ Nm2/C2

Substituting these values in the equation, we get

$$t=\frac{\left(1.6\times10^{-19}\right)^4\times\left(9\times10^9\right)^2}{\left(9.1\times10^{-31}\right)^2\times1.67\times10^{-27}\times\left(3\times10^8\right)^3\times6.67\times10^{-11}}$$

$$=\frac{(1.6)^4\times81}{9.1\times1.67\times27\times6.67}\times10^{-76+18+62+27-24+11}\text{ s}$$

$$=\frac{(1.6)^4\times81}{9.1\times1.67\times27\times6.67\times365\times24\times3600}\times10^{-76+18+62+27-24+11}\text{ years}$$

$$\approx6\times10^{-9}\times10^{18}\text{ years}$$

$$=6\text{ billion years}$$

Multiple Choice Questions :

1. The number of significant figures in 0.06900 is
(a) 5
(b) 4
(c) 2
(d) 3
Ans : b

2. The sum of the numbers 436.32, 227.2 and 0.301 in appropriate significant figures is
(a) 663.821
(b) 664
(c) 663.8
(d) 663.82
Ans : b

3. The mass and volume of a body are 4.237 g and 2.5 cm^3, respectively. The density of the material of the body in correct significant figures is
(a) 1.6048 g cm^{-3}
(b) 1.69 g cm^{-3}
(c) 1.7 g cm^{-3}
(d) 1.695 g cm^{-3}
Ans : c

4. The numbers 2.745 and 2.735 on rounding off to 3 significant figures will give
(a) 2.75 and 2.74
(b) 2.74 and 2.73
(c) 2.75 and 2.73
(d) 2.74 and 2.74
Ans : d

5. The length and breadth of a rectangular sheet are 16.2 cm and 10.1cm, respectively. The area of the sheet in appropriate significant figures and error is
(a) 164 ± 3 cm^2
(b) 163.62 ± 2.6 cm^2
(c) 163.6 ± 2.6 cm^2
(d) 163.62 ± 3 cm^2
Ans : a

6. Which of the following pairs of physical quantities does not have same dimensional formula?
(a) Work and torque.
(b) Angular momentum and Planck's constant.
(c) Tension and surface tension.
(d) Impulse and linear momentum.
Ans : c

7. Measure of two quantities along with the precision of respective measuring instrument is

A = 2.5 m s^{-1} ± 0.5 m s^{-1} B = 0.10 s ± 0.01 s. The value of A B will be
(a) (0.25 ± 0.08) m
(b) (0.25 ± 0.5) m
(c) (0.25 ± 0.05) m
(d) (0.25 ± 0.135) m
Ans : a

8. You measure two quantities as A = 1.0 m ± 0.2 m, B = 2.0 m ± 0.2 m. We should report correct value for $\sqrt{AB}$ as:
(a) 1.4 m ± 0.4 m
(b) 1.41m ± 0.15 m
(c) 1.4m ± 0.3 m
(d) 1.4m ± 0.2 m
Ans : d

9. Which of the following measurements is most precise?
(a) 5.00 mm
(b) 5.00 cm
(c) 5.00 m
(d) 5.00 km.
Ans : a

10 The mean length of an object is 5 cm. Which of the following measurements is most accurate?
(a) 4.9 cm
(b) 4.805 cm
(c) 5.25 cm
(d) 5.4 cm
Ans : a

11 Young's modulus of steel is 1.9×10^{11} N/m^2 . When expressed in CGS units of dynes/cm^2, it will be equal to (1N = 105 dyne, 1m^2 = 10^4 cm^2)
(a) 1.9×10^{10}
(b) 1.9×10^{11}
(c) 1.9×10^{12}
(d) 1.9×10^{13}
Ans : c

12 If momentum (P), area (A) and time (T) are taken to be fundamental quantities, then energy has the dimensional formula
(a) (P^1 A^{-1} T^1)
(b) (P^2 A^1 T^1)
(c) (P^1 $A^{-1/2}$ T^1)
(d) (P^1 $A^{1/2}$ T^{-1})
Ans : d

1. In an experiment, four quantities a, b, c, and d, are measured with percentage error 1%, 2%, 3% and 4 % respectively. Quantity P is calculated as follows : P = $\frac{a^3b^2}{cd}$. Percent of error in P is _______ (NEET 2013)
(a) 14% (b) 10% (c) 7% (d) 4 %
Ans : a; Percent Error = [3 × 1% + 2 × 2% + 1 × 3% + 1 × 4%] = [3% + 4% + 3% + 4%] = 14%

2.13 On the basis of dimensions, decide which of the following relations for the displacement of a particle undergoing simple harmonic motion is not correct:
(a) y = a sin 2πt/T
(b) y = a sin vt.
(c) y = $\frac{a}{T}$ sin (π/2)
(d) y = a$\sqrt{2}$ $[sin\frac{2\pi}{T} - \cos\frac{2\pi t}{T}]$
Ans : (b), (c)

2.14 If P, Q, R are physical quantities, having different dimensions, which of the following combinations can never be a meaningful quantity?
(a) (P – Q)/R
(b) PQ – R
(c) PQ/R
(d) (PR – Q^2)/R
(e) (R + Q)/P
Ans : (a), (e)

2.15 Photon is quantum of radiation with energy E = hν where ν is frequency and h is Planck's constant. The dimensions of h are the same as that of
(a) Linear impulse
(b) Angular impulse
(c) Linear momentum
(d) Angular momentum
Ans : (b), (d)

2.16 If Planck's constant (h) and speed of light in vacuum (c) are taken as two fundamental quantities, which one of the following can, in addition, be taken to express length, mass and time in terms of the three chosen fundamental quantities?
(a) Mass of electron (me)
(b) Universal gravitational constant (G)
(c) Charge of electron (e)
(d) Mass of proton (m_p)
Ans : (a), (b), (d)

2.17 Which of the following ratios express pressure?
(a) Force/ Area
(b) Energy/ Volume

(c) Energy/ Area
(d) Force/ Volume
Ans : (a), (b)

2.18 Which of the following are not a unit of time?
(a) Second
(b) Parsec
(c) Year
(d) Light year
Ans : (b), (d)

2.19 Why do we have different units for the same physical quantity?
Ans : Because, bodies differ in order of magnitude significantly in respect to the same physical quantity. For example, interatomic distances are of the order of angstroms, inter-city distances are of the order of km, and interstellar distances are of the order of light year.

2.20 The radius of atom is of the order of 1 Å and radius of nucleus is of the order of fermi. How many magnitudes higher is the volume of atom as compared to the volume of nucleus?
Ans : 10^{15}

2.21 Name the device used for measuring the mass of atoms and molecules
Ans : Mass spectrograph

2.22 Express unified atomic mass unit in kg.
Ans : 1 u = 1.67×10^{-27} kg

2.23 A function f (θ) is defined as:

$$f(\theta) = 1-\theta + \frac{\theta^2}{2!} - \frac{\theta^3}{3!} + \frac{\theta^4}{4!} \cdots$$

Why is it necessary for q to be a dimensionless quantity?
Ans : Since f(θ) is a sum of different powers of θ, it has to be dimensionless

2.24 Why length, mass and time are chosen as base quantities in mechanics?
Ans : Because all other quantities of mechanics can be expressed in terms of length, mass and time through simple relations.

2.25 The dimensional formula for the constant a in Vander Walls Gas equation $(P + (\frac{a}{V^2})\,(V-b)$ = RT is ____
(a) $[ML^4T^{-1}]$
(b) $[ML^3T^{-2}]$
(c) $[ML^4T^{-2}]$
(d) $[ML^5T^{-2}]$ [AIIMS 1999] Ans : (d)

2.26. What is the dimensional formula of gravitational constant _____

(a) $[ML^2T^{-1}]$

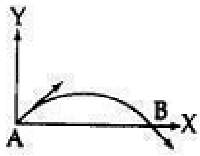
(b) $[ML^{-1}T^{-2}]$

(c) $[M^{-1}L^3T^{-2}]$

(d) None of these [AIIMS 2000] Ans : (c)

2.27

2. The velocity of a projectile at the initial point A is $(2\hat{i} + 3\hat{j})$ m/s. It's velocity (in m/s) at point B is :



(1) $-2\hat{i} - 3\hat{j}$

(2) $-2\hat{i} + 3\hat{j}$

(3) $2\hat{i} - 3\hat{j}$

(4) $2\hat{i} + 3\hat{j}$

2.28

3. A stone falls freely under gravity. It covers distances h_1, h_2 and h_3 in the first 5 seconds, the next 5 seconds and the next 5 seconds respectively. The relation between h_1, h_2 and h_3 is :

(1) $h_1 = 2h_2 = 3h_3$

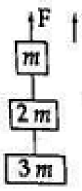
(2) $h_1 = \frac{h_2}{3} = \frac{h_3}{5}$

(3) $h_2 = 3h_1$ and $h_3 = 3h_2$

(4) $h_1 = h_2 = h_3$

2.29

4. Three blocks with masses m, 2 m and 3 m are connected by strings, as shown in the figure. After an upward force F is applied on block m, the masses move upward at constant speed v. What is the net force on the block of mass 2 m?
(g is the acceleration due to gravity)



(1) zero
(2) 2 m g
(3) 3 m g
(4) 6 m g

2.30

5. The upper half of an inclined plane of inclination θ is perfectly smooth while lower half is rough. A block starting from rest at the top of the plane will again come to rest at the bottom, if the coefficient of friction between the block and lower half of the plane is given by:

(1) $\mu = \frac{1}{\tan\theta}$

(2) $\mu = \frac{2}{\tan\theta}$

(3) $\mu = 2\tan\theta$

(4) $\mu = \tan\theta$

2.31

6. A uniform force of $(3\hat{i} + \hat{j})$ newton acts on a particle of mass 2 kg. Hence the particle is displaced from position $(2\hat{i} + \hat{k})$ meter to position $(4\hat{i} + 3\hat{j} - \hat{k})$ meter. The work done by the force on the particle is:

(1) 9 J
(2) 6 J
(3) 13 J
(4) 15 J

2.32

7. An explosion breaks a rock into three parts in a horizontal plane. Two of them go off at right angles to each other. The first part of mass 1 kg moves with a speed of 12 ms^{-1} and the second part of mass 2 kg moves with 8 ms^{-1} speed. If the third part flies off with 4 ms^{-1} speed, then its mass is :

2.33

43. In a n-type semiconductor, which of the following statement is true :

(1) Electrons are majority carriers and trivalent atoms are dopants.

(2) Electron are minority carriers and pentavalent atoms are dopants.

(3) Holes are minority carriers and pentavalent atoms are dopants.

(4) Holes are majority carriers and trivalent atoms are dopants.

CH 3 : Kinematics Part I

MOTION IN A STRAIGHT LINE
(8 Hours, 07 Marks (2M-1Q, 5M-1QLA))

Syllabus : **Frame of reference (inertial and non-inertial frames). Motion in a straight line: Position-time graph, speed and velocity. Elementary concepts of differentiation and integration for describing motion. Uniform and non-uniform motion, average speed and instantaneous velocity. Uniformly accelerated motion, velocity – time, position-time graphs, relations for uniformly accelerated motion (graphical treatment).**

2.1. Frame of reference (inertial and non-inertial frames) :
In order to specify position, we need to use a reference point and a set of axes. It is convenient to choose a rectangular coordinate system consisting of three mutually perpenducular axes, labelled X-, Y-, and Z- axes. The point of intersection of these three axes is called **origin (O)** and serves as the reference point. The coordinates (x, y. z) of an object describe the position of the object with respect to this coordinate system. To measure time, we position a clock in this system. This coordinate system along with a clock constitutes a **frame of reference**.
If one or more coordinates of an object change with time, we say that the object is in motion. Otherwise, the object is said to be at rest with respect to this frame of reference.

1. What is motion & rectilinear motion ?
Motion is change in position of an object with time. The study of motion of objects along a straight line, also known as rectilinear motion.

2. What is Kinematics ?
Kinematics is a study to describe motion without going into the causes of motion.

3. What do you mean by inertial and non-inertial frames ?

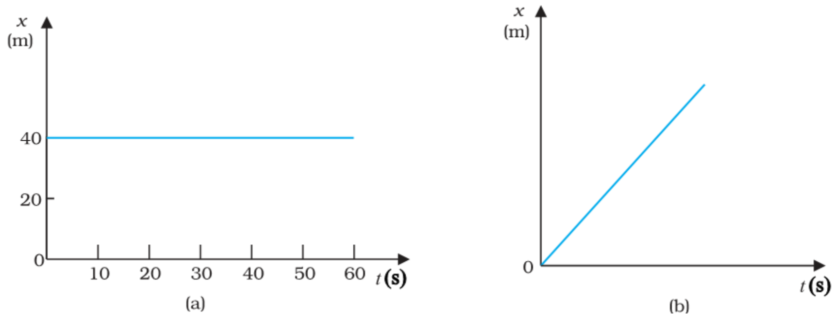
4. Explain path length during motion ?
The length of the distance covered by an object is called the path length.

5. Define the term displacement ?
Displacement of an object is defined as the change in position of an object.
Let x_1 and x_2 be the positions of an object at time t_1 and t_2. Then its displacement, denoted by Δx, in time $\Delta t = (t_2 - t_1)$, is given by the difference between the final and initial positions : $\Delta x = x_2 - x_1$. Displacement has both magnitude and direction. Such quantities are represented by vectors. The magnitude of displacement may or may not be equal to the path length traversed by an object. The magnitude of the displacement for a course of motion may be zero but the corresponding path length is not zero. Displacement can be positive or negative.
The area under the velocity-time curve between times t_1 and t_2 is equal to the displacement of the object during that interval of time.

2.2. Motion in a straight line: Position-time graph, speed and velocity.

If an object moving along the straight line covers equal distances in equal intervals of time, it is said to be in **uniform motion** along a straight line.

Motion of an object can be represented by a position-time graph as you have already learnt about it. Such a graph is a powerful tool to represent and analyse different aspects of motion of an object. For motion along a straight line, say X-axis, only x-coordinate varies with time and we have an x-t graph.



The position-time graph of (a) stationary object, and (b) an object in uniform motion is shown in Figure (1).

6. Explain average velocity ?

Average velocity is defined as the change in position or displacement (Δx) divided by the time intervals (Δt), in which the displacement occurs : Average velocity = $\frac{Displacement}{Time\ interval}$

$$\bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}$$

where x_2 and x_1 are the positions of the object at time t_2 and t_1, respectively. Here the bar over the symbol for velocity is a standard notation used to indicate an average quantity. The SI unit for velocity is m/s or m s^{-1}, although km h^{-1} is used in many everyday applications. Like displacement, average velocity is also a vector quantity.

The average velocity can be positive or negative depending upon the sign of the displacement. It is zero if the displacement is zero. Fig. 3.2 shows the x-t graphs for an object, moving with positive velocity (Fig. 3.2a), moving with negative velocity (Fig. 3.2b) and at rest (Fig. 3.2c).

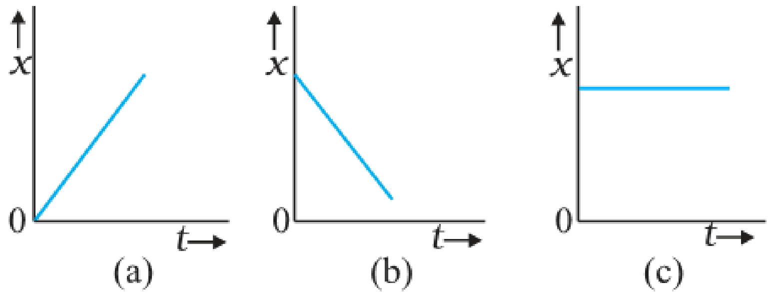


Fig. 3.2 : Position-time graph for an object (a) moving with positive velocity, (b) moving with negative velocity, and (c) at rest

2.3. Elementary concepts of differentiation and integration for describing motion.

In terms of derivatives, instantaneous velocity and acceleration are defined as

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} \qquad a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

2.4. Uniform and non-uniform motion, average speed and instantaneous velocity :

If an object moving along the straight line covers equal distances in equal intervals of time, it is said to be in **uniform motion** along a straight line.

7. Define average speed ?

To describe the rate of motion over the actual path, we introduce another quantity called average speed. Average speed is defined as the total path length travelled divided by the total time interval during which the motion has taken place :

Average speed = $\frac{Total\ path\ length}{Total\ time\ interval}$

Average speed has the same unit (m s^{-1}) as that of velocity. But it does not tell us in what direction an object is moving.

The average speed need not equal to the magnitude of the average velocity. This happens because the motion here involves change in direction so that the path length is greater than the magnitude of displacement. This shows that speed is, in general, greater than the magnitude of the velocity.

8. Explain instantaneous velocity ?

The velocity at an instant is defined as the limit of the average velocity as the time interval Δt becomes infinitesimally small. In other words,

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

Instantaneous velocity is the rate of change of position with respect to time, at that instant.

9. Explain instantaneous speed ?

Note that for uniform motion, velocity is the same as the average velocity at all instants. Instantaneous speed or simply speed is the magnitude of velocity.

2.5. Uniformly accelerated motion, velocity – time, position-time graphs, relations for uniformly accelerated motion (graphical treatment).

10. Define acceleration, uniform acceleration and instantaneous acceleration ?

Acceleration of a body is the rate of change of velocity with time.

The average acceleration a over a time interval is defined as the change of velocity divided by the time interval :

$$\overline{a} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}$$

where v_2 and v_1 are the instantaneous velocities or simply velocities at time t_2 and t_1. It is the average change of velocity per unit time. The SI unit of acceleration is m s^{-2} .

Instantaneous acceleration is defined in the same way as the instantaneous velocity :

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

The acceleration at an instant is the slope of the tangent to the v–t curve at that instant.

The acceleration of an object at a particular time is the slope of the velocity-time graph at that instant of time. For uniform motion, acceleration is zero and the x-t graph is a straight line inclined to the time axis and the v-t graph is a straight line parallel to the time axis. For motion with uniform acceleration, **x-t graph is a parabola** while the **v-t graph is a straight line** inclined to the time axis.

Since velocity is a quantity having both magnitude and direction, a change in velocity may involve either or both of these factors. Acceleration, therefore, may result from a change in speed (magnitude), a change in direction or changes in both. Like velocity, acceleration can also be positive, negative or zero. Position-time graphs for motion with positive, negative and zero acceleration are shown in Figs. 3.3 (a), (b) and (c), respectively.

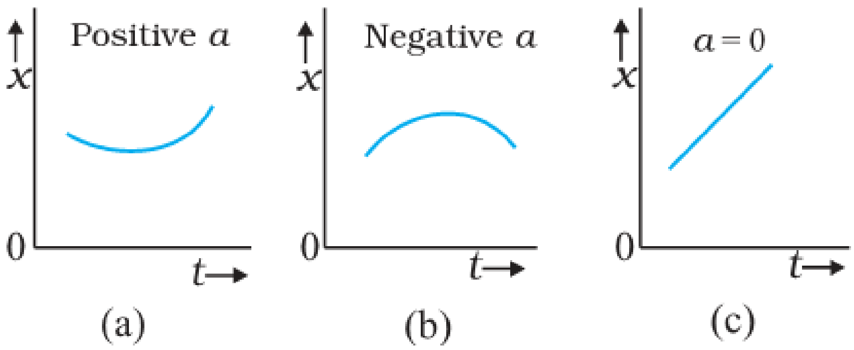


Fig. 3.3 : Position-time graph for motion with (a) positive acceleration; (b) negative acceleration, and (c) zero acceleration.

The Fig. 3.4 gives the velocity- time graph for motion with constant acceleration for the following cases :

(a) An object is moving in a positive direction with a positive acceleration, for example the motion of the car in Fig. 3.4 between t = 0 s and t = 10 s.

(b) An object is moving in positive direction with a negative acceleration, for example, motion of the car in Fig 3.4 between t = 18 s and 20 s.

(c) An object is moving in negative direction with a negative acceleration, for example the motion of a car moving from O in Fig. 3.4 in negative x-direction with increasing speed.

(d) An object is moving in positive direction till time t1, and then turns back with the same negative acceleration, for example the motion of a car from point O to point Q in Fig. 3.4 till time t_1 with decreasing speed and turning back and moving with the same negative acceleration.

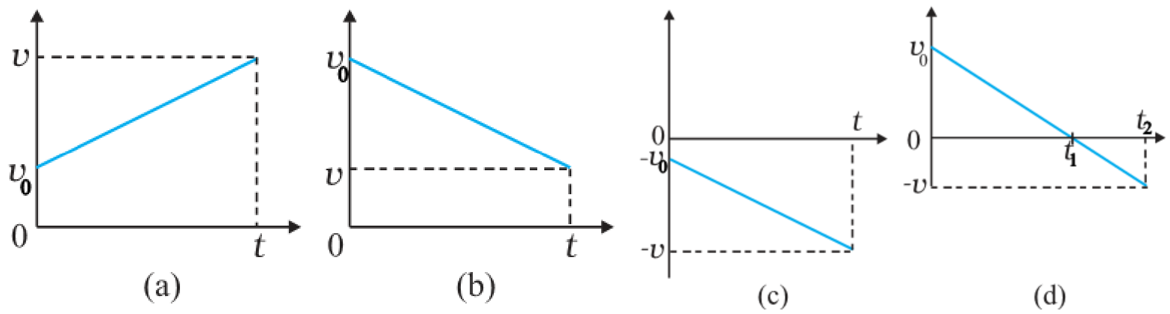


Fig. 3.4 : Velocity–time graph for motions with constant acceleration. (a) Motion in positive direction with positive acceleration, (b) Motion in positive direction with negative acceleration, (c) Motion in negative direction with negative acceleration, (d) Motion of an object with negative acceleration that changes direction at time t_1. Between times 0 to t_1, its moves in positive x - direction and between t_1 and t_2 it moves in the opposite direction.

An interesting feature of a velocity-time graph for any moving object is that the area under the curve represents the displacement over a given time interval.

11. Kinematic equations for uniformly accelerated motion :

For uniformly accelerated motion, we can derive some simple equations that relate displacement (x), time taken (t), initial velocity (v_0), final velocity (v) and acceleration (a). Equation (1) gives a relation between final and initial velocities v and v_0 of an object moving with uniform acceleration a : **$v = v_0 + at$ ---------- (1)**

This relation is graphically represented in Fig. 1. The area under this curve is :
Area between instants 0 and t
= Area of triangle ABC + Area of rectangle OACD
$= \frac{1}{2}(v - v_0)t + v_0 t$

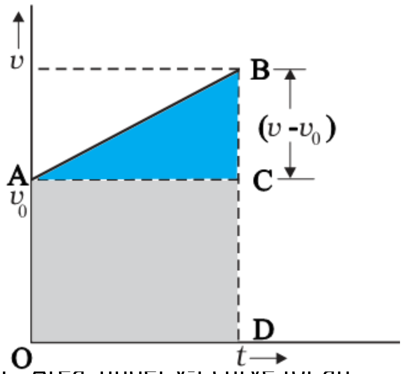


Fig. 1 : Area under v-t curve for an object with uniform acceleration.

We know that the area under v-t curve represents the displacement.
Therefore, the displacement x of the object is :
$x = \frac{1}{2}(v - v_0)t + v_0 t$ ------- (2)
But $(v - v_0) - at$, Therefore, $x = \frac{1}{2}at^2 + v_0 t$

or **$x = v_0 t + \frac{1}{2}at^2$ -------------- (3)**

Equation (2) can also be written as $x = (\frac{v + v_0}{2})t = \bar{v}t$ ------- (4)
From Equation (1) we can write $t = (v - v_0)/a$ and substituting this in Equation (4), we get

$$x = \bar{v}t = \left(\frac{v + v_0}{2}\right)\left(\frac{v - v_0}{a}\right) = \left(\frac{v^2 - v_0^2}{2a}\right)$$

Or **$v^2 = v_0^2 + 2ax$ ----------- (5)**

The Equations (1), (3), and (5) are connecting five quantities v_0, v, a, t and x and are called **kinematic equations** of rectilinear motion for constant acceleration.

If the position of the object at time t = 0 is 0. If the particle starts at $x = x_0$, x in above equations is replaced by $(x - x_0)$.

12. Relative velocity :

The concept of relative velocity is introduced to study the velocity of one object with respect to another moving object.

Consider two objects A and B moving uniformly with average velocities v_A and v_B in one dimension, say along x-axis. If $x_A(0)$ and $x_B(0)$ are positions of objects A and B, respectively at time t = 0, their positions $x_A(t)$ and $x_B(t)$ at time t are given by:

$x_A(t) = x_A(0) + v_A t$ ------------------- (6a)

$x_B(t) = x_B(0) + v_B t$ ------------------- (6b)

Then, the displacement from object A to object B is given by $x_{BA}(t) = x_B(t) - x_A(t)$

$= [x_B(0) - x_A(0)] + (v_B - v_A)t.$

It tells us that as seen from object A, object B has a velocity $v_B - v_A$ because the displacement from A to B changes steadily by the amount $v_B - v_A$ in each unit of time. We say that the velocity of object B relative to object A is $v_B - v_A$:

$v_{BA} = v_B - v_A$ ------------------- (7a)

Similarly, velocity of object A relative to object B is:

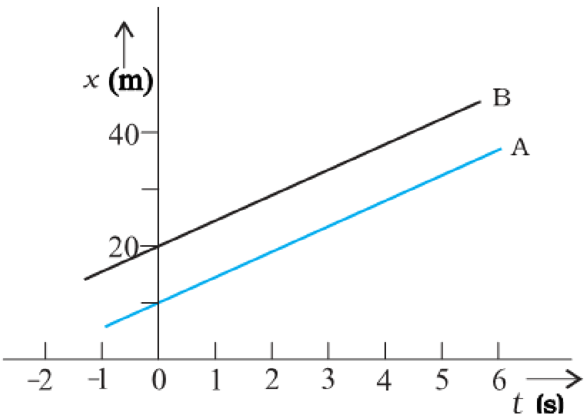
$\mathbf{v_{AB} = v_A - v_B}$ ------------------- **(7b)** **This shows : $v_{BA} = -v_{AB}$**

Special Cases :

(a) If $v_B = v_A$, $v_B - v_A = 0$. (Fig. 4 (a))

(b) (b) If $v_A > v_B$, $v_B - v_A$ is negative. One graph is steeper than the other and they meet at a common point. (Fig. 4 (b))

(c) Suppose v_A and v_B are of opposite signs. (Fig. 4 (c))



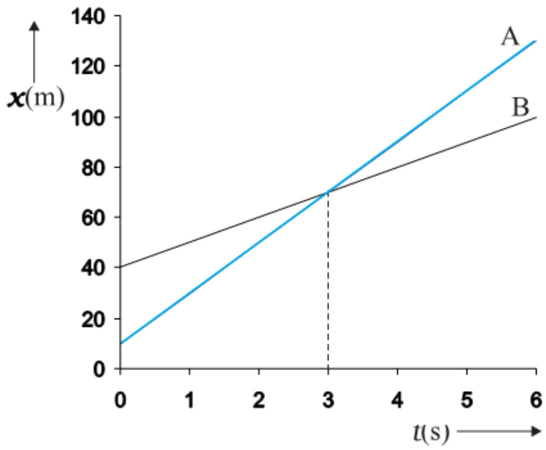


Fig. 4 (a): Position-time graphs of two objects with equal velocities. Fig. (b) : Position-time graphs of two objects with unequal velocities, showing the time of meeting.

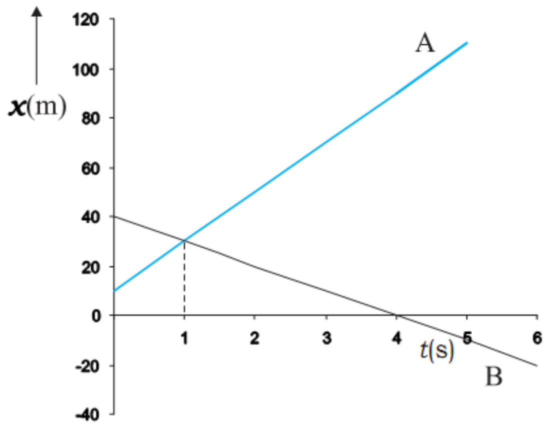


Fig. (c) : Position-time graphs of two objects with velocities in opposite directions, showing the time of meeting.

One mark questions

1. When is an object said to be in motion?

An object is said to be in motion if its position changes with time

2. What is rectilinear motion?

The motion of an object along a straight line is known as rectilinear motion

3. What is kinematics ?

Kinematics deals with the study of motion of bodies without considering the causes of motion.

4. What is required to specify the position of an object?

To specify the position of an object, a reference point called origin is required.

5. What is meant by path length?

The total distance traversed by an object is called path length.

6. What is displacement?

The change of position in a particular direction or the distance between the initial and final position of the object is called displacement.

7. What is meant by uniform motion?

If an object moving along the straight line covers equal distances in equal intervals of time, then it is said to have uniform motion.

8. What is the position - time graph?

A graph of position (along y - axis) against time (along x - axis) is known as position time graph.

9. Define average velocity.

Average velocity is defined as the displacement (Δx) divided by time interval (Δt).

$$\bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}$$

10. Define average speed.

Average speed is defined as the total path length traveled divided by the total time taken. Average speed = (path length / total time interval)

11. Define instantaneous velocity of a body in terms of its average velocity.

The instantaneous velocity is defined as the limit of the average velocity as the time interval Δt tends to zero.

$$v = \lim_{\Delta t \to 0} \bar{v} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

12. The average velocity of a body is equal to its instantaneous velocity. What do you conclude by this?

The body is moving with constant velocity.

13. What does the slope of position - time graph represent?

Velocity.

14. Define average acceleration.

Average acceleration is the change in velocity divided by the time interval during which the change occurs. $\vec{a} = \frac{\Delta v}{\Delta t}$

15. Define instantaneous acceleration of a body in terms of its average acceleration.

The instantaneous acceleration is defined as the limit of the average acceleration as the time interval Δt tends to zero.

$$a = \lim_{\Delta t \to 0} \bar{a} = \lim_{\Delta t \to 0} \frac{\Delta V}{\Delta t} = \frac{dV}{dt}.$$

16. What does the slope of velocity - time graph represent?

Acceleration.

17. What does area under velocity - time graph represent?

Displacement for a given time interval.

18. When is relative velocity of two moving objects zero?

Relative velocity is zero when the two objects move with same velocity in same direction.

19. What is the acceleration of a body moving with constant velocity?

Zero.

Two mark question

1. Distinguish between distance and displacement.

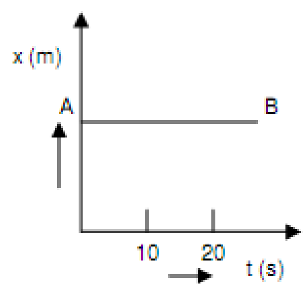
(i) The distance is the length of path traversed. The displacement is the change of position in a particular direction.

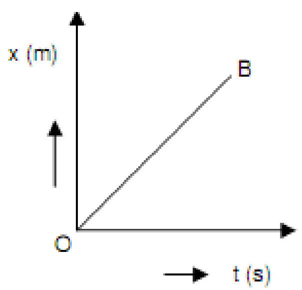
(ii) Distance is a scalar. But displacement is a vector.

(iii) When a body returns to initial position, then distance is not zero but displacement is zero.

2. Draw the position - time graph for an object (i) at rest (ii) with uniform motion.

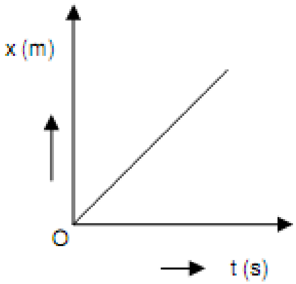
(i) rest (ii) uniform motion

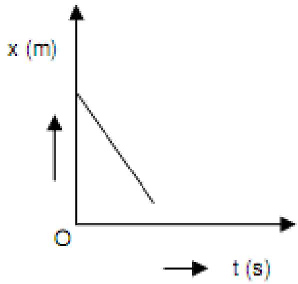




3. Draw the position - time graph for an object (a) moving with positive velocity and (b) moving with negative velocity.

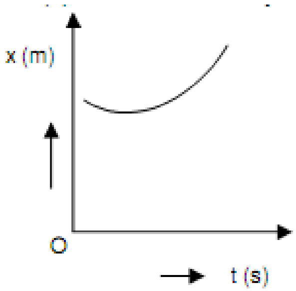
(a) Positive velocity (b) Negative velocity

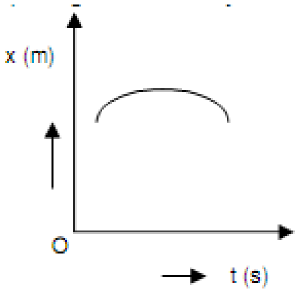


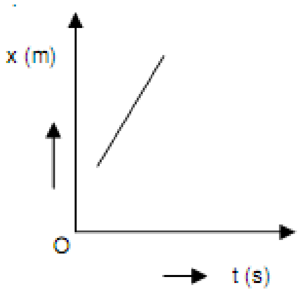


4. Draw position - time graph for motion with (a) positive acceleration (b) negative acceleration (c) zero acceleration.

(a) Positive velocity (b) Negative velocity (c) a = 0

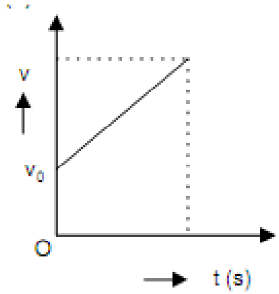


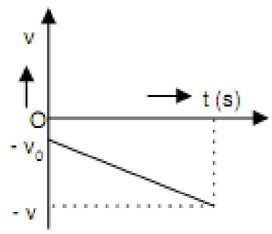




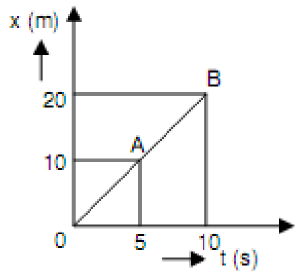
5. Draw velocity - time graphs for motion in (a) positive direction with positive acceleration (b) negative direction with negative acceleration.

(a) Positive acceleration (b) Negative acceleration





6. Find the velocity of the particle for the time interval t = 5 to t = 10 s from the following graph.



The velocity for the time interval $t_1 = 5s$ to $t_2 = 10s$ is given by

$$v = \frac{x_2 - x_1}{t_2 - t_1} = \frac{20 - 10}{10 - 5} = \frac{10}{5} = 2ms^{-1}$$

7. The displacement (in metre) of a particle moving along x - axis given by $x = 20t + 10t^2$. Calculate the instantaneous velocity at t = 2s.

We have $x = 20t + 10t^2$
Velocity $v = \frac{dx}{dt} = 20 + 20t$
Instantaneous velocity at t = 2s is v = 20 + 20 x 2 = 60 ms^{-1}.

8. A ball is thrown vertically upward and it reaches a height of 90 m. Find the velocity with which it was thrown.
v = 0, x = 90 m, g = 9.8 ms−2, v = ?
Using the equation $v^2 = v_0^2 + 2gx$ we get, $0 = v_0^2 - 2 \times 9.8 \times 90$
$\therefore \; v_0^2 = 2 \times 9.8 \times 90$
$v_0 = \sqrt{2 \times 9.8 \times 90} = 42 \text{ ms}^{-1}$

9. Define relative velocity with an example.
Relative velocity means velocity of one object w.r.t the other object.
Example : Consider two trains on parallel tracks with same velocity in same direction. Although both the trains are in motion w.r.t the ground, for an observer in one train, the other train does not appear to move. In this case the relative velocity becomes zero.

10. A car travels with a uniform velocity of 20 ms^{-1}. The driver applies the brakes and the car comes to rest in 10 second. Calculate the retardation.
$v_0 = 20 \text{ ms}^{-1}, v = 0, t = 10 \text{ s}.$

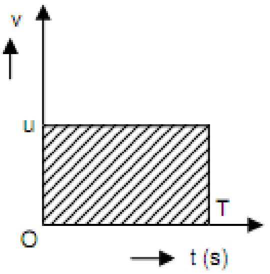
$$a = \frac{v - v_0}{t} = \frac{0 - 20}{10} = -2 \text{ ms}^{-2}$$

$\therefore$ Retardation = 2 ms^{-2}

Four Mark questions

1. a) What is the velocity - time graph?
b) Show that area under velocity - time graph is equal to displacement.

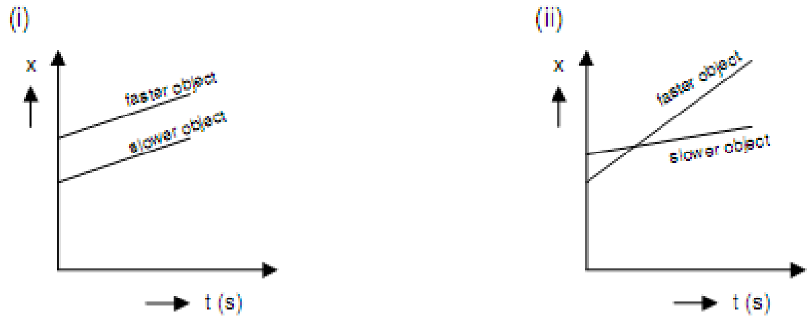
When instantaneous velocities of a particle in motion are plotted against time, the resultant graph is called velocity - time graph.



Area under the v - t graph is the area of the rectangle of height u and base T.
Therefore Area = u x T …………………… (1)
By definition, displacement during this time interval = u x T………….. (2)
From Equation equations (1) and (2), Area under velocity - time graph is equal to displacement.

2. a) Define relative velocity of an object w.r.t another.
b) Draw position - time graphs of two objects moving along a straight line when their relative velocity is (i) zero and (ii) non - zero

Relative velocity is the velocity of one object w.r.t another.
For example if A and B are two objects moving uniformly with average velocities v_A and v_B in one dimension, then the velocity of object B relative to object A is v_B - v_A. i.e. $v_{BA} = v_B - v_A$.
Similarly, velocity of object A relative to object B is
$v_{AB} = v_A - v_B$.



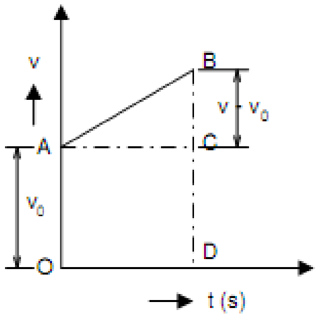
3. What is the significance of velocity - time graph?

Significance of velocity - time graph
(i) It represents the nature of motion of the particle.
(ii) Instantaneous velocity and instantaneous acceleration can be obtained from the curve.
(iii) Equations of motion along a straight line can be derived.
(iv) Area under velocity - time graph in a given time interval represents the distance traveled by the particle in that time interval.

Five marks theory questions :

1. Derive the equation of motion x = $v_0t + \frac{1}{2}at^2$ from v - t graph.

Consider an object moving with an initial velocity v_0 under constant acceleration 'a'. After 't' second, let v be its velocity and x the displacement. Let AB represent the velocity - time graph of the object. Here OA represents v_0, DB represents v and OD represents 't'.



The area under v - t graph represents the displacement.

∴ x = area under AB
= area of the rectangle OACD + area of Δ ABC.
= (OA × OD) + ½ (AC × BC)

$= v_0t + \frac{1}{2}t(v - v_0)$ ……… (1)

But $v = v_0 + at$

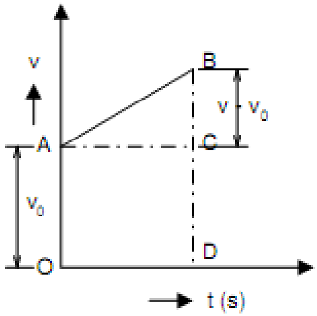
$\therefore v - v_0 = at$

Substituting this in equation (1) we have

$x = v_0t + \frac{1}{2}at^2$

2. Derive the equations of motion (i) at $v = v_0 + at$ and (ii) $v^2 = v_0^2 + 2ax$ from v - t graph.

Consider an object moving with an initial velocity v_0 under constant acceleration 'a'. After 't' second, 'v' be its velocity and 'x' the displacement. Let AB represent the velocity - time graph. Here OA represents v_0, DB represents 'v' and OD represents 't'.



(i) to derive $v = v_0 + at$

The slope of velocity - time graph represents uniform acceleration 'a'

$\therefore$ Acceleration = slope = $\frac{BC}{AC}$

$\therefore\ a = \frac{v - v_0}{t}$

$\therefore\ v - v_0 = at$

$\therefore\ v = v_0 + at$ .... (1)

(ii) The object has traveled distance 'x' in time 't' with average velocity $\bar{a}$ given as

$\bar{a} = \frac{v + v_0}{2}$

$x = \bar{a}t$

$= \left(\frac{v + v_0}{2}\right)t$ ....(2)

From equation (1) $t = \frac{v - v_0}{a}$

Substituting this in equation (1) we have

$x = \left(\frac{v + v_0}{2}\right)\left(\frac{v - v_0}{a}\right)$

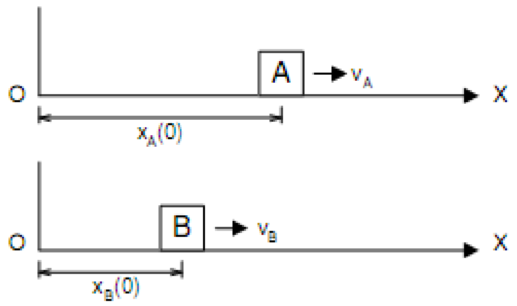
$= \frac{v^2 - v_0^2}{2a}$

$\therefore\ v^2 - v_0^2 = 2ax$

$\therefore\ v^2 = v_0^2 + 2ax$ .... (3)

3. Derive an expression for relative velocity between two moving objects.

Consider two objects A and B moving along x - axis uniformly with average velocities v_A and v_B respectively as shown in the figure.



Let $x_A(0)$ and $x_B(0)$ be their displacements from the origin 0 at t = 0

Let $x_A(t)$ and $x_B(t)$ be their displacements at time 't'.

$\therefore v_A = \frac{x_A(t) - x_A(0)}{t}$ and $v_A = \frac{x_B(t) - x_b(0)}{t}$

$\therefore x_A(t) - x_B(t) = [x_A(0) - x_B(0)] + [v_A(t) - v_B(t)]$

$= [x_A(0) - x_B(0)] + (v_A - v_B)t$ .... (1)

Change in displacement between the 2 bodies in time t

$= [x_A(t) - x_B(t)] - [x_A(0) - x_B(0)]$

$= [(x_A(0) - x_B(0)) + (v_A - v_B)t] - [(x_A(0) - x_B(0)]$ (because of equation (1))

$= (v_A - v_B)t$

Therefore relative velocity of A w.r.t B is

$v_{AB} = \frac{\text{change in displacement}}{\text{time}} = \frac{(v_A - v_B)t}{t}$

$\therefore v_{AB} = v_A - v_B$

Similarly we can show that relative velocity of B w.r.t A is $v_{BA} = v_B - v_A$.

Five mark problem :

1. A car moving along a straight highway with speed of 126 km h^{-1} to brought to stop within a distance of 200 m. What is the retardation of the car and how long does it take for the car to stop?

$$v_0 = 126 kmh^{-1}, = \frac{126 \;\; 1000}{3600} = 35 \;ms^{-1}, v = 0, x = 200 \;m$$

Applying $v^2 = v_0^2 + 2ax$

$$0 = 35^2 + 2a \;x\; 200$$

$$\therefore \; 0 = -\frac{35^2}{200 \;x\; 2} = -3.06 \;ms^{-2}$$

Applying $v = v_0 + at$

$$0 = 35 - 3.06 \;x\; t$$

$$t = \frac{35}{3.06} = 11.44 \text{ second}$$

2. The displacement (in metre) of a particle moving along x - axis is given by x = At^2 + B, where A = 2m and B = 3m. Calculate (i) average velocity between t = 3s and t = 5s. (ii) instantaneous velocity at t = 5s and (iii) instantaneous acceleration.

(i) Average velocity :

At $t_1 = 3s$, the displacement of the particle is

$$x_1 = 2.3^2 + 3 = 21 \;\; m$$

At $t_2 = 5s$, the displacement of the particle is

$$x_2 = 2.5^2 + 3 = 53 \;\; m$$

Average velocity $\bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{53 - 21}{5 - 3} = \frac{32}{2} = 16 \;\; ms^{-1}$

(ii) Instantaneous velocity :

$$v = \frac{dx}{dt} = \frac{d}{dt}\left(At^2 + B\right) = 2At$$

At t = 5s, v = 2 x 2 x 5 = 20 ms^{-1}

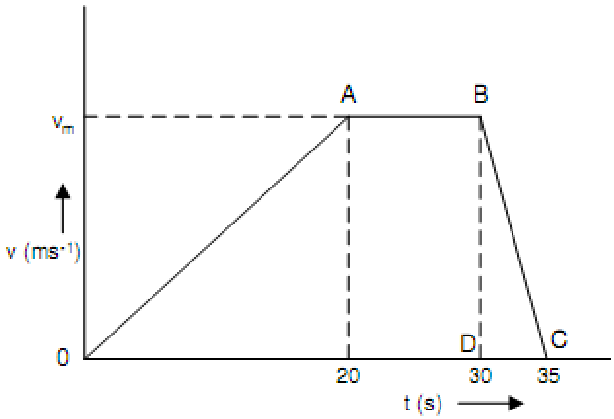
(iii) Instantaneous acceleration :

$$a = \frac{dv}{dt} = \frac{d}{dt}(2At) = 2A$$

$$= 2 \;x\; 2 = 4 \;\; ms^{-2}$$

3. A car starts from rest and accelerates uniformly at a rate of 2 ms^{-2} for 20 second. It then maintains a constant velocity for 10 second. The brakes are then applied and the car is uniformly retarded and comes to rest in 5 second. Draw the velocity - time graph for the motion and find : (i) the maximum velocity (ii) the retardation in the last 5 second (iii) total distance traveled and (iv) average velocity.

The velocity - time graph for the motion of the car is shown below



(i) Maximum velocity :
a = 2 ms^{-2}, v_0 = 0, t = 20 s
$v_m = v_0 + at$ = 0 + 2 x 20
= 40 ms^{-1}

(ii) Retardation :
Retardation is equal to the slope of the line BC

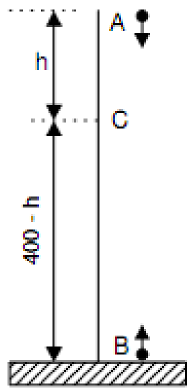
$$= -\frac{BC}{DC} = \frac{40}{5} = 8\,ms^{-2}$$

(iii) Total distance traveled : S = Area of trapezium OABC = ½ (AB + OC)BD

= ½ (10 + 35)40 = 900 m

(iv) Average velocity : = s/t = 900/35 = 25.71 ms^{-1}

4. A stone is dropped from the top of a tower 400 m high and at the same time another stone is projected upward vertically from the ground with a velocity of 100 ms^{-1}. Find where and when the two stones will meet.



Suppose the two stones meet after 't' second, when the stone from the top of the tower has covered a distance 'h' as shown in the figure. We have the equation x = $v_0 t + \frac{1}{2}at^2$

For downward motion, v_0 = 0, g = 9.8 ms^{-2},

h = $v_0 t + \frac{1}{2}gt^2$

h = $0 + \frac{1}{2}\,9.8\,t^2$ or h = $4.9\,t^2$ ------- (1)

For upward motion, v_0 = 100 ms^{-1}, g = - 9.8 ms^{-2},

$$400 - h = 100t - \frac{1}{2} \times 9.8t^2$$

$$400 - h = 100t - 4.9\ t^2$$

Substituting the values of h from equation (1) we have

$$400 - 49t^2 = 100t - 49t^2$$

$$\therefore 400 = 100\,t$$

$$\Rightarrow t = 4 \text{ second}$$

Substituting t = 4 s in equation (1) we have

$$h = 4.9 \times 4^2$$

$$= 78.4 \text{ m}$$

Therefore the two stones meet at 78.4 m below the top of the tower after 4 second.

5. Two trains are moving in opposite directions. Train A moves east with a speed of 10 ms^{-1} and train B moves west with a speed of 20 ms^{-1}. What is the (i) relative velocity of B w.r.t A and (ii) the relative velocity of ground w.r.t B. (iii) A dog is running on the roof of train A against its motion with a velocity of 5 ms^{-1} w.r.t train A. What is the velocity of the dog as observed by a man standing on the ground?

Let the direction of travel from west to the east be considered as positive direction.
Speed of train A w.r.t earth, $v_{AE} = 10\ ms^{-1}$
Speed of train B w.r.t earth, $v_{BE} = -20\ ms^{-1}$
(i) Relative velocity of train B w.r.t train A :

$$= v_{BA} = v_{BE} + v_{EA}$$

$$= v_{BE} - v_{AE}$$

$$= -20 - 10$$

$$= -30\ ms^{-1} \text{ from east to west}$$

(ii) Relative velocity of ground (i.e. earth) w.r.t B :

$$= v_{EB} = v_{EE} + v_{EB}$$

$$= v_{EE} - v_{BE}$$

$$= 0 - (-20)$$

$$= 20\ ms^{-1} \text{ from west to east}$$

(iii) Velocity of dog w.r.t train A is $v_{DA} = -5\ ms^{-1}$

$$v_{DA} = v_{DE} + v_{EA}$$

$$= v_{DE} - v_{AE}$$

$$\therefore v_{DE} = v_{DA} + v_{AE}$$

$$= -5 + 10$$

$$= 5\ ms^{-1} \text{ from west to east}$$

6. Two trains A and B of length 200 m each are moving on two parallel tracks with a uniform speed of 10 ms^{-1} in the same direction, with the train A ahead of B. The driver of

train B decides to overtake train A and accelerates by 1 ms^{-2}. If after 50 s, the guard of train B brushes past the driver of train A, what was the original distance between the two trains?

Let x be the original distance between the trains.
For train A: $v_0 = 10\ ms^{-1}$, $t = 50$ s, $a = 0$;
Distance traveled =

$$= x_A = v_0 t + \frac{1}{2} a t^2$$

$$= 10 \times 50 + 0$$

$$= 500\ m$$

For train B : $v_0 = 10$ ms, $t = 50$ s, 2 ms a^{-1}
Distance traveled =

$$= x_B = v_0 t + \frac{1}{2} a t^2$$

$$= 10 \times 50 + ½ \times 1 \times 50^2$$

$$= 500 + 1250$$

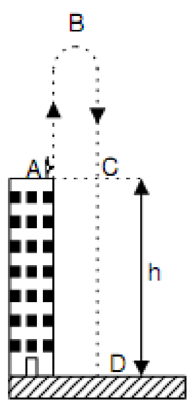
$$= 1750\ m$$

$$x_B = x + x_A + \text{length of train A} + \text{length of train B}$$

$$\therefore 1750 = x + 500 + 200 + 200$$

$$\therefore x = 1750 - 900 = 850\ m$$

7. A body is thrown vertically up from the top of a building with a velocity of 10 ms^{-1}. It reaches the ground in 5 s. Find the height of the building and the velocity with which the body reaches the ground. (g = 10 ms^{-2}).



Let the body be thrown vertically up with a velocity 10 ms^{-1} from the top of a building at a point A. It reaches the point B where its velocity is zero. Now $v_0 = 10\ ms^{-1}$. $v = 0$, $g = -10\ ms^{-2}$, $x = AB$.

Using $v^2 = v_0^2 + 2ax$

$$0^2 = 10^2 - 2 \times 10.AB$$

$$\therefore -20AB = 0^2 - 10^2$$

$$\therefore AB = \frac{100}{20} = 5\ m.$$

Let t be the time taken by the body to go from A to B.

Using $v = v_0 + at$

$0 = 10 - 10\,t$

$\therefore\ t = \frac{10}{10} = 1\ s$

Total time taken by body to reach the ground is 5 s.
Therefore the time taken by body to fall from B to D = 5 – 1 = 4 s.
BD = BC + CD = AB + CD = 5^{th}

Considering the body falling from B to D, velocity of the body at B is 0 $v_0 = 0$, $g = -10\ ms^{-2}$
Using equation of motion

$$x = v_0 t + \frac{1}{2}at^2$$

$$5^{th} = 0 + ½ \times 10\ (4)^2$$

$$= 80$$

$$\therefore h = 80 - 5 = 75\ m$$

The velocity with which the body reaches the ground is given by $v = v_0 + at$

$$v = 0 + 10 \times 4$$

$$v = 40\ ms^{-1}$$

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TEXTBOOK SOLUTIONS

3.1 In which of the following examples of motion, can the body be considered approximately a point object:
(a) a railway carriage moving without jerks between two stations.
(b) a monkey sitting on top of a man cycling smoothly on a circular track.
(c) a spinning cricket ball that turns sharply on hitting the ground.
(d) a tumbling beaker that has slipped off the edge of a table.

Answer: (a), (b)

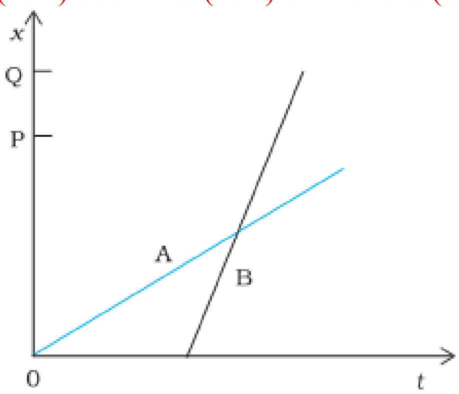
(a) The size of a carriage is very small as compared to the distance between stations. Therefore, the carriage can be treated as a point sized object.

(b) The size of a monkey is very small as compared to the size of a circular Therefore, the monkey can be considered as a point sized object on the trac

(c) The size of a spinning cricket ball is comparable to the distance through turns sharply on hitting the ground. Hence, the cricket ball cannot be consid point object.

(d) The size of a beaker is comparable to the height of the table from which slipped. Hence, the beaker cannot be considered as a point object.

3.2 The position-time (x-t) graphs for two children A and B returning from their school O to their homes P and Q respectively are shown in Fig. 3.19. Choose the correct entries in the brackets below ;
(a) (A/B) lives closer to the school than (B/A)
(b) (A/B) starts from the school earlier than (B/A)
(c) (A/B) walks faster than (B/A)
(d) A and B reach home at the (same/different) time
(e) (A/B) overtakes (B/A) on the road (once/twice)



Answer:

(a) A lives closer to school than B.

(b) A starts from school earlier than B.

(c) B walks faster than A.

(d) A and B reach home at the same time.

(e) B overtakes A once on the road.

Explanation:

(a) In the given x–t graph, it can be observed that distance OP < OQ. Hence distance of school from the A's home is less than that from B's home.

(b) In the given graph, it can be observed that for x = 0, t = 0 for A, wherea = 0, t has some finite value for B. Thus, A starts his journey from school ear B.

(c) In the given x–t graph, it can be observed that the slope of B is greater t of A. Since the slope of the x–t graph gives the speed, a greater slope mean the speed of B is greater than the speed A.

(d) It is clear from the given graph that both A and B reach their respective at the same time.

(e) B moves later than A and his/her speed is greater than that of A. From t graph, it is clear that B overtakes A only once on the road.

3.3 A woman starts from her home at 9.00 am, walks with a speed of 5 km h^{-1} on a straight road up to her office 2.5 km away, stays at the office up to 5.00 pm, and returns home by an auto with a speed of 25 km h^{-1}. Choose suitable scales and plot the x-t graph of her motion.

Answer:

Speed of the woman = 5 km/h

Distance between her office and home = 2.5 km

$$\text{Time taken} = \frac{\text{Distance}}{\text{Speed}}$$

$$= \frac{2.5}{5} = 0.5 \text{ h} = 30 \text{ min}$$

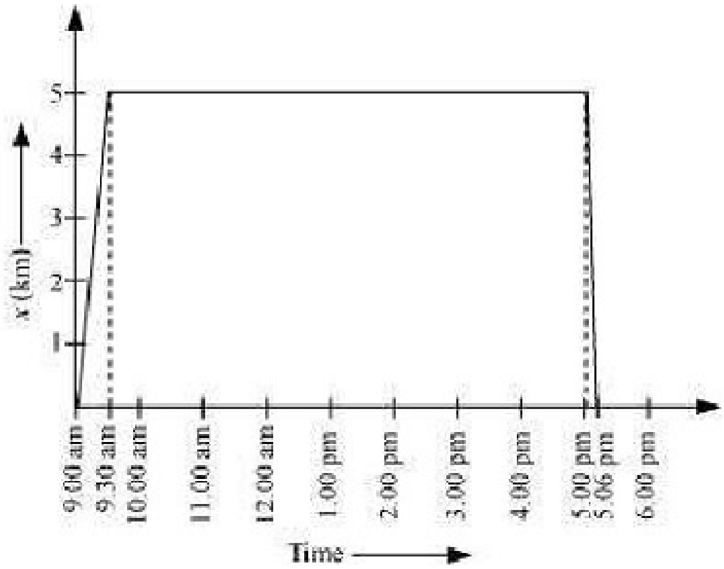
It is given that she covers the same distance in the evening by an auto.

Now, speed of the auto = 25 km/h

$$\text{Time taken} = \frac{\text{Distance}}{\text{Speed}}$$

$$= \frac{2.5}{25} = \frac{1}{10} = 0.1 \text{ h} = 6 \text{ min}$$

The suitable x-t graph of the motion of the woman is shown in the given figu



3.4 A drunkard walking in a narrow lane takes 5 steps forward and 3 steps backward, followed again by 5 steps forward and 3 steps backward, and so on. Each step is 1 m long and requires 1 s. Plot the x-t graph of his motion. Determine graphically and otherwise how long the drunkard takes to fall in a pit 13 m away from the start.

Answer:

Distance covered with 1 step = 1 m

Time taken = 1 s

Time taken to move first 5 m forward = 5 s

Time taken to move 3 m backward = 3 s

Net distance covered = 5 – 3 = 2 m

Net time taken to cover 2 m = 8 s

Drunkard covers 2 m in 8 s.

Drunkard covered 4 m in 16 s.

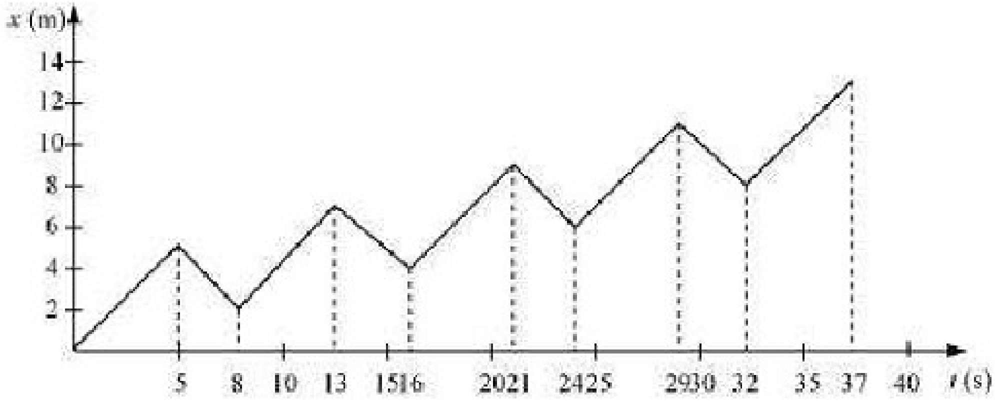
Drunkard covered 6 m in 24 s.

Drunkard covered 8 m in 32 s.

In the next 5 s, the drunkard will cover a distance of 5 m and a total distanc m and falls into the pit.

Net time taken by the drunkard to cover 13 m = 32 + 5 = 37 s

The x-t graph of the drunkard's motion can be shown as:



3.5 A jet airplane travelling at the speed of 500 km h^{-1} ejects its products of combustion at the speed of 1500 km h^{-1} relative to the jet plane. What is the speed of the latter with respect to an observer on the ground ?

Answer:

Speed of the jet airplane, vjet = 500 km/h

Relative speed of its products of combustion with respect to the plane,

vsmoke = – 1500 km/h

Speed of its products of combustion with respect to the ground = v′smoke

Relative speed of its products of combustion with respect to the airplane,

vsmoke = v′smoke – vjet

– 1500 = v′smoke – 500

v′smoke = – 1000 km/h

The negative sign indicates that the direction of its products of combustion i: opposite to the direction of motion of the jet airplane.

3.6 A car moving along a straight highway with speed of 126 km h^{-1} is brought to a stop within a distance of 200 m. What is the retardation of the car (assumed uniform), and how long does it take for the car to stop ?

Answer:

Initial velocity of the car, u = 126 km/h = 35 m/s

Final velocity of the car, v = 0

Distance covered by the car before coming to rest, s = 200 m

Retardation produced in the car = a

From third equation of motion, a can be calculated as:

$$v^2 - u^2 = 2as$$

$$(0)^2 - (35)^2 = 2 \times a \times 200$$

$$a = -\frac{35 \times 35}{2 \times 200} = -3.06 \text{ m/s}^2$$

From first equation of motion, time (t) taken by the car to stop can be obta

$$v = u + at$$

$$t = \frac{v-u}{a} = \frac{-35}{-3.06} = 11.44 \text{ s}$$

3.7 Two trains A and B of length 400 m each are moving on two parallel tracks with a uniform speed of 72 km h^{-1} in the same direction, with A ahead of B. The driver of B decides to overtake A and accelerates by 1 m s^{-2}. If after 50 s, the guard of B just brushes past the driver of A, what was the original distance between them ?

Answer:

For train A:

Initial velocity, u = 72 km/h = 20 m/s

Time, t = 50 s

Acceleration, aI = 0 (Since it is moving with a uniform velocity)

From second equation of motion, distance (sI)covered by train A can be obt

$$s_1 = ut + \frac{1}{2}a_1t^2$$

= 20 × 50 + 0 = 1000 m

For train B:

Initial velocity, u = 72 km/h = 20 m/s

Acceleration, a = 1 m/s2

Time, t = 50 s

From second equation of motion, distance (sII)covered by train A can be obt

as:

From second equation of motion, minimum acceleration (a) produced by car

obtained as:

$s = ut + \frac{1}{2} \times a \times (40)^2$ or $a = \frac{1,000}{1,000} = 1 \text{m/s}^2$

$$s_{II} = ut + \frac{1}{2}at^2$$

$$= 20 \times 50 + \frac{1}{2} \times 1 \times (50)^2 = 2250 \text{ m}$$

Hence, the original distance between the driver of train A and the guard of t

2250 −1000 = 1250 m.

3.8 On a two-lane road, car A is travelling with a speed of 36 km h^{-1}. Two cars B and C approach car A in opposite directions with a speed of 54 km h^{-1} each. At a certain instant, when the distance AB is equal to AC, both being 1 km, B decides to overtake A before C does. What minimum acceleration of car B is required to avoid an accident ?

Answer:

Velocity of car A, vA = 36 km/h = 10 m/s

Velocity of car B, vB = 54 km/h = 15 m/s

Velocity of car C, vC = 54 km/h = 15 m/s

Relative velocity of car B with respect to car A,

vBA = vB – vA

= 15 – 10 = 5 m/s

Relative velocity of car C with respect to car A,

vCA = vC – (– vA)

= 15 + 10 = 25 m/s

At a certain instance, both cars B and C are at the same distance from car A

s = 1 km = 1000 m

Time taken (t) by car C to cover 1000 m = $\frac{1000}{25} = 40\,\text{s}$

Hence, to avoid an accident, car B must cover the same distance in a maxim

40 s.

3.9 Two towns A and B are connected by a regular bus service with a bus leaving in either direction every T minutes. A man cycling with a speed of 20 km h–1 in the direction A to B notices that a bus goes past him every 18 min in the direction of his motion, and every 6 min in the opposite direction. What is the period T of the bus service and with what speed (assumed constant) do the buses ply on the
road?

Answer:

Let V be the speed of the bus running between towns A and B.

Speed of the cyclist, v = 20 km/h

Relative speed of the bus moving in the direction of the cyclist

= V – v = (V – 20) km/h

The bus went past the cyclist every 18 min i.e., $\frac{18}{60}\,\text{h}$ (when he moves in the direction of the bus).

Distance covered by the bus = $(V-20)\frac{18}{60}\,\text{km}$ ... (i)

Since one bus leaves after every T minutes, the distance travelled by the bu

equal to $V \times \frac{T}{60}$ ... (ii)

Both equations (i) and (ii) are equal.

$$(V-20)\times\frac{18}{60} = \frac{VT}{60} \qquad \text{... (iii)}$$

Relative speed of the bus moving in the opposite direction of the cyclist

= (V + 20) km/h

Time taken by the bus to go past the cyclist $= 6\,\text{min} = \frac{6}{60}\,\text{h}$

$$\therefore (V+20)\frac{6}{60}=\frac{VT}{60} \qquad \text{... (iv)}$$

From equations (iii) and (iv), we get

$$(V+20)\times\frac{6}{60}=(V-20)\times\frac{18}{60}$$

$$V+20=3V-60$$

$$2V=80$$

$$V=40 \text{ km/h}$$

Substituting the value of V in equation (iv), we get

$$(40+20)\times\frac{6}{60}=\frac{40T}{60}$$

$$T=\frac{360}{40}=9\text{min}$$

3.10 A player throws a ball upwards with an initial speed of 29.4 m s–1.
(a) What is the direction of acceleration during the upward motion of the ball ?
(b) What are the velocity and acceleration of the ball at the highest point of its motion ?
(c) Choose the x = 0 m and t = 0 s to be the location and time of the ball at its highest point, vertically downward direction to be the positive direction of x-axis, and give the signs of position, velocity and acceleration of the ball during its upward, and downward motion.
(d) To what height does the ball rise and after how long does the ball return to the player's hands ? (Take g = 9.8 m s–2 and neglect air resistance).

Answer:

(a) Downward

(b) Velocity = 0, acceleration = 9.8 m/s2

(c) x > 0 for both up and down motions, v < 0 for up and v > 0 for down m

> 0 throughout the motion

(d) 44.1 m, 6 s

Explanation:

(a) Irrespective of the direction of the motion of the ball, acceleration (which actually acceleration due to gravity) always acts in the downward direction t the centre of the Earth.

(b) At maximum height, velocity of the ball becomes zero. Acceleration due gravity at a given place is constant and acts on the ball at all points (includir highest point) with a constant value i.e., 9.8 m/s2.

(c) During upward motion, the sign of position is positive, sign of velocity is negative, and sign of acceleration is positive. During downward motion, the position, velocity, and acceleration are all positive.

(d) Initial velocity of the ball, u = 29.4 m/s

Final velocity of the ball, v = 0 (At maximum height, the velocity of the ball zero)

Acceleration, a = − g = − 9.8 m/s2

From third equation of motion, height (s) can be calculated as:

$$v^2 - u^2 = 2gs$$

$$s = \frac{v^2 - u^2}{2g}$$

$$= \frac{(0)^2 - (29.4)^2}{2 \times (-9.8)} = 44.1\,\text{m}$$

From first equation of motion, time of ascent (t) is given as:

$$v = u + at$$

$$t = \frac{v - u}{a} = \frac{-29.4}{-9.8} = 3\text{ s}$$

Time of ascent = Time of descent

Hence, the total time taken by the ball to return to the player's hands = 3 +

3.11 Read each statement below carefully and state with reasons and examples, if it is true or false ; A particle in one-dimensional motion

(a) with zero speed at an instant may have non-zero acceleration at that instant

(b) with zero speed may have non-zero velocity,

(c) with constant speed must have zero acceleration,

(d) with positive value of acceleration must be speeding up.

Answer:

(a) True

(b) False

(c) True

(d) False

Explanation:

(a) When an object is thrown vertically up in the air, its speed becomes zerc maximum height. However, it has acceleration equal to the acceleration due gravity (g) that acts in the downward direction at that point.

(b) Speed is the magnitude of velocity. When speed is zero, the magnitude (velocity along with the velocity is zero.

(c) A car moving on a straight highway with constant speed will have consta velocity. Since acceleration is defined as the rate of change of velocity, acce of the car is also zero.

(d) This statement is false in the situation when acceleration is positive and is negative at the instant time taken as origin. Then, for all the time before v becomes zero, there is slowing down of the particle. Such a case happens wl particle is projected upwards.

This statement is true when both velocity and acceleration are positive, at tl time taken as origin. Such a case happens when a particle is moving with pc acceleration or falling vertically downwards from a height.

3.12 A ball is dropped from a height of 90 m on a floor. At each collision with the floor, the ball loses one tenth of its speed. Plot the speed-time graph of its motion between t = 0 to 12 s.

Answer:

Ball is dropped from a height, s = 90 m

Initial velocity of the ball, u = 0

Acceleration, a = g = 9.8 m/s2

Final velocity of the ball = v

From second equation of motion, time (t) taken by the ball to hit the grounc obtained as:

$$s = ut + \frac{1}{2}at^2$$

$$90 = 0 + \frac{1}{2} \times 9.8t^2$$

$$t = \sqrt{18.38} = 4.29 \text{ s}$$

From first equation of motion, final velocity is given as:

v = u + at

= 0 + 9.8 × 4.29 = 42.04 m/s

Rebound velocity of the ball, ur = $\frac{9}{10}v = \frac{9}{10} \times 42.04 = 37.84 \text{ m/s}$

Time (t) taken by the ball to reach maximum height is obtained with the hel

equation of motion as:

v = ur + at′

0 = 37.84 + (− 9.8) t′

$$t' = \frac{-37.84}{-9.8} = 3.86 \text{ s}$$

Total time taken by the ball = t + t′ = 4.29 + 3.86 = 8.15 s

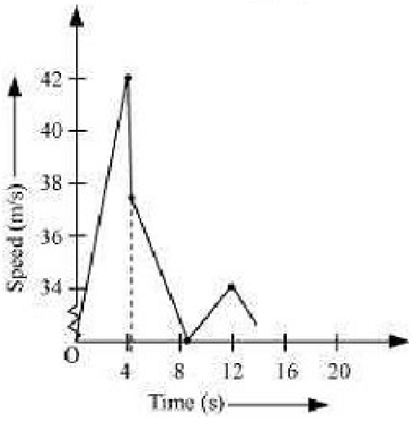
As the time of ascent is equal to the time of descent, the ball takes 3.86 s tc

back on the floor for the second time.

The velocity with which the ball rebounds from the floor $= \frac{9}{10} \times 37.84 = 34.05$

Total time taken by the ball for second rebound = 8.15 + 3.86 = 12.01 s

The speed-time graph of the ball is represented in the given figure as:



3.13 Explain clearly, with examples, the distinction between :
(a) magnitude of displacement (sometimes called distance) over an interval of time, and the total length of path covered by a particle over the same interval;
(b) magnitude of average velocity over an interval of time, and the average speed over the same interval. [Average speed of a particle over an interval of time is defined as the total path length divided by the time interval]. Show in both (a) and (b) that the second quantity is either greater than or equal to the first. When is the equality sign true ? [For simplicity, consider one-dimensional motion only].

Answer:

(a) The magnitude of displacement over an interval of time is the shortest c (which is a straight line) between the initial and final positions of the particle The total path length of a particle is the actual path length covered by the pa a given interval of time.

For example, suppose a particle moves from point A to point B and then, cor to a point, C taking a total time t, as shown below. Then, the magnitude of displacement of the particle = AC.

Whereas, total path length = AB + BC

It is also important to note that the magnitude of displacement can never be than the total path length. However, in some cases, both quantities are equa each other.

(b)

$$\text{Magnitude of average velocity} = \frac{\text{Magnitude of displacement}}{\text{Time interval}}$$

For the given particle,

$$\text{Average velocity} = \frac{AC}{t}$$

$$\text{Average speed} = \frac{\text{Total path length}}{\text{Time interval}}$$

$$= \frac{AB+BC}{t}$$

Since (AB + BC) > AC, average speed is greater than the magnitude of aver
velocity. The two quantities will be equal if the particle continues to move al
straight line.

3.14 A man walks on a straight road from his home to a market 2.5 km away with a speed of 5 km h^{-1}. Finding the market closed, he instantly turns and walks back home with a speed of 7.5 km h^{-1}. What is the (a) magnitude of average velocity, and (b) average speed of the man over the interval of time (i) 0 to 30 min, (ii) 0 to 50 min, (iii) 0 to 40 min ? [Note: You will appreciate from this exercise why it is better to define average speed as total path length divided by time, and not as magnitude of average velocity. You would not like to tell the tired man on his return home that his average speed was zero !]

Answer:

Time taken by the man to reach the market from home, $t_1 = \frac{2.5}{5} = \frac{1}{2}\text{ h} = 30$ m

Time taken by the man to reach home from the market, $t_2 = \frac{2.5}{7.5} = \frac{1}{3}\text{ h} = 20$ m

Total time taken in the whole journey = 30 + 20 = 50 min

$$\text{Average velocity} = \frac{\text{Displacement}}{\text{Time}} = \frac{2.5}{\frac{1}{2}} = 5\text{ km/h} \qquad \dots (a(i))$$

$$\text{Average speed} = \frac{\text{Distance}}{\text{Time}} = \frac{2.5}{\frac{1}{2}} = 5\text{ km/h} \qquad \dots (b(i))$$

Time = 50 min = $\frac{5}{6}$ h

Net displacement = 0

Total distance = 2.5 + 2.5 = 5 km

$$\text{Average velocity} = \frac{\text{Displacement}}{\text{Time}} = 0 \qquad \dots (a(ii))$$

$$\text{Average speed} = \frac{\text{Distance}}{\text{Time}} = \frac{5}{\left(\frac{5}{6}\right)} = 6\text{ km/h} \qquad \dots (b(ii))$$

Speed of the man = 7.5 km

Distance travelled in first 30 min = 2.5 km

Distance travelled by the man (from market to home) in the next 10 min

$= 7.5 \times \frac{10}{60} = 1.25\,\text{km}$

Net displacement = 2.5 − 1.25 = 1.25 km

Total distance travelled = 2.5 + 1.25 = 3.75 km

$$\text{Average velocity} = \frac{1.25}{\left(\frac{40}{60}\right)} = \frac{1.25 \times 3}{2} = 1.875\,\text{km/h} \qquad \dots (a(iii))$$

$$\text{Average speed} = \frac{3.75}{\left(\frac{40}{60}\right)} = 5.625\ \text{km/h} \qquad \dots (b(iii))$$

3.15 In Exercises 3.13 and 3.14, we have carefully distinguished between average speed and magnitude of average velocity. No such distinction is necessary when we consider instantaneous speed and magnitude of velocity. The instantaneous speed is always equal to the magnitude of instantaneous velocity. Why ?

Answer:

Instantaneous velocity is given by the first derivative of distance with respec

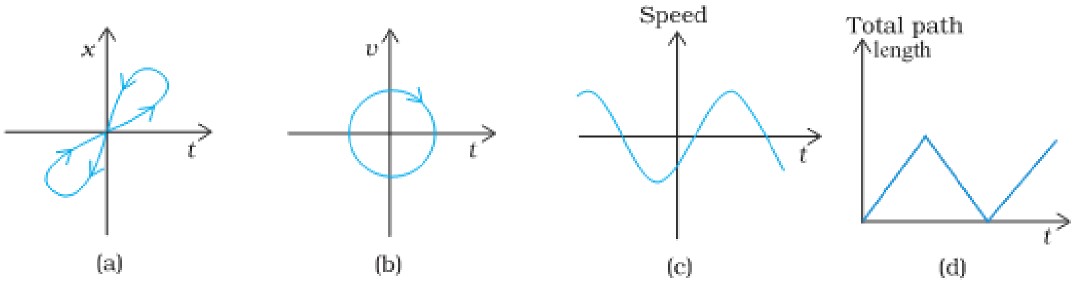
i.e.,

$$v_{\text{In}} = \frac{dx}{dt}$$

Here, the time interval dt is so small that it is assumed that the particle doe change its direction of motion. As a result, both the total path length and ma of displacement become equal is this interval of time.

Therefore, instantaneous speed is always equal to instantaneous velocity.

3.16 Look at the graphs (a) to (d) (Fig. 3.20) carefully and state, with reasons, which of these cannot possibly represent one-dimensional motion of a particle.



Answer:

(a) The given x-t graph, shown in (a), does not represent one-dimensional r the particle. This is because a particle cannot have two positions at the same of time.

(b) The given v-t graph, shown in (b), does not represent one-dimensional r the particle. This is because a particle can never have two values of velocity same instant of time.

(c) The given v-t graph, shown in (c), does not represent one-dimensional m the particle. This is because speed being a scalar quantity cannot be negativ

(d) The given v-t graph, shown in (d), does not represent one-dimensional r the particle. This is because the total path length travelled by the particle ca decrease with time.

3.17. Figure 3.21 shows the x-t plot of one-dimensional motion of a particle. Is it correct to say from the graph that the particle moves in a straight line for t < 0 and on a parabolic path for t >0 ? If not, suggest a suitable physical context for this graph.

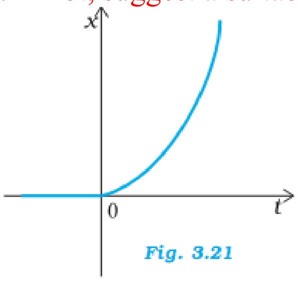


Fig. 3.21

Answer: No

The x-t graph of a particle moving in a straight line for t < 0 and on a parab for t > 0 cannot be shown as the given graph. This is because, the given par does not follow the trajectory of path followed by the particle as t = 0, x = C physical situation that resembles the above graph is of a freely falling body l sometime at a height

3.18 A police van moving on a highway with a speed of 30 km h^{-1} fires a bullet at a thief's car speeding away in the same direction with a speed of 192 km h^{-1}. If the muzzle speed of the bullet is 150 m s^{-1}, with what speed does the bullet hit the thief's car ? (Note: Obtain that speed which is relevant for damaging the thief's car).

Answer:

Speed of the police van, vp = 30 km/h = 8.33 m/s

Muzzle speed of the bullet, vb = 150 m/s

Speed of the thief's car, vt = 192 km/h = 53.33 m/s

Since the bullet is fired from a moving van, its resultant speed can be obtair

= 150 + 8.33 = 158.33 m/s

Since both the vehicles are moving in the same direction, the velocity with w

bullet hits the thief's car can be obtained as:

vbt = vb – vt

= 158.33 – 53.33 = 105 m/s

3.19 Suggest a suitable physical situation for each of the following graphs (Fig 3.22):

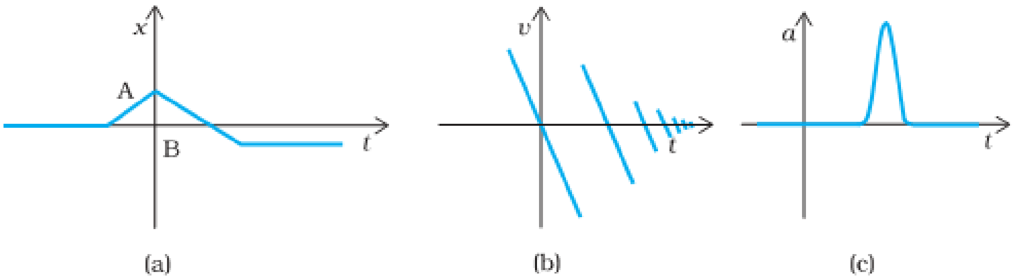


Fig. 3.22 .

Answer:

(a)The given x-t graph shows that initially a body was at rest. Then, its velc increases with time and attains an instantaneous constant value. The velocit reduces to zero with an increase in time. Then, its velocity increases with tir opposite direction and acquires a constant value. A similar physical situation when a football (initially kept at rest) is kicked and gets rebound from a rigi that its speed gets reduced. Then, it passes from the player who has kicked ultimately gets stopped after sometime.

(b)In the given v-tgraph, the sign of velocity changes and its magnitude dec with a passage of time. A similar situation arises when a ball is dropped on t floor from a height. It strikes the floor with some velocity and upon rebound velocity decreases by a factor. This continues till the velocity of the ball ever becomes zero.

(c)The given a-t graph reveals that initially the body is moving with a certair velocity. Its acceleration increases for a short interval of time, which again c zero. This indicates that the body again starts moving with the same constar velocity. A similar physical situation arises when a hammer moving with a ur velocity strikes a nail.

3.20 Figure 3.23 gives the x-t plot of a particle executing one-dimensional simple harmonic motion. (You will learn about this motion in more detail in Chapter14). Give the signs of position, velocity and acceleration variables of the particle at t = 0.3 s, 1.2 s, – 1.2 s.

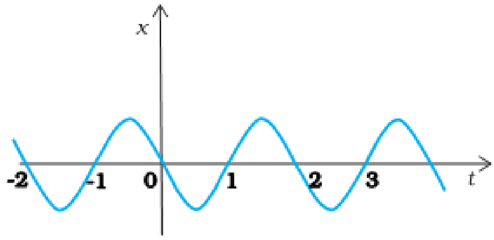


Fig. 3.23

Answer:

Negative, Negative, Positive (at t = 0.3 s)

Positive, Positive, Negative (at t = 1.2 s)

Negative, Positive, Positive (at t = −1.2 s)

For simple harmonic motion (SHM) of a particle, acceleration (a) is given by relation:

a = − ω2x ω □ angular frequency ... (i)

t = 0.3 s

In this time interval, x is negative. Thus, the slope of the x-t plot will also be negative. Therefore, both position and velocity are negative. However, using equation (i), acceleration of the particle will be positive.

t = 1.2 s

In this time interval, x is positive. Thus, the slope of the x-t plot will also be

Therefore, both position and velocity are positive. However, using equation (

acceleration of the particle comes to be negative.

t = − 1.2 s

In this time interval, x is negative. Thus, the slope of the x-t plot will also be

negative. Since both x and t are negative, the velocity comes to be positive.

equation (i), it can be inferred that the acceleration of the particle will be po

3.21 Figure 3.24 gives the x-t plot of a particle in one-dimensional motion. Three different equal intervals of time are shown. In which interval is the average speed greatest, and in which is it the least ? Give the sign of average velocity for each interval.

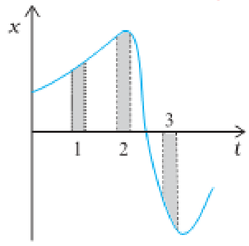


Fig. 3.24

Answer:

Interval 3 (Greatest), Interval 2 (Least)

Positive (Intervals 1 & 2), Negative (Interval 3)

The average speed of a particle shown in the x-t graph is obtained from the

the graph in a particular interval of time.

It is clear from the graph that the slope is maximum and minimum restively

intervals 3 and 2 respectively. Therefore, the average speed of the particle i

greatest in interval 3 and is the least in interval 2. The sign of average veloc

positive in both intervals 1 and 2 as the slope is positive in these intervals. H

it is negative in interval 3 because the slope is negative in this interval.

3.22 Figure 3.25 gives a speed-time graph of a particle in motion along a constant direction. Three equal intervals of time are shown. In which interval is the average acceleration greatest in magnitude ? In which interval is the average speed greatest ? Choosing the positive direction as the constant direction of motion, give the signs of v and a in the three intervals. What are the accelerations at the points A, B, C and D ?

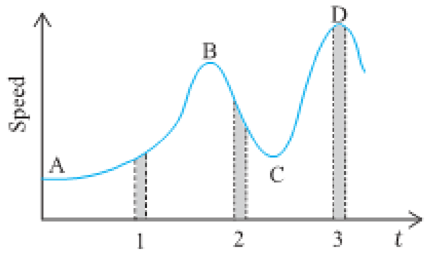


Fig. 3.25

Answer:

Average acceleration is greatest in interval 2

Average speed is greatest in interval 3

v is positive in intervals 1, 2, and 3

a is positive in intervals 1 and 3 and negative in interval 2

a = 0 at A, B, C, D

Acceleration is given by the slope of the speed-time graph. In the given case given by the slope of the speed-time graph within the given interval of time. Since the slope of the given speed-time graph is maximum in interval 2, ave acceleration will be the greatest in this interval.

Height of the curve from the time-axis gives the average speed of the partic clear that the height is the greatest in interval 3. Hence, average speed of tl particle is the greatest in interval 3.

In interval 1:

The slope of the speed-time graph is positive. Hence, acceleration is positive Similarly, the speed of the particle is positive in this interval.

In interval 2:

The slope of the speed-time graph is negative. Hence, acceleration is negati interval. However, speed is positive because it is a scalar quantity.

In interval 3:

The slope of the speed-time graph is zero. Hence, acceleration is zero in this However, here the particle acquires some uniform speed. It is positive in thi interval.

Points A, B, C, and D are all parallel to the time-axis. Hence, the slope is zer these points. Therefore, at points A, B, C, and D, acceleration of the particle

3.23 A three-wheeler starts from rest, accelerates uniformly with 1 m s^{-2} on a straight road for 10 s, and then moves with uniform velocity. Plot the distance covered by the vehicle during the nth second (n = 1,2,3….) versus n. What do you expect this plot to be during accelerated motion : a straight line or a parabola ?

Answer:

Straight line

Distance covered by a body in nth second is given by the relation

$$D_n = u + \frac{a}{2}(2n-1) \qquad \text{... (i)}$$

Where,

u = Initial velocity

a = Acceleration

n = Time = 1, 2, 3, ... ,n

In the given case,

u = 0 and a = 1 m/s2

$$\therefore D_n = \frac{1}{2}(2n-1) \qquad \text{... (ii)}$$

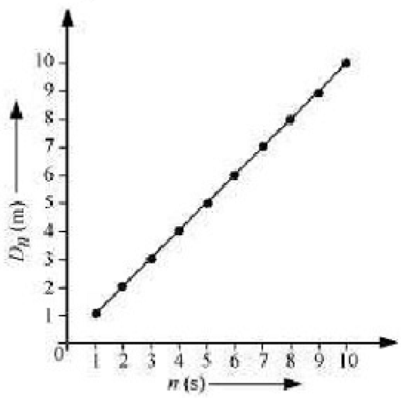
This relation shows that:

Dn □ n ... (iii)

Now, substituting different values of n in equation (iii), we get the following

n	1	2	3	4	5	6	7	8	9	10
Dn	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	8.5	9.5

The plot between n and Dn will be a straight line as shown:



Since the given three-wheeler acquires uniform velocity after 10 s, the line w parallel to the time-axis after n = 10 s.

3.24 A boy standing on a stationary lift (open from above) throws a ball upwards with the maximum initial speed he can, equal to 49 m s^{-1}. How much time does the ball take to return to his hands? If the lift starts moving up with a uniform speed of 5 m s^{-1} and the boy again throws the ball up with the maximum speed he can, how long does the ball take to return to his hands ?

Answer:

Initial velocity of the ball, u = 49 m/s

Acceleration, a = – g = – 9.8 m/s2

Case I:

When the lift was stationary, the boy throws the ball.

Taking upward motion of the ball,

Final velocity, v of the ball becomes zero at the highest point.

From first equation of motion, time of ascent (t) is given as:

$$v = u + at$$

$$t = \frac{v - u}{a}$$

$$= \frac{-49}{-9.8} = 5 \text{ s}$$

But, the time of ascent is equal to the time of descent.

Hence, the total time taken by the ball to return to the boy's hand = 5 + 5 =

Case II:

The lift was moving up with a uniform velocity of 5 m/s. In this case, the rel velocity of the ball with respect to the boy remains the same i.e., 49 m/s. Th in this case also, the ball will return back to the boy's hand after 10 s.

3.25 On a long horizontally moving belt (Fig. 3.26), a child runs to and fro with a speed 9 km h^{-1} (with respect to the belt) between his father and mother located 50 m apart on the moving belt. The belt moves with a speed of 4 km h^{-1}. For an observer on a stationary platform outside, what is the

(a) speed of the child running in the direction of motion of the belt ?.

(b) speed of the child running opposite to the direction of motion of the belt ?

(c) time taken by the child in (a) and (b) ?

Which of the answers alter if motion is viewed by one of the parents ?

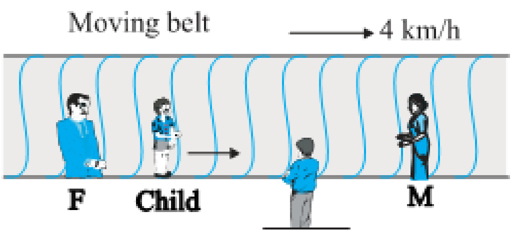


Fig. 3.26

Answer:

(a) Speed of the belt, vB = 4 km/h

Speed of the boy, vb = 9 km/h

Since the boy is running in the same direction of the motion of the belt, his s

(as observed by the stationary observer) can be obtained as:

vbB = vb + vB = 9 + 4 = 13 km/h

(b) Since the boy is running in the direction opposite to the direction of the r

the belt, his speed (as observed by the stationary observer) can be obtainec

vbB = vb + (– vB) = 9 – 4 = 5 km/h

(c) Distance between the child's parents = 50 m

As both parents are standing on the moving belt, the speed of the child in ei

direction as observed by the parents will remain the same i.e., 9 km/h = 2.5

Hence, the time taken by the child to move towards one of his parents is $\frac{50}{2.5}$

(d) If the motion is viewed by any one of the parents, answers obtained in (

(b) get altered. This is because the child and his parents are standing on the

belt and hence, are equally affected by the motion of the belt. Therefore, for

parents (irrespective of the direction of motion) the speed of the child remai

same i.e., 9 km/h.

For this reason, it can be concluded that the time taken by the child to reach

of his parents remains unaltered.

3.26 Two stones are thrown up simultaneously from the edge of a cliff 200 m high with initial speeds of 15 m s^{-1} and 30 m s^{-1}. Verify that the graph shown in Fig. 3.27 correctly represents the time variation of the relative position of the second stone with respect to the first. Neglect air resistance and assume that the stones do not rebound after hitting the ground. Take g = 10 m s^{-2}. Give the equations for the linear and curved parts of the plot.

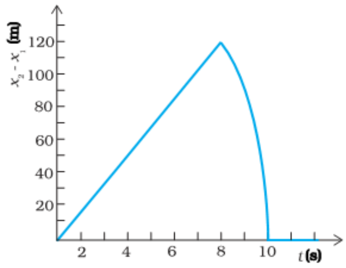


Fig. 3.27

Ans :

For first stone:

Initial velocity, uI = 15 m/s

Acceleration, a = −g = − 10 m/s2

Using the relation,

$$x_1 = x_0 + u_1 t + \frac{1}{2} a t^2$$

Where, height of the cliff, $x_0 = 200$ m

$$x_1 = 200 + 15t - 5t^2 \quad \text{... (i)}$$

When this stone hits the ground, x1 = 0

□− 5t2 + 15t + 200 = 0

t2 − 3t − 40 = 0

t2 − 8t + 5t − 40 = 0

t (t − 8) + 5 (t − 8) = 0

t = 8 s or t = − 5 s

Since the stone was projected at time t = 0, the negative sign before time is meaningless.

□t = 8 s

For second stone:

Initial velocity, uII = 30 m/s

Acceleration, a = −g = − 10 m/s2

Using the relation,

$$x_2 = x_0 + u_{II} t + \frac{1}{2} a t^2$$

$$= 200 + 30t - 5t^2 \quad \text{... (ii)}$$

At the moment when this stone hits the ground; x2 = 0

– 5t2 + 30 t + 200 = 0

t2 – 6t – 40 = 0

t2 – 10t + 4t + 40 = 0

t (t – 10) + 4 (t – 10) = 0

t (t – 10) (t + 4) = 0

t = 10 s or t = – 4 s

Here again, the negative sign is meaningless.

□t = 10 s

Subtracting equations (i) and (ii), we get

$$x_2 - x_1 = (200 + 30t - 5t^2) - (200 + 15t - 5t^2)$$

$$x_2 - x_1 = 15t \qquad \text{.... (iii)}$$

Equation (iii) represents the linear path of both stones. Due to this linear rel

between (x2 – x1) and t, the path remains a straight line till 8 s.

Maximum separation between the two stones is at t = 8 s.

(x2 – x1)max = 15× 8 = 120 m

This is in accordance with the given graph.

After 8 s, only second stone is in motion whose variation with time is given l

quadratic equation:

x2 – x1 = 200 + 30t – 5t2

Hence, the equation of linear and curved path is given by

x2 – x1 = 15t (Linear path)

x2 – x1 = 200 + 30t – 5t2 (Curved path)

3.27 The speed-time graph of a particle moving along a fixed direction is shown in Fig. 3.28. Obtain the distance traversed by the particle between (a) t = 0 s to 10 s, (b) t = 2 s to 6 s. What is the average speed of the particle over the intervals in (a) and (b) ?

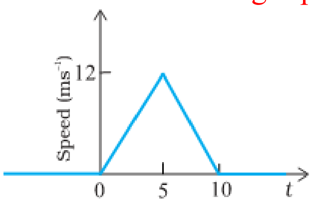


Fig. 3.28

Answer:

(a) Distance travelled by the particle = Area under the given graph

$$= \frac{1}{2} \times (10 - 0) \times (12 - 0) = 60 \text{ m}$$

Average speed = $\frac{\text{Distance}}{\text{Time}} = \frac{60}{10} = 6 \text{ m/s}$

(b) Let s1 and s2 be the distances covered by the particle between time t = 2 s to 5 s and t = 5 s to 6 s respectively.

Total distance (s) covered by the particle in time t = 2 s to 6 s

s = s1 + s2 ... (i)

For distance s1:

Let u′ be the velocity of the particle after 2 s and a′ be the acceleration of th

particle in t = 0 to t = 5 s.

Since the particle undergoes uniform acceleration in the interval t = 0 to t =

from first equation of motion, acceleration can be obtained as:

v = u + at

Where,

v = Final velocity of the particle

12 = 0 + a′ × 5

$$a' = \frac{12}{5} = 2.4 \text{ m/s}^2$$

Again, from first equation of motion, we have

v = u + at

= 0 + 2.4 × 2 = 4.8 m/s

Distance travelled by the particle between time 2 s and 5 s i.e., in 3 s

$$s_1 = u't + \frac{1}{2}a't^2$$

$$= 4.8 \times 3 + \frac{1}{2} \times 2.4 \times (3)^2$$

$$= 25.2 \text{ m} \qquad \text{... (ii)}$$

For distance s2:

Let a″ be the acceleration of the particle between time t = 5 s and t = 10 s.

From first equation of motion,

v = u + at (where v = 0 as the particle finally comes to rest)

0 = 12 + a″ × 5

$$a'' = \frac{-12}{5}$$

$$= -2.4 \text{ m/s}^2$$

Distance travelled by the particle in 1s (i.e., between t = 5 s and t = 6 s)

$$s_2 = u''t + \frac{1}{2}at^2$$

$$= 12 \times a + \frac{1}{2}(-2.4) \times (1)^2$$

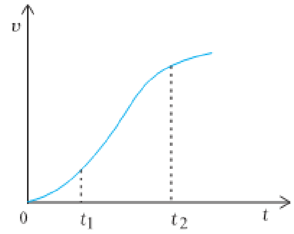
$$= 12 - 1.2 = 10.8 \text{ m} \qquad \text{... (iii)}$$

From equations (i), (ii), and (iii), we get

s = 25.2 + 10.8 = 36 m

$$\therefore \text{Average speed} = \frac{36}{4} = 9 \text{ m/s}$$

3.28 The velocity-time graph of a particle in one-dimensional motion is shown in Fig. 3.29 : Which of the following formulae are correct for describing the motion of the particle over the time-interval t_1 to t_2 :



(a) $x(t_2) = x(t_1) + v(t_1)(t_2 - t_1) + (½)\, a\, (t_2 - t_1)^2$

(b) $v(t_2) = v(t_1) + a\,(t_2 - t_1)$

(c) $v_{average} = (x(t_2) - x(t_1))/(t_2 - t_1)$

(d) $a_{average} = (v(t_2) - v(t_1))/(t_2 - t_1)$

(e) $x(t_2) = x(t_1) + v_{average}(t_2 - t_1) + (½)\, a_{average}\, (t_2 - t_1)^2$

(f) $x(t_2) - x(t_1)$ = area under the v-t curve bounded by the t-axis and the dotted line shown.

Answer:

The correct formulae describing the motion of the particle are (c), (d) and, (

The given graph has a non-uniform slope. Hence, the formulae given in (a),

(e) cannot describe the motion of the particle. Only relations given in (c), (d

are correct equations of motion.

Conceptual Questions :

1. Explain the difference between distance traveled, displacement, and displacement magnitude.
2. Explain the difference between speed and velocity.
3. On a graph of v_x versus time, what quantity does the area under the graph represent?
4. On a graph of v_x versus time, what quantity does the slope of the graph represent?
5. On a graph of a_x versus time, what quantity does the area under the graph represent?
6. On a graph of x versus time, what quantity does the slope of the graph represent?
7. What is the relationship between average velocity and instantaneous velocity? An object can have different instantaneous velocities at different times. Can the same object have different average velocities? Explain.
8. Can the velocity of an object be zero and the acceleration be nonzero at the same time? Explain.
9. You are bicycling along a straight north-south road. Let the x-axis point north. Describe your motion in each of the following cases. Example: $a_x > 0$ and $v_x > 0$ means you are moving north and speeding up. (a) $a_x > 0$ and $v_x < 0$. (b) $a_x = 0$ and $v_x < 0$. (c) $a_x < 0$ and $v_x = 0$. (d) $a_x < 0$ and $v_x < 0$. (e) Based on your answers, explain why it is not a good idea to use the expression "negative acceleration" to mean slowing down.
10. What is the distinction between a vector and a scalar quantity? Give two examples of each.

Conceptual Questions :

1. A ball is thrown straight up into the air. Neglect air resistance. While the ball is in the air its acceleration
(a) increases.
(b) is zero.
(c) remains constant.
(d) decreases on the way up and increases on the way down.
(e) changes direction.
2. Which car has a westward acceleration?
(a) a car traveling westward at constant speed
(b) a car traveling eastward and speeding up
(c) a car traveling westward and slowing down
(d) a car traveling eastward and slowing down
(e) a car starting from rest and moving toward the east
3. A toy rocket is propelled straight upward from the ground and reaches a height Δy. After an elapsed time Δt, measured from the time the rocket was first fired off, the rocket has fallen back down to the ground, landing at the same spot from which it was launched. The magnitude of the average velocity of the rocket during this time is

(a) zero (b) $2\frac{\Delta y}{\Delta t}$ (c) $\frac{\Delta y}{\Delta t}$ (d) $\frac{1}{2}\frac{\Delta y}{\Delta t}$

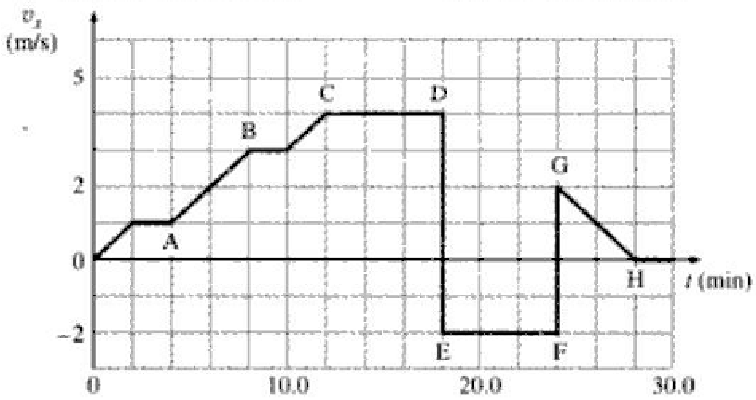
4. A toy rocket is propelled straight upward from the ground and reaches a height Δy. After an elapsed time Δt, measured from the time the rocket was first fired off, the rocket has fallen back down to the ground, landing at the same spot from which it was launched. The average speed of the rocket during this time is

(a) zero (b) $2\frac{\Delta y}{\Delta t}$ (c) $\frac{\Delta y}{\Delta t}$ (d) $\frac{1}{2}\frac{\Delta y}{\Delta t}$

5. A leopard starts from rest at $t = 0$ and runs in a straight line with a constant acceleration until $t = 3.0$ s. The distance covered by the leopard between $t = 1.0$ s and $t = 2.0$ s is

(a) the same as the distance covered during the first second.
(b) twice the distance covered during the first second.
(c) three times the distance covered during the first second.
(d) four timesthe distance covered during the first second.

Multiple-Choice Questions 6–15. A jogger is exercising along a long, straight road that runs north-south. She starts out heading north. A graph of $v_x(t)$ follows Question 10.



Multiple-Choice Questions 6–15

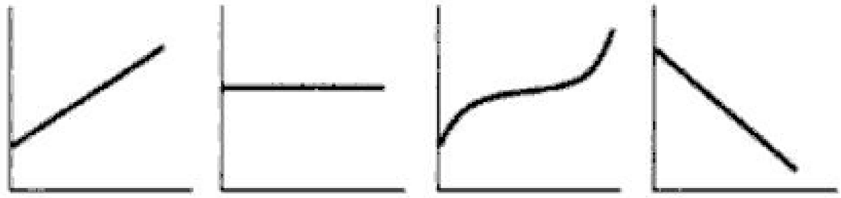
6. What distance does the jogger travel during the first 10.0 min ($t = 0$ to 10.0 min)?

(a) 8.5 m (b) 510 m (c) 900 m (d) 1020 m

7. What is the displacement of the jogger from $t = 18.0$ min to $t = 24.0$ min?

(a) 720 m, south (b) 720 m, north
(c) 2160 m, south (d) 3600 m, north

8. What is the displacement of the jogger for the entire 30.0 min?
   (a) 3120 m, south (b) 2400 m, north
   (c) 2400 m, south (d) 3840 m, north
9. What is the total distance traveled by the jogger in 30.0 min?
   (a) 3840 m (b) 2340 m
   (c) 2400 m (d) 3600 m
10. What is the average velocity of the jogger during the 30.0 min?
    (a) 1.3 m/s, north (b) 1.7 m/s, north
    (c) 2.1 m/s, north (d) 2.9 m/s, north
11. What is the average speed of the jogger for the 30 min?
    (a) 1.4 m/s (b) 1.7 m/s (c) 2.1 m/s (d) 2.9 m/s
12. In what direction is she running at time $t = 20$ min?
    (a) south (b) north (c) not enough information
13. In which region of the graph is a_x positive?
    (a) A to B (b) C to D (c) E to F (d) G to H
14. In which region is a_x negative?
    (a) A to B (b) C to D (c) E to F (d) G to H
15. In which region is the velocity directed to the south?
    (a) A to B (b) C to D (c) E to F (d) G to H
16. The figure below shows four graphs of x versus time. Which graph shows a constant, positive, nonzero velocity?
17. The four graphs below show v_x versus time. (a) Which graph shows a constant velocity? (b) Which graph shows a_x constant and positive? (c) Which graph shows a_x constant and negative? (d) Which graph shows a changing a_x that is always positive?



(a) (b) (c) (d)

Multiple Choice Questions A:

LEVEL I :

1. A car travels from A to B at a speed of 20 km h^{-1} and returns at a speed of 30 km h^{-1}. The average speed of the car for the whole journey is

(a) 5 km h^{-1} (b) 24 km h^{-1}

(c) 25 km h^{-1} (d) 50 km h^{-1}.

2. A body is thrown vertically upwards with a speed of 100 m s^{-1}. On the return journey, the speed in m s^{-1} at the starting point will be

(a) 100 m s^{-1} (b) 9.8 m s^{-1}

(c) 100×9.8 m s^{-1} (d) $\frac{100}{9.8}$ m s^{-1}.

3. If a body having initial velocity zero is moving with a uniform acceleration of 8 m s^{-2}, then the distance travelled by it in fifth second will be

(a) zero (b) 36 m

(c) 40 m (d) 100 m.

4. A pebble is dropped into a well of depth h. The splash is heard after time t. If c be the velocity of sound, then

(a) $t = \sqrt{\frac{gc}{2h}}$ (b) $t = c + gh$

(c) $t = c - v$ (d) $t = \sqrt{\frac{2h}{g}} + \frac{h}{c}$.

5. A ball released from a certain height falls 5 m in one second. In 4 s, it falls through

(a) 80 m (b) 40 m

(c) 1.25 m (d) 20 m.

6. The displacement-time graph of a moving particle is shown below. The instantaneous velocity of the particle is negative at the point

(a) C

(b) D

(c) E

(d) F.

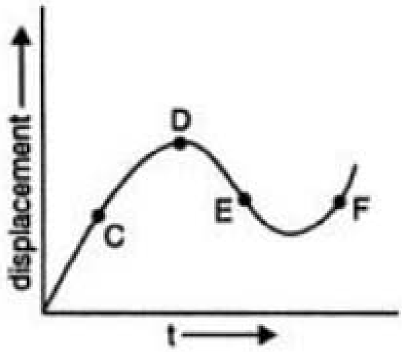


Fig. 1

7. The variation of velocity of a particle moving along a straight line is shown in the Fig. 2. The distance travelled by the particle in 4 s is

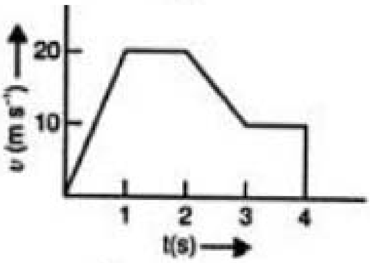


Fig. 2

(a) 25 m (b) 30 m

(c) 55 m (d) 60 m.

8. A particle starts from rest and moves along a straight line with constant acceleration. The variation of velocity v with displacement S is

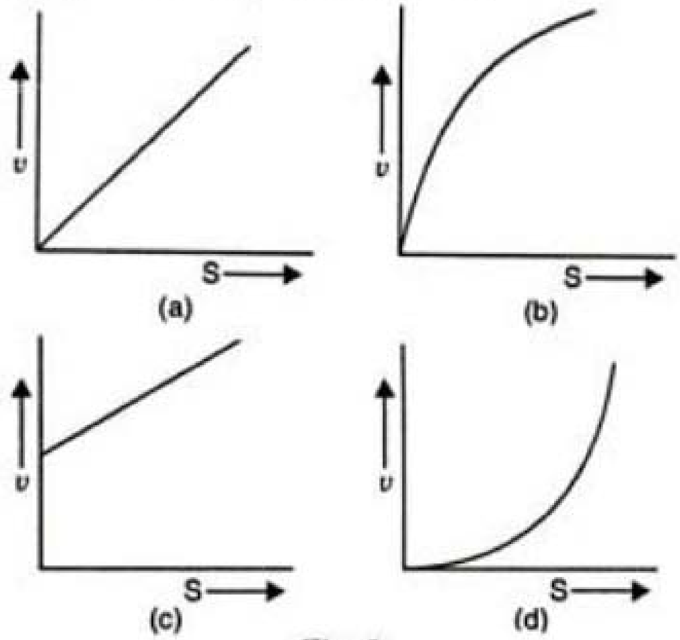


Fig. 3

9. A particle starts from rest at time t = 0 and moves on a straight line with acceleration as plotted in Fig. 4. The speed of the particle will be maximum after time

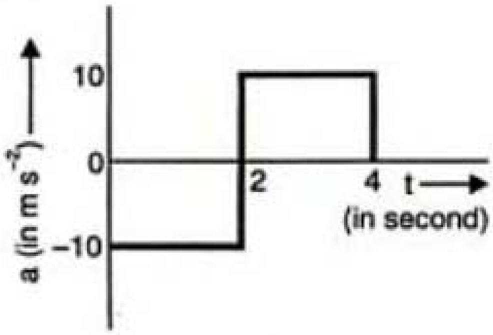


Fig. 4

(a) 1 s (b) 2 s

(c) 3 s (d) 4 s.

10. The displacement-time graphs of two moving particles make angles of 30° and 45° with the x-axis. The ratio of the two velocities is

(*a*) $\sqrt{3} : 1$
(*b*) 1 : 1
(*c*) 1 : 2
(*d*) $1 : \sqrt{3}$.

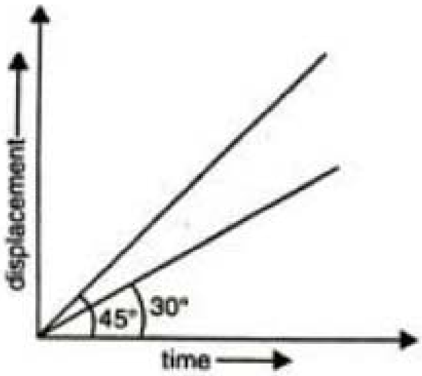


Fig. 5

11. The graph of displacement-time for a body travelling in a straight line is given. We can conclude that

(*a*) the velocity is constant.
(*b*) the velocity increases uniformly.
(*c*) the body is subjected to acceleration from O to A.
(*d*) the velocity of the body at A is zero.

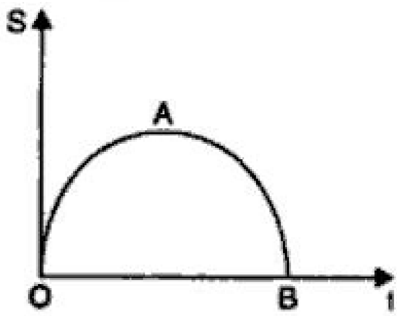


Fig. 6

12. A ball takes t second to fall from a height h_1 and $2t$ second to fall from a height h_2. Then h_1/h_2 is

(*a*) 2 (*b*) 4
(*c*) 0.5 (*d*) 0.25.

13. A stone is dropped from the top of the tower and reaches the ground in 3 s. Then the height of the tower is

(*a*) 18.6 m (*b*) 39.2 m
(*c*) 44.1 m (*d*) 98 m.

14. A body covers one half of its journey at 40 m s^{-1} and the next half at 50 m s^{-1}. Its average velocity is

(*a*) 44.4 m s^{-1} (*b*) 50 m s^{-1}
(*c*) 45 m s^{-1} (*d*) 40 m s^{-1}.

15. A car travels equal distances in the same direction with velocities 60 km h^{-1}, 20 km h^{-1} and 10 km h^{-1} respectively. The average velocity of the car over the whole journey of motion is

(*a*) 8 m s^{-1} (*b*) 7 m s^{-1}

(*c*) 6 m s^{-1} (*d*) 5 m s^{-1}.

16. A body starts from rest and is uniformly accelerated for 30 s. The distance travelled in the first 10 s is x_1, next 10 s is x_2 and the last 10 s is x_3. Then $x_1 : x_2 : x_3$ is the same as

(*a*) 1 : 2 : 4 (*b*) 1 : 2 : 5

(*c*) 1 : 3 : 5 (*d*) 1 : 3 : 9.

17. A ball is dropped from the top of a building. The ball takes 0.5 s to fall past the 3 m length of a window some distance from the top of the building. If the velocities of the ball at the top and at the bottom of the window are v_T and v_B respectively, then

(*a*) $v_T + v_B = 12$ m s^{-1} (*b*) $v_T - v_B = 4.9$ m s^{-1}

(*c*) $v_B v_T = 1$ m s^{-1} (*d*) $\frac{v_B}{v_T} = 1$ m s^{-1}.

18. In Q. 17, $\frac{v_T^2}{2g}$ gives the

(*a*) distance of the top of the building from the top of the window

(*b*) distance of the bottom of the window from the top of the building

(*c*) height of building

(*d*) distance of top of window from the ground.

19. Two cars are moving in the same direction with a speed of 30 km h^{-1}. They are separated from each other by 5 km. Third car moving in the opposite direction meets the two cars after an interval of 4 minutes. What is the speed of the third car ?

(*a*) 30 km h^{-1} (*b*) 35 km h^{-1}

(*c*) 40 km h^{-1} (*d*) 45 km h^{-1}.

20. The displacement-time graphs of two particles A and B are straight lines making angles of respectively 30° and 60° with the time- axis. If the velocity of A is v_A and that of B is v_B, then the value of $\frac{v_A}{v_B}$ is

(*a*) $\frac{1}{2}$ (*b*) $\frac{1}{\sqrt{3}}$

(*c*) $\sqrt{3}$ (*d*) $\frac{1}{3}$.

21. The position vector of a particle in SI units is given by

$$\vec{r} = 4t^2\,\hat{i} + 3t^2\,\hat{j} + 2\hat{k}$$

The acceleration of the particle is

(*a*) 4 m s^{-2} (*b*) 3 m s^{-2}

(*c*) 2 m s^{-2} (*d*) 10 m s^{-2}.

22. In Q. 21, the distance covered in the first 10 second is

(*a*) 200 m (*b*) 300 m

(*c*) 400 m (*d*) 500 m.

23. The velocity of a stone thrown vertically upwards is halved in 1.5 second. The maximum height attained by the stone is (Take g = 10 m s^{-2})

(*a*) 20 m (*b*) 25 m

(*c*) 30 m (*d*) 45 m.

24. A boy releases a ball from the top of a building. It will clear a window 2 m high at a distance 10 m below the top in nearly

(*a*) 1 s (*b*) 1.3 s

(*c*) 0.6 s (*d*) 0.13 s.

25. Two balls are dropped to the ground from different heights. One ball is dropped 2 s after the other but they both strike the ground at the same time, 5 s after the first was dropped. The difference in the heights at which they were dropped is (Given : g = 10 m s^{-2})

(*a*) 10 m (*b*) 20 m

(*c*) 40 m (*d*) 80 m.

26. A stone is allowed to fall from the top of a tower 100 metre high and at the same time another stone is projected vertically upwards from the ground with a velocity of 25 m s^{-1}. The two stones will meet after

(*a*) 4 s (*b*) 0.4 s

(*c*) 0.04 s (*d*) 40 s.

27. In Q. 26, the two stones meet at a height of

(*a*) 21.6 metre from the ground

(*b*) 50.0 metre from the ground

(*c*) 78.4 metre from the ground

(*d*) 19.6 metre from the ground.

28. A body starts from rest and moves along a straight line with uniform acceleration. It covers a distance of 150 m during 8th second of its motion. The acceleration of the body is

(*a*) 20 m s^{-2} (*b*) 10 m s^{-2}

(*c*) 30 m s^{-2} (*d*) 15 m s^{-2}.

29. The displacement of a body is given by $2s = gt^2$ where g is a constant. The velocity of the body at any time t is

(*a*) gt (*b*) $\frac{gt}{2}$

(*c*) $\frac{gt^2}{2}$ (*d*) $\frac{gt^3}{6}$.

30. A ball is thrown straight upward with a speed υ from a height h above the ground. The time taken for the ball to strike the ground is given by

(*a*) $-h = vt - \frac{1}{2}gt^2$ (*b*) $h = vt - \frac{1}{2}gt^2$

(*c*) $\frac{1}{2}gt^2$ (*d*) $\sqrt{\frac{2g}{h}}$.

31. The velocity of a body at the end of 5 s is 30 m s^{-1}, at the end of 12 s is 58 m s^{-1} and at the end of 22 s, it is 98 m s^{-1}. The body is moving with

(*a*) uniform velocity (*b*) uniform acceleration

(*c*) uniform displacement (*d*) uniform retardation.

QUESTION BANK (Level I)

1. (*b*)	**2.** (*a*)	**3.** (*b*)	**4.** (*d*)	**5.** (*a*)	**6.** (*c*)	**7.** (*c*)	**8.** (*b*)
9. (*b*)	**10.** (*d*)	**11.** (*d*)	**12.** (*d*)	**13.** (*c*)	**14.** (*a*)	**15.** (*d*)	**16.** (*c*)
17. (*a*)	**18.** (*a*)	**19.** (*d*)	**20.** (*d*)	**21.** (*d*)	**22.** (*d*)	**23.** (*d*)	**24.** (*d*)
25. (*d*)	**26.** (*a*)	**27.** (*a*)	**28.** (*a*)	**29.** (*a*)	**30.** (*a*)	**31.** (*b*).	

Multiple Choice Questions B:

1) A car, starting from rest, accelerates at the rate f through a distance S, then continues at constant speed for time t and then decelerates at the rate f / 2 to come to rest. If the total distance traversed is 15 S, then _____ (AIEEE 2005)

(a) $S = \frac{1}{6}ft^2$ (b) $S = ft$ (c) $S = \frac{1}{4}ft^2$ (d) $S = \frac{1}{72}ft^2$

2) The relation between time t and distance x is $t = a x^2 + b x$ where a and b are constants. The acceleration is _______ (AIEEE 2005)

(a) $2bv^3$ (b) $-2abv^2$ (c) $2av^2$ (d) $-2av^3$

3) A parachutist after bailing out falls 50 m without friction. When parachute opens, it decelerates at 2 m /s^2. He reaches the ground at the rate of 3 m /s. At what height, did he bail out ?

(a) 182 m (b) 91 m (c) 111 m (d) 293 m [AIEEE 2005]

(NOTE: Actually, frictional force in the upward direction is not constant but increases in proportion to the downward velocity. Hence, the downward deceleration of the parachutist keeps decreasing and he finally reaches a constant terminal velocity.)

4) A ball is released from the top of a tower of eight h metres. It takes T seconds to reach the ground. What s the position of the ball in T / 3 seconds ?

(a) h / 9 m from the ground (b) 7h / 9 m from the ground

(c) 8h / 9 m from the ground (d) 17h / 18 m from the ground [AIEEE 2004]

5) An automobile traveling with a speed of 60 km / hr can brake to stop within a distance of 20 m. If the car is going twice as fast, i.e., 120 km / hr, the stopping distance will be

(a) 20 m (b) 40 m (c) 60 m (d) 80 m [AIEEE 2004]

6) A car moving with a speed of 50 km / hr can be stopped by brakes in 6 m. If the same car is moving with a speed of 100 km / hr, then minimum stopping distance is

(a) 6 m (b) 12 m (c) 18 m (d) 24 m [AIEEE 2003]

7) Two cars 1 and 2, starting from rest are moving with speeds v_1 and v_2 m/s ($v_1 > v_2$). Car 2 is ahead of car 1 by 's' metres when the driver of car 1 sees car 2. What minimum retardation should be given to car 1 to avoid collision ?

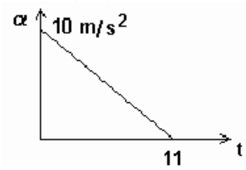
(a) $\frac{v_1 - v_2}{2}$ (b) $\frac{v_1 + v_2}{2}$ (c) $\frac{[v_1 + v_2]^2}{2s}$ (d) $\frac{[v_1 - v_2]^2}{2s}$ [AIEEE 2002]

8) A car moves along a straight line whose motion is given by $s = 12t + 3t^2 - 2t^3$, where (s) is in metres and (t) is in seconds. The velocity of the car at start will be

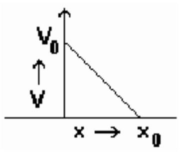
(a) 7 m/s (b) 9 m/s (c) 12 m/s (d) 16 m/s [AIEEE 2002]

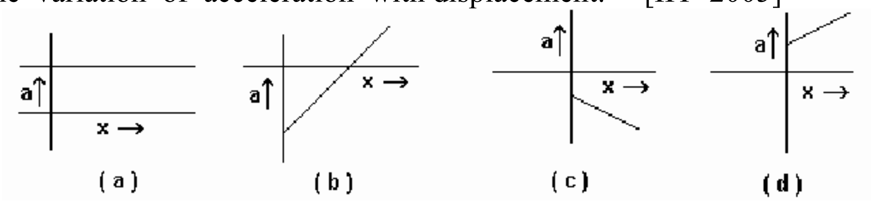
9) A particle starts from rest. Its acceleration (α) versus time (t) is as shown in the figure. The maximum speed of the particle will be ______

(a) 110 m/s (b) 55 m/s (c) 550 m/s (d) 660 m/s [IIT 2004]



10) If graph of velocity vs. distance is as shown, which of the following graphs correctly represents the variation of acceleration with displacement. [IIT 2005]





11) A particle of mass m moves on the x-axis as follows: it starts from rest at t = 0 from the point x = 0, and comes to rest at t = 1 at the point x = 1. No other information is available about its motion at intermediate times (0 < t < 1). If α denotes the instantaneous acceleration of the particle, then

(a) α cannot remain positive for all t in the interval $0 \le t \le 1$

(b) $|\alpha|$ cannot exceed 2 at any point in its path

(c) $|\alpha|$ must be ≥ 4 at some point or points in its path

(d) α must change sign during the motion, but no other assertion can be made with the information given. [IIT 1993]

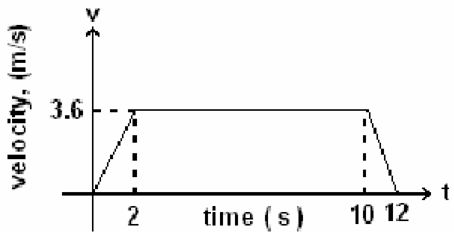
12) Four persons K, L, M and N are initially at the corners of a square of side of length d. If every person starts moving with velocity v such that K is always headed towards L, L towards M, M towards N and N towards K, then the four persons will meet after

(a) d/v s (b) $d\sqrt{2}/v$ s (c) $d/\sqrt{2}v$ s (d) $d/2v$ s [IIT 1984]

13) A lift is going up. The variation in the speed of the lift is as given in the graph. What is the height to which the lift takes the passengers ?

(a) 3.6 m (b) 28.8 m (c) 36 m

(d) cannot be calculated from the above graph [IIT 1970]



14) In the above graph, what will be the average velocity of the lift ?
(a) 1 m/s (b) 2.88 m/s (c) 3.24 m/s (d) 3 m/s [IIT 1970]

15) In the graph of question (12), the average acceleration of the lift is
(a) 1.8 m/s^2 (b) - 1.8 m/s^2 (c) 0.3 m/s^2 (d) zero [IIT 1970]

16) A car accelerates from rest at a constant velocity m for some time and then decelerates at a constant rate n to come to rest. If the total time of journey is t, then the maximum velocity acquired by the car is given by ________

(a) $\left(\frac{m+n}{mn}\right)t$ (b) $\left(\frac{mn}{m+n}\right)t$ (c) $\left(\frac{m^2 - n^2}{mn}\right)t$ (d) $\left(\frac{mn}{m-n}\right)t$

17) The coordinates of a moving particle at any time t are given by $x = at^2$ and $y = bt^2$. The 1 speed of the particle at time t is given by _____

(a) $2t(a + b)$ (b) $\sqrt{a^2 + b^2}$ (c) $2t\sqrt{a^2 + b^2}$ (d) $2t\sqrt{a^2 - b^2}$

18) The coordinates of a moving particle at any time t are given by $x = at^2$ and $y = bt^2$. The 1 speed of the particle at time t is given by ________
(a) 10 m (b) 15 m (c) 20 m (d) 25 m

19) If ‘a’, ‘b’ and ‘c’ are the distances travelled by a particle during xth, yth and zth second from the start, then which of the following relations is valid ?
(a) $a(y-z) + b(z-x) + c(x-y) = 0$
(b) $a(x-y) + b(y-z) + c(z-x) = 0$
(c) $a(z-x) + b(x-y) + c(y-z) = 0$
(d) $ax + by + cz = 0$

20) The distance-time graph of a particle at time t makes an angle of 45° with the time axis. After 1 second, it makes an angle of 60° with the time axis. The acceleration of the particle is _______.
(a) 3 - 1 3 + 1 (c) 3 (b) (d) 1

21) A stone is dropped into a well in which the level of water is at a depth h below the top of the well. If v is the velocity of sound, then the time t after which the splash of sound is heard after dropping the ball is ______

(a) $\sqrt{\frac{2h}{g}} + \frac{h}{v}$ (b) $\frac{2h}{v}$ (c) $\sqrt{\frac{h}{2g}} + \frac{h}{v}$ (d) $\sqrt{\frac{h}{2g}} + \frac{2h}{v}$

22) The relation between time t and distance x is t = ax2 + bx, where a and b are constants. If v represents the velocity, the retardation is
(a) $2av^2$ (b) $2bv^3$ (c) $2b^2v^3$ (d) $2ab\,v^3$ [NCERT 1982]

23) A particle moving with uniform acceleration has velocities u and v at points A and B in its path. The velocity of the body midway between A and B is _______

(a) $\frac{u+v}{2}$ (b) $\sqrt{\frac{u^2+v^2}{2}}$ (c) $\sqrt{uv}$ (d) None of these

24) A particle moves with uniform acceleration and v_1, v_2 and v_3 denote the average velocities in three successive intervals of time t_1, t_2 and t_3. Then $\frac{v_1-v_2}{v_2-v_2}$ is equal to _____

(a) $\frac{t_1-t_2}{t_2+t_3}$ (b) $\frac{t_1+t_2}{t_2+t_3}$ (c) $\frac{t_1-t_2}{t_2-t_3}$ (d) $\frac{t_1-t_2}{t_1-t_3}$

Answers :

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
d	d	d	c	d	d	d	c	b	b	c	a	c	d	d	b	c	b	a	a

21	22	23	24
a	a	c	b

Multiple Choice Questions C:

1. A particle moves half the distance with velocity 'u' and the other half with velocity 'v' in the same straight line, then the average velocity is given by

(a) $u + v$ (b) $\frac{u + v}{2}$

(c) $\frac{2(u + v)}{2}$ (d) $\frac{2uv}{u + v}$

Ans: (d)

Solution:

Let 2s be the total displacement moved by the particle

Let 't' be time taken to move.

1st, half distance moved is $t_1 = \frac{s}{u}$

2nd half distance moved is $t_2 = \frac{s}{u}$

$$\text{Average Velocity} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$$

$$= \frac{2s}{\frac{s}{u} + \frac{s}{v}} = \frac{2s}{s\left(\frac{1}{u} + \frac{1}{v}\right)}$$

$$= \frac{2s}{s\left(\frac{u+v}{uv}\right)}$$

$$\text{Average Velocity} = \frac{2uv}{u+v}.$$

2. A man walks at a speed of 6 km/hr for 1 km and 8 km/hr for the next 1 km. What is the average speed for the walk of 2 km?

(a) 6 km/hr (b) 7 km/hr

(c) 8 km/hr (d) 2 km/hr

Ans: (b)

Solution:

Distance travelled is 2 km

$$\text{Time taken} = \frac{\text{Distance}}{\text{Velocity}}$$

In the 1st case $t_1 = \dfrac{1\,\text{km}}{6\,\text{km / hr}}$

$$t_2 = \frac{1\,\text{km}}{8\,\text{km / hr}}$$

$$\text{Total time taken } T = t_1 + t_2 = \left(\frac{1}{6} + \frac{1}{8}\right)\text{hr}$$

$$T = \frac{7}{24}\text{ hr}$$

$$\text{Average speed} = \frac{\text{Distance travelled}}{\text{Time taken}}$$

$$= \frac{2}{\left(\frac{7}{24}\right)} = \frac{48}{7} = 7\text{km/hr}$$

3. **When a balloon is at a height of 80 m, ascending with a velocity of 10 m/s and acceleration 1.2 m/s², drops a packet. Find the time taken by the packet to reach ground? Take g = 10 m/s².**

(a) 5.2 sec (b) 5 sec

(c) 6.2 sec (d) 1.2 sec

Ans: (a)

Solution:

Velocity of the packet drops from a balloon
(u) = 10 m/s

Upwards at a height of 80 m

i.e., displacement = –80 m

$\therefore$ from equation of motion $s = ut - \frac{1}{2} gt^2$

$S = -80$, $u = 10$ m/s, $g = 10$ m/s^2

$-80 = 10\,t - \frac{1}{2} \times 10 \times t^2 = 1 \times 5 \times t^2$

$-80 = 10\,t - 5\,t^2$

$5\,t^2 = 10\,t - 80 = 0$

$t^2 - 2t - 16 = 0$

Solving equation $t \simeq 5.2$ sec.

4. A particle at rest starts moving in a horizontal straight line with uniform acceleration. The ratio of the distance covered during the fourth and the third second is

(a) $\frac{26}{9}$ (b) $\frac{7}{5}$

(c) $\frac{4}{3}$ (d) $\frac{2}{5}$

Ans: (b)

Solution:

$$S_n = u + \frac{a}{2}(2n - 1) \qquad \because u = 0$$

$$S_4 = 0 + \frac{a}{2}(2 \times 4 - 1)$$

$$S_3 = 0 + \frac{a}{2}.(2 \times 3 - 1)$$

$$\frac{S_4}{S_3} = \frac{\frac{7a}{2}}{\frac{5a}{2}} = \frac{7}{5}$$

5. **A cyclist of mass 'm' is taking a circular turn of radius 'R' on a frictional level road with a Velocity 'v". In order that the cyclist does not skid.**

(a) $\left(m\frac{v^2}{2}\right) > \mu mg$ (b) $\left(\frac{mv^2}{r}\right) > \mu mg$

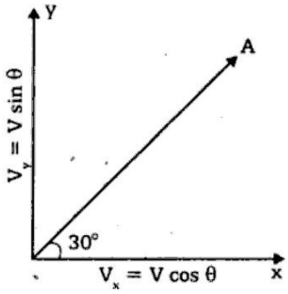
(c) $\left(\frac{mv^2}{r}\right) < \mu mg$ (d) $\left(\frac{v^2}{r}\right) = \mu g$

Ans: (c)

6. **A body is thrown into air with a Velocity 5 m/s making an angle 30° with the horizontal. If the vertical component of the Velocity is 5 m/s. What is the Velocity of the body? Also find the horizontal component of the velocity ?**

(a) 5 m/s, 0.5 m/s (b) 10 m/s, 0.5 m/s
(c) 10 m/s, 8.66 m/s (d) 10 m/s, 0.866 m/s

Ans: (c)



Solution:

$v_x \Rightarrow$ horizontal component of the Velocity

$v_x = v \cos\theta$

$v_y = v \sin\theta$

$v_y \Rightarrow$ vertical component of the Velocity

Given data:

$v_y = 5$ m/s

$5 = v \sin\theta$

$$\frac{5}{\sin\theta} = v$$

$$\frac{5}{\sin 30°} = \frac{5}{0.5} = v$$

$v = 10$ m/s

Vertical component of the Velocity

$= v_x = v \cos\theta$

$v_y = 10 \times \cos 30° = 10 \times 0.866$

$v_y = 8.66$ m/s

7. What happens if a Vector is multiplied by a number 2?

(a) the magnitude of the Vector is doubled and its direction is reversed.
(b) The magnitude of the Vector is doubled but its direction remains the same.
(c) Neither the magnitude nor the direction of the Vector undergo any change.
(d) the magnitude of the Vector is halved and its direction is reversed.

Ans: (b)

Solution:

When a Vector multiplied by a real positive number (n) it makes its magnitude 'n' times, but does not change the direction of the Vector.

Hence the correct choice is (b)

8. A particle starts from rest with a constant acceleration, At a time 't' second, the Velocity is found to be 200 m/s and one second later the Velocity becomes 250 m/s. Calculate the acceleration and the distance travelled during the $(t + 1)^{th}$ second?

(a) 125 m/s^2, 60 m (b) 120 m/s^2 100 m
(c) 70 m/s^2, 125 m (d) 50 m/s^2, 225 m

Ans: (d)

Solution:

1. Velocity at 't' = 200 m/s
   (v_1)
2. Velocity at (t + 1) second = 250 m/s
   (v_2)

$v_1 = a\,t$

200 m/s = a t .....(1)

$v_2 = a\,(t + 1)$

250 m/s = a (t + 1) ......(2)

subtracting (1) from (2) we get,

250 – 200 = at + a – at

50 = a m/s^2

acceleration 'a' = 50 m/s^2 'a'

Consider the interval 't' second to (t + 1) second time elapsed = 1 second

initial velocity = 200 m/s

final velocity = 250 m/s

According to equation of motion,

$v^2 = u^2 + 2as$

$(250)^2 = (200)^2 + 2\,(50)\,s$

62500 = 40000 + 100 s

62500 – 40000 = 100 s

s = 225 m

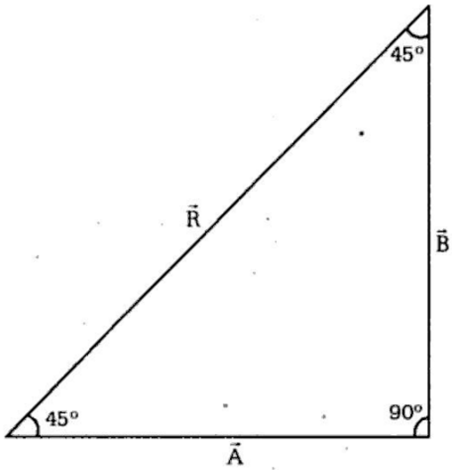
9. The resultant of two Vectors A and B subtends at angle of 45° with either of this. The magnitude of the resultant is

(a) $\frac{1}{\sqrt{2}}A$ (b) $\sqrt{2}A^2$

(c) $\sqrt{2}A$ (d) Zero

45°

Ans: (c)



Solution:

The angle between the two vectors $\vec{A}$ and $\vec{B}$ is 90°

$\therefore A = B$

Therefore the magnitude of the resultant is given by

$R^2 = A^2 + B^2 + 2AB \cos\theta$

$= A^2 + A^2 + 2A^2 \cos 90^\circ$

$\cos 90^\circ = 0$

$R^2 = A^2 + A^2 + 0$

$R^2 = 2A^2$

$R = \sqrt{2A^2} = \sqrt{2}A$

10. **A Ball is thrown from a field with a speed of 15 m/s at an angle of 45° with the horizontal. At what distance will it hit the field again ?**

Take g = 10 m/s²

(a) 2.25 m (b) 225 m

(c) 22.5 m (d) 20.25 m

Ans: (c)

Solution:

Horizontal Range $(R_{max}) = \frac{u^2}{g}$

$R = \frac{15 \times 15}{10}$ When the range is maximum, the angle is 45°

$\therefore R = 22.5$ m

Thus, the ball hits the field at 22.5 m from the point of projection.

11. **A motor launch takes 50s to travel 100 m upstream and 25s to travel the same distance down stream. What is the speed of the current and launch?**

(a) 1 m/s, 3 m/s (b) 1 m/s, 4 m/s
(c) 3 m/s, 1 m/s (d) 4 m/s, 3 m/s

(a) 1 m/s, 3 m/s (b) 1 m/s, 4 m/s
(c) 3 m/s, 1 m/s (d) 4 m/s, 3 m/s

Ans: (a)

Solution:

Let v_c be the speed of the current

Let v_t be the speed of the launch

for upstream, the resultant Velocity is $v_t - v_c$

Displacement = Velocity × Time

$100 = (v_t - v_c) \times 50$

$$v_t - v_c = \frac{100}{50} = 2 \qquad .....(1)$$

For downstream, the resultant Velocity is

$v_t + v_c$

$\therefore 100 = (v_t + v_c)\, 25$

$$\frac{100}{50} = v_t + v_c$$

$(v_t + v_c) = 4 \qquad .....(2)$

solving equation (1) and (2)

$2\, v_t = 6$

$$v_t = \frac{6}{2} = 3 \text{ m/s}$$

$$v_c = 1 \text{ m/s}$$

16. **A particle travels 40 m in its last second of motion while it falls from a height. What is the height? ($g = 10 \text{ m/s}^2$)**

(a) 60 m (b) 40 m
(c) 61.25 m (d) 75 m

Ans: (c)

Solution:

$$S_{n^{th}} = u + \tfrac{1}{2}(2n-1)g$$

Given data:

$S_{n^{th}} = 40$ m

$u = 0$

$g = 10 \text{ m/s}^2$

$$40 = 0 + \frac{10}{2}(2n - 1)$$

$$40 = 5(2n - 1)$$

$$40 = 10n - 5$$

$$35 = 10\,n$$

$$n = \frac{35}{10} = 3.5$$

Using the equation of motion

$s = ut + \tfrac{1}{2}gt^2$

$u = 0$

$t = 3.5$

$$\therefore s = 0 + \frac{1}{2}(10) \times (3.5)^2$$

$s = 5 \times 12.25$

$s = 61.25$

height = 61.25 m

17. A stone drops from a height of 100 m and simultaneously a stone is thrown up 40m/s. Find the time and position when they cross each other

(a) 2.5 S, 68.75 m (b) 2S, 60 m
(c) 2.25, 40 m (d) None.

Ans: (a)

CH 4

Unit II: Kinematics Part II

Motion in a Plane

(12 Hours, 11 Marks (1M-1Q, 2M-1Q, 3M-1Q, 5M-1QNP))

July Exam 17M [1M-1Q, 2M-1Q, 4M-1Q, 5M-1Q (T), 5M-1Q(P)]

Syllabus : Scalar and vector quantities: Position and displacement vectors, general vectors and notation, equality of vectors, multiplication of vectors by a real number; addition and subtraction of vectors. Relative velocity. Unit vector; Resolution of a vector in a plane-rectangular components. Scalar and Vector product of vectors. Motion in a plane. Cases of uniform velocity and uniform acceleration-projectile motion. Uniform circular motion.

4.1 Scalar and vector quantities : general vectors and notation,

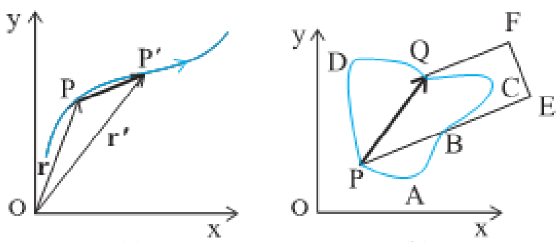
A scalar quantity is a quantity with magnitude only. It is specified completely by a single number, along with the proper unit. Examples are : the distance between two points, mass of an object, the temperature of a body and the time at which a certain event happened. The rules for combining scalars are the rules of ordinary algebra. Scalars can be added, subtracted, multiplied and divided just as the ordinary numbers.

A vector quantity is a quantity that has both a magnitude and a direction and obeys the triangle law of addition or equivalently the parallelogram law of addition. So, a vector is specified by giving its magnitude by a number and its direction. Some physical quantities that are represented by vectors are displacement, velocity, acceleration and force.

4.2 Position and displacement vectors :

Position of an object P with respect to origin O in Cartesian co-ordinate system, then OP = r is called **position vector** at a time t. The length of the vector r represents the magnitude of the vector and its direction is the direction in which P lies as seen from O.
If the object moves from P to P′, the vector PP′ (with tail at P and tip at P′) is called the **displacement vector** corresponding to motion from point P (at time t) to point P′ (at time t′).

It is important to note that displacement vector is the straight line joining the initial and final positions and does not depend on the actual path undertaken by the object between the two positions.



(a) (b)

Fig. 4.1 (a) Position and displacement vectors. (b) Displacement vector PQ and different courses of motion.

4.3 Equality of Vectors :

Two vectors A and B are said to be equal if, and only if, they have the same magnitude and the same direction.

4.4 Multiplication of vectors by a real number :

Multiplying a vector A with a positive number λ gives a vector whose magnitude is changed by the factor λ but the direction is the same as that of A :

$|\lambda \mathbf{A}| = \lambda |\mathbf{A}|$ if $\lambda > 0$.

For example, if A is multiplied by 2, the resultant vector 2A is in the same direction as A and has a magnitude twice of |A|.

Multiplying a vector A by a negative number λ gives a vector λA whose direction is opposite to the direction of A and whose magnitude is –λ times |A|.

4.5 Addition and subtraction of vectors :

Vectors, by definition, obey the triangle law or equivalently, the parallelogram law of addition. Two vectors A and B may be added graphically using head-to-tail method or parallelogram method.

Parallelogram law of vector addition :

If two vectors are represented in magnitude and direction by the two adjacent sides of a parallelogram drawn from a point, then their resultant is represented in magnitude and direction by the diagonal of the parallelogram passing through the point.

Let $\vec{A}$ and $\vec{B}$ are two vectors and are inclined at an angle θ. R = A + B

SN is normal to OP and PM is normal to OS.

From the geometry of the figure, $OS^2 = ON^2 + SN^2$

but ON = OP + PN = A + B cos θ

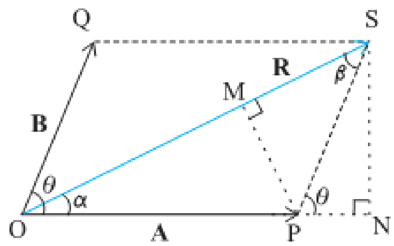
SN = B sin θ

$OS^2 = (A + B\cos\theta)^2 + (B\sin\theta)^2$

or, $R^2 = A^2 + B^2 + 2AB\cos\theta$ -------- (4.1) (Law of cosine)

The magnitude of the Resultant Vector (R) is given by $R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

α is the angle between R and Vector A.



In Δ OSN, SN = OS sinα = R sinα, and

in Δ PSN, SN = PS sin θ = B sin θ

Therefore, R sin α = B sin θ

$\frac{R}{\sin\theta} = \frac{B}{\sin\alpha}$ ------- (a)

Similarly, PM = A sin α = B sin β

$\frac{A}{\sin\beta} = \frac{B}{\sin\alpha}$ -------- (b)

Combining Eqns. (a) and (b)

$\frac{R}{\sin\theta} = \frac{A}{\sin\beta} = \frac{B}{\sin\alpha}$ -------- (c) (Law of sines)

Using above equation, we get, $\sin\alpha = \frac{B}{R}\sin\theta$

$\therefore$ **Tan α =** $\frac{SN}{OP+PN} = \frac{B\sin\theta}{A+B\cos\theta}$ --------- **(4.2)**

Special Cases :

(i) When the two vectors are in same direction : (θ= 0°)

R = $\sqrt{(A+B)^2}$ or R = A + B

$\therefore$ **Tan α = 0**

(ii) When θ = 180° ; R = $\sqrt{(A-B)^2}$; R = A - B

$\therefore$ **Tan α = 0**

(iii) When θ = 90° R = $\sqrt{A^2 + B^2}$

$\therefore$ **Tan α = B/A**

Vector addition is commutative : A + B = B + A

It also obeys the associative law : (A + B) + C = A + (B + C)

A null or zero vector is a vector with zero magnitude. Since the magnitude is zero, we don't have to specify its direction. It has the properties :

A + 0 = A

λ0 = 0

0. A = 0

The subtraction of vector B from A is defined as the sum of A and –B : A – B = A+ (–B)

4.6. Unit vector :

A unit vector is a vector of unit magnitude and points in a particular direction. It has no dimension and unit. It is used to specify a direction only. Unit vectors along the x-, y- and z-axes of a rectangular coordinate system are denoted by $\hat{\imath}$, $\hat{\jmath}$, $\hat{k}$, respectively.

$$\left|\hat{\imath}\right| = \left|\hat{\jmath}\right| = \left|\hat{k}\right| = 1$$

(1) A unit vector associated with a vector A has magnitude *one* and is along the vector A :

$\hat{n} = \frac{A}{|A|}$

The unit vectors $\hat{\imath}$, $\hat{\jmath}$, $\hat{k}$ are vectors of unit magnitude and point in the direction of the x-, y-, and z-axes, respectively in a right-handed coordinate system.

(2) A unit vector $\hat{n}$ which is perpendicular to $\vec{A}$ and $\vec{B}$ is given by $\hat{n} = \frac{\vec{A}\times\vec{B}}{|\vec{A}\times\vec{B}|}$

4.7. Resolution of a vector in a plane-rectangular components.

A vector A can be resolved into component along two given vectors a and b lying in the same plane : **A = λ a + μ b** where λ and μ are real numbers.

Let us resolve a general vector A into three components along x-, y-, and z-axes in three dimensions. If α, β, and γ are the angles between A and the x-, y-, and z-axes, respectively Fig. 4.9(d), we have,

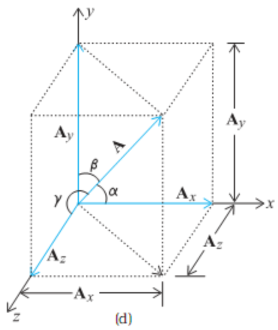
A_x = A cos α, A_y = A cos β, A_z = A cos γ

In general, we have

$A = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$

The magnitude of vector A is $A = \sqrt{A_x^2 + A_y^2 + A_z^2}$

Note that $A_y = A \cos \beta = A \cos (90 - \alpha) = A \sin \alpha$



4.8. Scalar and Vector product of vectors.

Scalar or DOT product:

It is defined as the product of the magnitud of 'A' and 'B' and the cos θ between them

i.e., $A.B = AB \cos \theta$

The Scalar product of two vectors is a Scal Quantity.

Properties of Scalar product:

1. The Scalar product is commutative

   $A.B = B.A$

2. $A.A = A^2$ or $A = \sqrt{A.A}$. This is true because in this case $\theta = 0°$

3. If two vectors A and B are perpendicular to each other

   $A. B = AB \cos 90° = 0$

For unit vectors $\hat{i}, \hat{j}$ and $\hat{k}$ along the three rectangular axes x, y and z, we have

$\hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k} = 1$

and

$\hat{i}.\hat{j} = \hat{j}.\hat{k} = \hat{k}.\hat{i} = 0$

Examples of Scalar product:

(i) Magnetic flux

$(\phi) = A.B$

Where B $\Rightarrow$ magnetic induction vector

A $\Rightarrow$ Area vector

(ii) Electric current

$(I) = J.A$

Where J $\Rightarrow$ current density vector

A $\Rightarrow$ Area vector

(iii) $W = F.S$

F $\Rightarrow$ Force vector

S $\Rightarrow$ Displacement vector

Vector or Cross product:

The Vector product of Vectors 'A' and 'B' is defined as

$$A \times B = (AB \sin\theta)\,\hat{n}$$

Where θ is the angle between the vectors 'A' and 'B'

Where $\hat{n}$ is the unit vector perpendicular to the plane containing 'A' and 'B'

$$A \times B = C$$

The magnitude of vector 'C' is given by

$$C = A\,B \sin\theta$$

Properties of a Vector product:

1. Vector product is anti commutative

   $(A \times B) = -(B \times A)$

2. $A \times A = AA \sin\theta = 0$

   i.e. the vector product of a vector by itself is zero.

   This is because, $\theta = 0$

   $\therefore \sin\theta = 0$

3. The distributive law holds good for both Scalar and Vector product.

   e.g. $A.(B + C) = A.B + A.C$

   $A \times (B + C) = (A \times B) + (A \times C)$

4. The right hand rule gives

$$\hat{i} \times \hat{j} = -\hat{j} \times \hat{i} = k$$

$$\hat{j} \times \hat{k} = -\hat{k} \times \hat{j} = \hat{i}$$

$$\hat{k} \times \hat{i} = -\hat{i} \times \hat{k} = \hat{j}$$

$$\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0$$

Where $\hat{i}$, $\hat{j}$ and k are the three mutually perpendicular unit vectors.

5. $$(A \times B) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$= \hat{i}\left(A_y B_z - A_z B_y\right) + \hat{j}\left(A_z B_x - A_x B_z\right) + \hat{k}\left(A_x B_y - A_y B_x\right)$$

Examples of vector product:

Angular momentum

$$\left(\vec{L}\right) = \vec{r} \times \vec{p}$$

Where $\vec{r} \Rightarrow$ position vector

p = Linear momentum vector

Torque $(\tau) = \vec{r} \times \vec{F}$

$F = \Rightarrow$ Force vector

$r = \Rightarrow$ Position vector.

4.9. Motion in a plane with constant acceleration :

Suppose that an object is moving in x-y plane and its acceleration a is constant. Over an interval of time, the average acceleration will equal this constant value. Now, let the velocity of the object be v0 at time t = 0 and v at time t.

Then, by definition

$$\mathbf{a} = \frac{\mathbf{v} - \mathbf{v_0}}{t - 0} = \frac{\mathbf{v} - \mathbf{v_0}}{t} \quad \text{or} \quad \mathbf{v} = \mathbf{v_0} + \mathbf{a}t$$

In terms of components :

$$v_x = v_{ox} + a_x t$$

$$v_y = v_{oy} + a_y t$$

If an object is moving in a plane with constant acceleration $a = |a| = \sqrt{a_x^2 + a_y^2}$ and its position vector at time t = 0 is r_o, then at any other time t, it will be at a point given by :

$$\mathbf{r} = \mathbf{r_o} + \mathbf{v_o}t + \frac{1}{2}\mathbf{a}t^2$$

and its velocity is given by : $v = v_o + a\,t$

where v_o is the velocity at time t = 0. In component form :

$$x = x_o + v_{ox}t + \frac{1}{2}a_x t^2$$

$$y = y_o + v_{oy}\,t + \frac{1}{2}a_y t^2$$

$$v_x = v_{ox} + a_x t$$

$$v_y = v_{oy} + a_y t$$

Motion in a plane can be treated as superposition of two separate simultaneous one-dimensional motions along two perpendicular directions.

Equations of Motion

(i) $v = u + at$

(ii) $S = ut + \frac{1}{2} at^2$

(iii) $v^2 = u^2 + 2\,as$

(iv) $S = u + \frac{1}{2}\,a\,(2n-1)$

4.10. Cases of uniform velocity and uniform acceleration-projectile motion.

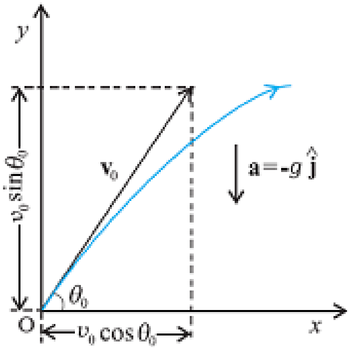
An object that is in flight after being thrown or projected is called a projectile. Such a projectile might be a football, a cricket ball, a baseball or any other object. The motion of a projectile may be thought of as the result of two separate, simultaneously occurring components of motions. One component is along a horizontal direction without any acceleration and the other along the vertical direction with constant acceleration due to the force of gravity. It was **Galileo** who first stated this independency of the horizontal and the vertical components of projectile motion in his Dialogue on the great world systems (1632).

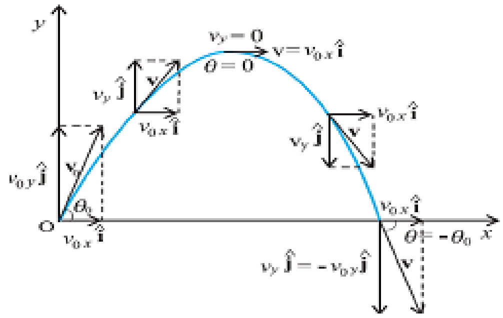
(a) Equation of path of a projectile :

Suppose that the projectile is launched with velocity v_0 that makes an angle θ_0 with the x-axis as shown in Fig. 4.2. Here, the air-resistance is neglected. After the object has been projected, the acceleration acting on it is that due to gravity which is directed vertically downward:

$\vec{a} = -g\hat{j}$

Therefore, $a_x = 0$; $a_y = -g$





The components of initial velocity v_0 are :

$v_{ox} = v_o \cos\theta_o$

$v_{oy} = v_o \sin\theta_o$

If we take the initial position to be the origin of the reference frame as shown in Fig. (a), we have :

$x_o = 0,\ y_o = 0$

Then

$$x = v_{ox} t = (v_o \cos\theta_o)\, t$$

and $$y = (v_o \sin\theta_o)\, t - (\tfrac{1}{2}) g t^2$$

The components of velocity at time t can be written as :

$$v_x = v_{ox} = v_o \cos\theta_o$$

$$v_y = v_o \sin\theta_o - g t \qquad \text{--------------------- (4.4)}$$

$$y = (\tan\theta_o) x - \frac{g}{2\,(v_o \cos\theta_o)^2} x^2 \qquad \text{------------------ (4.5)}$$

Now, since g, θ_0 and v_0 are constants, Eq. (4.5) is of the form $y = a\,x + b\,x^2$, in which a and b are constants. This is the equation of a parabola, i.e. the path of the projectile is a **parabola** as shown in Fig. (b).

(b) Maximum height of the projectile :

Consider a projectile moving in a direction making an angle θ with the horizontal.

Let v_0 - velocity of the projectile.

The velocity v_0 of the projectile resolved into

$V_X = v_0 \cos\theta$ along horizontal (x-axis)

$V_Y = v_0 \sin\theta$ along vertical (y-axis)

After the object has been projected, the acceleration acting on it due to gravity and is directed vertically upwards

$\vec{a} = -g\hat{j}$

Therefore, $a_x = 0$; $a_y = -g$

The **maximum height** h_m reached by the projectile can be calculated by substituting $t = t_m$

$$y = h_m = (v_0 \sin\theta_0)\left(\frac{v_0 \sin\theta_0}{g}\right) - \frac{g}{2}\left(\frac{v_0 \sin\theta_0}{g}\right)^2$$

$$h_m = \frac{(v_0 \sin\theta_0)^2}{2g} \quad \text{-----------} \quad \textbf{(4.6)}$$

(c) Time of maximum height :

Let this time be denoted by t_m. Since at this point, $v_y = 0$, Then Eqn. (4.4) becomes

$$v_y = v_o \sin\theta_o - g\,t_m = 0$$

Or, $t_m = v_o \sin\theta_o / g$ ----------- (4.7a)

The total time T_f during which the projectile is in flight can be obtained by putting $y = 0$ in Eq. (4.3). We get :

$$T_f = 2\,(v_o \sin\,\theta_o)/g \quad \text{------------} \quad \textbf{(4.7)}$$

T_f is known as the time of flight of the projectile. We note that $T_f = 2t_m$, which is expected because of the symmetry of the parabolic path.

(d) Horizontal range of a projectile :

The horizontal distance travelled by a projectile from its initial position (x = y = 0) to the position where it passes y = 0 during its fall is called the horizontal range, R. It is the distance travelled during the time of flight T_f. Therefore, the **range R** is

$$R = (v_o \cos\,\theta_o)\,(T_f)$$

$$= (v_o \cos\,\theta_o)\,\,(2\,v_o \sin\,\theta_o)/g$$

Or, $$R = \frac{v_0^2 \sin 2\theta_0}{g} \quad \text{----------} \quad \textbf{(4.8a)}$$

Above Equation shows that for a given projection velocity v_0, R is maximum when $\sin 2\theta_0$ is maximum, i.e., when $\theta_0 = 45^0$. The **maximum horizontal range** is, therefore,

$$R_m = \frac{v_0^2}{g} \quad \text{-----------------} \quad \textbf{(4.8)}$$

4.11. Uniform circular motion.

When an object follows a circular path at a constant speed, the motion of the object is called uniform circular motion. Suppose an object is moving with uniform speed v in a circle of radius R. Since the velocity of the object is changing continuously in direction, the object undergoes acceleration.

The magnitude of acceleration a is, by definition, given by :

$$|\mathbf{a}| = \lim_{\Delta t \to 0} \frac{|\Delta \mathbf{v}|}{\Delta t}$$

Let the angle between position vectors r and r′ be Δθ. Since the velocity vectors v and v′ are always perpendicular to the position vectors, the angle between them is also Δθ .

Therefore, the centripetal acceleration a_c is : $a_c = \left[\frac{v}{R}\right] v = \frac{v^2}{R}$ ------------ **(a)**

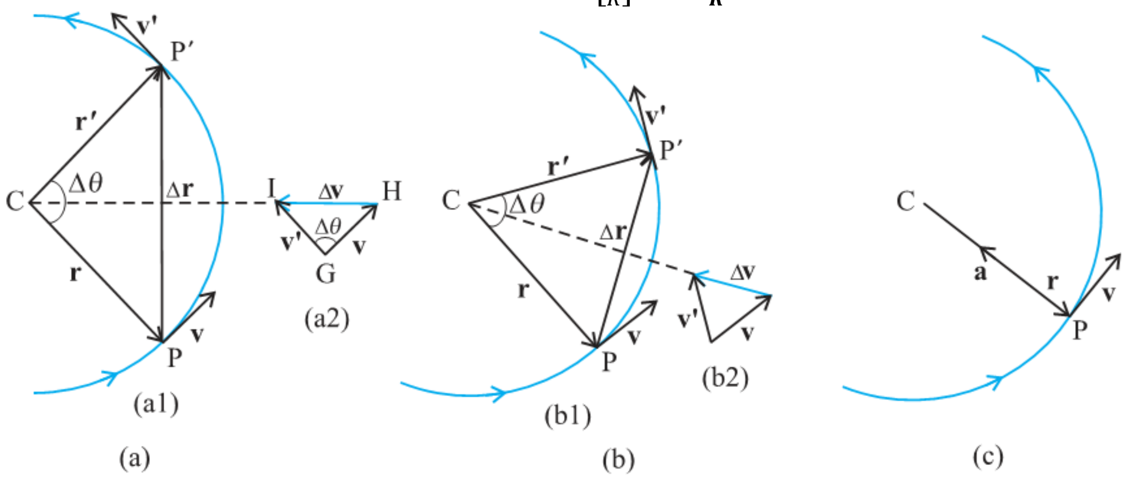


Fig. 4.4 : Velocity and acceleration of an object in uniform circular motion. The time interval Δt decreases from (a) to (c) where it is zero. The acceleration is directed, at each point of the path, towards the centre of the circle.

Thus, the acceleration of an object moving with speed v in a circle of radius R has a magnitude v^2 /R and is always directed towards the centre. This is why this acceleration is called centripetal acceleration. Since v and R are constant, the magnitude of the centripetal acceleration is also constant. However, the direction changes — pointing always towards the centre. Therefore, a centripetal acceleration is not a constant vector.

We can also describe the velocity and the acceleration of an object in uniform circular motion in another way :

As the object moves from P to P′ in time Δt (= t′ – t), the line CP (Fig. 4.19) turns through an angle Δθ as shown in the figure. Δθ is called angular distance. We define the angular speed ω (Greek letter omega) as the time rate of change of angular displacement :

$$\boldsymbol{\omega} = \frac{\Delta\boldsymbol{\theta}}{\Delta \boldsymbol{t}} \quad \text{-------------} \quad (3.\ 5)$$

Now, if the distance travelled by the object during the time Δt is Δs, i.e. PP′ is Δs, then :

$$\mathbf{v} = \frac{\Delta s}{\Delta t} \quad \text{but } \Delta s = R\ \Delta\theta. \text{ Therefore : } \quad v = R\frac{\Delta\theta}{\Delta t} = R\omega$$

$$\mathbf{v = R}\boldsymbol{\omega} \quad \text{----------} \quad (3.6)$$

We can express centripetal acceleration a_c in terms of angular speed :

$$a_c = \frac{v^2}{R} = \frac{\omega^2 R^2}{R} = \omega^2 R \qquad \text{or} \qquad \boldsymbol{a_c = \omega^2}\mathbf{R} \quad \text{-------------} \quad (3.7)$$

The time taken by an object to make one revolution is known as its **time period** T and the number of revolution made in one second is called its frequency ν (=1/T). However, during this time the distance moved by the object is s = 2πR. Therefore, v = 2πR/T =2πRν

In terms of frequency ν, we have $\omega = 2\pi\nu$, $v = 2\pi R\nu$

$$\mathbf{a_c = 4\pi^2 \nu^2 R} \quad \text{-----------} \quad (3.8)$$

ONE MARK QUESTIONS

1. What is scalar quantity ?

Ans : Physical quantity which have magnitude but no direction.

2. Give an example for scalar quantity.

Ans : Distance, speed, mass, temp etc.

3. Does the scalar addition obey ordinary addition rules?

Ans : Yes.

4. What is vector quantity ?

Ans : Physical quantity which have both magnitude and direction.

5. Give an example for vector quantity.

Ans : Displacement, velocity, acceleration, etc.

6. Does the vector addition obey ordinary addition rules ?

Ans : No.

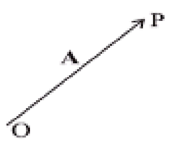
7. How does vector is different from scalar ?

Ans : Vector is having both magnitude and direction but scalar has only magnitude.

8. Is displacement a vector or a scalar ?

Ans : Vector.

9. Give the graphical representation of vector.

Ans : 

10. Define null vector

Ans : Vector having zero magnitude.

11. Define unit vector

Ans : It is a vector whose magnitude is unity.

12. What is position vector ?

Ans : A vector which gives the position of a particle with reference to the origin of a co-ordinate system.

13. What is negative of a vector ?

Ans : The negative of a vector is a vector having the same magnitude but opposite direction.

14. What are equal vectors ?

Ans : Two vectors of equal magnitudes and same direction.

15. What are parallel vectors ?

Ans : The vectors whose lines of action are parallel.

16. What are concurrent vectors (co-initial vectors) ?

Ans : The vectors having same initial point.

17. What are co-planar vectors ?

Ans : The vectors acting in the same plane

18. Does the vector addition obey the commutative law ?

Ans : Yes.

19. Does the vector addition obey the associative law ?

Ans : Yes.

20. Give the mathematical representation of unit vector.

Ans : $\hat{n} = \frac{\vec{A}}{|\vec{A}|}$

21. Mention any one law of addition of vector.

Ans : Law of triangle of vector.

22. State law of triangle of vectors.

Ans : The law states that if the two vectors acting at a point represents the two sides of a triangle taken in order, then the third side of the triangle taken in reverse order gives the resultant.

23. State the law parallelogram of vectors.

Ans : It states that if two vectors acting on a particle at the same time are presented in magnitude and direction by the two adjacent sides of a parallelogram drawn from a point, their resultant vector is represented in magnitude and direction by the diagonal of parallelogram drawn from the same point.

24. What is resultant vector ?

Ans : The resultant vector is a single vector whose effect is the same as the effect produced by the individual vectors together.

25. What are components of a vector ?

Ans : Effects of a vector in different directions are called components of a vector.

26. What is resolution of a vector ?

Ans : Splitting up of a vector in different directions.

27. What are rectangular components of a vector ?

Ans : The components of vector in two mutually perpendicular direction are called rectangular components.

28. The magnitude of the resultant of the two equal vectors is equal to the magnitude of the either vector, what is the angle between two vectors ?

Ans : 120^0

29. When the magnitude of the resultant of the two vectors is maximum ?

Ans : When angle between two vectors is 0^0

30. When the magnitude of the resultant of the two vectors is minimum ?

Ans : When angle between two vectors is 180^0

31. If 1 2 2 23 and 4 2 2 2 3 are acting at right angles to each other, what is the magnitude of their resultant ?

Ans : R = $\sqrt{A^2 + B^2}$

32. If two equal vectors acting at right angles to each other, what is the magnitude of their resultant ?

Ans : R = $\sqrt{2}$ (magnitude of the individual vector)

33. Vector 1 = 3 units, acting along east, and vector 4 2 2 2 3 = 4 units, acting along north. What is the magnitude of their resultant ?

Ans : R = $\sqrt{P^2 + Q^2} = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$

34. Is scalar multiplied by a vector a vector or a scalar.

Ans : Vector.

35. Give an example for scalar multiplied by a vector.

Ans : 1. $\vec{F} = m\vec{a}$ 2. $\vec{p} = m\vec{v}$

36. What is the magnitude of 1 3 = 3 7 -9 :

Ans : $|\vec{A}| = \sqrt{3^2 + (-1)^2} = \sqrt{9+1} = \sqrt{10}$

37. What is the unit vector of → P = 3i ˆ− 4 j ˆ

Ans : $|\vec{P}| = \frac{\vec{P}}{|\vec{P}|} = \frac{3\hat{i} - 4\hat{j}}{\sqrt{9+16}} = \frac{3\hat{i} - 4\hat{j}}{5}$

38. What is two dimensional motion ?

Ans : Motion of the particle in a plane.

39. Give an example for motion in two dimension .

Ans : Motion of a Javelin.

40. What is a projectile ?

Ans : Any particle moving in a direction making an angle θ with the horizontal under the action of gravity of the earth.

41. Give an example for a projectile .

Ans : A cricket ball thrown by a fielder.

42. Define projectile velocity ?

Ans : Velocity with which the projectile is projected.

43. Define angle of projection of a projectile.

Ans : The angle made by the projectile with the horizontal.

44. Define time of flight of a projectile.

Ans : Time taken by the projectile to reach the maximum height and then to the ground. OR The time during which the projectile is in air.

45. Define range of a projectile.

Ans : Horizontal distance travelled by the projectile is called range of the projectile.

46. What is the nature of path (trajectory) of projectile ?

Ans : Parabola.

47. What is the maximum height of a projectile ?

Ans : The maximum vertical distance travelled by the projectile is called maximum height.

48. Write the horizontal component of velocity of projectile.

Ans : $V_X = v_0 \cos \theta_0$

49. Write the vertical component of velocity of projectile.

Ans : $V_Y = v_0 \sin \theta_0$

50. Which component of velocity of a projectile is constant ?

Ans : The horizontal component .

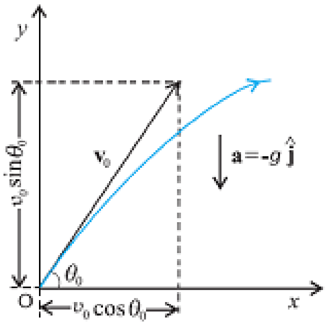
51. Which component of velocity of a projectile is zero at maximum height ?

Ans : The vertical component.

52. Which component of acceleration of a projectile is zero?

Ans : The horizontal component .

53. Draw the graphical representation for a projectile motion .

Ans : 

54. Write the expression for the path of a projectile.

Ans :

$$y = (\tan\theta_0)\, x - \frac{g}{2(v_0 \cos\theta_0)^2} x^2$$

55. Give the expression for the maximum height of a projectile.

Ans : $h_m = \frac{(v_0 \sin\theta_0)^2}{2g}$

56. Give the expression for the time of flight of a projectile.

Ans : $T_f = 2\,(v_0 \sin\theta_0)\,/\,g$

57. Give the expression for the range of a projectile.

Ans : $R = \frac{v_0^2 \sin 2\theta_0}{g}$

58. How does the maximum height depend upon velocity of projectile ?

Ans : Directly proportional to the square of the velocity.

59. How does the time of flight depend upon velocity of projectile ?

Ans : Directly proportional to the velocity.

60. How does the range of projectile depend upon velocity ?

Ans : Directly proportional to the square of the velocity.

61. If velocity of projectile is doubled what happens to the maximum height of projectile?

Ans : Increases by 4 times.

62. When the range of projectile is maximum ?

Ans : When $\theta = 45^0$.

63. Three athletes A,B and C participating in a long jump event jump by making angles 300, 450 and 600with the ground. Who will be the winner ?

Ans : Athlete B.

64. For what two angles of projection , the range of projectile is same?

Ans : θ and $(90- \theta)$

65. What is uniform circular motion?

Ans : Motion of the projectile in a circular path with uniform speed.

66. Which physical quantity remains constant for uniform circular motion?

Ans : Speed (angular velocity).

67. Is velocity of particle constant for a particle in a uniform circular motion?

Ans : No.

68. What is the direction of velocity of a particle in a uniform circular motion?

Ans : Along tangential direction.

69. What is the direction of acceleration of a particle in a uniform circular motion?

Ans : Towards the centre along the radius.

70. Give expression for centripetal acceleration.

Ans : $a = \frac{v^2}{r}$ OR $a = vw$

TWO MARK QUESTIONS :

1. Distinguish between scalar and vector with suitable example for each.

Scalar	**Vector**
Physical quantities which are having only magnitude. Ex : mass, length, time.	Physical quantities which are having both magnitude and direction. Ex : displacement, velocity.

2. Classify the following into scalars and vectors : Distance, displacement, speed, velocity, acceleration, mass, volume, time, linear momentum.

Scalars: Distance, speed, mass.
Vectors: displacement , velocity, acceleration , volume, linear momentum .

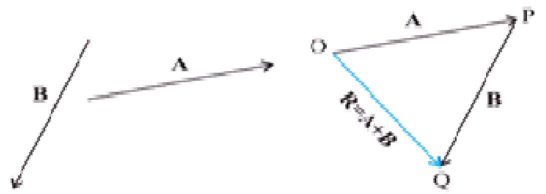
3. Pick out the scalar quantities among the following: Force, work, angular momentum, heat, torque.
Scalar quantities : work, heat.

4. Pick out the vector quantities among the following: Density, moment of force, temperature, electric field.

Vector quantities : Density, moment of force , electric field.

5. State and explain the law of triangle of vectors.

The law of triangle of vectors : It states that if two vectors can be represented in magnitude and direction by the two sides of a triangle taken in the same order , then the resultant is represented completely by the third side of the triangle taken in the reverse order.



Let two vectors $\vec{A}$ and $\vec{B}$ be represented both in magnitude and direction by the sides AB and BC of the triangle ABC taken in the same order, then the resultant $\vec{R}$ is by the third side AC taken in the opposite order.

6. Write the expression for the magnitude and direction of the resultant of two vectors acting at a point.

The expressions for the magnitude and direction of the resultant of two vectors acting at a point are : Magnitude $R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

Direction, $\tan\alpha = \frac{B\sin\theta}{A+B\cos\theta}$

7. The horizontal and vertical component of a vector are 3 units and 4 units respectively. What is the magnitude of the vector.

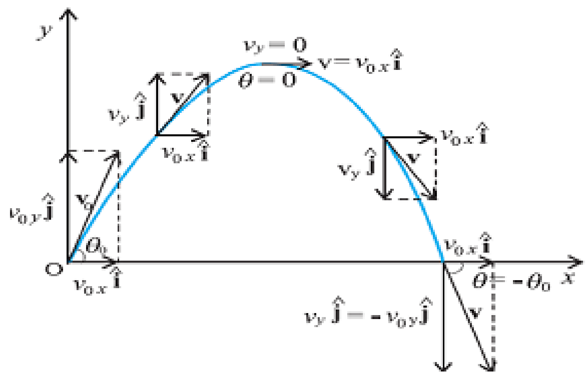
$$|\vec{A}| = \sqrt{a_x^2 + a_y^2} = \sqrt{3^2 + 4^2} = \sqrt{9+16} = 5 \text{ units}$$

8. A vector of 10 units acts at a point making an angle 300with the horizontal. what are the horizontal and vertical components of the vector ?

$A_x = A\cos\theta. = 10\cos 30^0 = 10 \text{ X } \frac{\sqrt{3}}{2} = 5\sqrt{3}$ units

$A_y = A\sin\theta. = 10\sin 30^0 = 10 \text{ X } \frac{1}{2} = 5$ units

9. Draw the diagram for the path of the projectile. And indicate the range and angle of the projectile.



10. Write the expression for the path (trajectory) of the projectile and explain the terms.

$$y = (\tan\theta_0)\, x - \frac{g}{2(v_0\cos\theta_0)^2}\, x^2$$

11. Write the expression for the maximum height of the projectile and explain the terms.

$$h_m = \frac{(v_0 \sin\theta_0)^2}{2g}.$$

θ_0 is the angle of projection and g is the acceleration due to gravity at the given place.

12. Write the expression for the time of flight of the projectile and explain the terms.

$$T_f = 2\,(v_0 \sin\theta_0)/g.$$

13. Write the expression for the range of the projectile and explain the terms.

$$R = \frac{v_0^2 \sin 2\theta_0}{g}.$$

14. For particle in a uniform circular motion speed is uniform but its velocity is not uniform, explain.

Because, the direction of the velocity, given by the tangent changes at each and every point on the circumference of the circle.

15. What is centripetal acceleration? Give the expression for it.

In a cicrular motion, the acceleration of a particle is always directed towards the centre. This accerleration is called centripetal acceleration.

$$a_c = \left(\frac{v}{R}\right) v = v^2/R.$$

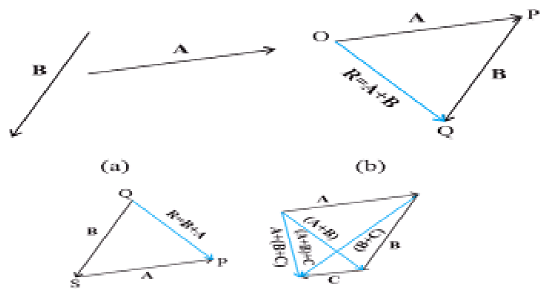
16. Write the expression for the centripetal acceleration and explain the terms.

$a_c = 4\pi^2 v^2 R$ where **v** - speed of the object; R- Radius of the circle.

III. 5 MARK QUESTIONS :

1. State and explain : I. Law of triangle of vectors & II. Law of parallelogram of vectors

I. Triangle method of vector addition OR Tail to tip method of vector addition.



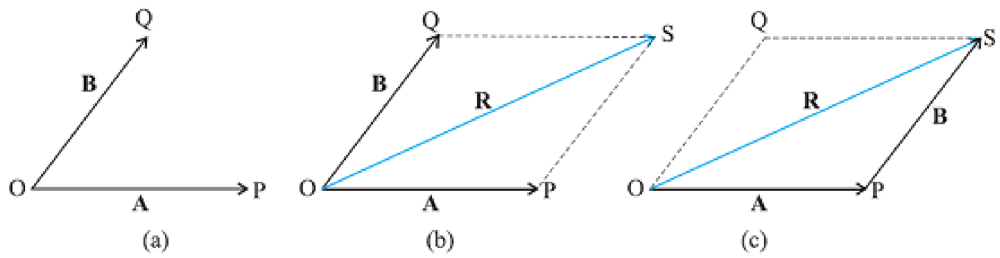
Explanation: To add $\vec{A}$ *with* $\vec{B}$, translate $\vec{B}$, by drawing parallel to itself so that the origin or initial point of $\vec{B}$ is at the tip of vector a. $\vec{A}$ and $\vec{B}$ are two vectors represented by two sides of a triangle taken in the same sense (direction). The vector sum of $\vec{A}$ and $\vec{B}$ (also called resultant of $\vec{A}$ and $\vec{B}$) is represented by the third side of the triangle taken in opposite sense (direction).

Statement: Triangle law of vector addition states that if two vectors can be represented in magnitude and direction by two sides of a triangle taken in the same order, then their resultant is represented completely by the third side of the triangle taken in opposite order.

II. Law of parallelogram of vectors

To add two vectors placed with common initial point, the parallelogram method of vector is used.

Illustration:

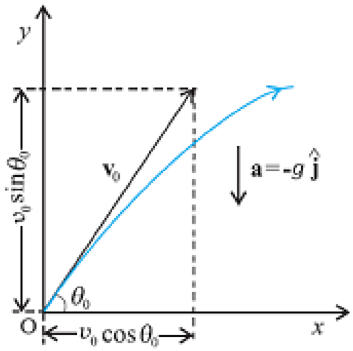


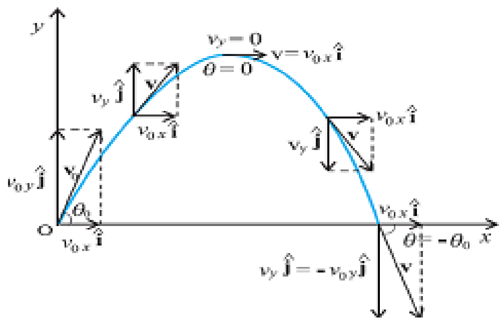
Explanation: To add vector $\vec{B}$ with $\vec{A}$ inclined at an angle θ, draw equal vector of $\vec{A}$ at the tip of $\vec{B}$. By law of triangle method of vector addition u 2 3= 4 2 3+ 1 3 Again by law of triangle method of vector addition $\vec{R} = \vec{A} + \vec{B}$. Note that $\vec{A} + \vec{B} = \vec{B} + \vec{A}$, that is vector addition follows commutative rule. $\vec{R}$, the diagonal of the completed parallelogram represents the vector sum of $\vec{A}$ and $\vec{B}$ completely both in magnitude and direction.

Statement of parallelogram law of vector addition:
" It states that if two vectors acting at a point can be represented both in magnitude and direction by the two adjacent sides of a parallelogram drawn from that point, the resultant is represented completely by the diagonal of the parallelogram passing through that point".

2. Derive the expression for trajectory of a projectile. OR show the trajectory (path)of a projectile is a parabola.

Equation of path(trajectory) of a projectile (parabola) :





Consider a projectile moving in a direction making an angle θ with the horizontal.
Let v_0 - velocity of the projectile.
The velocity v_0 of the projectile resolved into
$V_{0X} = v_0 \cos\theta$ along horizontal (x-axis)
$V_{0Y} = v_0 \sin\theta$ along vertical (y-axis)
After the object has been projected, the acceleration acting on it due to gravity and is directed vertically upwards.
Suppose that the projectile is launched with velocity v_0 that makes an angle θ_0 with the x-axis as shown in Fig. 4.2. Here, the air-resistance is neglected. After the object has been projected, the acceleration acting on it is that due to gravity which is directed vertically downward:

$\vec{a} = -g\hat{j}$

Therefore, $a_x = 0$; $a_y = -g$

The components of initial velocity v_0 are :

$v_{ox} = v_o \cos\theta_o$

$v_{oy} = v_o \sin\theta_o$

If we take the initial position to be the origin of the reference frame as shown in Fig. (a), we have :

$x_o = 0,\ y_o = 0$

Then

$$x = v_{ox} t = (v_o \cos\theta_o)\, t$$

and $$y = (v_o \sin\theta_o)\, t - (\tfrac{1}{2}) g t^2$$

The components of velocity at time t can be written as :

$$v_x = v_{ox} = v_o \cos\theta_o$$

$$v_y = v_o \sin\theta_o - g t \qquad \text{--------------------- (4.4)}$$

$$y = (\tan\theta_o) x - \frac{g}{2\,(v_o \cos\theta_o)^2} x^2 \qquad \text{------------------ (4.5)}$$

Now, since g, θ_0 and v_0 are constants, Eq. (4.5) is of the form $y = a x + b x^2$, in which a and b are constants. This is the equation of a parabola, i.e. the path of the projectile is a **parabola** as shown in Fig. (b).

3. Derive the expression for maximum height and time of flight of projectile.

(1) Maximum height of the projectile :

Consider a projectile moving in a direction making an angle e with the horizontal.

Let v_o - velocity of the projectile.

The velocity v_o of the projectile resolved into

$V_x = v_o \cos e$ along horizontal (x-axis)

$V_y = v_0 \sin e$ along vertical (y-axis)

After the object has been projected ,the acceleration acting on it due to gravity and is directed vertically upwards

$\vec{a} = -g\hat{j}$

Therefore, $a_x = 0$; $a_y = -g$

The **maximum height** h_m reached by the projectile can be calculated by substituting $t = t_m$

$$y = h_m = (v_0 \sin\theta_0)\left(\frac{v_0 \sin\theta_0}{g}\right) - \frac{g}{2}\left(\frac{v_0 \sin\theta_0}{g}\right)^2$$

$$h_m = \frac{(v_0 \sin\theta_0)^2}{2g} \qquad \text{-------- (4.6)}$$

(2) Time of maximum height :

Let this time be denoted by t_m. Since at this point, $v_y = 0$, Then Eqn. (4.4) becomes

$$v_y = v_o \sin\theta_o - g\, t_m = 0$$

Or, $$t_m = v_o \sin\theta_o / g \qquad \text{----------- (4.7a)}$$

The total time T_f during which the projectile is in flight can be obtained by putting y = 0 in Eq. (4.3). We get :

$$T_f = 2\,(v_o \sin\theta_o)/g \qquad \text{------------ (4.7)}$$

T_f is known as the time of flight of the projectile. We note that $T_f = 2\, t_m$, which is expected because of the symmetry of the parabolic path.

4. Derive the expression for time of flight and range of projectile.

(1) Time of maximum height :

Let this time be denoted by t_m. Since at this point, $v_y = 0$, Then Eqn. (4.4) becomes

$$v_y = v_o \sin\theta_o - g\, t_m = 0$$

Or, $t_m = v_o \sin\theta_o / g$ ---------- (4.7a)

The total time T_f during which the projectile is in flight can be obtained by putting y = 0 in Eq. (4.3). We get :

$T_f = 2\,(v_o \sin\theta_o)/g$ ------------ **(4.7)**

T_f is known as the time of flight of the projectile. We note that $T_f = 2\, t_m$, which is expected because of the symmetry of the parabolic path.

(2) Horizontal range of a projectile :

The horizontal distance travelled by a projectile from its initial position (x = y = 0) to the position where it passes y = 0 during its fall is called the horizontal range, R. It is the distance travelled during the time of flight T_f. Therefore, the **range R** is

$$R = (v_o \cos\theta_o)\,(T_f)$$

$$=(v_o \cos\theta_o)\;(2\, v_o \sin\theta_o)/g$$

Or, $R = \dfrac{v_0^2 \sin 2\theta_0}{g}$ --------- **(4.8a)**

Above Equation shows that for a given projection velocity v_o, R is maximum when $\sin 2\theta_0$ is maximum, i.e., when $\theta_0 = 45^0$. The **maximum horizontal range** is, therefore,

$R_m = \dfrac{v_0^2}{g}$ ----------------- **(4.8)**

5. Derive the expression for the centripetal acceleration.

When an object follows a circular path at a constant speed, the motion of the object is called uniform circular motion. Suppose an object is moving with uniform speed v in a circle of radius R. Since the velocity of the object is changing continuously in direction, the object undergoes acceleration.

The magnitude of acceleration a is, by definition, given by :

$$|\mathbf{a}| = \lim_{\Delta t \to 0} \frac{|\Delta \mathbf{v}|}{\Delta t}$$

Let the angle between position vectors r and r′ be $\Delta\theta$. Since the velocity vectors v and v′ are always perpendicular to the position vectors, the angle between them is also $\Delta\theta$.

Therefore, the centripetal acceleration a_c is : $a_c = \left[\frac{v}{R}\right] v = \frac{v^2}{R}$ ------------ **(a)**

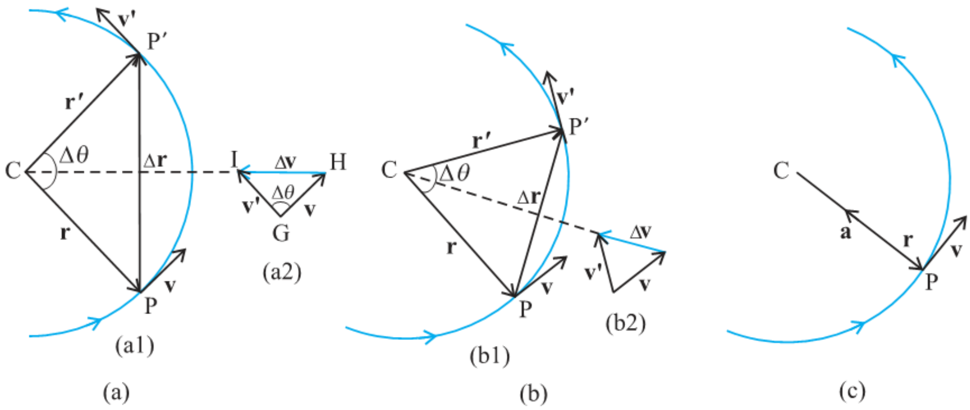


Fig. 3.4 : Velocity and acceleration of an object in uniform circular motion. The time interval Δt decreases from (a) to (c) where it is zero. The acceleration is directed, at each point of the path, towards the centre of the circle.

Thus, the acceleration of an object moving with speed v in a circle of radius R has a magnitude v^2 /R and is always directed towards the centre. This is why this acceleration is called centripetal acceleration. Since v and R are constant, the magnitude of the centripetal acceleration is also constant. However, the direction changes — pointing always towards the centre. Therefore, a centripetal acceleration is not a constant vector.

The angular speed ω is the rate of change of angular distance . It is related to velocity v by $v = \omega R$. The acceleration is $a_c = \omega^2 R$

If T is the time period of revolution of the object in circular motion and ϑ is its frequency. We have $\omega = 2\pi\vartheta$ $v = 2\pi R\vartheta,$ $a_c = 4\pi^2\vartheta^2 R$ ------- (b)

Multiple Choice Questions :

1. The angle between $\vec{P} + \vec{Q}$ and $\vec{P} - \vec{Q}$ will be ______
(a) 90° only
(b) Between 0° to 180°
(c) 180° only
(d) None of the above [AIIMS 1999] Ans : (b)

2. Two projectiles are projected same velocity. If one is projected at an angle of 30° and the other at 60° to the horizontal, the ratio of maximum heights reached is _______
(a) 1 : 3
(b) 9 : 1
(c) 3 : 1
(d) 1 : 9 [AIIMS 2001] Ans : (a)

3. Two equal vectors have a resultant equal to either of them. Then the angle between them will be ______
(a) 30°

(b) 60°
(c) 120°
(d) 180° [AIIMS 2001] Ans (c)

Solutions to Textbook Problems

4.1 State, for each of the following physical quantities, if it is a scalar or a vector : volume, mass, speed, acceleration, density, number of moles, velocity, angular frequency, displacement, angular velocity.

Answer:

Scalar: Volume, mass, speed, density, number of moles, angular frequency

Vector: Acceleration, velocity, displacement, angular velocity

A scalar quantity is specified by its magnitude only. It does not have any dire associated with it. Volume, mass, speed, density, number of moles, and angu frequency are some of the scalar physical quantities.

A vector quantity is specified by its magnitude as well as the direction associa with it. Acceleration, velocity, displacement, and angular velocity belong to th category.

4.2 Pick out the two scalar quantities in the following list : force, angular momentum, work, current, linear momentum, electric field, average velocity, magnetic moment, relative velocity.

Answer

Work and current are scalar quantities.

Work done is given by the dot product of force and displacement. Since the d product of two quantities is always a scalar, work is a scalar physical quantity

Current is described only by its magnitude. Its direction is not taken into accc Hence, it is a scalar quantity.

4.3 Pick out the only vector quantity in the following list : Temperature, pressure, impulse, time, power, total path length, energy, gravitational potential, coefficient of friction, charge.

Answer :

Impulse

Impulse is given by the product of force and time. Since force is a vector qua product with time (a scalar quantity) gives a vector quantity.

4.4 State with reasons, whether the following algebraic operations with scalar and vector physical quantities are meaningful : (a) adding any two scalars, (b) adding a scalar to a vector of the same dimensions, (c) multiplying any vector by any scalar, (d) multiplying any two scalars, (e) adding any two vectors, (f) adding a component of a vector to the same vector.

Answer:

(a) Meaningful

(b) Not Meaningful

(c) Meaningful

(d) Meaningful

(e) Meaningful

(f) Meaningful

Explanation:

(a)The addition of two scalar quantities is meaningful only if they both repres same physical quantity.

(b)The addition of a vector quantity with a scalar quantity is not meaningful.

(c) A scalar can be multiplied with a vector. For example, force is multiplied w time to give impulse.

(d) A scalar, irrespective of the physical quantity it represents, can be multipl another scalar having the same or different dimensions.

(e) The addition of two vector quantities is meaningful only if they both repre same physical quantity.

(f) A component of a vector can be added to the same vector as they both ha same dimensions.

4.5 Read each statement below carefully and state with reasons, if it is true or false : (a) The magnitude of a vector is always a scalar, (b) each component of a vector is always a scalar, (c) the total path length is always equal to the magnitude of the displacement vector of a particle. (d) the average speed of a particle (defined as total path length divided by the time taken to cover the path) is either greater or equal to the magnitude of average velocity of the particle over the same interval of time, (e) Three vectors not lying in a plane can never add up to give a null vector.

Answer:

(a) True

(b) False

(c) False

(d) True

(e) True

Explanation:

(a) The magnitude of a vector is a number. Hence, it is a scalar.

(b) Each component of a vector is also a vector.

(c) Total path length is a scalar quantity, whereas displacement is a vector qu
Hence, the total path length is always greater than the magnitude of displace
becomes equal to the magnitude of displacement only when a particle is mov
straight line.

(d) It is because of the fact that the total path length is always greater than c
to the magnitude of displacement of a particle.

(e) Three vectors, which do not lie in a plane, cannot be represented by the s
a triangle taken in the same order.

4.6 Establish the following vector inequalities geometrically or otherwise :

(a) $|\mathbf{a}+\mathbf{b}| \leq |\mathbf{a}| + |\mathbf{b}|$

(b) $|\mathbf{a}+\mathbf{b}| \geq ||\mathbf{a}| - |\mathbf{b}||$

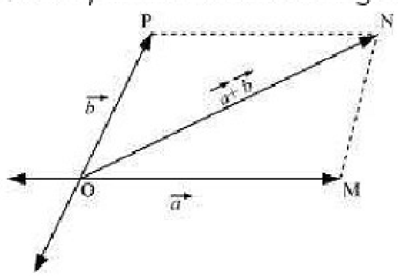
(c) $|\mathbf{a}-\mathbf{b}| \leq |\mathbf{a}| + |\mathbf{b}|$

(d) $|\mathbf{a}-\mathbf{b}| \geq ||\mathbf{a}| - |\mathbf{b}||$

When does the equality sign above apply?

Answer

(a) Let two vectors $\vec{a}$ and $\vec{b}$ be represented by the adjacent sides of a paralle
OMNP, as shown in the given figure.



Here, we can write:

$$\left|\overrightarrow{OM}\right| = \left|\vec{a}\right| \quad \ldots (i)$$

$$\left|\overrightarrow{MN}\right| = \left|\overrightarrow{OP}\right| = \left|\vec{b}\right| \quad \ldots (ii)$$

$$\left|\overrightarrow{ON}\right| = \left|\vec{a} + \vec{b}\right| \quad \ldots (iii)$$

In a triangle, each side is smaller than the sum of the other two sides.

Therefore, in ΔOMN, we have:

ON < (OM + MN)

$$\left|\vec{a}+\vec{b}\right| < \left|\vec{a}\right| + \left|\vec{b}\right| \qquad \ldots (iv)$$

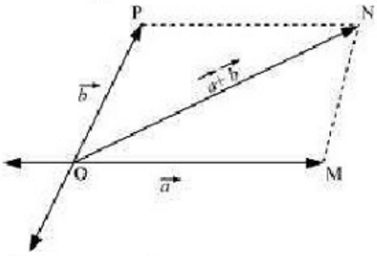
If the two vectors $\vec{a}$ and $\vec{b}$ act along a straight line in the same direction, ther write:

$$\left|\vec{a}+\vec{b}\right| = \left|\vec{a}\right| + \left|\vec{b}\right| \qquad \ldots (v)$$

Combining equations (iv) and (v), we get:

$$\left|\vec{a}+\vec{b}\right| \le \left|\vec{a}\right| + \left|\vec{b}\right|$$

(b) Let two vectors $\vec{a}$ and $\vec{b}$ be represented by the adjacent sides of a parallel OMNP, as shown in the given figure.



Here, we have:

$$\left|\overline{OM}\right| = \left|\vec{a}\right| \qquad \ldots (i)$$

$$\left|\overline{MN}\right| = \left|\overline{OP}\right| = \left|\vec{b}\right| \qquad \ldots (ii)$$

$$\left|\overline{ON}\right| = \left|\vec{a}+\vec{b}\right| \qquad \ldots (iii)$$

In a triangle, each side is smaller than the sum of the other two sides.

Therefore, in ΔOMN, we have:

$$ON + MN > OM$$

$$ON + OM > MN$$

$$\left|\overline{ON}\right| > \left|\overline{OM} - \overline{OP}\right| \qquad (\because OP = MN)$$

$$\left|\vec{a}+\vec{b}\right| > \left|\left|\vec{a}\right| - \left|\vec{b}\right|\right| \ldots (iv)$$

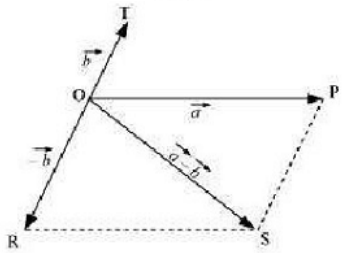
If the two vectors $\vec{a}$ and $\vec{b}$ act along a straight line in the same direction, ther write:

$$\left|\vec{a}+\vec{b}\right| = \left|\left|\vec{a}\right| - \left|\vec{b}\right|\right| \quad \ldots (v)$$

Combining equations (iv) and (v), we get:

$$\left|\vec{a}+\vec{b}\right| \geq \left|\left|\vec{a}\right| - \left|\vec{b}\right|\right|$$

(c) Let two vectors $\vec{a}$ and $\vec{b}$ be represented by the adjacent sides of a parallel PORS, as shown in the given figure.



Here we have:

$$\left|\overrightarrow{OR}\right| = \left|\overrightarrow{PS}\right| = \left|\vec{b}\right| \quad \ldots (i)$$

$$\left|\overrightarrow{OP}\right| = \left|\vec{a}\right| \quad \ldots (ii)$$

In a triangle, each side is smaller than the sum of the other two sides. Theref ΔOPS, we have:

$$OS < OP + PS$$

$$\left|\vec{a}-\vec{b}\right| < \left|\vec{a}\right| + \left|-\vec{b}\right|$$

$$\left|\vec{a}-\vec{b}\right| < \left|\vec{a}\right| + \left|\vec{b}\right| \quad \ldots (iii)$$

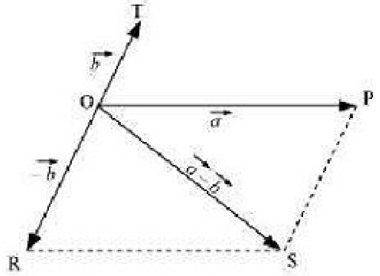
If the two vectors act in a straight line but in opposite directions, then we car

$$\left|\vec{a}-\vec{b}\right| = \left|\vec{a}\right| + \left|\vec{b}\right| \quad \ldots (iv)$$

Combining equations (iii) and (iv), we get:

$$\left|\vec{a}-\vec{b}\right| \leq \left|\vec{a}\right| + \left|\vec{b}\right|$$

(d) Let two vectors $\vec{a}$ and $\vec{b}$ be represented by the adjacent sides of a parallel PORS, as shown in the given figure.



The following relations can be written for the given parallelogram.

$$OS + PS > OP \quad \text{... (i)}$$

$$OS > OP - PS \quad \text{... (ii)}$$

$$\left|\vec{a}-\vec{b}\right| > \left|\vec{a}\right| - \left|\vec{b}\right| \quad \text{... (iii)}$$

The quantity on the LHS is always positive and that on the RHS can be positi negative. To make both quantities positive, we take modulus on both sides as

$$\left|\left|\vec{a}-\vec{b}\right|\right| > \left|\left|\vec{a}\right| - \left|\vec{b}\right|\right|$$

$$\left|\vec{a}-\vec{b}\right| > \left|\left|\vec{a}\right| - \left|\vec{b}\right|\right| \quad \text{... (iv)}$$

If the two vectors act in a straight line but in the opposite directions, then we write:

$$\left|\vec{a}-\vec{b}\right| = \left|\left|\vec{a}\right| - \left|\vec{b}\right|\right| \quad \text{... (v)}$$

Combining equations (iv) and (v), we get:

$$\left|\vec{a}-\vec{b}\right| \geq \left|\left|\vec{a}\right| - \left|\vec{b}\right|\right|$$

4.7 Given a + b + c + d = 0, which of the following statements are correct :
(a) a, b, c, and d must each be a null vector,
(b) The magnitude of (a + c) equals the magnitude of (b + d),
(c) The magnitude of a can never be greater than the sum of the magnitudes of b, c, and d,
(d) b + c must lie in the plane of a and d if a and d are not collinear, and in the line of a and d, if they are collinear ?

Answer:

(a) Incorrect

In order to make a + b + c + d = 0, it is not necessary to have all the four gi vectors to be null vectors. There are many other combinations which can give sum zero.

(b) Correct

a + b + c + d = 0

a + c = – (b + d)

Taking modulus on both the sides, we get:

| a + c | = | –(b + d)| = | b + d |

Hence, the magnitude of (a + c) is the same as the magnitude of (b + d).

(c) Correct

a + b + c + d = 0

a = (b + c + d)

Taking modulus both sides, we get:

| a | = | b + c + d |

$|\mathbf{a}| \leq |\mathbf{a}| + |\mathbf{b}| + |\mathbf{c}|$... (i)

Equation (i) shows that the magnitude of a is equal to or less than the sum o magnitudes of b, c, and d.

Hence, the magnitude of vector a can never be greater than the sum of the magnitudes of b, c, and d.

(d) Correct

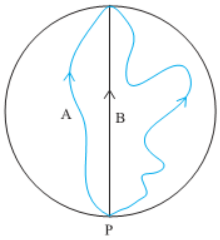
For a + b + c + d = 0

a + (b + c) + d = 0

The resultant sum of the three vectors a, (b + c), and d can be zero only if (b lie in a plane containing a and d, assuming that these three vectors are repre by the three sides of a triangle.

If a and d are collinear, then it implies that the vector (b + c) is in the line of d. This implication holds only then the vector sum of all the vectors will be ze

4.8 Three girls skating on a circular ice ground of radius 200 m start from a point P on the edge of the ground and reach a point Q diametrically opposite to P following different paths as shown in Fig. 4.20. What is the magnitude of the displacement vector for each ? For which girl is this equal to the actual length of path skate ?



Answer

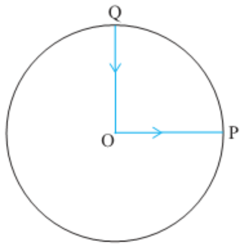
Displacement is given by the minimum distance between the initial and final positions of a particle. In the given case, all the girls start from point P and r point Q. The magnitudes of their displacements will be equal to the diameter ground.

Radius of the ground = 200 m

Diameter of the ground = 2 × 200 = 400 m

Hence, the magnitude of the displacement for each girl is 400 m. This is equa actual length of the path skated by girl B.

4.9 A cyclist starts from the centre O of a circular park of radius 1 km, reaches the edge P of the park, then cycles along the circumference, and returns to the centre along QO as shown in Fig. 4.21. If the round trip takes 10 min, what is the (a) net displacement, (b) average velocity, and (c) average speed of the cyclist ?



Answer

Displacement is given by the minimum distance between the initial and final positions of a body. In the given case, the cyclist comes to the starting point cycling for 10 minutes. Hence, his net displacement is zero.

(b) Average velocity is given by the relation:

$$\text{Average velocity} = \frac{\text{Net displacement}}{\text{Total time}}$$

Since the net displacement of the cyclist is zero, his average velocity will also zero.

(c) Average speed of the cyclist is given by the relation:

$$\text{Average speed} = \frac{\text{Total path length}}{\text{Total time}}$$

Total path length = OP + PQ + QO

$$= 1 + \frac{1}{4}(2\pi \times 1) + 1$$

$$= 2 + \frac{1}{2}\pi = 3.570 \text{ km}$$

Time taken = 10 min $= \frac{10}{60} = \frac{1}{6}$ h

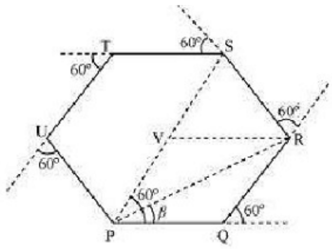
∴Average speed $= \frac{3.570}{\frac{1}{6}} = 21.42$ km/h

4.10 On an open ground, a motorist follows a track that turns to his left by an angle of 600 after every 500 m. Starting from a given turn, specify the displacement of the motorist at the third, sixth and eighth turn. Compare the magnitude of the displacement with the total path length covered by the motorist in each case.

Answer

The path followed by the motorist is a regular hexagon

the given figure



Let the motorist start from point P.

The motorist takes the third turn at S.

∴Magnitude of displacement = PS = PV + VS = 500 + 500 = 1000 m

Total path length = PQ + QR + RS = 500 + 500 +500 = 1500 m

The motorist takes the sixth turn at point P, which is the starting point.

∴Magnitude of displacement = 0

Total path length = PQ + QR + RS + ST + TU + UP

= 500 + 500 + 500 + 500 + 500 + 500 = 3000 m

The motorist takes the eight turn at point R

∴Magnitude of displacement = PR

$$= \sqrt{PQ^2 + QR^2 + 2(PQ)\cdot(QR)\cos 60^\circ}$$

$$= \sqrt{500^2 + 500^2 + (2 \times 500 \times 500 \times \cos 60^\circ)}$$

$$= \sqrt{250000 + 250000 + \left(500000 \times \frac{1}{2}\right)}$$

$$= 866.03 \text{ m}$$

$$\beta = \tan^{-1}\left(\frac{500 \sin 60^\circ}{500 + 500 \cos 60^\circ}\right) = 30^\circ$$

Therefore, the magnitude of displacement is 866.03 m at an angle of 30° witl

Total path length = Circumference of the hexagon + PQ + QR

= 6 × 500 + 500 + 500 = 4000 m

The magnitude of displacement and the total path length coresponding to the required turns is shown in the given table

Turn	Magnitude of displacement (m)	Total path length (m)
Third	1000	1500
Sixth	0	3000
Eighth	866.03; 30°	4000

4.11 A passenger arriving in a new town wishes to go from the station to a hotel located 10 km away on a straight road from the station. A dishonest cabman takes him along a circuitous path 23 km long and reaches the hotel in 28 min. What is (a) the average speed of the taxi, (b) the magnitude of average velocity ? Are the two equal ?

Answer

(a) Total distance travelled = 23 km

Total time taken = 28 min $= \frac{28}{60}$ h

∴Average speed of the taxi $= \frac{\text{Total distance travelled}}{\text{Total time taken}}$

$$= \frac{23}{\left(\frac{28}{60}\right)} = 49.29 \text{ km/h}$$

(b) Distance between the hotel and the station = 10 km = Displacement of tl

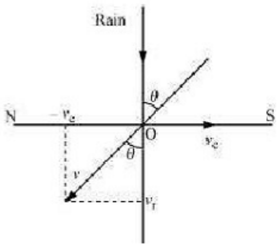
∴Average velocity $= \frac{10}{\frac{28}{60}} = 21.43$ km/h

Therefore, the two physical quantities (averge speed and average velocity) ar equal.

4.12 Rain is falling vertically with a speed of 30 m s^{-1}. A woman rides a bicycle with a speed of 10 m s^{-1} in the north to south direction. What is the direction in which she should hold her umbrella ?

Answer:

The described situation is shown in the given figure.



Here,

vc = Velocity of the cyclist

vr = Velocity of falling rain

In order to protect herself from the rain, the woman must hold her umbrella i

direction of the relative velocity (v) of the rain with respect to the woman.

$$v = v_r + (-v_c)$$

$$= 30 + (-10) = 20 \text{ m/s}$$

$$\tan\theta = \frac{v_c}{v_r} = \frac{10}{30}$$

$$\theta = \tan^{-1}\left(\frac{1}{3}\right)$$

$$= \tan^{-1}(0.333) \approx 18°$$

Hence, the woman must hold the umbrella toward the south, at an angle of n 18° with the vertical.

4.13 A man can swim with a speed of 4.0 km/h in still water. How long does he take to cross a river 1.0 km wide if the river flows steadily at 3.0 km/h and he makes his strokes normal to the river current? How far down the river does he go when he reaches the other bank ?

Answer

Speed of the man, vm = 4 km/h

Width of the river = 1 km

Time taken to cross the river $= \dfrac{\text{Width of the river}}{\text{Speed of the river}}$

$= \dfrac{1}{4}\text{ h} = \dfrac{1}{4} \times 60 = 15 \text{ min}$

Speed of the river, vr = 3 km/h

Distance covered with flow of the river = vr × t

$= 3 \times \dfrac{1}{4} = \dfrac{3}{4} \text{ km}$

$= \dfrac{3}{4} \times 1000 = 750 \text{ m}$

4.14 In a harbour, wind is blowing at the speed of 72 km/h and the flag on the mast of a boat anchored in the harbour flutters along the N-E direction. If the boat starts moving at a speed of 51 km/h to the north, what is the direction of the flag on the mast of the boat ?

Answer

Velocity of the boat, vb = 51 km/h

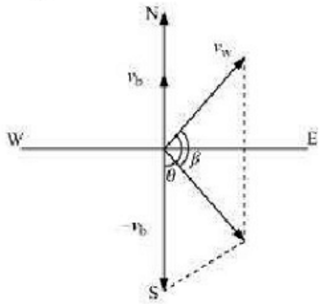
Velocity of the wind, vw = 72 km/h

The flag is fluttering in the north-east direction. It shows that the wind is blo

toward the north-east direction. When the ship begins sailing toward the nort

flag will move along the direction of the relative velocity (vwb) of the wind wi

respect to the boat.



The angle between vw and (−vb) = 90° + 45°

$$\tan\beta = \frac{51\sin(90+45)}{72+51\cos(90+45)}$$

$$= \frac{51\sin 45}{72+51(-\cos 45)} = \frac{51\times\frac{1}{\sqrt{2}}}{72-51\times\frac{1}{\sqrt{2}}}$$

$$= \frac{51}{72\sqrt{2}-51} = \frac{51}{72\times 1.414-51} = \frac{51}{50.800}$$

$\therefore \beta = \tan^{-1}(1.0038) = 45.11°$

Angle with respect to the east direction = 45.11° – 45° = 0.11°

Hence, the flag will flutter almost due east.

4.15 The ceiling of a long hall is 25 m high. What is the maximum horizontal distance that a ball thrown with a speed of 40 m s^{-1} can go without hitting the ceiling of the hall ?

Answer

Speed of the ball, u = 40 m/s

Maximum height, h = 25 m

In projectile motion, the maximum height reached by a body projected at ar

θ, is given by the relation:

$$h = \frac{u^2 \sin^2\theta}{2g}$$

$$25 = \frac{(40)^2 \sin^2\theta}{2 \times 9.8}$$

sin2 θ = 0.30625

sin θ = 0.5534

∴θ = sin−1(0.5534) = 33.60°

Horizontal range, R $= \frac{u^2 \sin 2\theta}{g}$

$$= \frac{(40)^2 \times \sin 2 \times 33.60}{9.8}$$

$$= \frac{1600 \times \sin 67.2}{9.8}$$

$$= \frac{1600 \times 0.922}{9.8} = 150.53 \text{ m}$$

4.16 A cricketer can throw a ball to a maximum horizontal distance of 100 m. How much high above the ground can the cricketer throw the same ball ?

Answer

Maximum horizontal distance, R = 100 m

The cricketer will only be able to throw the ball to the maximum horizontal di

when the angle of projection is 45°, i.e., θ = 45°.

The horizontal range for a projection velocity v, is given by the relation:

$$R = \frac{u^2 \sin 2\theta}{g}$$

$$100 = \frac{u^2}{g} \sin 90°$$

$$\frac{u^2}{g} = 100 \qquad \ldots (i)$$

The ball will achieve the maximum height when it is thrown vertically upward

such motion, the final velocity v is zero at the maximum height H.

Acceleration, a = −g

Using the third equation of motion:

$$v^2 - u^2 = -2gH$$

$$H = \frac{1}{2} \times \frac{u^2}{g} = \frac{1}{2} \times 100 = 50 \text{ m}$$

4.17 A stone tied to the end of a string 80 cm long is whirled in a horizontal circle with a constant speed. If the stone makes 14 revolutions in 25 s, what is the magnitude and direction of acceleration of the stone ?

Answer

Length of the string, l = 80 cm = 0.8 m

Number of revolutions = 14

Time taken = 25 s

Frequency, $$\nu = \frac{\text{Number of revolutions}}{\text{Time taken}} = \frac{14}{25} \text{ Hz}$$

Angular frequency, ω = 2πν

$$= 2 \times \frac{22}{7} \times \frac{14}{25} = \frac{88}{25} \text{ rad s}^{-1}$$

Centripetal acceleration, $a_c = \omega^2 r$

$$= \left(\frac{88}{25}\right)^2 \times 0.8$$

$$= 9.91 \text{ m/s}^2$$

The direction of centripetal acceleration is always directed along the string, tc the centre, at all points.

4.18 An aircraft executes a horizontal loop of radius 1.00 km with a steady speed of 900 km/h. Compare its centripetal acceleration with the acceleration due to gravity.

Answer

Radius of the loop, r = 1 km = 1000 m

Speed of the aircraft, v = 900 km/h $= 900 \times \frac{5}{18} = 250 \text{ m/s}$

Centripetal acceleration, $a_c = \frac{v^2}{r}$

$$= \frac{(250)^2}{1000} = 62.5 \text{ m/s}^2$$

Acceleration due to gravity, g = 9.8 m/s2

$$\frac{a_c}{g} = \frac{62.5}{9.8} = 6.38$$

$$a_c = 6.38 g$$

4.19 Read each statement below carefully and state, with reasons, if it is true or false :

(a) The net acceleration of a particle in circular motion is always along the radius of the circle towards the centre

(b) The velocity vector of a particle at a point is always along the tangent to the path of the particle at that point

(c) The acceleration vector of a particle in uniform circular motion averaged over one cycle is a null vector

Answer

(a) False

The net acceleration of a particle in circular motion is not always directed alor radius of the circle toward the centre. It happens only in the case of uniform motion.

(b) True

At a point on a circular path, a particle appears to move tangentially to the ci
path. Hence, the velocity vector of the particle is always along the tangent at

(c) True

In uniform circular motion (UCM), the direction of the acceleration vector poir
toward the centre of the circle. However, it constantly changes with time. The
average of these vectors over one cycle is a null vector.

4.20 The position of a particle is given by

$\mathbf{r} = 3.0t\,\hat{\mathbf{i}} - 2.0t^2\,\hat{\mathbf{j}} + 4.0\,\hat{\mathbf{k}}$ m

where t is in seconds and the coefficients have the proper units for r to be in metres.
(a) Find the v and a of the particle? (b) What is the magnitude and direction of velocity of the particle at t = 2.0 s ?

ANS:

(a) $\vec{v}(t) = (3.0\,\hat{\mathbf{i}} - 4.0t\,\hat{\mathbf{j}});\ \vec{a} = -4.0\,\hat{\mathbf{j}}$

The position of the particle is given by:

$$\vec{r} = 3.0t\,\hat{\mathbf{i}} - 2.0t^2\,\hat{\mathbf{j}} + 4.0\,\hat{\mathbf{k}}$$

Velocity $\vec{v}$, of the particle is given as:

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt}\left(3.0t\,\hat{\mathbf{i}} - 2.0t^2\,\hat{\mathbf{j}} + 4.0\,\hat{\mathbf{k}}\right)$$

$$\therefore \vec{v} = 3.0\,\hat{\mathbf{i}} - 4.0t\,\hat{\mathbf{j}}$$

Acceleration $\vec{a}$, of the particle is given as:

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left(3.0\,\hat{\mathbf{i}} - 4.0t\,\hat{\mathbf{j}}\right)$$

$$\therefore \vec{a} = -4.0\,\hat{\mathbf{j}}$$

(b) 8.54 m/s, 69.45° below the *x*-axis

We have velocity vector, $\vec{v} = 3.0\,\hat{\mathbf{i}} - 4.0t\,\hat{\mathbf{j}}$

At $t = 2.0$ s:

$$\vec{v} = 3.0\,\hat{\mathbf{i}} - 8.0\,\hat{\mathbf{j}}$$

The magnitude of velocity is given by:

$$|\vec{v}| = \sqrt{3^2 + (-8)^2} = \sqrt{73} = 8.54 \text{ m/s}$$

$$\text{Direction, } \theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{-8}{3}\right) = -\tan^{-1}(2.667) = -69.45°$$

The negative sign indicates that the direction of velocity is below the *x*-axis.

4.21 A particle starts from the origin at t = 0 s with a velocity of 10.0 j m/s and moves in the x-y plane with a constant acceleration of (8.0 i + 2.0 j) m s^{-2}. (a) At what time is the x- coordinate of the particle 16 m? What is the y-coordinate of the particle at that time? (b) What is the speed of the particle at the time ?

Answer

Velocity of the particle, $\vec{v} = 10.0\ \hat{\mathbf{j}}$ m/s

Acceleration of the particle $\vec{a} = \left(8.0\ \hat{\mathbf{i}} + 2.0\ \hat{\mathbf{j}}\right)$

Also,

But, $\vec{a} = \frac{d\vec{v}}{dt} = 8.0\ \hat{\mathbf{i}} + 2.0\ \hat{\mathbf{j}}$

$d\vec{v} = \left(8.0\ \hat{\mathbf{i}} + 2.0\hat{\mathbf{j}}\right)dt$

Integrating both sides:

$\vec{v}(t) = 8.0t\ \hat{\mathbf{i}} + 2.0t\ \hat{\mathbf{j}} + \vec{u}$

Where,

$\vec{u}$ = Velocity vector of the particle at t = 0

$\vec{v}$ = Velocity vector of the particle at time t

But, $\vec{v} = \frac{d\vec{r}}{dt}$

$d\vec{r} = \vec{v}dt = \left(8.0t\ \hat{\mathbf{i}} + 2.0t\ \hat{\mathbf{j}} + \vec{u}\right)dt$

Integrating the equations with the conditions: at t = 0; r = 0 andat t = t; r =

$$\vec{r} = \vec{u}t + \frac{1}{2}8.0t^2\ \hat{\mathbf{i}} + \frac{1}{2} \times 2.0t^2\ \hat{\mathbf{j}}$$

$$= \vec{u}t + 4.0t^2\,\hat{\mathbf{i}} + t^2\,\hat{\mathbf{j}}$$

$$= \left(10.0\,\hat{\mathbf{j}}\right)t + 4.0t^2\,\hat{\mathbf{i}} + t^2\,\hat{\mathbf{j}}$$

$$= \vec{u}t + 4.0t^2\,\hat{\mathbf{i}} + t^2\,\hat{\mathbf{j}}$$

$$= \left(10.0\,\hat{\mathbf{j}}\right)t + 4.0t^2\,\hat{\mathbf{i}} + t^2\,\hat{\mathbf{j}}$$

$$x\,\hat{\mathbf{i}} + y\,\hat{\mathbf{j}} = 4.0t^2\,\hat{\mathbf{i}} + \left(10t + t^2\right)\hat{\mathbf{j}}$$

Since the motion of the particle is confined to the x-y plane, on equating the coefficients of $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$, we get:

$$x = 4t^2$$

$$t = \left(\frac{x}{4}\right)^{\frac{1}{2}}$$

And $y = 10t + t^2$

(a) When x = 16 m:

$$t = \left(\frac{16}{4}\right)^{\frac{1}{2}} = 2 \text{ s}$$

∴y = 10 × 2 + (2)2 = 24 m

(b) Velocity of the particle is given by:

$$\vec{v}(t) = 8.0t\ \hat{\mathbf{i}} + 2.0t\ \hat{\mathbf{j}} + \vec{u}$$

at $t = 2$ s

$$\vec{v}(t) = 8.0 \times 2\ \hat{\mathbf{i}} + 2.0 \times 2\ \hat{\mathbf{j}} + 10\ \hat{\mathbf{j}}$$

$$= 16\ \hat{\mathbf{i}} + 14\ \hat{\mathbf{j}}$$

∴ Speed of the particle:

$$|\vec{v}| = \sqrt{(16)^2 + (14)^2}$$

$$= \sqrt{256 + 196} = \sqrt{452}$$

$$= 21.26 \text{ m/s}$$

4.22. i and j are unit vectors along x- and y- axis respectively. What is the magnitude and direction of the vectors i + j, and i − j ? What are the components of a vector A= 2 i + 3j along the directions of i + j and i – j ? [You may use graphical method].

Answer

Consider a vector $\vec{P}$, given as:

$$\vec{P} = \hat{\mathbf{i}} + \hat{\mathbf{j}}$$

$$P_x\ \hat{\mathbf{i}} + P_y\ \hat{\mathbf{j}} = \hat{\mathbf{i}} + \hat{\mathbf{j}}$$

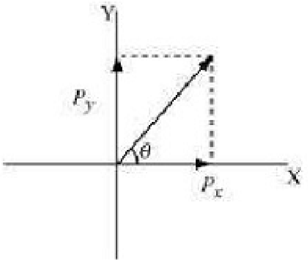
On comparing the components on both sides, we get:

$P_x = P_y = 1$

$|\vec{P}| = \sqrt{P_x^2 + P_y^2} = \sqrt{1^2 + 1^2} = \sqrt{2}$ ... (*i*)

Hence, the magnitude of the vector $\hat{\mathbf{i}} + \hat{\mathbf{j}}$ is $\sqrt{2}$.

Let θ be the angle made by the vector $\vec{P}$, with the x-axis, as shown in the fol figure.



$\therefore \tan\theta = \left(\frac{P_y}{P_x}\right)$

$\theta = \tan^{-1}\left(\frac{1}{1}\right) = 45°$ ... (*ii*)

Hence, the vector $\hat{\mathbf{i}} + \hat{\mathbf{j}}$ makes an angle of 45° with the *x*-axis.

Let $\vec{Q} = \hat{\mathbf{i}} - \hat{\mathbf{j}}$

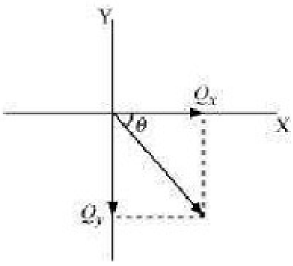
$Q_x\, \hat{\mathbf{i}} - Q_y\, \hat{\mathbf{j}} = \hat{\mathbf{i}} - \hat{\mathbf{j}}$

$Q_x = Q_y = 1$

$|\vec{Q}| = \sqrt{Q_x^2 + Q_y^2} = \sqrt{2}$ ... (*iii*)

Hence, the magnitude of the vector $\hat{\mathbf{i}} - \hat{\mathbf{j}}$ is $\sqrt{2}$.

Let θ be the angle made by the vector $\vec{Q}$, with the x- axis, as shown in the fo figure.



$\therefore \tan\theta = \left(\frac{Q_y}{Q_x}\right)$

$\theta = -\tan^{-1}\left(-\frac{1}{1}\right) = -45°$ ... (*iv*)

Hence, the vector $\hat{\mathbf{i}} - \hat{\mathbf{j}}$ makes an angle of $-45°$ with the x-axis.

It is given that:

$\vec{A} = 2\,\hat{\mathbf{i}} + 3\,\hat{\mathbf{j}}$

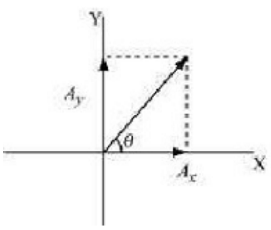
$A_x\,\hat{\mathbf{i}} + A_y\,\hat{\mathbf{j}} = 2\,\hat{\mathbf{i}} + 3\,\hat{\mathbf{j}}$

On comparing the coefficients of $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$, we have:

$A_x = 2$ and $A_y = 3$

$|\vec{A}| = \sqrt{2^2 + 3^2} = \sqrt{13}$

Let $\vec{A}_x$ make an angle θ with the x-axis, as shown in the following figure.



$\therefore \tan\theta = \left(\frac{A_y}{A_x}\right)$

$\theta = \tan^{-1}\left(\frac{3}{2}\right)$

$= \tan^{-1}(1.5) = 56.31°$

Angle between the vectors $\left(2\,\hat{\mathbf{i}} + 3\,\hat{\mathbf{j}}\right)$ and $\left(\hat{\mathbf{i}} + \hat{\mathbf{j}}\right)$, $\theta' = 56.31 - 45 = 11.31°$

Component of vector $\vec{A}$, along the direction of $\vec{P}$, making an angle θ'

$= \left(A\cos\theta'\right)\hat{P} = \left(A\cos 11.31\right)\frac{\left(\hat{\mathbf{i}} + \hat{\mathbf{j}}\right)}{\sqrt{2}}$

$= \sqrt{13} \times \frac{0.9806}{\sqrt{2}}\left(\hat{\mathbf{i}} + \hat{\mathbf{j}}\right)$

$= 2.5\left(\hat{\mathbf{i}} + \hat{\mathbf{j}}\right)$

$= \frac{25}{10} \times \sqrt{2}$

$= \frac{5}{\sqrt{2}}$ ... (v)

Let $\theta^{"}$ be the angle between the vectors $\left(2\,\hat{\mathbf{i}}+3\,\hat{\mathbf{j}}\right)$ and $\left(\hat{\mathbf{i}}-\hat{\mathbf{j}}\right)$.

$$\theta^{''}=45+56.31=101.31^{o}$$

Component of vector $\vec{A}$, along the direction of $\vec{Q}$, making an angle $\theta^{''}$

$$=\left(A\cos\theta^{''}\right)\vec{Q}=\left(A\cos\theta^{''}\right)\frac{\hat{\mathbf{i}}-\hat{\mathbf{j}}}{\sqrt{2}}$$

$$=\sqrt{13}\cos\left(901.31^{o}\right)\frac{\left(\hat{\mathbf{i}}-\hat{\mathbf{j}}\right)}{\sqrt{2}}$$

$$=-\sqrt{\frac{13}{2}}\sin 11.30^{o}\left(\hat{\mathbf{i}}-\hat{\mathbf{j}}\right)$$

$$=-0.5\left(\hat{\mathbf{i}}-\hat{\mathbf{j}}\right)$$

$$=-\frac{5}{10}\times\sqrt{2}$$

$$=-\frac{1}{\sqrt{2}} \qquad \dots(vi)$$

4.23 For any arbitrary motion in space, which of the following relations are true :

(a) $\mathbf{v}_{average} = (1/2)\ (\mathbf{v}\ (t_1) + \mathbf{v}\ (t_2))$

(b) $\mathbf{v}_{\ average} = [\mathbf{r}(t_2) - \mathbf{r}(t_1)\ ]\ /(t_2 - t_1)$

(c) $\mathbf{v}\ (t) = \mathbf{v}\ (0) + \mathbf{a}\ t$

(d) $\mathbf{r}\ (t) = \mathbf{r}\ (0) + \mathbf{v}\ (0)\ t + (1/2)\ \mathbf{a}\ t^2$

(e) $\mathbf{a}_{\ average} =[\ \mathbf{v}\ (t_2) - \mathbf{v}\ (t_1\ )]\ /(\ t_2 - t_1)$

(The 'average' stands for average of the quantity over the time interval t_1 to t_2)

Answer: (b) and (e)

(a)It is given that the motion of the particle is arbitrary. Therefore, the avera velocity of the particle cannot be given by this equation.

(b)The arbitrary motion of the particle can be represented by this equation.

(c)The motion of the particle is arbitrary. The acceleration of the particle may non-uniform. Hence, this equation cannot represent the motion of the particl space.

(d)The motion of the particle is arbitrary; acceleration of the particle may als non-uniform. Hence, this equation cannot represent the motion of particle in

(e)The arbitrary motion of the particle can be represented by this equation.

4.24 Read each statement below carefully and state, with reasons and examples, if it is true or false :
A scalar quantity is one that
(a) is conserved in a process
(b) can never take negative values
(c) must be dimensionless
(d) does not vary from one point to another in space
(e) has the same value for observers with different orientations of axes.

Answer

(a) False

Despite being a scalar quantity, energy is not conserved in inelastic collisions

(b) False

Despite being a scalar quantity, temperature can take negative values.

(c) False

Total path length is a scalar quantity. Yet it has the dimension of length.

(d) False

A scalar quantity such as gravitational potential can vary from one point to ar

in space.

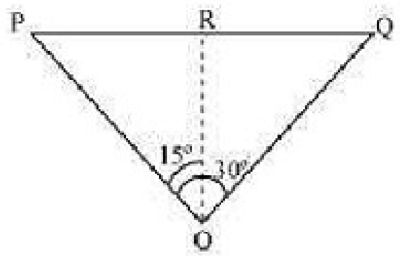
(e) True

The value of a scalar does not vary for observers with different orientations o

4.25 An aircraft is flying at a height of 3400 m above the ground. If the angle subtended at a ground observation point by the aircraft positions 10.0 s apart is 30°, what is the speed of the aircraft ?

Answer

The positions of the observer and the aircraft are shown in the given figure.



Height of the aircraft from ground, OR = 3400 m

Angle subtended between the positions, ∠POQ = 30°

Time = 10 s

In ΔPRO:

$$\tan 15° = \frac{PR}{OR}$$

$$\mathbf{PR} = OR \tan 15°$$
$$= 3400 \times \tan 15°$$

ΔPRO is similar to ΔRQO.

∴PR = RQ

PQ = PR + RQ

= 2PR = 2 × 3400 tan 15°

= 6800 × 0.268 = 1822.4 m

∴Speed of the aircraft $= \frac{1822.4}{10} = 182.24$ m/s

Additional Exercises

4.26 A vector has magnitude and direction. Does it have a location in space ? Can it vary with time ? Will two equal vectors a and b at different locations in space necessarily have identical physical effects ? Give examples in support of your answer.

Answer: No; Yes; No

Generally speaking, a vector has no definite locations in space. This is because a vector remains invariant when displaced in such a way that its magnitude and direction remain the same. However, a position vector has a definite location in space.

A vector can vary with time. For example, the displacement vector of a particle moving with a certain velocity varies with time.

Two equal vectors located at different locations in space need not produce the same physical effect. For example, two equal forces acting on an object at different points can cause the body to rotate, but their combination cannot produce an equal turning effect.

4.27 A vector has both magnitude and direction. Does it mean that anything that has magnitude and direction is necessarily a vector ? The rotation of a body can be specified by the direction of the axis of rotation, and the angle of rotation about the axis. Does that make any rotation a vector ?

Answer: No; No

A physical quantity having both magnitude and direction need not be conside vector. For example, despite having magnitude and direction, current is a sca quantity. The essential requirement for a physical quantity to be considered a is that it should follow the law of vector addition.

Generally speaking, the rotation of a body about an axis is not a vector quant does not follow the law of vector addition. However, a rotation by a certain sr angle follows the law of vector addition and is therefore considered a vector.

4.28 Can you associate vectors with (a) the length of a wire bent into a loop, (b) a plane area, (c) a sphere ? Explain.

Answer: No; Yes; No

(a) One cannot associate a vector with the length of a wire bent into a loop.

(b) One can associate an area vector with a plane area. The direction of this normal, inward or outward to the plane area.

(c) One cannot associate a vector with the volume of a sphere. However, an vector can be associated with the area of a sphere.

4.29 A bullet fired at an angle of 30° with the horizontal hits the ground 3.0 km away. By adjusting its angle of projection, can one hope to hit a target 5.0 km away ? Assume the muzzle speed to the fixed, and neglect air resistance.

Answer: No

Range, R = 3 km

Angle of projection, θ = 30°

Acceleration due to gravity, g = 9.8 m/s2

Horizontal range for the projection velocity u0, is given by the relation:

$$R = \frac{u_0^2 \sin 2\theta}{g}$$

$$3 = \frac{u_0^2}{g} \sin 60°$$

$$\frac{u_0^2}{g} = 2\sqrt{3} \qquad \ldots (i)$$

The maximum range (Rmax) is achieved by the bullet when it is fired at an an
45° with the horizontal, that is,

$$R_{max} = \frac{u_0^2}{g} \qquad \dots(ii)$$

On comparing equations (i) and (ii), we get:

$$R_{max} = 3\sqrt{3} = 2 \times 1.732 = 3.46 \text{ km}$$

Hence, the bullet will not hit a target 5 km away.

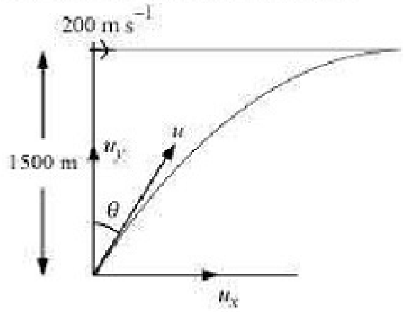
4.30 A fighter plane flying horizontally at an altitude of 1.5 km with speed 720 km/h passes directly overhead an anti-aircraft gun. At what angle from the vertical should the gun be fired for the shell with muzzle speed 600 m s-1 to hit the plane ? At what minimum altitude should the pilot fly the plane to avoid being hit ? (Take g = 10 m s^{-2}).

Answer

Height of the fighter plane = 1.5 km = 1500 m

Speed of the fighter plane, v = 720 km/h = 200 m/s

Let θ be the angle with the vertical so that the shell hits the plane. The situat
shown in the given figure.



Muzzle velocity of the gun, u = 600 m/s

Time taken by the shell to hit the plane = t

Horizontal distance travelled by the shell = uxt

Distance travelled by the plane = vt

The shell hits the plane. Hence, these two distances must be equal.

uxt = vt

$$u \sin\theta = v$$

$$\sin\theta = \frac{v}{u}$$

$$= \frac{200}{600} = \frac{1}{3} = 0.33$$

$$\theta = \sin^{-1}(0.33)$$

$$= 19.5°$$

In order to avoid being hit by the shell, the pilot must fly the plane at an altit higher than the maximum height achieved by the shell.

$$\therefore H = \frac{u^2 \sin^2(90 - \theta)}{2g}$$

$$= \frac{(600)^2 \cos^2 \theta}{2g}$$

$$= \frac{360000 \times \cos^2 19.5}{2 \times 10}$$

$$= 18000 \times (0.943)^2$$

$$= 16006.482 \text{ m}$$

$$\approx 16 \text{ km}$$

4.31 A cyclist is riding with a speed of 27 km/h. As he approaches a circular turn on the road of radius 80 m, he applies brakes and reduces his speed at the constant rate of 0.50 m/s every second. What is the magnitude and direction of the net acceleration of the cyclist on the circular turn ?

Answer

0.86 m/s2; 54.46° with the direction of velocity

Speed of the cyclist, $v = 27 \text{ km/h} = 7.5 \text{ m/s}$

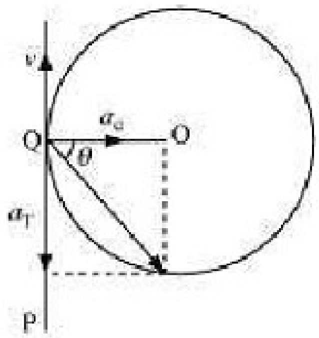
Radius of the circular turn, r = 80 m

Centripetal acceleration is given as:

$$a_c = \frac{v^2}{r}$$

$$= \frac{(7.5)^2}{80} = 0.7 \text{ m/s}^2$$

The situation is shown in the given figure:



Suppose the cyclist begins cycling from point P and moves toward point Q. At Q, he applies the breaks and decelerates the speed of the bicycle by 0.5 m/s2 This acceleration is along the tangent at Q and opposite to the direction of mo the cyclist.

Since the angle between a_c and a_T is 90°, the resultant acceleration a is given

$$a = \sqrt{a_c^2 + a_T^2}$$
$$= \sqrt{(0.7)^2 + (0.5)^2}$$
$$= \sqrt{0.74} = 0.86 \text{ m/s}^2$$
$$\tan\theta = \frac{a_c}{a_T}$$

Where θ is the angle of the resultant with the direction of velocity

$$\tan\theta = \frac{0.7}{0.5} = 1.4$$
$$\theta = \tan^{-1}(1.4)$$
$$= 54.46°$$

4.32 (a) Show that for a projectile the angle between the velocity and the x-axis as a function of time is given by

$$\theta(t) = \tan^{-1}\left(\frac{v_{0y} - gt}{v_{0x}}\right)$$

(b) Shows that the projection angle θ_0 for a projectile launched from the origin is given by

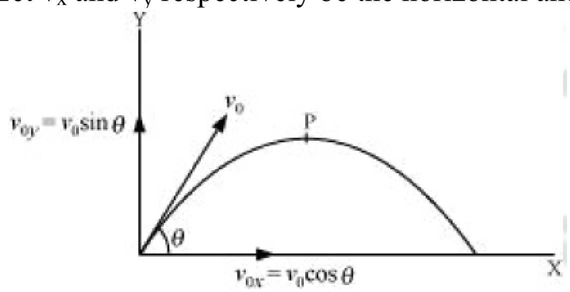
$$\theta_0 = \tan^{-1}\left(\frac{4h_m}{R}\right)$$

where the symbols have their usual meaning.

ANS:

(a) Let v_{0x} and v_{0y} respectively be the initial components of the velocity of the projectile along horizontal (*x*) and vertical (*y*) directions.

Let v_x and v_y respectively be the horizontal and vertical components of velocity at a point P.



Time taken by the projectile to reach point P = t

Applying the first equation of motion along the vertical and horizontal directic get:

$v_y = v_{0y} = gt$

And $v_x = v_{0x}$

$\therefore \tan\theta = \frac{v_y}{v_x} = \frac{v_{0y} - gt}{v_{0x}}$

$\theta = \tan^{-1}\left(\frac{v_{0y} - gt}{v_{0x}}\right)$

(b) Maximum vertical height, $h_m = \frac{u_0^2 \sin^2 2\theta}{2g}$ ... (i)

Horizontal range, $R = \frac{u_0^2 \sin^2 2\theta}{g}$ ... (ii)

Solving equations (i) and (ii), we get:

$$-\frac{h_m}{R} = \frac{\sin^2\theta}{2\sin^2\theta}$$

$$= \frac{\sin\theta \times \sin\theta}{2 \times 2\sin\theta\cos\theta}$$

$$= \frac{1}{4}\frac{\sin\theta}{\cos\theta} = \frac{1}{4}\tan\theta$$

$$\tan\theta = \left(\frac{4h_m}{R}\right)$$

$$\theta = \tan^{-1}\left(\frac{4h_m}{R}\right)$$

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CH 5

LAWS OF MOTION

(11 Hours, 10 Marks : 1Q -2M, 1Q-3M, 1Q-5M(LA))

Syllabus : Intuitive concept of force. Inertia, Newton's first law of motion; momentum and Newton's second law of motion; impulse; Newton's third law of motion. Law of conservation of linear momentum and its applications. Equilibrium of concurrent forces. Static and kinetic friction, laws of friction, rolling friction. Dynamics of uniform circular motion: Centripetal force, examples of circular motion (vehicle on level circular road, vehicle on banked road).

5.1. Intuitive concept of force :

It is seen in nature that, a force is required to put a stationary body in motion or stop a moving body, and some external agency is needed to provide this force. The external agency may or may not be in contact with the body.

Is an external force required to keep a body in uniform motion? To answer this question Aristotle developed a law of motion and accordingly : An external force is required to keep a body in motion. Aristotle (384 B.C– 322 B.C.), is a Greek thinker, held the view that if a body is moving, something external is required to keep it moving. According to this view, for example, an arrow shot from a bow keeps flying since the air behind the arrow keeps pushing it. The view was part of an elaborate framework of ideas developed by Aristotle on the motion of bodies in the universe. Most of the Aristotelian ideas on motion are now known to be wrong.

Aristotle's view that a force is necessary to keep a body in uniform motion is wrong. A force is necessary in practice to counter the opposing force of friction.

The answer is: a moving toy car comes to rest because the external force of friction on the car by the floor opposes its motion. To counter this force, the child has to apply an external force on the car in the direction of motion. When the car is in uniform motion, there is no net external force acting on it: the force by the child cancels the force (friction) by the floor.

The opposing forces such as friction (solids) and viscous forces (for fluids) are always present in the natural world. This explains why forces by external agencies are necessary to overcome the frictional forces to keep bodies in uniform motion.

5.2. Inertia, Newton's first law of motion :

Galileo extrapolated simple observations on motion of bodies on inclined planes, and arrived at the **law of inertia**. Objects (i) moving down an inclined plane accelerate, (ii) while those moving up retard. (iii) Motion on a horizontal plane is an intermediate situation. Galileo concluded that an object moving on a frictionless horizontal plane must neither have acceleration nor retardation, i.e. *it should move with constant velocity*.

Galileo thus, arrived at a new insight on motion that had eluded Aristotle and those who followed him. *The state of rest and the state of uniform linear motion (motion with constant velocity) are equivalent*. In both cases, there is no net force acting on the body. It is incorrect to assume that a net force is needed to keep a body in uniform motion. To maintain a body in uniform motion, we need to apply an external force to encounter the frictional force, so that the two forces sum up to zero net external force.

To summarize, if the net external force is zero, a body at rest continues to remain at rest and a body in motion continues to move with a uniform velocity. This property of the body is called **inertia**. Inertia means 'resistance to change'. A body does not change its state of rest or uniform motion, unless an external force compels it to change that state.
Newton built on Galileo's ideas and laid the foundation of mechanics in terms of three laws of motion that go by his name. Galileo's law of inertia was his starting point which he formulated as the First Law of motion:
Newton's first law of motion is the same law rephrased thus: "**Everybody continues to be in its state of rest or of uniform motion in a straight line, unless some external force act on it**". In simple terms, the First Law is "**If external force on a body is zero, its acceleration is zero. Acceleration can be non zero only if there is a net external force on the body**". The first law refers to the simple case when the net external force on a body is zero.
Example (1): Consider a book at rest on a horizontal surface. It is subject to two external forces : the force due to gravity (i.e. its weight W) acting downward and the upward force on the book by the table, the normal force R . R is a self-adjusting force. Since the book is observed to be at rest, the net external force on it must be zero, according to the first law. This implies that the normal force R must be equal and opposite to the weight W".
Example (2) : Consider the motion of a car starting from rest, picking up speed and then moving on a smooth straight road with uniform speed When the car is stationary, there is no net force acting on it. During pick-up, it accelerates. This must happen due to a net external force. It is the frictional force that accelerates the car as a whole. When the car moves with constant velocity, there is no net external force.
Example (3) : An astronaut accidentally gets separated out of his small spaceship accelerating in inter stellar space at a constant rate of 100 m s^{-2}. What is the acceleration of the astronaut the instant after he is outside the spaceship ? (Assume that there are no nearby stars to exert gravitational force on him.) Answer : Since there are no nearby stars to exert gravitational force on him and the small spaceship exerts negligible gravitational attraction on him, the net force acting on the astronaut, once he is out of the spaceship, is **zero**. By the first law of motion the acceleration of the astronaut is zero.

5.3. Momentum and Newton's second law of motion :

Momentum (p) of a body is the product of its mass (m) and velocity (v) : p = mv,
Momentum is clearly a vector quantity and its SI unit is Kg.m/s.

Newton's Second Law of Motion :

The rate of change of momentum of a body is proportional to the applied force and takes place in the direction in which the force acts. Thus $F \propto \frac{d}{dt}(mv); or\ F = k\frac{d}{dt}(mv) = k.m\frac{dv}{dt} = k.ma$, where F is the net external force on the body and a its acceleration. We set the constant of proportionality k = 1 in S.I. Then $\mathbf{F = m\frac{dv}{dt} = ma}$ ------- (1)
The SI unit of force is Newton : 1 N = 1 kg m s^{-2}.
(a) The second law is consistent with the First Law (F = 0 implies a = 0)
(b) It is a vector equation
(c) It is applicable to a particle, and also to a body or a system of particles, provided F is the total external force on the system and a is the acceleration of the system as a whole.
(d) F at a point at a certain instant determines **a** at the same point at that instant.

That is the Second Law is a local law; **a** at an instant does not depend on the history of motion.

Note : Consider a body of mass m initially moving with a velocity of u m/s. When a force of F Newton is applied on it, its velocity becomes v m/s after a time of t sec. Then according to Newton's second law, $F = \frac{Change\ in\ Momentum}{Time\ taken} = \frac{mv-mu}{t} = \frac{m\,(v-u)}{t} = ma$

Example 1: A bullet of mass 0.04 kg moving with a speed of 90 m s^{-1} enters a heavy wooden block and is stopped after a distance of 60 cm. What is the average resistive force exerted by the block on the bullet?

Answer : The retardation 'a' of the bullet (assumed constant) is given by $a = \frac{-u^2}{2s} = \frac{-90 \times 90}{2 \times 0.6}$ ms^{-2} = -6750 ms^{-2}. The retarding force, by the Second Law of motion, is = 0.04 kg x 6750 m s^{-2} = 270 N.

Example 2 : The motion of a particle of mass m is described by y = ut + ½gt^2. Find the force acting on the particle.

Answer : We know y = ut + ½gt^2. Now, $v = \frac{dy}{dt} = u + gt$, acceleration, $a = \frac{dv}{dt} =$ g

Then the force is F = ma = mg

Thus the given equation describes the motion of a particle under acceleration due to gravity and y is the position coordinate in the direction of g.

5.4. Impulse; Newton's third law of motion :

Impulse is the product of force and time which equals change in momentum.

Impulse = Force × time duration = Change in momentum

$$I = F \times t = \frac{P_2 - P_1}{t} \times t = P_2 - P_1$$

The notion of impulse is useful when a large force acts for a short time to produce a measurable change in momentum. Since the time of action of the force is very short, one can assume that there is no appreciable change in the position of the body during the action of the impulsive force.

6. Newton's third law of motion:

To every action, there is always an equal and opposite reaction.

In simple terms, the law can be stated thus :

Forces in nature always occur between pairs of bodies. Force on a body A by body B is equal and opposite to the force on the body B by A.

Action and reaction forces are simultaneous forces. There is no cause-effect relation between action and reaction. Any of the two mutual forces can be called action and the other reaction. Action and reaction act on different bodies and so they cannot be cancelled out. The internal action and reaction forces between different parts of a body do, however, sum to zero.

5.5. Law of conservation of linear momentum and its applications :

Law of Conservation of Momentum

The total momentum of an isolated system of particles is conserved. The law follows from the second and third law of motion. When external force is zero, i.e. $F = 0, \frac{dp}{dt} = 0, or\ p = constant,\ or\ mv = constant, or\ m_1 v_1 = m_2 v_2$.

Example : The application of the law of conservation of momentum is the collision of two bodies. Consider two bodies A and B, with initial momenta p_A and p_B. The bodies collide, get apart, with final momenta p'_A and p'_B respectively. By the Second Law :

$F_{AB}\Delta t = p'_A - p_A$ and $F_{BA}\Delta t = p'_B - p_B$

Since, $F_{AB} = -F_{BA}$ $(p'_A - p_A) = -(p'_B - p_B)$ i.e., $p'_A + p'_B = p_A + p_B$

which shows that the total final momentum of the isolated system equals its initial momentum.

5.6. Equilibrium of concurrent forces :

In mechanics there are several common forces acting on a body. Some of them are : Gravitational force, Contact forces, tension in a string, and the force due to spring.

A contact force on an object arises due to contact with some other object: solid or fluid. When bodies are in contact there are mutual contact forces (for each pair of bodies), satisfying the third law. The component of contact force normal to the surfaces in contact is called **normal reaction**. The component parallel to the surfaces in contact is called **friction**. Contact forces arise also when solids are in contact with fluids. For example, for a solid immersed in a fluid, there is an upward bouyant force equal to the weight of the fluid displaced. The viscous force, air resistance, etc are also examples of contact forces.

When a spring is compressed or extended by an external force, a **restoring force** is generated. This force is usually proportional to the compression or elongation (for small displacements). The spring force F is written as F = – k x where x is the displacement and k is the force constant. The negative sign denotes that the force is opposite to the displacement from the unstretched state.

5.7. Static and kinetic friction, laws of friction, rolling friction :

Friction :

Frictional force opposes (impending or actual) relative motion between two surfaces in contact. It is the component of the contact force along the common tangent to the surface in contact. Static friction f_s opposes impending relative motion; kinetic friction f_k opposes actual relative motion. They are independent of the area of contact and satisfy the following approximate laws :

$$f_s \leq (f_s)_{max} = \mu_s R$$

$$f_k = \mu_k R$$

Rolling friction : A body like a ring or a sphere rolling without slipping over a horizontal plane will suffer no friction, in principle. At every instant, there is just one point of contact between the body and the plane and this point has no motion relative to the plane. In this ideal situation, kinetic or static friction is zero and the body should continue to roll with constant velocity.

Rolling friction again has a complex origin, though somewhat different from that of static and sliding friction. During rolling, the surfaces in contact get momentarily deformed a little, and this results in a finite area (not a point) of the body being in contact with the surface. The net effect is that the component of the contact force parallel to the surface opposes motion.

In many practical situations, however, friction is critically needed. **Kinetic friction** that dissipates power is nevertheless important for quickly stopping relative motion. It is made use of by brakes in machines and automobiles. Similarly, **static friction** is important in daily life. We are able to walk because of friction. It is impossible for a car to move on a very slippery road. On an ordinary road, the friction between the tyres and the road provides the necessary external force to accelerate the car.

5.8. Dynamics of uniform circular motion: Centripetal force, examples of circular motion (vehicle on level circular road, vehicle on banked road) :

μ_s (co-efficient of static friction) and μ_k (co-efficient of kinetic friction) are constants characteristic of the pair of surfaces in contact. It is found experimentally that μ_k is less than μ_s.

(i) Circular motion of a vehicle on level circular road :

We know that acceleration of a body moving in a circle of radius R with uniform speed v is v^2/R directed towards the centre. According to the second law, the force f providing this acceleration is : f $= \frac{mv^2}{R}$ where m is the mass of the body. This force directed forwards the centre is called the **centripetal force**. For a stone rotated in a circle by a string, the centripetal force is provided by the tension in the string. The centripetal force for motion of a planet around the sun is the gravitational force on the planet due to the sun. For a car taking a circular turn on a horizontal road, the centripetal force is the force of friction.

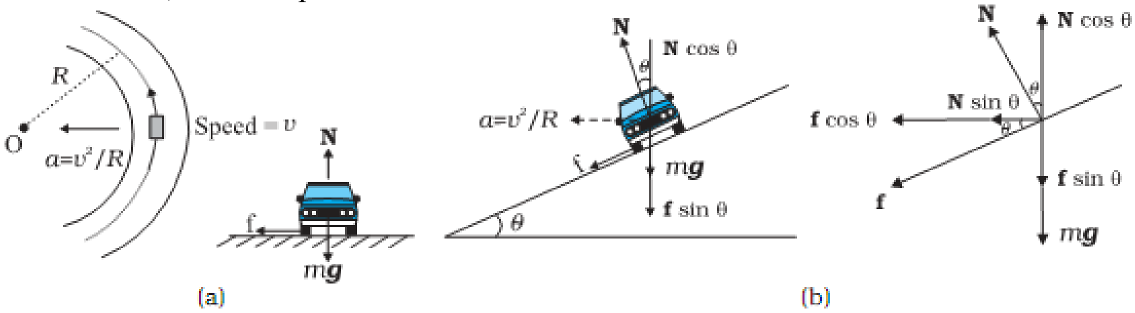


Fig. : Circular motion of a car on (a) a level road, (b) a banked road.

Three forces act on the car. They are (i) The weight of the car, mg, (ii) Normal reaction, N, (iii) Frictional force, f, As there is no acceleration in the vertical direction

N–mg = 0; N = mg ;

The centripetal force required for circular motion is along the surface of the road, and is provided by the component of the contact force between road and the car tyres along the surface. This by definition is the frictional force. The static friction that provides the centripetal acceleration. Static friction opposes the impending motion of the car moving away from the circle.

$$f \le \mu_s N = \frac{mv^2}{R}$$

Or $\quad v^2 \le \frac{\mu_s RN}{m} = \mu_s Rg \qquad [\mu_s = \text{mg}]$

which is independent of the mass of the car. This shows that for a given value of μ_s and R, there is a maximum speed of circular motion of the car possible, namely

$v_{max} = \sqrt{\mu_s Rg}$ ---------- (1)

(ii) Circular motion of a vehicle on a banked road :

We can reduce the contribution of friction to the circular motion of the car if the road is banked. Since there is no acceleration along the vertical direction, the net force along this direction must be zero. Hence, N cos θ = mg + f sin θ ------ (2)

The centripetal force is provided by the horizontal components of N and f.

N sin θ + f cos θ $= \frac{mv^2}{R}$ ----- (3)

But f ≤ μ_s N

Thus to obtain v_{max} we put $f = \mu_s N$
Then Eqs. (2) & (3) become N cos θ = mg + $\mu_s N$ sin θ
N sin θ + μ_sN cos θ = mv^2/R (4)
We obtain

$$N = \frac{mg}{cos\theta - \mu_s sin\theta}$$

Substituting value of N in Eq. (4), we get,

$$N = \frac{mg(sin\theta - \mu_s\, cos\theta)}{cos\theta - \mu_s sin\theta} = \frac{mv_{max}^2}{R} \quad \text{or} \quad v_{max} = \left(Rg\frac{\mu_s + \, tan\theta}{1 - \mu_s\, tan\theta}\right)^{\frac{1}{2}} \quad ------(5)$$

Comparing this with Eq. (1) we see that maximum possible speed of a car on a banked road is greater than that on a flat road. For $\mu_s = 0$ in Eq. (5),

$$v_o = (Rg\, tan\theta)^{½} \quad -------- \;(6)$$

At this speed, frictional force is not needed at all to provide the necessary centripetal force. Driving at this speed on a banked road will cause little wear and tear of the tyres. The same equation also tells you that for $v < v_o$, frictional force will be up the slope and that a car can be parked only if tan θ ≤ μ_s .

QUESTIONS :

1. What is Aristotle's fallacy?

An external force is required to keep the body in motion.

2. State Aristotlean law of motion

An external force is required to keep the body in motion.

3. Why uniformly moving body comes to rest?

Due to opposing force /frictional force.

4. What is uniform motion?

If a body covers equal distance in equal intervals of time, however small these intervals may be.

5. Who discovered Aristotlean law of motion?

Galileo galilei

6. What is the measure of inertia?

Gravitational mass

7. Give an example for inertia of rest.

A book kept on the stationary table

8. Give an example for inertia of motion

An object moving with uniform speed

9. State Newton's first law of motion.

Everybody continues to be in the state of rest or of uniform motion in a straight line unless compelled by an external force to act otherwise.

10. State the law of inertia.

Everybody continues to be in the state of rest or of uniform motion in a straight line unless compelled by an external force to act otherwise.

11. What is the acceleration of a body having uniform linear motion?

Zero

12. What is the force on a body moving with uniform speed?

Zero

13. Whose ideas did Newton make use of while framing his famous laws of motion?

Galileo Galilee's ideas

14. A space ship out in an interstellar space, far from all other objects and with all its rockets turned off, has zero acceleration but still in motion, Why?

Due to inertia of motion

15. State Newton's second law of motion
The rate of change of momentum of a body is directly proportional to the applied force and takes place in the direction in which the force acts.

16. Define linear momentum of a body.
Linear momentum of a body is defined as the product of the mass of the body and its velocity.

17. Is linear momentum of a body is scalar or a vector?
Vector

18. Write the S.I unit of linear momentum
Kg m s^{-1}

19. Write the dimensional formula for linear momentum.
$M^1L^1T^{-1}$

20. Why athletes run a few steps before taking a jump?
To gain the linear momentum, this enables the athlete to jump a longer distance.

21. Why a passenger standing in a bus fall backwards, when the bus suddenly starts moving?
Due to inertia of rest

22. Why a passenger getting down from a moving bus must run a few steps in the direction of the motion of the bus?
Due to inertia of motion

23. It is more difficult to catch a cricket ball than a tennis ball thrown with same velocity. Why?
Since the mass of a cricket ball is more than the tennis ball, the momentum of the cricket ball is more than the momentum of the tennis ball.

24. The rate of change of momentum of a body is 5 $Kgms^{-1}$. What is the force acting on the body?
Force = rate of change of momentum = 5 N

25. Why a cricketer does lower his hand soon after/while catching a cricket ball?
To increase the time interval to stop the ball and hence require a smaller force.

26. Give an example for an object having magnitude of momentum fixed but change in direction.
A stone tied to a string and whirled with uniform speed in a horizontal plane.

27. Define impulse of a force.
It is defined as the product of **force and time**. It is also equal to the change in momentum of the body.

28. What is an impulsive force?

Large force acting for a short interval of time to produce a finite change in momentum is called an impulsive force.

29. State Newton's third law of motion.
To every action, there is always an equal and opposite reaction.

30. State the law of conservation of linear momentum.
The total momentum of an isolated system of interacting particles is conserved.

31. What is meant by equilibrium of a particle?
A body is said to be in equilibrium if the net external force acting on it is zero.

32. What is frictional force?
The opposing force arising between the two surfaces in contact due to applied force.

33. What is meant by static friction?
The force of friction which opposes the applied force during the stationary state of a body is called static friction. $f_S = mg\sin\theta$

34. What is meant by normal reaction force?
Normal reaction force is a contact force exerted by one body on the other body in a direction perpendicular to the surface Momentum (p) of a body is the product of its mass (m) and velocity (v) : p = mv,
Momentum is clearly a vector quantity and its SI unit is Kg.m/s.
of contact. N = mg cosθ

35. What is meant by the limiting force of friction?
The maximum value of static frictional force which comes into play when a body just starts moving over the surface of another body is called as limiting force of friction.

36. What is meant by kinetic friction?
The force of friction which comes into play when one body moves over the surface of another body is called kinetic friction.

37. What is frictional force?
The opposing force arising between the two surfaces in contact due to applied force.

38. Define co-efficient of static friction.
The co-efficient of static friction is defined as the ratio of applied force to the normal force.

$\mu_S = \frac{f_S}{N}$ or $\mu_S = \tan\theta_{max}$

39. What is kinetic (sliding) friction?
Frictional force that opposes the relative motion between the surfaces in contact is called kinetic friction.

40. What is rolling friction?

The force of friction that comes into play when one body roles over the other body is called rolling friction.

41. What is centripetal force?
The force experienced by a body moving along a circular path and always directed towards the centre.

42. Is Aristotelian law correct? Justify you answer.
No. When the body is moving uniformly no external force is required to maintain the motion.

43. What is inertia? Who gave this concept?
An object continuing in the same state of rest or of uniform motion is called inertia. Galileo Galilee gave the concept of inertia.

44. A net external force of 5 N is acting on a body of mass 10 Kg. What is the acceleration produced?
F = ma => a = F/m = 5/10 = 0.5N

45. Compare the linear momenta of two bodies one of mass 5 g moving with a speed of 50 ms^{-1} and another body of mass 0.5 Kg moving with a speed of 0.5 ms^{-1}
We know that momentum p = mv

$P_1 = m_1v_1 = 5 \times 10^{-3} \times 50 = 0.25$

$P_2 = m_2v_2 = 0.5 \times 0.5 = 0.25$ Kg

Therefore $p_1 = p_2$
Both the bodies have the same momenta.

46. Mention any two advantages of friction.
Frictional force helps us to walk on the surface of earth Frictional force helps us to hold any object with hands. It helps to apply the brakes, vehicles to move without sliding etc.,

47. State Newton's second law of motion and hence derive F=ma
The rate of change of momentum of a body is directly proportional to the applied force and takes place in the direction in which the force acts.
Let us consider a body of mass 'm' moving with velocity 'v' under the action of force 'F' changes to v+Δv in a time 'Δt'.

$$\text{From II law of motion } F \propto \frac{\Delta p}{\Delta t} \text{ or } F = K\frac{\Delta p}{\Delta t} \text{ where k = constant}$$

$$\lim_{\nabla p \to 0} \frac{\Delta p}{\Delta t} = \frac{dp}{dt} \text{ Therefore } F = K\frac{dp}{dt}$$

$$\text{For a body of constant mass } \frac{dp}{dt} = \frac{d(mv)}{dt} = M\frac{dv}{dt} = ma$$

Therefore, F = K ma, If K =1, $F = \frac{dp}{dt} = ma$

F = ma

48. Write the S.I unit and dimensional formula for force.
Newton --- S.I unit

$M^1L^1T^{-2}$ ---- dimensional formula

49. Define Newton the S.I unit of force.
Force is said to be one newton if it causes an acceleration of $1ms^{-2}$ to a mass of 1 Kg.

50. What is the force acting on a body of mass 0.05 Kg if it accelerates the body by $3ms^{-2}$?
We know that $F = ma = 0.05 \times 3 = 0.15N$

51. What is impulse?
Impulse is product of force and time $I = F \times t$

52. What is impulsive force?
A large force acting for a short time to produce a finite change in momentum is called an impulsive force.

53. Write the S.I unit and dimensional formula of impulse.
S.I unit---- N.s
Dimensional formula-----MLT^{-1}

54. State Newton's third law of motion.
To every action, there is always an equal and opposite reaction.

55. State the law of conservation of linear momentum.
The total momentum of an isolated system of interacting particles is conserved.

56. Prove the law of conservation of momentum.
Let us consider two bodies A & B with initial momenta p_A & p_B collide, get apart with final momenta $p_A{}^1$ & $p_B{}^1$ respectively.

From Newton's second law of motion

$$F_{AB}\Delta t = p_A{}^1 - p_A \qquad \& \qquad F_{AB}\Delta t = p_B{}^1 - p_B$$

Where Δt = time for which the bodies are in contact.
From Newton's III law of motion

$$F_{AB} = -F_{BA} \rightarrow p_A{}^1 - p_A = -(p_B{}^1 - p_B)$$

$$F_{AB} = -F_{BA} \rightarrow p_A{}^1 - p_A = -(p_B{}^1 - p_B)$$

$$p_A{}^1 + p_B{}^1 = p_A + p_B$$

57. Mention the common forces in mechanics.
a) frictional force b) viscous force c) spring force d) gravitational force.

58. What is the change in momentum of a particle in uniform circular motion at diametrically opposite points?

p - (-p) = p + p = 2p

59. Write the expression for the spring force and explain the terms.

F = -Kx where k = spring constant, x = displacement

60. In mechanics we come across so many contact forces, their origin is electrical force though the particles are neutral. Explain.

At the microscopic level, all bodies are made of charged constituents (nuclei and electrons). Various contact forces are arising due to elasticity of bodies, molecular collisions and impacts, etc., These forces are due to electrical forces of charged constituents of different bodies.

61. Write the important points to be noted about the Newton's third law of motion with regard to the usage of the terms 'action & reaction'.

1. Action and reaction are nothing but force.
2. Forces always occur in pairs. $F_{AB} = -F_{BA}$ that is force on A by B is equal to negative force on B by A.
3. There is no cause and effect relation implied in third law.
4. The force on A by B and the force on B by A act at the same instant.
5. Action and reaction forces act on different bodies and not on the same body.
6. By considering system of two bodies as a whole F_{AB} & F_{BA} are internal forces of the system (A+B). They add up to give a null force.

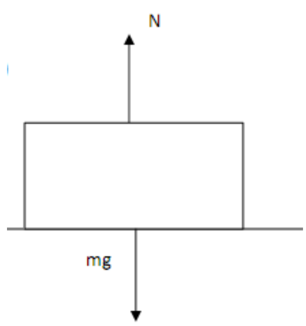
62. When do we say that the particle is in equilibrium under the action of 'n' number of forces say F_1, F_2, F_3 ------- F_n ?

A particle is said to be in equilibrium under the action of 'n' number of forces say F_1, F_2, F_3 ------- F_n if they can be represented by the sides of a closed n-sided polygon with arrows taken in order.

63. Derive the expression for maximum speed of circular motion of a car on (i) a level road (ii) on a banked road.

(i)

In order to have circular motion centripetal force should be balanced by static frictional force between car tyres and road.



Centripetal force = $\frac{mv^2}{R}$ where R= radius of the circle

frictional force $\int s \le \mu_s N$ But N = mg

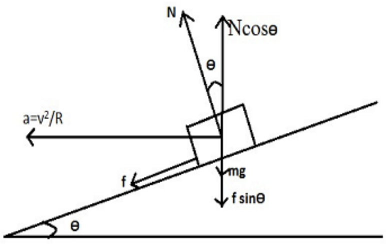
$\int s \le \mu_s N mg$

$\frac{mv^2}{R} \le \mu_s N mg$

$v^2 \leq \mu_s Rg$ or $\boldsymbol{\mu_s \geq \frac{v^2}{Rg}}$

From the above equation we know that the velocity is independent of mass of the car. Therefore for a given value of μ_s and R maximum speed of circular motion. Note that $2\pi(\frac{n}{t}) = \omega = \frac{v}{r}$

(ii)



From figure we have,

$$N\,cos\theta = mg + f\,sin\theta \quad \text{---------} \quad (1)$$

$$Nsin\theta + f\,cos\theta = {mv^2}/{R} \quad \text{---------} \quad (2)$$

Centripetal force is provided by the horizontal components of 'f' & 'N' and velocity is maximum when f = $\mu_s N$

$$N\cos\theta = mg + \mu_s N \sin\theta$$

$$N\cos\theta - \mu_s N \sin\theta = mg$$

$$N(\cos\theta - \mu_s \sin\theta) = mg$$

$$N = \frac{mg}{\cos\theta - \mu_s \sin\theta}$$

Substituting the value of N and 'f' in equation (2) we get

$$\frac{mg}{(\cos\theta - \mu_s \sin\theta)}(\sin\theta + \mu_s \cos\theta) = \frac{mv^2}{R}$$

$$v^2 = \frac{(\sin\theta + \mu_s \cos\theta)}{(\cos\theta - \mu_s \sin\theta)} Rg$$

Dividing the numerator and denominator by cosθ we get

$$v^2 = \frac{(\tan\theta + \mu_s)}{(1 - \mu_s \tan\theta)} Rg$$

$$V = \sqrt{\frac{(\tan\theta + \mu_s)}{(1 - \mu_s \tan\theta)}}\,Rg \qquad if\ \ \mu_s = 0, \qquad then\ \ v = \sqrt{\tan\theta\ Rg}$$

Also $\tan\theta = \frac{v^2}{Rg}$

64. Write the steps to be followed to solve problems in mechanics.

a) Using the given data a free body diagram should be drawn.
b) One of the convenient part should be chosen as one system
c) A separate diagram which shows this system and the forces acting on it is written
d) The magnitude and the directions of all the forces are represented.
e) The rest should be treated as unknown to be determined using Newton's laws of motion
f) The remaining part of the problem can be solved by considering another part of the diagram and Newton's third law of motion.

MULTIPLE CHOICE QUESTIONS :

1. A ball is travelling with uniform translatory motion. This means that ______
(a) it is at rest.
(b) the path can be a straight line or circular and the ball travels with uniform speed.
(c) all parts of the ball have the same velocity (magnitude and direction) and the velocity is constant.
(d) the centre of the ball moves with constant velocity and the ball spins about its centre uniformly.

2. A metre scale is moving with uniform velocity. This implies ________
(a) the force acting on the scale is zero, but a torque about the centre of mass can act on the scale.
(b) the force acting on the scale is zero and the torque acting about centre of mass of the scale is also zero.
(c) the total force acting on it need not be zero but the torque on it is zero.
(d) neither the force nor the torque need to be zero.

3. A cricket ball of mass 150 g has an initial velocity u = (3i + 4j) ms^{-1} and a final velocity v = (-3i + 4j) ms^{-1} after being hit. The change in momentum (final momentum – initial momentum) is (in kg m s^{-1})
(a) zero
(b -(0.45i + 0.6j)
(c) (0.9 1.2) + i j
(d) -5(i + j)

4. A force (3i + 4j)N acts on a body and displaced it by (3i + 4j) m. The work done by the force is _____
(a) 5 J (b) 25 J (c) 75 J (d) 100 J [AIIMS 2001] Ans (b)

5. A stone is tied to the end of a string of 80 cm long, is whirled in a horizontal circle, with a constant speed. If the stone makes 25 revolutions in 14 sec., then the magnitude of acceleration of the same will be _____

(a) 990 cm/s^2 (b) 780 cm/s^2 (c) 790 cm/s^2 (d) 950 cm/s^2
[AIIMS 2001] Ans (a)

Five Marks Questions :

EXERCISES

(For simplicity in numerical calculations, take g = 10 m s^{-2})

5.1 Give the magnitude and direction of the net force acting on (a) a drop of rain falling down with a constant speed,
(b) a cork of mass 10 g floating on water,
(c) a kite skillfully held stationary in the sky,
(d) a car moving with a constant velocity of 30 km/h on a rough road,
(e) a high-speed electron in space far from all material objects, and free of electric and magnetic fields.

Answer
(a) Zero net force
The rain drop is falling with a constant speed. Hence, it acceleration is zero Newton's second law of motion, the net force acting on the rain drop is zero.

(b) Zero net force
The weight of the cork is acting downward. It is balanced by the buoyant force by the water in the upward direction. Hence, no net force is acting on the floating.

(c) Zero net force
The kite is stationary in the sky, i.e., it is not moving at all. Hence, as per new law of motion, no net force is acting on the kite.

(d) Zero net force
The car is moving on a rough road with a constant velocity. Hence, its accelerate zero. As per Newton's second law of motion, no net force is acting on the car.

(e) Zero net force
The high speed electron is free from the influence of all fields. Hence, no ne acting on the electron.

5.2 A pebble of mass 0.05 kg is thrown vertically upwards. Give the direction and magnitude of the net force on the pebble,
(a) during its upward motion,
(b) during its downward motion,
(c) at the highest point where it is momentarily at rest. Do your answers change if the pebble was thrown at an angle of 45° with the horizontal direction? Ignore air resistance.

Answer
0.5 N, in vertically downward direction, in all cases

Acceleration due to gravity, irrespective of the direction of motion of an object acts downward. The gravitational force is the only force that acts on the peb three cases. Its magnitude is given by Newton's second law of motion as:
F= m X a

Where,

F= Net force

m= Mass of the pebble = 0.05 kg

$a = g = 10\ m/s^2$

:. $F = 0.05 \times 10 = 0.5$ N

The net force on the pebble in all three cases is 0.5 N and this force ac downward direction.
If the pebble is thrown at an angle of 45° with the horizontal, it will have horizontal and vertical components of velocity. At the highest point, only the component of velocity becomes zero. However, the pebble will have the component of velocity throughout its motion. This component of velocity pro effect on the net force acting on the pebble.

5.3 Give the magnitude and direction of the net force acting on a stone of mass 0.1 kg,
(a) just after it is dropped from the window of a stationary train,
(b) just after it is dropped from the window of a train running at a constant velocity of 36 km/h,
(c) just after it is dropped from the window of a train accelerating with 1 m s^{-2},
(d) lying on the floor of a train which is accelerating with 1 m s-2, the stone being at rest relative to the train. Neglect air resistance throughout.

Answer
(a) 1 N; vertically downward
Mass of the stone, m = 0.1 kg
Acceleration of the stone, a = g = 10 m/s^2
As per Newton's second law of motion, the net force acting on the stone,
$F = ma = mg$
= 0.1 X 10 = 1 N
Acceleration due to gravity always acts in the downward direction.
(b) 1 N; vertically downward
The train is moving with a constant velocity. Hence, its acceleration is zero direction of its motion, i.e.; in the horizontal direction. Hence, no force is actir stone in the horizontal direction. The net force acting on the stone is because of acceleration due to gravity and acts vertically downward. The magnitude of this force is 1 N.
(c) 1 N; vertically downward
It is given that the train is accelerating at the rate of $1 m/s^2$.
Therefore, the net force acting on the stone, $F' = ma = 0.1 \times 1 = 0.1$ N

This force is acting in the horizontal direction. Now, when the stone is drop horizontal force F', stops acting on the stone. This is because of the fact that acting on a body at an instant depends on the situation at that instant and not situations.
Therefore, the net force acting on the stone is given only by acceleration due to $F=mg=1$ N
This force acts vertically downward.
(d) 0.1 N; in the direction of motion of the train
The weight of the stone is balanced by the normal reaction of the floor.
Acceleration is provided by the horizontal motion of the train.
Acceleration of the train, a = 0.1 m/s^2
The net force acting on the stone will be in the direction of the motion of the magnitude is given by;
$F=ma$
=0.1 X 1 = 0.1 N

5.4 One end of a string of length l is connected to a particle of mass m and the other to a small peg on a smooth horizontal table. If the particle moves in a circle with speed v the net force on the particle (directed towards the centre) is :

$(i)\ T, \quad (ii)\ T\frac{mv^2}{l}, \quad (iii)\ \ T+\frac{mv^2}{l}, \quad (iv)\ \ 0$

T is the tension in the string. [Choose the correct alternative].

Answer
(i) When a particle connected to a string resolves in a circular path around a centripetal force is provided by the tension produced in the string. Hence, in case, the net force on the particle is the tension T, i.e.,

$$F = T = \frac{mv^2}{l}$$

Where F is the net force acting on the particle.

5.5 A constant retarding force of 50 N is applied to a body of mass 20 kg moving initially with a speed of 15 m s^{-1}. How long does the body take to stop ?

Answer
Retarding force, F = - 50 N

Mass of the body, m =20 kg

Initial velocity of the body, u =15 m/s

Final velocity of the body, v = 0

Using Newton's second law motion, the acceleration (a) produced in the body calculated as:

$F= ma$

-50 = 20 X a

$$\therefore \quad a = \frac{-50}{20} = -2.5\, m/s^2$$

Using the first equation of motion, the time (t) taken by the body to come to recalculated as:

v = u + at

$$\therefore \quad t = \frac{-u}{a} = \frac{-15}{-2.5} = 6s$$

5.6 A constant force acting on a body of mass 3.0 kg changes its speed from 2.0 m s^{-1} to 3.5 m s^{-1} in 25 s. The direction of the motion of the body remains unchanged. What is the magnitude and direction of the force ?

Answer
0.18 N; in the direction of motion of the body

Mass of the body, m = 3kg

Initial speed of the body, u =2 m/s

Final speed of the body, v= 3.5 m/s

Time, t = 25 s

Using the first equation of motion, the acceleration (a) produced in the body calculated as:

v = u + at

$$\therefore a = \frac{v-u}{l}$$

$$= \frac{3.5-2}{25} = \frac{1.5}{25} = 0.06\ m/s^2$$

As per Newton's second law of motion, force is given as:
F= ma

=3 X 0.06 = 0.18 N

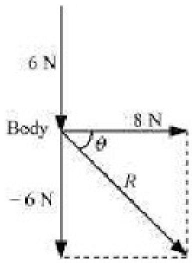
Since the application of force does not change the direction of the body, the acting on the body is in the direction of its motion.

5.7 A body of mass 5 kg is acted upon by two perpendicular forces 8 N and 6 N. Give the magnitude and direction of the acceleration of the body.
Answer

2 m/s^2, at an angle of 37° with a force of 8 N

Mass of the body, m = 5 kg

The given situation can be represented as follows:



The resultant of two forces is given as:

$$R = \sqrt{(8)^2 + (-6)^2} = \sqrt{64 + 36} = 10\,N$$

θ is the angle made by R with the force of 8 N

$$\therefore \quad \theta = tan^{-1}\left(\frac{-6}{8}\right) = -36.87^0$$

The negative sign indicates that θ is in the clockwise direction with respect to of magnitude 8 N. As per Newton's second law of motion, the acceleration (a) of the body is given

F = ma

$$\therefore \quad \theta \quad a = \frac{F}{m} = \frac{10}{5} = 2\,m/s^2$$

5.8 The driver of a three-wheeler moving with a speed of 36 km/h sees a child standing in the middle of the road and brings his vehicle to rest in 4.0 s just in time to save the child. What is the average retarding force on the vehicle ? The mass of the three-wheeler is 400 kg and the mass of the driver is 65 kg.

Answer

Initial speed of the three-wheeler, u = 36 km/h

Final speed of the three-wheeler, v = 10 m/s

Time, t = 4s

Mass of the three-wheeler, m = 400 kg

Mass of the driver, m′ = 65 kg

Total mass of the system, M = 400 + 65 = 465 kg

Using the first law of motion, the acceleration (a) of the three-wheeler can be as:

v = u + at

$$\therefore \quad \theta \quad a = \frac{v-u}{t} = \frac{0-10}{4} = 2.5\,\mathrm{m/s^2}$$

The negative sign indicates that the velocity of the three-wheeler is decrease time.

Using Newton's second law motion, the net force acting on the three-wheeler calculated as:

$F = Ma$

$= 465 \times (-2.5) = -1162.5$ N

The negative sign indicates that the force is acting against the direction of motion three-wheeler.

5.9 A rocket with a lift-off mass 20,000 kg is blasted upwards with an initial acceleration of 5.0 m s^{-2}. Calculate the initial thrust (force) of the blast.

Answer

Mass of the rocket, m = 20,000 kg

Initial acceleration, a = 5 m/s^2

Acceleration due to gravity, g = 10 m/s^2

Using Newton's second law of motion, the net force (thrust) acting on the rocket by the relation:

F - mg =ma

F = m (g + a)

= 20000 x (10 + 5)

= 20000 x 15 = 3 x 10^5N

5.10 A body of mass 0.40 kg moving initially with a constant speed of 10 m s^{-1} to the north is subject to a constant force of 8.0 N directed towards the south for 30 s. Take the instant the force is applied to be t = 0, the position of the body at that time to be x = 0, and predict its position at t = –5 s, 25 s, 100 s.

Answer

Mass of the body, m =0.40 kg

Initial speed of the body, u = 10 m/s due north

Force acting on the body, F = -8.0 N

$$a = \frac{F}{m} = \frac{-8.0}{0.40} = -20\ \mathrm{m/s^2}$$

Acceleration produced in the body,

(i) <u>At *t* = -5 s</u>

Acceleration, a′ = 0 and u = 10 m/s

$$s = ut + \frac{1}{2}a'\,t^2$$

$$= 10 \times (-5) \quad = \quad -50\,m$$

(ii) At t = 25 s

Acceleration, a″ = -20 m/s^2 and u = 10 m/s

$$s' = ut' + \frac{1}{2}a'' \mathrm{t}^2$$

$$= 10 \times 25 + \frac{1}{2} \times (-20) \times (25)^2$$

$$= 250 + 6250 = -6000m$$

(iii) At t = 100 s

For $0 \leq \mathrm{t} \leq 30$ s

$$a = -20\, m/s^2$$

$$u = 10\, m/s$$

$$s_1 = ut + \frac{1}{2}a'' \mathrm{t}^2$$

$$= 10 \times 30 + \frac{1}{2} \times (-20) \times (30)^2$$

$$= 300 - 9000$$

$$= -8700\, m$$

For $30' < t \leq 100$ s

As per the first equation of motion, for t = 30 s, final velocity is given as:

$$v = u + at$$

$$= 10 + (-20) \times 30 = -590\, m/s$$

Velocity of the body after 30s = -590 m/s

For motion between 30 s to 100 s, i.e., in 70 s:

$$s_2 = vt + \frac{1}{2}a'' \mathrm{t}^2$$

$= -590 \times 70 = -41300\ m$

$\therefore$ Total distance, $s'' = s_1 + s_2 = -8700 - 41300 = -50000\ m$

5.11 A truck starts from rest and accelerates uniformly at 2.0 m s-2. At t = 10 s, a stone is dropped by a person standing on the top of the truck (6 m high from the ground). What are the (a) velocity, and (b) acceleration of the stone at t = 11s ? (Neglect air resistance.)

Answer

(a) 22.36 m/s, at an angle of 26.57^0 with the motion of the truck

(b) 10 m/s^2

(a) Initial velocity of the truck, u = 0

Acceleration, a = 2 m/s^2

Time, t = 10 s

As per the first equation of motion, final velocity is given as:

v= u + at

= 0 + 2 X 10 = 20 m/s

The final velocity of the truck and hence, of the stone is 20 m/s.

At t = 11 s, the horizontal component (v_x) of velocity, in the absence of air remains unchanged, i.e.,

v_x = 20 m/s

The vertical component (v_y) of velocity of the stone is given by the first eq motion as:

$v_y = u + a_y \delta t$

$where, \quad \delta t = 11 - 10 = 1s \ and \ a_y = g = 10\ m/s^2$

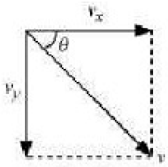
$\therefore v_y = 0 + 10 \times 1 = 10\ m/s$

The resultant velocity (v) of the stone is given as:

$v = \sqrt{v_x^2 + v_y^2}$

$= \sqrt{20^2 + 10^2} = \sqrt{400 + 100}$

$= \sqrt{500} = 22.36\ m/s$



Let θ be the angle made by the resultant velocity with the horizontal comp velocity, v_x

$$\therefore \tan \theta = \left(\frac{v_y}{v_x}\right)$$

$$\theta = tan^{-1}\left(\frac{10}{20}\right)$$

$$= tan^{-1}(0.5)$$

$$= 26.57^0$$

(b) When the stone is dropped from the truck, the horizontal force acting on it zero. However, the stone continues to move under the influence of gravity. H acceleration of the stone is 10 m/s^2 and it acts vertically downward.

5.12 A bob of mass 0.1 kg hung from the ceiling of a room by a string 2 m long is set into oscillation. The speed of the bob at its mean position is 1 m s^{-1}. What is the trajectory of the bob if the string is cut when the bob is (a) at one of its extreme positions, (b) at its mean position.

Answer

(a) Vertically downward

(b) Parabolic path

(a) At the extreme position, the velocity of the bob becomes zero. If the string this moment, the bob will fall vertically on the ground.

(b) At the mean position, the velocity of the bob is 1 m/s. The direction of this tangential to the arc formed by the oscillating bob. If the bob is cut at the mean then it will trace a projectile path having the horizontal component of veloc. Hence, it will follow a parabolic path.

5.13 A man of mass 70 kg stands on a weighing scale in a lift which is moving
(a) upwards with a uniform speed of 10 m s^{-1},
(b) downwards with a uniform acceleration of 5 m s^{-2},
(c) upwards with a uniform acceleration of 5 m s^{-2}. What would be the readings on the scale in each case?
(d) What would be the reading if the lift mechanism failed and it hurtled down freely under gravity ?

Answer

(a) Mass of the man, m = 70 kg

Acceleration, a = 0

Using Newton's second law of motion, we can write the equation of motion as:

R- mg = ma

Where, ma is the net force acting on the man.

As the lift is moving at a uniform speed, acceleration a=0

$\therefore R = mg$

$= 70 \times 10 = 700\ N$

$\therefore$ Reading on the weighing scale $= \frac{700}{g} = \frac{700}{10} = 70$ kg

(b) Mass of the man, m = 70 kg

Acceleration, $a = 5\ m/s^2$ downward

Using Newton's second law of motion, we can write the equation of motion as:

$R + mg = ma$

$R = m(g - a)$

$= 70\ (10 - 5) = 70 \times 5$

$= 350\ N$

(c) Mass of the man, m = 70 kg

Acceleration, $a = 5\ m/s^2$ upward

Using Newton's second law of motion, we can write the equation of motion as:

$R - mg = ma$

$R = m(g + a)$

$= 70\ (10 + 5) = 70 \times 15$

$= 1050\ N$ $\therefore$ Reading on the weighing scale $= \frac{350}{g} = \frac{350}{10} = 35$ kg

$\therefore$ Reading on the weighing scale $= \frac{1050}{g} = \frac{1050}{10} = 105$ kg

(d) When the lift moves freely under gravity, acceleration a = g

Using Newton's second law of motion, we can write the equation of motion as:

$R + mg = ma$

$R = m(g - a)$

$= \ m(g-g) \ = 0$

$\therefore$ Reading on the weighing scale $= \dfrac{0}{g} = \ 0 \ \ \text{kg}$

The man will be in a state of weightlessness.

5.14 Figure 5.16 shows the position-time graph of a particle of mass 4 kg. What is the (a) force on the particle for t < 0, t > 4 s, 0 < t < 4 s? (b) impulse at t = 0 and t = 4 s ? (Consider one-dimensional motion only).

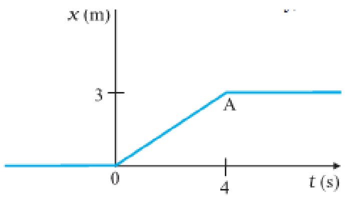


Fig. 5.16

Answer

(a) For t < 0

It can be observed from the given graph that the position of particle is with the time axis. It indicates that the displacement of the particle in this time is zero. Hence, the force acting on the particle is zero.

For t > 4 s

It can be observed from the given graph that the position of particle is para time axis. It indicates that the particle is at rest at a dist 3 m from the origin. Hence, no force acting on the particle.

For 0 < t < 4

It can be observed that the given position-time graph has a constant slope, hence acceleration produced in the particle is zero. Therefore, the force acting on the zero

(b) At t = 0

Impulse = Change in momentum

= *mv* - *mu*

Mass of the particle, m = 4 kg

Initial velocity of the particle, u = 0

$Final\ velocity\ of\ the\ particle, \ \ v = \dfrac{3}{4} \ \ m/s$

$\therefore$ Impulse $= 4\left(\frac{3}{4} - 0\right) = 3$ kg m/s

At t = 4 s

Initial velocity of the particle, $u = \frac{3}{4}\ m/s$

Final velocity of the particle, v = 0

$\therefore$ Impulse $= 4\left(0 - \frac{3}{4}\right) = -3$ kg m/s

5.15 Two bodies of masses 10 kg and 20 kg respectively kept on a smooth, horizontal surface are tied to the ends of a light string. a horizontal force F = 600 N is applied to (i) A, (ii) B along the direction of string. What is the tension in the string in each case?

Horizontal force, F = 600 N

Mass of body A_1 M_1 = 10 kg

Mass of body B_1 M_2 = 20 kg

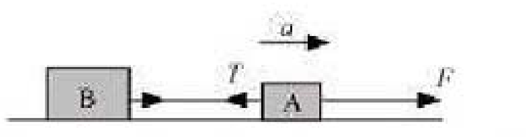
Total mass of the system, $m = m_1 + m_2$ = 30 kg

Using Newton's second law of motion, the acceleration (a) produced in the sy be calculated as:

$F = ma$

$\therefore\ a = \frac{F}{m} = \frac{600}{30} = 20\ \mathrm{m/s^2}$

When force F is applied on body A:



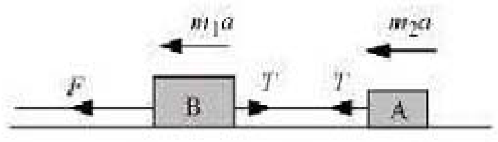
The equation of motion can be written as:

$F - T = m_1 a$

$\therefore\ T = F - m_1 a$

$= 600 - 10 \times 20 = 400$ N ……. (i)

When force F is applied on body B:



The equation of motion can be written as:

$F - T = m_2 a$

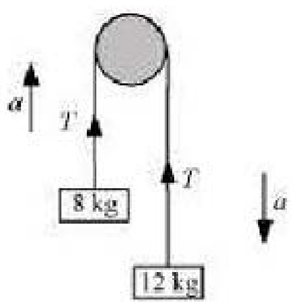
$T = F - m_2 a$

$\therefore\ T = 600 - 20 \times 20 = 200$ N ……..(ii)

5.16 Two masses 8 kg and 12 kg are connected at the two ends of a light inextensible string that goes over a frictionless pulley. Find the acceleration of the masses, and the tension in the string when the masses are released.

Answer

The given system of two masses and a pulley can be represented as shown following figure:



Smaller mass, $m_1 =$ 8 kg

Larger mass, $m_2 =$ 12 kg

Tension in the string = T

Mass m_2, owing to its weight, moves downward with acceleration a, and mass t upward.

Applying Newton's second law of motion to the system of each mass:

For mass m_1 :

The equation of motion can be written as:

$T - m_1 g = ma$ …..(i)

For mass m_2 :

The equation of motion can be written as:

$m_2 g - T = m_2 a$ …..(ii)

Adding equations (i) and (ii), we get:

$(m_2 - m_1) g = (m_1 + m_2) a$

$$\therefore \quad a = \left(\frac{m_2 - m_1}{m_1 + m_2}\right) g \qquad \ldots..(iii)$$

$$= \left(\frac{12 - 8}{12 + 8}\right) \times 10 = \frac{4}{20} \times 10 = 2\, m/s^2$$

Therefore, the acceleration of the masses is $2\ m/s^2$.

Substituting the value of a in equation (ii), we get:

$$m_2 g - T = m_2 \left(\frac{m_2 - m_1}{m_1 + m_2}\right) g$$

$$T = \left(m_2 - \frac{m_2^2 - m_1 m_2}{m_1 + m_2}\right) g$$

$$= \left(\frac{2m_1 m_2}{m_1 + m_2}\right) g$$

$$= \left(\frac{2m_1 m_2}{m_1 + m_2}\right) g$$

$$= \left(\frac{2 \times 12 \times 8}{12 + 8}\right) \times 10$$

$$= \frac{2 \times 12 \times 8}{20} \times 10 = 96\, N$$

Therefore, the tension in the string is 96 N.

5.17 A nucleus is at rest in the laboratory frame of reference. Show that if it disintegrates into two smaller nuclei the products must move in opposite directions.

Answer

Let $m,\ m_1,$ and m_2

be the respective masses of the parent nucleus and the two nuclei. the parent nucleus is at rest.

Initial momentum of the system (parent nucleus) = 0

Let v_1 and v_2 be the respective velocities of the daughter nuclei having masse m_2.

Total linear momentum of the system after disintegration = $m_1 v_1 + m_2 v_2$

According to the law of conservation of momentum:

Total initial momentun = Total final momentum

$$0 = m_1 v_1 + m_2 + v_2$$

$$v_1 = \frac{-m_2 v_2}{m_1}$$

Here, the negative sign indicates that the fragments of the parent nucleus directions opposite to each other.

5.18 Two billiard balls each of mass 0.05 kg moving in opposite directions with speed 6 m s^{-1} collide and rebound with the same speed. What is the impulse imparted to each ball due to the other ?

Answer
Mass of each ball = 0.05 kg

Initial velocity of each ball = 6 m/s

Magnitude of the initial momentum of each ball, p_i = 0.3 kg m/s

After collision, the balls change their directions of motion without chan magnitudes of their velocity.

Final momentum of each ball, p_f = - 0.3 kg m/s

Impulse imparted to each ball = Change in the momentum of the system

= $p_f - p_i$

= -0.3 – 0.3 = - 0.6 kg m/s

The negative sign indicates that the impulses imparted to the balls are op direction.

5.19 A shell of mass 0.020 kg is fired by a gun of mass 100 kg. If the muzzle speed of the shell is 80 m s^{-1}, what is the recoil speed of the gun ?
Answer

Mass of the gun, M = 100 kg

Mass of the shell, m = 0.020 kg

Muzzle speed of the shell, v = 80 m/s

Recoil speed of the gun =V

Both the gun and the shell are at rest initially.

Initial momentum of the system = 0

Final momentum of the system = mv – MV

Here, the negative sign appears because the directions of the shell and the opposite to each other.

According to the law of conservation of momentum:

Final momentum = Initial momentum

mv - MV = 0

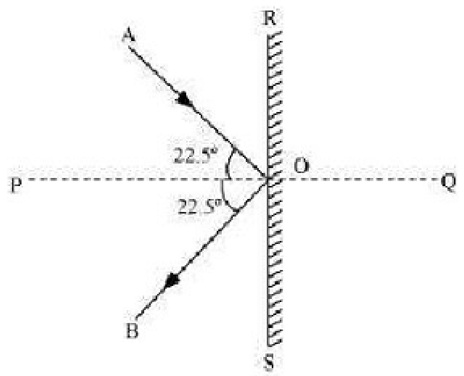
$$\therefore \; V = \frac{mv}{M}$$

$$\therefore \; \frac{0.020 \times 80}{100 \times 1000} = 0.016\, m/s$$

5.20 A batsman deflects a ball by an angle of 45° without changing its initial speed which is equal to 54 km/h. What is the impulse imparted to the ball ? (Mass of the ball is 0.15 kg.)

Answer

The given situation can be represented as shown in the following figure.



Where,

AO = Incident path of the bell

OB = Path followed by the ball after deflection

Δ AOB = Angle between the incident and deflected paths of the ball = 45^0

Δ AOP = Δ BOP = 22.5^0 = θ

Initial and final velocities of the ball = v

Horizontal component of the initial velocity = vcos θ along RO

Vertical component of the initial velocity = vsin θ along PO

Horizontal component of the final velocity = vcos θ along OS

Vertical component of the final velocity = vsin θ along OP

The horizontal component of velocities suffer no change. The vertical components velocities are in the opposite directions.

$\therefore$ Impulse imparted to the ball = Change in the linear momentum of the ball

= mvcos θ – (– mvcos θ)

= 2mvcos θ

Mass of the ball, m = 0.15 kg

Velocity of the ball, v = 54 km/h = 15 m/s

$\therefore$ Impulse $= 2 \times 0.15 \times 15 \cos 22.5^0 = 4.16$ kg m/s

5.21 A stone of mass 0.25 kg tied to the end of a string is whirled round in a circle of radius 1.5 m with a speed of 40 rev./min in a horizontal plane. What is the tension in the string ? What is the maximum speed with which the stone can be whirled around if the string can withstand a maximum tension of 200 N ?

Answer

Mass of the stone, m = 0.25 kg

Radius of the circle, r = 1.5 m

Number of revolution per second, $n \frac{40}{60} = \frac{2}{3}$ rps

Angular velocity, $\omega = \frac{v}{r} = 2 \pi n$ ... (i)

The centripetal force for the stone is provided by the tension T, in the string, i.e.

$$T = F_{Centipetal}$$

$$= \frac{mv^2}{r} = mr\omega^2 = mr(2xn)^2$$

$$= 0.25 \times 1.5 \times \left(2 \times 3.14 \times \frac{2}{3}\right)^2$$

$$= 6.57\ N$$

Maximum tension in the string, $T_{max} = 200$ N

$$T_{max} = \frac{mv_{max}^2}{r}$$

$$\therefore V_{max} = \sqrt{\frac{T_{max} \times r}{m}}$$

$$= \sqrt{\frac{200 \times 1.5}{0.25}}$$

$$= \sqrt{200} = 34.64 \text{ m/s}$$

Therefore, the maximum speed of the stone is 34.64 m/s.

5.22 If, in Exercise 5.21, the speed of the stone is increased beyond the maximum permissible value, and the string breaks suddenly, which of the following correctly describes the trajectory of the stone after the string breaks :
(a) the stone moves radially outwards,
(b) the stone flies off tangentially from the instant the string breaks,
(c) the stone flies off at an angle with the tangent whose magnitude depends on the speed of the particle ?
Answer

(b) When the string breaks, the stone will move in the direction of the velocity instant. According to the first law of motion, the direction of velocity vector is to the path of the stone at that instant. Hence, the stone will fly off tangentially instant the string breaks.

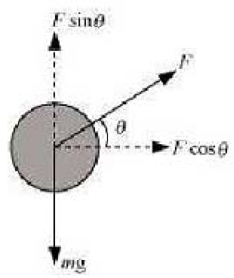
5.23 Explain why
(a) a horse cannot pull a cart and run in empty space,
(b) passengers are thrown forward from their seats when a speeding bus stops suddenly,
(c) it is easier to pull a lawn mower than to push it,
(d) a cricketer moves his hands backwards while holding a catch.
Answer

(a) In order to pull a cart, a horse pushes the ground backward with some fi ground in turn exerts an equal and opposite reaction force upon the feet of tl. This reaction force causes the horse to move forward.

An empty space is devoid of any such reaction force. Therefore, a horse can cart and run in empty space.

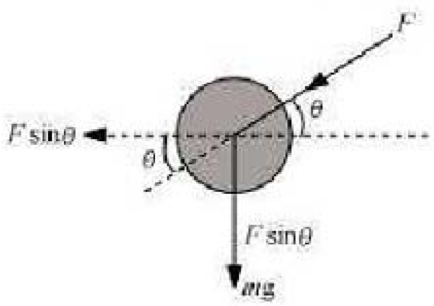
(b) When a speeding bus stops suddenly, the lower portion of a passenger's both is in contact with the seat, suddenly comes to rest. However, the upper portion remain in motion (as per the first law of motion). As a result, the passenge body is thrown forward in the direction in which the bus was moving.

(c) While pulling a lawn mower, a force at an angle θ is applied on it, as shown following figure.



The vertical component of this applied force acts upward. This reduces the weight of the mower.

On the other hand, while pushing a lawn mower, a force at an angle θ is applied shown in the following figure.



In this case, the vertical component of the applied force acts in the direction weight of the mower. This increases the effective weight of the mower.

Since the effective weight of the lawn mower is lesser in the first case, pulling mower is easier than pushing it.

(d) According to Newton's second law of motion, we have the question of motion

$$F = ma = m\frac{\Delta v}{\Delta t} \qquad \ldots..(i)$$

Where,

F= Stopping force experienced by the cricketer as he catches the ball

M = Mass of the ball

Δt = Time of impact of the ball with the hand

It can be inferred from equation (i) that the impact force is inversely proportion impact time, i.e.,

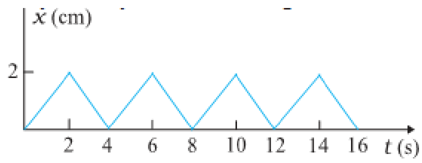
$$F \propto \frac{1}{\Delta t} \qquad \ldots..(ii)$$

Equation (ii) shows that the force experienced by the cricketer decreases if the impact increases and vice versa.

While taking a catch, a cricketer moves his hand backward so as to increase the impact (Δt). This is turn results in the decrease in the stopping force, thereby the hands of the cricketer from getting hurt.

Additional Exercises

5.24 Figure 5.17 shows the position-time graph of a body of mass 0.04 kg. Suggest a suitable physical context for this motion. What is the time between two consecutive impulses received by the body ? What is the magnitude of each impulse ?



Answer

A ball rebounding between two walls located between at $x = 0$ and $x = 2$ cm; af 2 s, the ball receives an impulse of magnitude $0.08 \times 10^{-2}\ kg\ m/s$ from the wall. The given graph shows that a body changes its direction of motion after e physically, this situation can be visualized as a ball rebounding to and fro beth stationary walls situated between positions $x = 0$ and $x = 2$ cm. Since the slope t graph reverses after every 2 s, the ball collides with a wall after every 2 s. T ball receives an impulse after every 2 s.

Mass of the ball, m = 0.04 kg

The slope of the graph gives the velocity of the ball. Using the graph, we can initial velocity (u) as:

$$\mathrm{u} = \frac{(2-0) \times 10^{-2}}{(2-0)} = 10^{-2}\ \mathrm{m/s}$$

Velocity of the ball before collision, $u = 10^{-2}\ \mathrm{m/s}$

Velocity of the ball after collision, $v = -10^{-2}\ \mathrm{m/s}$

(Here, the negative sign arises as the ball reverses its direction of motion.)

Magnitude of impulse = Change in momentum

$= |mv - mu|$

$= |0.04(v - u)|$

$= |0.04(-10^{-2} - 10^{-2})|$

$= 0.08 \times 10^{-2}\ \mathrm{kg\ m/s}$

5.25 Figure 5.18 shows a man standing stationary with respect to a horizontal conveyor belt that is accelerating with 1 m s^{-2}. What is the net force on the man? If the coefficient of static friction between the man's shoes and the belt is 0.2, up to what acceleration of the belt can the man continue to be stationary relative to the belt ? (Mass of the man = 65 kg.)

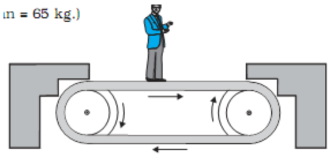


Fig. 5.18

Answer

Mass of the man, m = 65 kg

Acceleration of the belt, $a = 1\ m/s^2$

Coefficient of static friction, $\mu\ = 0.2$

The net force F, acting on the man is given by Newton's second law of motion as

$F_{net} = ma \quad = \ 65\ \times 1 \quad = 65\ N$

The man will continue to be stationary with respect to the conveyor belt unt force on the man is less than or equal to the frictional force f_{s1}exerted by the be

$F'_{net} = f_s$

$ma' = \mu mg$

$\therefore a' = 0.2\ \times 10\ = 2\ m/s^2$

Therefore, the maximum acceleration of the belt up to which the man c stationary is 2 m/s^2.

5.26 A stone of mass m tied to the end of a string revolves in a vertical circle of radius R. The net forces at the lowest and highest points of the circle directed vertically downwards are : [Choose the correct alternative]

	Lowest Point	Highest point
(a)	$mg - T_1$	$mg + T_2$
(b)	$mg + T_1$	$mg - T_2$
(c)	$mg + T_1 - \ (mv_1^2)/R$	$mg - T_2 + \ (mv_1^2)/R$
(d)	$mg - T_1 - \ (mv_1^2)/R$	$mg + T_2 + \ (mv_1^2)/R$

T_1 and v_1 denote the tension and speed at the lowest point. T_2 and v_2 denote corresponding values at the highest point.

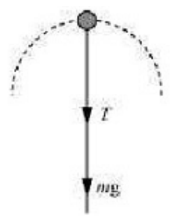
Answer

(a) The free body diagram of the stone at the lowest point is shown in the fi'gure.

According to Newton's second law of motion, the net force acting on the stor point is equal to the centripetal force, i.e,

$$F_{net} = T - mg = \frac{mv_1^2}{R} \quad \ldots..(i)$$

Where, v_1 = Velocity at the lowest point
The free body diagram of the stone at the highest point is shown in the following:



Using Newton's second law of motion, we have:

$$T + mg = \frac{mv_2^2}{R} \quad \ldots..(ii)$$

Where, v_2 =Velocity at the highest point

It is clear from equations (i) and (ii) that the net force acting at the lowest highest points are respectively (T –mg) and (T+mg).

5.27 A helicopter of mass 1000 kg rises with a vertical acceleration of 15 m s^{-2}. The crew and the passengers weigh 300 kg. Give the magnitude and direction of the (a) force on the floor by the crew and passengers,
(b) action of the rotor of the helicopter on the surrounding air,
(c) force on the helicopter due to the surrounding air.

Answer

(a) Mass of the helicopter, $m_h = 1000\ kg$

Mass of the crew and passengers, $m_p = 300\ kg$

Total mass of the system, m=1300 kg

Acceleration of the helicopter, $a = 15\ m/s^2$

Using Newton''s second law of motion, the reaction force R_1 on the system by can be calculated as:

$R - m_p g = ma$

$= m_p(g + a)$

$= 300(10 + 15) = 300 \times 25$

$= 7500\ N$

Since the helicopter is accelerating vertically upward, the reaction force will directed upward. Therefore, as per Newton's third law of motion, the force on by the crew and passengers is 7500 N, directed downward.

(b) Using Newton's second law of motion, the reaction force R_1' experience helicopter can be calculated as:

$R' - mg = ma$

$= m(g + a)$

$= 1300(10 + 15) = 1300 \times 25$

$= 32500\ N$

The reaction force experienced by the helicopter 'from the surrounding air upward. Hence, as per Newton's third law of motion, the action of the rot surrounding air will be 32500 N, directed downward.

(c) The force on the helicopter due to the surrounding air is 32500 N, directed

5.28 A stream of water flowing horizontally with a speed of 15 m s^{-1} gushes out of a tube of cross-sectional area 10^{-2} m^2, and hits a vertical wall nearby. What is the force exerted on the wall by the impact of water, assuming it does not rebound ?

Answer

Speed of the water stream, v =15 m/s

Cross-sectional area of the tube, $A = 10^{-2} m^2$

Volume of water coming out from the pipe per second,

$V = AV = 15 \times 10^{-2} m^3/s$

Density of water, $P = 10^3 \; kg/m^3$

Mass of water flowing out through the pipe per second = $p \times V = 150 \; kg/s$

The water strikes the wall and does not rebound. Therefore, the force exerted water on the wall is given by Newton's second law of motion as:

$$F = Rate\ of\ change\ of\ momentum = \frac{\Delta P}{\Delta t}$$

$$= \frac{mv}{t}$$

$$= 150 \times 15 = 2250\ N$$

5.29 Ten one-rupee coins are put on top of each other on a table. Each coin has a mass m. Give the magnitude and direction of
(a) the force on the 7th coin (counted from the bottom) due to all the coins on its top,
(b) the force on the 7th coin by the eighth coin,
(c) the reaction of the 6th coin on the 7th coin.

Answer

(a) Force on the seventh coin is exerted by the weight of the three coins on its

Weight of one coin $= mg$

Weight of three coins $= 3mg$

Hence, the force exerted on the 7th coin by the three coins on its top is 3mg. 1 acts vertically downward.

(b) Force on the seventh coin by the eighth coin is because of the weight of the coin and the other two coins (ninth and tenth) on its top.

Weight of the eighth coin $= mg$

Weight of the ninth coin $= mg$

Weight of the tenth coin $= mg$

Total weight of these three coins =

Hence, the force exerted on the 7th coin by the eighth coin is $3mg$. This vertically downward.

(c) The 6th 'coin experiences a downward force because of the weight of the (7th ,8th ,9th and 10th) on its top.

Therefore, the total downward force experienced by the 6th coin is 4mg.

As per Newton's third law of motion, the 6th coin will produce an equal reaction the 7th coin, but in the opposite direction. Hence, the reaction force of the 6th cc 7th coin is of magnitude 4*mg*. this force acts in the upward direction.

5.30 An aircraft executes a horizontal loop at a speed of 720 km/h with its wings banked at 15°. What is the radius of the loop ?

Answer

$$720 \times \frac{5}{18} = 200\, m/s$$

Speed of the aircraft, $v = 720\ km/h$

Acceleration due to gravity $g = 10\, m/s^2$

Angle of banking, $\theta = 15^o$

For radius r, of the loop, we have the relation:

$$\tan\theta = \frac{v^2}{rg}$$

$$r = \frac{v^2}{g\tan\theta}$$

$$= \frac{200 \times 200}{10 \times \tan 15} = \frac{4000}{0.268}$$

$$= 14925.37\ m$$

$$= 14.92\ km$$

5.31 A train runs along an unbanked circular track of radius 30 m at a speed of 54 km/h. The mass of the train is 106 kg. What provides the centripetal force required for this purpose — The engine or the rails ? What is the angle of banking required to prevent wearing out of the rail ?

Answer

Radius of the circular track, r =30 m

Speed of the train, v = 54 km/h = 15 m/s

Mass of the train, m = 10^6 kg

The centripetal force is provided by the lateral thrust of the rail on the wheel Newton's third law of motion, the wheel exerts an equal and opposite force or

This reaction force is responsible for the wear and rear of the rail

The angle of banking θ, is related to the radius (r) and speed (v) by the relation

$$\tan\theta = \frac{v^2}{rg}$$

$$= \frac{(15)^2}{3 \times 10} = \frac{225}{300}$$

$$\theta = \tan^{-1}(0.75) = 36.87^o$$

Therefore, the angle of banking is about 36.87^o.

5.32 A block of mass 25 kg is raised by a 50 kg man in two different ways as shown in Fig. 5.19. What is the action on the floor by the man in the two cases ? If the floor yields to a normal force

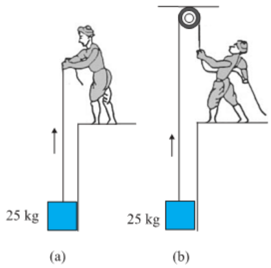


Fig. 5.19

Answer

750 N and 250 N in the respective cases; Method (b)

Mass of the block, *m* =25 kg

Mass of the man, *M* =50 kg

Acceleration due to gravity, g = 10 m/s^2

Force applied on the block, *F*= 25 X 10 = 250 N

Weight of the man, *W* = 50 X 10 = 500 N

<u>Case (a) : When the man lifts the block directly</u>

In this case the man applies a force in the upward direction. This increases his weight.

∴ Action on the floor by the man = 250 + 500 = 750 N

<u>Case (b) : When the man lifts the block using a pulley</u>

In this case, the man applies a force in the downward direction. This decreases apparent weight.

∴ Action on the floor by the man = 500 - 250 = 250 N

If the floor can yield to a normal force of 700 N, then the man should adopt the method to easily lift the block by applying lesser force.

5.33 A monkey of mass 40 kg climbs on a rope (Fig. 5.20) which can stand a maximum tension of 600 N. In which of the following cases will the rope break: the monkey
(a) climbs up with an acceleration of 6 m s^{-2}
(b) climbs down with an acceleration of 4 m s^{-2}
(c) climbs up with a uniform speed of 5 m s^{-1}
(d) falls down the rope nearly freely under gravity? (Ignore the mass of the rope).

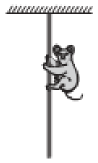


Fig. 5.20

Answer

Case (a)

Mass of the monkey, m = 40 kg

Acceleration due to gravity, g =10 m/s

Maximum tension that the rope can bear, T_{max} = 600 N

Acceleration of the monkey, a = 6 m/s^2 upward

Using Newton's second law of motion, we can write the equation of motion as:

$T - mg = ma$

$\therefore T = m(g + a)$

$= 40(10 + 6)$

$= 640\ N$

Since $T > T_{max}$ the rope will break in this case.

Case (b)

Acceleration of the monkey, a = 4 m/s^2 downward

Using Newton's second law of motion, we can write the equation of motion as:

$mg - T = ma$

$\therefore T = m(g - a)$

$= 40(10 - 4)$

$= 240\ N$

Since $T < T_{max}$ the rope will not break in this case.

Case (c)
The monkey is climbing with a uniform speed of 5 m/s. Therefore, its acceleration zero, i.e., a = 0.

Using Newton's second law of motion, we can write the equation of motion as:

$T - mg = ma$

$T - mg = 0$

$\therefore T = mg$

$= 40 \times 10$

$= 400\ N$

Since $T < T_{max}$ the rope will not break in this case.

Case (d)
When the monkey falls freely under gravity, it will acceleration become equal acceleration due to gravity, i.e., $a = g$

Using Newton's second law of motion, we can write the equation of motion as:

$mg - T = mg$

$\therefore T = m(g - g) = 0$

Since $T < T_{max}$ the rope will not break in this case.

5.34 Two bodies A and B of masses 5 kg and 10 kg in contact with each other rest on a table against a rigid wall (Fig. 5.21). The coefficient of friction between the bodies and the table is 0.15. A force of 200 N is applied horizontally to A. What are (a) the reaction of the partition (b) the action-reaction forces between A and B ? What happens when the wall is removed? Does the answer to (b) change, when the bodies are in motion? Ignore the difference between μ_s and μ_k.

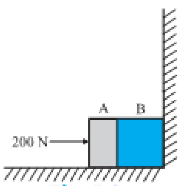


Fig. 5.21

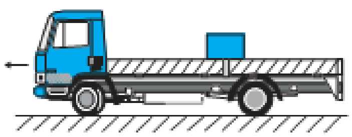


Fig. 5.22

Answer

(a) Mass of body $A, m_A = 5\ kg$

Mass of body $B, m_B = 10\ kg$

Applied force, $F = 200$ N

Coefficient of friction, $\mu_s = 0.15$

The force of friction is given by the relation:

$f_s = \mu\,(m_A + m_B)g$

$= 0.15\,(5 + 10) \times 10$

$= 1.5 \times 15 = 22.5$ N leftward

Net force acting on the partition = 200 – 22.5 = 177.5 N rightward

As per Newton's third law of motion, the reaction force of the partition will direction opposite to the net applied force.

Hence, the reaction of the partition will be 177.5 N, in the leftward direction.

(b) Force of the friction on mass A:

$f_A = \mu m_A g$

$= 0.15 \times \div 5 \times 10 = 7.5$ N leftward

Net force exerted by mass A on mass B = 200 - 7.5 = 192.5 N rightward

As per Newton's third law of motion, an equal amount of reaction force will be equal mass B on mass A, i.e., 192.5 N acting leftward.

When the wall is removed, the two bodies will move in the direction of the applied

Net force acting on the moving system = 177.5 N

The equation of motion for the system of acceleration a, can be written as:

Net force $= (m_A + m_B)a$

$$\therefore a = \frac{\text{Net force}}{m_A + m_B}$$

$$= \frac{177.5}{5 + 10} = \frac{177.5}{15} = 11.83 \ \text{m/s}^2$$

Net force causing mass A to move:

$F_A = m_A a$

$= 5 \times 11.83 = 59.15$ N

Net force exerted by mass A on mass B = 192.5 – 59.15 = 133.35 N

This force will act in the direction of motion. As per Newton's third law of motion equal amount of force will be exerted by mass B on mass A, i.e., 133.3 opposite to the direction of motion.

5.35 A block of mass 15 kg is placed on a long trolley. The coefficient of static friction between the block and the trolley is 0.18. The trolley accelerates from rest with 0.5 m s^{-2} for 20 s and then moves with uniform velocity. Discuss the motion of the block as viewed by (a) a stationary observer on the ground, (b) an observer moving with the trolley.

Answer

(a) Mass of the block, m = 15 kg

Coefficient of static friction, $\mu = 0.18$

Acceleration of the trolley, a = 0.5 m/s^2

As per Newton's second law of motion, the force (F) on the block caused by the of the trolley is given by the relation:

$F = ma = 15 \times 0.5 = 7.5$ N

This force is acted in the direction of motion of the trolley.

Force of static friction between the block and the trolley:

$f = \mu mg$

$= 0.18 \times 15 \times 10 = 27$ N

The force of static friction between the block and the trolley is greater than the external force. Hence, for an observer on the ground, the block will appear to be. When the trolley moves with uniform velocity there will be no applied extern. Only the force of friction will act on the block in this situation.

(b) An observer, moving with the trolley, has some acceleration. This is the case inertial frame of reference. The frictional force, acting on the trolley back opposed by a pseudo force of the same magnitude. However, this force ac opposite direction. Thus, the trolley will appear to be at rest for the observe with the trolley.

5.36 The rear side of a truck is open and a box of 40 kg mass is placed 5 m away from the open end as shown in Fig. 5.22. The coefficient of friction between the box and the surface below it is 0.15. On a straight road, the truck starts from rest and accelerates with 2 m s^{-2}. At what distance from the starting point does the box fall off the truck? (Ignore the size of the box).

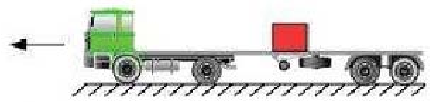


Fig. 5.22

Answer

Mass of the box, m = 40 kg

Coefficient of friction, $\mu = 0.15$

Initial velocity, $u = 0$

Acceleration, $a = 2 \text{ m/s}^2$

Distance of the box 'from the end of the truck, $s' = 5\, m$

As per Newton's second law of motion, the force on the box caused by the ac motion of the truck is given by:

$F = ma$

$= 10 \times 2 = 80$ N

As per Newton's third law of motion, a reaction force of 80 N is acting on the backward direction. The backward motion of the box is opposed by the force of acting between the box and the floor of the truck. This force is given by:

$f = \mu mg$

$= 0.15 \times 40 \times 10 = 60$ N

∴ Net force acting on the block:

$F_{net} = 80 - 60 = 20$ N backward

The backward acceleration produced in the box is given by:

$$a_{back} = \frac{F_{net}}{m} = \frac{20}{40} = 0.5 \ m/s^2$$

Using the second equation of motion, time *t* can be calculated as:

$$s' = ut + \frac{1}{2} a_{back} t^2$$

$$5 = 0 + \frac{1}{2} \times 0.5 \times t^2$$

$$\therefore \ t = \sqrt{20} \text{ s}$$

Hence, the box will fall from the truck after $\sqrt{20}$ s from start.
The distance s, travelled by the truck in $\sqrt{20}$ s is given by the relation:

$$s = ut + \frac{1}{2} at^2$$

$$= 0 + \frac{1}{2} \times 2 \times \left(\sqrt{20}\right)^2$$

$= 20$ m

5.37 A disc revolves with a speed of $33\frac{1}{3}$ rev/min, and has a radius of 15 cm. Two coins are placed at 4 cm and 14 cm away from the centre of the record. If the co-efficient of friction between the coins and the record is 0.15, which of the coins will revolve with the record ?

Answer
Coin placed at 4 cm from the centre
Mass of each coin = m
Radius of the disc, r = 15 cm = 0.15 m
Frequency of revolution, v = $33\frac{1}{3}$ rev/min $= \frac{100}{3x60} = \frac{5}{9}\, rev/s$
Coefficient of friction, $\mu = 0.15$

In the given situation, the coin having a force of friction greater than or equal centripetal force provided by the rotation of the disc will revolve with the disc, not the case, then the coin will slip from the disc.

Coin placed at 4 cm:
Radius of revolution, r^1 = 4 cm = 0.04 m
Angular frequency, $\omega = 2\pi v$ $= 2x\frac{22}{7}\, x\, \frac{5}{9} = 3.49 s^{-1}$
Frictional force, f = $\mu mg = 0.15\, x\, m\, x\, 10 = 1.5m\, N$
Centripetal force on the coin:
$F_{cent.} = mr'\, \omega^2$
= m x 0.04 x $(3.49)^2$
= 0.49m N

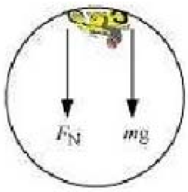
Since f > F_{cent}, the coin will revolve along with the record
<u>Coin placed at 14 cm</u>
Radius, r'' = 14 cm = 0.14 m
Angular frequency, $\omega = 3.49\, s^{-1}$
Frictional force, f = 1.5 m N
Centripetal force is given as :
F_{cent} = , $mr''\, \omega^2$
= m x 0.14 x $(3.49)^2$
= 1.7 m N

Since f < F_{cent}, the coin will slip from the surface of the record

5.38 You may have seen in a circus a motorcyclist driving in vertical loops inside a 'death-well' (a hollow spherical chamber with holes, so the spectators can watch from outside). Explain clearly why the motorcyclist does not drop down when he is at the uppermost point, with no support from below. What is the minimum speed required at the uppermost position to perform a vertical loop if the radius of the chamber is 25 m ?

Answer

In a death – well, motorcyclist does not fall at the top point of a vertical loop both the force of normal reaction and the weight of the motorcyclist act down are balanced by the centripetal force. This situation is shown in the following figure



The net force acting on the motorcyclist is the sum of the normal force (F_N) and due to gravity (F_g = mg).

The equation of motion for the centripetal acceleration a_c, can be written as :

$F_{net} = ma_c$

$F_N + F_g = ma_e$

$F_N + mg = \frac{mv^2}{r}$

Normal reaction is provided by the speed of the motorcyclist. At the minimum

(V_{min}), $F_N = 0$

$mg = \frac{m{v_{min}}^2}{r}$

$\therefore V_{min} = \sqrt{rg}$

$= \sqrt{25x10 = 15.8m/s}$

5.39 A 70 kg man stands in contact against the inner wall of a hollow cylindrical drum of radius 3 m rotating about its vertical axis with 200 rev/min. The coefficient of friction between the wall and his clothing is 0.15. What is the minimum rotational speed of the cylinder to enable the man to remain stuck to the wall (without falling) when the floor is suddenly removed ?

Answer

Mass the man, m = 70 kg

Radius of the drum, r = 3m

Coefficient of friction, $\mu = 0.15$

$= \frac{200}{60} = \frac{10}{3}\ rev/s$

Frequency of rotation, v=v 200 rev/min

The necessary centripetal force required for the rotation of the main is provide normal Force (F_N).

When the floor revolves, the man sticks to the wall of the drum. Hence, the weigh man (mg) acting downward is balanced by the frictional ($f = \mu F_N$) acting upward.

Hence, the man will not fall until:

$mg < f$

$mg < \mu F_{N=}$ umrω^2

$g < \mu r\omega^2$

$$\omega > \sqrt{\frac{g}{\mu r}}$$

The minimum angular speed is given as :

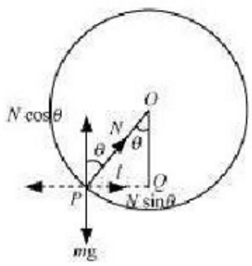
$$\omega_{min} = \sqrt{\frac{g}{\mu r}}$$

$$= \sqrt{\frac{10}{0.15x3}} = 4.71\, rad\, s^{-1}$$

5.40 A thin circular loop of radius R rotates about its vertical diameter with an angular frequency ω . Show that a small bead on the wire loop remains at its lowermost point for $\omega \le \sqrt{g/R}$. What is the angle made by the radius vector joining the centre to the bead with the vertical downward direction for $\omega = \sqrt{2g/R}$? Neglect friction.

Answer

Let the radius vector joining the bead with the centre make an angle θ with the downward direction



OP = R = Radius of the circle

N = Normal reaction

The respective vertical and horizontal equations of forces can be written as:

$mg = Ncos\theta$ (i)

$mlw^2 = Nsin\theta$ (ii)

In ΔOPQ, we have:

$$\sin\theta = \frac{1}{R}$$

$l = Rsin\theta$ ….. (iii)

Substituting equation (iii) in equation (ii), we get:

$m(Rsin\theta)\omega^2 = Nsin\theta$

$mR\,\omega^2 = N \ldots\ldots\ldots(iv)$

Substituting equation (iv) in equation (i), we get :

$mg = mR\,\omega^2\ \cos\theta$

$\cos\theta = \dfrac{g}{R\omega^2} \qquad \ldots.(v)$

Since $\cos\theta\ \le 1,$ the bread will remain at its lowermost point for $\frac{g}{R\omega^2} \le 1,$

$\omega \le \sqrt{\dfrac{g}{R}}$

For $\omega = \sqrt{\frac{2g}{R}}\ or\ \omega^2 = \frac{2g}{R}$ ………….(vi)

On equating equations (v) and (vi), we get :

$\dfrac{2g}{R} = \dfrac{g}{R\cos\theta}$

$\cos\theta = \dfrac{1}{2}$

$\therefore\ \theta = \cos^{-1}(0.5) = 60^0$

****

PROBLEMS FOR PRACTICE

CH 6

Work, Energy and Power

(11 Hours, 9 Marks (1M-1Q, 3M-1Q, 5MNP-1Q)

Syllabus : Work done by a constant force and a variable force; kinetic energy, work-energy theorem, power. Notion of potential energy, potential energy of a spring, conservative forces: conservation of mechanical energy (kinetic and potential energies); non-conservative forces: motion in a vertical circle, elastic and inelastic collisions in one and two dimensions.

6.1. Work done by a constant force and a variable force;

1. What is scalar product ?

The scalar product or dot product of any two vectors A and B, denoted as A.B (read A dot B) is defined as

$\vec{A}.\vec{B}$ = A B cos θ --------- (1) where θ is the angle between the two vectors.

Thus, scalar product of two vectors is equal to the product of magnitude of one vector with the magnitude of projection of second vector on the direction of the first vector.

Scalar product follows the commutative law : $\vec{A}.\vec{B} = \vec{B}.\vec{A}$

Scalar product obeys the distributive law : A. (B + C) = A.B + A.C

Further, A. (λ B) = λ (A.B) where λ is a real number.

In terms of unit vectors, scalar product is

$$\mathbf{A}\cdot\mathbf{B} = \left(A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} + A_z\hat{\mathbf{k}}\right)\cdot\left(B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}}\right)$$
$$= A_xB_x + A_yB_y + A_zB_z$$

From the definition of scalar product and, we have :

(i) $\mathbf{A}\cdot\mathbf{A} = A_xA_x + A_yA_y + A_zA_z$

Or, $A^2 = A_x^2 + A_y^2 + A_z^2$

since $\mathbf{A}\cdot\mathbf{A} = |\mathbf{A}|\,|\mathbf{A}| \cos 0 = A^2$.

(ii) $\mathbf{A}\cdot\mathbf{B} = 0$, if **A** and **B** are perpendicular.

6.2. Kinetic energy, work-energy theorem, power.

(1) Motivation for kinetic energy and work :

We know that the expression for rectilinear motion under constant acceleration **a** is : $v^2 + u^2 = 2as$ where u and v are the initial and final speeds and s the distance traversed. Multiplying both sides by m/2, we have $\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = mas = Fs$ ------ (1) where the last step follows from Newton's Second Law. We also know that $v^2 - u^2 = 2$ a.d,

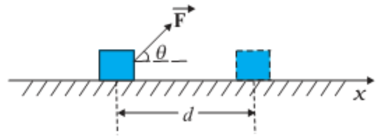
Multiplying both sides by m/2. We get, $\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = ma.d = F.d$ ------- (2)

The above equation provides a motivation for the definitions of work and kinetic energy. The left side of the equation is the difference in the quantity 'half the mass times the square of the speed' from its initial value to its final value. We call each of these quantities the **'kinetic energy'**, denoted by K. The right side is a product of the displacement and the component of the force along the displacement. This quantity is **called 'work'** and is denoted by W. Eq. (2) is then $K_f - K_i = W$ -------- (3), where K_i and K_f are respectively the initial and final kinetic energies of the object. Work refers to the force and the displacement over which it acts. Work is done by a force on the body over a certain displacement.

(2) Explain Work with examples ?

Work refers to the force and the displacement over which it acts. **Work is done by a force on the body over a certain displacement.**

Consider a constant force F acting on an object of mass m. The object undergoes a displacement d in the positive x-direction as shown in Fig. 6.2.



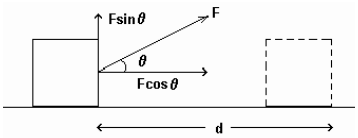


Fig. 6.2 An object undergoes a displacement d under the influence of the force F.

The work done by the force is defined to be the product of component of the force in the direction of the displacement and the magnitude of this displacement. Thus

W = (F cos θ).d = *F*.d --------- (6.4)

W = (the component of the force in the direction of displacement) x (the magnitude of displacement)

If there is no displacement, there is no work done even if the force is large. Thus, when you push hard against a rigid brick wall, the force you exert on the wall does no work. Yet your muscles are alternatively contracting and relaxing and internal energy is being used up and you do get tired. Thus, the meaning of work in physics is different from its usage in everyday language.

No work is done if :

(i) the displacement is zero as seen in the example above. A weightlifter holding a 150 kg mass steadily on his shoulder for 30 s does no work on the load during this time.

(ii) the force is zero. A block moving on a smooth horizontal table is not acted upon by a horizontal force (since there is no friction), but may undergo a large displacement.

(iii) the force and displacement are mutually perpendicular. This is so since, for θ = π/2 rad (= 90°), cos (π/2) = 0. For the block moving on a smooth horizontal table, the gravitational force mg does no work since it acts at right angles to the displacement. If we assume that the moon's orbits around the earth is perfectly circular then the earth's gravitational force does no work. The moon's instantaneous displacement is tangential while the earth's force is radially inwards and θ = π/2.

Work can be both **positive** and **negative**. If θ is between 0° and 90°, cos θ in Eq. (6.4) is positive.

If θ is between 90° and 180°, cos θ is negative.

In many examples the frictional force opposes displacement and θ = 180°. Then the work done by friction is negative (cos 180° = –1).

The dimension of both work and energy is $[ML^2T^{-2}]$. The SI unit of these is joule (J), named after the famous British physicist James Prescott Joule.

Example : A cyclist comes to a skidding stop in 10 m. During this process, the force on the cycle due to the road is 200 N and is directly opposed to the motion. (a) How much work does the road do on the cycle ? (b) How much work does the cycle do on the road ?

Answer : Work done on the cycle by the road is the work done by the stopping (frictional) force on the cycle due to the road.

(a) The stopping force and the displacement make an angle of 180° (π rad) with each other.

Thus, work done by the road, W_r = Fd cosθ = 200 × 10 × cos π = – 2000 J

It is this negative work that brings the cycle to a halt in accordance with WE theorem.

(b) From Newton's Third Law an equal and opposite force acts on the road due to the cycle. Its magnitude is 200 N. However, the road undergoes no displacement. Thus, work done by cycle on the road is zero.
In the above example it is understood that though the force on a body A exerted by the body B is always equal and opposite to that on B by A (Newton's Third Law); the work done on A by B is not necessarily equal and opposite to the work done on B by A.

(3) Explain Kinetic Energy ?

If an object of mass m has velocity v, its kinetic energy K is

$$K = \frac{1}{2}m\,\mathbf{v}.\mathbf{v} = \frac{1}{2}mv^2 \qquad \text{-------- (6.5)}$$

Kinetic energy is a scalar quantity. **The kinetic energy of an object is a measure of the work an object can do by the virtue of its motion.** This notion has been intuitively known for a long time. The kinetic energy of a fast flowing stream has been used to grind corn. Sailing ships employ the kinetic energy of the wind.

Example : In a ballistics demonstration a police officer fires a bullet of mass 50.0 g with speed 200 m s^{-1} on soft plywood of thickness 2.00 cm. The bullet emerges with only 10% of its initial kinetic energy. What is the emergent speed of the bullet ?

Answer : The initial kinetic energy of the bullet is $mv^2/2$ = 1000 J. It has a final kinetic energy of 0.1× 1000 = 100 J. If v_f is the emergent speed of the bullet, $\frac{1}{2}\,mv_f^2 = 100$ J and $v_f = \sqrt{\frac{2\times 100\ J}{0.05\ kg}}$ = 63.2 ms^{-1}

The speed is reduced by approximately 68% (not 90%).

(4) Obtain an expression for work done by a variable force ?

In practice, the constant force on an object is very rare. Most of the forces are variable forces. 'A force varying with position or time is known as the **variable force**.' Fig. 6.2 is a plot of a varying force in one dimension. If the displacement Δx is small, we can take the force F (x) as approximately constant and the work done is then ΔW = F (x) Δx.

This is illustrated in Fig. 6.3(a). Adding successive rectangular areas in Fig. 6.3(a) we get the total work done as

$$W \cong \sum_{x_i}^{x_f} F(x)\Delta x \qquad \text{------- (6.6)}$$

where the summation is from the initial position x_i to the final position x_f.

If the displacements are allowed to approach zero, then the number of terms in the sum increases without limit, but the sum approaches a definite value equal to the area under the curve in Fig. 6.3(b).

Then the work done is

$$W = \lim_{\Delta x \to 0} \sum_{x_i}^{x_f} F(x)\Delta x \qquad = \int_{x_i}^{x_f} F(x)\,dx \qquad \text{------- (6.7)}$$

where 'lim' stands for the limit of the sum when Δx tends to zero. Thus, for a varying force the work done can be expressed as a definite integral of force over displacement.

(5) State and prove Work-Energy Theorem ?

The time rate of change of kinetic energy is

$$\frac{dK}{dt} = \frac{d}{dt}\left(\frac{1}{2}m\,v^2\right) \quad = m\frac{dv}{dt}v \quad = F\,v \text{ (from Newton's Second Law)} \quad = F\frac{dx}{dt}$$

Thus dK = Fdx

Integrating from the initial position (x_i) to final position (x_f), we have

$$\int_{K_i}^{K_f} dK = \int_{x_i}^{x_f} F dx$$

where, K_i and K_f are the initial and final kinetic energies corresponding to x_i and x_f.

$$\text{or} \quad K_f - K_i = \int_{x_i}^{x_f} F dx \quad \text{--------- (6.8a)}$$

From Eq. (6.7), it follows that $\mathbf{K_f - K_i = W}$ --------- (6.8b)

Thus, the WE theorem is proved for a variable force.

While the WE theorem is useful in a variety of problems, it does not, in general, incorporate the complete dynamical information of Newton's Second Law. It is an integral form of Newton's second law. Newton's second law is a relation between acceleration and force at any instant of time. Work-energy theorem involves an integral over an interval of time. In this sense, the temporal (time) information contained in the statement of Newton's second law is 'integrated over' and is not available explicitly. Another observation is that Newton's second law for two or three dimensions is in **vector form** whereas the **work-energy theorem is in scalar form**.

(6) Explain the concept of power and obtain an expression for it ?

Power is defined as the time rate at which work is done or energy is transferred. The average power of a force is defined as the ratio of the work, W, to the total time t taken, $P_{av} = \frac{W}{t}$.

The instantaneous power is defined as the limiting value of the average power as time interval approaches zero, P = $\frac{dW}{dt}$ ------- (a)

The work dW done by a force F for a displacement d**r** is dW = **F**.d**r**. The instantaneous power can also be expressed as $P = F\frac{dr}{dt}$ $= Fv$ ----- (c)

where v is the instantaneous velocity when the force is F.

Power, like work and energy, is a scalar quantity. Its dimensions are $[ML^2T^{-3}]$. In the SI, its unit is called a watt (W). The watt is 1 J s^{-1}. The unit of power is named after James Watt, one of the innovators of the steam engine in the eighteenth century.

There is another unit of power, namely the horse-power (hp), 1 hp = 746 W. This unit is still used to describe the output of automobiles, motorbikes, etc.

We encounter the unit watt when we buy electrical goods such as bulbs, heaters and refrigerators. A 100 watt bulb which is on for 10 hours uses 1 kilowatt hour (kWh) of energy. 100 (watt) × 10 (hour) = 1000 watt hour = 1 kilowatt hour (kWh) = 103 (W) × 3600 (s) = 3.6 × 106 J

Our electricity bills carry the energy consumption in units of kWh. Note that kWh is a unit of energy and not of power.

6.3. Notion of potential energy, potential energy of a spring,

(7) Explain potential energy. Obtain an expression for potential energy under gravity ?

Potential energy is the 'stored energy' by virtue of the position or configuration of a body. The body left to itself releases this stored energy in the form of kinetic energy.

The gravitational force on a ball of mass m is mg. g may be treated as a constant near the earth surface. By 'near' we imply that the height h of the ball above the earth's surface is very small compared to the earth's radius R_E ($h \ll R_E$) so that we can ignore the variation of g near the earth's surface. In what follows we have taken the upward direction to be positive. Let us raise the ball up to a height h. The work done by the external agency against the gravitational force is mgh. This work gets stored as potential energy.

Gravitational potential energy of an object, as a function of the height h, is denoted by V(h) and it is the negative of work done by the gravitational force in raising the object to that height. V (h) = mgh

If h is taken as a variable, it is easily seen that the gravitational force F equals the negative of the derivative of V(h) with respect to h. Thus,

$$F = -\frac{\mathrm{d}}{\mathrm{d}h}V(h) = -m\,g$$

The negative sign indicates that the gravitational force is downward. When released, the ball comes down with an increasing speed.

Just before it hits the ground, its speed is given by the kinematic relation, $v^2 = 2gh$

This equation can be written as $\frac{1}{2}mv^2$ **= mgh** **---------** (6.8)

which shows that the gravitational potential energy of the object at height h, when the object is released, manifests itself as kinetic energy of the object on reaching the ground.

Mathematically, (for simplicity, in one dimension) the potential energy V(x) is defined if the force F(x) can be written as

$$F(x) = -\frac{\mathrm{d}V}{\mathrm{d}x}$$

This implies that

$$\int_{x_i}^{x_f} F(x)\mathrm{d}x = -\int_{V_i}^{V_f} \mathrm{d}V = V_i - V_f$$

The work done by a conservative force such as gravity depends on the initial and final positions only. In the previous chapter we have worked on examples dealing with inclined planes. If an object of mass m is released from rest, from the top of a smooth (frictionless) inclined plane of height h, its speed at the bottom is $\sqrt{2gh}$ irrespective of the angle of inclination.

Thus, at the bottom of the inclined plane it acquires a kinetic energy, mgh. If the work done or the kinetic energy did depend on other factors such as the velocity or the particular path taken by the object, the force would be called non-conservative.

The dimensions of potential energy are $[ML^2T^{-2}]$ and the unit is joule (J), the same as kinetic energy or work. To reiterate, the change in potential energy, for a conservative force, ΔV is equal to the negative of the work done by the force **ΔV = − F(x) Δx** -------- (6.9)

(8) Using the concept of potential energy, explain the principle of conservation of mechanical energy ?

Suppose that a body undergoes displacement Δx under the action of a conservative force F. Then from the WE theorem we have,

ΔK = F(x) Δx

If the force is conservative, the potential energy function V(x) can be defined such that −ΔV = F(x) Δx

The above equations imply that $\Delta K + \Delta V = 0$
$\Delta(K + V) = 0$ ---------- (6.10)

which means that K + V, the sum of the kinetic and potential energies of the body is a constant.
Over the whole path, x_i to x_f, this means that $K_i + V(x_i) = K_f + V(x_f)$ ------------ (6.11)
The quantity K + V(x), is called the total mechanical energy of the system. Individually the kinetic energy K and the potential energy V(x) may vary from point to point, but the sum is a constant. The aptness of the term 'conservative force' is now clear.
Let us consider some of the definitions of a conservative force.

- A force F(x) is conservative if it can be derived from a scalar quantity V(x) by the relation given by Eq. (6.9).
- The work done by the conservative force depends only on the end points. This can be seen from the relation, $W = K_f - K_i = V(x_i) - V(x_f)$ which depends on the end points.
- A third definition states that the work done by this force in a closed path is zero. This is once again apparent from Eq. (6.11) since $x_i = x_f$.

Thus, the principle of conservation of total mechanical energy can be stated as The total mechanical energy of a system is conserved if the forces, doing work on it, are conservative.

The above discussion can be made more concrete by considering the example of the gravitational force once again. Fig. 6.5 depicts a ball of mass m being dropped from a cliff of height H.

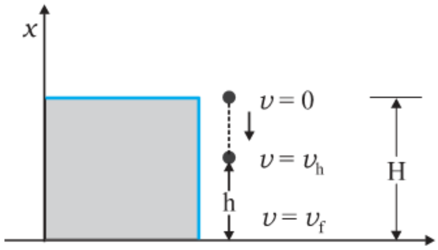


Fig. 6.5 The conversion of potential energy to kinetic energy for a ball of mass m dropped from a height H.

The total mechanical energies E_0, E_h, and E_H of the ball at the indicated heights zero (ground level), h and H, are

$$E_H = mgH \qquad (6.11\ a)$$

$$E_h = mgh + \frac{1}{2}mv_h^2 \qquad (6.11\ b)$$

$$E_0 = (1/2)\ mv^2 \qquad (6.11\ c)$$

The constant force is a special case of a spatially dependent force F(x). Hence, the mechanical energy is conserved. Thus $E_H = E_0$

$$\text{or,} \quad mgH = \frac{1}{2}mv_f^2$$

$$v_f = \sqrt{2gH}$$

a result that was obtained for a freely falling body.
Further, $E_H = E_h$ which implies, $v_h^2 = 2g(H - h)$ -------------- (6.11 d)
and is a familiar result from kinematics.
At the height H, the energy is purely potential. It is partially converted to kinetic at height h and is fully kinetic at ground level. This illustrates the conservation of mechanical energy.

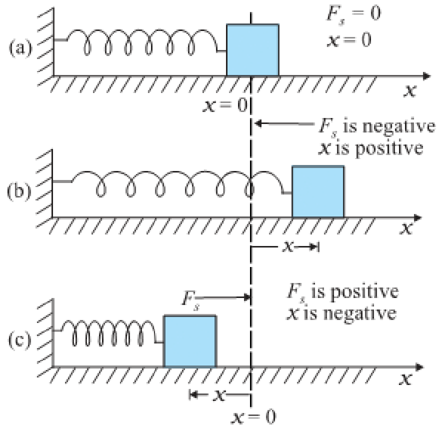
(9) Obtain an expression for potential energy of a spring ?

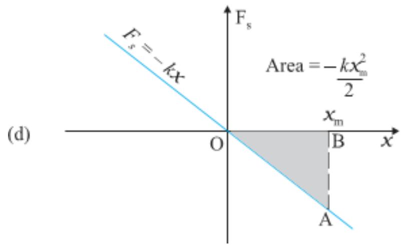
The spring force is an example of a variable force which is conservative.

Fig. 6.7 shows a block attached to a spring and resting on a smooth horizontal surface. The other end of the spring is attached to a rigid wall. The spring is light and may be treated as massless. In an ideal spring, the spring force F_s is proportional to x where x is the displacement of the block from the equilibrium position. The displacement could be either positive [Fig. 6.7(b)] or negative [Fig. 6.7(c)]. This force law for the spring is called Hooke's law and is mathematically stated as

$F_s = - kx$

The constant k is called the spring constant. Its unit is N m^{-1}. The spring is said to be stiff if k is large and soft if k is small.





Suppose that we pull the block outwards as in Fig. 6.7(b). If the extension is x_m, the work done by the spring force is

$$W_s = \int_0^{x_m} F_s \, dx = -\int_0^{x_m} kx \, dx = -\frac{k x_m^2}{2} \qquad \text{---------} \ (6.15)$$

Note that the work done by the external pulling force F is positive since it overcomes the spring force.

$$W = +\frac{k x_m^2}{2} \qquad \text{-------} \ (6.16)$$

The same is true when the spring is compressed with a displacement x_c (< 0). The spring force does work W_s = - $kx_c^2/2$ while the external force F does work + $kx_c^2/2$. If the block is moved from an initial displacement x_i to a final displacement x_f , the work done by the spring force W_s is

$$W_s = -\int_{x_i}^{x_f} k\,x\,dx = \frac{k x_i^2}{2} - \frac{k x_f^2}{2}$$

Thus the work done by the spring force depends only on the end points. Specifically, if the block is pulled from x_i and allowed to return to x_i ;

$$W_s = -\int_{x_i}^{x_i} k\,x\,dx = \frac{k x_i^2}{2} - \frac{k x_i^2}{2} = 0$$

The work done by the spring force in a cyclic process is zero. We have explicitly demonstrated that the spring force (i) is position dependent only as first stated by Hooke, ($F_s = - kx$); (ii) does work which only depends on the initial and final positions, e.g. Eq. (6.17). Thus, the spring force is a conservative force.

We define the potential energy V(x) of the spring to be zero when block and spring system is in the equilibrium position. For an extension (or compression) x the above analysis suggests that

$$V(x) = \frac{kx^2}{2}$$

If the block of mass m in Fig. 6.7 is extended to x_m and released from rest, then its total mechanical energy at any arbitrary point x, where x lies between $-x_m$ and $+x_m$, will be given by

$$\frac{1}{2}k\,x_m^2 = \frac{1}{2}k\,x^2 + \frac{1}{2}m\,v^2$$

where we have invoked the conservation of mechanical energy. This suggests that the speed and the kinetic energy will be maximum at the equilibrium position, x = 0, i.e.,

$$\frac{1}{2}m\,v_m^2 = \frac{1}{2}k\,x_m^2$$

where v_m is the maximum speed.

$$\text{or} \qquad v_m = \sqrt{\frac{k}{m}}\,x_m$$

Note that k/m has the dimensions of $[T^{-2}]$ and our equation is dimensionally correct. The kinetic energy gets converted to potential energy and vice versa, however, the total mechanical energy remains constant. This is graphically depicted in Fig. 6.8.

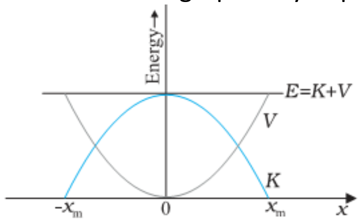


Fig. 6.8 Parabolic plots of the potential energy V and kinetic energy K of a block attached to a spring obeying Hooke's law. The two plots are complementary, one decreasing as the other increases. The total mechanical energy E = K + V remains constant.

Summary :

(i) Information on time is absent from the above discussions. In the example considered above, we can calculate the compression, but not the time over which the compression occurs. A solution of Newton's Second Law for this system is required for temporal information.

(ii) Not all forces are conservative. Friction, for example, is a non-conservative force. The principle of conservation of energy will have to be modified in this case.

(iii) The zero of the potential energy is arbitrary. It is set according to convenience. For the spring force we took V(x) = 0, at x = 0, i.e. the unstretched spring had zero potential energy. For the constant gravitational force mg, we took V = 0 on the earth's surface.

6.4. Conservative forces: conservation of mechanical energy (kinetic and potential energies);

(10) Deduce principle of conservation of total mechanical energy with an illustration ?

Suppose that a body undergoes displacement Δx under the action of a conservative force F. Then from the WE theorem we have, $\Delta K = F(x)\,\Delta x$

If the force is conservative, the potential energy function V(x) can be defined such that $-\Delta V = F(x)\,\Delta x$.

The above equations imply that

$\Delta K + \Delta V = 0$

$\Delta(K + V) = 0$

which means that K + V, the sum of the kinetic and potential energies of the body is a constant. Over the whole path, x_i to x_f, this means that $K_i + V(x_i) = K_f + V(x_f)$ --------- (6.11)

The quantity K + V(x), is called the total mechanical energy of the system. Individually the kinetic energy K and the potential energy V(x) may vary from point to point, but the sum is a constant.

Let us consider some of the definitions of a conservative force.

- A force F(x) is conservative if it can be derived from a scalar quantity V(x) by the relation given by Eq. (6.9).
- The work done by the conservative force depends only on the end points. This can be seen from the relation, $W = K_f - K_i = V(x_i) - V(x_f)$ which depends on the end points.
- A third definition states that the work done by this force in a closed path is zero. This is once again apparent from Eq. (6.11) since $x_i = x_f$.

Thus, the principle of conservation of total mechanical energy can be stated as The total mechanical energy of a system is conserved if the forces, doing work on it, are conservative.

Illustration : Consider a Fig. 6.5 which depicts a ball of mass m being dropped from a cliff of height H.

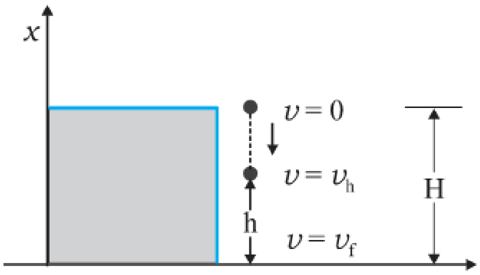


Fig. 6.5 The conversion of potential energy to kinetic energy for a ball of mass m dropped from a height H.

The total mechanical energies E_0, E_h, and E_H of the ball at the indicated heights zero (ground level), h and H, are

$E_H = mgH$ (6.11 a)

$E_h = mgh + \frac{1}{2}mv_h^2$ (6.11 b)

$E_0 = (1/2)\, mv^2$ (6.11 c)

The constant force is a special case of a spatially dependent force F(x). Hence, the mechanical energy is conserved. Thus $E_H = E_0$ or $mgH = \frac{1}{2}mv_f^2$ or $v_f = \sqrt{2gH}$

Further, $E_H = E_h$ which implies, $v_h^2 = 2g(H-h)$ ---------- (6.11 d)

and is a familiar result from kinematics. At the height H, the energy is purely potential. It is partially converted to kinetic at height h and is fully kinetic at ground level. This illustrates the conservation of mechanical energy.

6.5. Non-conservative forces: motion in a vertical circle, elastic and inelastic collisions in one and two dimensions :

(11) What are elastic and inelastic collisions. Obtain the expression for final velocities of two bodies colliding in one dimension?

Consider two masses m_1 and m_2. The particle m_1 is moving with speed v_{1i}, the subscript 'i' implying initial. We can cosider m_2 to be at rest. In this situation the mass m_1 collides with the stationary mass m_2 and this is depicted in Fig. 6.10.

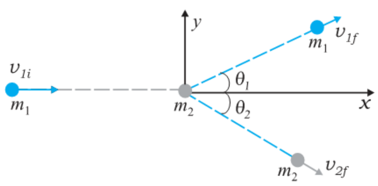


Fig. 6.10 Collision of mass m_1, with a stationary mass m_2.

In all collisions the total linear momentum is conserved; the initial momentum of the system is equal to the final momentum of the system. When two objects collide, the mutual impulsive forces acting over the collision time Δt cause a change in their respective momenta :

$\Delta p_1 = F_{12}\,\Delta t$

$\Delta p_2 = F_{21}\,\Delta t$

where F_{12} is the force exerted on the first particle by the second particle. F_{21} is likewise the force exerted on the second particle by the first particle. Now from Newton's Third Law, $F_{12} = -F_{21}$. This implies $\Delta p_1 + \Delta p_2 = 0$

Since the third law is true at every instant, the total impulse on the first object is equal and opposite to that on the second. On the other hand, the total kinetic energy of the system is not necessarily conserved. The impact and deformation during collision may generate heat and sound. Part of the initial kinetic energy is transformed into other forms of energy. A useful way to visualise the deformation during collision is in terms of a 'compressed spring'. If the 'spring' connecting the two masses regains its original shape without loss in energy, then the initial kinetic energy is equal to the final kinetic energy but the kinetic energy during the collision time Δt is not constant. Such a collision is called an **elastic collision**. On the other hand the deformation may not be relieved and the two bodies could move together after the collision. A collision in which the two particles move together after the collision is called a completely inelastic collision. The intermediate case where the deformation is partly relieved and some of the initial kinetic energy is lost is more common and is appropriately called an **inelastic collision**.

Collisions in One Dimension :

Consider first a completely inelastic collision in one dimension. Then, in Fig. 6.10,

$$\theta_1 = \theta_2 = 0$$

$$m_1 v_{1i} = (m_1 + m_2) v_f \quad \text{(momentum conservation)}$$

$$v_f = \frac{m_1}{m_1 + m_2} v_{1i} \qquad (6.23)$$

The loss in kinetic energy on collision is

$$\Delta K = \frac{1}{2} m_1 v_{1i}^2 - \frac{1}{2}(m_1 + m_2) v_f^2$$

$$= \frac{1}{2} m_1 v_{1i}^2 - \frac{1}{2} \frac{m_1^2}{m_1 + m_2} v_{1i}^2 \quad \text{[using Eq. (6.23)]}$$

$$= \frac{1}{2} m_1 v_{1i}^2 \left[1 - \frac{m_1}{m_1 + m_2}\right] \qquad = \frac{1}{2} \frac{m_1 m_2}{m_1 + m_2} v_{1i}^2$$

which is a positive quantity as expected.

Consider next an elastic collision. Using the above nomenclature with $\theta_1 = \theta_2 = 0$, the momentum and kinetic energy conservation equations are

$$m_1 v_{1i} = m_1 v_{1f} + m_2 v_{2f} \qquad (6.24)$$

$$m_1 v_{1i}^2 = m_1 v_{1f}^2 + m_2 v_{2f}^2 \qquad (6.25)$$

From Eqs. (6.24) and (6.25) it follows that,

$$m_1 v_{1i}(v_{2f} - v_{1i}) = m_1 v_{1f}(v_{2f} - v_{1f})$$

or, $$v_{2f}(v_{1i} - v_{1f}) = v_{1i}^2 - v_{1f}^2$$

$$= (v_{1i} - v_{1f})(v_{1i} + v_{1f})$$

Hence, $\therefore\ v_{2f} = v_{1i} + v_{1f}$ -------(6.26)

Substituting this in Eq. (6.24), we obtain

$$v_{1f} = \frac{(m_1 - m_2)}{m_1 + m_2} v_{1i} \qquad (6.27)$$

and $$v_{2f} = \frac{2 m_1 v_{1i}}{m_1 + m_2} \qquad (6.28)$$

Thus, the 'unknowns' $\{v_{1f}, v_{2f}\}$ are obtained in terms of the 'knowns' $\{m_1, m_2, v_{1i}\}$. Special cases of our analysis are interesting.

Case I : If the two masses are equal $v_{1f} = 0$ $v_{2f} = v_{1i}$
The first mass comes to rest and pushes off the second mass with its initial speed on collision.
Case II : If one mass dominates, e.g. $m_2 >> m_1$ $v_{1f} \approx -v_{1i}$ $v_{2f} \approx 0$
The heavier mass is undisturbed while the lighter mass reverses its velocity.

(12) Describe briefly the collisions in two-dimension?

In Fig. 6.10 the collision of a moving mass m_1 with the stationary mass m_2. Linear momentum is conserved in such a collision. Since momentum is a vector this implies three equations for the three directions {x, y, z}. Consider the plane determined by the final velocity directions of m_1 and m_2 and choose it to be the x-y plane. The conservation of the z-component of the linear momentum implies that the entire collision is in the x-y plane. The x- and y-component equations are

$$m_1 v_{1i} = m_1 v_{1f} \cos\theta_1 + m_2 v_{2f} \cos\theta_2 \qquad (6.29)$$

$$0 = m_1 v_{1f} \sin\theta_1 - m_2 v_{2f} \sin\theta_2 \qquad (6.30)$$

One knows $\{m_1, m_2, v_{1i}\}$ in most situations. There are thus four unknowns $\{v_{1f}, v_{2f}, \theta_1$ and $\theta_2\}$, and only two equations. If $\theta_1 = \theta_2 = 0$, we regain Eq. (6.24) for one dimensional collision.
If, further the collision is elastic,

$$\frac{1}{2} m_1 v_{1i}{}^2 = \frac{1}{2} m_1 v_{1f}{}^2 + \frac{1}{2} m_2 v_{2f}{}^2 \qquad (6.31)$$

We obtain an additional equation. That still leaves us one equation short. At least one of the four unknowns, say θ_1, must be made known for the problem to be solvable. For example, θ_1 can be determined by moving a detector in an angular fashion from the x to the y axis. Given $\{m_1, m_2, v_{1i}, \theta_1\}$ we can determine $\{v_{1f}, v_{2f}, \theta_2\}$ from Eqs. (6.29)-(6.31).

In our everyday world, collisions take place only when two bodies touch each other. But consider a comet coming from far distances to the sun, or alpha particle coming towards a nucleus and going away in some direction. Here we have to deal with forces involving action at a distance. Such an event is called **scattering.** The velocities and directions in which the two particles go away depend on their initial velocities as well as the type of interaction between them, their masses, shapes and sizes.

One mark questions

1. Define scalar product of two vectors.
The scalar product of to vectors is defined as the product of the magnitudes of the two vectors and the cosine of the angle between them.

2. Write the mathematical equation for dot product of two vectors.
If A and B are the two vectors, and j is the angle between them, then their dot product is given by
$\vec{A} \cdot \vec{B}$ = AB cosθ

3. What is the condition for the two vectors to be perpendicular to each other.
The two vectors are perpendicular to each other, if their dot product is zero i.e., if $\vec{A} \cdot \vec{B}$ = 0, then A and B are perpendicular to each other.

4. If $\vec{A} \cdot \vec{B}$ = 0, then what is the angle between $\vec{A}$ & $\vec{B}$
If $\vec{A} \cdot \vec{B}$ = 0 ; then the angle between them is 90°

5. In the equation $\vec{A} \cdot \vec{B}$ = A(B cos θ), what does 'B cos θ' represent?
It is the projection of B along A

6. If $\vec{A} \cdot \vec{B}$ =AB, then what is the angle between $\vec{A}$ & $\vec{B}$
The angle between $\vec{A}$ and $\vec{B}$ is zero (0). (Two vectors are parallel)

7. If $\vec{A} \cdot \vec{B}$ = -AB, then what is the angle between $\vec{A}$ & $\vec{B}$
The angle between A and B is 180° (Two vectors are antiparallel)

8. What is the value of scalar product of a vector with itself ?
Square of its magnitude ($\vec{A} \cdot \vec{A}$ = A A cos0 = A^2)

9. What is the value of dot product of unit vector with itself ?
The dot product of unit vector with itself is unity
$\hat{\imath} \cdot \hat{\imath} = 1$ or $\hat{\jmath} \cdot \hat{\jmath} = 1$ ($\theta = 0$)

10.Define work –energy theorem.
It states that workdone by a force on a body is equal to the change in its kinetic energy.

11.What do you mean by work done by a force?
Work is said to be done when a force applied on a body displaces it through a certain distance.

12.What is the work done by the tension in the string of a simple pendulum.
Zero (Tension (force) and displacement of a bob are perpendicular to each other)

13. Mention the dimensions of the work done .
[W] = [ML^2 T^{-2}]

14. What is the nature of the work done by frictional force.

Negative (The motion is opposed by the frictional force)

15. Define the term energy.
The capacity to do work is called energy

16. What is the nature of the work done by applied force.
Positive

17. Define Kinetic energy of a body.
The ability of a body to do work due to its motion is called kinetic energy.

18. What type of energy possessed by a man standing in a moving train?
Kinetic energy

19. What dose the area under ' force-displacement' curve represent.
Work done

20. Define potential energy of a body.
The ability of the body to do work due to its configuration or position is called potential energy.

21. Out of joule, calorie, kilowatt and electron volt, which one is not the unit of energy ?
Kilowatt

22. Can potential energy of on object be negative?
Yes, It is negative, when forces involved are attractive

23. If an object of mass 'm' is released from rest form the top of a frictionless inclined plane of height 'h' what is its speed at the bottom of the inclined plane.
$\sqrt{2gh}$; g = acceleration due to gravity

24. Whether the spring force is conservative or non –conservative?
Conservative force

25. Mention the S.I unit of spring constant.
Its unit is Newton per metre (Nm^{-1})

26. If the spring constant of a given spring is large, what it represent?
The spring is said to be stiff

27. If the spring constant of a given spring is small, what it represent?
The spring is said to be soft (or smooth)

28. Mention the expression for the work done by a spring force.
$W_s = -\frac{1}{2} kx^2$: K = spring constant, X = extension produced in the spring

29. What is the energy associated with 1 kg of mass.
Energy associated with 1 kg can be calculated using the
relation $E = mc^2$ $= 1 \text{ kg} \times (3\times10^8)^2$ $E = 9 \times 10^{16}$ J

30. What type of nuclear reaction takes place in nuclear power plant?
Controlled nuclear fission reaction

31. What type of nuclear reaction takes place in nuclear weapons
Un controlled nuclear fission reaction

32. How does an arrow gains K.E, when it is shot from a bow?
It gains K.E. From the configuration of the bow or P.E. of the bow

33. What kind of energy transformation take place at a thermoelectric power station?
The heat energy is converted into electrical energy

34. Which type of energy is responsible for the formation of molecules form the atoms and polymers from the molecules.
Chemical energy

35. What is mass 'defect'?
The difference between the sum of the masses of the nucleons forming the nucleus and rest mass of the nucleus is called ' mass defect'.

36. State the law of conservation of energy.
Energy can neither be created, nor destroyed i.e., the total energy of an isolated system remains constant.

37. What is power?
The time rate at which work is done or energy transferred is called power.

38. What is average power?
The ratio of the work (W) to the total time taken (t) is called average power.

$P_{av} = W/t$

39. What is instantaneous power?
The limiting value of the average power when time tends to zero is called instantaneous power

It is given by $P = dw/dt$

40. Give the practical unit of power.
The practical unit of power is horse power (Hp)

41. What is the unit used to describe the out put of automobiles and motorbikes?
Horse power (hp) (1 hp = 746 watt)

42. Convert 1.K.Wh in joule.
1 Kwh = 1000 x 60 x 60 watt S

$= 10^3 \times 3600$ Joule x S

1 Kwh = 3.6 x 106 J.S

43. The energy associated with the daily food intake of a human adult is 10^7 J express it in Kilo calories.
Given average human consumption in a day = 10^7 J

energy consumption in K. Calorie = $10^7/4.2 = 0.238 \times 10^7$cal

$= 0.24 \times 10^7$ cal

= 2400×10^3 cal

(i.e. 1 calorie = 4.2 J) 10^7 J =2400 K cal

44. What is elastic collision?

The collision in which both the momentum and kinetic energy of the system remains conserved is called elastic collision.

45. What is inelastic collision ?

The collision in which only momentum is conserved but kinetic energy is not conserved is called inelastic collision.

46. Give an example for elastic collision.

Examples : Collision of atoms, collision of subatomic particles like proton and electron, collision of molecules of gas (any one)

47. Give an example for inelastic collision.

Examples : Collision of mud on the wall, bullet striking a block of wood collision of plastic bodies (any one)

48. What is perfectly inelastic collision?

If two bodies stick together after colliding, the collision is perfectly inelastic

OR

A collision in which the two particles move together after the collision is called perfectly inelastic collision.

49. In which type of collision mechanical energy is not transformed into any other form of energy?

Elastic collision

50. In which type of collision whole mechanical energy may be transformed into other form?

Inelastic collision

51. What is head on collision?

If the initial velocities and final velocities of both the colliding bodies are along the same straight line then it is called head-on collision (one dimensional collision)

Two mark questions

1. Mention the two types of multiplication of vectors.
   i) Scalar product
   ii) Vector product

2. Explain how commutative law holds good in dot product.

Form definition $\vec{A}\,.\,\vec{B}$ = AB cosθ

$\vec{B}\,.\,\vec{A}$ = BA cosθ = ABcosθ

Hence $\vec{A}\,.\,\vec{B} = \vec{B}\,.\,\vec{A}$

Hence commutative law holds good in dot product

3. Explain how distributive law holds good in dot product.

If A. B & C are the vectors then

A.(B+c) = A . B + A . C

Hence distributive law holds good for dot product .

4. Find the value of 'n' so that vector (4i-6j+2k) may be perpendicular to the vector (6î+8j+nk)

Let $\vec{A}=4\hat{i}-6\hat{j}+2\hat{k}$

$\vec{B}=6\hat{i}+8\hat{j}+n\hat{k}$

Two vectors perpendicular to each other only If $\vec{A} \cdot \vec{B}=O$

$(4\hat{i}-6\hat{j}+2\hat{k}) \cdot (6\hat{I} + 8\hat{j} + n\hat{k}) = 0$

$24\, \hat{i}.\hat{i} - 48\, \hat{j}.\hat{j} + 2n\, \hat{k}.\hat{k} = 0$ ($\hat{i}.\hat{j} = \hat{j}.\hat{k} = \hat{k}.\hat{i} = 0$)

$24 - 48 + 2n = 0$ ($\hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k} = 1$)

$-24 + 2n = 0$

$2n = 24$

$n= 12$

5. Find the angle between the vectors A=2i-4j-5k and B =2i+2j-4k

WKT $\vec{A} \cdot \vec{B} = AB\cos\theta$ and $\vec{A} \cdot \vec{A} = A^2$ and $\vec{B} \cdot \vec{B} = B^2$

$$\cos\theta = \frac{\vec{A} \cdot \vec{B}}{AB}$$

$\vec{A} \cdot \vec{B} = 2 \times 2 + 2\times(-4) + (-4) \times (-5)$

$= 4 - 8 + 20$

$= -4 + 20$

$\vec{A}.\vec{B}. = 16$

$A^2 = \vec{A}. \vec{A}.$

$= 2x2+(-4)x(-4) + (-5)x(-5)$

$= 4 + 16 + 25 = 45$

$A = \sqrt{45}$

$B^2 = \vec{B} \cdot \vec{B}$

$= 2x2+(2)(2) + (-4)(-4)$

$= 4+4+16$

$= 24$

$B = \sqrt{24}$

$$\cos\theta = \frac{16}{\sqrt{45}\sqrt{24}} = \frac{16}{6.7082 \times 4.8989}$$

$$= \frac{16}{32.8628}$$

$\cos\theta = 0.4868$

$\theta = \cos(0.4868)$

$\theta = 60^{\circ} 52^{1}$

6. Define work done by the force.what is value of work done by the centripetal force?

The work done by the force is defined as the product of component of the force along the direction of the displacement and the magnitude of the displacement, Work done by the centripetal force is zero,

(force and displacement are perpendicular to each other)

7. Under what conditions the work done by a force is maximum and minimum?

Work done by a force is maximum, when the force and displacements are in the same direction ($\theta=0°$) and minimum when they are perpendicular to each other ($\theta=90°$)

8. State any two conditions under which a force does no work.

A force does no work when
[i] The Displacement is Zero.
[iii] The displacement is perpendicular to the direction of force.

9. Name the largest and smallest practical unit of enegry.

Largest practical unit is kilowatt hour [K wh]
Smallest practical unit is electron volt (eV)

10. What is non conservative force ? Give an example.

If the work done by the force depends on the path fallowed by the body is called non-conservative force Ex: frictional force.

11. What is conservative force ?Give an example.

If the work done by the force depends only on the initial and final potations of the body.
Ex: Gravitational force.

12. Write down the expression for spring force and explain the terms.

Spring force $F_s = - Kx$ Where K = - Spring constant
X = - displacement from the equilibrium position

13. If $A=Ax_i + Ay_j + Az_k$ and $B = Bx_i + By_j + Bz_k$ are the two vectors in rectangular components, then find out their scalar product and find out

$$\vec{A} \cdot \vec{B} = (Ax\hat{i} + Ay\hat{j} + Az\hat{k}) \cdot (Bx\hat{i} + By\hat{j} + Bz\hat{k})$$

$$= Ax\,Bx\,\hat{i}.\hat{i} + Ay\,By\,\hat{j}.\hat{j} + Az\,Bz\,\hat{k}.\hat{k}$$

$$\vec{A}\,\vec{B} = Ax\,Bx + Ay\,By + Az\,Bz$$

$$(\hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k}. = 1 \text{ and } \hat{i}.\hat{j} = \hat{j}.\hat{k} = \hat{k}.\hat{i} = 0)$$

$$\vec{A} \cdot \vec{A} = (Ax\hat{i} + Ay\hat{j} + Az\hat{k}) \cdot (Ax\hat{i} + Ay\hat{j} + Az\hat{k})$$

$$= Ax\,Ax\,\hat{i}.\hat{i} + Ay\,Ay\,\hat{j}.\hat{j} + Az\,Az\,\hat{k}.\hat{k}$$

$$\vec{A} \cdot \vec{A} = A^2x + Ay^2 + A^2z$$

14. What is collision ? mention its two types.

An event in which two bodies come in physical contact with other or path of one body is affected by the force due to the other body is called collisoin, [Physical contact between the two colliding bodies is not necessary, if a body can causes change in the velocity of another body without contact, collision may occur]. There are two types of collisions, they are elastic collision and inelastic collision.

The collision in which there is a conservation of both momentum and energy is called **elastic Collision**. Ex:- Collision between billiard balls marbles, ivory balls etc.

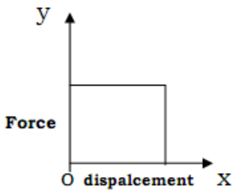
The collision energy in which there is no conservation of kinetic energy and only momentum is onserved is called **inelastic collision**.

Ex :- (1) A bullet hitting a hard target get embedded into it.

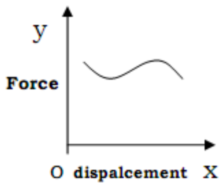
(2) Collision between a person and electron.

15. How do you represent graphically work done by a constant force and by a variable force ?

i. Work done by a constant force



ii. Work done by a Variable force



16. Distinguish between elastic collision & inelastic collision.

	Elastic collision	Inelastic collision
1	The collisions in which both the momentum and K.E of the system remains conserved are called elastic collisions	The collisions in which only momentum is conserved, but KE is not conserved are called inelastic collisions
2	There is no loss of KE during elastic collisions	There is a loss of KE during inelastic collisions
3	The forces involved are conservative in nature	The forces involved are non-conservative in nature
4	Mechanical energy is not transformed into any other form of energy	Whole Mechanical energy may be transformed into other forms

Questions carrying 4 mark and 5mark

1) Prove that for a particle in rectilinear motion under constant acceleration the change in kinetic energy of a particle is equal to the work done on it by the net force?

Consider a particle in rectilinear motion with constant acceleration 'a' then equation of motion is

$$v^2 - u^2 = 2as \text{ -------- } (1)$$

Where ' u ' and ' v ' are initial and final speeds. S the distance traveled

On multiplying equation (1) by m/2

We have, $\frac{1}{2} mv^2 - \frac{1}{2} mu^2 = mas$ -------- (2)

From Newton's II law, ma = F

ie $\frac{1}{2} mv^2 - \frac{1}{2} mu^2 = FS$ --------- (3)

in general, for 3 – Dimensions $\frac{1}{2} mv^2 - \frac{1}{2} mu^2 = F.d$ --------- (4)

F is the force d is the displacement

but $\frac{1}{2} mv^2 = K_f$ → Final kinetic energy $\frac{1}{2} mu^2 = K_i$ → initial kinetic energy

F – d = W-work done

Then, $\mathbf{K_f - K_i = W}$ ---------- (5)

Hence, equation (5) is the special ease of work- Energy theorem.

i.e., " The change in kinetic energy of a particle is equal to the work done on it by the net force.

2) Obtain graphically the work done by a variable force?

A graph of variable force F (X) Versus Displacement ' X ' is as shown in fig.

The area below the Curve gives total work done,

Refer Study material section

3) Prove the work-energy theorem for a variable force?

Refer Study material section

4) Describe the conservation of mechanical energy of a system?

Refer Study material section

5) Give an illustration for the conservation of mechanical energy in case of a ball dropped from a cliff of height 'H'?

Refer Study material section

6) Give an illustration for the conservation of mechanical energy in case of a bob suspended by a light string completes a semi-circular trajectory in the vertical plane?

Refer Study material section

7) Obtain en expression for potential energy of a spring?

Refer Study material section

8) Define power obtain the expression for instantaneous power? Mention the units of power?

Refer Study material section

9) What are elastic and inelastic collisions obtain the expression for final velocities of two bodies colliding in one dimension?

Refer Study material section

10) Describe briefly the collisions in two-dimension?

Refer Study material section

PROBLEMS :

1) A variable force given by F=x+8 acts on a particle. Calculate the work done by the force during the displacement of the particle from X = 1m to X= 3m

Solution : given F = X+8 The work done during a small Displacement dx is dw = F. dx

Total work done

$$W = \int_{X=1}^{X=3} dW = \int_{1}^{3} F dx \qquad W = \int_{1}^{3} (X+8)\,dx$$

$$W = \left.\frac{X^2}{2}\right]_1^3 + \left.8X\right]_1^3 \qquad W = \left(\frac{9}{2} - \frac{1}{2}\right) + \left(8 \times 3 - 8 \times 1\right)$$

W = 4 + 16 = 20 J

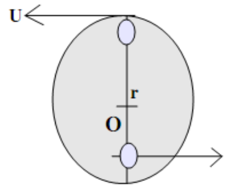
2) A bullet of mass 50g strikes a wooden plank with a velocity of 200ms^{-1} and energy out with a velocity of 50ms^{-1}. Calculate the work done by the bullet against the resistive force offered by the plank.

Given : m = 50g = 50 x 10^{-3} kg U = 200mS^{-1} V = 50mS^{-1}

Work done = loss of kinetic energy $W = 1/2Mu^2 - ½Mv^2$ $W = 1/2M(u^2 - v^2)$

W = 1/2x50x10-3 (4x104 – 0.25x104) W = 25 x 10^{-3} (3.75) x 10^4 W = 93.75 x 10 or W = 937.5 J

3) A metal bob is tied to one of an inextensible string of negligible mass and is rotated in a vertical circle of radius 8m. If the speed of the sphere at the highest point of the circle is 80mS^{-1}. Calculate its speed at the lowest point of the circle (g= 10mS^{-2})



Ep = O

Solution :

Let m be the mass of the metal sphere, Radius of the circle r = 8m

Speed at the highest point U = 80 mS^{-1}

Let -V be the speed at the lowest point of the circle, According to the law of conservation of energy.

$$(E_K + E_P)\text{ lowest point} = (E_{1K} + E_{1P})\text{ highest point}$$

$E_P = 0$ at lowest point.

(½ Mv^2 + 0) = ½ Mu^2 + mg (2r) (: . h = 2r)

. : $V^2 = U^2 + 4gr$

$V^2 = 80^2 + 4 \times 10 \times 8$

$V^2 = 6720$ or $V = \sqrt{6720}$ or V = 82 mS^{-1}

4) Two bodies of masses 0.2kg and 0.1 kg moving in the same direction on a straight line with the velocities 0.6ms^{-1} & 0.4ms^{-1}. Respectively suffer head–on collision, calculate their velocities after collision.

Given : M_1 = 0.2kg. M_2 = 0.1kg. U_1 = 0.6ms^{-1}. U_2 = 0.4ms^{-1}. V_1 = ? U_2 = ?

We know that

$$U_1 = \frac{m_1 - m_2}{m_1+m_2} U_1 + \frac{2m_2}{m_1+m_2} U_2 \quad \text{and} \quad U_2 = \frac{m_1 - m_2}{m_1+m_2} U_2 + \frac{2m_1}{m_1+m_2} U_1$$

and

$$V_1 = \left(\frac{0.2 - 0.1}{0.1 + 0.2}\right) 0.6 + \left(\frac{2 \times 0.1}{0.1 + 0.2}\right) 0.4$$

$V_1 = 0.47mS^{-1}$

and $V_2 = \left[\frac{0.1-0.2}{0.1+0.2}\right] 0.4 + \left[\frac{2 \times 0.2}{0.1+0.2}\right] 0.6$

$V_2 = 0.67mS^{-1}$

5) A Pump on the ground floor of a building can pump mp water to fill a tank of volume 30m^{-3} in 15 min if the tank is 40m above the ground, and the efficiency of the pump is 30% how much electric power is consumed by the pump ?

Solution : given volume V = 30m^{-3}. T =15min, L = 40m, efficiency 30% = 0.03

Work done by the pump = E_p (potential energy)

E_p = mgh

E_p = V x S x g x h Ep = 30 x 10^3 x 10 x 40

E_p = 3 x 40 x 10^5

E_p = 12 x 10^{6}J = power out put

Efficiency = $\frac{P_{output}}{P_{input}}$

Power consumed by the pump = P_{input}

P_{input} = P_{output} = 12 x 10^6 = 4 x 10^8 W, efficiency = 0.03

MCQ

6.1 An electron and a proton are moving under the influence of mutual forces. In calculating the change in the kinetic energy of the system during motion, one ignores the magnetic force of one on another. This is because,
(a) the two magnetic forces are equal and opposite, so they produce no net effect.
(b) the magnetic forces do no work on each particle.
(c) the magnetic forces do equal and opposite (but non-zero) work on each particle.
(d) the magnetic forces are necessarily negligible.

6.2 A proton is kept at rest. A positively charged particle is released from rest at a distance d in its field. Consider two experiments; one in which the charged particle is also a proton and in another, a positron. In the same time t, the work done on the two moving charged particles is
(a) same as the same force law is involved in the two experiments.
(b) less for the case of a positron, as the positron moves away more rapidly and the force on it weakens.
(c) more for the case of a positron, as the positron moves away a larger distance.
(d) same as the work done by charged particle on the stationary proton.

6.3 A man squatting on the ground gets straight up and stand. The force of reaction of ground on the man during the process is :
(a) constant and equal to mg in magnitude.
(b) constant and greater than mg in magnitude.
(c) variable but always greater than mg.
(d) at first greater than mg, and later becomes equal to mg.

6.4 A bicyclist comes to a skidding stop in 10 m. During this process, the force on the bicycle due to the road is 200N and is directly opposed to the motion. The work done by the cycle on the road is :
(a) + 2000J
(b) – 200J
(c) zero
(d) – 20,000 J

6.5 A body is falling freely under the action of gravity alone in vacuum. Which of the following quantities remain constant during the fall?
(a) Kinetic energy.
(b) Potential energy.
(c) Total mechanical energy.
(d) Total linear momentum.

6.6 During inelastic collision between two bodies, which of the following quantities always remain conserved?
(a) Total kinetic energy.
(b) Total mechanical energy.
(c) Total linear momentum.
(d) Speed of each body.

6.7 Two inclined frictionless tracks, one gradual and the other steep meet at A from where two stones are allowed to slide down from rest, one on each track as shown in Fig. 6.1. Which of the following statement is correct :

(a) Both the stones reach the bottom at the same time but not with the same speed.

(b) Both the stones reach the bottom with the same speed and stone I reaches the bottom earlier than stone II.

(c) Both the stones reach the bottom with the same speed and stone II reaches the bottom earlier than stone I.

(d) Both the stones reach the bottom at different times and with different speeds.

6.8 The potential energy function for a particle executing linear SHM is given by 2 1 () Vx kx = where k is the force constant of the oscillator (Fig. 6.2). For k = 0.5N/m, the graph of V(x) versus x is shown in the figure. A particle of total energy E turns back when it reaches m xx =± . If V and K indicate the P.E. and K.E., respectively of the particle at x = +xm, then which of the following is correct?

(a) V = O, K = E

(b) V = E, K = O

(c) V < E, K = O

(d) V = O, K < E.

6.9 Two identical ball bearings in contact with each other and resting on a frictionless table are hit head-on by another ball bearing of the same mass moving initially with a speed V as shown in Fig. 6.3.

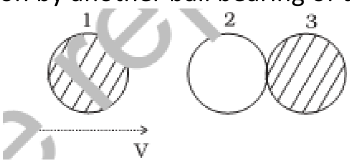


Fig. 6.3

If the collision is elastic, which of the following (Fig. 6.4) is a possible result after collision?

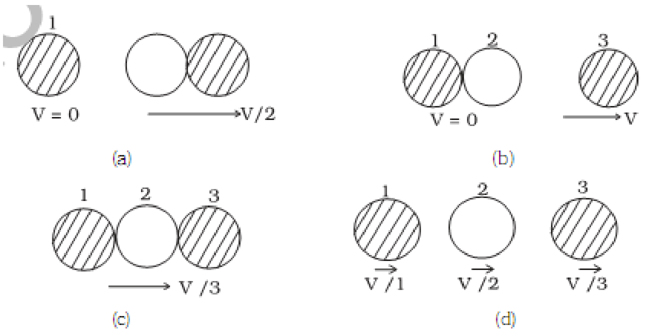


Fig. 6.4

6.10 A body of mass 0.5 kg travels in a straight line with velocity v = a x3/2, where a = 5 m–1/2 s–1. The work done by the net force during its displacement from x = 0 to x = 2 m is

(a) 1.5 J

(b) 50 J

(c) 10 J

(d) 100 J

6.11 A body is moving unidirectionally under the influence of a source of constant power supplying energy. Which of the diagrams shown in Fig. 6.5 correctly shows the displacement-time curve for its motion?

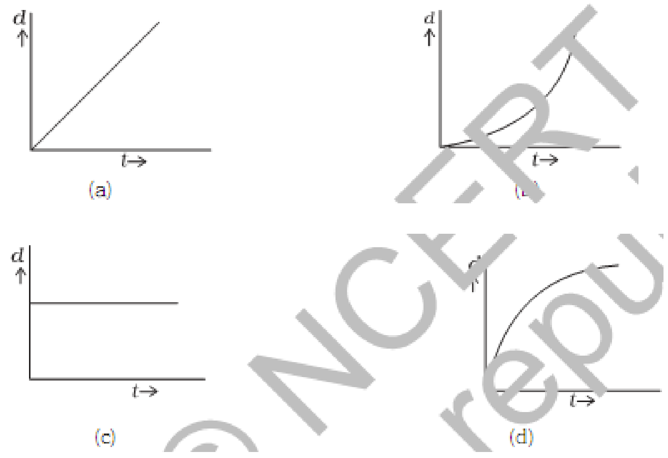
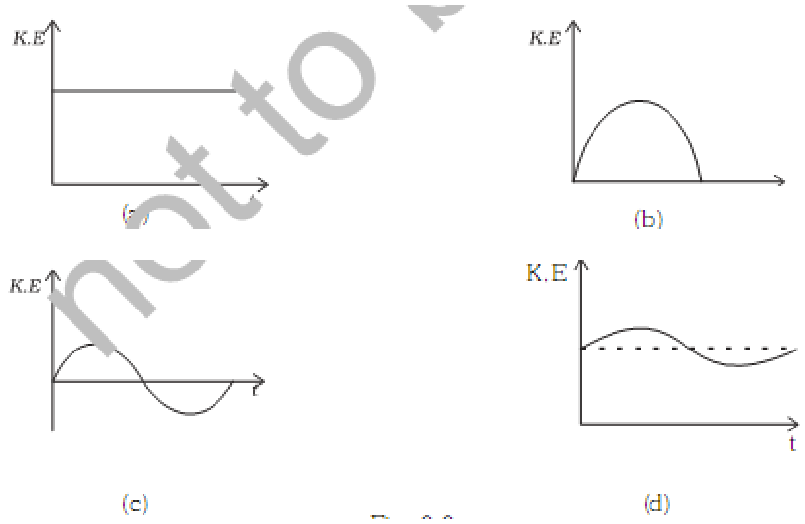
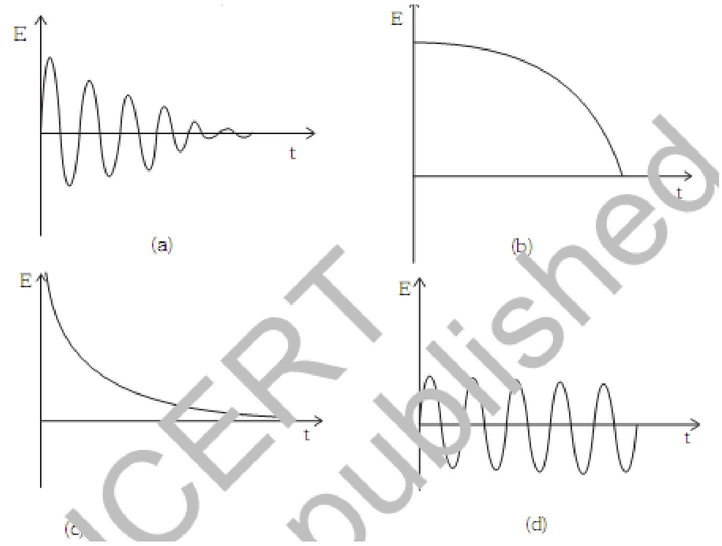


Fig. 6.5

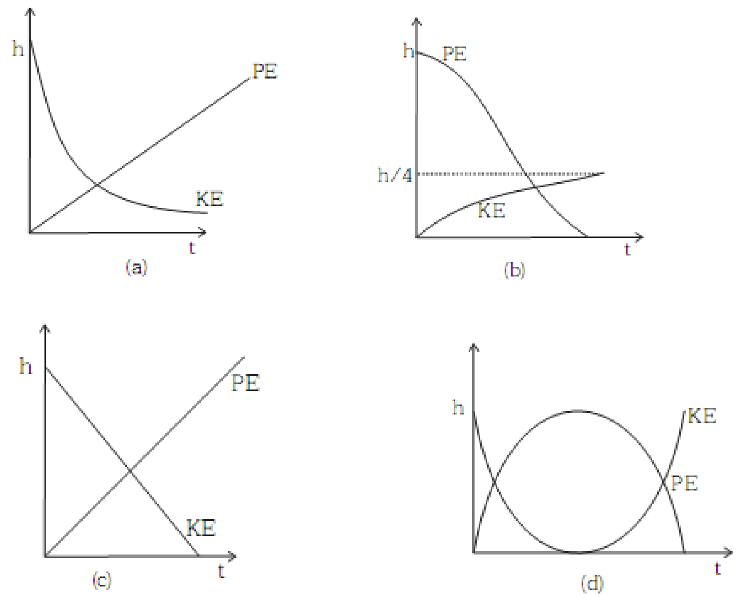
6.12 Which of the diagrams shown in Fig. 6.6 most closely shows the variation in kinetic energy of the earth as it moves once around the sun in its elliptical orbit?



13 Which of the diagrams shown in Fig. 6.7 represents variation of total mechanical energy of a pendulum oscillating in air as function of time?



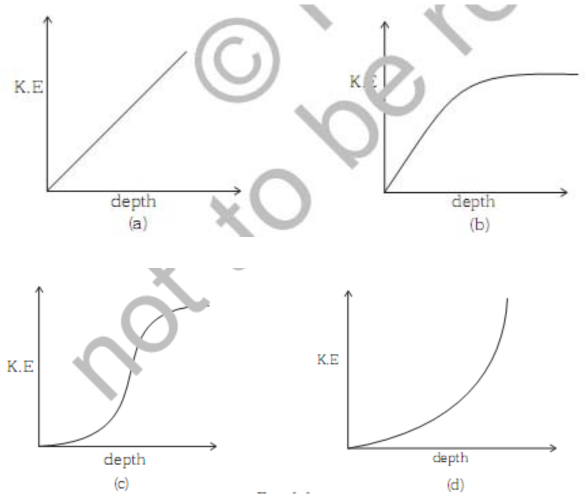
6.14 A mass of 5 kg is moving along a circular path of radius 1 m. If the mass moves with 300 revolutions per minute, its kinetic energy would be
(a) $250\pi^2$
(b) $100\pi^2$
(c) $5\pi^2$
(d) 0
6.15 A raindrop falling from a height h above ground, attains a near terminal velocity when it has fallen through a height (3/4)h. Which of the diagrams shown in Fig. 6.8 correctly shows the change in kinetic and potential energy of the drop during its fall up to the ground?



6.16 In a shotput event an athlete throws the shotput of mass 10 kg with an initial speed of 1m s^{-1} at 45° from a height 1.5 m above ground. Assuming air resistance to be negligible and acceleration due to gravity to be 10 m s^{-2}, the kinetic energy of the shotput when it just reaches the ground will be (a) 2.5 J

(b) 5.0 J
(c) 52.5 J
(d) 155.0 J

6.17 Which of the diagrams in Fig. 6.9 correctly shows the change in kinetic energy of an iron sphere falling freely in a lake having sufficient depth to impart it a terminal velocity ?



6.18 A cricket ball of mass 150 g moving with a speed of 126 km/h hits at the middle of the bat, held firmly at its position by the batsman. The ball moves straight back to the bowler after hitting the bat. Assuming that collision between ball and bat is completely elastic and the two remain in contact for 0.001s, the force that the batsman had to apply to hold the bat firmly at its place would be
(a) 10.5 N
(b) 21 N
(c) 1.05×10^4 N
(d) 2.1×10^4 N

EXERCISES

6.1 The sign of work done by a force on a body is important to understand. State carefully if the following quantities are positive or negative:
(a) work done by a man in lifting a bucket out of a well by means of a rope tied to the bucket.
(b) work done by gravitational force in the above case,
(c) work done by friction on a body sliding down an inclined plane,
(d) work done by an applied force on a body moving on a rough horizontal plane with uniform velocity,
(e) work done by the resistive force of air on a vibrating pendulum in bringing it to rest.

Answer

(a) Positive

In the given case, force and displacement are in the same direction. Hence, th work done is positive. In this case, the work is done on the bucket.

(b) Negative

In the given case, the direction of force (vertically downward) and displ (vertically upward) are opposite to each other. Hence, the sign of work done is n

(c) Negative

Since the direction of frictional force is opposite to the direction of motion, the w by frictional force is negative in this case.

(d) Positive

Here the body is moving on a rough horizontal plane. Frictional force opposes th of the body. Therefore, in order to maintain a uniform velocity, a uniform force applied to the body. Since the applied force acts in the direction of motion of t the work done is positive.

(e) Negative

The resistive force of air acts in the direction opposite to the direction of motic pendulum. Hence, the work done is negative in this case.

6.2 A body of mass 2 kg initially at rest moves under the action of an applied horizontal force of 7 N on a table with coefficient of kinetic friction = 0.1. Compute the (a) work done by the applied force in 10 s,
(b) work done by friction in 10 s,
(c) work done by the net force on the body in 10 s,
(d) change in kinetic energy of the body in 10 s, and interpret your results.

Answer

Mass of the body, m = 2 kg

Applied force, F = 7 N

Coefficient of kinetic friction, μ = 0.1

Initial velocity, u = 0

Time, t = 10 s

The acceleration produced in the body by the applied force is given by Newton' law of motion as:

$$a' = \frac{F}{m} = \frac{7}{2} = 3.5 \text{ m/s}^2$$

Frictional force is given as:

$f = \mu mg$

= 0.1 × 2 × 9.8 = – 1.96 N

The acceleration produced by the frictional force:

$$a'' = -\frac{1.96}{2} = -0.98 \text{ m/s}^2$$

Total acceleration of the body:

$$a = a' + a''$$
$$= 3.5 + (-0.98) = 2.52 \text{ m/s}^2$$

The distance travelled by the body is given by the equation of motion:

$$s = ut + \frac{1}{2}at^2$$

$$= 0 + \frac{1}{2} \times 2.52 \times (10)^2 = 126 \text{ m}$$

(a) Work done by the applied force, $W_a = F \times s$ = 7 × 126 = 882 J

(b) Work done by the frictional force, $W_f = F \times s$ = −1.96 × 126 = −247 J

(c) Net force = 7 + (−1.96) = 5.04 N

Work done by the net force, W_{net}= 5.04 ×126 = 635 J

(d) From the first equation of motion, final velocity can be calculated as:

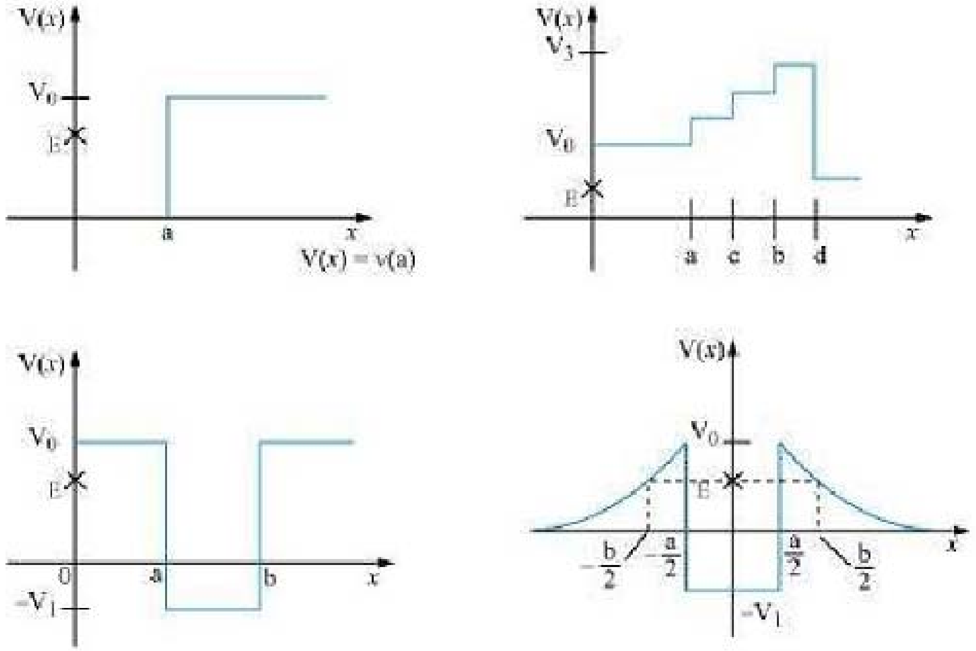
$v = u + at$

= 0 + 2.52 × 10 = 25.2 m/s

Change in kinetic energy $= \frac{1}{2}mv^2 - \frac{1}{2}mu^2$

$$= \frac{1}{2} \times 2\left(v^2 - u^2\right) = (25.2)^2 - 0^2 = 635 \text{ J}$$

6.3 Given in Fig. 6.11 are examples of some potential energy functions in one dimension. The total energy of the particle is indicated by a cross on the ordinate axis. In each case, specify the regions, if any, in which the particle cannot be found for the given energy.
Also, indicate the minimum total energy the particle must have in each case. Think of simple physical contexts for which these potential energy shapes are relevant.



Answer

(a) $x > a$; 0

Total energy of a system is given by the relation:

E = P.E. + K. E.

∴K.E. = E − P.E.

Kinetic energy of a body is a positive quantity. It cannot be negative. There

particle will not exist in a region where K.E. becomes negative.

In the given case, the potential energy (V_0) of the particle becomes greater tl

energy (E) for $x > a$. Hence, kinetic energy becomes negative in this region. Tl

the particle will not exist is this region. The minimum total energy of the particle

(b) All regions

In the given case, the potential energy (V_0) is greater than total energy (
regions. Hence, the particle will not exist in this region.

(c) $x > a$ and $x < b$; $-V_1$

In the given case, the condition regarding the positivity of K.E. is satisfied on
region between $x > a$ and $x < b$.

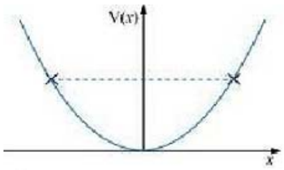
The minimum potential energy in this case is $-V_1$. Therfore, K.E. $= E - (-V_1) =$
Therefore, for the positivity of the kinetic energy, the totaol energy of the parti
be greater than $-V_1$. So, the minimum total energy the particle must have is $-V_1$

(d) $-\frac{b}{2} < x < \frac{a}{2}$; $\frac{a}{2} < x < \frac{b}{2}$; $-V_1$

In the given case, the potential energy (V_0) of the particle becomes greater
total energy (E) for $-\frac{b}{2} < x < \frac{b}{2}$ and $-\frac{a}{2} < x < \frac{a}{2}$. Therefore, the particle will not
these regions.

The minimum potential energy in this case is $-V_1$. Therfore, K.E. $= E - (-V_1) =$
Therefore, for the positivity of the kinetic energy, the totaol energy of the parti
be greater than $-V_1$. So, the minimum total energy the particle must have is $-V_1$

6.4 The potential energy function for a particle executing linear simple harmonic motion is given by $V(x) = kx^2/2$, where k is the force constant of the oscillator. For $k = 0.5$ N m^{-1}, the graph of V(x) versus x is shown in Fig. 6.12. Show that a particle of total energy 1 J moving under this potential must 'turn back' when it reaches $x = \pm 2$ m.



Answer

Total energy of the particle, $E = 1$ J

Force constant, $k = 0.5$ N m^{-1}

Kinetic energy of the particle, $K = \frac{1}{2}mv^2$

According to the conservation law:

$E = V + K$

$$1 = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$$

At the moment of 'turn back', velocity (and hence K) becomes zerc

$$\therefore \; 1 = \frac{1}{2}kx^2$$

$$\frac{1}{2} \times 0.5x^2 = 1$$

$$x^2 = 4$$

$$x = \pm 2$$

Hence, the particle turns back when it reaches $x = \pm 2$ m.

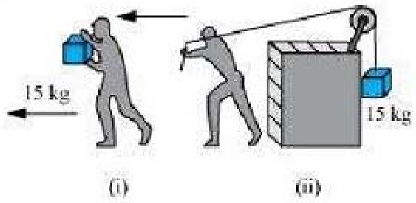
6.5 Answer the following :

(a) The casing of a rocket in flight burns up due to friction. At whose expense is the heat energy required for burning obtained? The rocket or the atmosphere?

(b) Comets move around the sun in highly elliptical orbits. The gravitational force on the comet due to the sun is not normal to the comet's velocity in general. Yet the work done by the gravitational force over every complete orbit of the comet is zero. Why ?

(c) An artificial satellite orbiting the earth in very thin atmosphere loses its energy gradually due to dissipation against atmospheric resistance, however small. Why then does its speed increase progressively as it comes closer and closer to the earth ?

(d) In Fig. 6.13(i) the man walks 2 m carrying a mass of 15 kg on his hands. In Fig. 6.13(ii), he walks the same distance pulling the rope behind him. The rope goes over a pulley, and a mass of 15 kg hangs at its other end. In which case is the work done greater ?



Answer

(a) Rocket

The burning of the casing of a rocket in flight (due to friction) results in the red the mass of the rocket.

According to the conservation of energy:

$$\text{Total Energy (T.E.)} = \text{Potential energy (P.E.)} + \text{Kinetic energy (K.E.)}$$
$$= mgh + \frac{1}{2}mv^2$$

The reduction in the rocket's mass causes a drop in the total energy. Therefore, energy required for the burning is obtained from the rocket.

(b) Gravitational force is a conservative force. Since the work done by a con: force over a closed path is zero, the work done by the gravitational force ov complete orbit of a comet is zero.

(c) When an artificial satellite, orbiting around earth, moves closer to earth, its energy decreases because of the reduction in the height. Since the total energy system remains constant, the reduction in P.E. results in an increase in K.E. He velocity of the satellite increases. However, due to atmospheric friction, the tota of the satellite decreases by a small amount.

(d) In the second case

Case (i)

Mass, m = 15 kg

Displacement, s = 2 m

Work done, $W = Fs\cos\theta$

Where, θ = Angle between force and displacement

$= mgs\cos\theta = 15 \times 2 \times 9.8\cos 90°$

$= 0 \qquad (\because \cos 90° = 0)$

Case (ii)

Mass, m = 15 kg

Displacement, s = 2 m

Here, the direction of the force applied on the rope and the direction of the displ of the rope are same.

Therefore, the angle between them, $\theta = 0°$

Since $\cos 0° = 1$

Work done, $W = Fs\cos\theta = mgs$

$= 15 \times 9.8 \times 2 = 294$ J

Hence, more work is done in the second case.

6.6 Underline the correct alternative :

(a) When a conservative force does positive work on a body, the potential energy of the body increases/decreases/remains unaltered.

(b) Work done by a body against friction always results in a loss of its kinetic/potential energy.

(c) The rate of change of total momentum of a many-particle system is proportional to the external force/sum of the internal forces on the system.
(d) In an inelastic collision of two bodies, the quantities which do not change after the collision are the total kinetic energy/total linear momentum/total energy of the system of two bodies.

Answer

(a) Decreases

(b) Kinetic energy

(c) External force

(d) Total linear momentum

Explanation:

(a) A conservative force does a positive work on a body when it displaces the the direction of force. As a result, the body advances toward the centre of decreases the separation between the two, thereby decreasing the potential e the body.

(b) The work done against the direction of friction reduces the velocity of a body there is a loss of kinetic energy of the body.

(c) Internal forces, irrespective of their direction, cannot produce any change in momentum of a body. Hence, the total momentum of a many- particle s proportional to the external forces acting on the system.

(d) The total linear momentum always remains conserved whether it is a collision or an inelastic collision.

6.7 State if each of the following statements is true or false. Give reasons for your answer.
(a) In an elastic collision of two bodies, the momentum and energy of each body is conserved.
(b) Total energy of a system is always conserved, no matter what internal and external forces on the body are present.
(c) Work done in the motion of a body over a closed loop is zero for every force in nature.
(d) In an inelastic collision, the final kinetic energy is always less than the initial kinetic energy of the system.

Answer

(a) False

(b) False

(c) False

(d) True

Explanation:

(a) In an elastic collision, the total energy and momentum of both the bodies, a each individual body, is conserved.

(b) Although internal forces are balanced, they cause no work to be done on a is the external forces that have the ability to do work. Hence, external forces ar change the energy of a system.

(c) The work done in the motion of a body over a closed loop is zero for a cons force only.

(d) In an inelastic collision, the final kinetic energy is always less than the initi energy of the system. This is because in such collisions, there is always a loss c in the form of heat, sound, etc.

6.8 Answer carefully, with reasons :

(a) In an elastic collision of two billiard balls, is the total kinetic energy conserved during the short time of collision of the balls (i.e. when they are in contact) ?

(b) Is the total linear momentum conserved during the short time of an elastic collision of two balls ?

(c) What are the answers to (a) and (b) for an inelastic collision ?

(d) If the potential energy of two billiard balls depends only on the separation distance between their centres, is the collision elastic or inelastic ? (Note, we are talking here of potential energy corresponding to the force during collision, not gravitational potential energy).

Answer

(a) No

In an elastic collision, the total initial kinetic energy of the balls will be equal to final kinetic energy of the balls. This kinetic energy is not conserved at the ins two balls are in contact with each other. In fact, at the time of collision, th energy of the balls will get converted into potential energy.

(b) Yes

In an elastic collision, the total linear momentum of the system always conserved.

(c) No; Yes

In an inelastic collision, there is always a loss of kinetic energy, i.e., the tota energy of the billiard balls before collision will always be greater than that after c The total linear momentum of the system of billiards balls will remain conservec the case of an inelastic collision.

(d) Elastic

In the given case, the forces involved are conservation. This is because they de the separation between the centres of the billiard balls. Hence, the collision is ela

6.9 A body is initially at rest. It undergoes one-dimensional motion with constant acceleration. The power delivered to it at time t is proportional to

(i) $t^{\frac{1}{2}}$ (ii) t (iii) $t^{\frac{3}{2}}$ (iv) t^2

Answer

(ii) t

Mass of the body = m

Acceleration of the body = a

Using Newton's second law of motion, the force experienced by the body is give equation:

$F = ma$

Both m and a are constants. Hence, force F will also be a constant.

$F = ma$ = Constant ... (i)

$F = ma$ = Constant ... (i)

For velocity v, acceleration is given as,

$$a = \frac{dv}{dt} = \text{Constant}$$

$$dv = \text{Constant} \times dt$$

$$v = \alpha t \qquad \ldots(ii)$$

Where, α is another constant

$$v \propto t \qquad \ldots(iii)$$

Power is given by the relation:

$P = F.v$

Using equations (i) and (iii), we have:

$P \propto t$

Hence, power is directly proportional to time.

6.10 A body is moving unidirectionally under the influence of a source of constant power. Its displacement in time t is proportional to

(i) $t^{\frac{1}{2}}$ (ii) t (iii) $t^{\frac{3}{2}}$ (iv) t^2

Answer

(iii) $t^{\frac{3}{2}}$

Power is given by the relation:

$P = Fv$

$$= mav = mv\frac{dv}{dt} = \text{Constant (say, } k\text{)}$$

$$\therefore vdv = \frac{k}{m}dt$$

Integrating both sides:

$$\frac{v^2}{2} = \frac{k}{m}t$$

$$v = \sqrt{\frac{2kt}{m}}$$

For displacement x of the body, we have:

$$v = \frac{dx}{dt} = \sqrt{\frac{2k}{m}}t^{\frac{1}{2}}$$

$$dx = k't^{\frac{1}{2}}dt$$

Where $k' = \sqrt{\frac{2k}{3}}$ = New constant

On integrating both sides, we get:

$$x = \frac{2}{3}k't^{\frac{3}{2}} \qquad \therefore x \propto t^{\frac{3}{2}}$$

6.11 A body constrained to move along the z-axis of a coordinate system is subject to a constant force F given by F = $-\hat{\imath} + 2\hat{\jmath} + 3\hat{k}N$ where $\hat{\imath}$, $\hat{\jmath}$, $\hat{k}$ are unit vectors along the x-, y- and z-axis of the system respectively. What is the work done by this force in moving the body a distance of 4 m along the z-axis ?

Answer

Force exerted on the body, $\mathbf{F} = -\hat{\mathbf{i}} + 2\,\hat{\mathbf{j}} + 3\,\hat{\mathbf{k}}$ N

Displacement, $s = 4\,\hat{\mathbf{k}}$ m

Work done, W = F.s

$$= \left(-\hat{\mathbf{i}} + 2\,\hat{\mathbf{j}} + 3\,\hat{\mathbf{k}}\right).\left(4\,\hat{\mathbf{k}}\right)$$

$$= 0 + 0 - 3 \times 4$$

$$= 12\ \text{J}$$

Hence, 12 J of work is done by the force on the body.

6.12 An electron and a proton are detected in a cosmic ray experiment, the first with kinetic energy 10 keV, and the second with 100 keV. Which is faster, the electron or the proton ? Obtain the ratio of their speeds. (electron mass = 9.11×10^{-31} kg, proton mass = 1.67×10^{-27} kg, 1 eV = 1.60×10^{-19} J).

Answer

Electron is faster; Ratio of speeds is 13.54 : 1

Mass of the electron, $m_e = 9.11 \times 10^{-31}$ kg

Mass of the proton, $m_p = 1.67 \times 10^{-27}$ kg

Kinetic energy of the electron, E_{Ke} = 10 keV = 10^4 eV

$= 10^4 \times 1.60 \times 10^{-19}$

$= 1.60 \times 10^{-15}$ J

Kinetic energy of the proton, E_{Kp} = 100 keV = 10^5 eV = 1.60×10^{-14} J

For the velocity of an electron v_e, its kinetic energy is given by the relation:

$$E_{Ke} = \frac{1}{2} m v_e^2$$

$$\therefore v_e = \sqrt{\frac{2 \times E_{Ke}}{m}}$$

$$= \sqrt{\frac{2 \times 1.60 \times 10^{-15}}{9.11 \times 10^{-31}}} = 5.93 \times 10^7\ \text{m/s}$$

For the velocity of a proton v_p, its kinetic energy is given by the relation:

$$E_{Kp} = \frac{1}{2} m v_p^2$$

$$v_p = \sqrt{\frac{2 \times E_{Kp}}{m}}$$

$$\therefore v_p = \sqrt{\frac{2 \times 1.6 \times 10^{-14}}{1.67 \times 10^{-27}}} = 4.38 \times 10^6 \text{ m/s}$$

Hence, the electron is moving faster than the proton.

The ratio of their speeds:

$$\frac{v_e}{v_p} = \frac{5.93 \times 10^7}{4.38 \times 10^6} = 13.54 : 1$$

6.13 A rain drop of radius 2 mm falls from a height of 500 m above the ground. It falls with decreasing acceleration (due to viscous resistance of the air) until at half its original height, it attains its maximum (terminal) speed, and moves with uniform speed thereafter. What is the work done by the gravitational force on the drop in the first and second half of its journey ? What is the work done by the resistive force in the entire journey if its speed on reaching the ground is 10 m s^{-1} ?

Answer

Radius of the rain drop, r = 2 mm = 2×10^{-3} m

Volume of the rain drop, $V = \frac{4}{3}\pi r^3$

$$= \frac{4}{3} \times 3.14 \times \left(2 \times 10^{-3}\right)^3 \text{ m}^{-3}$$

Gravitational force, $F = mg$

$$= \frac{4}{3} \times 3.14 \times \left(2 \times 10^{-3}\right)^3 \times 10^3 \times 9.8 \text{ N}$$

The work done by the gravitational force on the drop in the first half of its journe

$W_I = Fs$

$$= \frac{4}{3} \times 3.14 \times \left(2 \times 10^{-3}\right)^3 \times 10^3 \times 9.8 \times 250$$

= 0.082 J

This amount of work is equal to the work done by the gravitational force on th

the second half of its journey, i.e., W_{II}, = 0.082 J

As per the law of conservation of energy, if no resistive force is present, then

energy of the rain drop will remain the same.

∴Total energy at the top:

$E_T = mgh + 0$

$= \frac{4}{3} \times 3.14 \times \left(2 \times 10^{-3}\right)^3 \times 10^3 \times 9.8 \times 500 \times 10^{-5}$

$= 0.164$ J

Due to the presence of a resistive force, the drop hits the ground with a veloc

m/s.

∴Total energy at the ground:

$$E_G = \frac{1}{2}mv^2 + 0$$

$$= \frac{1}{2} \times \frac{4}{3} \times 3.14 \times \left(2 \times 10^{-3}\right)^3 \times 10^3 \times 9.8 \times (10)^2$$

$$= 1.675 \times 10^{-3} \text{ J}$$

∴Resistive force $= E_G - E_T = -0.162$ J

6.14 A molecule in a gas container hits a horizontal wall with speed 200 m s^{-1} and angle 30° with the normal, and rebounds with the same speed. Is momentum conserved in the collision ? Is the collision elastic or inelastic ?

Answer

Yes; Collision is elastic

The momentum of the gas molecule remains conserved whether the collision is

inelastic.

The gas molecule moves with a velocity of 200 m/s and strikes the stationary w

container, rebounding with the same speed.

It shows that the rebound velocity of the wall remains zero. Hence, the tot

energy of the molecule remains conserved during the collision. The given collis

example of an elastic collision.

6.15 A pump on the ground floor of a building can pump up water to fill a tank of volume 30 m^3 in 15 min. If the tank is 40 m above the ground, and the efficiency of the pump is 30%, how much electric power is consumed by the pump ?

Answer

Volume of the tank, $V = 30\ m^3$

Time of operation, $t = 15\ min = 15 \times 60 = 900\ s$

Height of the tank, $h = 40\ m$

Efficiency of the pump, $\eta = 30\%$

Density of water, $\rho = 10^3\ kg/m^3$

Mass of water, $m = \rho V = 30 \times 10^3\ kg$

Output power can be obtained as:

$$P_0 = \frac{\text{Work done}}{\text{Time}} = \frac{mgh}{t}$$

$$= \frac{30 \times 10^3 \times 9.8 \times 40}{900} = 13.067 \times 10^3\ W$$

For input power P_i, efficiency η, is given by the relation:

$$\eta = \frac{P_0}{P_i} = 30\%$$

$$P_i = \frac{13.067}{30} \times 100 \times 10^3$$

$$= 0.436 \times 10^5\ W$$

$$= 43.6\ kW$$

6.16 Two identical ball bearings in contact with each other and resting on a frictionless table are hit head-on by another ball bearing of the same mass moving initially with a speed V. If the collision is elastic, which of the following (Fig. 6.14) is a possible result after collision ?

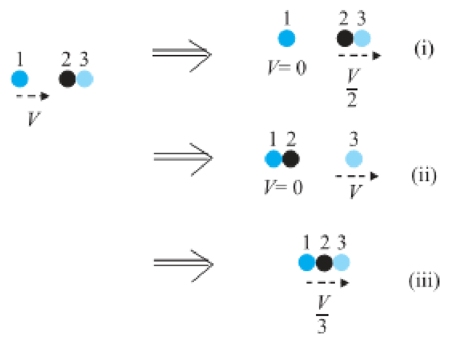


Fig. 6.14

Answer

Case (ii)

It can be observed that the total momentum before and after collision in eacl constant.

For an elastic collision, the total kinetic energy of a system remains conserve and after collision.

For mass of each ball bearing *m*, we can write:

Total kinetic energy of the system before collision:

$$= \frac{1}{2}mV^2 + \frac{1}{2}(2m)0$$

$$= \frac{1}{2}mV^2$$

Case (i)

Total kinetic energy of the system after collision:

$$= \frac{1}{2}m \times 0 + \frac{1}{2}(2m)\left(\frac{V}{2}\right)^2$$

$$= \frac{1}{4}mV^2$$

Hence, the kinetic energy of the system is not conserved in case (i).

Case (ii)

Total kinetic energy of the system after collision:

$$= \frac{1}{2}(2m) \times 0 + \frac{1}{2}mV^2$$

$$= \frac{1}{2}mV^2$$

Hence, the kinetic energy of the system is conserved in case (ii).

Case (iii)

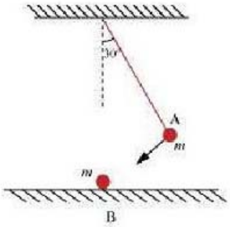
Total kinetic energy of the system after collision:

$$= \frac{1}{2}(3m)\left(\frac{V}{3}\right)^2$$

$$= \frac{1}{6}mV^2$$

Hence, the kinetic energy of the system is not conserved in case (iii).

6.17 The bob A of a pendulum released from 30° to the vertical hits another bob B of the same mass at rest on a table as shown in Fig. 6.15. How high does the bob A rise after the collision ? Neglect the size of the bobs and assume the collision to be elastic.



Answer

Bob A will not rise at all

In an elastic collision between two equal masses in which one is stationary, \
other is moving with some velocity, the stationary mass acquires the same
while the moving mass immediately comes to rest after collision. In this
complete transfer of momentum takes place from the moving mass to the s
mass.

Hence, bob A of mass m, after colliding with bob B of equal mass, will come
while bob B will move with the velocity of bob A at the instant of collision.

6.18 The bob of a pendulum is released from a horizontal position. If the length of the pendulum is 1.5 m, what is the speed with which the bob arrives at the lowermost point, given that it dissipated 5% of its initial energy against air resistance ?

Answer

Length of the pendulum, l = 1.5 m

Mass of the bob = m

Energy dissipated = 5%

According to the law of conservation of energy, the total energy of the system
constant.

At the horizontal position:

Potential energy of the bob, $E_P = mgl$

Kinetic energy of the bob, $E_K = 0$

Total energy = mgl ... (*i*)

At the lowermost point (mean position):

Potential energy of the bob, $E_P = 0$

Kinetic energy of the bob, $E_K = \frac{1}{2}mv^2$

Total energy $E_x = \frac{1}{2}mv^2$ ... (*ii*)

As the bob moves from the horizontal position to the lowermost point, 5% of it gets dissipated.

The total energy at the lowermost point is equal to 95% of the total energ horizontal point, i.e.,

$$\frac{1}{2}mv^2 = \frac{95}{100} \times mgl$$

$$\therefore v = \sqrt{\frac{2 \times 95 \times 1.5 \times 9.8}{100}}$$

$$= 5.28 \text{ m/s}$$

6.19 A trolley of mass 300 kg carrying a sandbag of 25 kg is moving uniformly with a speed of 27 km/h on a frictionless track. After a while, sand starts leaking out of a hole on the floor of the trolley at the rate of 0.05 kg s^{-1}. What is the speed of the trolley after the entire sand bag is empty ?

Answer

The sand bag is placed on a trolley that is moving with a uniform speed of 27 kı external forces acting on the system of the sandbag and the trolley is zero. V sand starts leaking from the bag, there will be no change in the velocity of th This is because the leaking action does not produce any external force on the This is in accordance with Newton's first law of motion. Hence, the speed of tł will remain 27 km/h.

6.20 A body of mass 0.5 kg travels in a straight line with velocity $v = ax^{3/2}$ where a = 5 $m^{-1/2}$ s^{-1}. What is the work done by the net force during its displacement from x = 0 to x = 2 m ?

Answer

Mass of the body, m = 0.5 kg

Velocity of the body is governed by the equation, $v = a\,x^{\frac{3}{2}}$ with $a = 5\ \mathrm{m}^{\frac{-1}{2}}\ \mathrm{s}^{-1}$

Initial velocity, u (at x = 0) = 0

Final velocity v (at x = 2 m) $= 10\sqrt{2}$ m/s

Work done, W = Change in kinetic energy

$$= \frac{1}{2} m\left(v^2 - u^2\right)$$

$$= \frac{1}{2} \times 0.5\left[\left(10\sqrt{2}\right)^2 - (0)^2\right]$$

$$= \frac{1}{2} \times 0.5 \times 10 \times 10 \times 2 \qquad = 50\ \mathrm{J}$$

6.21 The blades of a windmill sweep out a circle of area A. (a) If the wind flows at a velocity v perpendicular to the circle, what is the mass of the air passing through it in time t ? (b) What is the kinetic energy of the air ? (c) Assume that the windmill converts 25% of the wind's energy into electrical energy, and that A = 30 m^2, v = 36 km/h and the density of air is 1.2 kg m^{-3}. What is the electrical power produced ?

Answer

Area of the circle swept by the windmill = A

Velocity of the wind = v

Density of air = ρ

(a) Volume of the wind flowing through the windmill per sec = Av

Mass of the wind flowing through the windmill per sec = ρAv

Mass m, of the wind flowing through the windmill in time t = ρAvt

(b) Kinetic energy of air $= \frac{1}{2} mv^2$

$$= \frac{1}{2}(\rho Avt)v^2 = \frac{1}{2}\rho Av^3 t$$

(c) Area of the circle swept by the windmill = A = 30 m^2

Velocity of the wind = v = 36 km/h

Density of air, ρ = 1.2 kg m^{-3}

Electric energy produced = 25% of the wind energy

$$= \frac{25}{100} \times \text{Kinetic energy of air}$$

$$= \frac{1}{8} \rho A v^3 t$$

$$\text{Electrical power} = \frac{\text{Electrical energy}}{\text{Time}}$$

$$= \frac{1}{8} \frac{\rho A v^3 t}{t} = \frac{1}{8} \rho A v^3$$

$$= \frac{1}{8} \times 1.2 \times 30 \times (10)^3$$

$$= 4.5 \times 10^3 \text{ W} = 4.5 \text{ kW}$$

6.22 A person trying to lose weight (dieter) lifts a 10 kg mass, one thousand times, to a height of 0.5 m each time. Assume that the potential energy lost each time she lowers the mass is dissipated. (a) How much work does she do against the gravitational force ? (b) Fat supplies 3.8 × 107 J of energy per kilogram which is converted to mechanical energy with a 20% efficiency rate. How much fat will the dieter use up?

Answer

(a) Mass of the weight, m = 10 kg

Height to which the person lifts the weight, h = 0.5 m

Number of times the weight is lifted, n = 1000

∴Work done against gravitational force:

$$= n(mgh)$$

$$= 1000 \times 10 \times 9.8 \times 0.5$$

$$= 49 \times 10^3 \text{ J} = 49 \text{ kJ}$$

(b) Energy equivalent of 1 kg of fat = 3.8×10^7 J

Efficiency rate = 20%

Mechanical energy supplied by the person's body:

$$= \frac{20}{100} \times 3.8 \times 10^7 \text{ J}$$

$$= \frac{1}{5} \times 3.8 \times 10^7 \text{ J}$$

Equivalent mass of fat lost by the dieter:

$$= \frac{1}{\frac{1}{5} \times 3.8 \times 10^7} \times 49 \times 10^3$$

$$= \frac{245}{3.8} \times 10^{-4}$$

$$= \mathbf{6.45 \times 10^{-3}\ kg}$$

6.23 A family uses 8 kW of power. (a) Direct solar energy is incident on the horizontal surface at an average rate of 200 W per square meter. If 20% of this energy can be converted to useful electrical energy, how large an area is needed to supply 8 kW? (b) Compare this area to that of the roof of a typical house.

Answer

(a) 200 m^2

(a) Power used by the family, P = 8 kW = 8×10^3 W

Solar energy received per square metre = 200 W

Efficiency of conversion from solar to electricity energy = 20 %

Area required to generate the desired electricity = A

As per the information given in the question, we have:

$$8 \times 10^3 = 20\ \% \times (A \times 200)$$

$$= \frac{20}{100} \times A \times 200$$

$$\therefore A = \frac{8 \times 10^3}{40} = 200 \text{ m}^2$$

(b) The area of a solar plate required to generate 8 kW of electricity is almost e

to the area of the roof of a building having dimensions 14 m × 14 m.

Additional Exercises

6.24 A bullet of mass 0.012 kg and horizontal speed 70 m s^{-1} strikes a block of wood of mass 0.4 kg and instantly comes to rest with respect to the block. The block is suspended from the ceiling by means of thin wires. Calculate the height to which the block rises. Also, estimate the amount of heat produced in the block.

Answer

Mass of the bullet, m = 0.012 kg

Initial speed of the bullet, u_b = 70 m/s

Mass of the wooden block, M = 0.4 kg

Initial speed of the wooden block, u_B = 0

Final speed of the system of the bullet and the block = v

Applying the law of conservation of momentum:

$$mu_b + Mu_B = (m + M)v$$

$$0.012 \times 70 + 0.4 \times 0 = (0.012 + 0.4)v$$

$$\therefore v = \frac{0.84}{0.412} = 2.04 \text{ m/s}$$

For the system of the bullet and the wooden block:

Mass of the system, m' = 0.412 kg

Velocity of the system = 2.04 m/s

Height up to which the system rises = h

Applying the law of conservation of energy to this system:

Potential energy at the highest point = Kinetic energy at the lowest point

$$m'gh = \frac{1}{2}m'v^2$$

$$\therefore h = \frac{1}{2}\left(\frac{v^2}{g}\right)$$

$$= \frac{1}{2} \times \frac{(2.04)^2}{9.8}$$

= 0.2123 m

The wooden block will rise to a height of 0.2123 m.

Heat produced = Kinetic energy of the bullet – Kinetic energy of the system

$$= \frac{1}{2}mu^2 - \frac{1}{2}m'v^2$$

$$= \frac{1}{2} \times 0.012 \times (70)^2 - \frac{1}{2} \times 0.412 \times (2.04)^2$$

= 29.4 – 0.857 = 28.54 J

6.25 Two inclined frictionless tracks, one gradual and the other steep meet at A from where two stones are allowed to slide down from rest, one on each track (Fig. 6.16). Will the stones reach the bottom at the same time? Will they reach there with the same speed? Explain. Given θ_1 = 30°, θ_2 = 60°, and h = 10 m, what are the speeds and times taken by the two stones ?

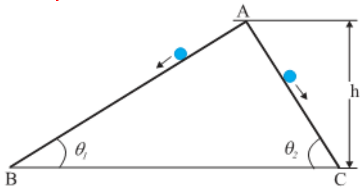


Fig. 6.16

Answer

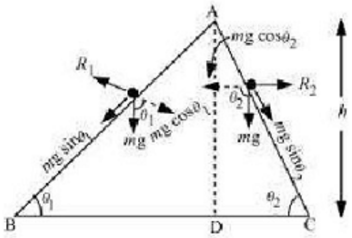
No; the stone moving down the steep plane will reach the bottom first

Yes; the stones will reach the bottom with the same speed

$v_B = v_C$ = 14 m/s

t_1 = 2.86 s; t_2 = 1.65 s

The given situation can be shown as in the following figure:



Here, the initial height (AD) for both the stones is the same (h). Hence, both the same potential energy at point A.

As per the law of conservation of energy, the kinetic energy of the stones at poir C will also be the same, i. e.,

$$\frac{1}{2}mv_1^2 = \frac{1}{2}mv_2^2$$

$v_1 = v_2 = v$, say

Where,

m = Mass of each stone

v = Speed of each stone at points B and C

Hence, both stones will reach the bottom with the same speed, v.

For stone I:

Net force acting on this stone is given by:

$$F_{net} = ma_1 = mg\sin\theta_1$$

$$a_1 = g\sin\theta_1$$

For stone II:

$$a_2 = g\sin\theta_2$$

$$\because \theta_2 > \theta_1$$

$$\therefore \sin\theta_2 > \sin\theta_1$$

$$\therefore a_2 > a_1$$

Using the first equation of motion, the time of slide can be obtained as:

$$v = u + at$$

$$\therefore t = \frac{v}{a} \qquad (\because u = 0)$$

For stone I:

$$t_1 = \frac{v}{a_1}$$

For stone II:

$$t_2 = \frac{v}{a_2}$$

$$\because a_2 > a_1 \qquad \therefore t_2 < t_1$$

Hence, the stone moving down the steep plane will reach the bottom first.

The speed (v) of each stone at points B and C is given by the relation obtained law of conservation of energy.

$$mgh = \frac{1}{2}mv^2$$

$$\therefore v = \sqrt{2gh}$$
$$= \sqrt{2 \times 9.8 \times 10}$$
$$= \sqrt{196} = 14 \text{ m/s}$$

The times are given as:

$$t_1 = \frac{v}{a_1} = \frac{v}{g \sin\theta_1} = \frac{14}{9.8 \times \sin 30} = \frac{14}{9.8 \times \frac{1}{2}} = 2.86 \text{ s}$$

$$t_2 = \frac{v}{a_2} = \frac{v}{g \sin\theta_2} = \frac{14}{9.8 \times \sin 60} = \frac{14}{9.8 \times \frac{\sqrt{3}}{2}} = 1.65 \text{ s}$$

6.26 A 1 kg block situated on a rough incline is connected to a spring of spring constant 100 N m^{-1} as shown in Fig. 6.17. The block is released from rest with the spring in the unstretched position. The block moves 10 cm down the incline before coming to rest. Find the coefficient of friction between the block and the incline. Assume that the spring has a negligible mass and the pulley is frictionless.

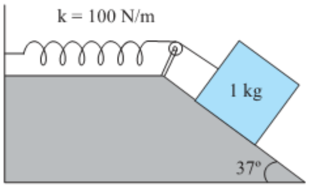
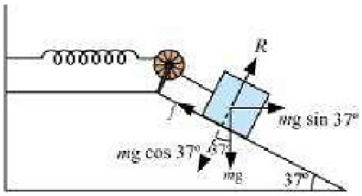


Fig. 6.17



Answer

Mass of the block, m = 1 kg

Spring constant, k = 100 N m^{-1}

Displacement in the block, x = 10 cm = 0.1 m

The given situation can be shown as in the following figure.

At equilibrium:

Normal reaction, $R = mg \cos 37°$

Frictional force, $f = \mu R = mg \sin 37°$

Where, μ is the coefficient of friction

Net force acting on the block = $mg \sin 37° - f$

$= mg\sin 37° - \mu mg\cos 37°$

$= mg(\sin 37° - \mu\cos 37°)$

At equilibrium, the work done by the block is equal to the potential energy of th

i.e.,

$$mg(\sin 37° - \mu \cos 37°)x = \frac{1}{2}kx^2$$

$$1 \times 9.8(\sin 37° - \mu \cos 37°) = \frac{1}{2} \times 100 \times 0.1$$

$$0.602 - \mu \times 0.799 = 0.510$$

$$\therefore \mu = \frac{0.092}{0.799} = 0.115$$

6.27 A bolt of mass 0.3 kg falls from the ceiling of an elevator moving down with an uniform speed of 7 m s^{-1}. It hits the floor of the elevator (length of the elevator = 3 m) and does not rebound. What is the heat produced by the impact ? Would your answer be different if the elevator were stationary ?

Answer

Mass of the bolt, m = 0.3 kg

Speed of the elevator = 7 m/s

Height, h = 3 m

Since the relative velocity of the bolt with respect to the lift is zero, at the impact, potential energy gets converted into heat energy.

Heat produced = Loss of potential energy

$= mgh = 0.3 \times 9.8 \times 3$

= 8.82 J

The heat produced will remain the same even if the lift is stationary. This is be

the fact that the relative velocity of the bolt with respect to the lift will remain ze

6.28 A trolley of mass 200 kg moves with a uniform speed of 36 km/h on a frictionless track. A child of mass 20 kg runs on the trolley from one end to the other (10 m away) with a speed of 4 m s^{-1} relative to the trolley in a direction opposite to the its motion, and jumps out of the trolley. What is the final speed of the trolley ? How much has the trolley moved from the time the child begins to run ?

Answer

Mass of the trolley, M = 200 kg

Speed of the trolley, v = 36 km/h = 10 m/s

Mass of the boy, m = 20 kg

Initial momentum of the system of the boy and the trolley

$= (M + m)v$

$= (200 + 20) \times 10$

= 2200 kg m/s

Let v' be the final velocity of the trolley with respect to the ground.

Final velocity of the boy with respect to the ground $= v' - 4$

Final momentum $= Mv' + m(v' - 4)$

$= 200v' + 20v' - 80$

$= 220v' - 80$

As per the law of conservation of momentum:

Initial momentum = Final momentum

$2200 = 220v' - 80$

$$\therefore v' = \frac{2280}{220} = 10.36 \text{ m/s}$$

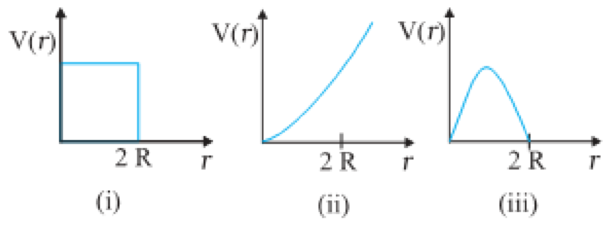
Length of the trolley, l = 10 m

Speed of the boy, v'' = 4 m/s

Time taken by the boy to run, $t = \frac{10}{4} = 2.5$ s

∴Distance moved by the trolley $= v'' \times t = 10.36 \times 2.5 = 25.9$ m

6.29 Which of the following potential energy curves in Fig. 6.18 cannot possibly describe the elastic collision of two billiard balls ? Here r is the distance between centres of the balls.



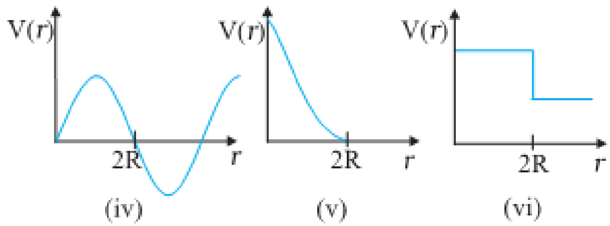


Fig. 6.18

Answer

(i), (ii), (iii), (iv), and (vi)

The potential energy of a system of two masses is inversely proportiona separation between them. In the given case, the potential energy of the syste two balls will decrease as they come closer to each other. It will become zero (= 0) when the two balls touch each other, i.e., at r = 2R, where R is the radius billiard ball. The potential energy curves given in figures (i), (ii), (iii), (iv), an not satisfy these two conditions. Hence, they do not describe the elastic between them.

6.30 Consider the decay of a free neutron at rest : $n \rightarrow p + e^-$
Show that the two-body decay of this type must necessarily give an electron of fixed energy and, therefore, cannot account for the observed continuous energy distribution in the β-decay of a neutron or a nucleus (Fig. 6.19).

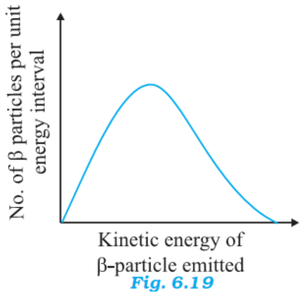


Fig. 6.19

Answer

The decay process of free neutron at rest is given as:

$$n \rightarrow p + e^-$$

From Einstein's mass-energy relation, we have the energy of electron as Δmc^2

Where,

Δm = Mass defect = Mass of neutron − (Mass of proton + Mass of electron)

c = Speed of light

Δm and c are constants. Hence, the given two-body decay is unable to exp
continuous energy distribution in the β-decay of a neutron or a nucleus. The pre
neutrino von the LHS of the decay correctly explains the continuous energy distr

PROBLEMS FOR PRACTICE

1) The velocity vectors of three particles of masses 5 kg, 10 kg and 15 kg are respectively (5, 3, 0), (1, 1, 2) and (1, 2, 4). Find the velocity of the centre of mass. The velocity vector components are in m / s. [March, 1999, March, 1998]
[Ans: (5/3, 11/6, 8/3)]

2) A system is made up of three particles. The respective linear momenta of particles are $10\,\hat{i}$; $20\,\hat{j} + 10\,\hat{k}$ and $10\,\hat{k}$. If the centre of mass of the system is moving with velocity $30\,\hat{i} + 60\,\hat{j} + 90\,\hat{k}$, calculate the total mass of the system.
[Oct., 1997, similar in Oct., 1989]
(Ans: There is a printing error in the problem. The linear momentum of the third particle should be corrected as $20\,\hat{k}$. Then the answer will be mass = 1 /3 unit.)

3) Find out the centre of mass of a system of two bodies 20 cm apart. The masses of the bodies are 2 kg and 8 kg. [October, 1996]
(Ans: At 4 cm from the 8 kg mass on a line joining the masses.)

4) The forces of 1 N, 2 N, 3 N and 4 N are acting on the particles of a system of four particles, in the direction of west, east, south and north respectively. Find the force and the direction of the force acting on the centre of mass of the system. [March, 1996]
(Ans: 2 N in the north-east direction)

5) Three particles of mass m_1 = 1.0 kg, m_2 = 2.0 kg and m_3 = 3.0 kg are placed at the vertices of an equilateral triangle of side 1 m. Find the coordinates of the centre of mass of the system with reference to the particle of mass m_1 [March, 1994]
(Ans: (2 / 3, $\sqrt{6}$ / 3) with m_1 at origin, m_3 on $+^{ve}$ x-axis and m_2 in the first quadrant.)

6) Two bodies of masses 10 kg and 2 kg moving with velocities $2\,\hat{i} \quad 7\,\hat{j} + 3\,\hat{k}$ and $10\,\hat{i} + 35\,\hat{j} \quad 3\hat{k}$ m /s respectively. Find the velocity of the centre of mass.
(Ans: $2\,\hat{k}$ m /s) [October, 1993]

7) The coordinates of particles of mass 2 gm, 3 gm and 5 gm are (3, 3, 0), (3, 3, 5) and (5, 3, 1) respectively. Find the coordinates of its centre of mass. [March, 1988]

[Ans: (4, 3, 2)]

8) A chemical bomb of 80 kg mass at rest suddenly explodes without any external force on it. It splits into two parts of 60 kg and 20 kg. The velocity of the bigger part is 1.5 m / s. Find the velocity of the smaller part. What is the velocity of the centre of mass of the bomb after the explosion ? [March, 1991]
(Ans: − 4.5 m / s, zero)

9) A light and a heavy body have equal kinetic energies of translation. Which one has the larger momentum ?
(Ans: Heavy body)

10) A dog of mass 10 kg is standing on board floating frictionless in water. Initially, the dog and the board are stationary and the dog is 20 m away from the bank. Now the dog travels a distance of 8 m on the board towards the bank. If the mass of the boat is 40 kg, find the distance of the dog at the end of this motion.
(Ans: 13.6 m)

11) Two bodies of masses 100 gm and 400 gm are moving towards each other with speeds 100 cm / s and 10 cm / s respectively. They suffer a head on collision and stick together.
(i) In which direction will the combined mass move after the collision ? and (ii) What will be the distance traveled by the combined mass after 10 s ?
(Ans: (i) in the direction of motion of 100 gm mass, (ii) 120 cm)

12) A sphere moving with velocity 2 9 m /s collides with another identical sphere at rest. After collision both the spheres are moving in a direction making angle of 45° with the velocity of the first sphere. Find the velocities of the spheres after collision.
(Ans: 9 m /s)

13) A 15 gm bullet fired from a gun has a velocity of 10 m /s when it enters a stationary wooden block of mass 985 gm. If the bullet comes to rest in the block, find the velocity of the block and the percentage loss of kinetic energy of the system of bullet and the block.
(Ans: 0.15 m /s, 98.5 %)

14) A bomb is dropped from a height of 1000 m from the surface of the earth. After 10 seconds, it explodes into two pieces of equal masses. One piece moves towards the earth with a speed of 300 m /s. To what maximum height from the surface of the earth will the other piece reach ?
(Ans: 1000 m)

CHAPTER 7

System of particles and rotational motion

(12 Hours, 11 Marks) (1Q-1M, 1Q-2M,1Q-3M,1Q – 5M(LA))

Syllabus :

Centre of mass of a two-particle system, momentum conservation and centre of mass motion. Centre of mass of a rigid body; centre of mass of uniform rod. Moment of a force, torque, angular momentum, conservation of angular momentum with some examples. Equilibrium of rigid bodies, rigid body rotation and equations of rotational motion, comparison of linear and rotational motions; moment of inertia, radius of gyration. Values of moments of inertia, for simple geometrical objects (no derivation). Statement of parallel and perpendicular axes theorems and their applications.

7.1. Centre of mass of a two-particle system, momentum conservation and centre of mass motion. Centre of mass of a rigid body; centre of mass of uniform rod.

1. What do you mean by rigid body ?

Ideally a rigid body is a body with a perfectly definite and unchanging shape. The distances between different pairs of such a body do not change. In practice we can consider bodies such as wheels, tops, steel beams, molecules and planets on the other hand, we can ignore that they warp, bend or vibrate and treat them as rigid.

2. What kind of motion can a rigid body have ?

(1) In a rigid body, during the motion, if all particles of the rigid body move together, i.e. they have the same velocity at any instant of time, then the motion of the rigid body is said to be **pure translational motion**.

(2) Rigid bodies can also have **rotational motion** about a fixed axis, every particle of the body moves in a circle, which lies in a plane perpendicular to the axis and has its centre on the axis.

(3) Rigid bodies can also have **oscillatory motion** from a fixed point.

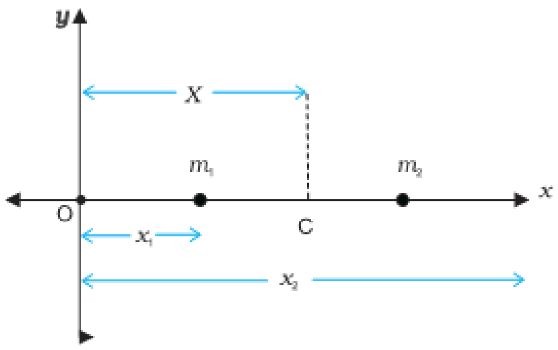
(4) Rigid bodies can also have both **translational and rotational motion**.

3. What is centre of mass ? Obtain an expression for centre of mass of two particles of equal masses ? Extend it to n particles along a straight line ?

In multi-particle or multi-body system, the centre of mass is a point of balance.

(a) Expression for centre of mass of two particles of equal masses :

Consider two particles of masses m_1 and m_2 located at distances x_1 and x_2 meters from the origin O along x-axis of Cartesian coordinate system as shown in following figure.



The centre of mass of the system is that point C which is at a distance X from O, where X is given by

$$X = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

Here, X can be regarded as the mass-weighted mean of x_1 and x_2. If the two particles have the same mass $m_1 = m_2 = m$, then

$$X = \frac{mx_1 + mx_2}{2m} = \frac{x_1 + x_2}{2} \qquad \text{-------- (7.1)}$$

Thus, for two particles of equal mass the centre of mass X_{CM} lies exactly **midway** between them. This equation (7.1) gives the **position vector of the centre of mass** of a system.

(2) Expression for centre of mass of n particles along a straight line :

If we have n particles of masses m_1, m_2, ...m_n respectively, along a straight line taken as the x-axis, then by definition the position of the centre of the mass of the system of particles is given by

$$X = \frac{m_1 x_1 + m_2 x_2 + \dots + m_n x_n}{m_1 + m_2 + \dots + m_n} = \frac{\sum m_i x_i}{\sum m_i} \qquad \text{----------- (7.2)}$$

where x_1, x_2,...x_n are the distances of the particles from the origin; X is also measured from the same origin. The symbol $\sum$ (the Greek letter sigma) denotes summation, in this case over n particles. The sum

$\sum m_i = M$

is the total mass of the system.

Suppose that we have three particles, not lying in a straight line. We may define x and y-axes in the plane in which the particles lie and represent the positions of the three particles by coordinates (x_1,y_1), (x_2,y_2) and (x_3,y_3) respectively.

Let the masses of the three particles be m_1, m_2 and m_3 respectively. The centre of mass C of the system of the three particles is defined and located by the coordinates (X, Y) given by

$$X = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3} \quad \text{----------} \quad (7.3a)$$

$$Y = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3} \quad \text{-------------} \quad (7.3b)$$

For the particles of equal mass $m = m_1 = m_2 = m_3$,

$$X = \frac{m(x_1 + x_2 + x_3)}{3m} = \frac{x_1 + x_2 + x_3}{3}$$

$$Y = \frac{m(y_1 + y_2 + y_3)}{3m} = \frac{y_1 + y_2 + y_3}{3}$$

Thus, for three particles of equal mass, the centre of mass coincides with the centroid of the triangle formed by the particles.

Results of Eqs. (7.3a) and (7.3b) are generalised easily to a system of n particles, not necessarily lying in a plane, but distributed in space. The centre of mass of such a system is at (X, Y, Z), where

$$X = \frac{\sum m_i x_i}{M}$$

$$Y = \frac{\sum m_i y_i}{M}$$

$$\text{and} \quad Z = \frac{\sum m_i z_i}{M} \quad \text{--------------} \quad (7.4)$$

Here $M = \sum m_i$ is the total mass of the system. The index i runs from 1 to n; m_i is the mass of the i^{th} particle and the position of the i^{th} particle is given by (x_i, y_i, z_i).

The above 3 equations can be combined into one equation using the notation of position vectors. Let r_i be the position vector of the i^{th} particle and **R be the position vector of the centre of mass:** $r_i = x_i\hat{\imath} + y_i\hat{\jmath} + z_i\hat{k}$ and $R = X\hat{\imath} + Y\hat{\jmath} + Z\hat{k}$

Then $\mathbf{R} = \frac{\sum \boldsymbol{m_i r_i}}{\boldsymbol{M}}$ --------- (7.5)

The sum on the right hand side is a vector sum.

If M is the total mass of the body. The coordinates of the centre of mass now are

$$X = \frac{1}{M}\int x\,dm, \quad Y = \frac{1}{M}\int y\,dm \quad \text{and} \quad Z = \frac{1}{M}\int z\,dm$$

The vector expression equivalent to these three scalar expressions is

$$\mathbf{R} = \frac{1}{M} \int \boldsymbol{r}\, \boldsymbol{dm} \quad \text{----------} \quad (7.6)$$

4. Obtain an expression for centre of mass of uniform rod ?

Let us consider a thin rod, whose width and breath (in case the cross section of the rod is rectangular) or radius (in case the cross section of the rod is cylindrical) is much smaller than its length. Taking the origin to be at the geometric centre of the rod and x-axis to be along the length of the rod, we can say that on account of reflection symmetry, for every element dm of the rod at x, there is an element of the same mass dm located at –x (Fig. 7.8).

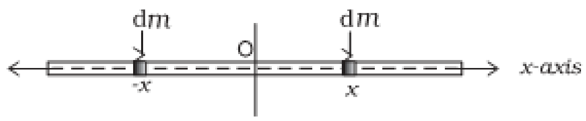


Fig. 7.8 Determining the CM of a thin rod.

The net contribution of every such pair to the integral and hence the integral ∫x dm itself is zero. From Eq. (7.6), the point for which the integral itself is zero, is the centre of mass. Thus, the centre of mass of a homogenous thin rod coincides with its **geometric centre**. This can be understood on the basis of reflection symmetry.

5. Obtain an expression for motion of center of mass of a system and explain it ?

The centre of mass of a system of particles moves as if all the mass of the system was concentrated at the centre of mass and all the external forces were applied at that point.

Proof : Consider a system of particles. R be the position vector of the centre of mass**.** Then

$$\mathbf{R} = \frac{\sum m_i r_i}{M}$$

or $$M\mathbf{R} = \sum m_i \mathbf{r}_i = m_1 \mathbf{r}_1 + m_2 \mathbf{r}_2 + \ldots + m_n \mathbf{r}_n$$

Differentiating the two sides of the equation with respect to time we get

$$M \frac{d\mathbf{R}}{dt} = m_1 \frac{d\mathbf{r}_1}{dt} + m_2 \frac{d\mathbf{r}_2}{dt} + \ldots + m_n \frac{d\mathbf{r}_n}{dt}$$

or

$$M\,\mathbf{V} = m_1 \mathbf{v}_1 + m_2 \mathbf{v}_2 + \ldots + m_n \mathbf{v}_n \quad \text{--------} \quad (7.7)$$

Differentiating Eq.(7.7) with respect to time, we obtain

$$M\frac{d\mathbf{V}}{dt} = m_1\frac{d\mathbf{v}_1}{dt} + m_2\frac{d\mathbf{v}_2}{dt} + \ldots + m_n\frac{d\mathbf{v}_n}{dt}$$

or

$$M\mathbf{A} = m_1\mathbf{a}_1 + m_2\mathbf{a}_2 + \ldots + m_n\mathbf{a}_n \qquad \text{---------} \quad (7.8)$$

Where a_n is acceleration of n^{th} particle and A (= dV/dt) is the acceleration of the centre of mass of the system of particles. Using Newtons law we can write as :

$MA = F_1 + F_2 + \ldots\ldots\ldots + F_n$ --------- (7.9)

Thus, the total mass of a system of particles times the acceleration of its centre of mass is the vector sum of all the forces acting on the system of particles.

Among these forces on each particle there will be external forces exerted by bodies outside the system and also internal forces exerted by the particles on one another. We know from Newton's third law that these internal forces occur in equal and opposite pairs and in the sum of forces of Eq. (7.8), their contribution is zero. Only the external forces contribute to the equation. We can then rewrite Eq. (7.9) as

$MA = F_{ext}$ ------- (7.10)

where F_{ext} represents the sum of all external forces acting on the particles of the system.

The above eqn (7.9) states that the centre of mass of a system of particles moves as if all the mass of the system was concentrated at the centre of mass and all the external forces were applied at that point.

6. Prove the law of conservation of the total linear momentum of a system of particles. ?

Let us recall that the linear momentum of a particle is defined as

m = pv ------------ (7.11)

Let us also recall that Newton's second law written in symbolic form for a single particle is F = dP/dt ---------- (7.12)

where F is the force on the particle. Let us consider a system of n particles with masses m_1, m_2,...m_n respectively and velocities v_1, v_2, …… v_n respectively. The particles may be interacting and have external forces acting on them. The linear momentum of the first particle is $m_1 v_1$, of the second particle is $m_2 v_2$ and so on.

For the system of n particles, the linear momentum of the system is defined to be the vector sum of all individual particles of the system,

$P = p_1 + p_2 + p_3 \ldots\ldots\ldots + p_n$

$= m_1v_1 + m_2v_2 + m_3v_3 \ldots\ldots\ldots\ldots + m_nv_n$ ------------ (7.13)

Comparing this with Eq. (7.9)

M = PV (7.14)

Thus, the total momentum of a system of particles is equal to the product of the total mass of the system and the velocity of its centre of mass. Differentiating Eq. (7.14) with respect to time.

$$\frac{d\mathbf{P}}{dt} = M\frac{d\mathbf{V}}{dt} = M\mathbf{A} \qquad (7.16)$$

Comparing Eq.(7.16) and Eq. (7.11),

$$\frac{d\mathbf{P}}{dt} = \mathbf{F}_{ext} \qquad (7.17)$$

This is the statement of Newton's second law extended to a system of particles. Suppose now, that the sum of external forces acting on a system of particles is zero. Then from Eq.(7.17)

$$\frac{d\mathbf{P}}{dt} = 0 \quad \text{or} \quad \mathbf{P} = \text{Constant} \qquad (7.18a)$$

Thus, when the total external force acting on a system of particles is zero, the total linear momentum of the system is constant. This is the law of conservation of the total linear momentum of a system of particles.

7.2 Moment of a force, torque, angular momentum, conservation of angular momentum with some examples.

1. What is Moment of force (Torque) ? Explain ?

We know that

$$\frac{\boldsymbol{dl}}{\boldsymbol{dt}} = \frac{d}{dt}(r \times p) = \frac{dr}{dt} \times p + r \times \frac{dp}{dt} = \mathrm{v} \times \mathrm{mv} + r \times F = 0 + (r \times F) = \boldsymbol{\tau}$$

$$\therefore \quad \frac{\boldsymbol{dl}}{\boldsymbol{dt}} = \boldsymbol{\tau}$$

Thus, the time rate of change of the angular momentum of a particle is equal to **the torque** acting on it.

2. Define Angular momentum of a particle ? Obtain an expression for it ?

The quantity angular momentum is the rotational analogue of linear momentum. It is a vector quantity. Consider a particle of mass m and linear momentum p at a position r relative to the origin O. The angular momentum l of the particle with respect to the origin O is defined to be l = r × p -------- (7.25). The magnitude of the angular momentum vector is :

I = r.p.sin θ ------- (1.26a)

where p is the magnitude of p and θ is the angle between r and p. We may write :

I = $r_\perp$p ----------- (7.26b)

where $r_\perp$ (= r sin θ) is the perpendicular distance of the directional line of p from the origin and $p_\perp$ (= p sin θ) is the component of p in a direction perpendicular to r. We expect the angular momentum to be zero (l = 0), if the linear momentum vanishes (p = 0), if the particle is at the origin (r = 0), or if the directional line of p passes through the origin θ = 0° or 180°.

If we differentiate l = r × p with respect to time, we get :

$$\frac{dl}{dt} = \frac{d}{dt}(r \times p)$$

Applying the product rule for differentiation to the right hand side, we get,

$$\frac{d}{dt}(r \times p) = \frac{dr}{dt} \times p + r \times \frac{dp}{dt} \quad \text{----------} \quad (7.27)$$

$= \mathrm{v} \times \mathrm{mv} + r \times F$ [note : $v \times mv = 0$, as the vector product of two parallel vectors vanishes]

$= 0 + \tau$

$$\frac{d}{dt}(r \times p) = \tau \quad \text{or} \quad \frac{\boldsymbol{dl}}{\boldsymbol{dt}} = \boldsymbol{\tau} \quad \text{------------} \quad (7.28)$$

Thus, the time rate of change of the angular momentum of a particle is equal to the torque acting on it.

3. Explain conservation of angular momentum with some examples ?

The total angular momentum of a system of particles about a given point is L and is given by

L = $\sum r_i \, p_i$ and $\frac{dL}{dt} == \sum \tau_i$

The total torque is sum of total internal torque of the system and total external torque.

$$\tau = \tau_{\text{int}} + \tau_{\text{ext}}$$

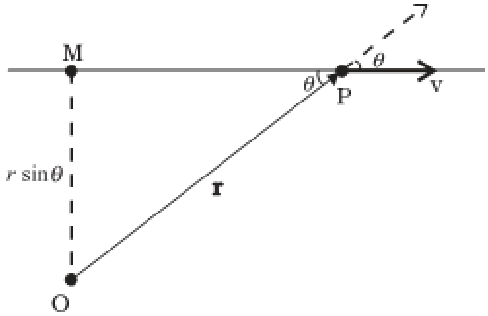
The contribution of internal forces to the total torque of a system is zero, since the torque resulting from each action reaction pair of force is zero. Therefore, $\frac{\boldsymbol{dL}}{\boldsymbol{dt}} = \boldsymbol{\tau} = \boldsymbol{\tau}_{\text{ext}}$

Conservation of Angular momentum :

If total external torque of a system is zero, i.e., τext = 0, then $\frac{\boldsymbol{dL}}{\boldsymbol{dt}} = \mathbf{0},$ or L = constant.

Thus the total external torque on a system of particles is zero, then the total angular momentum of the system is conserved, I.e., remains constant.

Ex : Show that the angular momentum about any point of a single particle moving with constant velocity remains constant throughout the motion.



Answer : Let the particle with velocity v be at point P at some instant t. We want to calculate the angular momentum of the particle about an arbitrary point O.

The angular momentum is l = r × mv. Its magnitude is mvr sinθ, where θ is the angle between r and v as shown in Fig. Although the particle changes position with time, the line of direction of v remains the same and hence OM = r sin θ. is a constant. Further, the direction of l is perpendicular to the plane of r and v. It is into the page of the figure. This direction does not change with time. Thus, l remains the same in magnitude and direction and is therefore conserved.

7.3 Equilibrium of rigid bodies, rigid body rotation and equations of rotational motion, comparison of linear and rotational motions; moment of inertia, radius of gyration.

1. Write a note on Equilibrium of rigid bodies ?

A rigid body is said to be in mechanical equilibrium, if both its linear momentum and angular momentum are not changing with time, or equivalently, the body has neither linear acceleration nor angular acceleration. This means :

(1) the total force, i.e. the vector sum of the forces, on the rigid body is zero;

$$\mathbf{F}_1 + \mathbf{F}_2 + \ldots + \mathbf{F}_n = \sum_{i=1}^{n} \mathbf{F}_i = \mathbf{0}$$

2) The total torque, i.e. the vector sum of the torques on the rigid body is zero,

$$\boldsymbol{\tau}_1 + \boldsymbol{\tau}_2 + \ldots + \boldsymbol{\tau}_n = \sum_{i=1}^{n} \boldsymbol{\tau}_i = \mathbf{0}$$

If the total torque on the rigid body is zero, the total angular momentum of the body does not change with time. Eq. (7.30 b) gives the condition for the rotational equilibrium of the body.

In a number of problems all the forces acting on the body are coplanar. Then we need only three conditions to be satisfied for mechanical equilibrium. Two of these conditions correspond to translational equilibrium; the sum of the components of the forces along any two perpendicular axes in the plane must be zero. The third condition corresponds to rotational equilibrium. The sum of the components of the torques along any axis perpendicular to the plane of the forces must be zero.

A body may be in partial equilibrium, i.e., it may be in translational equilibrium and not in rotational equilibrium, or it may be in rotational equilibrium and not in translational equilibrium.

A pair of equal and opposite forces with different lines of action is known as a **couple**. A couple produces rotation without translation. When we open the lid of a bottle by turning it, our fingers are applying a couple to the lid. **Another known example** is a compass needle in the earth's magnetic field. The earth's magnetic field exerts equal forces on the north and south poles. The force on the North Pole is towards the north, and the force on the South Pole is toward the south. Except when the needle points in the north-south direction; the two forces do not have the same line of action. Thus there is a couple acting on the needle due to the earth's magnetic field.

2. Explain rigid body rotation ? Obtain an expression for momentum of couple ?

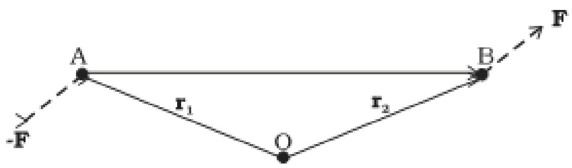
Consider a couple as shown in Fig. acting on a rigid body. The forces F and -F act respectively at points B and A. These points have position vectors r_1 and r_2 with respect to origin O. Let us take the moments of the forces about the origin. The moment of the couple = sum of the moments of the two forces making the couple.

$= \mathbf{r}_1 \times (-\mathbf{F}) + \mathbf{r}_2 \times \mathbf{F}$

$= \mathbf{r}_2 \times \mathbf{F} - \mathbf{r}_1 \times \mathbf{F}$

$= (\mathbf{r}_2 - \mathbf{r}_1) \times \mathbf{F}$

But $\mathbf{r}_1 + \mathbf{AB} = \mathbf{r}_2$, and hence $\mathbf{AB} = \mathbf{r}_2 - \mathbf{r}_1$.



The **moment of the couple**, therefore, is **AB** × **F**. Clearly this is independent of the origin, the point about which we took the moments of the forces.

The product of the forces forming the couple and the arm of the couple is called the moment of the couple or torque.

Torque = one of the forces × perpendicular distance between the forces.

The torque in rotational motion plays the same role as the force in translational motion. A quantity that is a measure of this rotational effect produced by the force is called torque.

In vector notation, $\vec{\tau} = \vec{r} \times \vec{F}$

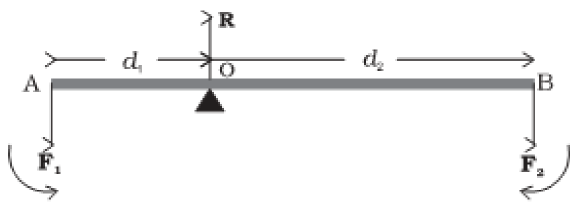
The torque is maximum when θ = 90° (i.e.) when the applied force is at right angles to $\vec{r}$.

Examples of couple are :

1. Forces applied to the handle of a screw press,
2. Opening or closing a water tap.
3. Turning the cap of a pen.
4. Steering a car.

3. Explain Principle of moments of a lever ?

An ideal lever is essentially a light (i.e. of negligible mass) rod pivoted at a point along its length. This point is called the fulcrum. A seesaw on the children's playground is a typical example of a lever. Two forces F_1 and F_2, parallel to each other and usually perpendicular to the lever, as shown here, act on the lever at distances d_1 and d_2 respectively from the fulcrum as shown in Fig.



The lever is a system in mechanical equilibrium. Let R be the reaction of the support at the fulcrum; R is directed opposite to the forces F_1 and F_2. For translational equilibrium,

$R - F_1 - F_2 = 0$ ----------- (i)

For considering rotational equilibrium we take the moments about the fulcrum; the sum of moments must be zero,

$d_1F_1 - d_2F_2 = 0$ ---------- (ii)
Normally the anticlockwise (clockwise) moments are taken to be positive (negative). Note R acts at the fulcrum itself and has zero moment about the fulcrum.
In the case of the lever force F_1 is usually some weight to be lifted. It is called the load and its distance from the fulcrum d_1 is called the load arm. Force F_2 is the effort applied to lift the load; distance d_2 of the effort from the fulcrum is the effort arm. Eq. (ii) can be written as
$d_1F_1 = d_2 F_2$ -------------- (a)
or load arm × load = effort arm × effort
The above equation expresses the principle of moments for a lever. Incidentally the ratio F_1/F_2 is called the Mechanical Advantage (M.A.);
M.A. $= \frac{F_1}{F_2} = \frac{d_2}{d_2}$ --------- (b)
If the effort arm d_2 is larger than the load arm, the mechanical advantage is greater than one. Mechanical advantage greater than one means that a small effort can be used to lift a large load. There are several examples of a lever around us besides the see-saw. The beam of a balance is a lever.
One can show that the principle of moment holds even when the parallel forces F_1 and F_2 are not perpendicular, but act at some angle, to the lever.

4. What is Centre of gravity ? Obtain the value of the total gravitational torque about the centre of gravity ?
Centre of Gravity (CG) of a body as that point where the total gravitational torque on the body is zero.
Take an irregular-shaped cardboard and a narrow tipped object like a pencil. You can locate by trial and error a point G on the cardboard where it can be balanced on the tip of the pencil. (The cardboard remains horizontal in this position.) This point of balance is the centre of gravity (CG) of the cardboard. The tip of the pencil provides a vertically upward force due to which the cardboard is in mechanical equilibrium. As shown in the Fig. 7.24, the reaction of the tip is equal and opposite to Mg, the total weight of (i.e., the force of gravity on) the cardboard and hence the cardboard is in translational equilibrium. It is also in rotational equilibrium; if it were not so, due to the unbalanced torque it would tilt and fall. There are torques on the card board due to the forces of gravity like m_1g, m_2g …. etc, acting on the individual particles that make up the cardboard.

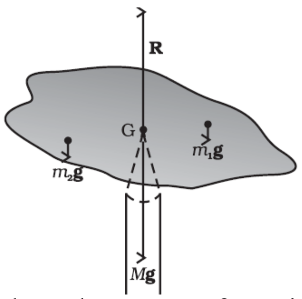


Fig. 7.24 : *Balancing a cardboard on the tip of a pencil. The point of support,* G, *is the centre of gravity.*

The CG of the cardboard is so located that the total torque on it due to the forces m_1g, m_2g …. etc. is zero. If r_i is the position vector of the i^{th} particle of an extended body with respect to its CG, then the torque about the CG, due to the force of gravity on the particle is $\tau_i = r_i \times m_ig$. The total gravitational torque about the CG is zero, i.e.

$$\boldsymbol{\tau}_g = \sum \boldsymbol{\tau}_i = \sum \mathbf{r}_i \times m_i\mathbf{g} = 0$$

Thus, the centre of gravity of the body coincides with the centre of mass. We note that this is true because the body being small, g does not vary from one point of the body to the other. If the body is so extended that g varies from part to part of the body, then the centre of gravity and centre of mass will not coincide. Basically, the two are different concepts. The centre of mass has nothing to do with gravity. It depends only on the distribution of mass of the body.

5. Explain moment of inertia of a rotating body ? hence, obtain an expression for radius of gyration ?

We know that for a body rotating about a fixed axis, each particle of the body moves in a circle with linear velocity. For a particle at a distance from the axis, the linear velocity is $v_i = r_i \omega$. The kinetic energy of motion of this particle is

$$k_i = \frac{1}{2}m_i v_i^2 = \frac{1}{2}m_i r_i^2 \omega^2$$

where m_i is the mass of the particle. The total kinetic energy K of the body is then given by the sum of the kinetic energies of individual particles,

$$K = \sum_{i=1}^{n} k_i = \frac{1}{2}\sum_{i=1}^{n}(m_i r_i^2 \omega^2)$$

Here n is the number of particles in the body. Note ω is the same for all particles. Hence, taking ω out of the sum,

$$K = \frac{1}{2}\omega^2 (\sum_{i=1}^{n} m_i r_i^2)$$

We define a new parameter characterizing the rigid body, called the moment of inertia I, given by

$$I = \sum_{i=1}^{n} m_i r_i^2$$

With this definition

$$K = \frac{1}{2}I\omega^2 \quad \text{-------------} \quad (7.35)$$

Note that the parameter I is independent of the magnitude of the angular velocity. It is a characteristic of the rigid body and the axis about which it rotates.

Compare Eq. (7.35) for the kinetic energy of a rotating body with the expression for the kinetic energy of a body in linear (translational) motion,

$$K = \frac{1}{2}m\,v^2$$

Here m is the mass of the body and v is its velocity. We have already noted the analogy between angular velocity ω (in respect of rotational motion about a fixed axis) and linear velocity v (in respect of linear motion). It is then evident that the parameter, moment of inertia I, is the desired rotational analogue of mass. In rotation (about a fixed axis), the moment of inertia plays a similar role as mass does in linear motion.

Examples to calculate Moment of Inertia :

(a) Consider a thin ring of radius R and mass M, rotating in its own plane around its centre with angular velocity ω. Each mass element of the ring is at a distance R from the axis, and moves with a speed Rω. The kinetic energy is therefore,

$$K = \frac{1}{2}Mv^2 = \frac{1}{2}MR^2\omega^2$$

Comparing with Eq. (7.35) we get $I = MR^2$ for the ring.

(b) Next, take a rigid massless rod of length l with a pair of small masses, rotating about an axis through the centre of mass perpendicular to the rod (Fig. 7.28). Each mass M/2 is at a distance l/2 from the axis. The moment of inertia of the masses is therefore given by

$(M/2)(l/2)^2 + (M/2)(l/2)^2$

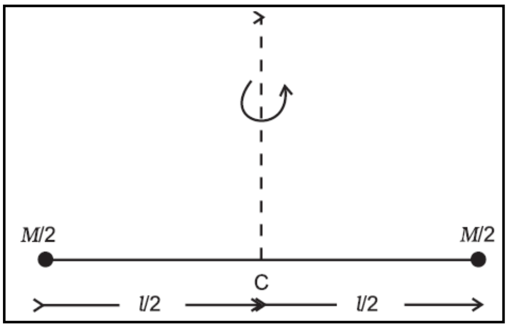


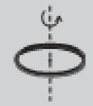
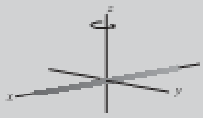
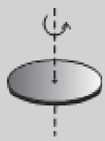
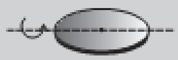
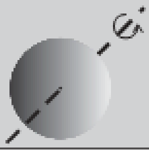
Fig. 7.28 : *A light rod of length l with a pair of masses rotating about an axis through the centre of mass of the system and perpendicular to the rod. The total mass of the system is M.*

Thus, for the pair of masses, rotating about the axis through the centre of mass perpendicular to the rod

$I = Ml^2 / 4$

Table 7.1 gives the moment of inertia of various familiar regular shaped solids about specific axes :

Table 7.1 Moments of Inertia of some regular shaped bodies about specific axes

Z	Body	Axis	Figure	I
(1)	Thin circular ring, radius R	Perpendicular to plane, at centre		$M R^2$
(2)	Thin circular ring, radius R	Diameter		$M R^2/2$
(3)	Thin rod, length L	Perpendicular to rod, at mid point		$M L^2/12$
(4)	Circular disc, radius R	Perpendicular to disc at centre		$M R^2/2$
(5)	Circular disc, radius R	Diameter		$M R^2/4$
(6)	Hollow cylinder, radius R	Axis of cylinder		$M R^2$
(7)	Solid cylinder, radius R	Axis of cylinder		$M R^2/2$
(8)	Solid sphere, radius R	Diameter		$2 M R^2/5$

As the mass of a body resists a change in its state of linear motion, it is a measure of its inertia in linear motion. Similarly, as the moment of inertia about a given axis of rotation resists a change in its rotational motion, it can be regarded as a measure of rotational inertia of the body; it is a measure of the way in which different parts of the body are distributed at different distances from the axis. Unlike the mass of a body, the moment of inertia is not a fixed quantity but depends on the orientation and position of the axis of rotation with respect to the body as a whole. As a measure of the way in which the mass of a rotating rigid body is distributed with respect to the axis of rotation, we can define a new parameter, the radius of gyration. It is related to the moment of inertia and the total mass of the body.

In general for moment of inertia, we can write $I = Mk^2$, where k has the dimension of length. For a rod, about the perpendicular axis at its midpoint, i.e. $k^2 = L^2/12$. Similarly, $k = R/2$ for the circular disc about its diameter. The length k is a geometric property of the body and axis of rotation. It is called the **radius of gyration**. $k = \sqrt{\frac{I}{M}}$. The radius of gyration of a body about an axis may be defined as the distance from the axis of a mass point whose mass is equal to the mass of the whole body and whose moment of inertia is equal to the moment of inertia of the body about the axis.

Thus, the moment of inertia of a rigid body depends on the mass of the body, its shape and size; distribution of mass about the axis of rotation, and the position and orientation of the axis of rotation.

The dimensions of moments of inertia we ML^2 and its SI units are kg m^2. The property of this extremely important quantity I as a measure of rotational inertia of the body has been put to a great practical use.

The machines, such as steam engine and the automobile engine, etc., that produce rotational motion have a disc with a large moment of inertia, called a flywheel. Because of its large moment of inertia, the flywheel resists the sudden increase or decrease of the speed of the vehicle. It allows a gradual change in the speed and prevents jerky motions, thereby ensuring a smooth ride for the passengers on the vehicle.

The **radius of gyration of a body** about an axis may be defined as the distance from the axis of a mass point whose mass is equal to the mass of the whole body and whose moment of inertia is equal to the moment of inertia of the body about the axis. $k = \sqrt{\frac{I}{M}}$.

6. Obtain the following equations of rotational motion (a) Kinematic equations for rotational motion and (b) Dynamics of rotational motion about a fixed axis ?

(a) Kinematic equations for rotational motion :

In case of kinamatics of rotation about fixed axis, the motion involves only one degree of freedom, i.e., needs only one independent variable to describe the motion. This in translation corresponds to linear motion.

For specifying the angular displacement of the rotating body consider any particle like P (Fig.7.33) of the body. Its angular displacement θ in the plane it moves is the angular displacement of the whole body; θ is measured from a fixed direction in the plane of motion of P, which we take to be the x′ - axis, chosen parallel to the x-axis. Note, as shown, the axis of rotation is the z – axis and the plane of the motion of the particle is the x - y plane. Fig. 7.33 also shows θ_0, the angular displacement at t = 0.
We know that the angular velocity is the time rate of change of angular displacement, **ω = dθ/dt**. Note since the axis of rotation is fixed, there is no need to treat angular velocity as a vector. Further, the angular acceleration, **α = dω /dt.**

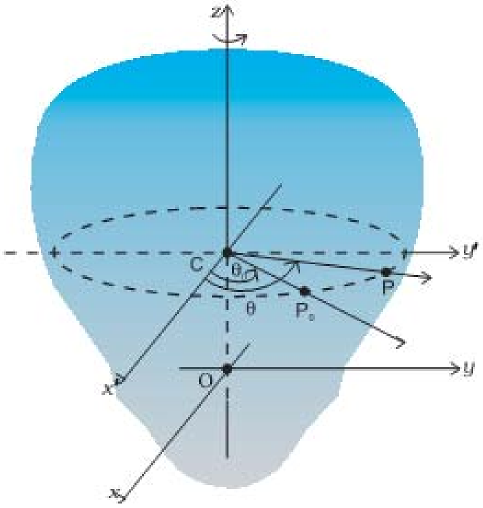


Fig.7.33 Specifying the angular position of a rigid body.

The kinematical quantities in rotational motion, angular displacement (θ), angular velocity (ω) and angular acceleration (α) respectively correspond to kinematic quantities in linear motion, displacement (x), velocity (v) and acceleration (a). We know the kinematical equations of linear motion with uniform (i.e. constant) acceleration:

$$v = v_0 + at \quad \text{(a)}$$

$$x = x_0 + v_0 t + \frac{1}{2}at^2 \quad \text{(b)}$$

$$v^2 = v_0^2 + 2ax \quad \text{(c)}$$

where x_0 = initial displacement and v_0 = initial velocity. The word ‘initial’ refers to values of the quantities at t = 0 The corresponding kinematic equations for rotational motion with uniform angular acceleration are:

$$\omega = \omega_0 + \alpha t$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

and $\omega^2 = \omega_0{}^2 + 2\alpha(\theta - \theta_0)$

where θ_0 = initial angular displacement of the rotating body, and ω_0 = initial angular velocity of the body.

(b) Dynamics of rotational motion about a fixed axis :

In the case of rotational motion about a fixed axis, we have to consider following two conditions :

1) We need to consider only those forces that lie in planes perpendicular to the axis. Forces which are parallel to the axis will give torques perpendicular to the axis and need not be taken into account.

(2) We need to consider only those components of the position vectors which are perpendicular to the axis. Components of position vectors along the axis will result in torques perpendicular to the axis and need not be taken into account.

Work done by a torque :

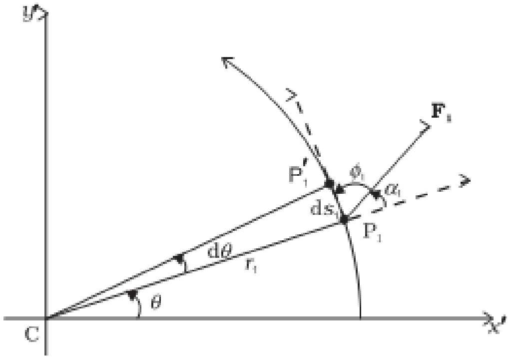


Fig. 7.34 : Work done by a force F1 acting on a particle of a body rotating about a fixed axis; the particle describes a circular path with centre C on the axis; arc $P_1P'_1(ds_1)$ gives the displacement of the particle.

Figure 7.34 shows a cross-section of a rigid body rotating about a fixed axis, which is taken as the z-axis. Here, we need to consider only those forces which lie in planes perpendicular to the axis. Let F_1 be one such typical force acting as shown on a particle of the body at point P_1 with its line of action in a plane perpendicular to the axis. For convenience we call this to be the $x' - y'$ plane (coincident with the plane of the page). The particle at P_1 describes a circular path of radius r_1 with centre C on the axis; $CP_1 = r_1$.
In time Δt, the point moves to the position P_1' . The displacement of the particle ds_1, therefore, has magnitude $ds_1 = r_1 d\theta$ and direction tangential at P_1 to the circular path as shown. Here $d\theta$ is the angular displacement of the particle, $d\theta = \angle P_1CP_1'$. The work done by the force on the particle is $dW_1 = F_1 . ds_1 = F_1 ds_1 \cos\varphi_1 = F_1(r_1 \, d\theta)\sin\alpha_1$ where φ_1 is the angle between F_1 and the tangent at P_1, and α_1 is the angle between F_1 and the radius vector OP_1; $\varphi_1 + \alpha_1 = 90°$.
The torque due to F_1 about the origin is $OP_1 \times F_1$. Now $OP_1 = OC + CP_1$. [Refer to Fig. 7.17(b).] Since OC is along the axis, the torque resulting from it is excluded from our consideration. The effective torque due to F_1 is $\tau_1 = CP_1 \times F_1$; it is directed along the axis of rotation and has a magnitude $\tau_1 = r_1F_1 \sin\alpha$, Therefore,
$dW_1 = \tau_1 d\theta$
If there are more than one forces acting on the body, the work done by all of them can be added to give the total work done on the body. Denoting the magnitudes of the torques due to the different forces as $\tau_1, \tau_2, \ldots$ etc,
$dW = (\tau_1 + \tau_2 + \tau_3 \, \ldots\ldots) \, d\theta$
Remember, the forces giving rise to the torques act on different particles, but the angular displacement $d\theta$ is the same for all particles. Since all the torques considered are parallel to the fixed axis, the magnitude τ of the total torque is just the algebraic sum of the magnitudes of the torques, i.e., $\tau = \tau_1 + \tau_2 + \ldots..$
We, therefore, have
$dW = \tau d\theta$ ------------- (7.41)
This expression gives the work done by the total (external) torque τ which acts on the body rotating about a fixed axis. Its similarity with the corresponding expression
$dW = F \, ds$

for linear (translational) motion is obvious.
Dividing both sides of Eq. (7.41) by dt gives

$P = \frac{dW}{dt} = \tau \frac{d\theta}{dt} = \tau\, \omega$ or $P = \tau\, \omega$ -------- (7.42)

This is the instantaneous power. Compare this expression for power in the case of rotational motion about a fixed axis with the expression for power in the case of linear motion,
P = Fv
In a perfectly rigid body there is no internal motion. The work done by external torques is therefore, not dissipated and goes on to increase the kinetic energy of the body. The rate at which work is done on the body is given by Eq. (7.42).
This is to be equated to the rate at which kinetic energy increases. The rate of increase of kinetic energy is

$$\frac{d}{dt}\left(\frac{I\omega^2}{2}\right) = I\frac{(2\omega)}{2}\frac{d\omega}{dt}$$

We assume that the moment of inertia does not change with time. This means that the mass of the body does not change, the body remains rigid and also the axis does not change its position with respect to the body.

Since $\alpha = d\omega/dt =$ we get

$$\frac{d}{dt}\left(\frac{I\omega^2}{2}\right) = I\,\omega\,\alpha$$

Equating rates of work done and of increase in kinetic energy, $\tau\omega = I\,\omega\,\alpha$
$\tau = I\,\alpha$ ------- (7.43)
Eq. (7.43) is similar to Newton's second law for linear motion expressed symbolically as
F = ma
Just as force produces acceleration, torque produces angular acceleration in a body. The angular acceleration is directly proportional to the applied torque and is inversely proportional to the moment of inertia of the body. Eq.(7.43) can be called Newton's second law for rotation about a fixed axis.

6. Comparison of linear and rotational motions ?

Ans :

	Linear Motion	Rotational Motion about a Fixed Axis
1	Displacement x	Angular displacement θ
2	Velocity $v = dx/dt$	Angular velocity $\omega = d\theta/dt$
3	Acceleration $a = dv/dt$	Angular acceleration $\alpha = d\omega/dt$
4	Mass M	Moment of inertia I
5	Force $F = Ma$	Torque $\tau = I\,\alpha$
6	Work $dW = F\,ds$	Work $W = \tau\, d\theta$
7	Kinetic energy $K = Mv^2/2$	Kinetic energy $K = I\omega^2/2$
8	Power $P = F\,v$	Power $P = \tau\omega$
9	Linear momentum $p = Mv$	Angular momentum $L = I\omega$

7.4 Values of moments of inertia, for simple geometrical objects (no derivation). Statement of parallel and perpendicular axes theorems and their applications

1. Explain on what factors the values of moments of inertia depends ? What is its unit and Dimension ?

The moment of inertia of a rigid body depends on the mass of the body, its shape and size; distribution of mass about the axis of rotation, and the position and orientation of the axis of rotation. From the definition of moment of inertia, I $=\sum_{i=1}^{n} m_i\, r_i^2$ [Eq. (7.34)], we can infer that the dimensions of moments of inertia we ML^2 and its SI units are kg m^2.

The machines, such as steam engine and the automobile engine, etc., that produce rotational motion have a disc with a large moment of inertia, called a flywheel. Because of its large moment of inertia, the flywheel resists the sudden increase or decrease of the speed of the vehicle. It allows a gradual change in the speed and prevents jerky motions, thereby ensuring a smooth ride for the passengers on the vehicle.

2. What is rolling motion ? Obtain an expression for kinetic energy of rolling motion ?

Rolling of a wheel or disc without slipping is called rolling motion. Rolling motion is a combination of rotation and translation.

The kinetic energy of a rolling body can be separated into kinetic energy of translation and kinetic energy of rotation. This is a special case of a general result for a system of particles, according to which the kinetic energy of a system of particles (K) can be separated into the kinetic energy of motion of the centre of mass (translation) ($MV^2/2$) and kinetic energy of rotational motion about the centre of mass of the system of particles (K′). Thus,

$K = K' + ½ MV^2$ ------ (1)

In our notation, the kinetic energy of the centre of mass, i.e., the kinetic energy of translation, of the rolling body is $mv_{cm}^2/2$, where m is the mass of the body and v_{cm} is the centre of the mass velocity. Since the motion of the rolling body about the centre of mass is rotation, K′ represents the kinetic energy of rotation of the body; $K' = I\omega^2/2$, where I is the moment of inertia about the appropriate axis, which is the symmetry axis of the rolling body. The kinetic energy of a rolling body, therefore, is given by

$K = ½ I\omega^2 + ½ mv_{cm}^2$ ---------- (2)

Substituting $I = mk^2$ where k = the corresponding radius of gyration of the body and $v_{cm} = R\omega$, we get

$$K = \frac{1}{2}\frac{mk^2 v_{cm}^2}{R^2} + \frac{1}{2}mv_{cm}^2$$

$$\text{or} \quad K = \frac{1}{2}mv_{cm}^2\left(1+\frac{k^2}{R^2}\right) \qquad \text{-------} \quad (3)$$

Equation (3) is an expression for kinetic energy of rolling motion and applies to any rolling body: a disc, a cylinder, a ring or a sphere.

3. Compare translational motion and rotational motion

Comparison of translational motion and rotational motion :

1	Linear displacement	$\vec{d}$	Angular displacement	θ
2	Linear velocity	$\vec{v}$	Angular velocity	$\vec{\omega}$
3	Linear acceleration	$\vec{a} = \frac{d\vec{v}}{dt}$	Angular acceleration	$\vec{\alpha} = \frac{d\vec{\omega}}{dt}$
4	Mass	m	Moment of inertia	I
5	Linear momentum	$\vec{p} = m\vec{v}$	Angular momentum	$\vec{L} = I\vec{\omega}$
6	Force	$\vec{F} = m\vec{a}$	Torque	$\vec{\tau} = I\vec{\alpha}$
7	Newton's Second Law of motion	$\vec{F} = \frac{d\vec{P}}{dt}$	Result similar to Newton's Second Law	$\vec{\tau} = \frac{d\vec{L}}{dt}$
8	Translational kinetic energy	$K = \frac{1}{2}mv^2$	Rotational kinetic energy	$K = \frac{1}{2}I\omega^2$
9	Work	$W = \vec{F}\cdot\vec{d}$	Work	$W = \tau\theta$
10	Power	$P = Fv$	Power	$P = \tau\omega$
11	Equations of linear motion with constant linear acceleration	$v = v_0 + at$ $d = v_0 t + \frac{1}{2}at^2$ $2ad = v^2 - v_0^2$	Equations of rotational motion with constant angular acceleration	$\omega = \omega_0 + \alpha t$ $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$ $2\alpha\theta = \omega^2 - \omega_0^2$

4. Moment of inertia and radius of gyration for some symmetric bodies :

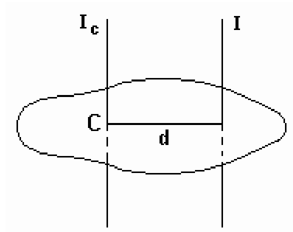
	Body	Axis	I	K
1	Thin rod of length L	Passing through its centre and perpendicular to its length	$\frac{1}{12}ML^2$	$\frac{L}{2\sqrt{3}}$
2	Ring of radius R → Thin-walled hollow cylinder of radius R →	Passing through its centre and perpendicular to its plane Geometric axis	MR^2	R
3	Ring of radius R → Circular disc of radius R → Solid cylinder of radius R →	Any diameter Passing through its centre and perpendicular to its plane Geometric axis	$\frac{1}{2}MR^2$	$\frac{R}{\sqrt{2}}$
4	Circular disc of radius R →	Any diameter	$\frac{1}{4}MR^2$	$\frac{R}{2}$
5	Thin-walled hollow sphere of radius R →	Any diameter	$\frac{2}{3}MR^2$	$\sqrt{\frac{2}{3}}R$
6	Solid sphere of radius R →	Any diameter	$\frac{2}{5}MR^2$	$\sqrt{\frac{2}{5}}R$
7	Solid right circular cone of radius R →	Geometric axis	$\frac{3}{10}R^2$	$\sqrt{\frac{3}{10}}R$

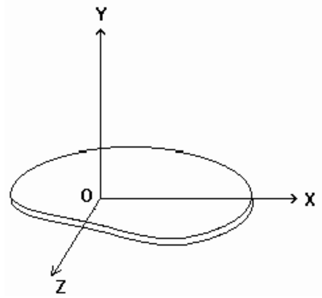
5. State and explain parallel and perpendicular axes theorems of moment of inertia?

(i) Parallel axes theorem :

The moment of inertia (I) of a body about a given axis is equal to the sum of its moment of inertia I_C about a parallel axis passing through its centre of mass and the product of its mass and square of perpendicular distance (d) between the two axes.

$I = I_C + Md^2$ ------------ (1)





(ii) Perpendicular axes theorem :

(a) For laminar bodies : For laminar bodies, the moment of inertia I_Z about Z-axis normal to its plane is equal to the sum of its moments of inertia about X-axis, I_X and Y-axis, I_Y. $I_Z = I_X + I_Y$ --------- (2)

(b) For three-dimensional bodies : The sum of moments of inertia of a three dimensional body about any three mutually perpendicular axes drawn through the same point is equal to twice the moment of inertia of the body about that point I_0.

$I_Z + I_X + I_Y = 2I_0$ ----------- (3)

6. Calculate moment of inertia of certain symmetric objects :

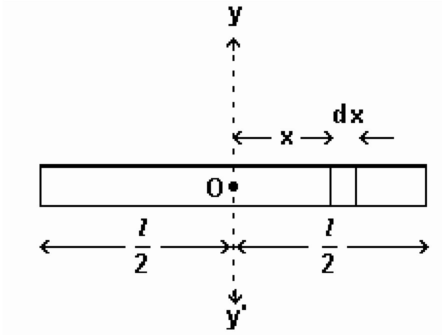
Calculation of moment of inertia of certain symmetric objects :

(a) Moment of inertia of a thin uniform rod about an axis, passing through its centre and perpendicular to its length :

To calculate moment of inertia of a thin rod of length l and mass M about an axis yy' passing through its centre O and perpendicular to its length, consider O as origin and X-axis along the length of the rod. A small element of length dx of the rod is at a distance x from O.

The moment of inertia of this element about yy' is

$dI = \frac{M}{l}\,dx.x^2$ $\therefore$ Moment of inertia of rod is = I =



$$I = \int_{-\frac{l}{2}}^{+\frac{l}{2}} \frac{M}{l}\,dx \cdot x^2 = \frac{M}{l}\left[\frac{x^3}{3}\right]_{-\frac{l}{2}}^{+\frac{l}{2}}$$

$$= \frac{M}{3l}\left[\frac{l^3}{8} + \frac{l^3}{8}\right] = \frac{Ml^2}{12}$$

(b) Moment of inertia of a thin ring or a thin walled hollow cylinder or a thin walled hollow sphere :

As the entire mass, M, of a thin ring is at the same distance, equal to the radius R of the ring from its centre, the moment of inertia of a thin ring about an axis passing through its centre and perpendicular to its plane is MR^2. Similarly, the moments of inertia of a thin walled cylinder about its geometric axis or of a thin walled hollow sphere about its centre are also given by MR^2, where M represents their mass and R their radii.

(c) Moment of inertia of a disc or a solid cylinder :

To calculate moment of inertia of a disc of uniform thickness t, radius R and mass M about an axis passing through its centre and perpendicular to its plane, consider an element of the disc in the form of a thin ring of thickness dx at a distance x from its centre. Mass of this ring is $2\pi x\,dx \cdot t \cdot \rho$, where ρ is the density of the material of the ring. Therefore, the moment of inertia of the ring about an axis passing through the centre, O, of the disc and perpendicular to its plane is

$$dI = (2\pi x\,dx \cdot t \cdot \rho)\,x^2 = (2\pi t \rho)\,x^3\,dx$$

$\therefore$ moment of inertia of the disc about an axis passing through its centre and perpendicular to its plane is

$$I = \int_0^R (2\pi t\rho)\,x^3\,dx = (2\pi t\rho)\left[\frac{x^4}{4}\right]_0^R = 2\pi t\rho\,\frac{R^4}{4} = \frac{1}{2}\left(\pi R^2 t\rho\right)R^2$$

$$= \frac{1}{2}MR^2$$

Similarly, moment of inertia of a solid cylinder about its axis is also $\frac{1}{2}MR^2$.

(d) Moment of inertia of a thin walled hollow sphere about its diameter :

Moment of inertia of a thin walled hollow sphere about its centre is $I_0 = MR^2$.

By perpendicular axes theorem for three dimensional bodies, $2\, I_0 = I\,x + I\,y + I\,z$.
Now, $Ix = Iy = Iz$ = Moment of inertia, I, of the hollow sphere about its diameter.

$$\therefore \quad I = \frac{2}{3} I_0 = \frac{2}{3} MR^2.$$

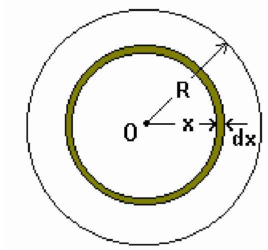
(e) Moment of inertia of a solid sphere about its centre :

Let the solid sphere of mass M be of radius R. Consider a thin spherical shell of radius x and of thickness dx. The mass of this shell is $4\pi x^2 \cdot dx\,\rho$, where ρ is the density of the material of the sphere. Hence the moment of inertia of the shell about the centre O is : $\mathbf{d\,I_0 = 4\,\pi\,x^2 \cdot dx\,\rho \cdot x^2}$

Therefore, the moment of inertia of the solid sphere about its centre is

$$I_0 = 4\pi\rho \int_0^R x^4\,dx = 4\pi\rho\left[\frac{x^5}{5}\right]_0^R = \frac{4\pi\rho}{5} R^5$$

$$= \frac{4}{3}\pi R^3 \rho \cdot \frac{3R^2}{5} = \frac{3}{5} MR^2$$



(f) Moment of inertia of a solid sphere about its diameter :

The moments of inertia of the solid sphere about three mutually perpendicular axes passing through its centre given by Ix, Iy and Iz are all equal and represent the moment of inertia, I, of the solid sphere about the diameter.

$\therefore \quad I = Ix = Iy = Iz$

By theorem of perpendicular axes in three dimensions, $2\, I_0 = Ix + Iy + Iz = 3I$

$$\therefore \quad I = \frac{2}{3} I_0 = \frac{2}{3} \times \frac{3}{5} MR^2 = \frac{2}{5} MR^2$$

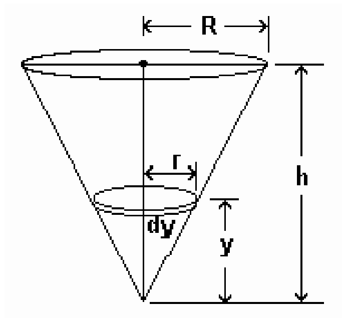
(g) Moment of inertia of a solid cone about its geometric axis :

Consider a disc of radius r and thickness dy at a height y from the vertex of the cone.
Mass of disc = dm = volume × density = $\pi r^2\, dy \cdot \rho$.

$$\therefore \quad \text{M. I. of disc} = d\,I = \frac{(dm)\,r^2}{2} = \frac{\pi r^4 \rho\, dy}{2}$$

From the geometry of the figure, $\frac{r}{y} = \frac{R}{h} \quad \therefore \quad r = \frac{R}{h}\, y$

$$\therefore \quad d\,I = \frac{\pi R^4 \rho}{2\,h^4}\, y^4\, dy$$



$$\therefore \quad I = \frac{\pi R^4 \rho}{2h^4} \int_0^h y^4\, dy = \frac{\pi R^4 \rho}{2h^4} \left[\frac{y^5}{5} \right]_0^h$$

$$= \frac{\pi R^4 \rho}{2h^4} \frac{h^5}{5} \qquad = \frac{3}{10} \left[\frac{1}{3} \pi R^2 h \rho \right] R^2$$

$$= \frac{3}{10} M R^2$$

One mark Questions

1. What is a Rigid body?
A Rigid body is one for which the distances between different particles of the body do not change even though there are forces on them.

2. What type of motion a rigid body may have when it is fixed along an axis.
Rotational motion.

3. When does a rigid body said to have rotational motion?
A Rigid body is said to have rotational motion about a fixed axis if every particle of the body moves in a circle which lies in a plane perpendicular to the axis and has its centre on the axis.

4. What you mean precession of a spinning top?
The movement of the axis of the top around the vertical is called precession of the spinning top.

5. What is centre of mass of a system of particles?
Centre of mass of a system of particles is the point where the entire mass of the system can be assumed to be concentrated.

6. What is the location of the centre of mass of a lamina of triangular shape.
At the centroid of the triangle.

7. Give the location of centre of mass of sphere of uniform mass density?
At the geometric centre.

8. Give the location of centre of mass of cylinder of uniform mass density.
At the centre of its axis of symmetry

9. Give the location of centre of mass of ring of uniform mass density?
At the centre of ring.

10. Give the location of centre mass of a cube of uniform mass density?
At its geometrical centre.

11. Does the centre of mass of a body necessarily lie inside the body?
No, (it may lie outside the body also)

12. Give the expression for moment of inertia about an axis passing through the centre perpendicular to its plane.
$I = Mr^2$

13. Give an example for a body whose centre of mass lies inside the body.
Solid sphere or solid cube.

14. Give an example for a body whose centre of mass lies outside the body
Ring

15. Give the expression for moment of Inertia of a thin rod about an axis perpendicular to the rod and passing through its mid point.

I = $\frac{ML^2}{12}$

16. Write the expression for the moment of inertia of a circular disc of radius R about an axis perpendicular to it and passing through its centre.

I = $\frac{MR^2}{2}$

17. Write the expression for the moment of inertia of a circular disc of radius R about its diameter.

I = $\frac{MR^2}{4}$

18. Give the expression for moment inertia of a hollow cylinder of radius R about its axis.

I = MR^2

19. Give the expression for moment of inertia of a solid cylinder of radius R about its axis.

I = $\frac{MR^2}{2}$

20. Give and expression for the moment of inertia of a solid sphere of radius R about its diameter.

I = $\frac{2MR^2}{5}$

21. Define linear momentum of a system of particles.

Total momentum of a system of particles is equal to the product of the total mass of the system and the velocity of its centre of mass.

22. What is the total external force on a system of particles when its total momentum is constant.

Zero

23. What will be the nature of motion of centre of mass of a system when total external force acting on the system is zero.

Moves uniformly in a straight line.

24. A moving Radium nucleus decays into Radon and an α particle. The two particles produced during decay move in different directions. What is the direction of motion of the centre of mass after decay?

The centre of mass moves along the original path.

25. Mention any one rule to find the direction of vector product of two vectors.

Rule of Right handed screw or Rule of the right hand.

26. What is the vector product of two parallel vectors?

Zero.

27. What is the angle between a ൈ b and b ൈ a ?
180°

28. Write the relation between linear velocity and angular velocity?
$v = \omega r$

29. Write the S.I. unit of angular velocity
Rad S^{-1}

30. Write the dimension of angular velocity
$[M^0L^0T^{-1}]$

31. Define angular acceleration
The time rate of change of angular velocity ($\alpha = \frac{d\omega}{dt}$)

32. Is moment of force a vector or a scalar?
Vector.

33. What is the mechanical advantage of a lever.
Using small effort one can lift large load.

34. What is meant by a mechanical advantage of a lever?
The Mechanical advantage of a lever is the ratio of load to the effort.

35. Write the expression for the torque in terms of position vector and force.
$\tau = r \times F$

36. What is the S.I. unit of Torque.
Nm

37. Write the dimensions of torque
$[M^1L^2T^{-2}]$

38. Write the expression for the angular momentum in terms of linear momentum and position vector.
$l = r \times p$

39. Write the expression for angular momentum in terms of moment of inertia and angular velocity
$L = I\,\omega$

40. Define angular momentum.
Moment of momentum ($L = \sum_{i=1}^{n} r_i \times p_i$)

41. The time rate of change of angular momentum of a particle is equal to the torque acting on it. Is it true or false?
TRUE

42. What is the torque acting on a system when total angular momentum of a system is constant.
ZERO

43. Define a couple.
A pair of equal and opposite forces with different lines of action is known as a couple.

44. Define moment of a couple.
The moment of a couple is equal to the sum of the moments of the two forces making the couple.

45. The mechanical advantage of a lever is greater than one what does it mean?
A small effort is enough to lift a large load.

46. Write the expression for moment of inertia.
($I = \sum_{i=1}^{n} m_i r_i^2$)

47. Is moment inertia a vector or a scalar?
Scalar

48. Write the SI unit of moment of inertia.
Kgm^2

49. Give the dimensions of moment of inertia.
$[M^1L^2T^0]$

50. Define the term Radius of gyration
The radius of gyration of a body about an axis may be defined as the distance from the axis of a mass point where mass is equal to the moss of the whole body and whose moment of inertia is equal to the moment of inertia of the body about the axis.

51. How is Torque related to the angular momentum.
$$\tau = \frac{dl}{dt}$$
or Torque is proportional to the time rate of change of angular momentum.

52. What is the magnitude of torque acting in a body rotating with a constant angular momentum.
ZERO

53. Name the physical quantity which is equal to the time rate of change of angular momentum.
Torque.

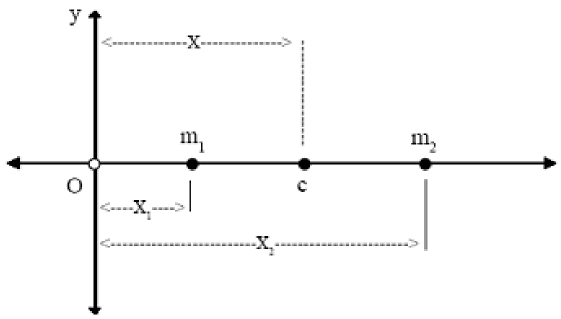
54. Does the moment of inertia of a thin rod change with change of the axis of rotation?

YES

55. Three bodies, a ring, a solid cylinder and a solid sphere roll down the same in lined plane without slipping. They start from rest. The radius of the bodies are identical. Which body has the greatest rotational kinetic energy while reaching the bottom of the inclined plane?
Sphere has the greatest rotational kinetic energy while reaching the bottom of the inclined plane.

Two marks Questions

1. Two particles of equal mass are at a distances of X_1 and X_2 from the origin of a co-ordinate system. Find the distance of their centre mass from the origin.



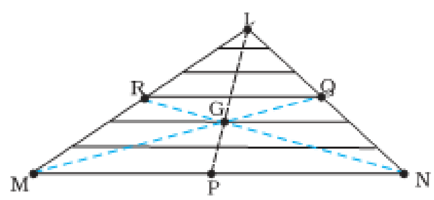
Let 'C' be the centre of mass of the system which is at a distance X from the origin O.

We have $X = \frac{M_1X_1 + M_2X_2}{M_1 + M_2}$

Since the two particles have the same mass $m_1 = m_2 = m$
$\therefore X = \frac{mx_1 + mx_2}{m+m} = \frac{x_1 + x_2}{2}$

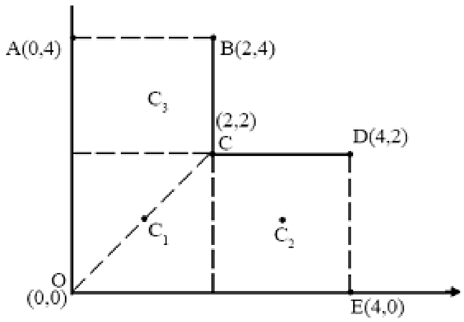
Thus for two particles of equal mass the centre of mass lies exactly midway between them.

2. How do you find the centre of mass of a triangular lamina.
Subdivide the lamina (LMN) into narrow strips each parallel to the base MN as shown in the figure. By symmetry each strip has its centre of mass at mid point. Join the midpoint of all the strips, we get a median LP. Therefore the centre of mass of the triangle as a whole must lie on the median LP.
Similarly it must lie on the median MQ and NR. This means that the centre of mass lies on the point of concurrence of the median, i.e. on the centroid G of the triangle. Thus centroid of the triangle itself is the centre of mass of the triangular lamina.



3. Find the centre of mass of a L- shaped uniform lamina of mass 3 kg?
We can think of the L – shape to consist of 3 squares each of length 2m. The mass of each square is 1kg, since the lamina is uniform. The centre of mass c, c1 and c3 of the squares are by

symmetry, Their geometric centers and have coordinates (1,1), (3,1), 1,3) respectively. We take the masses of the squares to be concentrated at these points. The centre of mass of these points.



Hence,

$$x = \frac{[1(1)+1\,(3)+1\,(1)]kgm}{1+1+1} = \frac{5}{3}\,m = 1.66m$$

$$y = \frac{[1(1)+1\,(1)+1\,(3)]kgm}{1+1+1} = \frac{5}{3}\,m = 1.66m$$

Thus centre of mass of the L – Shape lies on the line OC.

4. Write the expression for the position vector of the centre of mass of a system consisting of three objects in terms of their masses and position vectors.

$$R = \frac{\sum m_i r_i}{M}.$$

$$\therefore R = \frac{m_1r_1+m_2r_1+m_3r_3}{m_1+m_2+m_3}$$

5. Name two examples for vector product.

a. Moment of a force
b. Angular momentum.

6. Define vector product of two vectors.

a. A vector product of two vector a and b is a vector C such that, magnitude of C = c = ab sin θ. Where a and b are magnitudes of a & b and θ is the angle between the two vectors.
C is perpendicular to the plane containing a and b.

7. Vector product is not commutative why?

a. The magnitude of both a × b and b × a is the same (ab sin θ.); also, both of them are perpendicular to the plane of a and b. But the rotation of the right handed screw in case of a × b is from a to b, Where in case of b × a it is from b to a.
b. This means the two vectors are in opposite directions.
c. We have a × b = - b × a

8. a × b does not change sign under reflection. Explain why?

a. Under reflection we have

b. $x \rightarrow -x$, $y \rightarrow -y$, $z \rightarrow -z$

c. As a result all the components of a vector changes sign and then, $a \rightarrow -a$, $b \rightarrow -b$

$$\therefore a \times b \rightarrow (-a) \times (-b) = a \times b$$

Thus, $a \times b$ does not change sign under reflection.

9. Distinguish between vector product and scalar product of two vectors

	Vector product	Scalar product
1 2	Vector product of two vectors is a vector It is not commutative eg: angular momentum	Scalar product of 2 vector is a scalar. It is commutative eg: work

10. Find the scalar product and vector product of two vectors

$a = (3\hat{i} - 4\hat{j} + 5\hat{k})$ and $b = (-2\hat{i} + \hat{j} - 3\hat{k})$

$a.b = (3\hat{i} - 4\hat{j} + 5\hat{k}).(-2\hat{i} + \hat{j} - 3\hat{k}) = -6 - 4 - 15 = -25$

$$a \times b = \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -4 & 5 \\ -2 & 1 & -3 \end{bmatrix} = (-7\hat{i} - \hat{j} - 5\hat{k})$$

11. A particle is moving in a circular path with a uniform speed, What is the direction of (i) ω and (ii) V ?

i. ω is directed along the fixed axis of rotation.

ii. V is perpendicular to both ω and r and is directed along the tangent to the circle described by the particle.

12. Define torque. Is it a vector or a scalar?

The moment of a force or torque acting on the particle with respect to the origin is defined on the vector product of position vector and the force acting on the particle $\tau = r \times F$

It is a vector.

13. Write the dimensions and SI unit of torque.

a. Torque has dimension $M L^2 T^{-2}$

b. Its SI unit is newtonmetre (Nm)

14. Define angular momentum. Write the expression for it.

The angular momentum l of the particle with respect to the origin O is the vector product of position vector and the linear momentum of the rotating particle.

$l = r \times p$

15. Is angular momentum a scalar or a vector ? Write the dimension of l. Angular momentum is a vector.

It has dimensions $M^1L^2T^{-1}$

16. Show that the total gravitational torque about the centre of gravity of the body is Zero.

We have $\tau = r \times F$

Since position vector (r) = 0

(gravitational force acts at the centre of gravity of the body r = 0)

$\therefore \tau = 0$ (total gravitational torque about the C.G is zero)

17. Mention the two factors on which torque of a rotating body depends.

a. Magnitude of the force.

b. Perpendicular distance of the point of application of the force from the origin or axis of rotation.

18. Write the relation between angular momentum and torque. What is the torque acting on a body rotating with constant angular momentum.?

a. $\frac{dL}{dt} = \tau_{ext}$

b. Zero

19. Find the torque of a force 7 $\hat{i}$ + 3 $\hat{j}$ -5 $\hat{k}$ about the origin. The force acts on a particle whose position vector is $\hat{i}$ - $\hat{j}$ + $\hat{k}$.

Here $\hat{i} - \hat{j} + \hat{k}$ and $F = 7\hat{i} + 3\hat{j} - 5\hat{k}$

We shall use the determinant rule to find the torque $\tau = r \times F$

$$\tau = \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 7 & 3 & -5 \end{bmatrix} = (5\text{-}3)\,\hat{i} - (-5\text{-}7)\,\hat{j} + [3\text{-}(-7)]\,\hat{k}$$

$$= 2\hat{i} + 12\hat{j} + 10\hat{k}$$

20. State and explain the principle of conservation of angular momentum.

We know that the time rate of the total angular momentum of as system of particles about a point is equal to the sum of the external torques acting on the system taken about the same point.

i.e: $\frac{dL}{dt} = \tau_{ext}$

$$if \quad \tau_{ext} = 0 = \frac{dL}{dt} = 0$$

or L = constant

Thus if the total external torque on a system of particles is zero, the total angular momentum of the system is conserved. i.e. remains constant.

21. Give the general conditions of equilibrium of a rigid body.

A rigid body is said to be in mechanical equilibrium if both its linear momentum and angular momentum are not changing with time or equivalently, the body has neither linear acceleration nor angular acceleration This means The total force. i.e: the vector sum of the force on the rigid body is zero.

a. $F_1 + F_2 + F_3 + \ldots + F_n = \sum_{i=1}^{n} F_i = 0$ (Condition for translational equilibrium.)

b. The total torque i.e., the vector sum of the torques on the rigid body is Zero

$$\tau_1 + \tau_2 + \tau_3 + \ldots + \tau_n = \sum_{i=1}^{n} \tau_i = 0$$ (¨condition for rotational equilibrium)

22. Write the expression for work done by a torque and explain the terms.

$d\omega = \tau \, d\theta$ *where* $d\theta$ is the angular displacement of the particle, τ is the external torque.

23. What are the factors on which the moment of inertia of a body depend?

The moment of inertia of a rigid body depends on the mass of the body, its shape and size; distribution of mass about the axis of rotation and the position and orientation of the axis of rotation.

24. Why a fly wheel is used in a engine of a train (Vehicle)?

i. A fly wheel has **large moment of inertia**. Because of its large moment of inertia, it resist the sudden increase or decrease of the speed of the vehicle.

ii. It allows a gradual change in the speed and prevents jerky motions, thereby ensuring a smooth ride for passengers on the vehicle.

Three marks Questions

1. Write three Kinematic equations of rotational motion of a body with a uniform angular acceleration and explain the terms.

$$\omega = \omega_0 + \propto t$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2} \propto t^2$$

$$\omega^2 = \omega_0^2 + 2 \propto (\theta - \theta_0)$$

ω_0 is the initial angular velocity, ω *is* angular velocity after `t' seconds, α- angular acceleration, θ_0 - initial angular displacement, θ - angular displacement in ‘t’ seconds.

2. The angular speed of a motor wheel is increased from 1200rpm to 3120 rpm in 16 seconds. What is its angular acceleration, assuming the acceleration to be uniform.?

We shall use $\omega = \omega_0 + \alpha t$

ω_0 initial angular speed in rad/sec.

$= 2\pi \times$ angular speed in rev/sec

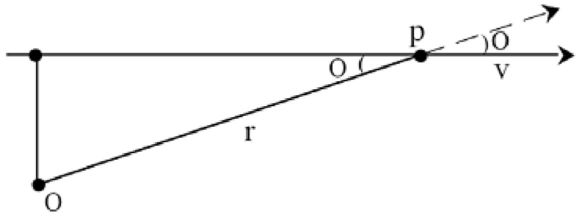
$= \frac{2\pi \text{ angular speed in rev/min}}{60 \ sec/min} = \frac{2\pi \times 1200}{60} = 40\pi$ rad/sec.

Similarly ω = final angular speed in rad/sec. $= \frac{2\pi \times 3120}{60} = 2\pi \times 52$ rad/sec.

ω = 104π rad/sec

∴ angular acceleration $\alpha = \frac{\omega - \omega_0}{t} = \frac{104\pi - 40\pi}{16} = \frac{64\pi}{16} = 4\pi$ rad/sec.

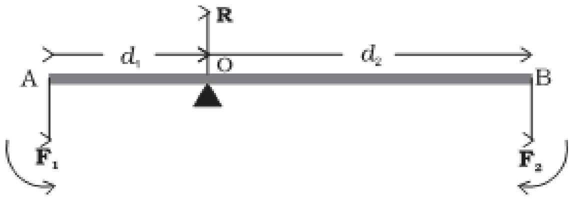
3. Show the angular momentum about any point of a single particle moving with constant velocity remains constant throughout the motion.



Consider a particle of mass m moving with velocity V at P.
O be an arbitrary point about which angular momentum has to be calculated.

We have angular momentum $l = r \times mv$
Its magnitude is $mvr \sin\theta$ where θ is the angle between r & v. Although the particle changes position with time, the line of direction of □ remains the same and hence OM = r sin θ is a constant.
Further, the direction of l is perpendicular to the plane of r and v. This direction does not change with time. Thus, l remains the same in magnitude and direction and is therefore conserved.

4. Explain the principles of moments for a lever.



Consider an ideal lever as shown in the figure.
Two forces F_1 and F_2, parallel to each other and usually perpendicular to the lever, as shown in the figure, act on the lever at distances. d_1 and d_2 respectively from the fulcrum.
If R is the reaction of the support at fulcrum
There for translational equilibrium $R-F_1-F_2 = 0$ and for rotational equilibrium the sum of the moments $d_1F_1 - d_2F_2 = 0$ …….. (1)
In the case of the lever force F_1 is usually some weight to be lifted. It is called the load and its distance from the fulcrum d_1 is called the load arm. Force F_2 is the effort applied to lift the load. distance d_2 of the effort arm from the fulcrum is the effort arm.
From equation (1) : $d_1F_1 = d_2 F_2$
or
Load arm × load = effort arm × effort. This is the principle of moments for a lever.

The ratio F_1/F_2 is called the mechanical advantage.

$$\text{M.A.} = \frac{F_1}{F_2} = \frac{D_2}{D_1}$$

If the effort arm d_2 is larger than the load arm d_1, the mechanical advantage is greater than one. i.e.: a small effort can be used to lift a large load.

5. Starting from the definition of moment of inertia obtain an expression for moment of inertia of a thin ring.

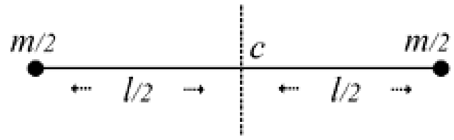
Consider a thin ring of radius R and Mass M, rotating in its own plane around its centre with angular velocity ω. Each mass element of the ring is at a distance R form the axis and moves with a speed R ω, The Kinetic energy is therefore

$k = \frac{1}{2}Mv^2 = \frac{1}{2}MR^2\omega^2$ .... (1)

But $k = I\omega^2$ ....(2)

Comparing equation (1) and (2) we get $I = \frac{1}{2}MR^2$

6. Obtain an expression for M.I. of a rotating pair of small masses attached to the two ends of a rigid mass less rod of length l rotating about and axis through the centre of mass perpendicular to the rod.



From the figure each mass m/2 is at distance $l/2$ from the axis. The momentum is therefore

: $\left(\frac{m}{2}\right)\left(\frac{l}{2}\right)^2+\left(\frac{m}{2}\right)\left(\frac{l}{2}\right)^2$

Therefore for the pair of masses, rotating about the axis through the centre of mass perpendicular to the rod $I = \frac{ml^2}{4}$

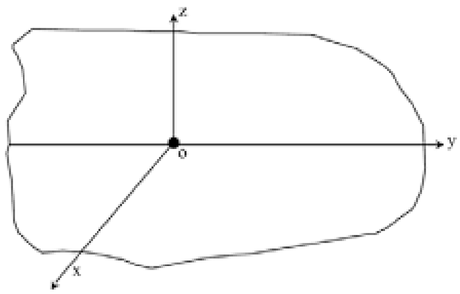
Four marks questions

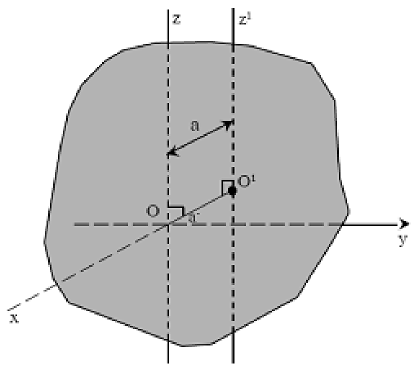
1. State and explain perpendicular axis theorem and parallel axis theorem.

(a). Perpendicular axis theorem : It states that the moment of inertia of a planer body (lamina) about an axis perpendicular to its plane is equal to the sum of its moment of inertia about two perpendicular axes concurrent with perpendicular axis and lying in the plane of the body.

The figure shows a planar body An axis perpendicular to the body through a point O is taken as the Z axis. Two mutually perpendicular axis lying in the plane of the body and concurrent with Z axis. i.e., passing through O, are taken as the x and y axes.

The theorem states that Iz = Ix + Iy.





(b). Theorem of parallel axis : The moment of inertia of a body about any axis is equal to the sum of the moment of inertia of the body about a parallel axis passing through its centre of mass and the product of its mass and the square of the distance between the two parallel axes.
Z and Z1 are two parallel axes separated by a distance a. The z axis passes through the centre of mass o of the rigid body. Then according to the theorem of parallel axis $I'_Z = I_Z + Ma^2$

Where I_Z and I'_Z are the moments of inertia of the body about the Z and Z' axes respectively, M is the total mass of the body and **a** is the perpendicular distance between the two parallel axes.

2. Using perpendicular axis theorem obtain the expression for moment of inertia of a disc about its diameter. Assume the expression for moment of inertia about a perpendicular axis passing through the centre.

Moment of inertia of the disc about an axis perpendicular to it and through its centre = $\frac{MR^2}{2}$.
Where M is the mass of the disc and R is the radius. The disc can be considered to be a planar body. Hence the theorem of perpendicular axis is applicable to it.
Consider three concurrent axis through the centre of the disc, O as the x, y, z axes. x and y axes lie in the plane of the disc and z is perpendicular to it.
Consider three concurrent axis through the centre of the disc, O as the x, y, z axes. x and y axes lie in the plane of the disc and z is perpendicular to it.
By the theorem of perpendicular axis
$I_z = I_X + I_y$
Here x and y axes are along two diameters of the disc and by symmetry the moment of inertia of the disc is same about any diameter.
Hence, $I_x = I_y$
And $I_z = 2\,I_x$

$$I_z = \frac{MR^2}{2}$$

$$I_x = \frac{I_z}{2} = \frac{MR^2}{4}$$

3. Using perpendicular axis theorem obtain expression for moment of inertia of a ring about its diameter. Assume the expression for MI about a perpendicular axis passing through the centre.

Moment of inertia of a circular ring about an axis perpendicular to it and through its centre $I = MR^2$
Where M is the mass of the disc and R is its radius. The ring can be considered to be a planar body. Hence the theorem of perpendicular axis is applicable to it. Consider three concurrent axis through the centre of the ring O as the x, y, z axes. x and y axis lie in the plane of the disc and z is perpendicular to it.
By the theorem of perpendicular axes $I_Z = I_x + I_y$. Here x & y axes are along two diameters of the ring and by symmetry the M.I. of the ring is the same about any diameter.
Hence $I_x = I_y$ and $I_z = 2I_X$
But $I_z = MR^2/2$

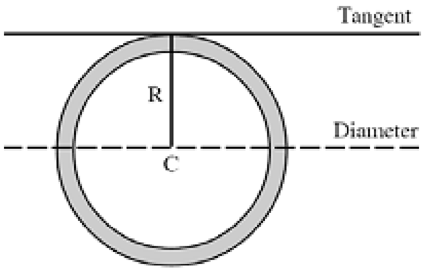
$$I_X = \frac{I_Z}{2} = \frac{MR^2}{2}$$

4. Using parallel axis theorem obtain the expression for M.I. of a ring about a tangent to the circumference of the ring. Assume the expression for M.I. of a ring about its diameter.

The tangent to the ring in the plane of the ring is parallel to one of the diameters of the ring. The distance between these two parallel axes is R, the radius of the ring. Using the parallel axis theorem

$$I_{tangent} = I_{diameter} + MR^2 = \frac{MR^2}{2} + MR^2$$

a. $$= \frac{3}{2}MR^2$$



5. Derive an expression for the kinetic energy of a rolling body.

Consider a body of mass M rolling with a velocity v. The total kinetic energy of a rolling body is the sum of the Kinetic energy of the body due to the motion of the centre of mass $\frac{mv^2}{2}$ and Kinetic energy of rotational motion about the centre of mass of the system of particles (K^1).

Thus $K = K^1 + \frac{MV^2}{2}$ --------- (1)

Here the kinetic energy of the centre of mass, i.e., the K.E. of translation of the rolling body is $\frac{Mv_{cm}^2}{2}$, where m is the mass of the body and v_{cm} is the Velocity of the centre of mass.

Since the motion of the rolling body about the centre of mass is rotation, K^1 is the kinetic energy of rotation of the body; $K^1 = \frac{I\omega^2}{2}$ where I is the moment of inertia about the appropriate axis, which is the symmetry axis of the rolling body.

$\therefore$ the kinetic energy of a rolling body $K = \frac{1}{2}I\omega^2 + \frac{1}{2}mv_{cm}^2$

Substituting $I = mK^2$

Where K is the corresponding radius of gyration of the body and $v_{cm} = \omega R$, We get

$$\therefore K = \frac{mk^2 v_{cm}^2}{R^2} + \frac{1}{2}mv_{cm}^2$$

$$\therefore K = \frac{1}{2}mv_{cm}^2\left[1 + \frac{k^2}{R^2}\right]$$

6. State and explain the principle of conservation of angular momentum in case of (i) swivel chair (ii) an acrobat.

In the absence of the external torque, the total angular momentum of a body rotating about a fixed axis remains constant.

If the external torque is Zero, $I_Z = I\omega$ = constant.

Swivel chair: Sit on a swivel chair with arms folded and feet not resting on. i.e., away from the ground. Now rotate the chair rapidly. While the chair is rotating with considerable angular speed stretch the arms horizontally. The angular speed reduces now. If the arms are folded back, the angular speed increases again. This is because of the law of conservation of angular momentum. If the friction of the rotational mechanism is neglected, there is no external torque about the axis of rotation of the chair and hence $I\omega$ is constant. Stretching the arms increases I about the axis of rotation, resulting decreasing the angular speed ω. Bringing the arms closer to body has the opposite effect.

An acrobat make use of this principle during the course of his performance. Sometimes he stretches out his hands and legs to increase the M.I. of the body and to decrease the angular speed. On the other hand to increase the angular speed he brings hands and legs near his body, the moment of inertia decreases. i.e., principle of angular momentum can be used to perform somer-saults in air by an acrobat.

7. Three equal masses are kept at P(x_1,y_1), Q(x_2,y_2) and R (x_3,y_3) in a co-ordinate system. Show that, their centre of mass coincides with the centroid of the triangle PQR.

a. Let the masses of the three particles be m_1, m_2 and m_3 respectively, the centre of mass C of the system of the three particles is defined and located by the co-ordinates (x, y) given by

$$X = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3}$$

$$Y = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3}$$

For the particles of equal mass $m_1 = m_2 = m_3 = m$

$$x = \frac{m(x_1 + x_2 + x_3)}{3m} = \frac{x_1 + x_2 + x_3}{3}$$

$$y = \frac{m(y_1 + y_2 + y_3)}{3m} = \frac{y_1 + y_2 + y_3}{3}$$

Thus, for three particle of equal mass, the centre of mass coincides with the centroid of the triangle formed by the particle.

8. Show that torque is equal to rate of change of angular momentum of a particle.

We have $l = r \times p$

Where r – position vector, p – momentum. Differentiating the above equation

$$\frac{dl}{dt} = \frac{d}{dt}(r \times p)$$

Using product rule, on R.H. S.

$$\frac{d}{dt}(r \times p) = \frac{dr}{dt} \times p + r \times \frac{dp}{dt}$$

Now the velocity of the particle is $v = \frac{dr}{dt}$ and $p = mv$

$\therefore, \frac{dr}{dt} \times p = v \times mv = 0$

$since \frac{dp}{dt} = F$

$r \times \frac{dp}{dt} = r \times F = \tau$

$\therefore \frac{dl}{dt} = \tau$

Five marks questions

1. A cord of negligible mass is wound round the rim of a flywheel of mass 40 kg and radius 40 cm. A steady pull of 25 N is applied on the cord as shown in the figure. The flywheel is mounted on a horizontal axle with frictionless bearings. Compute the angular acceleration of the wheel. Find work done by the pull, when two metre of cord is unwound.
Find also the kinetic energy of the wheel at this point assume that the wheel starts from rest.

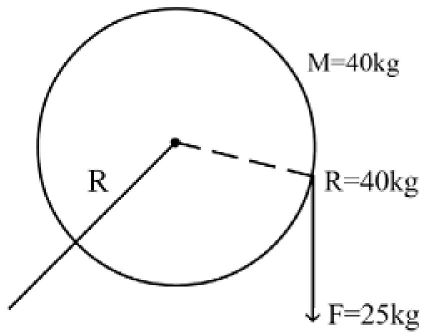
(a) We have $I\alpha = \tau$

The torque $\tau = FR$ = 25 x 0.40 Nm = 10 N

I = M.I. of fly wheel about its axis $\frac{MR^2}{2}$

$= \frac{40 \times (0.40)^2}{2} = 20 \times 0.16 = 3.2 kgm^2$

$\propto = angular\ acceleration = \frac{10Nm}{3.2\ kgm^2} = 3.125S^{-2}$



(b) work done by the pull unwinding 2m of the cord

= 25 × 2m = 50 j

(c) Let ω be the final angular velocity. The kinetic energy gained $K = \frac{1}{2}I\omega^2$

Since the wheel starts from rest,

$\omega^2 = \omega_0^2 + 2\propto\theta$

$\omega_0 = 0$

The angular displacement

$$\theta = \frac{length\ of\ un\ wound\ string}{radius\ of\ wheel}$$

$$= \frac{2m}{0.4} = 5\ rad$$

$$\therefore \omega^2 = 2 \times 3.125 \times 5 = 31.25\ (\text{rad/s})^2$$

$$\therefore \text{K.E gained} = \frac{1}{2} \times 3.2 \times 31.25 \quad (K = \frac{1}{2} I\omega^2)$$

$$= 1.6 \times 31.25$$

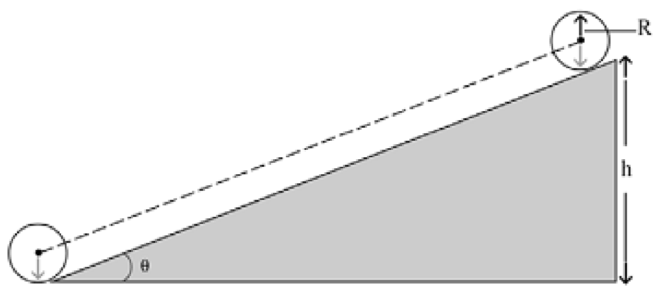
$$= 50\ \text{J}$$

2. Three bodies, a ring, a solid cylinder and a solid sphere roll down the same inclined plane without slipping. They starts from rest. The radii of the bodies are identical which of the bodies reaches the ground with maximum velocity?

Since energy of a rolling body is conserved potential energy lost by the body in rolling down the inclined plane (mgh) Must be equal to K.E. gained. Since the bodies start from rest the K.E. gained = the final K.I. of the bodies. We have K.E. of a rolling body

$$K = \frac{1}{2} mv^2 \left[1 + \frac{k^2}{R^2}\right] \ldots\ldots\ldots\ldots.(1)$$

Where ϑ is the final velocity of the body. Equating the equation (1) with the potential energy lost weight



$$mgh = \frac{1}{2} mv^2 \left[1 + \frac{k^2}{R^2}\right]$$

$$\text{or } v^2 = \frac{2gh}{\left[1 + \frac{k^2}{R^2}\right]}$$

for a ring $K^2 = R^2$

$$v_{\text{ring}} = \sqrt{\frac{2gh}{1+1}} = \sqrt{gh}$$

for a solid cylinder $k^2 = \frac{R^2}{2}$

$$v_{\text{ring}} = \sqrt{\frac{2gh}{1+1}} = \sqrt{gh}$$

for a solid cylinder $k^2 = \frac{R^2}{2}$

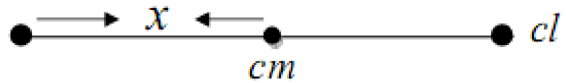
$\therefore v_{\text{disc}} = \sqrt{\frac{2gh}{1+\frac{1}{2}}} = \sqrt{\frac{4gh}{3}}$

For a solid sphere $k^2 = \frac{2R^2}{5}$

$\therefore v_{\text{sphere}} = \sqrt{\frac{2gh}{1+\frac{2}{5}}} = \sqrt{\frac{10gh}{7}}$

From the result obtained it is clear that among the three bodies the sphere has the greatest and the ring has the least velocity of the centre of mass at the bottom of the inclined plane.

3. In the HCl molecule, the separation between the nuclei of the two atoms is about 1.27 Å (1 Å $=10^{-10}$ m). Find the approximate location of the CM of the molecule, given that a chlorine atom is about 35.5 times as massive as a hydrogen atom and nearly all the mass of and atom is concentrated in its nucleus.



Let us consider hydrogen nucleus as the origin for measuring distance. If `m' is the mass of the hydrogen atom, then mass of the chlorine atom = 35.5 m. Distance of the centre of moss of HCl molecule from the origin is given by

$$x = \frac{m_1x_1+m_2x_2}{m_1+m_2}$$

Here $x_1 = 0,\ x_2 = 1.27 \times 10^{-10}\ metre$

$$\therefore x = \frac{m\times o+35.5\text{m} \times 1.27\times10^{-10}}{m+35.5m}$$

$$\therefore x = \frac{35.5 \times 1.27\times10^{-10}}{36.5}$$

$$= 1.23\ \times 10^{-10m} = 1.235A^0$$

4. Torques of equal magnitude are applied to a hollow cylinder and a solid sphere, both having the same mass and radius. The cylinder is free to rotate about its standard axis of symmetry, and the sphere is free to rotate about an axis passing through its centre. Which of the two will acquire a greatest angular acceleration after a given time?

Moment of inertia of a cylinder about its axis of symmetry $I_c = MR^2$
Moment of inertia of a sphere about its diameter $I_s = \frac{2}{5}\ M\ R^2$
Angular acceleration $\alpha = \frac{\tau}{I}$

$\propto_c = \frac{\tau}{I_c} = \frac{\tau}{MR^2}$

also $\propto_s = \frac{\tau}{I_s} = \frac{\tau}{\frac{2}{5}MR^2} = 2.5\frac{\tau}{MR^2} = 2.5 \propto_c$

the sphere will acquire a greatest angular acceleration.

5. A solid cylinder of mass 20 kg rotates about its axis with angular speed 100 rad. S^{-1}. The radius of the cylinder is 0.25m. What is the kinetic energy associated with the rotation of the cylinder? What is the magnitude of angular momentum of the cylinder about its axis?

Sol. (i) Kinetic energy of rotation $K = \frac{1}{2} I\omega^2$

Here, $I = \frac{1}{2} MR^2$

$\therefore I = \frac{20}{2} \times (0.25)^2 = 0.625 kgm^2$

and $\omega = 100 radS^{-1}$

$\therefore$ Kinetic energy of rotation $\frac{1}{2} \times 0.625 \times 100^2 = 3125$ J

Also angular momentum $L = I\omega = 0.625 \times 100 = 62.5$ Kg m^2s^{-1}

6. (a) A child stands at the centre of a turn table with his arm outstretched. The turn table is set rotating with an angular speed of 40 rev/min. how much is the angular speed of the child, if he folds his hand back and thereby reduces his moment of inertia to 2/5 times the initial value? Assume that the turntable rotates without friction.

(b) Show that the child's new K.E. of rotation is more than the initial K.E. of rotation. How do you account for this increase in Kinetic. Energy ?

Given (a) $I_{final} = \frac{2}{5} I_{initial} = \quad \omega_i = 40\, rev/min$. Using the principle of conservation of angular momentum. We get $I_i\omega_i = I_F\omega_F$

Or $\omega_F = \frac{I_i\omega_i}{I_F} = \frac{I_i \times 40}{\frac{2}{5}I_i} = 100\, rev\, min^{-1}$

(b) $\frac{Final\ K.E. of\ rotation}{initia\ K.E. of\ rotation} = \frac{\frac{1}{2} I_F\omega_F^2}{\frac{1}{2} I_i\omega_i^2}$

$$= \left(\frac{I_F}{I_i}\right)\left(\frac{\omega_F}{\omega_i}\right)^2 = \frac{2}{5} \times \left(\frac{100}{40}\right)^2 = \frac{5}{2} = 2.5$$

$\therefore Final\ kinetic = 2.5 \times$ initial K.E.

Final K.E. is more than initial K.E. because the child uses his internal energy when the folds his hands.

7. A rope of negligible mass is wound round a hollow cylinder of mass 3kg and radius 40cm. What is the angular acceleration of the cylinder if the rope is pulled with a force of 30N? What is the linear acceleration of the rope? Assume that there is no slipping.

Given: M = 3Kg, R = 40 cm, = 0.4 m

MI of the hollow cylinder about its axis I = $MR^2 = 3 \times (0.4)^2 = 0.48\ Kgm^2$

When the force of 30N is applied over the rope wound round the cylinder, the torque will act on the cylinder.

$\tau = R \times F = 30 \times 0.4 = 12$ Nm

If α be the angular acceleration produced, then $\tau = I\alpha$

$$\text{OR } \alpha = \frac{\tau}{I} = \frac{12}{0.48} = 25 radS^{-2}$$

8. To maintain a rotor at a uniform angular speed of 200 rad/sec an engine needs to transmit a torque of 180 Nm. What is the power required by the engine?

Given :

$\tau = 180\text{Nm},\ \omega = 200 radS^{-1}$

Power $p = \tau\omega$

$p = 180 \times 200$

$= 36000\ Watt.$

9. A metre stick is balanced on a knife edge at its centre. When two Coins, each of mass 5g are put on the top of the other at 12.0 cm mark, the stick is found to be balanced at 45.0 cm. What is the mass of the metre stick?

Let m be the mass of the metre stick. It is concentrated at C the 50 cm mark for equilibrium about C^1, at 45cm mark

$$10\,g\,(45 - 12) = mg(50 - 45)$$

$$10g \times 33 = mg \times 5$$

$$m = 10 \times \frac{33}{5} = 2 \times 33 = 66g$$

10. A solid sphere rolls down two different inclined planes of the same heights but different angles of inclination. Will it reach the bottom with the same speed in each case? Will it take longer to roll down one plane than the other? If so, which one and why?

The kinetic energy of a rolling body $K = \frac{1}{2}I\omega^2 + \frac{1}{2}\,mv^2$

Where ν is the velocity of the centre of mass at the bottom of the inclined plane. According to the principle of conservation of energy

$\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = mgh$ (P.E. lost by the body)

For a sphere $I = \frac{2}{5}mR^2 \;\therefore\; \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}mR^2\right)\omega^2 = mgh$

as $R\omega = v$

$$\frac{1}{2}mv^2 + \frac{1}{5}mv^2 = mgh$$

$$v = \sqrt{\frac{10}{7}gh}$$

Since the inclined planes have the same height, ν must be same and also the time.

11. A loop of radius 2m weighs 100 kg. It rolls along a horizontal floor. So that its centre of mass has a speed of 20 cm/s. How much work is done to stopit?

Given: R = 2m, M = 100 Kg., V = 20 cm/s = 0.2 m/s

Total energy of the loop $= \frac{1}{2}\,mv^2 + \frac{1}{2}I\omega^2 \quad = \frac{1}{2}\,mv^2 + \frac{1}{2}MR^2\omega^2$

$= \frac{1}{2}\,mv^2 + \frac{1}{2}\,mv^2$ since Rω = v.

$= mv^2$

∴ Work required to stop the loop = total energy of the loop. $= 100 \times (0.2)^2 = 4$ J.

12. The oxygen molecule has a mass of 5.30×10^{-26} Kg and a M.I. of $I = 1.94 \times 10^{-46}$ Kgm^2, about and axis through its centre perpendicular to the lines joining the two atoms. Suppose the mean speed of such a molecule in a gas is 500 m/s and that its K.E. of rotation is two thirds of its K.E. of translation. Find the average angular velocity of the molecule.

Given : $m = 5.30 \times 10^{-26}$ Kg; $I = 1.94 \times 10^{-46}$ Kg m^2, v = 500 m/s

An oxygen molecule contains two atoms. If m is the mass of the oxygen molecule then mass of each atom of oxygen = m/2

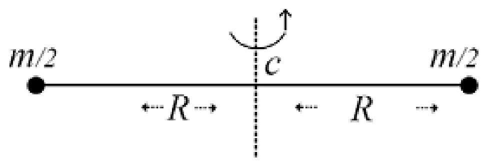
If 2R is distance between the two atoms.

Then moment of inertia of oxygen molecule $I = \frac{1}{2}mR^2 + \frac{1}{2}mR^2 = mR^2$

$mR^2 = 1.94 \times 10^{-46}$

$$R^2 = \frac{1.94\times10^{-46}}{m} = \frac{1.94\times10^{-46}}{5.30\times10^{-23}}$$

$$R = \sqrt{\frac{1.94\times10^{-24}}{5.3}} = 0.61 \times 10^{-10}m$$



But K.E of rotation = $\frac{2}{3}$ K.E. of translation

$$\frac{1}{2}I\omega^2 = \frac{2}{3}\times\frac{1}{2}mv^2$$

$$I = mR^2$$

$$\therefore \frac{1}{2}mR^2\omega^2 = \frac{1}{3}mv^2$$

$$\text{OR } \omega = \sqrt{\frac{2}{3}} \times \frac{v}{R} = \sqrt{\frac{2}{3}} \times \frac{500}{0.61\times10^{-10}}$$

$$= 6.7 \times 10^{12}\ rad/S$$

13. A solid cylinder rolls up an inclined plane at the bottom of the inclined plane, the centre of mass of the cylinder has a speed of 5m/s. How far will the cylinder go up the plane?

Given : v = 5m/s

If `h` is the height attained by the cylinder.

$$\text{Then } \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = mgh$$

$$\frac{1}{2}mv^2 + \frac{1}{2}(\frac{1}{2}mR^2)\omega^2 = mgh$$

$$i.e;\ \frac{3}{4}mv^2 = mgh$$

$$h = \frac{3v^2}{4g} = \frac{3\times5^2}{4\times9.8} = 1.913m$$

14. A man stands on a rotating plat form, with his arms stretched horizontally holding a 5 kg weight in each hand. The angular speed of the platform is 30 revolutions per minute. The man then brings his arms close to his body with the distance of each weight form the axis changing from 90cm to 20cm. The moment of inertia of the man together with the platform may be taken to be constant and equal to 76 Kg/m^2.

(a). What is his new angular speed? (neglect friction)
(b) Is kinetic energy conserved in the process? If not, from where does the change come from? given ωi = 30 rmp.

Ans: (a) Initial = inertia of the man together with the platform + moment inertia of the out stretched weight.

$$= 7.6 + 2(MR^2)$$

$$= 7.6 \times 2 \times 5 \times (0.9)^2$$

$$= 7.6 \times 10 \times 0.81$$

$$= 15.7\ kgm^2$$

$$I_{\text{Final}} = 7.6 + 2 \times (MR^2)$$

$$= 7.6 \times 2 \times 5 \times (0.2)^2 = 8.0 kgm^2$$

Using the principle of conservation of angular momentum

$$I_1\omega_1 = I_2\omega_2$$

$$\omega_2 = \frac{I_1\omega_1}{I_2} = \frac{15.7 \times 30}{8} = 58.88 rpm$$

(b) Kinetic energy is not conserved. As the moment of inertia decreases, the K.E. of rotation increases. This change comes from the work done by the man in bringing his arms close to his body.

Solutions of Textbook Problems :

Question 7.1:

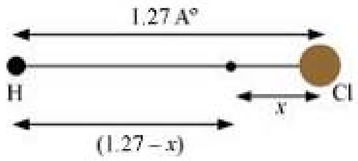
Give the location of the centre of mass of a (i) sphere, (ii) cylinder, (iii) ring, and (iv) cube, each of uniform mass density. Does the centre of mass of a body necessarily lie inside the body?

ANS:

Geometric centre; No

The centre of mass (C.M.) is a point where the mass of a body is supposed to be concentrated. For the given geometric shapes having a uniform mass density, the C.M. lies at their respective geometric centres.
The centre of mass of a body need not necessarily lie within it. For example, the C.M. of bodies such as a ring, a hollow sphere, etc., lies outside the body.

Question 7.2:
In the HCl molecule, the separation between the nuclei of the two atoms is about 1.27 Å (1 Å = 10^{-10} m). Find the approximate location of the CM of the molecule, given that a chlorine atom is about 35.5 times as massive as a hydrogen atom and nearly all the mass of an atom is concentrated in its nucleus.
ANS:
The given situation can be shown as:



Distance between H and Cl atoms = 1.27Å
Mass of H atom = m
Mass of Cl atom = 35.5m
Let the centre of mass of the system lie at a distance x from the Cl atom.
Distance of the centre of mass from the H atom = (1.27 – x)
Let us assume that the centre of mass of the given molecule lies at the origin.
Therefore, we can have:

$$\frac{m(1.27-x)+35.5mx}{m+35.5m}=0$$

$$m(1.27-x)+35.5mx=0$$

$$1.27-x=-35.5x$$

$$\therefore x=\frac{-1.27}{(35.5-1)}=-0.037\ \text{Å}$$

Here, the negative sign indicates that the centre of mass lies at the left of the molecule. Hence, the centre of mass of the HCl molecule lies 0.037Å from the Cl atom.

Question 7.3:
A child sits stationary at one end of a long trolley moving uniformly with a speed V on a smooth horizontal floor. If the child gets up and runs about on the trolley in any manner, what is the speed of the CM of the (trolley + child) system ?
ANS:
No change.
The child is running arbitrarily on a trolley moving with velocity v. However, the running of the child will produce no effect on the velocity of the centre of mass of the trolley. This is because the force due to the boy's motion is purely internal. Internal forces produce no effect on the motion of the bodies on which they act. Since no external force is involved in the boy–trolley

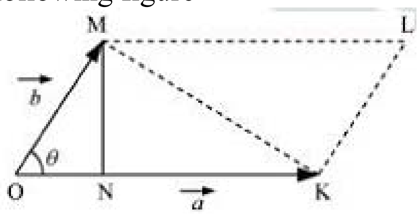
system, the boy's motion will produce no change in the velocity of the centre of mass of the trolley.

Question 7.4:

Show that the area of the triangle contained between the vectors $\vec{a}$ and $\vec{b}$ is one half of the magnitude of $|\vec{a} \times \vec{b}|$.

ANS:

Consider two vectors $\overrightarrow{OK} = |\vec{a}|$ and $\overrightarrow{OM} = |\vec{b}|$, inclined at an angle θ, as shown in the following figure



In ΔOMN, we can write the relation:

$$\sin\theta = \frac{MN}{OM} = \frac{MN}{|\vec{b}|}$$

$$MN = |\vec{b}|\sin\theta$$

$$|\vec{a} \times \vec{a}| = |\vec{a}|\,|\vec{b}|\sin\theta \qquad = OK \cdot MN \times \frac{2}{2}$$

= 2 × Area of ΔOMK

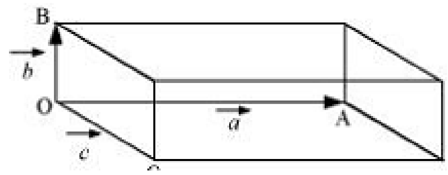
∴ Area of ΔOMK = ½$|\vec{a} \times \vec{b}|$

Question 7.5:

Show that a. (b × c) is equal in magnitude to the volume of the parallelepiped formed on the three vectors, a, b and c.

ANS:

A parallelepiped with origin O and sides a, b, and c is shown in the following figure.



Volume of the given parallelepiped = abc

$\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = \vec{b}$, $\overrightarrow{OC} = \vec{c}$

Let $\hat{n}$ be a unit vector perpendicular to both b and c. Hence, $\hat{n}$ and a have the same direction.

$\therefore \vec{b} \times \vec{c} = bc \sin\theta\ \hat{n} \quad = bc \sin 90°\ \hat{n} \quad = bc\,\hat{n}$

$\vec{a} \,.\, (\vec{b} \times \vec{c}) = a \,.\, (bc\,\hat{n})$

= abc cosθ $\hat{n}$

= abc cos 0°

= abc

= Volume of the parallelepiped

Question 7.6:
Find the components along the x, y, z axes of the angular momentum l of a particle, whose position vector is r with components x, y, z and momentum is p with components p_x, p_y and p_z. Show that if the particle moves only in the x-y plane the angular momentum has only a z-component.

ANS:

$l_x = yp_z - zp_y$

$l_y = zp_x - xp_z$

$l_z = xp_y - yp_x$

Linear momentum of the particle, $\vec{p} = p_x\hat{\imath} + p_y\hat{\jmath} + p_z\hat{k}$

Position vector of the particle, $\vec{r} = x\,\hat{\imath} + y\hat{\jmath} + z\,\hat{k}$

Angular momentum, $\vec{l} = \vec{r} \times \vec{p}$

$= (x\,\hat{\imath} + y\hat{\jmath} + z\,\hat{k}) \times (p_x\hat{\imath} + p_y\hat{\jmath} + p_z\hat{k})$

$$= \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ x & y & z \\ p_x & p_y & p_z \end{vmatrix}$$

$$l_x\hat{\mathbf{i}} + l_y\hat{\mathbf{j}} + l_z\hat{\mathbf{k}} = \hat{\mathbf{i}}\left(yp_z - zp_y\right) - \hat{\mathbf{j}}\left(xp_z - zp_x\right) + \hat{\mathbf{k}}\left(xp_y - zp_x\right)$$

Comparing the coefficients of $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$ we get :

$$\left.\begin{aligned} l_x &= yp_z - zp_y \\ l_y &= xp_z - zp_x \\ l_z &= xp_y - yp_x \end{aligned}\right\} \qquad \dots (i)$$

The particle moves in the x-y plane. Hence, the z-component of the position vector and linear momentum vector becomes zero, i.e., $z = p_z = 0$

Thus, equation (i) reduces to:

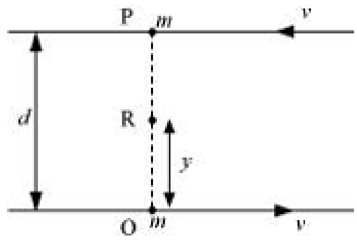
$$\left.\begin{aligned} l_x &= 0 \\ l_y &= 0 \\ l_z &= xp_y - yp_x \end{aligned}\right\}$$

Therefore, when the particle is confined to move in the x-y plane, the direction of angular momentum is along the z-direction.

Question 7.7:
Two particles, each of mass m and speed v, travel in opposite directions along parallel lines separated by a distance d. Show that the vector angular momentum of the two particle system is the same whatever be the point about which the angular momentum is taken.

ANS:

Let at a certain instant two particles be at points P and Q, as shown in the following figure.



Angular momentum of the system about point P:

$\vec{L}_P = mv \times 0 + mv \times d$

$= mvd$ ... (i)

Angular momentum of the system about point Q :

$\vec{L}_Q = mv \times d + mv \times 0$

$= mvd$ ... (ii)

Consider a point R, which is at a distance y from point Q, i.e.,
QR = y
∴ PR = d – y
Angular momentum of the system about point R:

$\vec{L}_R = mv \times (d - y) + mv \times y$

$= mvd - mvy + mvy$

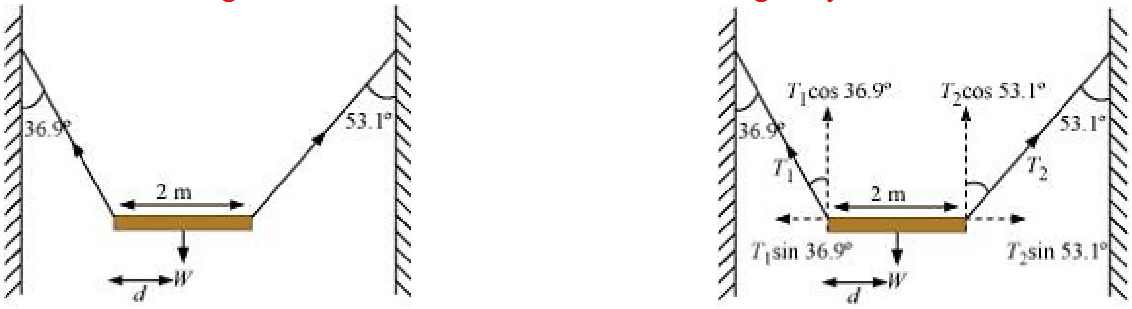
$= mvd$ ... (iii)

Comparing equations (i), (ii), and (iii), we get:

$\vec{L}_P = \vec{L}_Q = \vec{L}_R$ ... (iv)

We infer from equation (iv) that the angular momentum of a system does not depend on the point about which it is taken.

Question 7.8:
A non-uniform bar of weight W is suspended at rest by two strings of negligible weight as shown in Fig.7.39. The angles made by the strings with the vertical are 36.9° and 53.1° respectively. The bar is 2 m long. Calculate the distance d of the centre of gravity of the bar from its left end.



ANS:
The free body diagram of the bar is shown in the above figure.
Length of the bar, l = 2 m
T_1 and T_2 are the tensions produced in the left and right strings respectively.
At translational equilibrium, we have:

$$T_1 \, Sin\, 36.9° = T_2 \, Sin\, 53.1°$$

or $\frac{T_1}{T_2} = \frac{Sin\ 53.1°}{Sin\ 36.9°} = \frac{0.800}{0.600} = \frac{4}{3}$

=> $T_1 = \frac{4}{3}T_2$

For rotational equilibrium, on taking the torque about the centre of gravity, we have:

$$T_1 \cos 36.9 \times d = T_2 \cos 53.1(2-d)$$

$$T_1 \times 0.800d = T_2\ 0.600(2-d)$$

$$\frac{4}{3} \times T_2 \times 0.800d = T_2[0.600 \times 2 - 0.600d]$$

$$1.067d + 0.6d = 1.2$$

$$\therefore d = \frac{1.2}{1.67} = 0.72 \text{ m}$$

Hence, the C.G. (centre of gravity) of the given bar lies 0.72 m from its left end.

Question 7.9:

A car weighs 1800 kg. The distance between its front and back axles is 1.8 m. Its centre of gravity is 1.05 m behind the front axle. Determine the force exerted by the level ground on each front wheel and each back wheel.

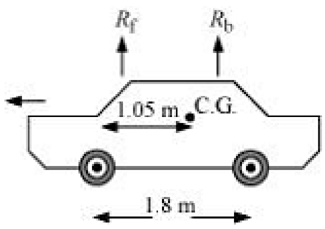
ANS:

Mass of the car, m = 1800 kg

Distance between the front and back axles, d = 1.8 m

Distance between the C.G. (centre of gravity) and the back axle = 1.05 m

The various forces acting on the car are shown in the following figure.



R_f and R_b are the forces exerted by the level ground on the front and back wheels respectively.

At translational equilibrium:

$R_f + R_b = mg$

$= 1800 \times 9.8$

$= 17640$ N … (i)

For rotational equilibrium, on taking the torque about the C.G., we have:

$$R_f(1.05) = R_b(1.8 - 1.05)$$

$$R_f \times 1.05 = R_b \times 0.75$$

$$\frac{R_f}{R_b} = \frac{0.75}{1.05} = \frac{5}{7} \qquad \frac{R_b}{R_f} = \frac{7}{5} \qquad R_b = 1.4\ R_f$$

Solving equations (i) and (ii), we get:

$$1.4R_f + R_f = 17640$$

$R_f = \frac{17640}{2.4} = 7350$ N

∴ $R_b = 17640 - 7350 = 10290$ N

Therefore, the force exerted on each front wheel = 7350/2 = 3675 N, and

The force exerted on each back wheel = 10290/2 = 5145 N

Question 7.10:

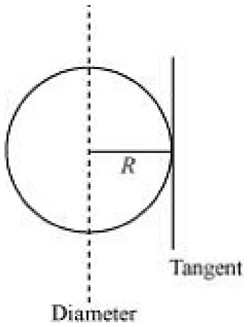
(a) Find the moment of inertia of a sphere about a tangent to the sphere, given the moment of inertia of the sphere about any of its diameters to be $2MR^2/5$, where M is the mass of the sphere and R is the radius of the sphere.

(b) Given the moment of inertia of a disc of mass M and radius R about any of its diameters to be $MR^2/4$, find its moment of inertia about an axis normal to the disc and passing through a point on its edge.

ANS:

(a) $\frac{7}{5}MR^2$

The moment of inertia (M.I.) of a sphere about its diameter = $\frac{2}{5}MR^2$



M.I. = $\frac{2}{5}MR^2$

According to the theorem of parallel axes, the moment of inertia of a body about any axis is equal to the sum of the moment of inertia of the body about a parallel axis passing through its centre of mass and the product of its mass and the square of the distance between the two parallel axes.

The M.I. about a tangent of the sphere = $\frac{2}{5}MR^2 + MR^2 = \frac{7}{5}MR^2$

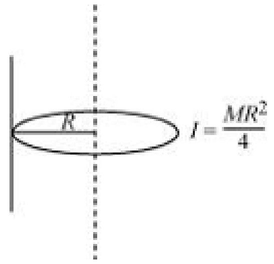
(b) $\frac{3}{5}MR^2$

The moment of inertia of a disc about its diameter = $\frac{1}{4}MR^2$

According to the theorem of perpendicular axis, the moment of inertia of a planar body (lamina) about an axis perpendicular to its plane is equal to the sum of its moments of inertia about two perpendicular axes concurrent with perpendicular axis and lying in the plane of the body.

The M.I. of the disc about its centre = $\frac{1}{4}MR^2 + \frac{1}{4}MR^2 = \frac{1}{2}MR^2$

The situation is shown in the given figure.



Applying the theorem of parallel axes:
The moment of inertia about an axis normal to the disc and passing through a point on its edge
$\frac{1}{2}MR^2 + MR^2 = \frac{3}{2}MR^2$

Question 7.11:
Torques of equal magnitude are applied to a hollow cylinder and a solid sphere, both having the same mass and radius. The cylinder is free to rotate about its standard axis of symmetry, and the sphere is free to rotate about an axis passing through its centre. Which of the two will acquire a greater angular speed after a given time?
ANS:
Let m and r be the respective masses of the hollow cylinder and the solid sphere. The moment of inertia of the hollow cylinder about its standard axis, $I_1 = mr^2$.

The moment of inertia of the solid sphere about an axis passing through its centre,

$$I_1 = mr^2$$

We have the relation:
$\tau = I\alpha$
Where, α = Angular acceleration, τ = Torque, I = Moment of inertia
For the hollow cylinder, $\tau_1 = I_1\alpha_1$
For the solid sphere, $\tau_{11} = I_{11}\alpha_{11}$

As an equal torque is applied to both the bodies, $\tau_1 = \tau_2$
$\therefore \frac{\alpha_2}{\alpha_1} = \frac{I_1}{I_2} = \frac{mr^2}{\frac{2}{5}mr^2} = \frac{2}{5}$
$\alpha_{11} > \alpha_1$ ---------- (1)

Now, using the relation:
$\omega = \omega_0 + \alpha t$
Where,
ω_0 = Initial angular velocity
t = Time of rotation
ω = Final angular velocity
For equal ω_0 and t, we have:
$\omega \propto \alpha$ … (ii)
From equations (i) and (ii), we can write:
$\omega_{II} > \omega_I$
Hence, the angular velocity of the solid sphere will be greater than that of the hollow cylinder.

Question 7.12:
A solid cylinder of mass 20 kg rotates about its axis with angular speed 100 rad s^{-1}. The radius of the cylinder is 0.25 m. What is the kinetic energy associated with the rotation of the cylinder? What is the magnitude of angular momentum of the cylinder about its axis?
ANS:
Mass of the cylinder, m = 20 kg
Angular speed, ω = 100 rad s^{-1}
Radius of the cylinder, r = 0.25 m
The moment of inertia of the solid cylinder:

$$I = \frac{mr^2}{2}$$

$$= \frac{1}{2} \times 20 \times (0.25)^2 \quad = \quad 6.25 \text{ Kgm}^2$$

$\therefore$ Kinetic energy $= \frac{1}{2} I\omega^2 \quad = \frac{1}{2} \times 0.625 \times (100)^2 = 3125$ J
∴ Angular momentum, L = Iω
= 6.25 × 100
= 62.5 Js

Question 7.13:
(a) A child stands at the centre of a turntable with his two arms outstretched. The turntable is set rotating with an angular speed of 40 rev/min. How much is the angular speed of the child if he folds his hands back and thereby reduces his moment of inertia to 2/5 times the initial value? Assume that the turntable rotates without friction.
(b) Show that the child's new kinetic energy of rotation is more than the initial kinetic energy of rotation. How do you account for this increase in kinetic energy?
ANS:
(a) 100 rev/min
Initial angular velocity, ω_1= 40 rev/min
Final angular velocity = ω_2
The moment of inertia of the boy with stretched hands = I_1
The moment of inertia of the boy with folded hands = I_2
The two moments of inertia are related as :

$$I_2 = \frac{2}{5} I_1$$

Since no external force acts on the boy, the angular momentum L is a constant. Hence, for the two situations, we can writ $I_2\omega_2 = I_1\omega_1$

$$\omega_2 = \frac{I_1}{I_2}\omega_1$$

$$= \frac{I_1}{\frac{2}{5} I_1} \times 40 = \frac{5}{2} \times 40$$

= 100 rev/min.
(b) Final K.E. = 2.5 Initial K.E.
Final kinetic rotation, $E_F = \frac{1}{2} I_2\omega_2^2$

Initial kinetic rotation, $E_I = \frac{1}{2} I_1\omega_1^2$

$$\frac{E_F}{E_I} = \frac{\frac{1}{2} I_2\omega_2^2}{\frac{1}{2} I_1\omega_1^2}$$

$$= \frac{2}{5}\frac{I_1}{I_1}\frac{(100)^2}{(40)^2} \qquad = \frac{2}{5} \times \frac{100 \times 100}{40 \times 40} \qquad = \frac{5}{2} = 2.5$$

$$\therefore E_F = 2.5E_I$$

The increase in the rotational kinetic energy is attributed to the internal energy of the boy.

Question 7.14:
A rope of negligible mass is wound round a hollow cylinder of mass 3 kg and radius 40 cm. What is the angular acceleration of the cylinder if the rope is pulled with a force of 30 N? What is the linear acceleration of the rope? Assume that there is no slipping.
ANS:
Mass of the hollow cylinder, m = 3 kg
Radius of the hollow cylinder, r = 40 cm = 0.4 m
Applied force, F = 30 N
The moment of inertia of the hollow cylinder about its geometric axis:

$I = mr^2$
$= 3 \times (0.4)^2 \qquad = 0.48$ kg m^2
Torque $\tau = F \times r \quad = \quad 30 \times 0.4 \; = 12$ Nm
For angular acceleration α, torque is also given by the relation:

$\tau = I\,\alpha$ or $\alpha = \tau / I \; = \; (12 / 0.48) \; = 25$ rad.s^{-2}

Linear acceleration = r α = 0.4 × 25 = 10 m s^{-2}

Question 7.15:
To maintain a rotor at a uniform angular speed of 200 rad s^{-1}, an engine needs to transmit a torque of 180 Nm. What is the power required by the engine? (Note: uniform angular velocity in the absence of friction implies zero torque. In practice, applied torque is needed to counter frictional torque). Assume that the engine is 100 % efficient.
ANS:
Angular speed of the rotor, ω = 200 rad/s
Torque required, τ = 180 Nm
The power of the rotor (P) is related to torque and angular speed by the relation:

P = τ ω
$= 180 \times 200 = 36 \times 10^3$
= 36 kW
Hence, the power required by the engine is 36 kW.

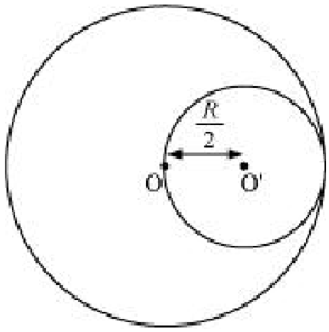
Question 7.16:

From a uniform disk of radius R, a circular hole of radius R/2 is cut out. The centre of the hole is at R/2 from the centre of the original disc. Locate the centre of gravity of the resulting flat body.

ANS:

R/6; from the original centre of the body and opposite to the centre of the cut portion. Mass per unit area of the original disc = σ, Radius of the original disc = R, Mass of the original disc, M = $\pi R^2 \sigma$.

The disc with the cut portion is shown in the following figure:



Radius of the smaller disc = R/2

Mass of the smaller disc, M'= $\pi \left(\frac{R}{2}\right)^2 \sigma = \frac{1}{4}\pi R^2 \sigma = \frac{M}{4}$

Let O and O′ be the respective centres of the original disc and the disc cut off from the original. As per the definition of the centre of mass, the centre of mass of the original disc is supposed to be concentrated at O, while that of the smaller disc is supposed to be concentrated at O′.

It is given that: OO′= R/2

After the smaller disc has been cut from the original, the remaining portion is considered to be a system of two masses. The two masses are:

M (concentrated at O), and –M′ (= $\frac{M}{4}$) concentrated at O′

(The negative sign indicates that this portion has been removed from the original disc.)

Let x be the distance through which the centre of mass of the remaining portion shifts from point O.

The relation between the centre of masses of two masses is given as:

$$x = \frac{m_1 r_1 + m_2 r_2}{m_1 + m_2}$$

For the given system, we can write :

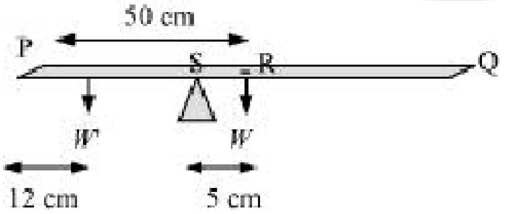
$$x = \frac{M \times 0 - M' \times \left(\frac{R}{2}\right)}{M + (-M')}$$

$$= \frac{\frac{-M}{4} \times \frac{R}{2}}{M - \frac{M}{4}} = \frac{-MR}{8} \times \frac{4}{3M} = \frac{-R}{6}$$

(The negative sign indicates that the centre of mass gets shifted toward the left of point O.)

Question 7.17:
A metre stick is balanced on a knife edge at its centre. When two coins, each of mass 5 g are put one on top of the other at the 12.0 cm mark, the stick is found to be balanced at 45.0 cm. What is the mass of the metre stick?

ANS:
Let W and W′ be the respective weights of the metre stick and the coin



The mass of the metre stick is concentrated at its mid-point, i.e., at the 50 cm mark. Mass of the meter stick = m' Mass of each coin, m = 5 g
When the coins are placed 12 cm away from the end P, the centre of mass gets shifted by 5 cm from point R toward the end P. The centre of mass is located at a distance of 45 cm from point P.
The net torque will be conserved for rotational equilibrium about point R.

$$10 \times g(45 - 12) - m'g(50 - 45) = 0$$

$$\therefore m' = \frac{10 \times 33}{5} = 66 \text{ g}$$

Hence, the mass of the metre stick is 66 g.

Question 7.18:
A solid sphere rolls down two different inclined planes of the same heights but different angles of inclination. (a) Will it reach the bottom with the same speed in each case? (b) Will it take longer to roll down one plane than the other? (c) If so, which one and why?
ANS:
(a) Yes (b) Yes (c) On the smaller inclination

(a) Mass of the sphere = m, Height of the plane = h,
Velocity of the sphere at the bottom of the plane = v, At the top of the plane, the total energy of the sphere = Potential energy = mgh, At the bottom of the plane, the sphere has both translational and rotational kinetic energies.

Hence, total energy = $\frac{mv^2}{2} + \frac{1}{2}I\omega^2$

Using the law of conservation of energy, we can write: $\frac{mv^2}{2} + \frac{1}{2}I\omega^2$ = mgh ------- (1)

For a solid sphere, the moment of inertia about its centre I = $\frac{2}{5}\ mr^2$

Hence, equation (i) becomes:

$$\frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}mr^2\right)\omega^2 = mgh$$

$$\frac{1}{2}v^2 + \frac{1}{5}r^2\omega^2 = gh$$

But we know the relation, v – rω

$$\therefore \frac{1}{2}v^2 + \frac{1}{5}v^2 = gh$$

$$v^2\left(\frac{7}{10}\right) = gh$$

$$v = \sqrt{\frac{10}{7}gh}$$

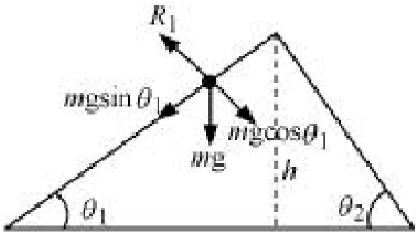
Hence, the velocity of the sphere at the bottom depends only on height (h) and acceleration due to gravity (g). Both these values are constants. Therefore, the velocity at the bottom remains the same from whichever inclined plane the sphere is rolled.

(b) & (c)

Consider two inclined planes with inclinations θ_1 and θ_2, related as: $\theta_1 < \theta_2$.

The acceleration produced in the sphere when it rolls down the plane inclined at θ_1 is: $g \sin \theta_1$

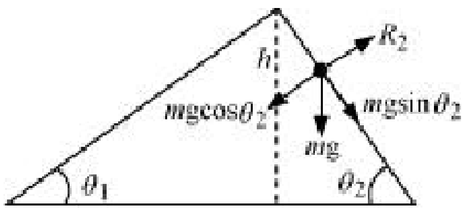
The various forces acting on the sphere are shown in the following figure.



R_1 is the normal reaction to the sphere.

Similarly, the acceleration produced in the sphere when it rolls down the plane inclined at θ_2 is: $g \sin \theta_2$

The various forces acting on the sphere are shown in the following figure.



R_2 is the normal reaction to the sphere.

$\theta_2 > \theta_1$; $\sin \theta_2 > \sin \theta_1$ ... (i)

$\therefore a_2 > a_1$ ... (ii)

Initial velocity, u = 0, Final velocity, v = Constant. Using the first equation of motion, we can obtain the time of roll as:

v = u + at

$\therefore$ t $\propto \frac{1}{\alpha}$

For inclination θ_1 : $t_1 \propto \frac{1}{\alpha_1}$

For inclination θ_2 : $t_2 \propto \frac{1}{\alpha_2}$ ------- (iii)

From equations (ii) and (iii), we get:

$t_2 < t_1$

Hence, the sphere will take a longer time to reach the bottom of the inclined plane having the smaller inclination.

Question 7.19:

A hoop of radius 2 m weighs 100 kg. It rolls along a horizontal floor so that its centre of mass has a speed of 20 cm/s. How much work has to be done to stop it?

ANS:

Radius of the hoop, r = 2 m

Mass of the hoop, m = 100 kg

Velocity of the hoop, v = 20 cm/s = 0.2 m/s

Total energy of the hoop = Translational KE + Rotational KE

$E_T = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

Moment of inertia of the hoop about its centre, I = mr^2

$E_T = \frac{1}{2}mv^2 + \frac{1}{2}(mr^2)\omega^2$

But we have the relation v = rω

$\therefore E_T = \frac{1}{2}mv^2 + \frac{1}{2}mr^2\omega^2 = \frac{1}{2}mv^2 + \frac{1}{2}mv^2 = mv^2$

The work required to be done for stopping the hoop is equal to the total energy of the hoop.

$\therefore$ Required work to be done, W = $mv^2 = 100 \times (0.2)^2 = 4$ J

Question 7.20:

The oxygen molecule has a mass of 5.30×10^{-26} kg and a moment of inertia of 1.94×10^{-46} kg m^2 about an axis through its centre perpendicular to the lines joining the two atoms. Suppose the mean speed of such a molecule in a gas is 500 m/s and that its kinetic energy of rotation is two thirds of its kinetic energy of translation. Find the average angular velocity of the molecule.

ANS:

Mass of an oxygen molecule, m = 5.30×10^{-26} kg

Moment of inertia, I = 1.94×10^{-46} kg m^2, Velocity of the oxygen molecule, v = 500 m/s, The separation between the two atoms of the oxygen molecule = 2r

Mass of each oxygen atom = m/2

Hence, moment of inertia I, is calculated as: $\frac{1}{2}mr^2 + \frac{1}{2}mr^2 = mr^2$

$r = \sqrt{\frac{I}{m}} = \sqrt{\frac{1.94 \times 10^{-46}}{5.36 \times 10^{-26}}} = 0.60 \times 10^{-10}\, m$

It is given that:

$$KE_{rot} = \frac{2}{3} KE_{trans}$$

$$\frac{1}{2} I\omega^2 = \frac{2}{3} \times \frac{1}{2} \times mv^2$$

$$mr^2\omega^2 = \frac{2}{3} mv^2$$

$$\omega = \sqrt{\frac{2}{3}} \frac{v}{r}$$

$$= \sqrt{\frac{2}{3}} \times \frac{500}{0.6 \times 10^{-10}} \qquad = 6.80 \times 10^{12} \text{ rad/s}$$

Question 7.21:

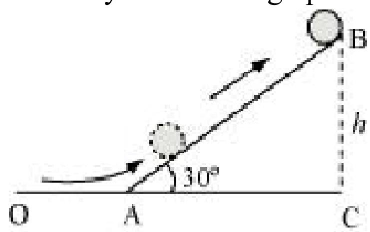
A solid cylinder rolls up an inclined plane of angle of inclination 30°. At the bottom of the inclined plane the centre of mass of the cylinder has a speed of 5 m/s.

(a) How far will the cylinder go up the plane?

(b) How long will it take to return to the bottom?

ANS:

A solid cylinder rolling up an inclination is shown in the following figure.



Initial velocity of the solid cylinder, v = 5 m/s

Angle of inclination, θ = 30°

Height reached by the cylinder = h

(a) Energy of the cylinder at point A:

$K.E._{rot} = K.E._{trans}$

$\frac{1}{2}I\omega^2 = \frac{1}{2}mv^2$

Energy of the cylinder at point B = mgh

Using the law of conservation of energy, we can write:

$\frac{1}{2}I\omega^2 = \frac{1}{2}mv^2$ = mgh

Moment of inertia of the solid cylinder, $I = \frac{1}{2}mr^2$

$$\therefore \frac{1}{2}\left(\frac{1}{2}mr^2\right)\omega^2 + \frac{1}{2}mv^2 = mgh$$

$$\frac{1}{4}mr^2\omega^2 + \frac{1}{2}mv^2 = mgh$$

But we have the relation v = rω

$$\therefore \frac{1}{4}v^2 + \frac{1}{2}v^2 = gh$$

$$\frac{3}{4}v^2 = gh$$

$$\therefore h = \frac{3}{4}\frac{v^2}{g} \qquad = \frac{3}{4} \times \frac{5 \times 5}{9.8} = 1.91 \text{ m}$$

In ΔABC:

Sin $\theta = \frac{BC}{AC}$

Sin 30° $= \frac{h}{AB}$

AB $= \frac{1.91}{0.5} = 3.82$ m

Hence, the cylinder will travel 3.82 m up the inclined plane.

(b) For radius of gyration K, the velocity of the cylinder at the instance when it rolls back to the bottom is given by the relation:

$$v = \left(\frac{2gh}{1 + \frac{K^2}{R^2}}\right)^{\frac{1}{2}}$$

$$\therefore v = \left(\frac{2gAB\sin\theta}{1 + \frac{K^2}{R^2}}\right)^{\frac{1}{2}}$$

For the solid cylinder, $K^2 = R^2/2$

$$\therefore v = \left(\frac{2gAB\sin\theta}{1 + \frac{1}{2}}\right)^{\frac{1}{2}}$$

$$= \left(\frac{4}{3}gAB\sin\theta\right)^{\frac{1}{2}}$$

The time taken to return to the bottom is:

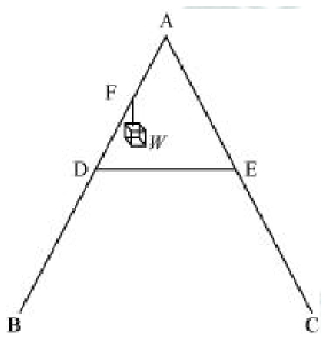
t $= \frac{AB}{v}$

$$= \frac{AB}{\left(\frac{4}{3}gAB\sin\theta\right)^{\frac{1}{2}}} = \left(\frac{3AB}{4g\sin\theta}\right)^{\frac{1}{2}} \qquad = \left(\frac{11.46}{19.6}\right)^{\frac{1}{2}} = 0.764 \text{ s}$$

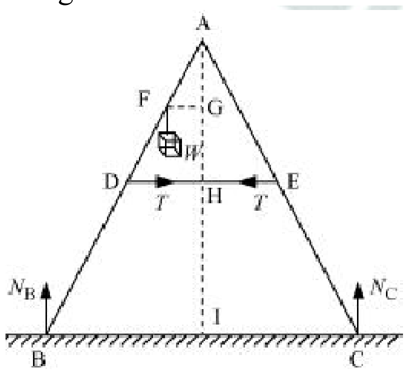
Therefore, the total time taken by the cylinder to return to the bottom is (2 × 0.764) 1.53 s.

Question 7.22:

As shown in Fig.7.40, the two sides of a step ladder BA and CA are 1.6 m long and hinged at A. A rope DE, 0.5 m is tied half way up. A weight 40 kg is suspended from a point F, 1.2 m from B along the ladder BA. Assuming the floor to be frictionless and neglecting the weight of the ladder, find the tension in the rope and forces exerted by the floor on the ladder. (Take g = 9.8 m/s^2) (Hint: Consider the equilibrium of each side of the ladder separately.)



ANS:
The given situation can be shown as:



NB = Force exerted on the ladder by the floor point B
NC = Force exerted on the ladder by the floor point C
T = Tension in the rope
BA = CA = 1.6 m
DE = 0. 5 m
BF = 1.2 m
Mass of the weight, m = 40 kg
Draw a perpendicular from A on the floor BC. This intersects DE at mid-point H.
ΔABI and ΔAIC are similar
∴BI = IC
Hence, I is the mid-point of BC.
DE || BC
BC = 2 × DE = 1 m
AF = BA – BF = 0.4 m … (i)
D is the mid-point of AB.
Hence, we can write: AD = $\frac{1}{2}$BA = 0.8 m ------ (ii)
Using equations (i) and (ii), we get:
FE = 0.4 m

Hence, F is the mid-point of AD.
FG||DH and F is the mid-point of AD. Hence, G will also be the mid-point of AH.
ΔAFG and ΔADH are similar

$$\therefore \frac{FG}{DH} = \frac{AF}{AD}$$

$$\frac{FG}{DH} = \frac{0.4}{0.8} = \frac{1}{2} \qquad FG = \frac{1}{2}DH \qquad = \frac{1}{2} \times 0.25 = 0.125 \text{ m}$$

In ΔADH:

$$AH = \sqrt{AD^2 - DH^2}$$

$$= \sqrt{(0.8)^2 - (0.25)^2} = 0.76 \text{ m}$$

For translational equilibrium of the ladder, the upward force should be equal to the downward force.

$N_C + N_B = mg = 392 \quad \ldots \quad$ (iii)

For rotational equilibrium of the ladder, the net moment about A is:

$$-N_B \times BI + mg \times FG + N_C \times CI + T \times AG - T \times AG = 0$$

$$-N_B \times 0.5 + 40 \times 9.8 \times 0.125 + N_C \times (0.5) = 0$$

$$(N_C - N_B) \times 0.5 = 49$$

$$N_C - N_B = 98 \qquad \ldots (iv)$$

Adding equations (iii) and (iv), we get:
$N_C = 245$ N
$N_B = 147$ N

For rotational equilibrium of the side AB, consider the moment about A

$$-N_B \times BI + mg \times FG + T \times AG = 0$$

$$-245 \times 0.5 + 40 + 9.8 \times 0.125 + T \times 0.76 = 0$$

$$0.76T = 122.5 - 49$$

$$\therefore T = 96.7 \text{ N}$$

Question 7.23:
A man stands on a rotating platform, with his arms stretched horizontally holding a 5 kg weight in each hand. The angular speed of the platform is 30 revolutions per minute. The man then brings his arms close to his body with the distance of each weight from the axis changing from 90cm to 20cm. The moment of inertia of the man together with the platform may be taken to be constant and equal to 7.6 kg m^2.
(a) What is his new angular speed? (Neglect friction.)
(b) Is kinetic energy conserved in the process? If not, from where does the change come about?
ANS:
(a) 58.88 rev/min (b) No
(a) Moment of inertia of the man-platform system = 7.6 kg m^2
Moment of inertia when the man stretches his hands to a distance of 90 cm:
$2 \times m\, r^2$

$= 2 \times 5 \times (0.9)^2$
$= 8.1$ kg m^2
Initial moment of inertia of the system, $I_i = 7.6 + 8.1 = 15.7$ Kgm^2
Angular speed, $\omega_i = 30$ rev/min
Angular momentum, $L_i = I_i\omega_i = 15.7 \times 30$ ------- (i)

Moment of inertia when the man folds his hands to a distance of 20 cm: $2 \times mr^2$
$= 2 \times 5\ (0.2)^2 = 0.4$ kg m^2

Final moment of inertia, $I_f = 7.6 + 0.4 = 8.0$ Kgm^2

Final angular speed = $\omega_j = ?$

Final angular momentum, $L_f = I_f\omega_f = 0.79\ \omega_f$ ------- (ii)
From the conservation of angular momentum, we have: $I_i\omega_i = I_f\omega_f$
$\therefore \omega_f = \frac{15.7 \times 30}{8} = 58.88$ rev/min

(b) Kinetic energy is not conserved in the given process. In fact, with the decrease in the moment of inertia, kinetic energy increases. The additional kinetic energy comes from the work done by the man to fold his hands toward himself.

Question 7.24:
A bullet of mass 10 g and speed 500 m/s is fired into a door and gets embedded exactly at the centre of the door. The door is 1.0 m wide and weighs 12 kg. It is hinged at one end and rotates about a vertical axis practically without friction. Find the angular speed of the door just after the bullet embeds into it. (Hint: The moment of inertia of the door about the vertical axis at one end is $ML^2/3$.)
Answer
Mass of the bullet, m = 10 g = 10×10^{-3} kg
Velocity of the bullet, v = 500 m/s
Thickness of the door, L = 1 m
Radius of the door, r = ½ m
Mass of the door, M = 12 kg
Angular momentum imparted by the bullet on the door: α = mvr
$= (10 \times 10^{-3}) \times (500) \times ½ = 2.5$ kg m^2 s^{-1} ------- (i)

Moment of inertia of the door : $I = \frac{1}{3}ML^2 = \frac{1}{3} \times 12 \times (1)^2 = 4$ Kgm^2
But $\alpha = I\omega$ $\therefore \omega = \frac{\alpha}{I} = \frac{2.5}{4} = 0.625$ rad s^{-1}

Question 7.25:
Two discs of moments of inertia I_1 and I_2 about their respective axes (normal to the disc and passing through the centre), and rotating with angular speeds ω_1 and ω_2 are brought into contact face to face with their axes of rotation coincident. (a) What is the angular speed of

the two-disc system? (b) Show that the kinetic energy of the combined system is less than the sum of the initial kinetic energies of the two discs. How do you account for this loss in energy? Take $\omega_1 \neq \omega_2$.

Answer

(a) :

Moment of inertia of disc I $= I_1$

Angular speed of disc I $= \omega_1$

Angular speed of disc II $= I_2$

Angular momentum of disc II $= \omega_1$

Angular momentum of disc I, $L_1 = I_1\omega_1$

Angular momentum of disc II, $L_2 = I_2\omega_2$

Total initial angular momentum, $L_i = I_1\omega_1 + I_2\omega_2$

When the two discs are joined together, their moments of inertia get added up.

Moment of inertia of the system of two discs, $I = I_1 + I_2$

Let ω be the angular speed of the system.

Total final angular momentum, $L_f = (I_1 + I_2)\omega$

Using the law of conservation of angular momentum, we have:

$$L_i = L_f$$

$$I_1\omega_1 + I_2\omega_2 = (I_1 + I_2)\omega$$

$$\therefore \omega = \frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2}$$

(b) Kinetic energy of disc I, $E_1 = \frac{1}{2}I_1\omega_1^2$

Kinetic energy of disc II, $E_2 = \frac{1}{2}I_2\omega_2^2$

Total initial kinetic energy, $E_i = \frac{1}{2}\left(I_1\omega_1^2 + I_2\omega_2^2\right)$

When the discs are joined, their moments of inertia get added up.

Moment of inertia of the system, $I = I_1 + I_2$

Angular speed of the system = ω

Final kinetic energy E_f:

$$= \frac{1}{2}(I_1 + I_2)\omega^2$$

$$= \frac{1}{2}(I_1 + I_2)\left(\frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2}\right)^2 = \frac{1}{2}\frac{(I_1\omega_1 + I_2\omega_2)^2}{I_1 + I_2}$$

$$\therefore E_i - E_f$$

$$= \frac{1}{2}\left(I_1\omega_1^2 + I_2\omega_2^2\right) - \frac{\left(I_1\omega_1 + I_2\omega_2\right)^2}{2\left(I_1 + I_2\right)}$$

$$= \frac{1}{2}I_1\omega_1^2 + \frac{1}{2}I_2\omega_2^2 - \frac{1}{2}\frac{I_1^2\omega_1^2}{\left(I_1 + I_2\right)} - \frac{1}{2}\frac{I_2^2\omega_2^2}{\left(I_1 + I_2\right)} - \frac{1}{2}\frac{2I_1I_2\omega_1\omega_2}{\left(I_1 + I_2\right)}$$

$$= \frac{1}{\left(I_1 + I_2\right)}\left[\frac{1}{2}I_1^2\omega_1^2 + \frac{1}{2}I_1I_2\omega_1^2 + \frac{1}{2}I_1I_2\omega_2^2 + \frac{1}{2}I_2^2\omega^2 - \frac{1}{2}I_1^2\omega_1^2 - \frac{1}{2}I_2^2\omega_2^2 - I_1I_2\omega_1\omega_2\right]$$

$$= \frac{I_1I_2}{2\left(I_1 + I_2\right)}\left[\omega_1^2 + \omega_2^2 - 2\omega_1\omega_2\right]$$

$$= \frac{I_1I_2\left(\omega_1 - \omega_2\right)^2}{2\left(I_1 + I_2\right)}$$

All the quantities on RHS are positive.

$$\therefore E_i - E_f > 0$$

$$E_i > E_f$$

The loss of KE can be attributed to the frictional force that comes into play when the two discs come in contact with each other.

Question 7.26:

(a) Prove the theorem of perpendicular axes.

(Hint: Square of the distance of a point (x, y) in the x–y plane from an axis through the origin perpendicular to the plane is $x^2 + y^2$).

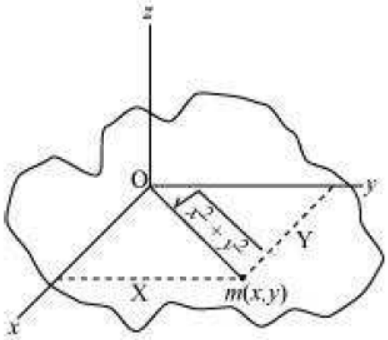
(b) Prove the theorem of parallel axes.

(Hint: If the centre of mass is chosen to be the origin $\sum m_i r_i = 0$)

Answer :

(a) The theorem of perpendicular axes states that the moment of inertia of a planar body (lamina) about an axis perpendicular to its plane is equal to the sum of its moments of inertia about two perpendicular axes concurrent with perpendicular axis and lying in the plane of the body.

A physical body with centre O and a point mass m, in the x–y plane at (x, y) is shown in the following figure.



Moment of inertia about x-axis, $I_x = mx^2$

Moment of inertia about y-axis, $I_y = my^2$

Moment of inertia about z-axis, $I_z = m\left(\sqrt{x^2+y^2}\right)^2$

$I_x + I_y = mx^2 + my^2$

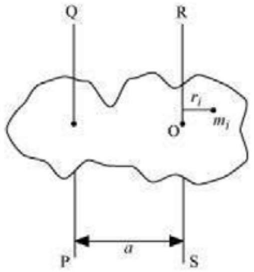
$= m(x^2 + y^2)$

$= m\left(\sqrt{x^2+y^2}\right)^2$

$I_x + I_y = I_z$

Hence the theorem is proved.

(b) The theorem of parallel axes states that the moment of inertia of a body about any axis is equal to the sum of the moment of inertia of the body about a parallel axis passing through its centre of mass and the product of its mass and the square of the distance between the two parallel axes.



Suppose a rigid body is made up of n particles, having masses m_1, m_2, m_3, … , m_n, at perpendicular distances r_1, r_2, r_3, … , r_n respectively from the centre of mass O of the rigid body. The moment of inertia about axis RS passing through the point O:

$$I_{RS} = \sum_{i=1}^{n} m_i r_i^2$$

The perpendicular distance of mass mi, from the axis QP = a + r_i

Hence, the moment of inertia about axis QP:

$$I_{QP} = \sum_{i=1}^{n} m_i (a + r_i)^2$$

$$= \sum_{i=1}^{n} m_i (a^2 + r_i^2 + 2ar_i)^2$$

$$= \sum_{i=1}^{n} m_i a^2 + \sum_{i=1}^{n} m_i r_i^2 + \sum_{i=1}^{n} m_i 2ar_i$$

$$= I_{RS} + \sum_{i=1}^{n} m_i a^2 + 2\sum_{i=1}^{n} m_i ar_i^2$$

Now, at the centre of mass, the moment of inertia of all the particles about the axis passing through the centre of mass is zero, that is,

$$2\sum_{i=1}^{n} m_i a r_i = 0$$

$\because a \neq 0$

$\therefore \sum m_i r_i = 0$

Also,

$$\sum_{i=1}^{n} m_i = M; \quad M = \text{Total mass of the rigid body}$$

$\therefore I_{QP} = I_{RS} + Ma^2$

Hence, the theorem is proved.

Question 7.27:

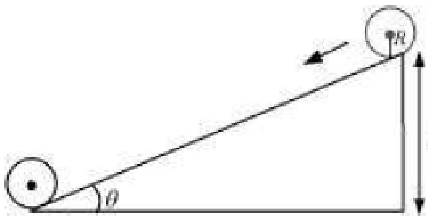
Prove the result that the velocity v of translation of a rolling body (like a ring, disc, cylinder or sphere) at the bottom of an inclined plane of a height h is given by

$$v^2 = \frac{2gh}{\left(1 + k^2/R^2\right)}.$$

Using dynamical consideration (i.e. by consideration of forces and torques). Note k is the radius of gyration of the body about its symmetry axis, and R is the radius of the body. The body starts from rest at the top of the plane.

Answer :

A body rolling on an inclined plane of height h,is shown in the following figure:



m = Mass of the body

R = Radius of the body

K = Radius of gyration of the body

v = Translational velocity of the body

h =Height of the inclined plane

g = Acceleration due to gravity

Total energy at the top of the plane, $E_1 = mgh$

Total energy at the bottom of the plane, $E_b = KE_{Rot} + KE_{Trans}$

$$= \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$$

But $I = mk^2$ and $\omega = \dfrac{v}{R}$

$$\therefore E_b = \frac{1}{2}\left(mk^2\right)\left(\frac{v^2}{R^2}\right) + \frac{1}{2}mv^2$$

$$= \frac{1}{2} mv^2 \frac{k^2}{R^2} + \frac{1}{2} mv^2$$

$$= \frac{1}{2} mv^2 \left(1 + \frac{k^2}{R^2}\right)$$

From the law of conservation of energy, we have: $E_T = E_b$

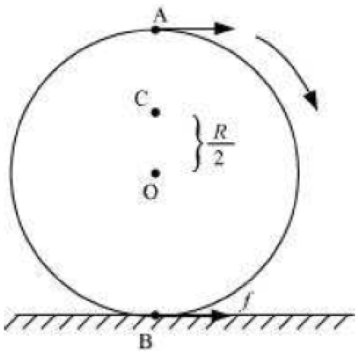
mgh = $\frac{1}{2} mv^2 [1 + \frac{k^2}{R^2}]$

$$\therefore v = \frac{2gh}{\left(1 + k^2 / R^2\right)}$$

Hence, the given result is proved.

Question 7.28:

A disc rotating about its axis with angular speed ω_o is placed lightly (without any translational push) on a perfectly frictionless table. The radius of the disc is *R*. What are the linear velocities of the points A, B and C on the disc shown in Fig. 7.41? Will the disc roll in the direction indicated?



Answer :

$v_A = R\omega_o$; $v_B = R\omega_o$; $v_c = \left(\frac{R}{2}\right)\omega_o$; The disc will not roll

Angular speed of the disc = ω_o

Radius of the disc = R

Using the relation for linear velocity, $v = \omega_o R$

For point A:

$v_A = R\omega_o$; in the direction tangential to the right

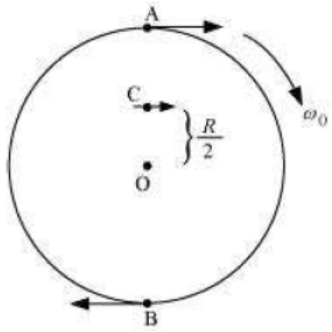
For point B:

$v_B = R\omega_o$; in the direction tangential to the left

For point C:

$v_c = \left(\frac{R}{2}\right)\omega_o$; in the direction same as that of v_A

The directions of motion of points A, B, and C on the disc are shown in the following figure



Since the disc is placed on a frictionless table, it will not roll. This is because the presence of friction is essential for the rolling of a body.

Question 7.29:

Explain why friction is necessary to make the disc in Fig. 7.41 roll in the direction indicated.

(a) Give the direction of frictional force at B, and the sense of frictional torque, before perfect rolling begins.

(b) What is the force of friction after perfect rolling begins?

Answer :

A torque is required to roll the given disc. As per the definition of torque, the rotating force should be tangential to the disc. Since the frictional force at point B is along the tangential force at point A, a frictional force is required for making the disc roll.

(a) Force of friction acts opposite to the direction of velocity at point B. The direction of linear velocity at point B is tangentially leftward. Hence, frictional force will act tangentially rightward. The sense of frictional torque before the start of perfect rolling is perpendicular to the plane of the disc in the outward direction.

(b) Since frictional force acts opposite to the direction of velocity at point B, perfect rolling will begin when the velocity at that point becomes equal to zero. This will make the frictional force acting on the disc zero.

Question 7.30:

A solid disc and a ring, both of radius 10 cm are placed on a horizontal table simultaneously, with initial angular speed equal to 10 π rad s^{-1}. Which of the two will start to roll earlier? The co-efficient of kinetic friction is $\mu_k = 0.2$.

Answer :

Disc :

Radii of the ring and the disc, r = 10 cm = 0.1 m

Initial angular speed, $\omega 0 = 10\pi$ rad s^{-1}

Coefficient of kinetic friction, $\mu_k = 0.2$

Initial velocity of both the objects, $u = 0$
Motion of the two objects is caused by frictional force. As per Newton's second law of motion, we have frictional force, $f = ma$
$\mu_k mg = ma$
Where,
a = Acceleration produced in the objects
m = Mass
$\therefore a = \mu_k g$ … (i)
As per the first equation of motion, the final velocity of the objects can be obtained as:
$v = u + at$
$= 0 + \mu_k gt$
$= \mu_k gt$ … (ii)
The torque applied by the frictional force will act in perpendicularly outward direction and cause reduction in the initial angular speed.
Torque, $\tau = -I\alpha$
α = Angular acceleration
$\mu_x mgr = -I\alpha$
Using the first equation of rotational motion to obtain the final angular speed:

$$\omega = \omega_0 + \alpha t$$

$$= \omega_0 + \frac{-\mu_k mgr}{I}t \qquad \dots (iv)$$

Rolling starts when linear velocity, $v = r\omega$

$$\therefore v = r\left(\omega_0 - \frac{\mu_k gmrt}{I}\right) \qquad \dots (v)$$

Equating equations (ii) and (v), we get:

$$\mu_k gt = r\left(\omega_0 - \frac{\mu_k gmrt}{I}\right)$$

$$= r\omega_0 - \frac{\mu_k gmr^2 t}{I} \qquad \dots (vi)$$

For the ring: $I = mr^2$

$$\therefore \mu_k gt = r\omega_0 - \frac{\mu_k gmr^2 t}{mr^2} \qquad = r\omega_0 - \mu_k gmt_r$$

$$2\mu_k gt = r\omega_0$$

$$\therefore t_r = \frac{r\omega_0}{2\mu_k g}$$

$$= \frac{0.1 \times 10 \times 3.14}{2 \times 0.2 \times 9.8} = 0.80 \text{ s} \qquad \dots (vii)$$

For the disc: $I = \frac{1}{2}mr^2$

$$\therefore \mu_k g t_d = r\omega_0 \frac{\mu_k g m r^2 t}{\frac{1}{2}mr^2} \qquad = r\omega_0 - 2\mu_k g t$$

$$3\mu_k g t_d = r\omega_0$$

$$\therefore t_d = \frac{r\omega_0}{3\mu_k g}$$

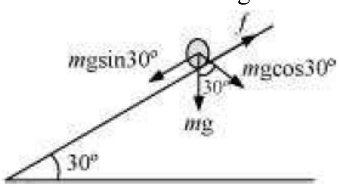
$$= \frac{0.1 \times 10 \times 3.14}{3 \times 0.2 \times 9.8} = 0.53 \text{ s} \qquad \dots (viii)$$

Since $t_d > t_r$, the disc will start rolling before the ring.

Question 7.31:

A cylinder of mass 10 kg and radius 15 cm is rolling perfectly on a plane of inclination 30°. The coefficient of static friction $\mu_S = 0.25$.
(a) How much is the force of friction acting on the cylinder?
(b) What is the work done against friction during rolling?
(c) If the inclination θ of the plane is increased, at what value of θ does the cylinder begin to skid, and not roll perfectly?

Answer :
Mass of the cylinder, m = 10 kg
Radius of the cylinder, r = 15 cm = 0.15 m
Co-efficient of kinetic friction, $\mu_S = 0.25$
Angle of inclination, $\theta = 30°$
Moment of inertia of a solid cylinder about its geometric axis, $I = \frac{1}{2} mr^2$
The various forces acting on the cylinder are shown in the following figure:



The acceleration of the cylinder is given as:

$$a = \frac{mg\sin\theta}{m + \frac{I}{r^2}} \qquad = \frac{mg\sin\theta}{m + \frac{1}{2}\frac{mr^2}{r^2}} = \frac{2}{3}g\sin 30^\circ \qquad = \frac{2}{3} \times 9.8 \times 0.5 = 3.27 \text{ m/s}^2$$

(a) Using Newton's second law of motion, we can write net force as:

$$f_{net} = ma$$

$$mg\sin 30^\circ - f = ma$$

$$f = mg\sin 30^\circ - ma$$

$$= 10 \times 9.8 \times 0.5 - 10 \times 3.27$$

$$= 49 - 32.7 = 16.3 \text{ N}$$

(b) During rolling, the instantaneous point of contact with the plane comes to rest. Hence, the work done against frictional force is zero.

(c) For rolling without skid, we have the relation:

$$\mu = \frac{1}{3}\tan\theta$$

$$\tan\theta = 3\mu = 3 \times 0.25$$

$$\therefore \theta = \tan^{-1}(0.75) = 36.87°$$

Question 7.32:

Read each statement below carefully, and state, with reasons, if it is true or false;

(a) During rolling, the force of friction acts in the same direction as the direction of motion of the CM of the body.

(b) The instantaneous speed of the point of contact during rolling is zero.

(c) The instantaneous acceleration of the point of contact during rolling is zero.

(d) For perfect rolling motion, work done against friction is zero.

(e) A wheel moving down a perfectly *frictionless* inclined plane will undergo slipping (not rolling) motion.

Answer

(a) False

Frictional force acts opposite to the direction of motion of the centre of mass of a body. In the case of rolling, the direction of motion of the centre of mass is backward. Hence, frictional force acts in the forward direction.

(b) True

Rolling can be considered as the rotation of a body about an axis passing through the point of contact of the body with the ground. Hence, its instantaneous speed is zero.

(c) False

When a body is rolling, its instantaneous acceleration is not equal to zero. It has some value.

(d) True

When perfect rolling begins, the frictional force acting at the lowermost point becomes zero. Hence, the work done against friction is also zero.

(e) True

The rolling of a body occurs when a frictional force acts between the body and the surface. This frictional force provides the torque necessary for rolling. In the absence of a frictional force, the body slips from the inclined plane under the effect of its own weight.

Question 7.33:

Separation of Motion of a system of particles into motion of the centre of mass and motion about the centre of mass:

(a) Show $\mathbf{p}_i = \mathbf{p}'_i + m_i\mathbf{V}$

Where $\mathbf{p}_i$ is the momentum of the ith particle (of mass mi) and $\mathbf{p}'_i = m_i\,\mathbf{v}'_i$. Note $\mathbf{v}'_i$ is the velocity of the ith particle relative to the centre of mass.

Also, prove using the definition of the centre of mass $\sum_i P'_i = 0$

(b) Show $K = K' + ½MV^2$

Where K is the total kinetic energy of the system of particles, K' is the total kinetic energy of the system when the particle velocities are taken with respect to the centre of mass and $MV^2/2$ is the kinetic energy of the translation of the system as a whole (i.e. of the centre of mass motion of the system). The result has been used in Sec. 7.14.

(c) Show $\mathbf{L} = \mathbf{L}' + \mathbf{R} \times M\mathbf{V}$

Where $\mathbf{L}' = \sum_i \boldsymbol{r}'_i\,\boldsymbol{p}'_i$ is the angular momentum of the system about the centre of mass with velocities taken relative to the centre of mass. Remember $\mathbf{r}'i = \mathbf{r}i - \mathbf{R}$; rest of the notation is the standard notation used in the chapter. Note **L′** and $M\mathbf{R} \times \mathbf{V}$ can be said to be angular momenta, respectively, about and of the centre of mass of the system of particles.

(d) Show $\frac{d\mathbf{L}'}{dt} = \sum_i \mathbf{r}'_i \times \frac{d}{dt}(\mathbf{p}'_i)$

Further, show that

$$\frac{d\mathbf{L}'}{dt} = \tau'_{ext}$$

where τ'_{ext} is the sum of all external torques acting on the system about the centre of mass. (Hint: Use the definition of centre of mass and Newton's Third Law. Assume the internal forces between any two particles act along the line joining the particles.)

Answer

(a) Take a system of i moving particles.

Mass of the ith particle = m_i

Velocity of the ith particle = $\mathbf{v}_i$

Hence, momentum of the ith particle, $\mathbf{p}_i = m_i \mathbf{v}_i$

Velocity of the centre of mass = $\mathbf{V}$

The velocity of the ith particle with respect to the centre of mass of the system is given as:

$\mathbf{v}'_i = \mathbf{v}_i - \mathbf{V}$ … (1)

Multiplying m_i throughout equation (1), we get:

$m_i \mathbf{v}'_i = m_i \mathbf{v}_i - m_i \mathbf{V}$

$\mathbf{p}'_i = \mathbf{p}_i - m_i \mathbf{V}$

Where,

$\mathbf{p}_i' = m_i\mathbf{v}_i'$ = Momentum of the ith particle with respect to the centre of mass of the system

$\therefore \mathbf{p}_i = \mathbf{p}'_i + m_i \mathbf{V}$

We have the relation: $\mathbf{p}'_i = m_i\mathbf{v}_i'$

Taking the summation of momentum of all the particles with respect to the centre of mass of the system, we get:

$$\sum_i \mathbf{p}'_i = \sum_i m_i \mathbf{v}'_i = \sum_i m_i \frac{d\mathbf{r}'_i}{dt}$$

Where,

$\mathbf{r}'_i$ = Position vector of ith particle with respect to the centre of mass

$$\mathbf{v}'_i = \frac{d\mathbf{r}'_i}{dt}$$

As per the definition of the centre of mass, we have:

$$\sum_i m_i \mathbf{r}'_i = 0$$

$$\therefore \sum_i m_i \frac{d\mathbf{r}'_i}{dt} = 0$$

$$\sum_i \mathbf{p}'_i = 0$$

(b) We have the relation for velocity of the i^{th} particle as:

$\mathbf{v}_i = \mathbf{v'}_i + \mathbf{V}$

$$\sum_i m_i \mathbf{v}_i = \sum_i m_i \mathbf{v'}_i + \sum_i m_i \mathbf{V} \quad \ldots (2)$$

Taking the dot product of equation (2) with itself, we get:

$$\sum_i m_i \mathbf{v}_i . \sum_i m_i \mathbf{v}_i = \sum_i m_i (\mathbf{v'}_i + \mathbf{V}) . \sum_i m_i (\mathbf{v'}_i + \mathbf{V})$$

$$M^2 \sum_i v_i^2 = M^2 \sum_i v_i'^2 + M^2 \sum_i \mathbf{v}_i . \mathbf{v'}_i + M^2 \sum_i \mathbf{v'}_i . \mathbf{v}_i + M^2 V^2$$

Here, for the centre of mass of the system of particles, $\sum_i \mathbf{v}_i . \mathbf{v'}_i = -\sum_i \mathbf{v'}_i . \mathbf{v}_i$

$$M^2 \sum_i v_i^2 = M^2 \sum_i v_i'^2 + M^2 V^2$$

$$\frac{1}{2} M \sum_i v_i^2 = \frac{1}{2} M \sum_i v'^2_i + \frac{1}{2} M V^2$$

$$K = K' + \frac{1}{2} M V^2$$

Where,

$K = \frac{1}{2} M \sum_i v_i^2$ = Total kinetic energy of the system of particles

$K' = \frac{1}{2} M \sum_i v'^2_i$ = Total kinetic energy of the system of particles with respect to the centre of mass

$= \frac{1}{2} m v^2$ = Kinetic energy of the translation of the system as a whole.

(c) Position vector of the *i*th particle with respect to origin = $\mathbf{r}_i$

Position vector of the *i*th particle with respect to the centre of mass = $\mathbf{r'}_i$

Position vector of the centre of mass with respect to the origin = $\mathbf{R}$

It is given that:

$\mathbf{r'}_i = \mathbf{r}_i - \mathbf{R}$

$\mathbf{r}_i = \mathbf{r'}_i + \mathbf{R}$

We have from part **(a)**,

$\mathbf{p}_i = \mathbf{p'}_i + m_i \mathbf{V}$

Taking the cross product of this relation by $\mathbf{r}_i$, we get:

$$\sum_i \mathbf{r}_i \times \mathbf{p}_i = \sum_i \mathbf{r}_i \times \mathbf{p'}_i + \sum_i \mathbf{r}_i \times m_i \mathbf{V}$$

$$\mathbf{L} = \sum_i (\mathbf{r'}_i + \mathbf{R}) \times \mathbf{p'}_i + \sum_i (\mathbf{r'}_i + \mathbf{R}) \times m_i \mathbf{V}$$

$$= \sum_i \mathbf{r'}_i \times \mathbf{p'}_i + \sum_i \mathbf{R} \times \mathbf{p'}_i + \sum_i \mathbf{r'}_i \times m_i \mathbf{V} + \sum_i \mathbf{R} \times m_i \mathbf{V}$$

$$= \mathbf{L'} + \sum_i \mathbf{R} \times \mathbf{p'}_i + \sum_i \mathbf{r'}_i \times m_i \mathbf{V} + \sum_i \mathbf{R} \times m_i \mathbf{V}$$

Where,

$$\mathbf{R} \times \sum_i \mathbf{p'}_i = 0 \text{ and}$$

$$\left(\sum_i \mathbf{r'}_i\right) \times M\mathbf{V} = \mathbf{0}$$

$$\sum_i m_i = M$$

$$\therefore \mathbf{L} = \mathbf{L'} + R \times M\mathbf{V}$$

(d) We have the relation:

$$\mathbf{L'} = \sum_i \mathbf{r'}_i \times \mathbf{p'}_i$$

$$\frac{d\mathbf{L'}}{dt} = \frac{d}{dt}\left(\sum_i \mathbf{r'}_i \times \mathbf{p'}_i\right)$$

$$= \frac{d}{dt}\left(\sum_i \mathbf{r'}_i\right) \times \mathbf{p'}_i + \sum_i \mathbf{r'}_i \times \frac{d}{dt}(\mathbf{p'}_i)$$

$$= \frac{d}{dt}\left(\sum_i m_i \mathbf{r'}_i\right) \times \mathbf{v'}_i + \sum_i \mathbf{r'}_i \times \frac{d}{dt}(\mathbf{p'}_i)$$

Where, $\mathbf{r'}_i$ is the position vector with respect to the centre of mass of the system of particles.

$$\therefore \sum_i m_i \mathbf{r'}_i = 0$$

$$\therefore \frac{d\mathbf{L'}}{dt} = \sum_i \mathbf{r'}_i \times \frac{d}{dt}(\mathbf{p'}_i)$$

We have the relation:

$$\frac{d\mathbf{L'}}{dt} = \sum_i \mathbf{r'}_i \times \frac{d}{dt}(\mathbf{p'}_i)$$

$$= \sum_i \mathbf{r'}_i \times m_i \frac{d}{dt}(\mathbf{v'}_i)$$

Where, $\frac{d}{dt}(\mathbf{v'}_i)$ is the rate of change of velocity of the ith particle with respect ot the centre of mass of the system

Therefore, according to Newton's third law of motion, we can write:

$$m_i \frac{d}{dt}(\mathbf{v'}_i) = \text{Extrenal force acting on the } i\text{th particle} = \sum_i (\tau'_i)_{\text{ext}}$$

i.e., $\sum_i \mathbf{r'}_i \times m_i \frac{d}{dt}(\mathbf{v'}_i) = \tau'_{\text{ext}}$ = External torque acting on the system as a whole

$$\therefore \frac{d\mathbf{L'}}{dt} = \tau'_{\text{ext}}$$

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Problems for Practice

1) The minute's hand of a watch is 2 cm long. Find (a) the angular velocity, (b) linear velocity and (c) the radial acceleration. [April, 2002]
[Ans: (a) π / 1800 rad /s, (b) π / 900 cm /s, (c) (π^2 / 162) × 10^{-4} cm /s^2]
(Note: Linear velocity and radial acceleration are for a particle on the tip of the minute's hand of the watch.)

2) The length of a simple pendulum is one metre and mass of its bob is 10 gram. If the angular displacement is 36 °, find the angular acceleration. [October, 1998]
(Ans: 5.76 rad /s 2)

3) A force $\vec{F} = 2\hat{i} + 2\hat{j} + \hat{k}$ is acting at a point $\vec{r} = 2\hat{i} + 2\hat{j} + \hat{k}$ away from the axis of rotation. Find the torque acting on it. [March, 1998]
(Ans: zero)

4) The position vector of a particle is $\vec{r} = 2\hat{i} + 2\hat{j} + \hat{k}$ and the force acting on is $\vec{F} = 5\hat{i} + 6\hat{j} + 7\hat{k}$. Calculate the torque acting on the particle. [October, 1997]
(Ans: $8\hat{i} - 9\hat{j} + 2\hat{k}$)

5) What should be the maximum speed of a vehicle when it passes on the inclined curved path ? The radius of the curved path is 200 metre and slope θ = 11 °32 '. (The path is frictionless). [March, 1997] (Ans: 20 m / s)
(Note: The problem is incorrectly framed. The path cannot be frictionless. The car cannot run on a frictionless path. At a very low speed, the car will have a tendency to slide inwards on the inclined path resulting in outward radial frictional force on its tyres. As the speed increases, this outward radial frictional force reduces and becomes zero at a definite speed. The answer given is this definite speed at which there is no radial frictional force on the tyres of the car and which the author intends to ask. If the speed of the car increases further, car will have a tendency to skid outwards and inward radial frictional force acts on its tyres which increases with speed of the car. At a certain maximum speed of the car, this frictional force reaches a maximum value. If the speed of the car exceeds this maximum value, it will skid outwards, assuming that the outward toppling speed is higher than this speed which is true for a vehicle like car having less height compared its width.)

6) A body is performing rotational motion. The motion is described by the equation $\theta = a\,t + b\,t^2$. Calculate the values of a and b in terms of initial angular speed (ω_0) and constant angular acceleration (α) derivating the above equation with respect to time. If a = 2 rad /s and b = 5 rad /s^2, compute ω_0 and α . [October, 1996]
(Ans: $a = \omega_0$, $b = \alpha / 2$, $\omega_0 = 2$ rad /s , $\alpha = 10$ rad /s 2)

7) A sphere, in space, revolves by 1 revolution in 10 seconds with respect to axis passing through its centre. Suddenly the diameter of the sphere becomes four times during the motion. How many revolutions will it perform in next 10 seconds ? [March, 1996]
(Ans: 1 / 16 revolution)

8) A circular disc of mass m and radius r has its moment of inertia $m\,r^2 / 2$. The disc is set rolling on a table. If ω is the angular velocity, show that its total kinetic energy is given by $(3 / 4)\, m\, r^2\, \omega^2$. [March, 1995]

9) A rigid body is rotating with uniform angular acceleration. Its angular velocity and angular displacement after 2 seconds are 84 rad /s and 134 rad respectively. Find its angular displacement after 8 seconds. [October, 1994]
(Ans: 944 rad)

10) The angular velocity of a flywheel increases from 1200 rev /min in 10 seconds. Obtain its angular acceleration and the number of revolutions made during this time. [Oct., 1993]
(Ans: 3 rev /s 2, 350 revolutions)

11) A circular ring of radius 10 cm and mass 1 kg is rotating with angular velocity of 10 rev /s about the axis at right angles to its plane and passing through its centre. Find the work that must be done by a torque to increase the rate to 20 rev /s . [March, 1994]
(Ans: 59.2 J)

12) A flywheel takes 4 seconds to rotate through an angle of 240 rad. If its angular velocity at the end of this time becomes 80 rad /s, calculate its constant angular acceleration.
(Ans: 10 rad /s^2) [March, 1993]

13) Linear speed of a vehicle is 36 km /hr when angular acceleration of its wheel is 5 rad /s^2. Calculate its linear speed at the end of 5 s. The diameter of its wheel is 40 cm.
(Ans: 15 m /s) [October, 1992]

14) Find the angular velocity and acceleration of second, minute and hour hands of a clock.
(Ans: π / 30 rad /s, π / 1800 rad /s, π / 21600 rad /s ; α = 0 for all) [March, 1992]

15) If the radius of the earth suddenly changes to X times the present value, find the period of revolution in terms of X. Also find the ratio of final angular energy to the initial angular kinetic energy in terms of X. Moment of inertia of earth = (2 / 5) MR^2.
(Ans: 24 X 2 hours, 1 / X 2) [October, 1991]

16) A solid cylinder of 5 kg and diameter 80 cm rolls down without slipping from the top of the frictionless inclined plane of height 4.9 m. Find its moment of inertia and velocity at the bottom of the slope. G = 9.8 m / s^2. [March, 1991]
(Ans: 0.40 Kg-m^2 about its axis of rolling, 8 m /s. Note: The statement 'frictionless plane' is incorrect as the cylinder cannot roll without slipping if the plane is frictionless.)

17) A ring of 5 metre diameter and 50000 gm mass is rotating about an axis which passes through its centre and perpendicular to its plane. The angular velocity of this ring is found to be increased from 5 rad /s to 25 rad /s in 5 seconds. Calculate the torque acting on this ring in M. K. S. [March, 1990]
(Ans: 1250 Nm)

18) A flywheel starts its motion with angular velocity of 2 rev /s and in 4 seconds it attains the angular velocity of 4 rev /s. Find its constant angular acceleration and angular displacement during this time. [October, 1989]
(Ans: 0.5 rev /s 2, 12 revolutions)

19) Moment of inertia of a flywheel about an axis passing through its centre and perpendicular to its plane is 62.5 kg-m 2. If its frequency is increased by 18 Hz in 5 seconds, calculate the torque acting on the flywheel. [March, 1989]
(Ans: 1413 Nm)

20) A ring of radius 40 cm and mass 60 kg rotates about an axis passing through its centre and perpendicular to its plane. The angular velocity of the ring increases from 5 rad /s to 25 rad /s in 5 sec. Calculate the work done by the torque in 5 sec. [October, 1988, similar problem was asked in Oct., 1987 and May, 1986]
(Ans: 2880 J)

21) A ring of mass 1 kg and diameter 1 m slips from the top of a frictionless surface of the inclined plane of height 2 m. Find its moment of inertia and velocity at the bottom of the slope.
[March, 1988]
(Ans: 0.25 kg-m^2 about an axis passing through its centre and perpendicular to its plane, 6.3 m /s)

22) Find the angular and linear velocity of the earth due to its revolution around the sun. Radius of earth's orbit = 1.5×10^8 km.
(Ans: 1.99 × 10 − 7 rad /s, 2.98 × 10 4 m /s)

23) A turntable rotates about a fixed vertical axis making one revolution in 10 seconds. The moment of inertia of the turntable about this axis is 1200 kg-m^2. A man of mass 80 kg initially standing at the centre of the turntable runs out along a radius. What is the angular velocity of the turntable when the man is 2 m from the centre.
(Ans: 0.5 rad /s)

24) A small block rests on a turntable at 0.65 m from the centre. It is rotated in such a way that the block undergoes a constant tangential acceleration, a_t = 2 m /s^2. Determine (a) how long will it take for the block to start slipping on the turntable, (b) the speed of the block at that instant. μ s = 0.60.
[Ans: (a) 0.95 s, (b) 1.90 m /s]

25) An automobile track is so designed that when a car travels at 100 km /hour, the force between the car and the track acts normal to the surface of the track. Find the angle of banking the track assuming it to be a circle of radius 250 m.
(Ans: 17.46 °)

26) A car starts skidding when traveling at the speed of 60 km /hr on a horizontal road taking a curve of 50 m. At what angle should the road be banked so that the car can travel with the same speed of 60 km /hr without friction between the tyre and the road. What will be the maximum speed of the car on the banked road without skidding ?
(Ans: 29.55 °, 103 km /hr)

27) Length of a simple pendulum is 80 cm. Its maximum angular displacement in a vertical plane is 2 °. Mass of the bob is 0.1 kg. Determine the angular frequency, maximum velocity, maximum acceleration and maximum restoring force of the bob.
(Ans: 3.5 rad /s, 0.01 m /s, 0.34 m /s 2, 0.034 N)

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ROTATIONAL MOTION MCQ

1. Moment of inertia of a disc about an axis which is tangent and parallel to its plane is I. then the moment of inertia of disc about a tangent,, but perpendicular to its plane will be **(MHT-CET-2005)**
(a) $\frac{3I}{4}$ (b) $\frac{3I}{2}$
(c) $\frac{5I}{6}$ (d) $\frac{6I}{5}$

2. If radius of solid sphere is doubled by keeping its mass constant, then
(a) $\frac{I_1}{I_2} = \frac{1}{4}$ (b) $\frac{I_1}{I_2} = \frac{4}{1}$
(c) $\frac{I_1}{I_2} = \frac{3}{2}$ (d) $\frac{I_1}{I_2} = \frac{2}{3}$
Answer: (a)

3. Calculate the M.I. of a thin uniform ring about an axis tangent to the ring and in a plane of the ring, if its M.I. about an axis passing through the centre and perpendicular to plane is 4 kg m^2. **(MHT-CET-2006)**
(a) 12 kg m^2 (b) 3 kg m^2
(c) 6 kg m^2 (d) 9 kg m^2
Answer: (c)

4. By keeping moment of inertia of a body constant, if we double the time period, then angular momentum of body **(MHT-CET-2005)**
(a) Remains constant (b) Becomes half
(c) Doubles (d) Quadruples
Answer: (b)

5. $\frac{L^2}{2I}$ represents **(MH-CET 2003)**
(a) Rotational kinetic energy of a particle.
(b) Potential energy of a particle
(c) Torque on a particle
(d) Power
Answer: (a)

6. The M.I. of a disc about an axis passing through its centre and perpendicular to plane is $\frac{MR^2}{2}$, then its M.I. about a tangent parallel to its diameter is [**MH-CET 2002**]
(a) $\frac{MR^2}{2}$ (b) $\frac{4}{5}MR^2$
(c) $\frac{5}{4}MR^2$ (d) $\frac{3}{4}MR^2$
Answer: (c)

7. The M.I. of disc about an axis perpendicular to its plane and passing through its centre is $\frac{MR^2}{2}$. Its M.I. about a tangent perpendicular to its plane will be **(MH-CET 2002)**
(a) $\frac{3}{2}MR^2$ (b) $\frac{3}{4}MR^2$

(c) $\frac{1}{2}MR^2$ (d) Cannot be determined

Answer: (b)

8. The torque acting is 2000Nm with an angular acceleration of 2 rad/s^2. the moment of inertia of body is **(MHT-CET-2004)**

(a) 1200 kgm^2 (b) 900 kgm^2

(c) 1000 kgm^2 (d) Can't say

Answer: (d)

9. Four solid spheres each of mass M and diameter 2r, are placed with their centers on the four corners of a square of side a (> 2r). the moment of inertia of the system about one side of square is **(DEC 92)**

(a) $\frac{2}{5}M(r^2 + 4a^2)$ (b) $\frac{2}{5}M(r^2 + 2a^2)$

(c) $\frac{2}{5}M(a^2 + 5a^2)$ (d) $\frac{2}{5}M(4r^2 + 5a^2)$

Answer: (d)

10. For increasing the angular velocity of a object by 10%, the kinetic energy has to be increased by

(MHT-CET-2001)

(a) 40% (b) 20%

(c) 10% (d) 21%

Answer: (d)

11. M.I. of a thin uniform circular ring about the tangent to the plane of the ring is **(CPMT 92)**

(a) $\frac{MR^2}{2}$ (b) MR^2

(c) $\frac{MR^2}{4}$ (d) $\frac{3}{2}MR^2$

Answer: (d)

12. A thin uniform ring of mass M and radius R passing through its centre and perpendicular to its plane. Then its M.I. is, **(CPMT 82)**

(a) $\frac{1}{2}MR^2$ (b) MR^2

(c) 2MR^2 (d) $\frac{3}{2}MR^2$

Answer: (b)

13. Ratio of rotational K.E. to rolling K.E. of a solid sphere is [**MH-CET 2002**]

(a) $\frac{2}{3}$ (b) $\frac{2}{5}$

(c) $\frac{2}{7}$ (d) ∞

Answer: (c)

14. A body of M.I. of 5 kg m^2, rotating with an angular velocity of 6 rad/s, has the same kinetic energy as a mass of 20 kg, moving with a velocity of

(a) 3 m/s (b) 2 m/s

(c) 4 m/s (d) 5 m/s

Answer: (a)

15. A thin uniform circular disc of mass M and radius R is rotating in a horizontal plane about an axis passing through its centre and perpendicular to the plane with angular velocityω. Another disc of same mass but half the radius is gently placed over it coaxially. The angular speed of the

composite disc will be **(IIT 86)**

(a) $\frac{5}{4}\bullet$ (b) $\frac{4}{5}\bullet$

(c) $\frac{2}{5}\bullet$ (d) $\frac{5}{2}\bullet$

Answer: (b)

16. A wheel having a moment of inertia of 2 kg m^2 about its vertical axis, is rotating at the rate of 60 r.p.m. about this axis. What is the retarding torque required to stop its rotation one minute?

(a) $\frac{\pi}{12}Nm$ (b) $\frac{\pi}{15}Nm$

(c) $\frac{\pi}{12}Nm$ (d) $\frac{\pi}{18}Nm$

Answer: (b)

17. Two bodies have their moments of inertia I and 2I respectively about their axis of rotation. If their kinetic energies of rotation are equal, their angular momenta will be in the ratio of

(a) 1:2 (b) 2:1

(c) $\sqrt{2}:1$ (d) $1:\sqrt{2}$

Answer: (d)

18. A particle of mass m is moving with a constant velocity along a line parallel to the +ve direction of the X-axis. The magnitude of its angular momentum w.r.t the origin

(a) Is zero
(b) Goes on increasing as x is increased
(c) Goes on decreasing as x is increased
(d) Remains constant for all positions of the particle

Answer: (d)

19. Torqueses of equal magnitude are applied to a hollow cylinder and a solid sphere, both having the same mass and radius. Both of them are free to rotate about their axis of symmetry. If α_c and α_s are the angular accelerations of the cylinder and the sphere respectively, then the ratio $\frac{\alpha_c}{\alpha_s}$ will be

(a) $\frac{5}{2}$ (b) $\frac{2}{5}$ (c) $\frac{4}{3}$ (d) $\frac{3}{4}$

Answer: (b)

20. A dancer on ice spins faster when she folds here arms. This is due to **(CPMT. PMT MP 86)**

(a) Increases in energy and increase in angular momentum
(b) Decrease in friction at the skates
(c) Constant angular momentum and increase in kinetic energy
(d) Increase in energy an decreases in angular momentum

Answer: (c)

21. The moment of inertia of a loop of radius R and mass M about any tangent line will be **(CPMT 92)**

(a) $\frac{3}{2}MR^2$ (b) $\frac{MR^2}{2}$ (c) MR^2 (d) $\frac{MR^2}{4}$

Answer: (a)

22. A mass is revolving in a circle which is in the plane of the paper. The direction of angular acceleration is

(CPMT 83)

(a) Upward to the radius
(b) Towards the radius

(c) Tangential
(d) At right angle to angular velocity
Answer: (c)

23. Angular momentum is **(CBSE 93)**
(a) A scalar
(b) A polar vector
(c) A scalar as well as vector
(d) An axial vector
Answer: (d)

24. Which is the wrong relation from the following?
(MH-CET 99)
(a) $\tau = I\,\alpha$ (b) $F = ma$
(c) $L = I\,\omega$ (d) $I = \tau\,\alpha$
Answer: (d)

25. Two circular discs A and B have equal masses and uniform thickness but have densities ρ_1 and ρ_2 such that $\rho_1 > \rho_2$. their moment of inertia is
(MHT-CET-2000)
(a) $I_1 > I_2$ (b) $I_1 >> I_2$
(c) $I_1 < I_2$ (d) $I_1 = I_2$
Answer: (c)

26. Radius of gyration of disc rotating about an axis perpendicular to its passing through its centre is
(MHT-CET-2003)
(a) $\frac{R}{2}$ (b) $\frac{R}{\sqrt{2}}$ (c) $\frac{R}{\sqrt{3}}$ (d) $\frac{R}{3}$
Answer: (b)

27. The moment of inertia of a body about a given axis is 1.2 kg $\times$ metre^2. Initially, the body is at rest. In order to produce a rotating kinetic energy of 1500 joules, an angular acceleration of 25 radian/sec^2 must be applied about that axis for a duration of **(CBSE 90)**
(a) 4 sec (b) 2 sec
(c) 8 sec (d) 10 sec
Answer: (b)

28. M.I. of thin uniform rod about the axis passing through its centre and perpendicular to its length is $\frac{ML^2}{12}$. The rod is cut transversely into two halves, which are then riveted end to end. M.I. of the composite rod about the axis passing through its centre and perpendicular to its length will be [**MH-CET 2001**]
(a) $\frac{ML^2}{3}$ (b) $\frac{ML^2}{12}$
(c) $\frac{ML^2}{48}$ (d) $\frac{ML^2}{6}$
Answer: (a)

29. A body having moment of inertia about its axis of rotation equal to 3 kg-m^2 is rotating with angular velocity equal to 3 rad/s. kinetic energy of this rotating body is the same as that of a

body of mass 27 kg moving with a speed of **(SCRA 94)**
(a) 1.0 m/s (b) 0.5 m/s
(c) 1.5 m/s (d) 2.0 m/s
Answer: (a)

30. For increasing the angular velocity of an object by 10%, the kinetic energy has to be increased by [**MH-CET 2001**]
(a) 40% (b) 20%
(c) 10% (d) 21%
Answer: (d)

31. When a torque acting upon a system is zero then the quantity which remains constant is **(CPMT 79)**
(a) Force (b) Linear impulse
(c) Linear momentum (d) Angular momentum
Answer: (a)

32. A tube of length L is filled completely with an incompressible liquid of mass M and closed at both the ends. The tune is then rotated in a horizontal plane about one of its end with a uniform angular velocityω. The force exerted by the liquid at the other end is, **(IIT 92)**
(a) $\frac{1}{2}M\omega^2 L$ (b) $M\omega^2 L$
(c) $\frac{1}{4}M\omega^2 L$ (d) $\frac{1}{2}M\omega^2 L^2$
Answer: (a)

33. Two discs has same mass rotates about the same axes. ρ_1 and ρ_2 are densities of two bodies ($r_1 > r_2$) then what is the relation between I_1 and I_2. **(MHT-CET-2008)**
(a) $I_1 > I_2$ (b) $I_1 < I_2$
(c) $I_1 = I_2$ (d) None of these
Answer: (b)

34. The momentum of inertia of a disc of mass M and radius R about a tangent in its plane is, **(MP. PMT 96)**
(a) $\frac{MR^2}{2}$ (b) $\frac{MR^2}{4}$
(c) MR^2 (d) $\frac{5MR^2}{4}$
Answer: (d)

35. The M.I. of uniform circular disc about a diameter is I. its M.I. about an axis perpendicular to its plane passing through a point on its rim will be **(CBSE 90, 91)**
(a) 4 I (b) 6 I
(c) 8 I (d) 9 I
Answer: (b)

36. Four point masses, each of value m, are placed at the corners of a square ABCD, having each side of length L. what is the moment of inertia of this system about an axis passing through A and parallel to the diagonal BD?
(a) $3mL^2$ (b) $2\ mL^2$

(c) $\sqrt{3}\, mL^2$ (d) mL^2
Answer: (a)

37. A thin uniform, circular ring is rolling down an inclined plane of inclination 30° without slipping. Its linear acceleration along the inclined plane will be **(CBSE 92)**
(a) g/2 (b) g/3
(c) g/4 (d) 2g/3
Answer: (a)

38. The moment of inertia of a thin circular disc of mass M and radius R about any diameter is **(MH-CET 99)**
(a) $\frac{MR^2}{4}$ (b) $\frac{MR^2}{2}$
(c) MR^2 (d) $2\ MR^2$
Answer: (a)

39. The moment of inertia of a copper disc, rotating about an axis passing through its centre and perpendicular to its plane
(a) Increases f its temperature is increased
(b) Changes if its axis of rotation is changed
(c) Increases if its angular velocity is increased
(d) Both (a) and (b) are correct
Answer: (d)

40. A constant torque of 31.4 N-m is applied to a pivoted wheel. If the angular acceleration of the wheel is 4π rad/s^2, then the moment of inertia of the wheel is
(a) 1.5 kg m^2 (b) 2.5 kg m^2
(c) 3.5 kg m^2 (d) 4.5 kg m^2
Answer: (b)

41. A rope is wound round a hollow cylinder of mass 5 kg and radius 0.5 m. what is the angular acceleration of the cylinder if the rope is pulled with a force of 20 N?
(a) 4 rad/s^2 (b) 5 rad/s^2
(c) 6 rad/s^2 (d) 8 rad/s^2
Answer: (d)

42. A solid cylinder of mass 20 kg, has length 1 metre and radius 0.5m. then its momentum of inertia in kg m^2 about its geometrical axis is
(a) 2.5 (b) 5
(c) 1.5 (d) 3
Answer: (a)

43. A wheel rotates with a constant angular acceleration of 2 rad/s^2. if the wheel start from rest the number of revolutions it makes in the first ten second will be approximately **(MP-PMT 94)**
(a) 8 (b) 16
(c) 24 (d) 32
Answer: (b)

44. The M.I. of a uniform semicircular disc of mass M and radius R about a line perpendicular to the plane of the disc and passing through the centre is

(a) $\frac{1}{2}MR^2$ (b) $\frac{1}{4}MR^2$

(c) MR^2 (d) $\frac{3}{4}MR^2$

Answer: (a)

45. Dimensions of angular momentum is **(MHT-CET-2004)**

(a) $[M^1L^2T^{-2}]$ (b) $[M^{-1}L^{-2}T^{-1}]$

(c) $[M^1L^2T^{-2}]$ (d) $[M^1L^0T^{-1}]$

Answer: (c)

46. If a body is rotating about an axis, passing through its centre of mass then its angular momentum is directed along its **(MNR 77, NCERT 82, PMT MP 86)**

(a) Radius (b) Tangent

(c) Circumference (d) Axis of rotation

Answer: (d)

47. If a horizontal cylindrical tube, partly filled with water is rapidly rotated about a vertical axis passing through its centre, the moment of inertia of the water about its axis will

(a) Decrease

(b) Increase

(c) Not change

(d) Increase or decrease depending upon clock wise or anticlockwise sense of rotation

Answer: (b)

48. A sphere of mass 0.5 kg and diameter 1 m rolls without sliding with a constant velocity of5 m/s. what is the ratio of the rotational K.E. to the total kinetic energy of the sphere? **(MHT-CET-2002)**

(a) $\frac{7}{10}$ (b) $\frac{5}{7}$

(c) $\frac{2}{7}$ (d) $\frac{1}{2}$

Answer: (c)

49. $\frac{L^2}{2I}$ represents **(MHT-CET-2003)**

(a) Rotational kinetic energy of a particle

(b) Potential energy of a particle

(c) Torque on a particle

(d) Power

Answer: (a)

50. The total energy of rolling ring of mass 'm' and radius 'R' **(MHT-CET-2007)**

(a) $3/2\ mv^2$ (b) $1/2\ mv^2$

(c) mv^2 (d) $5/2\ mv^2$

Answer: (c)

51. If I, α and τ are the moment of inertia, angular acceleration and torque respectively of a body

rotating about any axis with angular velocity ω, then
(CPMT 82)
(a) $\tau = I\alpha$ (b) $\tau = I\omega$
(c) $I = \tau\omega$ (d) $\alpha = I\omega$
Answer: (a)

52. The moment of inertia of a circular ring about an axis passing through its centre and normal to its plane is 200 gm $\times$ cm^2. then its moment of inertia about a diameter is **(PMT 87 MP)**
(a) 400 gm $\times$ cm^2 (b) 300 gm $\times$ cm^2
(c) 200 gm $\times$ cm^2 (d) 100 gm $\times$ cm^2
Answer: (d)

53. Two particles A and B, initially at rest, moves towards each other under a mutual force of attraction. At the instant when the speed of A is υ and the speed of B is 2 υ, the speed of centre of mass is, **(IIT 82)**
(a) Zero (b) υ
(c) 1.5 υ (d) 3 υ
Answer: (a)

54. A body of M.I. 3 kg m^2 rotating with an angular velocity 2 rad/s has the same K.E. as a mass of 12 kg moving with a velocity of **(MH-CET 99)**
(a) 1 m/s (b) 2 m/s
(c) 4 m/s (d) 8 m/s
Answer: (a)

55. Moment of inertia of a disc about the tangent parallel to its plane is I. The moment of inertia of the disc tangent and perpendicular to its plane is **(MH-CET 2005)**
(a) $\frac{3I}{4}$ (b) $\frac{3I}{2}$
(c) $\frac{5I}{6}$ (d) $\frac{6I}{5}$
Answer: (d)

56. The moment of inertia of a disc about its geometrical axis is I. then its M.I. about its diameter will be
(a) I (b) 2I
(c) $\frac{I}{2}$ (d) $\frac{I}{4}$
Answer: (c)

57. A particle moves for 20 s with velocity 3 m/s and then moves with velocity 4 m/s for another 20 s and finally moves with velocity 5 m/s for next 20 s. what is the average velocity of the particle? **(MHT-CET-2004)**
(a) 3 m/s (b) 4 m/s
(c) 5 m/s (d) Zero
Answer: (b)

58. The term moment of momentum is called
(C.P.M.T.74, MH-CET 99)
(a) Momentum (b) Force
(c) Torque (d) Angular momentum
Answer: (d)

59. When a mass is rotating in a plane about a fixed point its angular momentum is directed along. **(NCERT 82, MNR 87, MP 86)**
(a) The radius
(b) The tangent to orbit
(c) The line at an angle of 45° to the plane of rotation
(d) The axis of rotation
Answer: (d)

60. A mass M is moving with a constant velocity parallel to the X-axis. Its angular momentum with respect to the origin
(a) Is zero
(b) Remains constant
(c) Goes on increasing
(d) Goes on decreasing
Answer: (b)

61. The torque acting is 2000 Nm with an angular acceleration of 2 rad/s^2. The moment of inertia of body is **(MH-CET 2004)**
(a) 1200 kgm^2 (b) 900 kgm^2
(c) 1000 kgm^2 (d) Can't say
Answer: (c)

62. By keeping moment of inertia of a body is constant, if we double the time period, then angular momentum of body – **(MH-CET 2005)**
(a) Remains constant (b) Doubles
(c) Becomes half (d) Quadruples
Answer: (c)

63. A disc of moment of inertia I_1 is rotating with angular velocity ω_1 about an axis perpendicular to its plane and passing through its centre. If another disc of moment of inertia I_2 about the same axis is gently placed over it, then the new angular velocity of the combined disc will be
(a) $\frac{(I_1+I_2)\omega_1}{I_1}$ (b) $\frac{I_1\omega_1}{I_1+I_2}$
(c) ω_1 (d) $\frac{I_2\omega_1}{I_1+I_2}$
Answer: (b)

64. The moment of inertia of a disc about a tangent axis in its plane is **(MHT-CET-2002)**
(a) $\frac{mR^2}{4}$ (b) $\frac{3MR^2}{2}$
(c) $\frac{5}{4}MR^2$ (d) $\frac{7MR^2}{4}$
Answer: (c)

65. The centre of mass of a system of two particles divides. The distance between them **(MHT-CET-2004)**
(a) Inverse into of square of masses of particle
(b) Direct ratio of square of masses of particle
(c) Inverse ratio of masses of particle
(d) Direct ratio of masses of particle

Answer: (c)

66. A uniform disc of mass 2 kg is rotated about an axis perpendicular to the plane of the disc. If radius of gyration is 50 cm, then the M.I. of disc about same axis is **(MHT-CET-2006)**
(a) 0.25 kg m^2 (b) 0.5 kg m^2
(c) 2 kg m^2 (d) 1 kg m^2
Answer: (b)

67. A rod length is l density of material is D and area of cross section A. it is rotates about its axes perpendicular to the length passing through its centre then find its kinetic energy is **(MHT-CET-2008)**
(a) $\frac{Al^3D\omega^2}{3}$ (b) $\frac{Al^3D\omega^2}{12}$
(c) $\frac{Al^3D\omega^2}{6}$ (d) $\frac{Al^3D\omega^2}{24}$
Answer: (d)

68. The moment of inertia of a thin rod of mass M and length l about an axis passing through one of its and perpendicular to length is. **(PMT MP 95)**
(a) Ml^2 (b) $\frac{Ml^2}{3}$
(c) $\frac{Ml^2}{2}$ (d) $\frac{Ml^2}{12}$
Answer: (b)

69. Constant torque acting on a uniform circular wheel changes it angular momentum from A to 4 A in 4 seconds. The magnitude of this torque is **(MP PMT 97)**
(a) $\frac{3}{4}A$ (b) A
(c) 4 A (d) 12 A
Answer: (a)

70. A spherical solid ball of a kg mass and radius 3 cm is rotating about an axis passing through its centre with an angular velocity of 50 radian/s. the kinetic energy of rotation is **(CPMT 89)**
(a) 4500 J (b) 90 J
(c) 910 J (d) $\frac{9}{20}$ J
Answer: (d)

71. When a steady torque is acting on a body, the body **(NCERT 73)**
(a) Continues in its state of rest or uniform motion along a straight line
(b) Gets linear acceleration
(c) Gets angular acceleration
(d) Rotates at a constant speed
Answer: (d)

72. The M.I. of a solid cylinder of mass M and radius R about a line parallel to the axis of the cylinder and ling on the surface of the cylinder is **(MP-PMT 94)**
(a) $\frac{2}{5}MR^2$ (b) $\frac{3}{5}MR^2$

(c) $\frac{3}{2}MR^2$ (d) $\frac{5}{2}MR^2$

Answer: (c)

73. The moment of inertia of a body comes into play
(AFMC (pune) 79)
(a) In motion along a curved path
(b) In linear motion
(c) In rotational motion
(d) None of the above
Answer: (c)

74. The speed of a homogeneous, solid sphere after rolling down in the inclined plane of vertical height h, from rest without sliding is **(CBSE 92)**

(a) $\sqrt{\left(\frac{10}{7}gh\right)}$ (b) $\sqrt{gh}$

(c) $\sqrt{\left(\frac{6}{3}gh\right)}$ (d) $\sqrt{\left(\frac{4}{3}gh\right)}$

Answer: (a)

75. Two disc with same mass but different radii are moving with same K.E. one of them rolls and other slides without friction. Then [**MH-CET 2000**]
(a) Rolling disc has greater velocity
(b) Sliding disc has greater velocity
(c) Both have same velocity
(d) The disc with greater radius will have greater velocity.
Answer: (b)

76. Centre of mass of two body system divides the distance between two bodies, is proportional to **(MH-CET 2004)**
(a) Inverse of square of the mass
(b) Inverse of mass
(c) The ratio of the square of mass
(d) The ratio of mass
Answer: (b)

77. A thin circular ring of mass M and radius R is rotating about an axis passing through its centre and perpendicular to its plane with a constant angular velocity ω_1. Two small bodies each of mass m are attached gently to the opposite ends of a diameter of ring. The new angular velocity ω_2 of the ring will be

(a) $\frac{M+2m}{M\omega_1}$ (b) $\frac{M\omega_1}{M+2m}$

(c) $\frac{\omega_1(M+2m)}{M}$ (d) $\frac{\omega_1(m+2M)}{2m}$

Answer: (b)

78. A solid sphere, a hollow sphere and a disc are released from the top of a frictionless inclined plane so that they slide down the inclined plane (without rolling). The maximum acceleration down the plane is
(a) For the solid sphere
(b) For the hollow sphere
(c) For the disc
(d) The same for all bodies
Answer: (d)

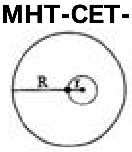
79. The kinetic energy of a body is 4 joule and its moment of inertia is 2 kg m^2 then angular momentum is
(MHT-CET-2008)
(a) 4 kg m^2/sec (b) 5 kg m^2 /sec
(c) 6 kg m^2/sec (d) 7 kg m^2/sec
Answer: (a)

80. Moment of inertia depends on **(MHT-CET-2002)**
(a) Distribution of particles
(b) Mass
(c) Position of axis of rotation
(d) All of these
Answer: (d)

81. A disc of moment of inertia $9.8/\pi^2$ kg m^2 is rotating at 600 rpm. If the frequency of rotation charges from 600 rpm to 300 rpm, then what is the work done?
(MHT-CET-2004)
(a) 1470 J (b) 1452 J
(c) 1567 J (d) 1632 J
Answer: (a)

82. A disc of mass 2 kg and diameter 2m is performing rotational motion. Find the work done, if the disc is rotating from 300 rpm to 600 rpm. **(MH-CET 2004)**
(a) 1479 J (b) 14.79 J
(c) 147.9 J (d) 1.479
Answer: (a)

83. What will be distance of centre of mass of the disc (see fig.) from its geometrical centre? **(MHT-CET-2001)**



(a) $\frac{r}{\left(\frac{R^2}{r^2}-1\right)}$ to left (b) R + r, to left

(c) $\frac{r}{R+r}$, to left (d) $\frac{r}{(r^2+R^2)}$, to left

Answer: (a)

84. The moment of inertia of uniform circular disc about an axis passing its centre is $6kgm^2$. its M.I. about an axis perpendicular to its plane and just touching the rim will be

(a) 18 kg m^2 (b) 30 kg m^2

(c) 15 kg m^2 (d) 3 kg m^2

Answer: (a)

85. A spherical ball rolls on a table without slipping. Then the fraction of its total energy associated with rotation is **(PMT, 87 MP)**

(a) $\frac{2}{5}$ (b) $\frac{2}{7}$

(c) $\frac{3}{5}$ (d) $\frac{3}{7}$

Answer: (b)

86. The dimensions of angular momentum are – **(MH-CET 2004)**

(a) $[M^1 L^2 T^{-1}]$ (b) $[M^1 L^1 T^{-1}]$

(c) $[M^1 L^1 T^{-2}]$ (d) $[M^1 L^2 T^{-2}]$

Answer: (a)

87. A cylinder of mass 10 kg and radius 20 cm is free to rotate about its axis. It receives an angular impulse of 4 kg m^2/s. what is the angular speed of the cylinder if the cylinder is initially at rest?

(a) 20 rad/s (b) 15 rad/s

(c) 10 rad/s (d) 5 rad/s

Answer: (a)

88. The moment of inertia of an electron in n^{th} orbit will be **(MHT-CET-2001)**

(a) MR_n^2 (b) $\frac{MR_n^2}{2}$

(c) $\frac{1}{2}MR_n^2$ (d) $\frac{2}{3}MR_n^2$

Answer: (a)

89. The M.I. of a body does not depends upon **(CPMT 75)**

(a) Angular velocity of a body

(b) Axis of rotation of the body

(c) The mass of the body

(d) The distribution of the body

Answer: (a)

90. What is the moment of inertia of a solid sphere of radius R and density ρ about its diameter?

(a) $\frac{8}{3}\pi R^3\rho$ (b) $\frac{8}{15}\pi R^4\rho$

(c) $\frac{8}{3}\pi R^5\rho$ (d) $\frac{15}{8}\pi R^3\rho^2$

Answer: (c)

91. M.I. of a thin uniform rod about the axis passing through its centre and perpendicular to its length is $ML^2/12$. The rod is cut transversely into two halves, which are then riveted end to end. M.I. of the composite rod about the axis passing through its centre and perpendicular to its length will be

(MHT-CET-2001)

(a) $\frac{ML^2}{3}$ (b) $\frac{ML^2}{12}$

(c) $\frac{ML^2}{48}$ (d) $\frac{ML^2}{6}$

Answer: (b)

92. The radius of gyration of a disc of mass 100 gm and radius 5 cm about an axis passing through its centre of gravity and perpendicular to the plane is

(MHT-CET-2000)

(a) 0.5 cm (b) 2.5 cm

(c) 3.54 cm (d) 6.54 cm

Answer: (c)

93. A thin circular ring of mass M and radius r is rotating about its axis passing through its centre and perpendicular to its plane with a constant angular velocity ω two objects each of mass m are attached gently to the opposite ends of a diameter of the ring. The ring will now rotate with an angular velocity of **(I.I.T. 83)**

(a) $\frac{\omega(M-2m)}{(M+2m)}$ (b) $\frac{\omega M}{(M+2m)}$

(c) $\frac{\omega M}{(M+m)}$ (d) $\frac{\omega(M+2m)}{M}$

Answer: (b)

94. A solid cylinder of mass M and radius R rolls down an inclined plane without slipping. The speed of its centre of mass when it reaches the bottom is

(EAMCET 85, PEN 85 MP)

(a) $\sqrt{2gh}$ (b) $\sqrt{\frac{4gh}{3}}$

(c) $\sqrt{\frac{3gh}{4}}$ (d) $\sqrt{\frac{4g}{h}}$

Answer: (b)

95. The position vector of a particle of mass 10 g, about the origin is $4\hat{i}+3\hat{j}\,m$. If it moves with a linear velocity of $4\hat{i}$ m/s, then its angular momentum will be

(a) $12\hat{k}$ Js (b) $0.2\hat{k}$ Js

(c) $-0.12\hat{k}$ Js (d) $-12\hat{k}$ Js

Answer: (c)

96. Moment of inertia depends upon the **(MH-CET 2002)**

(a) Mass of the body

(b) Distribution of mass of the body

(c) Position of axis of rotation

(d) All of these

Answer: (d)

97. A body of moment of inertia of 3 kgm^2 rotating with an angular velocity of 2rad/s has the same kinetic energy as a mass of 12 kg moving with a velocity of
(MHT-CET-1999)

(a) 1 m/s (b) 2 m/s

(c) 4 m/s (d) 8 m/s

Answer: (a)

98. If a gymnast, sitting on a rotating stool, with his arms outstretched, suddenly lowers his arms **(NCERT 78)**

(a) The angular velocity decreases

(b) His moment of inertia decreases

(c) The angular velocity remains constant

(d) The angular momentum increases

Answer: (b)

99. The moment of inertia of a thin circular disc of mass M and radius R about any diameter is **(MHT-CET-1999)**

(a) $\frac{MR^2}{4}$ (b) $\frac{MR^2}{2}$

(c) MR^2 (d) 2MR^2

Answer: (a)

100. A disc of radius R rotating about its axis has a moment of inertia I about that axis. When it is rotating about that axis at a constant angular velocity ω a heavy particle of mass m is placed gently at the rim of the disc. The resulting angular velocity of the system is
(CPMT 91)
(a) ω (b) $I\omega\,(I + mR)$
(c) $(I + mR)/I\omega$ (d) $I\omega/(I + mR^2)$
Answer: (d)

CH 8

Gravitation

[9 Hours, 8 Marks (1Q -3M, 1Q-5M(NP))]

Syllabus :

Keplar's laws of planetary motion. The universal law of gravitation. Acceleration due to gravity and its variation with altitude and depth. Gravitational potential energy; gravitational potential. Escape velocity. Orbital velocity of a satellite. Geo-stationary satellites.

8.1 Keplar's laws of planetary motion. The universal law of gravitation :

1. State and Explain Keplar's laws of planetary motion

Kepler's laws of planetary motion :-

(i) Law of orbits :- All planets move in elliptical orbits with the sun situated at one of the foci of the ellipse.

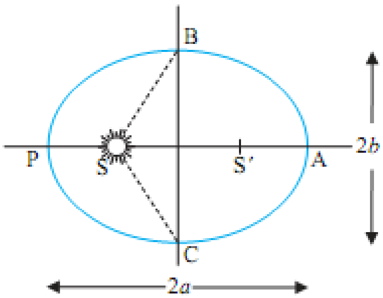
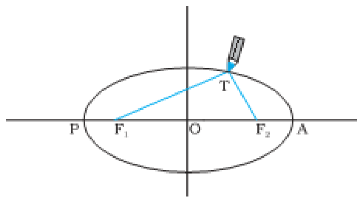


Fig. 8.1(a) An ellipse traced out by a planet around the sun. The closest point is P and the farthest point is A, P is called the perihelion and A the aphelion. The semi-major axis is half the distance AP.



This law was a deviation from the Copernican model which allowed only circular orbits. Circle is a closed curve and is a special case of ellipse.

The midpoint of the line PA is the centre of the ellipse O and the length PO = AO is called the semi-major axis of the ellipse. For a circle, the two foci merge into one and the semi-major axis becomes the radius of the circle.

(ii). Law of areas :- The line that joins any planet to the sun sweeps equal areas in equal intervals of time.

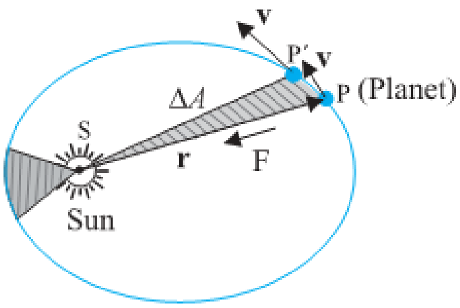


Fig. 8.2 The planet P moves around the sun in an elliptical orbit. The shaded area is the area ΔA swept out in a small interval of time Δt.

This law comes from the observations that planets appear to move slower when they are farther from the sun than when they are nearer.
ΔA is the area swept out in a small interval of time Δt.

(iii) Kepler's law of periods :- The square of the time period of revolution of a planet is proportional to the cube of the semi - major axis of the ellipse traced out by the planet.
If T is the period of revolution of the planet and a is semi - major axis of the elliptical path, then **$T^2 \alpha\, a^3$**.

2. State and explain universal law of gravitation ?

Every body in the universe attracts every other body with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.

The magnitude of the force $\vec{F}$ on a point mass m_2 due to another point mass m_1 at a distance r is given by,

$$|\vec{F}| = F = G\,(m_1 m_2)/r^2$$

8.2 Acceleration due to gravity and its variation with altitude and depth :

3. What is acceleration due to gravity ? Obtain an expression for it ?

The acceleration experienced by an object of mass m in earths surface located at a distance R_E from earths centre due to earths gravitational attraction is called acceleration due to gravity (g).

If the mass m is situated on the surface of earth, then $r = R_E$ and the gravitational force on it is,

$$F = \frac{G\,M_E m}{R_E^2}$$

The acceleration experienced by the mass m, which is usually denoted by the symbol g is related to F by Newton's 2nd law by relation F = mg. Thus

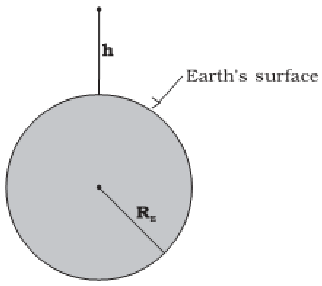
$$g = \frac{F}{m} = \frac{G\,M_E}{R_E^2}$$

Acceleration g is readily measurable. R_E is a known quantity. The measurement of G by Cavendish's experiment (or otherwise), combined with knowledge of g and R_E enables one to estimate M_E.

4. How is the acceleration due to gravity varies above & below the surface of earth ?

Case (1) : Acceleration due to gravity above the surface of earth :

Consider a point mass m at a height h above the surface of the earth as shown in Fig. 8.8(a). The radius of the earth is denoted by R_E. Since this point is outside the earth,



its distance from the centre of the earth is (R_E + h). If F (h) denoted the magnitude of the force on the point mass m , we get from Eq. (8.5) :

$$F(h) = \frac{GM_E m}{(R_E + h)^2}$$

The acceleration experienced by the point mass is F(h)/m = g (h) and we get

$$g(h) = \frac{F(h)}{m} = \frac{GM_E}{(R_E + h)^2} \quad \text{-------} \quad (a)$$

This is clearly less than the value of g on the surface of earth : $g = \frac{G\,M_E}{R_E^2}$

For h << R_E, we can expand the RHS of Eq. (a) :

$$g(h) = \frac{GM}{R_E^2\,(1+\frac{h}{R_E})^2} = g\,(1 + h/R_E)^{-2} \quad \text{-------} \quad (b)$$

For h/R_E << 1, using binomial expression, we can write,

$$g(h) = g\left(1 - \frac{2h}{R_E}\right) \quad \text{---------} \quad (c)$$

Thus Eq. (c) tells us that for small heights h above earth ground surface, **the value of g decreases** by a factor (1- 2h /R_E).

Case (2) : Acceleration due to gravity below the surface of earth :

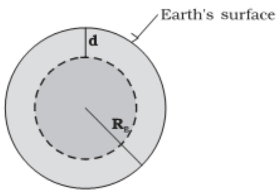
Consider a point object mass m at a depth d below the surface of the earth (Fig. (b)), so that its distance from the center of the earth is (R_E - d) as shown in the figure.

If M_S is the mass of the smaller sphere, then, M_S / M_E = $(R_E - d)^3$ / R_E^3

Since mass of a sphere is proportional to be cube of its radius.

Thus the force on the point mass is F(d) = G M_S m / $(R_E – d)^2$

Substituting for M_S from above , we get F(d) = G M_E m (R_E – d) / R_E^3 --------- (d)



and hence the acceleration due to gravity at a depth d, g(d) = F(d)/m is

$$g(d) = F(d)/m = \frac{GM_E}{R_E^3}(R_E - d)$$

$$= g\,\frac{R_E - d}{R_E} = g\left(1 - \frac{d}{R_E}\right) \quad \text{---------} \quad (e)$$

Thus, as we go down below earth's surface, the **acceleration due gravity g decreases** by a factor $\left(1 - \frac{d}{R_E}\right)$.

Note : Acceleration due to earth's gravity (g) is maximum on its surface and decreases when we go up or down.

8.3 Gravitational potential energy; gravitational potential :

5. What is gravitational potential energy ? Obtain an expression for gravitational potential energy on the surface of earth ?

Gravitational potential energy of a body at a point on the surface of the earth is equal to the workdone while bringing the body from infinity (a point where g = 0) to that point.

$$V = W_{21} = \int_{R1}^{R2} F = \int_{R1}^{R2} \frac{GM_E m}{r^2}\,dr = -GM_E m\left[\frac{1}{R_2} - \frac{1}{R_1}\right]$$

If $R_2 = \infty$, R_1 = r, we get gravitation potential at a distance r from earth surface

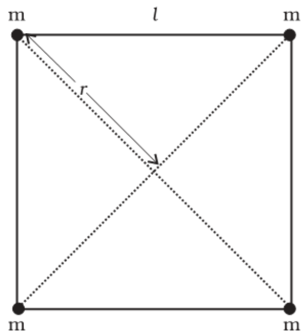
V(r) = $-GM_E m$ /r -------- (1)

Example : Find the potential energy of a system of four particles placed at the vertices of a square of side l. Also obtain the potential at the centre of the square.

Answer Consider four masses each of mass m at the corners of a square of side l ; See Fig. We have four mass pairs at distance l and two diagonal pairs at distance $\sqrt{2}\,l$. Hence,

$$W(r) = -4\frac{Gm^2}{l} - 2\frac{Gm^2}{\sqrt{2}\,l}$$

$$= -\frac{2Gm^2}{l}\left(2+\frac{1}{\sqrt{2}}\right) = -5.41\frac{Gm^2}{l}$$



The gravitational potential at the centre of the square (r = $\sqrt{2}\,l/2$ = is

$$U(r) = -4\sqrt{2}\,\frac{Gm}{l}$$

8.4 Escape velocity. Orbital velocity of a satellite :

6. What is escape velocity ? Obtain an expression for it ?

The minimum velocity required to project an object vertically upwards from the surface of the earth, so that it escapes from the gravitational influence of the earth and never returns to the earth, is called escape velocity.

Consider an object of mass m at a distance of r from the centre of the earth.

The gravitational force acting on the mass m is given by

F = GMm/r^2 ....(1)

This force acts towards the centre of the earth.

Let dr be the small distance covered by the object away from the centre of the earth. Therefore the work done on the object against the gravitational force of attraction of the earth is

dW = $\vec{F}\,.\,\vec{dr}$ = F dr cos 180° = - Fdr

From Eqn. (1) we get, dW = -[GMm/r^2] dr ------- (2)

Therefore total work done to displace the object from the surface of the earth [i.e., r = R] to [r = ∞] is calculated by integrating equation (2) between the limits R and ∞.

Therefore $\int_R^\infty dW = \int_R^\infty -[GMm/r^2]$ dr

$$W = -GMm\left[\frac{r^{-1}}{-1}\right]_R^\infty = GMm\left[\frac{1}{r}\right]_R^\infty = GMm\left[\frac{1}{\infty}-\frac{1}{R}\right]$$

Or W = $-\,GMm/R$ [since 1/∞ = 0] this work done is equal to the potential energy (V) of the object of mass m. That is V = -GMm/R. Let V_e is the escape speed of the object of mass m, then its K.E. is ½ mv_e^2.

If K.E. of the object = magnitude of potential energy of the object, then $\frac{1}{2}mv_e^2$ = GMm/R

or $v_e^2 = \frac{2GM}{R}$ or $v_e = \sqrt{\frac{2GM}{R}}$ (1)

But GM/R^2 = g or GM = gR^2 and putting R = R_E,

Therefore, $\boldsymbol{V_e = \sqrt{2gR}}$ ---------- (2) is the expression for escape velocity.

Using the value of g and R_E, numerically $V_{e\ \min}$ ≈ **11.2 km/s**.

7. Obtain an expression for period of rotation and energy of an orbiting earth satellite ?

Consider a satellite in a circular orbit of a distance (R_E + h) from the centre of the earth, where R_E = radius of the earth. If m is the mass of the satellite and V its speed, the centripetal force required for this orbit is

F(centripetal) = $\frac{mV^2}{(R_E+h)}$ -------- (1)

directed towards the center. This centripetal force is provided by the gravitational force, which is F(gravitation) = $\frac{GmM_E}{(R_E+h)^2}$ ------- (2)

Where M_E is the mass of the earth.

Equating R.H.S of Eqs. (1) and (2) and cancelling out m, we get

$V^2 = \frac{G\,M_E}{(R_E+h)}$ -------- (3)

Thus V decreases as h increases. From equation (3), the speed V for h = 0 is

V^2 (h = 0) = GM/R_E = gR_E -------- (4)

where we have used the relation g = GM/R_E^2.

(i) Expression for period of rotation of an orbiting earth satellite :

In every orbit, the satellite traverses a distance 2π(R_E + h) with speed V. It's time period T therefore is

T = $\frac{2\pi\,(R_E+h)}{V} = \frac{2\pi(R_E+h)^{3/2}}{\sqrt{GM_E}}$ ------------ (5)

on substitution of value of V from Eq. (3).

Squaring both sides of Eq. (5), we get

T^2 = k (R_E + h)^3 (where k = 4 π^2 / GM_E) ------------ (6)

which is Kepler's law of periods, as applied to motion of satellites around the earth. For a satellite very close to the surface of earth h can be neglected in comparison to R_E in Eq. (5). Hence, for such satellites, T is T_o, where

$\boldsymbol{T_o = 2\pi\sqrt{\frac{R_E}{g}}}$ --------- **(7)**

If we substitute the numerical values g ; 9.8 m s^{-2} and R_E = 6400 km., we get, $\boldsymbol{T_o = 2\pi\sqrt{\frac{6.4\times10^6}{9.8}}}$ **s** Which is approximately **85 minutes.**

Thus the period of rotation of satellite around the earth is about 85 minutes.

(ii) Expression for energy of an orbiting earth satellite :

Using Eq. (3), the kinetic energy of the satellite in a circular orbit with speed v is

K.E. = $\frac{1}{2}\,mv^2 = \frac{G\,M_E\,m}{2(R_E+h)}$ ----------- (8)

Considering gravitational potential energy at infinity to be zero, the potential energy at distance (R_e +h) from the center of the earth is

P.E. = $-\frac{GmM_E}{(R_E+h)}$ ------------ (9)

The K.E is positive whereas the P.E is negative. However, in magnitude the K.E. is half the P.E, so that the total E is E = P.E. + K.E. = $-\frac{GmM_E}{2(R_E+h)}$ --------- (10)

The total energy of an circularly orbiting satellite is thus negative, with the potential energy being negative but twice is magnitude of the positive kinetic energy. If the total energy is positive or zero, the object escapes to infinity. Satellites are always at finite distance from the earth and hence their energies cannot be positive or zero.

8.5 Geo-stationary satellites :

8. Write a note on Geostationary & Polar Satelites

(i) Geostationary Satelites :

If we arrange the value of (R_E + h) such that T in Eq. (5) becomes equal to 24 hours. If the circular orbit is in the equatorial plane of the earth, such a satellite, having the same period as the period of rotation of the earth about its own axis would appear stationery viewed from a point on earth. The (R_E + h) for this purpose works out to be large as compared to R_E :

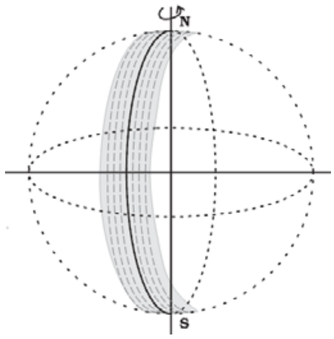
$$R_E + h = \left(\frac{T^2\, G\, M_E}{4\pi^2}\right)^{1/3}$$

and for T = 24 hours, h works out to be **35,800 km**. which is much larger than R_E. Satellites in a circular orbits around the earth in the equatorial plane with **T = 24 hours** are called **Geostationary Satellites**. Clearly, since the earth rotates with the same period, the satellite would appear fixed from any point on earth. It takes very powerful rockets to throw up a satellite to such large heights above the earth but this has been done in view of the several benefits of many practical applications.

A Geostationary satellite, appearing fixed above the broadcasting station can however receive high frequency TV signals and broadcast them back to a wide area on earth. The INSAT group of satellites sent up by India are one such group of Geostationary satellites widely used for telecommunications in India.

(ii) Polar satellites : These are low altitude (h ≈ 500 to 800 km) satellites, but they go around the poles of the earth in a north-south direction whereas the earth rotates around its axis in an east-west direction. Since its time period is around 100 minutes it crosses any altitude many times a day. However, since its height h above the earth is about 500-800 km, a camera fixed on it can view only small strips of the earth in one orbit. Adjacent strips are viewed in the next orbit, so that in effect the whole earth can be viewed strip by strip during the entire day. These satellites can view polar and equatorial regions at close distances with good resolution.

Information gathered from such satellites is extremely useful for remote sensing, meterology as well as for environmental studies of the earth.



9. Write a note on weightlessness ?

Weight of an object is the force with which the earth attracts it. When an object is in free fall under gravity, it is weightless and this phenomenon is usually called the phenomenon of weightlessness. In a satellite around the earth, every part and parcel of the satellite has an acceleration towards the center of the earth which is exactly the value of earth's acceleration due to gravity at that position. Thus in the satellite everything inside it is in a state of free fall. This is just as if we were falling towards the earth from a height. Thus, in a manned satellite, people inside experience no gravity. Gravity for us defines the vertical direction and thus for them there are no horizontal or vertical directions, all directions are the same. Pictures of astronauts floating in a satellite reflect show this fact.

10. Why moon has no atmosphere ?

Moon is the only natural satellite of the earth with a near circular orbit with a time period of approximately 27.3 days which is also roughly equal to the rotational period of the moon about its own axis. The escape speed for the moon turns out to be 2.3 km/s, about five times smaller than earth. This is the reason that moon has **no atmosphere**. Gas molecules if formed on the surface of the moon having velocities larger than this will escape the gravitational pull of the moon.

Important Formulas :

S. No.	Formula	Related Formula
1	**Kepler's law of periods :** If T is the period of revolution of the planet and a is semi - major axis of the elliptical path, then **$T^2 \propto a^3$**.	$\omega = 2\pi/T$
2	Gravitational Force $\lvert \vec{F} \rvert = F = G(m_1m_2)/r^2$	**Gravitational force of earth,** $F = \frac{G M_E m}{R_E^2}$ **Acceleration due to Gravity** $g = \frac{F}{m} = \frac{G M_E}{R_E^2}$ Gravitational Field Intensity $I = \frac{G M_E}{R_E^2}$
3	For small height h above earth ground surface, **the value of g decreases** by a factor $(1 - 2h/R_E)$.	For below earth's surface of depth d , the **acceleration due gravity g decreases** by a factor $\left(1 - \frac{d}{R_E}\right)$.
4	Gravitational potential of earth at a distance r is	

	V(r) = $-\boldsymbol{GM_E m}$ /r	
5	Escape velocity of earth $\boldsymbol{V_e = \sqrt{2gR}}$	$For\ the\ earth, V_{e\ min} \approx$ **11.2 km/s**.
6	Period of rotation of an orbiting earth satellite, $\boldsymbol{T_o = 2\pi\sqrt{\frac{R_E}{g}}}$	Period of rotation of an orbiting earth satellite, T_o = 85 minutes
7	Total energy of an circularly orbiting satellite = K.E + P.E = E $= -\frac{GmM_E}{2(R_E+h)}$	K.E. $= \frac{1}{2}\,mv^2 = \frac{G\,M_E\,m}{2(R_E+h)}$ P.E. $= -\frac{GmM_E}{(R_E+h)}$
8	$T = \frac{2\pi\,(R_E+h)}{V} = \frac{2\pi(R_E+h)^{3/2}}{\sqrt{GM_E}}$	$R_E + h = \left(\frac{T^2\,G\,M_E}{4\pi^2}\right)^{1/3}$ To make T= 24 hours, R_E + h = R_E +**35,800 km** **i.e. For geo-stationary satellites, h = 35,800 Km.**
9	Escape speed for the moon turns out to be 2.3 km/s, about five times smaller than earth	Hence moon has **no atmosphere**

I . One mark Questions:

1. State Kepler's law of orbits.

All planets move in elliptical orbits with the sun situated at one of the foci of the ellipse.

2. State Kepler's law of areas.

The line that joins any planet to the sun sweeps equal areas in equal intervals of time.

3. State Kepler's law of periods.

The square of the time period of revolution of a planet is proportional to the cube of the semi – major axis of the ellipse traced out by the planet.

4. Which physical quantity is conserved in the case of law of areas?

"Angular momentum" is conserved in the case of law of areas.

5. State universal law of gravitation.

Every body in the universe attracts every other body with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.

6. Express universal law of gravitation in mathematical form.

The magnitude of the force $\vec{F}$ on a point mass m_2 due to another point mass m_1 at a distance r is given by,

$$|\vec{F}| = F = G\,(m_1 m_2)/r^2$$

7. Express universal law of gravitation in vector form.

The force $\vec{F}$ on a point mass m_2 due to another point mass m_1 at a distance r is given by

$$|\vec{F}| = F = G\,[(m_1 m_2)/r^2](-\hat{r}) = -G\,(m_1 m_2/r^2)\,\hat{r}, \quad \vec{F} = -[(G\,m_1 m_2)/|\vec{r}|^3]\,\vec{r}$$

8. What is the value of gravitational constant?

$G = 6.67 \times 10^{-11}$ N m^2/kg^2

9. Name the experiment that has given the value of gravitational constant.

Cavendish's experiment.

10. Write the relation between g and G.

$g = GM_E / R_E^2$ for earth,

Where, g is acceleration due to gravity on the surface of the earth.

G is gravitational constant,

ME is mass of earth,

RE is radius of the earth.

11. Write the expression for acceleration due to gravity at a point above the surface of the earth.

$G(h) = g(1 - 2h/R_E)$ where h is height of the point, R_E is radius of the earth, g is acceleration due to gravity on the surface of the earth.

12. Write the expression for acceleration due to gravity at a point below the surface of the earth.

$g(d) = g(R_E - d)/R_E = g(1 - d/R_E)$

where d is depth of the point, R_E is the radius of the earth, g is acceleration due to gravity on the surface of the earth.

13. Write the expression for gravitational potential energy of a particle at a point due to the earth.

$W(r) = - GM_Em/r$

Where G is gravitational constant,

M_E is mass of the earth,

m is mass of the particle,

r is distance of the particle from the surface of the earth.

14. Write the expression for gravitational potential energy between two masses separated by a distance.

$V = - Gm_1m_2/r$

Where G is gravitational constant,

m_1 & m_2 are masses,

r is distance between the masses,

15. What is escape speed?

Escape speed is the minimum speed required for an object to escape from the earth (i.e.to reach infinity (or) zero gravitational potential energy.)

16. What is a satellite?

A satellite is an object which revolves around the planet in circular or elliptical orbit.

17. What is the value of period of moon?

The period of revolution around the earth (≈ rotation about its own axis) of moon is 27.3 days.

18. What are geostationary satellites?

Geostationary satellites are the satellites in circular orbits around the earth in the equatorial plane with the period T = 24 hrs.

19. What are polar satellites?

Polar satellites are low altitude ($h \approx 500$ to 800 km) satellites that go around the poles of the earth in a north-south direction where as the earth rotates around its axis in an east-west direction.

20. Give the period of geostationary satellites?
The period of geostationary satellite T is 24 hrs.

21. Name the group of the geostationary satellites sent up by India?
The group of the geostationary satellites sent up by India is INSAT group of satellites.

22. Give an important use of geostationary satellites.
Geostationary satellites are widely used for telecommunications in India.

23. Write the dimensional formula for gravitational constant.
The dimensional formula for G is $[M^{-1} L^{3} T^{-2}]$.

24. Define orbital speed of a satellite around the earth.
Orbital speed of a satellite around the earth is the speed required to put a satellite into its orbit.

25. Name the force that provides the necessary centripetal force for the earth around the sun in an approximately circular orbit.
Gravitational force between earth and the sun provides the necessary centripetal force for the earth around the sun in an approximately circular orbit.

26. How does the escape velocity of a body varies with the mass of the earth?
The escape speed of a body is proportional to the square root of the mass of the earth.

27. How does speed of the earth changes when it is nearer to the sun?
The speed of the earth increases when it is nearer to the sun.

28. What are central forces?
Central forces are always directed towards or away from a fixed point, that is along the position vector of the point of application of the force with respect to the fixed point.

29. Give the value of escape speed for moon.
Escape speed on the surface of the moon is 2.3 km/s.

30. The Newton's law of gravitation is said to be universal law. Why?
Newton's law of gravitation is independent of the nature of the interacting bodies in nature. Therefore it is called as universal law.

31. Differentiate gravitation and gravity.
The force of attraction between any two bodies in nature is gravitation where as the force of attraction between a body and the earth is gravity.

32. Give the relation between escape and orbital speed.
Escape speed, $v_e = \sqrt{2}\, v_o$. Where v_o is orbital speed.

33. Write the dimensional formula of "g'.
The dimensional formula of g is $[M^0L^1T^{-2}]$.

II. Two mark questions:

1. State and explain Kepler's law of orbits.
Kepler's law of orbits:- All planets move in elliptical orbits with the sun situated at one of the foci of the ellipse. Ellipse is a closed curve.

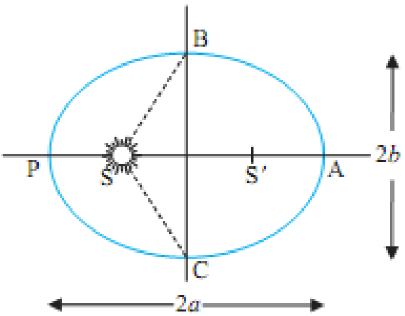


Fig. 8.1(a) An ellipse traced out by a planet around the sun. The closest point is P and the farthest point is A, P is called the perihelion and A the aphelion. The semimajor axis is half the distance AP.

Figure shows an ellipse traced out by a planet around the sun. The closest point is P (perihelion) and the farthest point is A (aphelion).

2. State and explain Kepler's law of areas.
Kepler's Law of areas:- The line that joins any planet to the sun sweeps equal areas in equal intervals of time. Planets move slower when they are farther from the sun than when they are nearer.

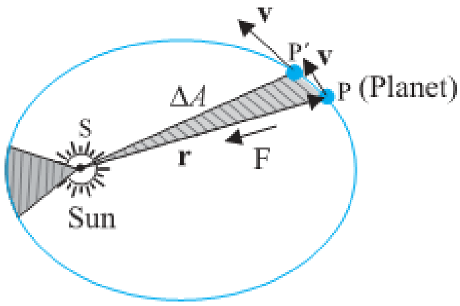


Fig. 8.2 The planet P moves around the sun in an elliptical orbit. The shaded area is the area ΔA swept out in a small interval of time Δt.
The planet P moves around the sun in an elliptical orbit. The shaded area is the area Δ A swept out in a small interval of time Δ t.

3. State and explain Kepler's law of periods.
Kepler's law of periods:- The square of the time period of revolution of a planet is proportional to the cube of the semi - major axis of the ellipse traced out by the planet.

If T is the period of revolution of the planet and a is semi - major axis of the elliptical path, then $T^2 \alpha a^3$.

4. State universal law of gravitation. Express it in mathematical form.

Every body in the universe attracts every other body with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between them. The magnitude of the force on a point mass m_1 due to another point mass m_2 is;

$\vec{F} = F = G\, m_1 m_2/r^2$

Where r is the distance between m_1 and m_2.

5. Moon has no atmosphere. Why?

Escape speed of a body on the surface of moon is 2.3 km/s. This value is five times smaller than the escape speed on the surface of the earth. Gas molecules if formed on the surface on the moon having velocities larger than this will escape from the gravitational pull of the moon. Because of this reason moon has no atmosphere.

6. Define gravitational potential energy of a body. Give an expression for it.

Potential energy of a body arising out of the force of gravity is called the gravitational potential energy. The expression for gravitational potential energy of a body due to earth is

$V = W(r) = -G\, M_E\, m/r$

Where M_E is mass of the earth,

m is mass of the body,

r is the distance of the body from the centre of the earth

7. State and explain Newton's law of gravitation.

Every body in the universe attracts every other body with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between them. The force $\vec{F}$ on a point mass m_2 due to another point mass m_1 has the magnitude.

$|\vec{F}| = F = G\, m_1 m_2/r^2$

where G is the universal gravitational constant. The gravitational force is attractive. That is the force $\vec{F}_{12}$ on the body 1 due to 2 and $\vec{F}_{21}$ on the body 2 due to 1 are related as $\vec{F}_{12} = -\vec{F}_{21}$.

8. Derive the relation between gravitational constant and acceleration due to gravity.

Let m be the mass of the body situated on the surface of the earth of radius R_E and mass M_E. According to Newton's law of gravitation the force between the body close to the surface of the earth and the earth is;

$F = G\, mM_E/R_E^2$ .... (1)

Where G is gravitational constant.

But according to Newton's 2nd law of motion, the gravitational force exerted on the body by the earth;

$F = mg$.......(2)

Where g is acceleration due to gravity.

From equations (1)&(2), $mg = GmM_E/R_E^2$ (or) $g = GM_E/R_E^2$.

9. Write the expression for escape speed on the earth. Give its value in the case of earth.

Expression for escape speed on the earth ;

$v_e = \sqrt{\frac{2GM_E}{R_E^2}}$ (or) $v_e = \sqrt{2gR_E}$

where G is gravitational constant,

M_E is mass of the earth,

R_E is radius of the earth,

g is acceleration due to gravity.

The value of escape speed on the earth is 11.2 km/s.

10. Give two uses of polar satellites.

Polar satellites are useful for (a) remote sensing. (b) meteorology/ (c) Environmental studies of the earth.

11. Explain the state of weightlessness of a body.

Weight of a body is the force with which the earth attracts it. If there is no opposite force (support) exerted on the body it would fall down. During this free fall, the object appears to lose its weight and becomes weightless, since there is no upward force on the body. This state of a body is called weightlessness.

12. Astronauts in satellite experience weightlessness. Explain why?

The necessary centripetal force of the satellite around the earth is provided by the gravitational force of attraction between earth and satellite. Every part in a satellite around the earth has acceleration towards the centre of the earth which is acceleration due to gravity of the earth at that position. Therefore everything inside it is in a state of free fall.

This is as if we were falling towards the earth from a height. In a manned satellite astronauts inside the satellite experience no gravity. Therefore they experience a state of weightlessness.

13. A freely falling body is acted up on by a constant acceleration. Explain why?

The expression for acceleration due to gravity on the earth's surface is

$$g = \frac{GM_E}{R_E^2}$$

where G is gravitational constant,

M_E is mass of the earth,

R_E is radius of the earth.

An object released near the surface of the earth is accelerated downward under the influence of the force of gravity. The magnitude of acceleration due to gravity is g. If air resistance is neglected, the object is said to be in free fall. If the height through which the object falls is small compared to the earth's radius, g can be taken to be constant equal to 9.8m/s^2. Therefore a freely falling body is acted upon by a constant acceleration.

14. An object weighs more on the surface than at the centre of the earth. Why?

The weight of a body at a place on the surface of the earth is given by W = mg, since the value of g is maximum on the surface, the object weighs more on the surface. At the centre of the earth, the value of g is zero. Therefore the weight of the body at the centre of the earth is zero.

15. A body weighs more at pole than at equator of the earth. Explain why?

The weight of a body at a place on the surface of the earth is given by W = mg. Since the value of g is maximum at poles than at the equator, the weight of the body is more at the poles than at the equator.

16. "Cavendish weighed the earth". Why this statement is popular? Justify with the expression for the mass of the earth.

The acceleration experienced by the mass m due to the earth is called acceleration due to gravity g, which is related to the force F by Newton's 2nd law from the relation F =mg.

Therefore g = $\frac{F}{M} = \frac{GM_E}{R_E^2}$

Acceleration g is measurable. R_E is a known quantity. The measurement of G by Cavendish's experiment and with the knowledge of g and R_E, the mass of the earth M_E can be estimated. Because of this reason there is a popular statement regarding Cavendish, "Cavendish weighed the earth".

17. The radius and mass of a planet are two times that of the earth's values. Calculate the acceleration due to gravity on the surface of the planet.

Let mass of the earth is M_E and that of planet is $M_P = 2M_E$. Also radius of the earth is R_E and that of planet is $R_P = 2R_E$.

But we know that on the surface of the earth, $g_E = \frac{GM_E}{R_E^2}$ --------- (1)

and on the surface of the planet, $g_P = \frac{GM_P}{R_P^2}$ ----------- (2)

$\frac{(1)}{(2)} \Rightarrow \frac{g_E}{g_P} = 2$

Therefore, $G_P = G_E/2$,

But we know that $g_E = 9.8\ m/s^2$. And $g_P = 9.8/2 = 4.9 m/s^2$.

18. Who proposed 'Geocentric theory'? Give the brief account of the theory.

Geocentric theory was proposed by Ptolemy about 2000 years ago. According to this theory, all celestial objects, stars, the sun and the planets all revolved around the earth. The only motion that was thought to be possible for celestial objects was motion in a circle. Complicated schemes of motion were put forward by Ptolemy in order to describe the observed motion of the planets. The planets were described as moving in circles with the centre of the circles themselves moving in larger circles.

19. Who proposed 'Heliocentric theory'? Give the brief account of the theory.

Heliocentric theory was proposed by Nicolas Copernicus. According to this theory, sun was at the centre with the planets revolving around. Planets moved in circles around a fixed central sun. This theory was discredited by the church but Galileo supported it and faced prosecution from the State. Tycho Brahe recorded observations of the planets with naked eyes. This data was analyzed by Kepler and later formulated three laws called Kepler's laws of planetary motion, that support Helio centric theory.

20. The orbiting speed of an earth's satellite is 10 km s^{-1}. What is its escape speed?

Orbiting speed of earth's satellite, v_o = 10 km/s. Escape speed, v_e = ?

We have $v_e = \sqrt{2}\, v_o = \sqrt{2} \times 10 = 14.14$m/s.

21. Distinguish between 'Geostationary' and 'polar' satellites.

Geostationary satellites are the satellites in circular orbits around the earth in the equatorial plane with the period T = 24hrs.

Polar satellites are low altitude (h ≈ 500 to 800 km) satellites and go around the poles of the earth in a north-south direction with a time period of around 100 minutes where as the earth rotates around its axis in an east- west direction.

22. Give any two applications of artificial satellites.

Artificial satellites find applications in the fields like telecommunication, geophysics, meteorology.

II. Four/Five mark questions:

1. State Kepler's laws of planetary motion and explain law of orbits and law of areas.

Kepler's laws of planetary motion :-

(i) Law of orbits :- All planets move in elliptical orbits with the sun situated at one of the foci of the ellipse.

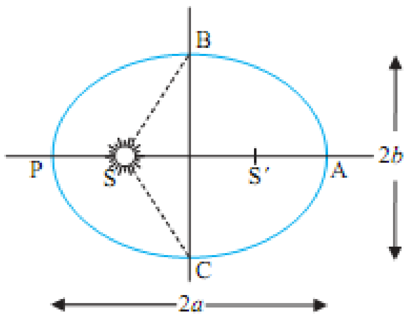


Fig. 8.1(a) An ellipse traced out by a planet around the sun. The closest point is P and the farthest point is A, P is called the perihelion and A the aphelion. The semimajor axis is half the distance AP.

This law was a deviation from the Copernican model which allowed only circular orbits. Circle is a closed curve and is a special case of ellipse.

The midpoint of the line PA is the centre of the ellipse O and the length PO = AO is called the semi-major axis of the ellipse. For a circle, the two foci merge into one and the semi-major axis becomes the radius of the circle.

(ii). Law of areas :- The line that joins any planet to the sun sweeps equal areas in equal intervals of time.

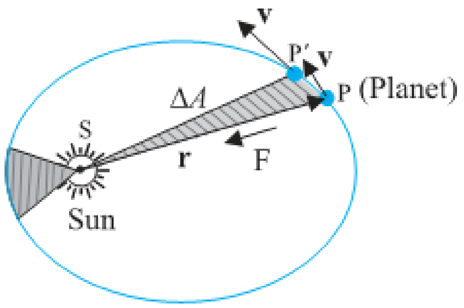


Fig. 8.2 The planet P moves around the sun in an elliptical orbit. The shaded area is the area ΔA swept out in a small interval of time Δt.

This law comes from the observations that planets appear to move slower when they are farther from the sun than when they are nearer.
ΔA is the area swept out in a small interval of time Δ t.

(iii) Kepler's law of periods :- The square of the time period of revolution of a planet is proportional to the cube of the semi - major axis of the ellipse traced out by the planet.
If T is the period of revolution of the planet and a is semi - major axis of the elliptical path, then $T^2 \alpha\, a^3$.

2. Derive $g=(GM_E)/R_E^2$, where the symbols have their usual meaning.

Let m be the mass of the body on the surface of the earth of radius R_E. The entire mass M_E of the earth is concentrated at the centre of the earth.
The magnitude of the force acting on the mass m is
$F=\frac{GmM_E}{R_E^2}$ ....(1)
If the entire earth is assumed to be of uniform density ρ, its mass is
$M_E = \frac{4}{3}\pi R_E^3 \rho$.....................(2) (since volume of the earth $V_E = \frac{4}{3}\pi R_E^3$)

Therefore from (1)and (2) $F = \frac{Gm(\frac{4}{3})\pi\rho R_E^3}{R_E^2}$ (But $(4/3)\pi\rho = \frac{M_E}{R_E^3}$)

Therefore $F = Gm(\frac{M_E}{R_E^3})\frac{R_E^3}{R_E^2}$

That is $F = \frac{GmM_E}{R_E^2}$.........................(3)

By Newton's 2nd law of motion, F = mg ....(4) is the force on the mass m and g is the acceleration experienced by the mass m.
Therefore from (3) and (4)

$g = \frac{F}{m} = \frac{GmM_E}{R_E^2 m}$

that is, $g = \frac{GM_E}{R_E^2}$.

3. Derive the expression for acceleration due to gravity at a point above the surface of the earth.

Consider a point mass m at a height h above the surface of the earth as shown in figure.

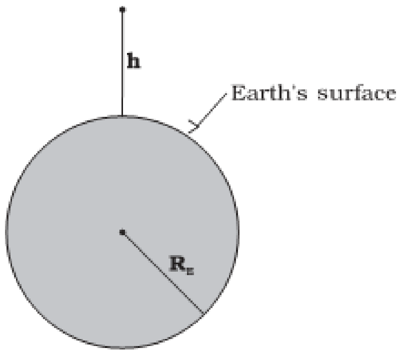


Fig. : g at a height h above the surface of the earth.

The radius of the earth is R_E. Since this point is outside the earth, its distance from the centre of the earth is $(R_E + h)$. If F (h) is the magnitude of the force on the point mass m,
Then $F(h) = GM_E m/(R_E+h)^2$ ....(1)
The acceleration experienced by the point mass is $F(h)/m = g(h)$ and we get
$g(h) = F(h)/m = GM_E/(R_E+h)^2$..(2)
This is clearly less than the value of g on the surface of earth, $g = GM_E/R_E^2$
For $h \ll R_E$ we can expand the RHS of equation (2), $g(h) = GM_E/[R_E^2(1+h/R_E)^2]$
$= g(1+h/R_E)^{-2}$
For $h/R_E \ll 1$, using binomial expression,
$g(h) = g(1-2h/R_E)$...........................(3)
Equation (3) tells that for small heights h above the surface of the earth, the value of g decreases by a factor $(1-2h/R_E)$.

4. Derive the expression for acceleration due to gravity at a point below the surface of the earth.

Consider a point mass m at a depth d below the surface of the earth as shown in figure.

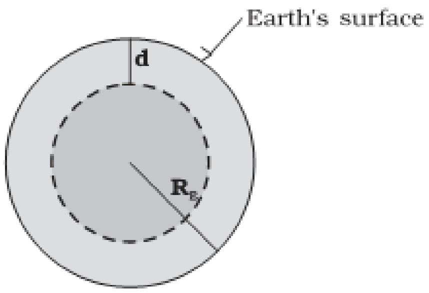


Fig. : g at a depth d. In this case only the smaller sphere of radius $(R_E–d)$ contributes to g

Its distance from the centre of the earth is (R_E-d). The earth can be thought of as being composed of a smaller sphere of radius $(R_E – d)$ and a spherical shell of thickness d. The force on m due to the outer shell of thickness d is zero. The point mass m is outside the smaller sphere of radius $(R_E – d)$. The force due to this smaller sphere is just as if the entire mass of the smaller sphere is concentrated at the centre. If M_S is the mass of the smaller sphere, then
$M_S/M_E = (R_E – d)^3/R_E^3$ ....(1)
(Since mass of a sphere is proportional to the cube of its radius).

Therefore the force on the point mass is $F(d) = GM_S m/(R_E – d)^2$ ……………………….(2)
Substituting for MS we get
$F(d) = GM_E m(R_E – d)/R_E^3$ ……………………………..(3)
Therefore the acceleration due to gravity at a depth d is, $g(d) = F(d)/m$
that is $g(d) = [GM_E/R_E^3](R_E – d)$
$= g (R_E – d)/R_E$

$g(d) = g(1 – d/R_E)$……………………………(4)

Therefore as we go down below earth's surface, the acceleration due to gravity decreases by a factor $(1 – d/R_E)$.

5. Derive the expression for gravitational potential energy of a particle at a point due to the earth.

Gravitational potential energy of a particle at a point due to the earth is defined as the amount of work done in displacing the particle from infinity to that point in the gravitational field. It is denoted by V. Consider a particle of mass m placed at a distance r from the centre of the earth of mass M . The gravitational force of attraction between the earth and the particle is,
$F = GMm/r^2$.
If the particle of mass m is displaced through a small distance dr towards the earth then, work done is given by, $dW = F\,dr = [GMm/r^2]\,dr$……………………………… (1)
Amount of work done in displacing the particle of mass m from infinity to a distance r with respect to the earth is given by

$$\int_{\infty}^{r} dW = \int_{\infty}^{r} [GMm/r^2]dr$$

$$= GMm \left[\frac{r^{-1}}{-1}\right]^{r}_{infinity}$$

$$= - GMm\,[1/r]^{r}_{infinity}$$

Therefore $W= -GMm[(1/r)-(1/\infty)] = - GMm/r$
The amount of work done is equal to the gravitational potential energy V.
Therefore $V = - GMm/r$.

6. Obtain the expression for escape speed.

The minimum speed required to project an object vertically upwards from the surface of the earth, so that it escapes from the gravitational influence of the earth and never returns to the earth, is called escape speed.
Consider an object of mass m at a distance of r from the centre of the earth.
The gravitational force acting on the mass m is given by
$F = GMm/r^2$ …………………………(1)
This force acts towards the centre of the earth.
Let dr be the small distance covered by the object away from the centre of the earth. Therefore the work done on the object against the gravitational force of attraction of the earth is

$dW = \vec{F}.\vec{dr}$ = F dr cos 180^0 = - F dr

From equation (1) we get, dW = - [GMm/r^2] dr.........................(2)

Therefore total work done to displace the object from the surface of the earth [i.e, r=R] to [r = ∞] is calculated by integrating equation(2) between the limits R and ∞.

Therefore $\int_R^{\infty} dW = \int_R^{\infty} -[GMm/r^2]$ dr

$$W = -GMm\left[\frac{r^{-1}}{-1}\right]^{\infty}_{R} = GMm\left[\frac{1}{r}\right]^{\infty}_{R} = GMm\left[\frac{1}{\infty} - \frac{1}{R}\right]$$

Or W = - GMm/R [since $\frac{1}{\infty}$ = 0] this work done is equal to the potential energy (V) of the object of mass m. That is V = - GMm/R . Let v_e is the escape speed of the object of mass m then its kinetic energy is K.E = $\frac{1}{2}mv_e^2$. If kinetic energy of the object = magnitude of potential energy of the object then,

$\frac{1}{2}mv_e^2$ = GMm/R

Or $v_e^2 = \frac{2GM}{R}$

$v_e = \sqrt{\frac{2GM}{R}}$

but $GM/R^2 = g$

or $GM = g R^2$

Therefore $v_e = \sqrt{2gR}$.

7. Derive the expression for orbital speed of a satellite/ period of a satellite around the earth.

The speed with which a satellite moves in its orbit around the earth is called orbital speed. The time taken by a satellite to complete one revolution in its orbit around the earth is called period of a satellite.

Let a satellite of mass m revolves around the earth in an orbit at a height of h from the surface of the earth. If R is the radius of the earth, then the radius of the orbit of the satellite is R + h. Let v_o be the orbital speed of the satellite.

The gravitational force of attraction between the earth and the satellite provides the necessary centripetal force to the satellite to move in a circular orbit around the earth. i.e, Gravitational force = centripetal force

i.e, Gravitational force = centripetal force

$$\frac{GMm}{(R+h)^2} = \frac{mv_o^2}{(R+h)}$$

That is $v_o^2 = \frac{GM}{(R+h)}$

Therefore $v_o = \sqrt{\frac{GM}{(R+h)}}$

But $\frac{GM}{R^2} = g$ or $GM = gR^2$

Therefore $v_o = \sqrt{\frac{gR^2}{(R+h)}}$

If the satellite is very close to the earth i.e, h<<

Then (R+h) ≈ R

Therefore $v_o = \sqrt{gR}$.

Period of a Satellite T = $\frac{Circumference\ of\ the\ orbit}{orbital\ speed}$

i.e, T = $\frac{2\pi(R+h)}{v_o}$

we know that $v_o = \sqrt{\frac{gR^2}{(R+h)}}$

Therefore T = $\frac{2\pi(R+h)}{\sqrt{\frac{gR^2}{(R+h)}}}$

i.e, T = $\frac{2\pi}{R}\sqrt{\frac{(R+h)^3}{g}}$.

If the satellite is very close to the earth, h<<

Then (R+h) ≈ R

Therefore T= $2\pi\sqrt{\frac{R}{g}}$

8. Obtain the expression for energy of an orbiting satellite.

The total energy of a satellite in Its orbit is equal to the sum of the potential energy and kinetic energy of the satellite. Potential energy of the satellite comes due to the gravitational force of attraction between the earth and the satellite. Kinetic energy of the satellite comes due to the orbital motion of the satellite around the earth.

Potential energy of the satellite is given by

P.E = $\frac{-GmM_E}{R_E+h}$..................................(1)

Where G is gravitational constant,

m is the mass of the satellite,

M_E is mass of the earth,

R_E is radius of the earth,

h is height of the satellite above earth's surface.

Kinetic energy of the satellite is given by

K.E = $\frac{1}{2}$m v_o^2....................................(2)

Where m is mass of the satellite,

v_o is orbital speed of the satellite.

The centripetal force on the satellite is equal to the gravitational force of attraction between earth and satellite.

That is,

$$\frac{mv_o^2}{R_E+h} = \frac{GmM_E}{(R_E+h)^2}$$

$$mv_o^2 = \frac{GmM_E}{(R_E+h)}\ \ldots\ldots(3)$$

Substituting equation (3) in (2),

K.E = $\frac{GmM_E}{2(R_E+h)}$....................................(4)

From the equations (1) and (4), the total energy of the satellite is,

E = P.E + K.E = $\frac{-GmM_E}{R_E+h} + \frac{GmM_E}{2(R_E+h)}$

Or E = $\frac{-GmM_E}{2(R_E+h)}$

Negative sign in the above expression implies that the satellite is bound to the earth.

9. Show that the law of areas follows from the law of conservation of angular momentum.

Law of areas :- The line that joins any planet to the sun sweeps equal areas in equal intervals of time.

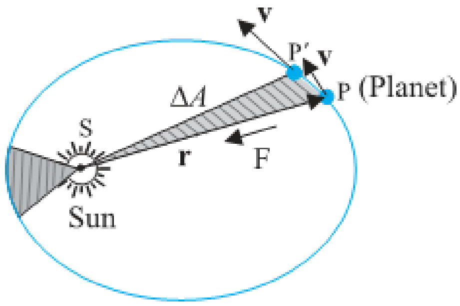


Fig. 8.2 The planet P moves around the sun in an elliptical orbit. The shaded area is the area ΔA swept out in a small interval of time Δt.

The law of areas can be understood as a consequence of conservation of angular momentum which is valid for any central force. A central force is such that the force on the planet is along the vector joining the sun and the planet. Let the sun be at the origin and let the position and momentum of the planet be denoted by $\vec{r}$ and $\vec{p}$ respectively. Then the area swept out by the planet of mass m in time interval Δt is $\Delta \vec{A}$ given by

$$\Delta \vec{A} = {}^1/_2 \, (\vec{r} \times \vec{v}\, \Delta t) \text{......................................}(1)$$

$$\frac{\Delta \vec{A}}{\Delta t} = {}^1/_2 \, (\vec{r} \times \vec{p})/m \qquad \left(\text{since } \vec{v} = \frac{\vec{p}}{m}\right)$$

$$= \vec{L}/(2m) \text{..}(2)$$

Where $\vec{v}$ is the velocity, $\vec{L} = \vec{r} \times \vec{p}$ is the angular momentum.

For a central force, which is directed along $\vec{r}$, $\vec{L}$ is a constant as the planet goes around. $\Delta\vec{A}/\Delta t$ is a constant according to the last equation. This is the law of areas. Gravitation is a central force and hence the law of areas follows.

10. Describe Cavendish's experiment to determine the value of gravitational constant G.

The value of the gravitational constant G entering the Universal law of gravitation can be determined experimentally and this was first done by English scientist Henry Cavendish in 1798, the apparatus used by him is as shown in figure.

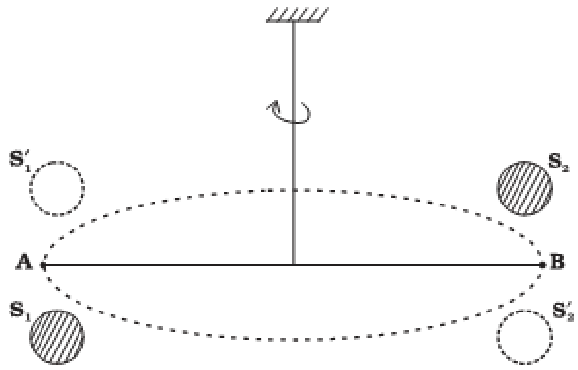


Fig 8.6: Schematic drawing of Cavendish's experiment. S_1 and S_2 are large spheres which are kept on either side (shown shades) of the masses at A or B. When the big spheres are taken to the other side of the masses (shown by dotted circles), the bar AB rotates a little since the torque reverses direction. The angle of rotation can be measured experimentally.

The bar AB has two small lead spheres attached at its ends. The bar is suspended from a rigid support by a fine wire. Two large lead spheres are brought close to the small ones but on opposite sides as shown. The big spheres attract the nearby small ones by equal and opposite force as shown. There is no net force on the bar but only a torque which is clearly equal to F times the length of the bar, where F is the force of attraction between a big sphere and its neighbouring small sphere. Due to this torque, the suspended wire gets twisted till such time as the restoring torque of the wire equals the gravitational torque . If ϴ is the angle of twist of the suspended wire, the restoring torque is proportional to ϴ equal to τ ϴ, where τ is the restoring couple per unit angle of twist. τ can be measured independently e.g. by applying a known torque and measuring the angle of twist. The gravitational force between the spherical balls is the same as if their masses are concentrated at their centres.

Thus if d is the separation between the centres of the big and its neighbouring small ball, M and m their masses, the gravitational force between the big sphere and the neighbouring small ball is $F = GMm/d^2$ ……………………………………(1)

If L is the length of the bar AB, then the torque arising out of F is, F multiplied by L. At equilibrium, this is equal to the restoring torque and hence

$FL = \tau \theta$.

$G (Mm/d^2) L = \tau \theta$ ………………………………(2)

Observation of ϴ thus enables one to calculate G from this equation.

From the above equation (2), G can be calculated and its value is $G = 6.67 \times 10^{-11}$ Nm^2/kg^2.

Problems of Question bank :

1) The orbital radius of the Neptune planet around the Sun is n times that of earth. The Neptune planet takes 164.3 years to complete one revolution. Find the value of n. (The planetary orbits are assumed to be circular) ?

Ans :

Given,

$$r_N = n\, r_E \quad \Rightarrow \quad \frac{r_N}{r_E} = n$$

$$n = ?$$

$$T_N = 164.3 year,\ T_E = 1\ year \quad \Rightarrow \quad \frac{T_N}{T_E} = 164.3$$

According to Kepler's laws,

$$T^2 \;\alpha\; r^3$$

$$\left(\frac{T_N}{T_E}\right)^2 = \left(\frac{r_N}{r_E}\right)^3$$

$$\left(\frac{164.3}{1}\right)^2 = (n)^3$$

$$\Rightarrow n = (164.3)^{2/3}$$

Using logs and simplifying, n = 30

2) The Planet Mars take 1.88 years to complete on revolution around the sun. The mean distance of the earth from the Sun is 1.5x10^8 km. Calculate that of planet Mars?

Ans :

$$T_M = 1.88 year,\; T_E = 1\; year \;\Rightarrow\; \frac{T_M}{T_E} = 1.88$$

$$r_E = 1.5\; X\; 10^8\; km\,,\quad r_M = ?$$

According to Kepler's laws,

$$T^2 \;\alpha\; r^3$$

$$\left(\frac{T_M}{T_E}\right)^2 = \left(\frac{r_M}{r_E}\right)^3$$

$$(1.88)^2 = \left(\frac{r_M}{r_E}\right)^3$$

$$\frac{r_M}{r_E} = (1.88)^{2/3}$$

$$r_M = (1.88)^{2/3}\; X\; r_E$$

$$r_M = (1.88)^{2/3} \text{ x } 1.5 \text{ x } 10^8\; km$$

$$r_M = 1.524 \text{ x } 1.5 \text{ x } 10^8\; km$$

$$\boldsymbol{r_M = 2.286 \text{ x } 10^8\; km}$$

3) A Satellite orbiting at a height of 2.5R above the earth's surface takes 6√2 hours to complete one revolution. Show that another satellite orbiting at a height of 6R from the earth's surface is a geostationary satellite. 'R' is the radius if the earth.

Ans :

Given, that the Satellite's orbit is at a height of 2.5R above the earth's surface. Therefore its radius is

$$r_1 = R + 2.5\,R = 3.5\,R$$

Similarly,

$r_2 = R + 6\,R = 7\,R$

$T_1 = 6\sqrt{2}\,hour\,, \qquad T_2 = ?$

Using to Kepler's laws,

$$T^2 \;\alpha\; r^3$$

$$\left(\frac{T_2}{T_1}\right)^2 = \left(\frac{r_2}{r_1}\right)^3$$

$$\frac{T_2}{T_1} = \left(\frac{7R}{3.5R}\right)^{3/2} = 2^{3/2} = 2\sqrt{2}$$

$$T_2 = 2\sqrt{2}\;\;T_1 = 2\sqrt{2}\,X 6\sqrt{2} = 24\,hours.$$

$$\boldsymbol{T_2 = 24\,hours.}$$

Therefore, it is a geostationary satellite.

4) The mass of the Planet Jupiter is 2 x 10^{27} kg and the mass of the Sun is 1000 times the mass of Jupiter. The mean distance between the Sun and Jupiter is 7.8 x 10^8 km. Calculate the value of gravitational constant G, given the force of gravitation between the Sun and Jupiter is 4.276 x 10^{23} N.

Ans :

Given,

$M_J = 2\,X\,10^{27}\,kg\,,$

$M_S = 1000\,M_J = 1000\,X\,2\,X\,10^{27}\,kg = 2\,X\,10^{30}\,kg\,,$

$r = 7.8\,X\,10^8\,km = 7.8\,X\,10^{11}\,m$

$F = 4.276\,X\,10^{23} N$

$G = ?$

From Newton's laws of universal Gravitation ,

$$F = \frac{G\,M_J\;M_S}{r^2} \;\Rightarrow G = \frac{Fr^2}{M_J\;M_S}$$

$$G = \frac{(4.276\,X\,10^{23}\,)X\,(7.8\,X\,10^{11}\,)^2}{(2\,X\,10^{27})\,X\,(2\,X\,10^{30})}$$

$$G = \frac{(4.276\,X\,10^{23}\,)X\,(60.84\,X\,10^{22}\,)}{4\,X\,10^{57}} = 65.037\,X\,10^{-12}$$

$$\boldsymbol{G = 6.5037\,X\,10^{-11}\,Nm^2kg^{-2}}$$

5) The gravitation force of attraction between Earth and Sun is 35.47 x 10^{21} N, calculate the mass of the Sun, given the mass of the earth is 5.98 x 10^{24} kg and the mean distance between Earth and Sun is 1.496 x 10^{11}.

Ans :

Given,

$F = 35.47\ X\ 10^{21} N$

$M_E = 5.98\ X\ 10^{24}\ kg\ ,\ \ M_S = ?$

$r = 1.496\ X\ 10^{11}\ m$

$G = 6.67\ X\ 10^{-11} Nm^2 kg^{-2}$

Using the Newton's laws of universal Gravitation,

$$F = \frac{G\ M_S\ M_E}{r^2} \Rightarrow M_S = \frac{Fr^2}{G\ M_E}$$

$$M_S = \frac{(35.47\ X\ 10^{21}) X\ (1.496\ X\ 10^{11})^2}{(6.67\ X\ 10^{-11})\ X\ (5.98\ X\ 10^{24})}$$

$$\boldsymbol{M_S = 1.99\ X\ 10^{30}\ kg}$$

6) The mass and diameter of a planet are three times that of the Earth. What is the acceleration due to gravity on the surface of that planet? Given g on earth's surface is 9.8 ms^{-2}.

Ans :

Given,

$M_P = 3\ M_E\ ,\ \ D_P = 3\ D_E\ \Rightarrow R_P = 3\ R_E$

$$g_E = \frac{G\ M_E}{{R_E}^2}\ and\ g_P = \frac{G\ M_P}{{R_P}^2} = \frac{G\ 3M_E}{9{R_E}^2}$$

$$\frac{g_P}{g_E} = \frac{1}{3} \Rightarrow g_P = \frac{g_E}{3} = \frac{9.8}{3} = 3.2666\ ms^{-2}$$

$$\Rightarrow \boldsymbol{g_P = 3.2666\ ms^{-2}}$$

7) Calculate the mean radius of the earth, given its mass is 5.98x1024 kg and acceleration due to the gravity on its surface is g=9.8ms-2. If the radius of the Earth were to shrink by 1% with its mass remaining same, what would be the value of acceleration due to gravity? Given G = 6.67 × 10^{-11} Nm2 kg^{-2} ?

Ans :

Given,

$M_E = 5.98 \ X \ 10^{24} \ kg$

$g = 9.8 \ ms^{-2}$

$G = \ 6.67 \ X \ 10^{-11} Nm^2 kg^{-2}$

R = ?

We have

$$g = \frac{G \ M}{R^2} \Rightarrow \ R = \sqrt{\frac{GM}{g}}$$

$$R = \sqrt{\frac{(6.67 \ X \ 10^{-11})(5.98 \ X \ 10^{24})}{9.8}}$$

$$\boldsymbol{R = \ 6.3796 \ X \ 10^6 m = \ 6379.6 \ km}$$

If radius is shrinks by 1 %

$$R' = \ R - 1\% \ of \ R = R - \left(1/_{100}\right) \ R \ = 0.99 \ R$$

$$R' = 0.99 \ R = 0.99 \ X \ 6.3796 \ X \ 10^6 = 6.3158 \ X \ 10^6 m$$

$$g' = \frac{G \ M}{R'^2}$$

$$g' = \frac{(6.67 \ X \ 10^{-11}) \ (5.98 \ X \ 10^{24} \ )}{(6.3158 \ X \ 10^6)^2} = \ 6.315 \ X \ 10^1 = 6.315 \ ms^{-2}$$

$$\boldsymbol{g' = 6.315 \ ms^{-2}}$$

8) Estimate the value of acceleration due to gravity at the peak of Mount Everest, which is 8848m above sea level. The value of g at sea level is 9.8 ms^{-2} and the mean radius of the earth is $6.37 x 10^6$ m.

Ans :

Given,

$h = 8848 \ m$

$g = 9.8 \ ms^{-2}$

$g(h) \ =?$

$R_E = \ 6.37 \ X \ 10^6 m$

We have

$$g(h) = g\left(1 - \frac{2h}{R_E}\right)$$

$$g(h) = 9.8\left(1 - \frac{2\ X\ 8848}{6.37\ X\ 10^6}\right)$$

$$g(h) = 9.8\ (1 - 0.002778)$$

$$g(h) = 9.8\ (0.997222)$$

$$\boldsymbol{g(h) = 9.773\ ms^{-2}}$$

9) At what height above the surface of the Earth will acceleration due to gravity becomes half its value at Earth's surface. Given the mean radius of Earth is 6.37x10^6 m.

Ans :

Given,

$$g(h) = {}^{g}/_{2}$$

$$h = ?$$

$$R_E = 6.37\ X\ 10^6 m$$

We have

$$g(h) = g\left(\frac{R_E}{R_E + h}\right)^2$$

$$\frac{g}{2} = g\left(\frac{R_E}{R_E + h}\right)^2 \Rightarrow \frac{1}{2} = \left(\frac{R_E}{R_E + h}\right)^2$$

Therefore,

$$\left(\frac{R_E}{R_E + h}\right) = \frac{1}{\sqrt{2}}$$

$$R_E + h = \sqrt{2}\,R_E \quad \Rightarrow \quad h = \sqrt{2}\,R_E - R_E = (\sqrt{2} - 1)\,R_E$$

$$h = (1.4142 - 1)\ 6.37\ X\ 10^6 = (0.4142)\ X\ 6.37\ X\ 10^6 = 2.6385\ X\ 10^6\ m$$

$$\boldsymbol{h = 2.6385\ X\ 10^6\ m = 2638.5\ km}$$

10) What is the value of acceleration due to the gravity at distance of 3000 km from the centre of Earth. Given its value on the surface is 9.8 ms^{-2} and the mean radius of the Earth is 6.37x10^6 m.

Ans :

The point under consideration lies inside the Earth's surface, because its distance from the centre of the earth x = 3000 km = 3×10^6 m is less then R_E = 6.37×10^6 m.

$g = 9.8\ ms^{-2}$

$g(d) = ?$

We have,

$$g(d) = g\left(1 - \frac{d}{R_E}\right)$$

$$g(d) = 9.8\left(1 - \frac{3.37\ X\ 10^6}{6.37\ X\ 10^6}\right)$$

$$g(d) = 9.8\,(1 - 0.529)$$

$$g(d) = 9.8\,(0.471)$$

$$\mathbf{g(d) = 4.6158\ ms^{-2}}$$

11) Four particles each of mass 1kg are packed at four vertices of a square of side 1m. Find the gravitational potential energy of this system. Given, G = 6.67 × 10^{-11} Nm^2kg^{-2}.

Given :

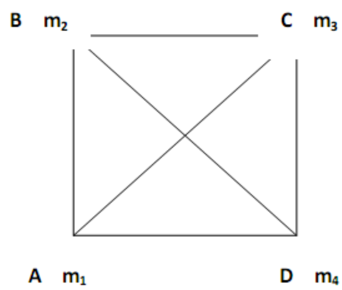
$m_1 = m_2 = m_3 = m_4 = m = 1\ kg$

$AB = BC = CD = DA = x = 1m$

$Diagonal\ AC = BD = \sqrt{2}\,x\ m$

$G = 6.67\ X\ 10^{-11} Nm^2kg^{-2}$

$V_T = ?$



The gravitational potential energy of a two particles is

$$V = \frac{-G\,m_1m_2}{r}$$

The total gravitational potential energy is

$$V_T = [V_{AB} + V_{BC} + V_{CD} + V_{DA}] + [V_{AC} + V_{BD}]$$

$$But\ V_{AB} = V_{BC} = V_{CD} = V_{DA} = \frac{-G\,m\,m}{x} = \frac{-G\ X\ 1\ X\ 1}{1} = -G\ \mathbf{J}$$

$$V_{AC} = V_{BD} = \frac{-Gmm}{\sqrt{2}\,x} = \frac{-G\ X\ 1\ X\ 1}{\sqrt{2}} = \frac{-G}{\sqrt{2}}\ \mathbf{J}.$$

$$V_T = 4(-G) + 2\left(\frac{-G}{\sqrt{2}}\right) = -4G - \frac{2G}{\sqrt{2}}$$

$$V_T = G\left(-4 - \frac{2}{\sqrt{2}}\right) = G(-4 - \sqrt{2}) = -5.4142\,G$$

$$V_T = -5.4142\ X\ 6.67\ X\ 10^{-11}$$

$$\mathbf{V_T = -36.11\ X\ 10^{-11} J}$$

12) Given the mass of the moon Mm=7.35x10^{22} kg and the radius of the moon Rm =1.7x10^{6} m, estimate the escape speed for moon . Given G = 6.67 × 10^{-11} Nm^{2}kg^{-2}.

Ans :

$M_m = 7.35\ X\ 10^{22}\ kg \qquad R_m = 1.7\ X\ 10^{6}\ m$

$G = 6.67\ X\ 10^{-11} Nm^2 kg^{-2}$

$v_e = ?$

We have

$$v_e = \sqrt{2\,g\,R}$$

But,

$$g = \frac{G\,M}{R^2} \Rightarrow v_e = \sqrt{\frac{2GM}{R}}$$

For moon,

$$v_e = \sqrt{\frac{2G\,M_m}{R_m}}$$

$$v_e = \sqrt{\frac{2X(6.67\ X\ 10^{-11})X(7.35\ X\ 10^{22})}{1.7\ X\ 10^{6}}}$$

$$\boldsymbol{v_e = 2401ms^{-1} = 2.401\ kms^{-1}}$$

13) Calculate the period of Earth's revolution around the sun. Given the mass of the Sun is Ms = 2x10^{30} kg, mean radius of the Earth's orbit, R=1.5x10^{11} m.

Ans :

Given :

$M_s = 2X\ 10^{30}\ kg \qquad R = 1.5\ X\ 10^{11}\ m$

$G = 6.67\ X\ 10^{-11} Nm^2 kg^{-2}$

$T = ?$

From the derivation of Kepler's 3rd law,

$$T^2 = \frac{4\pi^2 R^3}{GM_S} = \frac{4X3.14^2 X(1.5X10^{11})^3}{(6.67X10^{-11})(2X10^{30})}$$

$$T^2 = 9.977X10^{14}$$

$$T = 3.1587X10^{7}\ s$$

$$\Rightarrow T = \frac{3.1587X10^{7}}{60X60X24} days$$

$$\boldsymbol{T = 365.5\ days}$$

14) A satellite of mass 500 kg orbits the Earth at a height of 400 km above the surface. How much energy is required to shift it to height orbit at a height 600 km? Given the mean radius of the Earth, Re=6.4x10^{6} m, mass of the Earth, M$_E$=6x10^{24} kg, and G = 6.67 × 10^{-11} Nm2 Kg^{-2}.

Ans :

Given : m= 500 Kg.

$$R_E = 6.4X\ 10^6\ m$$

$$h_1 = 400\ km = 0.4\ X\ 10^6\ m \Rightarrow R_E + h_1 = 6.8\ X\ 10^6\ m$$

$$h_2 = 600\ km = 0.6\ X\ 10^6\ m \Rightarrow R_E + h_2 = 7\ X\ 10^6\ m$$

$$M_E = 6\ X\ 10^{24}\ kg$$

$$G = 6.67\ X\ 10^{-11} Nm^2 kg^{-2}$$

$$Total\ energy\quad E = \frac{-GmM}{2(R_E + h)}$$

Initially,

$$E_i = \frac{-GM_E m}{2(R_E + h_1)} = \frac{-GM_E m}{2\ X\ 6.8\ X\ 10^6}$$

Finally,

$$E_f = \frac{-GM_E m}{2(R_E + h_2)} = \frac{-GM_E m}{2\ X\ 7\ X\ 10^6}$$

Change in energy, $\Delta E = E_f - E_i$

$$\Delta E = \frac{-GM_E m}{2\ X\ 7\ X\ 10^6} + \frac{GM_E m}{2\ X\ 6.8\ X\ 10^6}$$

$$\Delta E = \frac{GM_E m}{2\ X\ 10^6}\left[\frac{1}{6.8} - \frac{1}{7}\right]$$

$$\Delta E = \frac{6.67\ X\ 10^{-11}\ X\ 6\ X\ 10^{24}\ X\ 500}{2\ X\ 10^6}\left[\frac{0.2}{(6.8)\ (7)}\right]$$

$$\boldsymbol{\Delta E = 42.04X10^7 J.}$$

NCERT Solutions :

Question 8.1:

Answer the following:

(a) You can shield a charge from electrical forces by putting it inside a hollow conductor. Can you shield a body from the gravitational influence of nearby matter by putting it inside a hollow sphere or by some other means?

(b) An astronaut inside a small space ship orbiting around the earth cannot detect gravity. If the space station orbiting around the earth has a large size, can he hope to detect gravity?

(c) If you compare the gravitational force on the earth due to the sun to that due to the moon, you would find that the Sun's pull is greater than the moon's pull. (You can check this yourself using the data available in the succeeding exercises).

However, the tidal effect of the moon's pull is greater than the tidal effect of sun. Why?

ANS:

(a) No (b) Yes

(a) Gravitational influence of matter on nearby objects cannot be screened by any means. This is because gravitational force unlike electrical forces is independent of the nature of the material medium. Also, it is independent of the status of other objects.

(b) If the size of the space station is large enough, then the astronaut will detect the change in Earth's gravity (g).

(c) Tidal effect depends inversely upon the cube of the distance while, gravitational force depends inversely on the square of the distance. Since the distance between the Moon and the Earth is smaller than the distance between the Sun and the Earth, the tidal effect of the Moon's pull is greater than the tidal effect of the Sun's pull.

Question 8.2:

Choose the correct alternative:

(a) Acceleration due to gravity increases/decreases with increasing altitude.

(b) Acceleration due to gravity increases/decreases with increasing depth. (assume the earth to be a sphere of uniform density).

(c) Acceleration due to gravity is independent of mass of the earth/mass of the body.

(d) The formula $-G\,Mm(1/r_2 - 1/r_1)$ is more/less accurate than the formula $mg(r_2 - r_1)$ for the difference of potential energy between two points r2and r1distance away from the centre of the earth.

ANS:

(a) Decreases

(b) Decreases

(c) Mass of the body

(d) More

Explanation:

(a) Acceleration due to gravity at depth h is given by the relation :

$$g_h = \left(1 - \frac{2h}{R_e}\right)g$$

Where,
R_e = Radius of the Earth
g = Acceleration due to gravity on the surface of the Earth
It is clear from the given relation that acceleration due to gravity decreases with an increase in height.
(b) Acceleration due to gravity at depth d is given by the relation:

$$g_d = \left(1 - \frac{d}{R_e}\right) g$$

It is clear from the given relation that acceleration due to gravity decreases with an increase in depth.
(c) Acceleration due to gravity of body of mass m is given by the relation: g = $\frac{GM}{R^2}$
Where,
G = Universal gravitational constant
M = Mass of the Earth
R = Radius of the Earth
Hence, it can be inferred that acceleration due to gravity is independent of the mass of the body.
(d) Gravitational potential energy of two points r_2 and r_1 distance away from the centre of the Earth is respectively given by:

$$V(r_1) = -\frac{GmM}{r_1}$$

$$V(r_2) = -\frac{GmM}{r_2}$$

$$\therefore \text{Difference in potential energy, } V = V(r_2) - V(r_1) = -GmM\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

Hence, this formula is more accurate than the formula $mg(r_2 - r_1)$

Question 8.3:
Suppose there existed a planet that went around the sun twice as fast as the earth. What would be its orbital size as compared to that of the earth?
ANS:
Lesser by a factor of 0.63
Time taken by the Earth to complete one revolution around the Sun,
T_e = 1 year
Orbital radius of the Earth in its orbit, Re = 1 AU
Time taken by the planet to complete one revolution around the Sun, $T_P = \frac{1}{2} T_e = \frac{1}{2}$ year.
Orbital radius of the planet = R_P
From Kepler's third law of planetary motion, we can write:

$$\left[\frac{R_P}{R_e}\right]^3 = \left[\frac{T_P}{T_e}\right]^2$$

$\left[\frac{T_P}{T_e}\right]^{\frac{2}{3}} = \left[\frac{\frac{1}{2}}{1}\right]^{\frac{2}{3}} = (0.5)^{\frac{2}{3}} = 0.63$

Hence, the orbital radius of the planet will be 0.63 times smaller than that of the Earth.

Question 8.4:

I_0 is one of the satellites of Jupiter, has an orbital period of 1.769 days and the radius of the orbit is 4.22×10^8m. Show that the mass of Jupiter is about one-thousandth that of the sun.

ANS :

Orbital period of I_0 is $T_0 = 1.769$ days

Orbital radius of I_0 is $R_0 = 4.22 \times 10^8$ m

Satellite I_0 is revolving around the Jupiter

Mass of the latter is given by the relation : $M_J = \frac{4\pi^2 R_0^3}{GT_0^2}$ --------- (1)

Where, M_J= Mass of Jupiter; G = Universal gravitational constant

Orbital period of earth = $T_e = 365.25$ days = $365.25 \times 24 \times 60 \times 60$ sec.

Orbital radius of earth = $R_e = 1$ AU = 1.496×10^{11} m

Mass of Sun is given as $M_S = \frac{4\pi^2 R_e^3}{GT_e^2}$ ------- (2)

$\therefore \frac{M_S}{M_J} =$

$$= \frac{4\pi^2 R_e^3}{GT_e^2} \times \frac{GT_{Io}^2}{4\pi^2 R_{Io}^3} = \frac{R_e^3}{R_{Io}^3} \times \frac{T_{Io}^2}{T_e^2} = \left(\frac{1.769 \times 24 \times 60 \times 60}{365.25 \times 24 \times 60 \times 60}\right)^2 \times \left(\frac{1.496 \times 10^{11}}{4.22 \times 10^8}\right)^3$$

= 1045.04

$$\therefore \frac{M_S}{M_J} \sim 1000$$

$M_S \approx 1{,}000\, M_J$

Hence, it can be inferred that the mass of Jupiter is about one-thousandth that of the Sun.

Question 8.5:

Let us assume that our galaxy consists of 2.5×10^{11} stars each of one solar mass. How long will a star at a distance of 50,000 ly from the galactic centre take to complete one revolution? Take the diameter of the Milky Way to be 10^5 ly.

ANS:

Mass of our galaxy Milky Way, M = 2.5×10^{11} solar mass

Solar mass = Mass of Sun = 2.0×10^{36} kg

Mass of our galaxy, M = $2.5 \times 10^{11} \times 2 \times 10^{36} = 5 \times 10^{41}$ kg

Diameter of Milky Way, d = 10^5 ly

Radius of Milky Way, r = 5×10^4 ly

1 ly = 9.46×10^{15} m

$\therefore r = 5 \times 10^4 \times 9.46 \times 10^{15} = 4.73 \times 10^{20}$ m

Since a star revolves around the galactic centre of the Milky Way, its time period is given by the relation :

$$T = \left(\frac{4\pi^2 r^3}{GM}\right)^{\frac{1}{2}} = \left(\frac{4\times(3.14)^2\times(4.73)^3\times10^{60}}{6.67\times10^{-11}\times5\times10^{41}}\right)^{\frac{1}{2}} = \left(\frac{39.48\times105.82\times10^{30}}{33.35}\right)^{\frac{1}{2}}$$

$$=\left(125.27\times10^{30}\right)^{\frac{1}{2}} = 1.12\times10^{16}\ \text{s}$$

$$1\ \text{year} = 365\times324\times60\times60\ \text{s}$$

$$1\text{s} = \frac{1}{365\times24\times60\times60}\ \text{years}$$

$$\therefore 1.12\times10^{16}\ \text{s} = \frac{1.12\times10^{16}}{365\times24\times60\times60}$$

$$= 3.55\times10^{8}\ \text{years}$$

Question 8.6:

Choose the correct alternative:

(a) If the zero of potential energy is at infinity, the total energy of an orbiting satellite is negative of its kinetic/potential energy.

(b) The energy required to launch an orbiting satellite out of earth's gravitational influence is more/less than the energy required to project a stationary object at the same height (as the satellite) out of earth's influence.

ANS:

(a) Kinetic energy

(b) Less

(a) Total mechanical energy of a satellite is the sum of its kinetic energy (always positive) and potential energy (may be negative). At infinity, the gravitational potential energy of the satellite is zero. As the Earth-satellite system is a bound system, the total energy of the satellite is negative. Thus, the total energy of an orbiting satellite at infinity is equal to the negative of its kinetic energy.

(b) An orbiting satellite acquires a certain amount of energy that enables it to revolve around the Earth. This energy is provided by its orbit. It requires relatively lesser energy to move out of the influence of the Earth's gravitational field than a stationary object on the Earth's surface that initially contains no energy.

Question 8.7:

Does the escape speed of a body from the earth depend on

(a) the mass of the body,

(b) the location from where it is projected,

(c) the direction of projection,

(d) the height of the location from where the body is launched?

ANS:

(a) No

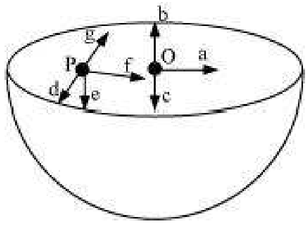
(b) No

(c) No
(d) Yes
Escape velocity of a body from the Earth is given by the relation: $V_{ESC} = \sqrt{2gR}$
g = Acceleration due to gravity
R = Radius of the Earth
It is clear from equation (i) that escape velocity v_{esc} is independent of the mass of the body and the direction of its projection. However, it depends on gravitational potential at the point from where the body is launched. Since this potential marginally depends on the height of the point, escape velocity also marginally depends on these factors.

Question 8.8:
A comet orbits the Sun in a highly elliptical orbit. Does the comet have a constant (a) linear speed, (b) angular speed, (c) angular momentum, (d) kinetic energy, (e) potential energy, (f) total energy throughout its orbit? Neglect any mass loss of the comet when it comes very close to the Sun.
ANS:
(a) No
(b) No
(c) Yes
(d) No
(e) No
(f) Yes
Angular momentum and total energy at all points of the orbit of a comet moving in a highly elliptical orbit around the Sun are constant. Its linear speed, angular speed, kinetic, and potential energy varies from point to point in the orbit.

Question 8.9:
Which of the following symptoms is likely to afflict an astronaut in space (a) swollen feet, (b) swollen face, (c) headache, (d) orientational problem ?
ANS:
(b), (c), and (d)
(a) Legs hold the entire mass of a body in standing position due to gravitational pull. In space, an astronaut feels weightlessness because of the absence of gravity. Therefore, swollen feet of an astronaut do not affect him/her in space.
(b) A swollen face is caused generally because of apparent weightlessness in space. Sense organs such as eyes, ears nose, and mouth constitute a person's face. This symptom can affect an astronaut in space.
(c) Headaches are caused because of mental strain. It can affect the working of an astronaut in space.
(d) Space has different orientations. Therefore, orientational problem can affect an astronaut in space.

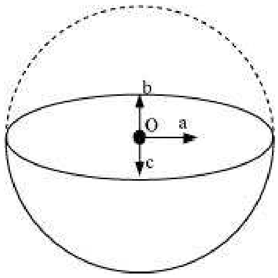
Question 8.10:
Choose the correct answer from among the given ones:
The gravitational intensity at the centre of a hemispherical shell of uniform mass density has the direction indicated by the arrow (see Fig 8.12) (i) a, (ii) b, (iii) c, (iv) O.



Answer :

(iii)

Gravitational potential (V) is constant at all points in a spherical shell. Hence, the gravitational potential gradient (dv/dr) is zero everywhere inside the spherical shell. The gravitational potential gradient is equal to the negative of gravitational intensity. Hence, intensity is also zero at all points inside the spherical shell. This indicates that gravitational forces acting at a point in a spherical shell are symmetric. If the upper half of a spherical shell is cut out (as shown in the given figure), then the net gravitational force acting on a particle located at centre O will be in the downward direction.



Since gravitational intensity at a point is defined as the gravitational force per unit mass at that point, it will also act in the downward direction. Thus, the gravitational intensity at centre O of the given hemispherical shell has the direction as indicated by arrow c.

Question 8.11:

Choose the correct answer from among the given ones:

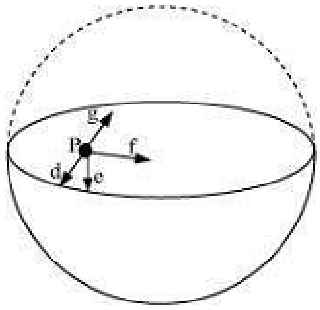
For the problem 8.10, the direction of the gravitational intensity at an arbitrary point P is indicated by the arrow (i) d, (ii) e, (iii) f, (iv) g.

Answer :

(ii)

Gravitational potential (V) is constant at all points in a spherical shell. Hence, the gravitational potential gradient (dv/dr) is zero everywhere inside the spherical shell. The gravitational potential gradient is equal to the negative of gravitational intensity. Hence, intensity is also zero at all points inside the spherical shell. This indicates that gravitational forces acting at a point in a spherical shell are symmetric.

If the upper half of a spherical shell is cut out (as shown in the given figure), then the net gravitational force acting on a particle at an arbitrary point P will be in the downward direction.



Since gravitational intensity at a point is defined as the gravitational force per unit mass at that point, it will also act in the downward direction. Thus, the gravitational intensity at an arbitrary point P of the hemispherical shell has the direction as indicated by arrow e.

Question 8.12:

A rocket is fired from the earth towards the sun. At what distance from the earth's centre is the gravitational force on the rocket zero? Mass of the sun = 2×10^{30} kg, mass of the earth = 6×10^{24} kg. Neglect the effect of other planets etc. (orbital radius = 1.5×10^{11} m).

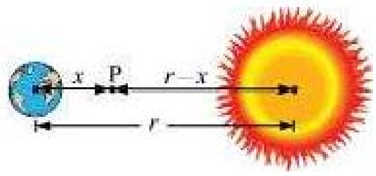
Answer :

Mass of the Sun, $M_S = 2 \times 10^{30}$ kg

Mass of the Earth, $M_e = 6 \times 10^{24}$ kg

Orbital radius, $r = 1.5 \times 10^{11}$ m

Mass of the rocket = m



Let x be the distance from the centre of the Earth where the gravitational force acting on satellite P becomes zero.

From Newton's law of gravitation, we can equate gravitational forces acting on satellite P under the influence of the Sun and the Earth as:

$$\frac{GmM_s}{(r-x)^2} = Gm\frac{M_e}{x^2}$$

$$\left(\frac{r-x}{x}\right)^2 = \frac{M_s}{M_e}$$

$$\frac{r-x}{x} = \left(\frac{2\times10^{30}}{60\times10^{24}}\right)^{\frac{1}{2}} = 577.35$$

$$1.5\times10^{11} - x = 577.35x$$

$$578.35x = 1.5\times10^{11}$$

$$x = \frac{1.5\times10^{11}}{578.35} = 2.59\times10^{8}\ \text{m}$$

Question 8.13:

How will you 'weigh the sun', that is estimate its mass? The mean orbital radius of the earth around the sun is 1.5×10^{8} km.

Answer :

Orbital radius of the Earth around the Sun, r = 1.5 × 10^{11} m
Time taken by the Earth to complete one revolution around the Sun,
T = 1 year = 365.25 days = 365.25 × 24 × 60 × 60 s
Universal gravitational constant, G = 6.67 × 10–11 $Nm^2\ kg^{-2}$
Thus, mass of the Sun can be calculated using the relation,

$$M = \frac{4\pi^2 r^3}{GT^2} \qquad = \frac{4\times(3.14)^2\times(1.5\times10^{11})^3}{6.67\times10^{-11}\times(365.25\times24\times60\times60)^2} \qquad = \frac{133.24\times10}{6.64\times10^4} = 2.0\times10^{30}\ \text{kg}$$

Hence, the mass of the Sun is 2 × 10^{30} kg.

Question 8.14:

A Saturn year is 29.5 times the earth year. How far is the Saturn from the sun if the earth is 1.50 ×10^8 km away from the sun?

Answer :
Distance of the Earth from the Sun, re = 1.5 × 10^8 km = 1.5 × 10^{11} m
Time period of the Earth = Te
Time period of Saturn, Ts = 29. 5 Te
Distance of Saturn from the Sun = r_s
From Kepler's third law of planetary motion, we have

$$T = \left(\frac{4\pi^2 r^3}{GM}\right)^{\frac{1}{2}}$$

For Saturn and Sun, we can write

$$\frac{r_s^3}{r_e^3} = \frac{T_s^2}{T_e^2} \quad \text{or} \quad r_s = r_e\left(\frac{T_s}{T_e}\right)^{\frac{2}{3}} \qquad = 1.5\times10^{11}\left(\frac{29.5T_e}{T_e}\right)^{\frac{2}{3}} \qquad = 1.5\times10^{11}(29.5)^{\frac{2}{3}}$$

$$= 1.5\times10^{11}\times9.55$$

$$= 14.32\times10^{11}\text{m}$$

Hence, the distance between Saturn and the Sun is 14.32 × 10^{11} m

Question 8.15:

A body weighs 63 N on the surface of the earth. What is the gravitational force on it due to the earth at a height equal to half the radius of the earth?

Answer:
Weight of the body, W = 63 N
Acceleration due to gravity at height h from the Earth's surface is given by the relation:

$$g' = \frac{g}{\left(\frac{1+h}{R_e}\right)^2}$$

Where,
g = Acceleration due to gravity on the Earth's surface

Re = Radius of the Earth

$$For\ h = \frac{R_e}{2}$$

$$g' = \frac{g}{\left(1 + \frac{R_e}{2 \times R_e}\right)^2} = \frac{g}{\left(1 + \frac{1}{2}\right)^2} = \frac{4}{9} g$$

Weight of a body of mass m at height h is given as:

$$W' = mg'$$

$$= m \times \frac{4}{9} g = \frac{4}{9} \times mg \qquad = \frac{4}{9} W \qquad = \frac{4}{9} \times 63 = 28 \text{ N}$$

Question 8.16:

Assuming the earth to be a sphere of uniform mass density, how much would a body weigh half way down to the centre of the earth if it weighed 250 N on the surface?

Answer :

Weight of a body of mass m at the Earth's surface, W = mg = 250 N

Body of mass m is located at depth,

Where, R_e = Radius of the Earth,

Acceleration due to gravity at depth g (d) is given by the relation:

$$g' = \left(1 - \frac{d}{R_e}\right) g \qquad = \left(1 - \frac{R_e}{2 \times R_e}\right) g = \frac{1}{2} g$$

Weight of the body at depth d,

$$W' = mg'$$

$$= m \times \frac{1}{2} g = \frac{1}{2}\ mg = \frac{1}{2} W \qquad = \frac{1}{2} \times 250 = 125 \text{ N}$$

Question 8.17:

A rocket is fired vertically with a speed of 5 km s^{-1} from the earth's surface. How far from the earth does the rocket go before returning to the earth? Mass of the earth = 6.0 × 10^{24} kg; mean radius of the earth = 6.4 × 10^6 m; G= 6.67 × 10^{-11} N m^2 kg^{-2}.

Answer :

8 × 10^6 m from the centre of the Earth

Velocity of the rocket, v = 5 km/s = 5 × 10^3 m/s

Mass of the Earth, M_e = 6.0 × 10^{24} kg

Radius of the Earth, R_e = 6.4 × 10^6 m

Height reached by rocket mass, m = h

At the surface of the Earth,

Total energy of the rocket = Kinetic energy + Potential energy

$$= \frac{1}{2} mv^2 + \left(\frac{-GM_e m}{R_e}\right)$$

At highest point h,

$v = 0$

And, Potential energy $= -\dfrac{GM_e m}{R_e + h}$ $\quad = 0 + \left(-\dfrac{GM_e m}{R_e + h}\right) = -\dfrac{GM_e m}{R_e + h}$

Total energy of the rocket :

From the law of conservation of energy, we have

Total energy of the rocket at the Earth's surface = Total energy at height h

$$\frac{1}{2}mv^2 + \left(-\frac{GM_e m}{R_e}\right) = -\frac{GM_e m}{R_e + h}$$

$$\frac{1}{2}v^2 = GM_e\left(\frac{1}{R_e} - \frac{1}{R_e + h}\right) \qquad = GM_e\left(\frac{R_e + h - R_e}{R_e(R_e + h)}\right)$$

$$\frac{1}{2}v^2 = \frac{GM_e h}{R_e(R_e + h)} \times \frac{R_e}{R_e}$$

$$\frac{1}{2} \times v^2 = \frac{gR_e h}{R_e + h}$$

Where $g = \dfrac{GM}{R_e^2} = 9.8 \text{ m/s}^2$ (Acceleration due to gravity on the Earth's surface)

$$\therefore v^2(R_e + h) = 2gR_e h$$

$$v^2 R_e = h(2gR_e - v^2)$$

$$h = \frac{R_e v^2}{2gR_e - v^2} \qquad = \frac{6.4\times10^6 \times (5\times10^3)^2}{2\times9.8\times6.4\times10^6 - (5\times10^3)^2}$$

$$h = \frac{6.4\times25\times10^{12}}{100.44\times10^6} = 1.6\times10^6 \text{ m}$$

Height achieved by the rocket with respect to the centre of the Earth

$= R_e + h$

$= 6.4\times10^6 + 1.6\times10^6$

$= 8.0\times10^6 \text{ m}$

Question 8.18:

The escape speed of a projectile on the earth's surface is 11.2 km s^{-1}. A body is projected out with thrice this speed. What is the speed of the body far away from the earth? Ignore the presence of the sun and other planets.

Answer :

Escape velocity of a projectile from the Earth, v_{esc} = 11.2 km/s

Projection velocity of the projectile, $v_p = 3v_{esc}$

Mass of the projectile = m

Velocity of the projectile far away from the Earth = v_f

Total energy of the projectile on the Earth

$$= \frac{1}{2}mv_p^2 - \frac{1}{2}mv_{esc}^2$$

Gravitational potential energy of the projectile far away from the Earth is zero.

Total energy of the projectile far away from the Earth = $\frac{1}{2}mv_f^2$

From the law of conservation of energy, we have

$$\frac{1}{2}mv_p^2 - \frac{1}{2}mv_{esc}^2 = \frac{1}{2}mv_f^2$$

$$v_f = \sqrt{v_p^2 - v_{esc}^2}$$

$$= \sqrt{(3v_{esc})^2 - (v_{esc})^2} \qquad = \sqrt{8}\, v_{esc} \qquad = \sqrt{8} \times 11.2 = 31.68 \text{ km/s}$$

Question 8.19:

A satellite orbits the earth at a height of 400 km above the surface. How much energy must be expended to rocket the satellite out of the earth's gravitational influence? Mass of the satellite = 200 kg; mass of the earth = 6.0 ×10^{24} kg; radius of the earth = 6.4 ×10^6 m; G = 6.67 × 10^{-11} N m^2 kg^{-2}.

Answer :

Mass of the Earth, M = 6.0 × 10^{24} kg

Mass of the satellite, m = 200 kg

Radius of the Earth, Re = 6.4 × 10^6 m

Universal gravitational constant, G = 6.67 × 10–11 Nm^2kg^{-2}

Height of the satellite, h = 400 km = 4 × 10^5 m = 0.4 ×10^6 m

Total energy of the satellite at height h =

$$= \frac{1}{2}mv^2 + \left(\frac{-GM_e m}{R_e + h}\right)$$

Orbital velocity of the satellite, v = $\sqrt{\frac{GM_e}{R_e + h}}$

Total energy of height, h =

$$= \frac{1}{2}m\left(\frac{GM_e}{R_e + h}\right) - \frac{GM_e m}{R_e + h} = -\frac{1}{2}\left(\frac{GM_e m}{R_e + h}\right)$$

The negative sign indicates that the satellite is bound to the Earth. This is called bound energy of the satellite.

Energy required to send the satellite out of its orbit = – (Bound energy)

$$= \frac{1}{2}\frac{GM_e m}{(R_e + h)} \qquad = \frac{1}{2} \times \frac{6.67 \times 10^{-11} \times 6.0 \times 10^{24} \times 200}{(6.4 \times 10^6 + 0.4 \times 10^6)} \qquad = \frac{1}{2} \times \frac{6.67 \times 6 \times 2 \times 10}{6.8 \times 10^6} = 5.9 \times 10^9 \text{J}$$

Question 8.20:

Two stars each of one solar mass (= 2× 10^{30} kg) are approaching each other for a head on collision. When they are a distance 10^9 km, their speeds are negligible. What is the speed

with which they collide? The radius of each star is 10^4 km. Assume the stars to remain undistorted until they collide. (Use the known value of G).

Answer :

Mass of each star, M = 2 × 10^{30} kg

Radius of each star, R = 10^4 km = 10^7 m

Distance between the stars, r = 10^9 km = 10^{12}m

For negligible speeds, v = 0 total energy of two stars separated at distance r

$$= \frac{-GMM}{r} + \frac{1}{2}mv^2$$

$$= \frac{-GMM}{r} + 0 \qquad \text{... (i)}$$

Now, consider the case when the stars are about to collide:

Velocity of the stars = v

Distance between the centers of the stars = 2R

Total kinetic energy of both stars =

$$= \frac{1}{2}Mv^2 + \frac{1}{2}Mv^2 = Mv^2$$

Total potential energy of both stars =

$$= \frac{-GMM}{2R}$$

Total energy of the two stars =

$$Mv^2 - \frac{GMM}{2R} \qquad \text{... (ii)}$$

Using the law of conservation of energy, we can write:

$$Mv^2 - \frac{GMM}{2R} = \frac{-GMM}{r}$$

$$v^2 = \frac{-GM}{r} + \frac{GM}{2R} = GM\left(-\frac{1}{r} + \frac{1}{2R}\right)$$

$$= 6.67\times10^{-11}\times2\times10^{30}\left[-\frac{1}{10^{12}} + \frac{1}{2\times10^7}\right]$$

$$= 13.34\times10^{19}\left[-10^{-12} + 5\times10^{-8}\right]$$

$$\sim 13.34\times10^{19}\times5\times10^{-8}$$

$$\sim 6.67\times10^{12}$$

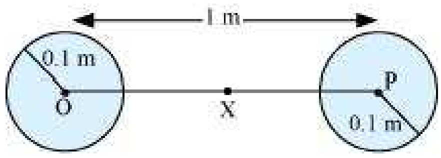
$$v = \sqrt{6.67\times10^{12}} = 2.58\times10^6 \text{m/s}$$

Question 8.21:

Two heavy spheres each of mass 100 kg and radius 0.10 m are placed 1.0 m apart on a horizontal table. What is the gravitational force and potential at the mid-point of the line joining the centers of the spheres? Is an object placed at that point in equilibrium? If so, is the equilibrium stable or unstable?

Answer :

0;
-2.7×10^{-8} J /kg;
Yes; Unstable
Explanation:
The situation is represented in the given figure:



Mass of each sphere, M = 100 kg
Separation between the spheres, r = 1m
X is the mid point between the spheres. Gravitational force at point X will be zero. This is because gravitational force exerted by each sphere will act in opposite directions. Gravitational potential at point X:

$$= \frac{-GM}{\left(\frac{r}{2}\right)} - \frac{GM}{\left(\frac{r}{2}\right)} = -4\frac{GM}{r} \qquad = \frac{4 \times 6.67 \times 10^{-11} \times 100}{1} \qquad = -2.67 \times 10^{-8} \text{ J/kg}$$

Any object placed at point X will be in equilibrium state, but the equilibrium is unstable. This is because any change in the position of the object will change the effective force in that direction.

Question 8.22:

As you have learnt in the text, a geostationary satellite orbits the earth at a height of nearly 36,000 km from the surface of the earth. What is the potential due to earth's gravity at the site of this satellite? (Take the potential energy at infinity to be zero). Mass of the earth = 6.0 × 1024 kg, radius = 6400 km.

Answer :
Mass of the Earth, M = 6.0×10^{24} kg
Radius of the Earth, R = 6400 km = 6.4×10^{6} m
Height of a geostationary satellite from the surface of the Earth, h = 36000 km = 3.6×10^{7} m
Gravitational potential energy due to Earth's gravity at height h,

$$= \frac{-GM}{(R+h)}$$

$$= -\frac{6.67 \times 10^{-11} \times 6.0 \times 10^{24}}{3.6 \times 10^{7} + 0.64 \times 10^{7}} \qquad = -\frac{6.67 \times 6}{4.24} \times 10^{13-7} \qquad = -9.4 \times 10^{6} \text{ J/kg}$$

Question 8.23:

A star 2.5 times the mass of the sun and collapsed to a size of 12 km rotates with a speed of 1.2 rev. per second. (Extremely compact stars of this kind are known as neutron stars. Certain stellar objects called pulsars belong to this category). Will an object placed on its equator remain stuck to its surface due to gravity? (Mass of the sun = 2 × 10^{30} kg).

Answer :
Yes
A body gets stuck to the surface of a star if the inward gravitational force is greater than the outward centrifugal force caused by the rotation of the star.

Gravitational force, $f_g = \frac{GMm}{R^2}$
Where,
M = Mass of the star = 2.5 × 2 × 10^{30} = 5 × 10^{30} kg
m = Mass of the body
R = Radius of the star = 12 km = 1.2 ×10^4 m

$$\therefore f_g = \frac{6.67\times10^{-11}\times5\times10^{30}\times m}{\left(1.2\times10^4\right)^2} = 2.31\times10^{11} m \text{ N}$$

Centrifugal force, $f_c = mr\omega^2$
ω = Angular speed = 2πν
ν = Angular frequency = 1.2 rev s–1
$f_c = mR\,(2\pi\nu)^2 = m \times (1.2\times10^4) \times 4 \times (3.14)^2 \times (1.2)^2 = 1.7\times10^5$ m N
Since $f_g > f_c$, the body will remain stuck to the surface of the star.

Question 8.24:
A spaceship is stationed on Mars. How much energy must be expended on the spaceship to launch it out of the solar system? Mass of the space ship = 1000 kg; mass of the Sun = 2 × 10^{30} kg; mass of mars = 6.4 × 10^{23} kg; radius of mars = 3395 km; radius of the orbit of mars = 2.28 × 10^8kg; G= 6.67 × 10^{-11} m^2kg^{-2}.

Answer :
Mass of the spaceship, m_s = 1000 kg
Mass of the Sun, M = 2 × 10^{30} kg
Mass of Mars, m_m = 6.4 × 10^{23} kg
Orbital radius of Mars, R = 2.28 × 10^8 kg =2.28 × 10^{11}m
Radius of Mars, r = 3395 km = 3.395 × 10^6 m
Universal gravitational constant, G = 6.67 × 10^{-11} m^2kg^{-2}

Potential energy of the spaceship due to the gravitational attraction of the Sun = $\frac{-GMm_s}{R}$
Potential energy of the spaceship due to the gravitational attraction of Mars = $\frac{-Gm_m m_s}{r}$
Since the spaceship is stationed on Mars, its velocity and hence, its kinetic energy will be zero.
Total energy of the spaceship = $\frac{-GMm_s}{R} + \frac{-GM_m m_s}{r}$
$= -Gm_s\left[\frac{M}{R} + \frac{m_m}{r}\right]$
The negative sign indicates that the system is in bound state.
Energy required for launching the spaceship out of the solar system = – (Total energy of the spaceship)

$$= Gm_s\left(\frac{M}{R}+\frac{m_m}{r}\right) \qquad = 6.67\times10^{-11}\times10^3\times\left(\frac{2\times10^{30}}{2.28\times10^{11}}+\frac{6.4\times10^{23}}{3.395\times10^6}\right)$$

$$= 6.67\times10^{-8}\left(87.72\times10^{17}+1.88\times10^{17}\right)$$

$$= 6.67 \times 10^{-8} \times 89.50 \times 10^{17}$$

$$= 596.97 \times 10^{9} \quad \approx \; 6 \times 10^{11} \text{ J}$$

Question 8.25:

A rocket is fired 'vertically' from the surface of mars with a speed of 2 km s^{-1}. If 20% of its initial energy is lost due to Martian atmospheric resistance, how far will the rocket go from the surface of mars before returning to it? Mass of mars = 6.4× 10^{23} kg; radius of mars = 3395 km; G = 6.67× 10^{-11} N m^2 kg^{-2}.

Answer :

Initial velocity of the rocket, v = 2 km/s = 2 × 10^3 m/s

Mass of Mars, M = 6.4 × 10^{23} kg

Radius of Mars, R = 3395 km = 3.395 × 10^6 m

Universal gravitational constant, G = 6.67× 10^{-11} N m^2 kg^{-2}

Mass of the rocket = m

Initial kinetic energy of the rocket = ½ mv^2

Initial potential energy of the rocket = - GMm/R

Total initial energy = ½ mv^2 – (GMm/R)

If 20 % of initial kinetic energy is lost due to Martian atmospheric resistance, then only 80 % of its kinetic energy helps in reaching a height.

Total initial energy available (80/20) [½ mv^2 – (GMm/R)] = 0.4 mv^2 – (GMm/R)

Maximum height reached by the rocket = h

At this height, the velocity and hence, the kinetic energy of the rocket will become zero.

Total energy of the rocket at height h = $-\frac{GMm}{(R+h)}$

Applying the law of conservation of energy for the rocket, we can write:

$$0.4mv^2 - \frac{GMm}{R} = \frac{-GMm}{(R+h)}$$

$$0.4v^2 = \frac{GM}{R} - \frac{GM}{R+h}$$

$$= GM\left(\frac{1}{R} - \frac{1}{R+h}\right) \qquad = GM\left(\frac{R+h-R}{R(R+h)}\right) \qquad = \frac{GMh}{R(R+h)}$$

$$\frac{R+h}{h} = \frac{GM}{0.4v^2 R}$$

$$\frac{R}{h} + 1 = \frac{GM}{0.4v^2 R}$$

$$\frac{R}{h}=\frac{GM}{0.4v^2R}-1$$

$$h=\frac{R}{\frac{GM}{0.4v^2R}-1}$$

$$=\frac{0.4R^2v^2}{GM-0.4v^2R} \qquad =\frac{0.4\times\left(3.395\times10^6\right)^2\times\left(2\times10^3\right)^2}{6.67\times10^{-11}\times6.4\times10^{23}-0.4\times\left(2\times10^3\right)^2\times\left(3.395\times10^6\right)}$$

$$=\frac{18.442\times10^{18}}{42.688\times10^{12}-5.432\times10^{12}}=\frac{18.442}{37.256}\times10^6 \qquad =495\times10^3\text{ m}=495\text{ km}$$

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Ex. 11.8. *Calculate the gravitational field intensity and the gravitational potential at the surface of the planet, Jupiter. The mass of Jupiter is 1.9 × 10^{27} kg and its radius is 6.9 × 10^7 m (G = 6.67 × 10^{-11} N m^2 kg^{-2}).*

Solution.

Let M be the mass and R be the radius of the planet Jupiter. The gravitational field intensity at the surface of Jupiter is given by,

$$I=\frac{GM}{R^2}=\frac{6.67\times10^{-11}\times1.9\times10^{27}}{(6.9\times10^7)^2}$$

$$=26.6\ \text{Nkg}^{-1}$$

The gravitational potential at its surface

$$V=-\frac{GM}{R}=\frac{6.67\times10^{-11}\times1.9\times10^{27}}{6.9\times10^7}$$

$$=1.84\times10^9\ \text{J/kg}$$

(i) If the Earth stops rotating about its axis i.e., if ω = 0, then the value of g will increase everywhere except at the poles. This increase will be maximum at the equator and will go on decreasing towards the poles.

(ii) If there is an increase in the angular velocity of Earth, then the value of g and hence the weight of a body will decrease at all places except at the poles.

23. Calculate the escape velocity for an atmospheric particle 1600 km above the earth's surface, given that the radius of the earth is 6400 km and acceleration due to gravity on the surface of earth is 9.8 m s^{-2}.

24. Two masses of 90 kg and 160 kg are at a distance 5 m apart. Compute the magnitude of intensity of the gravitational field at a point distance 8 m from the 90 kg and 4 m from the 160 kg mass. $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}.

25. A rocket is fired vertically with a speed of 5 km s^{-1} from the earth's surface. How far from the earth does the rocket go before returning to the earth ? Mass of the earth = 6×10^{24} kg. Mean radius of earth = 6.4×10^6 *m*, $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}. [*NCERT*]

26. What is the period of revolution of Neptune around the sun, given that the diameter of its orbit is 30 times the diameter of the earth's orbit around the sun; both orbits being assumed to be circular ?

27. What is the acceleration due to gravity at the bottom of a sea 30 km deep taking radius of the earth as 6.3×10^6 km ?

28. The weight of a body on the surface of earth is 250 N. Calculate its weight at distance equal to half the radius of earth below the surface of earth. (radius of earth 6400 km).

29. What is the gravitational potential and gravitational potential energy of a body of 0.2 kg at a height 1600 km above the surface of the earth ? ($G = 6.67 \times 10^{-11}$ kg^{-2}, mass of the earth = 6×10^{24} kg and radius of the earth = 6400 km).

30. A rocket is fired 'vertically' from the surface of mars with a speed of 2 km/s. If 20% of its initial energy is lost due to martial atmospheric resistance how far will the rocket go from the surface of mars before returning to it ? Mass of Mars = 6.4×10^{23} kg, radius of Mars = 3395 km, $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}).

31. Three mass points each of mass *m* are placed at the vertices of an equilateral triangle of side *l*. What is the gravitational field (*i.e.*, gravitational force on a unit mass) and potential due to the three masses at the centroid of the triangle ?

32. Jupiter has a mass 318 times that of the earth, and its radius is 11.2 times the earth's radius. Estimate the escape velocity of a body from Jupiter's surface given that the escape velocity from the earth's surface is 11.2 km s^{-1}.

33. Compare the period of rotation of planet Mars about the sun with that of the earth about it. The mean distance of Mars from the sun is 1.52 A.U.

34. Determine the speed with which the earth has to rotate on its axis so that a person on the equator weigh (3/5)th as much as at present. Take the equatorial radius as 6400 km.

35. A spaceship is launched into a circular orbit close to earth's surface. What additional velocity has to be imparted to the spaceship in the orbit to overcome the gravitational pull ? Radius of earth = 6400 km and g = 9.8 m s^{-2}.

36. Calculate the increase in the potential energy of an object of mass m raised from the surface of the earth to a height equal to the radius R of the earth.

37. A rocket is launched vertically from the surface of the earth with an initial speed of 10 km/s. How far above the surface of the earth would it go ? Ignore the atmospheric resistance. Radius of the earth = 6400 km.

1. The escape velocity for a body at earth's surface is 11.2 km /s. If mass of Jupiter is 318 times that of Earth and its radius is 11.2 times that of Earth, then find the escape velocity of the same object on Jupiter. [March, 2003]
(Ans: 59.7 km / s , Note: the phrase ' the same object ' is unnecessary)

2. **A satellite weighing 2000 kg is orbiting the earth at 1600 km height above the surface. Find (i) the binding energy of the satellite and (ii) its escape velocity. Mass of earth = 6 x 10^{24} kg and its radius = 6400 km. [April, 2002]**
(Ans: 5.0 x10^{10} J, 7.07 km /s, Note: This escape velocity is in addition to its orbital velocity and is in a direction perpendicular to it.)

3. If the earth were entirely made of iron with a uniform density 7.86 X 10^3 kg /m^3, what would be the value of acceleration due to gravity on its surface ? Radius of the earth = 6.37 x 10^6 metre (Take G = 6.67 x 10^{-11} Nm^2 /kg^2). [March, 2002]
(Ans: 13.98 m /s^2)

4. Find the value of 'G' so that the inertial mass (m_i) and gravitational mass (m_g) become equal on the surface of the earth. (Mass of earth = 5.98 x 10^{24} kg. Gravitational acceleration = 9.8 m /s^2. earth's radius = 6,400 km.) [October, 1997]
(Ans: 6.71 Nm^2 / kg^2)

5. A stone is projected with a velocity of 19.6 m / s and at 30 ° to the horizontal. Calculate (i) range, (ii) total time of motion and (iii) maximum height. [March, 1997]
(Ans: (i) 33.95 m, (ii) 2 s, (iii) 4.9 m)

6. Calculate the rate of change in gravitational acceleration (g) w. r. t. height from the surface of the earth.
(Radius of earth = 6400 km and g at the surface of earth = 980 cm /s^2)
(Ans: - 3.06 x 10^{-6} /s^2 near the surface of earth) [October, 1996]

7. If G = 6.67 x 10^{-11} MKS unit and radius of the earth is 6370 km, the find average density of the earth. [March, 1996]
(Ans: 5.5 x 10 3 kg /m^3)

8. **A sphere of mass 40 kg is attracted by second sphere of mass 15 kg when their centres are 2 m apart with a force equal to 10 - 3 dyne. Calculate the constant of gravitation.**
(Ans: 6.67 x 10 $^{-11}$ Nm^2 / kg^2)

9. The earth's mass is 90 times that of moon and their diameters are in the ratio 4 : 1. What is the value of g on moon ? g on earth = 9.8 m / s^2)
(Ans: 1.74 m /s^2)

10. How far from the earth does acceleration due to gravity become one percent of its value at the earth's surface ? Radius of earth = 6.38 x 10^6 m.
(Ans: 57.4 x 10^3 km)

11. What is the value of g at a height equal to the radius of the earth ? At what altitude above the earth's surface would the numerical value of g be half of that at the surface ? Radius of the earth = 6400 km.
(Ans: 2.45 m / s 2, 2650 km)

12. Moon revolves around earth 13 times in one year and the earth revolves around the sun once in a year. Compare the mass of sun and earth. The distance of earth from the sun is 390 times the distance of moon from the earth.
(Ans: 3.512 ´ 10 5)

13. Calculate the binding energy of the earth-sun system neglecting the effect of the presence of other planets and satellites. Mass of earth = 6 x 10^{24}, mass of sun = 3.3 x 10^5 times the mass of earth and the distance between earth and sun = 1.5 x 10 8 km.
(Ans: 5.28 ´ 10 33 J)

14. The mass of the moon is 7.36 x 10^{22} kg and its radius is 1.74 x 10^{6} m. What is the escape velocity from the surface of the moon ? G = 6.67 x 10^{-11} N m^2 / kg ^{2}.
(Ans: 2.38 km / s)

15. Two satellites of same mass, A and B, are orbiting the earth at altitudes R and 3R respectively, where R is the radius of the earth. Assuming the orbits to be circular, calculate the ratios of their potential and kinetic energies.
(Ans: 2 : 1, 2 : 1)

16. Prove that the escape velocity v_e for a body on a planet having density r and radius R is v_e = 2 R √ (2 p G r / 3)

17. How much work would have to be done to lift a body of mass 10 4 kg from the earth's surface to an altitude equal to the radius of the earth ?
(Ans: 3.13 x 10^{11} joule)

18. A football player kicks a football in the direction making an angle of 45° with the horizontal. The initial velocity of the ball in that direction is 50 m /s . Find (a) the horizontal displacement, (b) the maximum height attained and (c) the time of flight.
(Ans: (a) 256 m, (b) 63.8 m, (c) 7. 2 s)

19. A bomber flying upwards at an angle of 60° with the vertical releases a bomb at an altitude of 800 m. The bomb strikes the ground 20 seconds after its release. Find (a) the velocity of the bomber at the time of release of the bomb, (b) the maximum height attained by the bomb and (c) the horizontal distance traveled by the bomb before it strikes the ground. g = 10 m / s^2.
(Ans: (a) 120 m / s, (b) 980 m, (c) 2078 m)

20. What should be the initial velocity of a football kicked at an angle of 45° with horizontal to pass just touching the top of a pole of 0.8 m height kept at 1 m distance from initial position ? g = 9.8 m /s^2.
(Ans: 7 m / s)

21. A ball is thrown horizontally with a velocity of 15 m /s from the top of a tower of 25 m height. Find the time of flight of the ball and the horizontal distance from the tower to the point where the ball falls on the ground.
(Ans: 2.26 s, 33.8 m)

22. A body travels half the total distance in the last second during free fall. Find its height from the ground and the total time of free fall.
(Ans: 57 m, 2.4 s)

23) A stone is projected from the ground in a direction making an angle of 22.5° with the horizontal. It falls on the ground at a distance of 10 √2 m. Find the initial velocity of the stone, maximum height attained and the time of flight.
(Ans: 14 m / s, 1.4 m, 1.09 s)

24) Height of a tower is 39.2 m. A body is allowed to fall from the top of the tower. At the same time, another body is projected vertically upwards with velocity 19.6 m / s from its bottom. Where and when will they meet ? g = 9.8 m / s^2.
(Ans: at 19.6 m from the bottom of the tower, after 2 s)

25) A lift starts ascending from ground level with uniform acceleration of f m /s^2. A stone is dropped from it after t sec. Show that it reaches the ground after [√f + √(f + g)] t √f / g seconds.

26) A jungle native aims at a monkey, hanging on a tree, at a height of 300 m and 400 m horizontally away from him. If the shot is fired at the instant monkey lets go his hold on the tree, find the velocity of the shot when it leaves the gun, if the monkey is hit 123.6 m above the horizontal level of the gun.
(Ans: 83.3 m / s)

27) The distance between two bodies of mass M_1 and M_2 is d. Prove that the gravitational potential of the point between the two bodies at which the force of gravitational attraction is zero, is given by f = - G / d [M_1 + M_2 + 2 M_1 M_2] .

28) With what velocity should a body be projected in a vertically upward direction from the surface of the earth so that it can reach a height nR e from the surface of the earth. Re = radius of the earth.
[Ans: [2ngR e / (n + 1)] 1 / 2]

29) A body is projected from the top of a 73.5 m high hill with a velocity of 19.6 m /s in the upward direction making an angle of 30 ° with the horizontal. Find the time of flight, range and its downward velocity when it strikes the ground.
(Ans: 5 s, 84.9 m, 39.2 m /s downwards)

30) Find the magnitude of displacement of a particle projected at 100 m /s making an angle of 60° with the horizontal in the fourth second of its motion.
(Ans: 72 m)

MCQ

1) The change in the value of 'g' at a height 'h' above the surface of the earth is the same as at a depth 'd' below the surface of earth. When both 'd' and 'h' are much smaller than the radius of the earth, then which one of the following is correct ?
(a) d = 3h / 2 (b) d = h / 2 (c) d = h (d) d = 2h [AIEEE 2 05, 2003]

2) A particle of mass 10 g is kept on the surface of a uniform sphere of mass 100 kg and radius 10 cm. Find the work to be done against the gravitational force between them to take the particle far away from the surface. (G = 6.67 × 10 -11 Nm2/kg2)
(a) 3.33 × 10 - 10 J (b) 13.34 × 10 - 10 J
(c) 6.67 × 10 - 10 J (d) 6.67 × 10 - 9 J [AIEEE 2005]

3) Average density of the earth
(a) is a complex function of g (b) does not de end on g
(c) is inversely proportional to g (d) is directly p oport onal to g [AIEEE 2005]

4) A satellite of mass m revolves around the earth of radius R at a height x from its surface. If g is the acceleration due to gravity on the surface of the earth, the orbital speed of the satellite is
(a) gx (b) gR /(R - x) (c) gR 2 /(R + x) (d) [gR 2/(R + x)] 1/2 [AIEEE 2004]

5) The time-period of an earth satellite in ircu ar orbit is independent of
(a) the mass of the satellite (b) radius of its orbit
(c) both the mass and radius of th orbit
(d) neither the mass of th sat llite nor the radius of its orbit [AIEEE 2004]

6) If g is the acceleration d to gravity on the earth's surface, the gain in the potential energy of an object of mass m raised from the surface of the earth to a height equal to the radius R of the earth is
(a) 2 mgR (b (1 2) mgR (c) (1 /4) mgR (d) mgR [AIEEE 2004, IIT 1983]

7) Suppose the gra itational force varies inversely as the nth power of the distance. Then the time period of a planet in circular orbit of radius R around the sun will be proportional to
( ) R (n +) / 2 (b) R (n - 1) / 2 (c) R n (d) R (n - 2) / 2 [AIEEE 2004]

8) The time-period of a satellite of earth is 5 hours. If the separation between the earth and the satellite is increased to 4 times the previous value, the new time-period will become
(a) 10 hours (b) 20 hours (c) 40 hours (d) 80 hours [AIEEE 2003]

9) The escape velocity for a body projected vertically upwards from the surface of the earth is 11 km/s. If the body is projected at an angle of 45° with the vertical, the escape velocity will be
(a) 11 / √2 km/s (b) 11√2 km/s (c) 2 km/s (d) 11 km/s [AIEEE 2003]

10) A body weighs 500 N on the surface of the earth. How much would it weigh half way below the surface of the earth ?
(a) 1000 N (b) 500 N (c) 250 N (d) 125 N [AIEEE 2002]

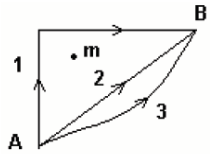
11) The time-period of revolution of planet A around the sun is 8 times that of B. The distance of A from the sun is how many times greater than that of B from the sun ?
(a) 2 (b) 3 (c) 4 (d) 5 [AIEEE 2002]

12) The angular velocity of rotation of a star (of mass M and radius R) at which the matter will start escaping from its equator is
(a) √ (2GR /M) (b) √ (2GM /R 3) (c) √ (2GM /R) (d) √ (2GM 2 /R) [AIEEE 2002]

13) Energy required to move a body of mass m from an orbit of radius 2R o 3R is
(a) GMm /(12R 2) (b) GMm /(3R 2) (c) GMm /(8R) (d) GMm /(6R) [AIEEE 2002]

14) An infinite number of identical point masses each equal to m are placed at points x = 1, x = 2, x = 4, x = 8m, … … The total gravitational potential at point at x = 0 is
(a) - Gm (b) - 2Gm (c) + 2Gm (d) infinite [AIEEE 2002]

15) If W1, W2 and W3 represent the work done in moving a particle from A to B along three different paths 1, 2 and 3 respectively in the gravitational field of a po t mass m as shown in the figure, find the correct relation between W1, W2 and W3.
(a) W1 > W2 > W3 (b) W1 = W2 = W3
(c) W1 < W2 < W3 (d) W2 > W1 > W3 [IIT 2003]



16) A geostationary satellite orbits around the earth in a circular orbit of radius 36000 km. Then, the time period of a spy satellite orbiting a few hundred km above the earth's surface (Rearth = 6400 m) ill approximately be
(a) 1 / 2 hr (b) 1 r (c) 2 hr (d) 4 hr [IIT 2002]

17) A satellite S is moving in an elliptical orbit around the earth. The mass of the satellite is very small compared to the mass of the earth.
(a) The acceleration of S is always directed towards the centre of the earth.
(b) The angular momentum of S about the centre of the earth changes in direction, but its magnitude remains constant.
(c) Th total mechanical energy of S varies periodically with time.
(d) The linear momentum of S remains constant in magnitude. [IIT 1998]

18) If the distance between the earth and the sun were half its present value, the number of day in a year would have been
(a) 64.5 (b) 129 (c) 182.5 (d) 730 [IIT 1996]

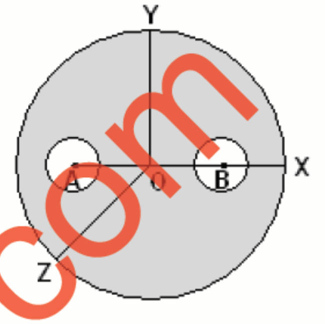
19) The magnitudes of the gravitational field at distances r1 and r2 from the centre of uniform sphere of radius R and mass M are F1 and F2 respectively. Then

(a) $\frac{F_1}{F_2} = \frac{r_1}{r_2}$ if $r_1 < R$ and $r_2 < R$ (b) $\frac{F_1}{F_2} = \frac{r_2^2}{r_1^2}$ if $r_1 > R$ and $r_2 > R$

(c) $\frac{F_1}{F_2} = \frac{r_1}{r_2}$ if $r_1 > R$ and $r_2 > R$ (d) $\frac{F_1}{F_2} = \frac{r_1^2}{r_2^2}$ if $r_1 < R$ and $r_2 < R$

[IIT 1994]

20) A solid sphere of uniform density and radius 4 units is located with its centre at the origin O of coordinates. Two spheres of equal radii 1 unit with their centres at A (- 2, 0, 0) and B (2, 0, 0) respectively, are taken out of the solid leaving behind spherical cavities as shown in the figure.



(a) the gravitational force due to this object at the origin is zero
(b) the gravitational force at the point B (2, 0, 0) is zero
(c) the gravitational potential is the same at all points of the circle $y^2 + z^2 = 36$
(d) the gravitational potential is the same at all points on the circle $y^2 + z^2 = 4$
[IIT 1993]

21) Imagine a light planet revolving around a very massive star in a circular orbit of radius R with a period of revolution T. If the gravitational force of attraction between the planet and the star is proportional to $R^{-5/2}$, then
(a) T^2 is proportional to R^2 (b) T^2 is proportional to $R^{7/2}$
(c) T^2 is proportional to $R^{3/2}$ (d) T^2 is proportional to $R^{3.75}$ [IIT 1989]

22) v_e and v_p denote the escape velocities from the earth and another planet having twice the radius and the same mean density as the earth, then
(a) $v_e = v_p$ (b) $v_e = 0.5\, v_p$ (c) $v_e = 2\, v_p$ (d) $v_e = 0.25\, v_p$ [NCERT 1974]

23) The ratio of the kinetic energy required to be given to the satellite to escape earth's gravitational field to the kinetic energy required to be given so that the satellite moves in a circular orbit just above earth's atmosphere is
(a) one (b) two (c) half (d) infinity [NCERT 1975]

24) g_e and g_p denote the acceleration due to gravity on the surface of the earth and another planet whose mass and radius are twice that of the earth, then
(a) $g_p = g_e$ (b) $g_p = 0.5 g_e$ (c) $g_p = 2g_e$ (d) $g_p = g_e / \sqrt{2}$ [NCERT 1973]

25) The weight of an object in the coal mine, sea level and at the top of the mountain are respectively W_1, W_2 and W_3, then
(a) $W_1 < W_2 > W_3$ (b) $W_1 = W_2 = W_3$
(c) $W_1 < W_2 < W_3$ (d) $W_1 > W_2 > W_3$ [EAMCET 1990]

26) With what angular velocity the earth should spin in order that a body lying at 60° latitude may become weightless
(a) $\sqrt{(g/R)}$ (b) $\sqrt{(2g/R)}$ (c) $2\sqrt{(g/R)}$ (d) $\sqrt{(g/2R)}$

27) A body is projected vertically from the surface of the earth of radius R with velocity equal to half of the escape velocity. The maximum height reached by the body is
(a) R (b) R / 2 (c) R / 3 (d) R / 4

28) The escape velocity from the earth's surface is 11 km/s. A planet has a radius twice that of the earth but its mean density is the same as that of the earth. The value of the escape velocity from this planet would be
(a) 22 km/s (b) 11 km/s (c) 5.5 km/s (d) 16.5 km/s [CPMT 1990]

29) If R is the radius of the earth and g the acceleration due to gravity on the earth's surface, the mean density of the earth is
(a) $(4\pi G)/(3gR)$ (b) $(3\pi R)/(4gG)$ (c) $(3g)/(4\pi RG)$ (d) $(\pi Rg)/(12G)$
[CPMT 1990]

30) The radius of the earth is 6400 km and g = 10 m/s^2. In order that a body of 5 kg weighs zero at the equator, the angular speed of the earth is
(a) 1 / 80 rad / s (b) 1 / 400 rad / s
(c) 1 / 800 rad / s (d) 1 / 1600 rad / s [MP, PMT 1985]

31) If the earth were at one-fourth its present distance from the un, the duration of the

year will be (a) half the present year (b) one-eighth the present year
(c) one-fourth the present year (d) one-sixth the present year

32) If two stars of masses m_1 and m_2 separated by a distance d rotate about their common centre of mass, then their common angular velocity is
(a) $\sqrt{(G m_1 m_2 / d^2)}$ (b) $\sqrt{(G m_1 m_2 / d)}$
(c) $\sqrt{[G(m_1 + m_2)/d^3]}$ (d) $\sqrt{(G m_1 m_2)}$

33) If the radius of the earth were to decrease by 1 %, its mass remaining the same, the acceleration due to gravity on the surface of the earth will
(a) increase by 1 % (b) decrease by 2 %
(c) decrease by 1 % (d) increase by 2 %

34) Three point masses each of mass m rotate in a circle of radius r with constant angular velocity ω due to their mutual gravitational attraction. If at any instant, the masses are on the vertices of an equilateral triangle of side a, then the value of ω is
(a) $\sqrt{(GM/a^3)}$ (b) $\sqrt{(3GM/a^3)}$ (c) $\sqrt{(GM/3a^3)}$ (d) none

35) Two bodies each of mass 66.7 kg are at a distance of 2 m. The escape velocity of a body midway between them is
(a) 13.34 m / s (b) 6.67 m / s (c) 33.35 m / s (d) zero

36) If a body of mass m is taken out from a point below the surface of earth equal to half the radius of earth, R, to a height R above the earth's surface, then work done on it will be (a) (5 / 6) mgR (b) (6 / 7) mgR (c) (7 / 8) mgR (d) (8 / 9) mgR

37) A satellite is revolving around the earth in a circular orbit with a velocity of 7.07 km / s. What minimum increase in its velocity is needed to make it escape gravitational pull of earth ?
(a) 4.23 km/s in the direction of its velocity
(b) 11.3 km/s in a direction perpendicular to its velocity
(c) 2.93 km/s in the direction of its velocity
(d) 4.23 km/s in a direction perpendicular to its velocity

38) The escape velocity of a body from the surface of the earth is v. It is given a velocity twice this velocity on the surface of the earth. What will be its velocity at infinity ?
(a) v (b) 2 v (c) $\sqrt{2}$ v (d) $\sqrt{3}$ v

39) A satellite is moving on a circular path of radius r around the earth with time-period T. If its radius slightly increases by Δr, the change in its time-period is
(a) (3T/2r) Δr (b) (T / r) Δr (c) $(T/r)^2 \Delta r$ (d) none of these

40) A satellite is orbiting a planet at a constant height in a circular orbit. If he mass of the planet is reduced to half, the satellite would
(a) fall on the planet (b) go to an orbit of higher radius
(c) escape from the planet (d) go to an orbit of smaller ra ius

41) A satellite of mass m is revolving in a circular orbit around the earth of mass M. If E is its total mechanical energy, then its angular momentum is
(a) $\sqrt{(E/mr^2)}$ (b) $E/(2mr^2)$ (c) $(2Emr^2)^{1/2}$ (d) $\sqrt{(2Emr)}$

42) A body of mass m is projected from the surface of the earth with a speed v (v < escape velocity). Its speed at a height equal to radius R of earth is
(a) $\sqrt{(gR)}$ (b) $\sqrt{(v^2 - 2gR)}$ (c) $\sqrt{(v^2 - gR)}$ (d) none of these

43) A body of mass m rises to height h = R / 5 from the earth's surface, where R is earth's radius. If g is acceleration due to gravity at earth's surface, the increase in potential energy of the body is
(a) mgh (b) (4 / 5) mgh (c) (5 / 6) mgh (d) (6 / 7) mgh

44) Two particles of equal mass m go round a circle of radius R under the action of their mutual gravitational attraction. The speed of each particle is
(a) $\sqrt{(Gm/2R)}$ (b) $\sqrt{(4Gm/R)}$ (c) $(1/2R)\sqrt{(1/Gm)}$ (d) $1/2\sqrt{(Gm/R)}$

45) A body weighs W in a train at rest at equator. If it runs from west to east around the equator with velocity v and earth's angular velocity is ω, then its weight will be
(a) W (b) $W(1+2v\omega/g)$ (c) $W(1-2v\omega/g)$ (d) $W(1+v^2/R)$

Answers

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
d	c	b	d	a	b	a	c	d	c	c	b	d	b	d	b	a	b	a,b	a,c,d	b

22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43
b	b	b	a	c	c	a	c	c	b	c	d	b	a	c	c	d	a	d	c	c	c

44	45															
d	c															

CH 9
Mechanical properties of solids
(5 Hours, 4 Marks (1M-1Q, 3M-1Q)

Syllabus :
Elastic behaviour, Stress-strain relationship, Hooke's law, Young's modulus, bulk modulus, shear, modulus of rigidity, poisson's ratio; elastic energy.

9.1 Introduction
A solid has definite shape and size. In order to change (or deform) the shape or size of a body, a force is required.
The property of a body, by virtue of which it tends to regain its original size and shape when the applied force is removed, is known as **elasticity** and the deformation caused is known as **elastic** deformation.
Why is glass brittle while brass is not? Answers to such questions begin with the study of how relatively simple kinds of loads or forces act to deform different solids bodies.

9.2 Elastic behaviour of solids
The elastic behaviour of solids can be explained in terms of microscopic nature of the solid. Robert Hooke, an English physicist (1635 - 1703 A.D) performed experiments on springs and found that the elongation (change in the length) produced in a body is proportional to the applied force or load. In 1676, he presented his law of elasticity, now called Hooke's law.

When a force is applied on body, it is deformed to a small or large extent depending upon the nature of the material of the body and the magnitude of the deforming force. The deformation may not be noticeable visually in many materials but it is there. When a body is subjected to a deforming force, a restoring force is developed in the body. This restoring force is equal in magnitude but opposite in direction to the applied force. The restoring force per unit area is known as **stress.** If F is the force applied and A is the area of cross section of the body,
Magnitude of the stress = F/A ----------- (9.1)
The SI unit of stress is N m^{-2} or pascal (Pa) and its dimensional formula is [$ML^{-1}T^{-2}$].

There are three ways in which a solid may change its dimensions when an external force acts on it. These are shown in Fig. 9.2.
In Fig.9.2(a), a cylinder is stretched by two equal forces applied normal to its cross-sectional area. The restoring force per unit area in this case is called **tensile stress**. If the cylinder is compressed under the action of applied forces, the restoring force per unit area is known as **compressive stress**. Tensile or compressive stress can also be termed as longitudinal stress.
In both the cases, there is a change in the length of the cylinder. The change in the length ΔL to the original length L of the body (cylinder in this case) is known as **longitudinal strain**.
Longitudinal strain $\Delta L/L$ ------- (9.2)

However, if two equal and opposite deforming forces are applied parallel to the cross-sectional area of the cylinder, as shown in Fig. 9.2(b), there is relative displacement between the opposite faces of the cylinder. The restoring force per unit area developed due to the applied tangential force is known as **tangential** or **shearing stress**.

As a result of applied tangential force, there is a relative displacement Δx between opposite faces of the cylinder as shown in the Fig. 9.2(b).
The strain so produced is known as **shearing strain** and it is defined as the ratio of relative displacement of the faces Δx to the length of the cylinder L.
Shearing strain $x/L = \tan\theta$ -------- (9.3)
where θ is the angular displacement of the cylinder from the vertical (original position of the cylinder). Usually θ is very small, tan θ is nearly equal to angle θ, (if θ = 10°, for example, there is only 1% difference between θ and tan θ).

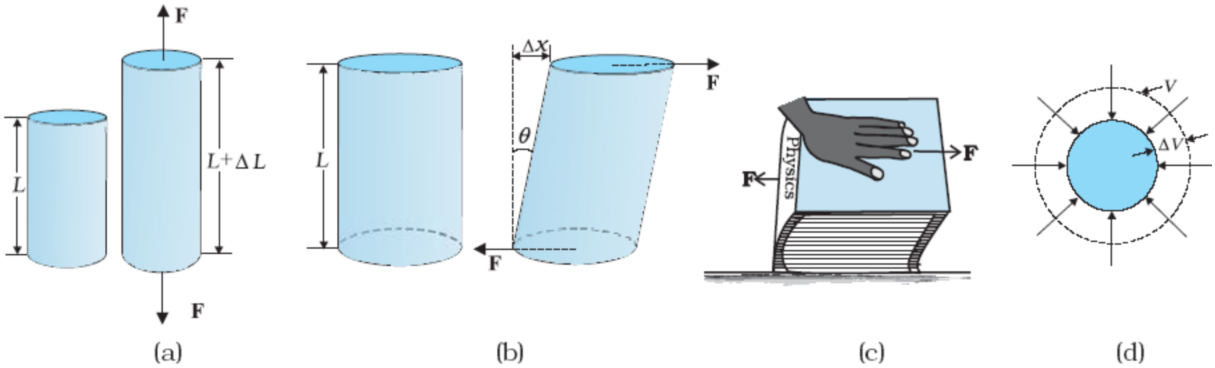


Fig. 9.2 *(a) Cylinder subjected to tensile stress stretches it by an amount ΔL. (b) A cylinder subjected to shearing (tangential) stress deforms by an angle θ .(c) A book subjected to a shearing stress (d) A solid sphere subjected to a uniform hydraulic stress shrinks in volume by an amount ΔV.*

It can also be visualized, when a book is pressed with the hand and pushed horizontally, as shown in Fig. 9.2 (c).
Thus, shearing strain = $\tan\theta \approx \theta$ (9.4)
In Fig. 9.2 (d), a solid sphere placed in the fluid under high pressure is compressed uniformly on all sides. The force applied by the fluid acts in perpendicular direction at each point of the surface and the body is said to be under hydraulic compression. This leads to decrease in its volume without any change of its geometrical shape.
The body develops internal restoring forces that are equal and opposite to the forces applied by the fluid (the body restores its original shape and size when taken out from the fluid). The internal restoring force per unit area in this case is known as **hydraulic stress** and in magnitude is equal to the hydraulic pressure (applied force per unit area).
The strain produced by a hydraulic pressure is called **volume strain** and is defined as the ratio of change in volume (ΔV) to the original volume (V).
Volume strain $\Delta V / V$ --------- (9.5)
Since the strain is a ratio of change in dimension to the original dimension, it has no units or dimensional formula.

9.3 Stress and strain Relationship & Hooke's law

For small deformations the **stress and strain are proportional to each other**. This is known as Hooke's law.

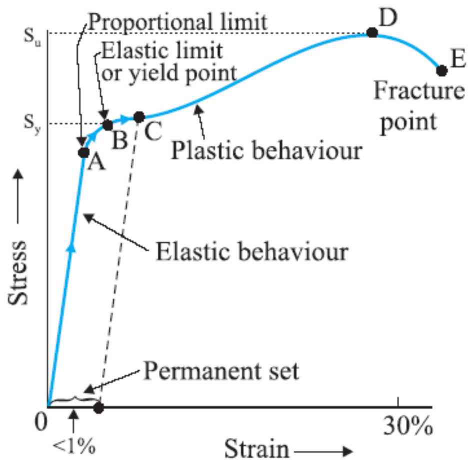
Thus, stress ∝ strain stress = k . strain -------- (9.6)

where k is the proportionality constant and is known as modulus of elasticity.

Hooke's law is an empirical law and is found to be valid for most materials. However, there are some materials which do not exhibits this linear relationship.

9.5 Stress-strain curve

The relation between the stress and the strain for a given material under tensile stress can be found experimentally. In a standard test of tensile properties, a test cylinder or a wire is stretched by an applied force. The fractional change in length (the strain) and the applied force needed to cause the strain are recorded.



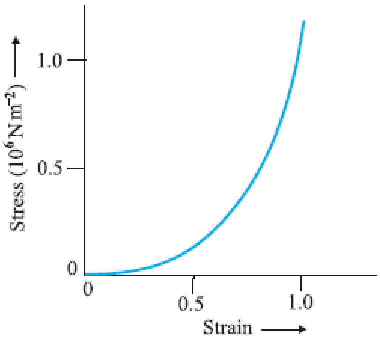


***Fig. 9.3** A typical stress-strain curve for a metal.* *(b) Stress-strain curve for the elastic tissue of Aorta*

The applied force is gradually increased in steps and the change in length is noted. A graph is plotted between the stress (which is equal in magnitude to the applied force per unit area) and the strain produced. A typical graph for a metal is shown in Fig. 9.3. Analogous graphs for compression and shear stress may also be obtained. The stress-strain curves vary from material to material. These curves help us to understand how a given material deforms with increasing loads. From the graph, we can see that in the region between O to A, the curve is linear. In this region, Hooke's law is obeyed.

The body regains its original dimensions when the applied force is removed. In this region, the solid behaves as an elastic body.

In the region from A to B, stress and strain are not proportional. Nevertheless, the body still returns to its original dimension when the load is removed. The point B in the curve is known as **yield point** (also known as **elastic limit**) and the corresponding stress is known as **yield strength** (S_Y) of the material.

If the load is increased further, the stress developed exceeds the yield strength and strain increases rapidly even for a small change in the stress. The portion of the curve between B and D shows this. When the load is removed, say at some point C between B and D, the body does not regain its original dimension. In this case, even when the stress is zero, the strain is not zero. The

material is said to have a **permanent set**. The deformation is said to be **plastic deformation**. The point D on the graph is the ultimate **tensile strength** (*Su*) of the material.
Beyond this point, additional strain is produced even by a reduced applied force and fracture occurs at point E. If the ultimate strength and fracture points D and E are close, the material is said to be brittle. If they are far apart, the material is said to be ductile.

As stated earlier, the stress-strain behaviour varies from material to material. For example, rubber can be pulled to several times its original length and still returns to its original shape. Fig. 9.4 shows stress-strain curve for the elastic tissue of aorta, present in the heart. Note that although elastic region is very large, the material does not obey Hooke's law over most of the region. Secondly, there is no well defined plastic region. Substances like tissue of aorta, rubber etc. which can be stretched to cause large strains are called **elastomers**.

9.6 Elastic moduli : Young's modulus, bulk modulus, shear, modulus of rigidity, poisson's ratio; elastic energy :

The proportional region within the elastic limit of the stress-strain curve (region OA in Fig. 9.3) is of great importance for structural and manufacturing engineering designs. The ratio of stress and strain, called **modulus of elasticity**, is found to be a characteristic of the material.

1. Young's Modulus

Experimental observation show that for a given material, the magnitude of the strain produced is same whether the stress is tensile or compressive. The ratio of tensile (or compressive) stress (σ) to the longitudinal strain (ε) is defined as **Young's modulus** and is denoted by the symbol *Y*.

$Y = \sigma / \varepsilon$ --------- (9.7)

From Eqs. (9.1) and (9.2), we have

$Y = (F/A) / (\Delta L/L)$

$= (F \,.\, L) / (A \,.\, \Delta L)$ --------- (9.8)

Since strain is a dimensionless quantity, the unit of Young's modulus is the same as that of stress *i.e.*, N m^{-2} or Pascal (Pa). Table 9.1 gives the values of Young's moduli and yield strengths of some materials.

For metals Young's moduli are large. Therefore, these materials require a large force to produce small change in length. To increase the length of a thin steel wire of 0.1 cm^2 cross-sectional area by 0.1%, a force of 2000 N is required. The force required to produce the same strain in aluminium, brass and copper wires having the same cross-sectional area are 690 N, 900 N and 1100 N respectively. It means that steel is more elastic than copper, brass and aluminium. It is for this reason that steel is preferred in heavy-duty machines and in structural designs. Wood, bone, concrete and glass have rather small Young's moduli.

2. Shear Modulus

The ratio of shearing stress to the corresponding shearing strain is called the ***shear modulus*** of the material and is represented by *G*. It is also called the ***modulus of rigidity***.

G = shearing stress (σ_s)/shearing strain

$G = (F/A)/(\Delta x/L)$

$= (F \,.\, L)/(A \,.\, \Delta x)$ ---------- (9.10)

Similarly, from Eq. (9.4)
$G = (F/A)/\ \theta$
$= F/(A\ .\ \theta)$ -------- (9.11)
The shearing stress σ_s can also be expressed as $\sigma_s = G\ .\ \theta$ -------- (9.12)
SI unit of shear modulus is N m^{-2} or Pa. The shear moduli of a few common materials are given in Table 9.2. It can be seen that shear modulus (or modulus of rigidity) is generally less than Young's modulus (from Table 9.1). For most materials $G \approx Y/3$.

3. Bulk Modulus
We have seen that when a body is submerged in a fluid, it undergoes a hydraulic stress (equal in magnitude to the hydraulic pressure). This leads to the decrease in the volume of the body thus producing a strain called volume strain [Eq. (9.5)].
The ratio of hydraulic stress to the corresponding hydraulic strain is called ***bulk modulus***. It is denoted by symbol B.

$B = -\ p/(\Delta V/V)$ ------------- (9.13)

The negative sign indicates the fact that with an increase in pressure, a decrease in volume occurs. That is, if p is positive, ΔV is negative. Thus for a system in equilibrium, the value of bulk modulus B is always positive. SI unit of bulk modulus is the same as that of pressure *i.e.*, N m–2 or Pa. The bulk moduli of a few common materials are given in Table 9.3.
The reciprocal of the bulk modulus is called *compressibility* and is denoted by k. It is defined as the fractional change in volume per unit increase in pressure.

$k = (1/B) = -\ (1/\Delta p)\ .\ (\Delta V/V)$ -------- (9.14)

It can be seen from the data given in Table 9.3 that the bulk moduli for solids are much larger than for liquids, which are again much larger than the bulk modulus for gases (air).

4. Poisson's ratio :
In general, a deforming force in one direction can produce strains in other directions also. The proportionality between stress and strain in such situations cannot be described by just one elastic constant. For example, for a wire under longitudinal strain, the lateral dimensions (radius of cross section) will undergo a small change, which is described by another elastic constant of the material called ***Poisson ratio.***

5. Determination of Young's Modulus of the Material of a Wire :
A typical experimental arrangement to determine the Young's modulus of a material of wire under tension is shown in Fig. 9.6. It consists of two long straight wires of same length and equal radius suspended side by side from a fixed rigid support. The wire A (called the *reference wire*) carries a millimetre main scale M and a pan to place a weight. The wire B (called the *experimental wire*) of uniform area of cross section also carries a pan in which known weights can be placed. A vernier scale V is attached to a pointer at the bottom of the experimental wire B, and the main scale M is fixed to the reference wire A. The weights placed in the pan exert a downward force and stretch the experimental wire under a tensile stress. The elongation of the wire (increase in length) is measured by the vernier arrangement. The reference wire is used to

compensate for any change in length that may occur due to change in room temperature, since any change in length of the reference wire due to temperature change will be accompanied by an equal change in experimental wire.

Both the reference and experimental wires are given an initial small load to keep the wires straight and the vernier reading is noted. Now the experimental wire is gradually loaded with more weights to bring it under a tensile stress and the vernier reading is noted again. The difference between two vernier readings gives the elongation produced in the wire. Let *r* and *L* be the initial radius and length of the experimental wire, respectively. Then the area of cross-section of the wire would be $\pi r2$. Let *M* be the mass that produced an elongation ΔL in the wire. Thus the applied force is equal to *Mg*, where *g* is the acceleration due to gravity. From Eq. (9.8), the Young's modulus of the material of the experimental wire is given by

$$Y = \frac{\sigma}{\varepsilon} = \frac{M_g}{\pi r^2} \cdot \frac{L}{\Delta L}$$

$$= M_g \; X \; L/(\pi r^2 \; X \; \Delta L)$$

Table 9.4 : Stress, strain and various elastic moduli :

Type of stress	Stress	Strain	Change in Shape	Change in Volume	Elastic modulus	Name of modulus	State of Mater
Tensile or Compressive	Two equal and opposite forces perpendicular to opposite faces $\sigma = \frac{F}{A}$	Elongation or compression parallel to force direction $(\Delta L/L)$ (longitudinal strain	Yes	No	Y = $(FxL)/$ (AxΔL)	Young's modulus	Solid
Shearing	Two equal and opposite forces parallel to opposite surfaces (forces in each case such that total force and total torque on the body vanishes $\sigma_s = F/A$	Pure shear. θ	Yes	No	G= $\frac{Fx\theta}{A}$	Shear modulus	Solid
Hydraulic	Forces perpendicular everywhere to the surface, force per unit area (pressure) same everywhere	Volume change (compression or elongation $(\Delta V/V)$	No	Yes	B = $-\frac{p}{\frac{\Delta V}{V}}$	Bulk modulus	Solid, liquid and gas

9.7 Applications of elastic behaviour of materials

ONE MARK QUESTIONS:

1. What is elasticity of a body?
The property of a body due to which it tends to regain its original size and shape when the deforming force is removed is called elasticity.

2. What is plasticity?
The property of a body due to which it does not regain its original size and shape when the deforming force is removed is called plasticity.

3. Which property of a body is responsible for regaining original shape and size of a body when deforming force acting on it is removed?
Elasticity

4. Elasticity is an internal property of a matter. Is it true or false?
True

5. What is the cause of the elasticity?
It is due to intermolecular forces between the molecules of the material

6. What is plastic substance?
Substance which does not regain its original size and shape when the deforming force is removed is called plastic substance.

7. Give one example for plastic substance.
Putty (mud)

8. Define stress.
The restoring force per unit area of the body is called stress

9. Write the expression for magnitude of the stress.
Magnitude of stress = F/A

10. Write the S.I unit of the stress.
Nm^{-2} or Pa

11. Write the dimensional formula of the stress.
[Stress] = $[L^{-1}MT^{-2}]$

12. Dimensional formula of stress is same as that of the pressure. Is it true or false?
True

13. Stress is a vector quantity. Is it true or false?
False

14. Define strain.

Strain produced in a body is defined as ratio of change in dimension to the original dimension.

15. Why strain is unit less and dimensionless physical quantity?
Because strain is ratio of same physical quantities

16. Define longitudinal strain.
Longitudinal strain is defined as ratio of change in the length to the original length.

17. Write the expression for longitudinal strain.
Longitudinal strain = $\Delta L/L$ where ΔL - change in the length, L - original length

18. Define volume strain.
Volume strain is defined as ratio of change in the volume to the original volume

19. Write the expression for volume strain.
Volume strain = $\Delta L /L$ where ΔL -change in the volume, V-original volume

20. Define shearing strain.
It is the angle through which a vertical face of a body displaced when tangential deforming force applied on it.

21. Define elastic limit. (yield strength.)
The maximum stress below which Hooke's law is applicable is called elastic limit OR it is the maximum stress up to which body regain its original shape and size.

22. Define ultimate tensile strength.
The minimum stress needed to cause the fracture of the material is known as ultimate tensile strength

23. What are elastomers ?
Material which can be stretched to a large value of strain without breaking is called elastomers

24. Elastomers does not obey Hooke`s law. Is it true or false?
True

25. Give one example for elastomers.
Rubber

26. Define modulus of elasticity.
Modulus of elasticity is defined as the ratio of stress acting on the body to the resulting strain in it.

27. Modulus of elasticity of a body is dependent on the dimensions of a body. Is it true or false?
False

28. Dimensional formula of modulus of elasticity is same as that of the stress. Is it true or false?

True

29. Define Young`s modulus.
Young's modulus is defined as ratio of longitudinal stress acting on the body to the longitudinal strain produced in it.

30. Write the expression for magnitude of the Young`s modulus.
Young's modulus= Y = FL / A ΔL

31. What is the S.I unit of the Young`s modulus?
Nm^{-2} or Pa

32. Write the dimensional formula of the Young`s modulus.
[Young's modulus] = $[L^{-1}MT^{-2}]$

33. Why steel is preferred in heavy duty machines and in structural design?
Because Young's modulus of steel is highest and is more elastic

34. Why springs are manufactured in steel instead of copper?
Because Young's modulus of steel is more than that of copper and hence steel is more elastic than copper.

35. Define shear modulus or modulus of rigidity.
The ratio of shearing stress acting on the body to the corresponding shearing strain is called rigidity modulus

36. Define bulk modulus.
The ratio of hydraulic stress acting on the body to the corresponding volume strain is called bulk modulus

37. What is compressibility?
The reciprocal of the bulk modulus is called compressibility

38. Solid are least are compressible since they have larger value of bulk modulus. Is it true or false?
True

39. Young`s modulus and shear modulus are relevant only for solid. Why?
Because only solid has length and definite shape

40. Young`s modulus of rubber is greater than that of steel. Is it true or false?
False
41. Rubber is more elastic than that of steel. Is it true or false?
False

42. What is the buckling of the material of the road?

The bending of beam under a load is called buckling.

43. Stretching of coil is measured by its shearing modulus. Is it true or false?
True

44. Why load baring bar has cross sectional shape of the type I.
This shape reduces the weight and cost of the beam and is much stronger.

45. Why pillars or columns of the bridges and buildings have distributed shape at their ends?
Pillars having distributed shape at the ends support more load than pillars with rounded ends.

46. What is the value of young`s modulus for a perfectly rigid body?
Infinity.

47. Why liquid and gas do not posses modulus of rigidity?
Liquid and gas has no definite shape.

48. Write the S.I for the compressibility.
$N^{-1}m^2$ OR Pa^{-1}

49. How does modulus of elasticity vary with increase of temperature of the body?
Modulus of elasticity decrease with increases of temperature.

50. The material for given load stretches to a little extent is a more elastic body. Is it true or false
True

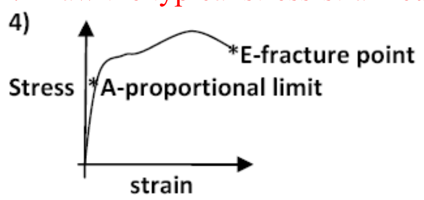
TWO MARK QUESTIONS:

1. State and explain Hooke's law.
Within elastic limit stress and strain are proportional to each other.
Stress $\propto$ strain OR Stress/strain = constant (modulus of elasticity).

2. Mention any two types of stress.
a) longitudinal stress b) shearing stress

3. Mention any two types of strain.
a) longitudinal strain b) shearing strain [c) bulk strain]

4. Draw the typical stress-strain curve for a metal.



5. Which is more elastic-steel or rubber? Why?
Steel.

Because Young`s modulus of steel is more than that of rubber. Hence steel is more elastic than rubber.

6. Explain elastic behavior of the solid.

When solid is deformed atoms or molecule are displaced from their equilibrium position resulting change in the interatomic distance.

Due to inter molecular attraction body regain its original shape and size when deforming force is removed.

7. Write the expression for Young`s modulus of the material of the wire under stretching. Explain the terms.

$$Y = \frac{MgL}{\pi r^2 \Delta L}$$

Where M -- mass attached to the wire. L , r—initial length and radius of the wire. ΔL- elongation in the wire.

8. Write the expression for rigidity modulus of the material. Explain the terms.

$$G = \frac{F}{A\theta}$$

Where F — tangential force. A — area of the face. θ - Shearing strain.

9. Write the expression for bulk modulus of the material. Explain the terms.

$$B = \frac{-PV}{\Delta V}$$

Where P - hydraulic stress, V- initial volume and ΔV- change in the volume

10. A square lead slab of side 50cm and thickness 10 cm subjected to shearing force of 9 X 10^4N.Calculate the shearing stress acting on the slab.

Area of the face on which force is acting = A = 50 cm x 10 cm = 0.5m x 0.1m = 0.05 m^2

$$\text{Shearing stress} = \frac{Tangential\ Force}{Area\ of\ the\ face}$$

$$\text{Stress} = \frac{9.4\,10^4}{0.05} = 1.8 \times 10^6\ Nm^{-2}$$

11. Write the expression for sag or depression of a bar when it is loaded at the middle. Explain the terms.

$$\delta = \frac{W L^3}{4 b d^3 y}$$

Where W - load, l - length of the span, b - breadth of the beam, d - depth (thickness) of the beam. Y - Young’s modulus

12. Write the expression for compressibility and explain the terms

$$K = 1/B = \frac{-\Delta V}{PV}$$

Where P- hydraulic stress, V- initial volume and ΔV- change in the volume

13. Distinguish between ductile material and brittle material.

Ductile material	Brittle material
1) The material showing large amount of plastic deformation b/n the elastic limit and the fracture point called ductile material	The material showing small amount of plastic deformation b/n the elastic limit and The fracture point is called brittle material 1 mark
2) They have permanent stretch without breaking	They fractured soon after the elastic limit is crossed 1 mark

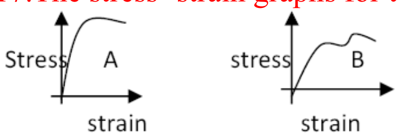
14. Write two application of elastic behavior of the material.
a)To estimate the maximum height of a mountain. b) In minimizing of the bending of loaded beam (c) In selecting metallic rope for crane) [any two]

15. Mention two methods of decreasing the depression in the beam which loaded at the middle with weight.
a) increasing the depth (thickness of the beam) b) increasing its breadth

16. A steel rod of area of cross section 3.14 x 10^{-4}m^2 is stretched by a force of 100 kN. Calculate the stress acting on the rod

longitudinal stress = Force / Area = F/A $= \dfrac{100 \times 10^3}{3.14 \times 10^{-4}} = 3.18 \times 10^8\ \mathrm{Nm^{-2}}$

17.The stress- strain graphs for two material A and B are shown in the figure

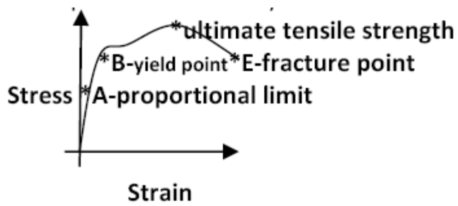


Which of the material has the greater Young's modulus? And which material is more elastic?
Material A has larger Young's modulus, (since it has larger slope)
Material A is more elastic (since it has large Young's modulus)

18.Compute the fractional change in the volume of glass sphere when subjected to a hydraulic pressure of 1.013 x 10^6 Nm^{-2}. Given bulk modulus of glass is 3.7x10^{10} Nm^{-2}
Compute the fractional change in the volume of glass sphere when subjected to a hydraulic pressure of 1.013 x 10^6 Nm^{-2}. Given bulk modulus of glass is 3.7 x 10^{10} Nm^{-2}.

FOUR MARK QUESTIONS :

1.Define terms stress and strain. Draw the stress verses strain graph for a metallic wire stretched upto the fracture point
The restoring force per unit area of the body is called stress
Strain produced in a body is defined as ratio of change in dimension to the original dimension



2. Define the following terms

a) Elastic limit b) permanent set c) plastic deformation d) fracture point

a) The maximum stress below which Hooke's law is applicable is called elastic limit

b) When a wire is stretched more, then it has permanent strain even when the stress is zero. Then wire is said to have permanent set.

c) When a wire is stretched too much ,then it has permanent strain even when the stress is zero. This behavior of the material is called plastic deformation

d) The stretched wire breaks for certain applied stress is fracture point

3. State and explain Hooke's law. Define modulus of elasticity and write its dimensional formula

Within elastic limit stress and strain are proportional to each other.

Stress $\propto$ strain OR Stress/strain = constant (modulus of elasticity).

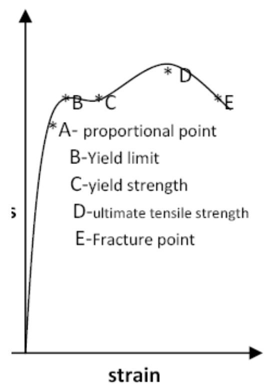
Modulus of elasticity is defined as ratio of stress acting on the body to the resulting strain in it.

[Modulus of elasticity]= $[L^{-1}MT^{-2}]$

FIVE MARK QUESTIONS :

1. Draw typical stress – strain graph for a metal and explain the important features in it.

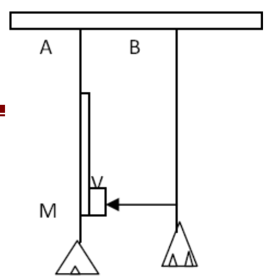
Stress and strain curve for metal is as shown in fig. When a metal Wire is stretched, for small value of load the elongation produced Is proportional to the load. Hence stress is directly proportional to the strain upto point A, obeying Hooke's law. Stress corresponding to the point A is called proportional limit. When stress increased beyond A, for a small stress change, there is a large strain up to point B so that stress is directly proportional to strain. But on removal of load the body is still regain its original shape and size, when applied load is less than certain limit. This limit is called elasticity limit.(point B). Metals shows elasticity behavior. If stress is increased beyond B strain further increase rapidly and if load is removed wire does not regain its original length i.e. the stain produced in the wire is permanent and it is said to have permanent set. Such a deformation is called is plastic deformation.



As stress increased further (beyond C) large strain is produced and wire breaks at E which is known as fracture point.

2. Explain the experiment to determine Young's modulus of the material wire under stretching.

Experimental arrangement to determine Young`s modulus of a material wire under tension as shown in the figure. Two identical wires of same length and radius suspended side by side from a fixed rigid support. Reference wire A carries millimetre main scale M and a pan to place the weight. Experimental wire B also carries a pan in which known weights can be placed. Vernier scale V is attached to a pointer at the bottom of the experimental wire helps to find elongation of wire.



Both wire are given an initial small load to keep straight and initial reading is noted. Now experimental wire is gradually loaded with more weights and reading is noted. The difference between two reading gives elongation ΔL of the wire of initial length L and radius r for a load mass M.

The Young`s modulus of the material of the experimental wire is given by Y = $\frac{MgL}{\pi r^2 \Delta L}$

FIVE MARK PROBLEMS:

1. A steel rod of radius 10mm and length 2m is stretched by a force of 100 kN along its length. The elongation in the wire is 3.2 mm. Find the stress and Young's modulus of the material of the rod.

Stress = $\frac{force}{area} = \frac{F}{A} = \frac{F}{\pi r^2} = \frac{100\ x\ 10^3}{3.14\ X(10-2)^2} = 3.18x108Nm^{-2}$

Y = $\frac{stress}{\frac{\Delta L}{L}} = \frac{3.14x10^8}{\frac{3.2x10^{-3}}{2}} = 1.96x10^{11}Nm^{-2}$

2. The upper face of a cube of edge 1m moves through a distance of 1mm relative to the lower fixed surface under action of a tangential force $1.5x10^8$N. Calculate tangential stress and rigidity modulus.

2) From figure $t\partial n\theta = \frac{BC}{AB} = \frac{1mm}{1m} = 0.001$ or

Shearing strain = $\theta = 0.001\ (\theta\ small)$

Shearing stress = $\frac{tangential\ force}{area\ of\ the\ face} = \frac{F}{A}$

$= \frac{1.5X10^8}{1} = \mathbf{1.5x10^8Nm^{-2}}$

Modulus of rigidity, G = $\frac{stress}{strain}$

$= \frac{1.5X10^8}{0.001} = 1.5X10^{11}Nm^{-2}$

3. When a rubber ball is taken in deep of 100m in sea its volume is decrease by 0.1% due to hydraulic stress. If the density of sea water is 1000 kgm^{-3}, calculate the bulk modulus and compressibility of the rubber.

3) $\Delta V = 0.1\%\ of$ Original volume V

$= \frac{0.1}{100} = 10^{-3}V$

P = ρgh = 100x9.8x100 =$\underline{9.8x10^5Nm^{-2}}$

B = $\frac{p}{\frac{\Delta V}{V}}$

$= \frac{9.8\,x10^5}{10^{-3}V} = 9.8$**x10^{8}Nm^{-2}**

Compressibility = K =1/B = **0.1x10^{-8}N^{-1}m^2**

4. A steel wire of length 5m and cross section 3x10-5m^2 stretched by the same amount as copper of length 3.7m and cross section 4x10^{-5} m^2 under given load. Find the ratio of Young's modulus of steel to that of copper.

Length of steel wire = Ls = 5m , area of cross section As = 3x10^{-5}m^2

Length of copper wire = Lc = 3.7m , area of cross section Ac = 4x10^{-5}m^2

Extension of steel wire = extension of copper wire = l

Young modulus = Y = $\frac{FL}{Al}$

For steel $Y_S = \frac{FLs}{Asl}$

For copper $Y_C = \frac{FLc}{Acl}$

$$\frac{Y_S}{Y_C} = \frac{\frac{FLs}{Asl}}{\frac{FLc}{Acl}} - \frac{L_sA_c}{L_cA_s}$$

$$= \frac{5x4x10^{-5}}{3.7x3x10^{-5}} \qquad = 1.8$$

5. Two wires of area of cross section 5x10^{-6} m^2, one made of steel and the other made of brass are loaded as shown. The unloaded length of steel wire is 1.5m and that of brass wire is 1m. Find the elongations in each wires. Y for steel is 2x10^{11} Nm^{-2} and for brass is 0.91x10^{11}Nm^{-2}

Total load on steel wire =Fs = 4+6=10Kgwt = 10 x 9.8 = 98N

Ls = 1.5m ΔLs=? Ys = 2 x 10^{11}Nm^{-2} A = 5x10^{-6} m^2

Load on brass wire = F_b = 6 kgwt = 6 x 9.8 = 58.8N

Lb=1m ΔLb = ? Yb = 0.91x1011 Nm-2 A = 5x10-6 m2

Young's modulus=Y=$\frac{FL}{A\Delta L}$

For steel $Y_S=\frac{FsLs}{A\Delta Ls}$ hence elongation for steel = $\Delta Ls = \frac{FsLs}{AYs} - \frac{98x1.5}{5x10^{-6}x2x10^{11}}$

= **<u>1.47x10^{-4}m</u>**

For brass Yb = $\frac{FbLb}{A\Delta Lb}$ hence elongation for brass = $\Delta Lb = \frac{FbLb}{AYb} = \frac{58.8x1}{5x10^{-6}x0.91x10^{11}}$

= **<u>1.29x10^{-4}m</u>**

6.Find the force required to stretch a wire of area of cross section $2x10^{-4}$ m^2 so that its length becomes 1.5 times original length . Young's modulus=$3.6x10^{11}$ Nm^{-2}.

6) A= $2x10^{-4}m^2$, Y = $3.6x10^{11}$ Nm^{-2}

Let L original length then ΔL = 1.5L – L = 0.5L

Young's modulus = Y = $\frac{FL}{A\Delta L}$

F= $\frac{YA\Delta L}{L} = \frac{3.6x10^{11}x2x10^{-4}x0.5L}{L}$

= $3.6x10^{7}$N

TEXTBOOK EXERCISES

9.1 A steel wire of length 4.7 m and cross-sectional area 3.0 □ 10^{-5} m^2 stretches by the same amount as a copper wire of length 3.5 m and cross-sectional area of 4.0 □ 10^{-5} m^2 under a given load. What is the ratio of the Young's modulus of steel to that of copper?

ANS:
Length of the steel wire, $L_1 = 4.7$ m
Area of cross-section of the steel wire, $A_1 = 3.0 \times 10^{-5}\ m^2$
Length of the copper wire, $L_2 = 3.5$ m
Area of cross-section of the copper wire, $A_2 = 4.0 \times 10^{-5}\ m^2$
Change in length = $\Delta L_1 = \Delta L_2 = \Delta L$
Force applied in both the cases = F
Young's modulus of the steel wire:

$$Y_1 = \frac{F_1}{A_1} x \frac{L_1}{\Delta L} = \frac{Fx4.7}{3.0x10^{-5}x\Delta L} \qquad \text{(i)}$$

Young's modulus of the copper wire:

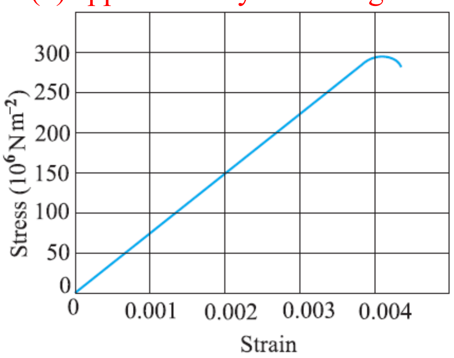
$$Y_2 = \frac{F_2}{A_2} x \frac{L_2}{\Delta L_2} = \frac{Fx3.5}{4.0x10^{-5}x\Delta L} \qquad \text{(ii)}$$

Dividing (*i*) by (*ii*), we get:

$$\frac{Y_1}{Y_2} = \frac{4.7x4.0x10^{-5}}{3.0x10^{-5}x3.5} = 1.79{:}1$$

The ratio of Young's modulus of steel to that of copper is 1.79 : 1.

9.2 Figure 9.11 shows the strain-stress curve for a given material. What are (a) Young's modulus and (b) approximate yield strength for this material?

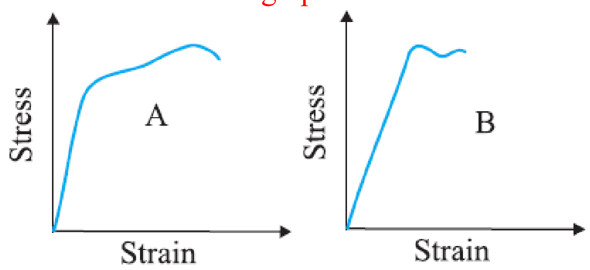


ANS:
(a) It is clear from the given graph that for stress 150×10^6 N/m^2, strain is 0.002.
∴ Young's modulus, Y = *Stress/Strain* =

$= \frac{150x10^6}{0.002} = 7.5x10^{10} N/m^2$

Hence, Young's modulus for the given material is 7.5 ×1010 N/m^2.
(b) The yield strength of a material is the maximum stress that the material can sustain without crossing the elastic limit.
It is clear from the given graph that the approximate yield strength of this material is
300×10^6 N/m^2 or 3×10^8 N/m^2.

9.3 The stress-strain graphs for materials *A* and *B* are shown in Fig. 9.12.



The graphs are drawn to the same scale.
(a) Which of the materials has the greater Young's modulus?
(b) Which of the two is the stronger material?
ANS:
(a) A (b) A
(a) For a given strain, the stress for material **A** is more than it is for material **B**, as shown in the two graphs.
Young's modulus Y = Stress/Strain
For a given strain, if the stress for a material is more, then Young's modulus is also greater for that material. Therefore, Young's modulus for material **A** is greater than it is for material **B**.

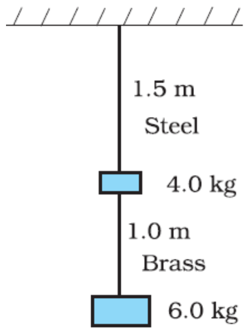
(b) The amount of stress required for fracturing a material, corresponding to its fracture point, gives the strength of that material. Fracture point is the extreme point in a stress-strain curve. It can be observed that material **A** can withstand more strain than material **B**. Hence, material **A** is stronger than material **B**.

9.4 Read the following two statements below carefully and state, with reasons, if it is true or false.
(a) The Young's modulus of rubber is greater than that of steel;
(b) The stretching of a coil is determined by its shear modulus.
Answer: (a) False **(b)** True
(a) For a given stress, the strain in rubber is more than it is in steel.
Young's modulus, Y = Stress/Strain
For a constant stress : $Y \propto 1/Strain$
Hence, Young's modulus for rubber is less than it is for steel.

(b) Shear modulus is the ratio of the applied stress to the change in the shape of a body. The stretching of a coil changes its shape. Hence, shear modulus of elasticity is involved in this process.

9.5 Two wires of diameter 0.25 cm, one made of steel and the other made of brass are loaded as shown in Fig. 9.13. The unloaded length of steel wire is 1.5 m and that of brass wire is 1.0 m. Compute the elongations of the steel and the brass wires.

ANS:



Elongation of the steel wire = 1.49 × 10–4 m
Elongation of the brass wire = 1.3×10^{-4} m
Diameter of the wires, d = 0.25 m
Hence, the radius of the wires, r = d/2 = 0.125 cm
Length of the steel wire, L_1 = 1.5 m
Length of the brass wire, L_2 = 1.0 m
Total force exerted on the steel wire: $F_1 = (4 + 6)\,g = 10 \times 9.8 = 98$ N
Young's modulus for steel:

$$Y_1 = \frac{\frac{F_1}{A_1}}{\frac{\Delta L_1}{L_1}}$$

Where,
ΔL_1 = Change in the length of the steel wire
A_1 = Area of cross-section of the steel wire = πr_1^2
Young's modulus of steel, $Y_1 = 2.0 \times 10^{11}$ Pa

$$\therefore \Delta L_1 = \frac{F_1 x L_1}{A_1 x Y_1} = \frac{F_1 x L_1}{\pi r_1^2 x Y_1} = \frac{98x1.5}{\pi(0.125x10^{-2})^2 x2x10^{n}} = 1.49x10^{-4} \text{ m}$$

Total force on the brass wire:
$F_2 = 6 \times 9.8 = 58.8$ N
Young's modulus for brass :

$$Y_2 = \frac{\frac{F_2}{A_2}}{\frac{\Delta L_2}{L_2}}$$

where

ΔL_2 = Change in length
A_2 = Area of cross-section of the basis wire

$$\therefore \Delta L_2 = \frac{F_2 x L_2}{A_2 x\, Y_2} = \frac{F_2 x L_2}{\pi r_2^2\; x\; Y_2} = \frac{58.8x1.0}{\pi x(0.125x10^{-2})x(0.91x10^{11})} = 1.3x10^{-4}m$$

Elongation of the steel wire = 1.49×10^{-4} m
Elongation of the brass wire = 1.3×10^{-4} m

9.6 The edge of an aluminium cube is 10 cm long. One face of the cube is firmly fixed to a vertical wall. A mass of 100 kg is then attached to the opposite face of the cube. The shear modulus of aluminium is 25 GPa. What is the vertical deflection of this face?

ANS:
Edge of the aluminium cube, L = 10 cm = 0.1 m
The mass attached to the cube, m = 100 kg
Shear modulus (η) of aluminium = 25 GPa = 25 × 109 Pa

$$= \frac{Shear\ stress}{Shear\ strain} = \frac{\frac{F}{A}}{\frac{L}{\Delta L}}$$

Shear modulus, η Where,
F = Applied force = mg = 100 × 9.8 = 980 N
A = Area of one of the faces of the cube = 0.1 × 0.1 = 0.01 m^2
ΔL = Vertical deflection of the cube

$$\therefore L = \frac{FL}{An} = \frac{980x0.1}{10^{-2}x(25x10^9)} = 3.92 \text{ x } 10^7\text{m}$$

The vertical deflection of this face of the cube is 3.92 ×10^{-7} m.

9.7 Four identical hollow cylindrical columns of mild steel support a big structure of mass 50,000 kg. The inner and outer radii of each column are 30 and 60 cm respectively. Assuming the load distribution to be uniform, calculate the compressional strain of each column.

ANS:
Mass of the big structure, M = 50,000 kg
Inner radius of the column, r = 30 cm = 0.3 m
Outer radius of the column, R = 60 cm = 0.6 m
Young's modulus of steel, $Y = 2 \times 10^{11}$ Pa
Total force exerted, $F = Mg$ = 50000 × 9.8 N
Stress = Force exerted on a single column = 50,000 □ 9.8 / 4 = 122500 N
Young's modulus, Y = *Stress / Strain*
Strain = (F/A)/Y

Where,
Area, $A = \pi (R^2 - r^2) = \pi ((0.6)^2 - (0.3)^2)$

Strain = $\frac{122500}{\pi[(0.6)^2 - (0.3)^2] x\ 2x10^n}$ = = 7.22×10^{-7}

Hence, the compressional strain of each column is 7.22×10^{-7}.

9.8 A piece of copper having a rectangular cross-section of 15.2 mm . 19.1 mm is pulled in tension with 44,500 N force, producing only elastic deformation. Calculate the resulting strain?

ANS:
Length of the piece of copper, l = 19.1 mm = 19.1×10^{-3} m
Breadth of the piece of copper, b = 15.2 mm = 15.2×10^{-3} m
Area of the copper piece:
$A = l \times b$
$= 19.1 \times 10^{-3} \times 15.2 \times 10^{-3}$
$= 2.9 \times 10^{-4}\ m^2$
Tension force applied on the piece of copper, F = 44500 N
Modulus of elasticity of copper, $\eta = 42 \times 109\ N/m^2$
Modulus of elasticity, η = Stress/Strain = (F/A)/Strain
 Strain = F/A

$= \frac{44500}{2.9x10^{-4}\ x\ 42x10^9} = 3.65 \times 10^{-3}$

9.9 A steel cable with a radius of 1.5 cm supports a chairlift at a ski area. If the maximum stress is not to exceed 108 N m^{-2}, what is the maximum load the cable can support ?

ANS:
Radius of the steel cable, r = 1.5 cm = 0.015 m
Maximum allowable stress = $10^8\ N\ m^{-2}$

Maximum stress = (Maximum Force / Area of Cross section)
∴ Maximum force = Maximum stress × Area of cross-section
$= 10^8 \times \pi (0.015)^2$
$= 7.065 \times 10^4$ N
Hence, the cable can support the maximum load of 7.065×10^4 N.

9.10 A rigid bar of mass 15 kg is supported symmetrically by three wires each 2.0 m long. Those at each end are of copper and the middle one is of iron. Determine the ratios of their diameters if each is to have the same tension.

ANS:
The tension force acting on each wire is the same. Thus, the extension in each case is the same. Since the wires are of the same length, the strain will also be the same.
The relation for Young's modulus is given as:

$$Y = \frac{Stress}{Strain} = \frac{\frac{F}{A}}{Strain} = \frac{\frac{4F}{\pi d^2}}{Strain}$$

Where,
F = Tension force
A = Area of cross-section
d = Diameter of the wire
It can be inferred from equation (i) that
Young's modulus for iron, $Y1$ = 190 × 109 Pa
Diameter of the iron wire = $d1$
Young's modulus for copper, $Y2$ = 110 × 109 Pa
Diameter of the copper wire = $d2$
Therefore, the ratio of their diameters is given as:

$$\frac{d_2}{d_1} = \sqrt{\frac{Y_1}{Y_2}} = \sqrt{\frac{190x10^9}{110x10^9}} = \sqrt{\frac{19}{11}} = 1.31:1$$

9.11 A 14.5 kg mass, fastened to the end of a steel wire of unstretched length 1.0 m, is whirled in a vertical circle with an angular velocity of 2 rev/s at the bottom of the circle. The cross-sectional area of the wire is 0.065 cm2. Calculate the elongation of the wire when the mass is at the lowest point of its path.

ANS:
Mass, m = 14.5 kg
Length of the steel wire, l = 1.0 m
Angular velocity, ω = 2 rev/s
Cross-sectional area of the wire, a = 0.065 cm^2
Let δl be the elongation of the wire when the mass is at the lowest point of its path.
When the mass is placed at the position of the vertical circle, the total force on the mass is:
$F = mg + ml\omega^2$
$= 14.5 \times 9.8 + 14.5 \times 1 \times (2)^2$
$= 200.1$ N

$$\text{Young's modulus} = \frac{Stress}{Strain}$$

$$Y = \frac{\frac{F}{A}}{\frac{\Delta l}{l}} = \frac{F}{A}\frac{l}{\Delta l} \qquad \therefore \Delta l = \frac{Fl}{AY}$$

Young's modulus for steel = 2 × 1011 Pa

$$\therefore \Delta l = \frac{200.1x1}{0.065x10^{-4}x2x10^{n}} = 1539.23 \text{ x } 10^7 \qquad = 1.539x10^{-4}\text{m}$$

Hence, the elongation of the wire is 1.539×10^{-4} m.

9.12 Compute the bulk modulus of water from the following data: Initial volume = 100.0 litre, Pressure increase = 100.0 atm (1 atm = 1.013 . 105 Pa), Final volume = 100.5 litre. Compare the bulk modulus of water with that of air (at constant temperature). Explain in simple terms why the ratio is so large.

ANS:

Initial volume, $V1$ = 100.0l = 100.0 × 10 –3 m3

Final volume, $V2$ = 100.5 l = 100.5 ×10 –3 m3

Increase in volume, ΔV = $V2$ – $V1$ = 0.5 × 10–3 m3

Increase in pressure, Δp = 100.0 atm = 100 × 1.013 × 105 Pa

Bulk modulus = $\frac{\Delta p}{\frac{\Delta V}{V_1}} = \frac{\Delta p x V_1}{\Delta V} = \frac{100x1.1013x10^5x100x10^{-3}}{0.5x10^{-3}} = 2.026x10^9\ Pa$

$\therefore \frac{Bulk\ modulus\ of\ water}{Bulk\ modulus\ of\ air} = \frac{2.026x10^9}{1.0x10^5}$ = 2.026x10^4

This ratio is very high because air is more compressible than water.

9.13 What is the density of water at a depth where pressure is 80.0 atm, given that its density at the surface is 1.03 . 103 kg m^{-3}?

ANS:

Let the given depth be h.

Pressure at the given depth, p = 80.0 atm = 80 × 1.01 × 10^5 Pa

Density of water at the surface, ρ_1 = 1.03 × 10^3 kg m^{-3}

Let ρ_2 be the density of water at the depth h.

Let V_1 be the volume of water of mass m at the surface.

Let V_2 be the volume of water of mass m at the depth h.

Let ΔV be the change in volume.

$\Delta V = V_1 - V_2$

= m $\left[\frac{1}{p_1} - \frac{1}{p_2}\right]$

9.14 Compute the fractional change in volume of a glass slab, when subjected to a hydraulic pressure of 10 atm.

ANS:

Hydraulic pressure exerted on the glass slab, p = 10 atm = 10 × 1.013 × 10^5 Pa

Bulk modulus of glass, B = 37 × 10^9 Nm^{-2}

Bulk modulus, B = $\frac{p}{\frac{\Delta V}{V}}$

Where, $\Delta V/V$ = = Fractional change in Volume

$\therefore \frac{\Delta V}{V} = \frac{p}{B} = \frac{10x1.013x10^5}{37x10^9}$ = = **2.73 □ 10-5**

$\therefore$ $Volumetric$ strain $= \frac{\Delta V}{V_1} = m\left(\frac{1}{p_1} - \frac{1}{p_2}\right) x \frac{P_1}{m}$

$\therefore \frac{\Delta V}{V_1} = 1 - \frac{p_1}{p_2}$ ....(i)

Bulk modulus, $B = \frac{pV_1}{\Delta V}$ $\frac{\Delta V}{V_1} = \frac{p}{B}$

Compressibility of water $= \frac{1}{B} =$ 45.8 x 10^{-11} Pa^{-1}

$\therefore \frac{\Delta V}{V_1} = 80$ x 1.013 x 10^5 x 45.8 x 10^{-11} = 3.71 x 10^{-3}

For equations (*i*) and (*ii*), we get:

$1 - \frac{\rho_1}{\rho_2} = 3.71 \, x \, 10^{-3}$ $\rho_2 = \frac{1.03 \, x 10^3}{1-(3.71 \, x \, 10^{-3})}$ = 1.034 x 10^3kg m^{-3}

Therefore, the density of water at the given depth (*h*) is 1.034 × 10^3 kg m^{-3}.

9.15 Determine the volume contraction of a solid copper cube, 10 cm on an edge, when subjected to a hydraulic pressure of 7.0 . 106 Pa.

ANS:

Length of an edge of the solid copper cube, l = 10 cm = 0.1 m

Hydraulic pressure, p = 7.0 ×10^6 Pa

Bulk modulus of copper, B = 140 × 10^9 Pa

Bulk modulus, $B = \frac{p}{\frac{\Delta V}{V}}$

Where,

= Volumetric strain

ΔV = Change in volume

V = Original volume.

$\Delta V = \frac{pV}{B}$

Original volume of the cube, $V = l^3$

$\therefore \Delta V = \frac{pl^3}{B} = \frac{7x10^6 x(0.1)^3}{140 \, x \, 10^9} =$5 x $10^{-8}m^3$ = 5 x 10^{-2} cm^{-3}

Therefore, the volume contraction of the solid copper cube is 5 × 10^{-2} cm^{-3}.

9.16 How much should the pressure on a litre of water be changed to compress it by 0.10%?
ANS:
Volume of water, $V = 1$ L
It is given that water is to be compressed by 0.10%.

$\therefore$ *Fractional change,* $\frac{\Delta V}{V} = \frac{0.1}{100x1} = 10^{-3}$

Bulk modulus, B= $\frac{\rho}{\frac{\Delta V}{V}}$ $\qquad p = B\,x\,\frac{\Delta V}{V}$

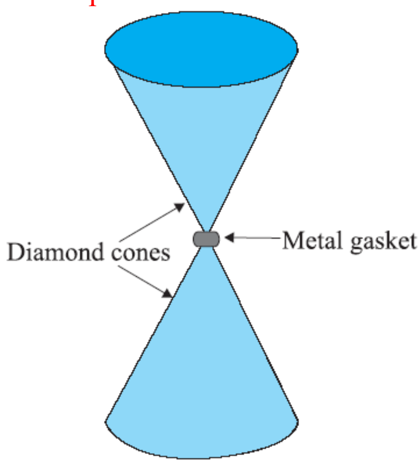
Bulk modulus of water, B= 2.2×10^{9} Nm^{-2}

$p = 2.2\,x\,10^{9}\,x\,10^{-3}$
$= 2.2\,x\,10^{6}\,Nm^{-2}$

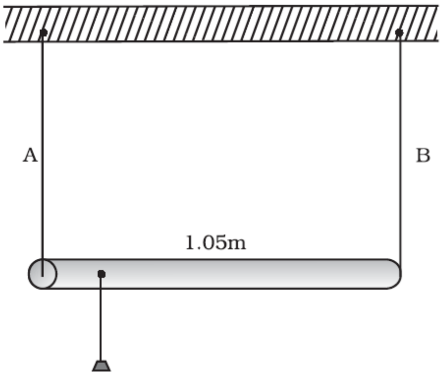
Therefore, the pressure on water should be 2.2×10^{6} Nm^{-2}.

Additional Exercises

9.17 Anvils made of single crystals of diamond, with the shape as shown in Fig. 9.14, are used to investigate behaviour of materials under very high pressures. Flat faces at the narrow end of the anvil have a diameter of 0.50 mm, and the wide ends are subjected to a compressional force of 50,000 N. What is the pressure at the tip of the anvil?



9.18 A rod of length 1.05 m having negligible mass is supported at its ends by two wires of steel (wire A) and aluminium (wire B) of equal lengths as shown in Fig. 9.15. The cross-sectional areas of wires A and B are 1.0 mm^2 and 2.0 mm^2, respectively. At what point along the rod should a mass m be suspended in order to produce (a) equal stresses and (b) equal strains in both steel and aluminium wires.



9.19 A mild steel wire of length 1.0 m and cross-sectional area 0.50 . 10^{-2} cm^2 is stretched, well within its elastic limit, horizontally between two pillars. A mass of 100 g is suspended from the mid-point of the wire. Calculate the depression at the midpoint.

9.20 Two strips of metal are riveted together at their ends by four rivets, each of diameter 6.0 mm. What is the maximum tension that can be exerted by the riveted strip if the shearing stress on the rivet is not to exceed 6.9 □ 10^7 Pa? Assume that each rivet is to carry one quarter of the load.

9.21 The Marina trench is located in the Pacific Ocean, and at one place it is nearly eleven km beneath the surface of water. The water pressure at the bottom of the trench is about 1.1 □ 10^8 Pa. A steel ball of initial volume 0.32 m^3 is dropped into the ocean and falls to the bottom of the trench. What is the change in the volume of the ball when it reaches to the bottom?

PRACTICE PROBLEMS

OBJECTIVE QUESTIONS

1) If 'S' is stress and 'Y' is Young's modulus of material of a wire, the energy stored in the wire per unit volume is
(a) S^2 / (2Y) (b) 2 S^2Y (c) S / 2Y (d) 2Y / S^2 [AIEEE 2005]

2) A 20 cm long capillary tube is dipped in water. The water rises upto 8 cm. If the entire arrangement is put in a freely falling elevator, the length of water column in the capillary tube will be (a) 10 cm (b) 8 cm (c) 20 cm (d) 4 cm [AIEEE 2005]

3) A wire fixed at the upper end stretches by length l by applying a force F. The work done in stretching is
(a) F /2l (b) Fl (c) 2 Fl (d) Fl / 2 [AIEEE 2004]

4) Spherical balls of radius R are falling in a viscous fluid of viscosity η with a velocity v. The retarding viscous force acting on the spherical ball is
(a) directly proportional to R but inversely proportional to v
(b) directly proportional R and velocity v
(c) inversely proportional to both radius R and velocity v.
(d) inversely proportional to R but directly proportional to velocity v [AIEEE 2004]

5) If two soap bubbles of different radii are connected by a tube
(a) air flows from the bigger bubble to the smaller bubble till sizes become equal
(b) air flows from the bigger bubble to the smaller bubble till the sizes are interchanged
(c) air flows from the smaller bubble to the bigger
(d) there is no flow of air [AIEEE 2004]

6) A wire suspended vertically from one of its ends is stretched by attaching a weight of 200 N to the lower end. The weight stretches the wire by 1 mm. The elastic energy stored in the wire is
(a) 0.1 J (b) 0.2 J (c) 10 J (d) 20 J [AIEEE 2003]

7) A spring of spring constant 5×10^3 N / m is stretched initially by 5 cm from the unstretched position. The work required to stretch it further by another 5 cm is
(a) 6.25 Nm (b) 12.50 Nm (c) 18.75 Nm (d) 25.00 Nm [AIEEE 2003]

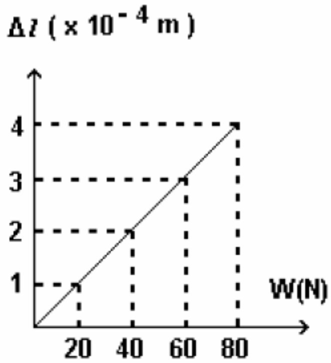
8) Rain drops are spherical in shape because of
(a) surface tension (b) capillarity
(c) downward motion (d) acceleration due to gravity [AIEEE 2002]

9) Which of the following affects the elasticity of a substance ?
(a) hammering and annealing (b) change in temperature
(c) impurity in substance (d) all of these [AIEEE 2002]

10) The normal density of iron is ρ and its bulk modulus is K. The increase in the density of an iron lump, when a pressure P is applied uniformly on all sides, will be
(a) Pρ / K (b) PK / ρ (c) P / ρK (d) K /ρP [AIEEE 2002]

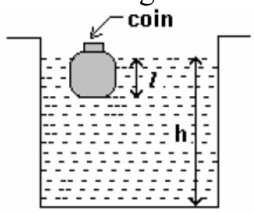
11) A vessel is filled with water upto a height of 3 m. There is a hole at a height of 52.5 cm from the bottom. Ratio of area of cross section of hole to vessel is 0.1. Then square of velocity of water coming out of hole in (m / s)^2 is
(a) 50 (b) 50.5 (c) 51 (d) 40 [IIT 2005]

12) The pressure of a medium is changed from 1.01×10^5 Pa to 1.165×10^5 Pa and change in volume is 10% keeping temperature constant. The Bulk modulus of the medium is
(a) 204.8×10^5 Pa (b) 102.4×10^5 Pa
(c) 51.2×10^5 Pa (d) 1.55×105 Pa [IIT 2005]



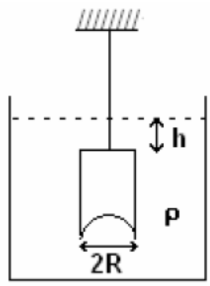
13) The adjacent graph shows the extension (Δl) of a wire of length 1 m suspended from the top of a roof at one end and with a load W connected to the other end. If the cross-sectional area of the wire is 10^{-6} m^2, the Young's modulus of the material of the wire is
(a) 2×10^{11} N / m^2 (b) 2×10^{-11} N / m^2
(c) 3×10^{12} N / m^2 (d) 2×10^{13} N / m^2
[IIT 2003]

14) A wooden block, with a coin placed on its top, floats in water as shown in the figure. The distance l and h are shown there. After some time the coin falls into the water. Then



(a) l decreases and h increases
(b) l increases and h decreases
(c) both l and h increase
(d) both l and h decrease [IIT 2002]

15) A hemispherical portion of radius R is removed from the bottom of a cylinder of radius R. The volume of the remaining cylinder is V and mass M. It is suspended by a string in a liquid of density ρ where it stays vertical. The upper surface of the cylinder is at a depth h below the liquid surface. The force on the bottom of cylinder by the liquid is:



(a) Mg (b) Mg - Vρg
(c) Mg + $\pi R^2 h\rho g$ (d) ρg (V + $\pi R^2 h$)
[IIT 2001]

16) When a block of iron floats in mercury at 0° C, a fraction k1 of its volume is submerged, while at the temperature 60° C, a fraction k_2 is seen to be submerged. If the coefficient of volume expansion of iron is γ_{Fe} and that of mercury is γ_{Hg} , then the ratio k_1 / k_2 can be expressed as

(a) $\frac{1+60\gamma_{Fe}}{1+60\gamma_{Hg}}$ (b) $\frac{1-60\gamma_{Fe}}{1+60\gamma_{Hg}}$ (c) $\frac{1+60\gamma_{Fe}}{1-60\gamma_{Hg}}$ (d) $\frac{1+60\gamma_{Hg}}{1+60\gamma_{Fe}}$

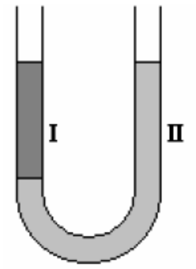
[IIT 2001]

17) A large open tank has two holes in the wall. One is a square hole of side L at a depth y from the top and the other is a circular hole of radius R at a depth 4y from the top. When the tank is completely filled with water, the quantities of water flowing out per second from both holes are the same. Then, R is equal to
(a) $\frac{□}{\sqrt{2□}}$ (b) 2 π L (c) L (d) L/2π
[IIT 2000]

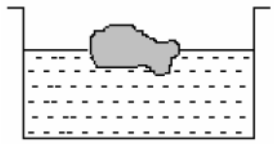
18) Water from a tap emerges vertically downwards with an initial speed of 1.0 m / s. The cross-sectional area of tap is 10 - 4 m 2. Assume that the pressure is constant throughout the stream of water and that the flow is steady. The cross-sectional area of stream 0.15 m below the tap is
(a) 5.0 x 10^{-4} m^2 (b) 1.0 x 10^{-4} m^2 (c) 5.0 x 10 - 5 m 2 (d) 2.0 x 10 - 5 m 2
[IIT 1998]

19) A vessel contain oil (density = 0.8 gm / cm^3) over mercury (density = 13.6 gm / cm^3). A homogeneous sphere floats with half its volume immersed in mercury and the other half in oil. The density of the material of the sphere in gm / cm^3 is
(a) 3.3 (b) 6.4 (c) 7.2 (d) 12.8 [IIT 1988]

20) A U-tube of uniform cross section (see figure) is partially filled with a liquid I. Another liquid II which does not mix with liquid I is poured into one side. It is found that the liquid levels of the two sides of the tube are the same, while the level of liquid II has risen by 2 cm. If the specific gravity of liquid I is 1.1, the specific gravity of liquid II must be
(a) 1.12 (b) 1.1 (c) 1.05 (d) 1.0
[IIT 1983]



21) A body floats in a liquid contained in a beaker. The whole system is as shown in the figure falls freely under gravity. The upthrust on the body due to the liquid is
(a) zero (b) equal to the weight of the liquid displaced
(c) equal to the weight of the body in air
(d) equal to the weight of the immersed portion of the body [IIT 1982]



22) The following four wires are made of the same material. Which of these will have the largest extension when the same tension is applied ?
(a) length = 50 cm, diameter = 0.5 mm (b) length = 100 cm, diameter = 1 mm
(c) length = 200 cm, diameter = 2 mm (d) length = 300 cm, diameter = 3 mm [IIT 1981]

23) When pressure is applied through a hole in the top of a closed tube containing water, the pressure is transmitted to
(a) only to the bottom of the container (b) all directions
(c) only the side faces and the bottom of the container
(d) only the side of the container [NCERT 1990]

24) When a large bubble rises from the bottom of a lake to the surface, its radius doubles. The atmospheric pressure is equal to that of a column of water of height H. The depth of the lake is
(a) H (b) 2H (c) 7H (d) 8H [NCERT 1979]

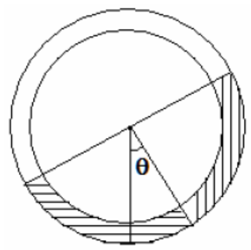
25) A raft of wood, density 600 kg/m3, of mass 120 kg floats in water. How much weight can be put on the raft to make it just sink ?
(a) 200 kg (b) 40 kg (c) 120 kg (d) 80 kg [NCERT 1979]

26) A body of volume 100 c.c. is immersed completely in water contained in a jar. The weight of water and jar before immersion of the body is 700 gm. After immersion, the weight of water and jar will be
(a) 500 gm (b) 700 gm (c) 100 gm (d) 800 gm [NCERT 1978]

27) Two rods of different materials having coefficients of linear expansion α_1 and α_2 and Young's modulii Y_1 and Y_2 respectively are fixed between two massive walls. The rods are heated such that they undergo the same increase in temperature. There is no bending of the rods. If $\alpha_1 : \alpha_2 = 2 : 3$, the thermal stresses developed in the two rods are equal provided $Y_1 : Y_2$ is equal to
(a) 2 : 3 (b) 1 : 1 (c) 3 : 2 (d) 4 : 9 [IIT 1989]

28) A small uniform tube is bent into a circle of radius r whose plane is vertical. Equal volumes of two immiscible liquids of densities d and d' fill half the circle. The angle between the radius passing through the interface and the vertical is given as

(a) $\tan^{-1}(d/d')$ (b) $\tan^{-1}(d'/d)$
(c) $\tan^{-1}\left(\frac{d-d'}{d+d'}\right)$ (d) $\tan^{-1}\left(\frac{d+d'}{d-d'}\right)$



29) Water squirts out of a small hole from a water-filled can. The hole is located at a distance y below the water surface. The height of the water in the can is h. The distance R from the base of the can, directly below the hole, where water strikes is given as
(a) $2\sqrt{yh}$ (b) $\sqrt{y}$ (h-y) (c) $\sqrt{2y(h-y)}$ (d) $2\sqrt{y(h-y)}$

30) The volume of an air bubble is doubled as it rises from the bottom of a lake to its surface. The atmospheric pressure is 75 cm of Hg and the ratio of density of mercury to that of lake water is 40/3. What is the depth of the lake?
(a) 10 m (b) 15 m (c) 20 m (d) 25 m

Answers

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
a	c	d	b	c	a	c	a	d	a	a	d	a	d	d	a	a	c	b	b

21	22	23	24	25	26	27	28	29	30
a	a	b	c	d	d	a	c	b	a

SURFACE TENSION MCQ

1.	One end of a towel dips into a bucket full of water and other end hangs over the bucket. It is found that after some time the towel becomes fully wet. It happens **(CPMT 86)** (a) Because viscosity of eater is high (b) Because of the capillary action of cotton threads (c) Because of gravitational force (d) Because of evaporation of water. **Answer: (b)**
2.	For tap water and clean glass, the angle of contact is (a) 0° (b) 90° (c) 140° (d) 8° **Answer: (d)**
3.	Water rises up to a height h_1 in a capillary tube of radius r. the mass of the water lifted in the capillary tube is M. if the radius of the capillary tube is doubled, the mass of water that will rise in the capillary tube will be (a) M (b) 2M (c) $\frac{M}{2}$ (d) 4M **Answer: (b)**
4.	Water rises through a height h in a capillary tube of internal radius (r). if T is the S.T. of water, then the pressure difference between the liquid level in the container and the lowest point of the concave meniscus is (a) $\frac{T}{r}$ (b) $\frac{r}{T}$ (c) $\frac{2T}{r}$ (d) $\frac{r}{2T}$ **Answer: (c)**
5.	A number of small drops of mercury coalesce adiabatically to form a single drop. The temperature of drop **(MHT-CET-2008)** (a) Increases (b) Is infinite (c) Remains unchanged (d) May decrease or increase depending upon size **Answer: (d)**
6.	The angle of contact between a glass capillary tube of length 10 cm and a liquid is 90°. If the capillary tube is dipped vertically in the liquid, then the liquid (a) Will rise in the tube (b) Will get depressed in the tube (c) Will rise up to 10 cm in the tube and will over flow (d) Will neither rise nor fall in the tube **Answer: (d)**
7.	When there are no external forces, the shape of a liquid drop is determined by (a) Surface tension of the liquid (b) Density of liquid

	(c) Viscosity of liquid (d) Temperature of air only **Answer: (a)**
8.	If T is surface tension of soap solution, the amount of work done in blowing a soap bubble from diameter D to a diameter 2D is **(PMT MP 90)** (a) $2 \pi D^2 T$ (b) $4 \pi D^2 T$ (c) $6 \pi D^2 T$ (d) $8 \pi D^2 T$ **Answer: (c)**
9.	Choose the wrong statement from the following. (a) Small droplets of a liquid are spherical due to surface tension (b) Oil rises through the wick due to capillarity (c) In drinking the cold drinks through a straw, we use the phenomenon of capillarity (d) Gum is used to stick two surfaces. In this process we use the property of Adhesion **Answer: (c)**
10.	If the surface of a liquid is plane, then the angle of contact of the liquid with the walls of container is **(MHT CET 2004)** (a) Acute angle (b) Obtuse angle (c) $90°$ (d) $0°$ **Answer: (d)**
11.	A capillary tube when immersed vertically in a liquid records a rise of 3 cm. if the tube is immersed in the liquid at an angle of 60° with the vertical, then length of the liquid column along the tube will be **(MHT-CET 1999)** (a) 2 cm (b) 3 cm (c) 6 cm (d) 9 cm **Answer: (c)**
12.	If sap bubbles of different radii are in communication with each other **(PMT MP 88, NCERT 80)** (a) Air flow from the larger bubble into the smaller one until the two bubbles are of equal size (b) The sizes of the bubbles remain unchanged. (c) Air flows from the smaller into the larger on and lager bubble grows at the expense of the smaller one (d) Air flows from the larger into the smaller one becomes equal to that of the larger one and the large one equal to that of the smaller one. **Answer: (c)**
13.	A capillary tube of radius r can support a liquid of weight 6.28×10^{-4} N. if the surface tension of the liquid is 5×10^{-2} N/m. the radius of capillary must be **(CPMT 88)** (a) 2.5×10^{-3} m (b) 2.0×10^{-4} m (c) 1.5×10^{-3} m (d) 2.0×10^{-3} m **Answer: (d)**
14.	The work done in blowing a soap bubble of radius R is W_1 and that to a radius 3R is W_2. the ratio of work done is (a) 1:3 (b) 3:1 (c) 1:9 (d) 9:1 **Answer: (c)**
15.	When the angle of contact between a solid and a liquid is 90°, then (a) Cohesive force > Adhesive force

(b) Cohesive force < Adhesive force
(c) Cohesive force = Adhesive force
(d) Cohesive force >> Adhesive force
Answer: (c)

16. Rain drops are spherical in shape because of
(MHT-CET 2000)
(a) Surface tension
(b) Capillary
(c) Downward motion
(d) Acceleration due to gravity
Answer: (a)

17. A sheet can be made water proof by coating it with a substance that changes the angle of contact
(a) to $\frac{\pi}{2}$
(b) To zero
(c) From acute to obtuse
(d) From obtuse to acute
Answer: (c)

18. Water rises in a capillary tube to a certain height such that the upward force due to surface tension is balanced by 75×10^{-4} N, forces due to the weight of the liquid. If the surface tension of water is 6×10^{-2} N/m, the inner-circumference of the capillary must be **(CPMT 88, 86)**
(a) 1.25×10^{-2} m (b) 0.50×10^{-2} m
(c) 6.5×10^{-2} m (d) 12.5×10^{-2} m
Answer: (d)

19. What is the change in surface energy, when a mercury drop of radius R splits up into 1000 droplets of radius r?
(a) $8 \pi R^2 T$ (b) $16 \pi R^2 T$
(c) $24 \pi R^2 T$ (d) $36 \pi R^2 T$
Answer: (d)

20. Which of the following is not based one the principle of capillarity **(MHT CET 2005)**
(a) Floating of wood on eater surface
(b) Ploughing of soil
(c) Rise of oil in wick of lantern
(d) Soaking of ink by bloating paper
Answer: (a)

21. The rise of a liquid in a capillary tube does not depend upon
(a) Angle of contact
(b) Density of the liquid
(c) Radius of the capillary tube
(d) Atmospheric pressure
Answer: (d)

22. The height of water in a capillary tube of radius 2 cm is 4 cm. what should be the radius of capillary, if the water rises to 8 cm in tube? **(MHT-CET-2001)**
(a) 1 cm (b) 0.1 cm
(c) 2 cm (d) 4 cm
Answer: (a)

23. The work done to get 'n' smaller equal size spherical drops from a bigger size spherical size drop of water is proportional to **(EAMCET 91)**

	(a) $\frac{1}{n^{\frac{2}{3}}}-1$ (b) $\frac{1}{n^{\frac{1}{3}}}-1$ (c) $n^{\frac{1}{3}}-1$ (d) $n^{\frac{4}{3}}-1$ **Answer: (c)**
24.	For a liquid, which is rising in a capillary tube, the angle of contact is (a) 90° (b) 180° (c) Acute (d) Obtuse **Answer: (c)**
25.	W is the work done, when a bubble of volume V is formed from a solution. How much work is required to be done to form a bubble of volume 2V? (a) 2 W (b) W (c) $2^{1/3}$ W (d) $4^{1/3}$ W **Answer: (d)**

26.	Two soap bubbles have radii in the ratio of 4:3. What is the ratio of work done to below these bubbles? **(MHT-CET-2003)** (a) 4:3 (b) 16:9 (c) 9:16 (d) 3:4 **Answer: (B)**
27.	A soap bubble (surface tension 30×10^{-3} N/m) has radius 2 cm. the work done in bobbling the radius is **(MNR 80)** (a) Zero (b) 1.1355×10^{-4} J (c) 2.261×10^{-4} J (d) 4.532×10^{-4} J **Answer: (d)**
28.	In a surface tension experiment with a capillary tube water rises up to 0.1 m. if the same experiment is repeated on an artificial satellite which is revolving around the earth. The rise of water in a capillary tube will be **(Rorkee 92)** (a) 0.1 m (b) 9.8 m (c) 0.98 (d) Full length of capillary tube **Answer: (d)**
29.	Surface tension of a soap solution is 1.9×10^{-2} N/m. work done in blowing a bubble of 2.0 cm diameter will be **(PMT MP 90)** (a) $7.6 \times 10^{-6}\,\pi$ J (b) $15.2 \times 10^{-6}\,\pi$ J (c) $1.9 \times 10^{-6}\,\pi$ J (d) $1 \times 10^{-4}\,\pi$ J **Answer: (b)**
30.	At critical temperature, the surface tension of a liquid **(A.I.I.M.S 80)** (a) Is zero (b) Is infinity (c) Is the same as that at any other temperature (d) Can not be determined

	Answer: (a)
31.	Out of the following. Which one is not an example of capillary action? (a) Ploughing of the field (b) Absorption of ink in a blotting paper (c) Floating of wood on the surface of water (d) Rise of oil in the wick of a lamp **Answer: (c)**
32.	A capillary tube is placed vertically in a liquid. If the cohesive force is less than the adhesive force, then (a) The meniscus will be convex upwards (b) The liquid will wet the solid (c) The angle of contact will be obtuse (d) The liquid will drip in the capillary tube **Answer: (b)**
33.	If the surface of a liquid is plane, then the angle of contact of the liquid with the walls of container is **(MHT-CET-2004)** (a) Acute angle (b) Obtuse angle (c) 90° (d) 0° **Answer: (d)**
34.	The surface of water in contact with glass wall is **(MHT-CET-2004)** (a) Plane (b) concave (c) convex (d) Both 'b' and 'c' **Answer: (b)**
35.	Water rises to a height of 10 cm in a capillary tube, and mercury falls to a depth of 3.42 cm in the same capillary tube. If the density of mercury is 13.6 gm/cm^3 and the angle of contact is 135°, the ratio of surface tension for water and mercury is **(PMT 88)** (a) 1:0.5 (b) 1:3 (c) 1:6.5 (d) 1.5:1 **Answer: (c)**
36.	Surface tension may be defined as **(CPMT 90)** (a) The work done per unit area in increasing the surface area of a liquid under isothermal condition (b) The work done per unit area in increasing the surface area of a liquid under adiabatic condition (c) The work done per unit area in increasing the surface is of a liquid under both isothermal and adiabatic conditions. (d) Free surface energy per init volume **Answer: (a)**
37.	The surface tension for pure water in a capillary tube experiment is **(MHT CET 2002)** (a) $\frac{3g}{2hr}$ (b) $\frac{3}{hr\,pg}$ (c) $\frac{r\,pg}{2h}$ (d) $\frac{hr\,pg}{2}$ **Answer: (d)**
37.	A liquid is kept in a glass vessel. If the liquid solid adhesive force between the liquid and the vessel is very weak as compared to the cohesive force in the liquid, then the shape of the liquid surface near the solid should be

(a) Concave (b) Convex
(c) Horizontal (d) Almost vertical
Answer: (b)

39. The height of a liquid in a fine capillary tube
(a) Increases with an increase in the density of a liquid
(b) Decreases with a decrease in the diameter of the tube
(c) Decreases with an increase in the surface tension
(d) Increases as the effective value of acceleration due to gravity is decreased
Answer: (d)

40. When a soap bubble is charged **(MNR 88)**
(a) It contracts
(b) It expands
(c) It does not undergo any change in size
(d) None of these
Answer: (b)

41. If common salt is dissolved in water, then the S.T. of salt water is
(a) Increased
(b) Decreased
(c) Not changed
(d) First decreases and then increases
Answer: (a)

42. In a capillary tube, fall of liquid is possible when angle of contact is **(MHT CET 2066)**
(a) Acute angle (b) Right angle
(c) Obtuse angle (d) None of these
Answer: (c)

43. Energy needed in breaking a drop of radius R into n drops of radius r, is **(CPMT 82)**
(a) $(4\pi r^2 n - 4\pi R^2)$ (b) $(\frac{4\pi}{3} n \pi r^2 \frac{4}{3} \pi R^2)$
(c) $(4\pi R^2 - 4\pi r^2)nT$ (d) $(4\pi R^2 - n\,4\pi r^2)T$
Answer: (a)

44. One thousand small water droplets of equal size combine to form a big drop. The ratio of the final surface energy to the initial surface energy is
(Surface tension of water = 70 dyne/cm)
(MHT CET 99)
(a) 10:1 (b) 1000:1
(c) 1:10 (d) 1:1000
Answer: (c)

45. A spherical water drop of radius R is split up into 8 equal droplets. If T is the surface tension of water, then the work done in this process is
(a) $4\pi R^2 T$ (b) $8\pi r^2 T$
(c) $3\pi R^2 T$ (d) $2\pi r^2 T$
Answer: (a)

46. Water can rise up to a height of 12 cm in a capillary tube. If the tube is lowered to keep only 9 cm above the water level then the water at the upper end of the capillary will **(MHT-CET 2000)**
(a) Overflow
(b) From a convex surface
(c) From a flat surface
(d) From a concave surface
Answer: (c)

47.	A square frame of length L is immersed in soap solution and taken out. The force experienced by the square plate is **(MHT-CET-2007)** (a) TL (b) 2TL (c) 4TL (d) 8TL **Answer: (d)**
48.	A drop of oil is placed on the surface of water. Which of the following statement is correct? **(NCERT 76)** (a) It will remain on it as a sphere (b) It will spread as a thin layer (c) It will partly be as spherical droplets and partly as thin film (d) It will float as distorted drop on the water surface. **Answer: (b)**
49.	A mercury drop of radius 1 cm is broken into 10^6 droplets of equal size. The work done is ($T = 35 \times 10^{-2}$ N/m) **(Roorkee 84)** (a) 4.35×10^{-2} J (b) 4.35×10^{-3} J (c) 4.35×10^{-6} J (d) 4.35×10^{-8} J **Answer: (a)**
50.	Plants get water through the roots because of **(CPMT 83)** (a) Capillarity (b) Viscosity (c) Gravity (d) Elasticity Answer: (a)

51.	A drop of liquid of diameter 2.8 mm beaks up into 125 identical drops. The change in energy is nearly (S.T. of liquid = 75×10^{-3} N/m) **(CPMT 89)** (a) Zero (b) 19×10^{-7} J (c) 46×10^{-7} J (d) 74×10^{-7} J **Answer: (d)**
52.	Amount of energy required to blow a bubble of radius 5 cm, is (Surface tension of soap is 30×10^{-2} N/m) **(MHT-CET-2002)** (a) 1.88 J (b) 1.88×10^{-1} J (c) 1.88×10^{-2} J (d) 1.88×10 J **Answer: (c)**
53.	The surface tension of a liquid is T. the increase in its surface energy on increasing the surface area by A is **(MPPET-91)** (a) AT^{-1} (b) AT (c) A^2T (d) A^2T^2 **Answer: (b)**
54.	A liquid does not wet the surface of a solid if the angle of contact is **(AFMC Pune 88)** (a) Zero (b) An acute one (c) 45° (d) An obtuse one **Answer: (d)**
55.	The pressure just below the meniscus of water **(NCERT 76)** (a) Is greater than just above it

	(b) Is less than just above it (c) Is same as just above it (d) Is always equal to atmospheric pressure. **Answer: (b)**
56.	5 g of water rises in the bore of capillary tube when it is dipped in water. If the radius of bore capillary tube is doubled, the mass of water that rises in the capillary tube above the outside water level is **(MHT-CET 2001)** (a) 1.5 g (b) 10 g (c) 5 g (d) 15 g **Answer: (b)**
57.	When a capillary tube is immersed vertically in water the capillary rise is 3 cm. if the same capillary tube is inclined at angle of 60° to the vertical, the length of the water column in the capillary tube above that of the outside level is **(MHT CET 2003)** (a) 6 cm (b) 1 cm (c) 8 cm (d) Zero **Answer: (a)**
58.	Water rises up to a height of 5 cm in a capillary tube of radius 2 mm. what is the radius of the radius of the capillary tube if the water rises up to a height of 10 cm in another capillary? (a) 4 mm (b) 1 mm (c) 3 mm (d) 1 cm **Answer: (b)**
59.	Water rises up to a height of 4 cm, in a capillary tube immersed vertically in water. What will be the length of water column in the capillary tube, if the tube is immersed in water, at an angle of 60° with the vertical? (a) 4 cm (b) 6 cm (c) 8 cm (d) 2 cm **Answer: (c)**
60.	Work done in blowing a liquid drop to radius R is W_1 and that to radius 3R is W_2. the ratio of work done is **(MHT-CET-2005)** (a) 1:3 (b) 1:4 (c) 1:2 (d) 1:9 **Answer: (d)**
61.	Potential energy of a molecule on the surface of a liquid is as compare to another molecule inside of the liquid is **(MHT-CET-2008)** (a) More (b) Less (c) Both 'a' and 'b' (d) None of these **Answer: (a)**
62.	Rain drops are spherical because of **(MHT CET 2002)** (a) Gravitational force (b) Surface tension (c) Air resistance (d) Low viscosity of water **Answer: (b)**
63.	Pressure inside two soap bubbles is 1.01 and 1.02 atmospheres. ratio between their volume is **(PMT 91)** (a) 102:101 (b) $(102)^3:(101)^3$ (c) 8:1 (d) 2:1

	Answer: (c)
64.	Excess pressure inside a bubble of radius r and of a liquid of surface tension T is **(MHT CET 2000)** (a) $\frac{T}{r}$ (b) $\frac{2T}{r}$ (c) $\frac{3T}{r}$ (d) $\frac{4T}{r}$ **Answer: (d)**
65.	A square frame of side l is dipped in a liquid soap when it is taken out of the liquid, a film is formed on it. If surface tension is T then force acting on it is **(MHT CET 99)** (a) 2T l (b) 8T l (c) 4T l (d) 16T l **Answer: (b)**
66.	Two soap bubbles with radii 3 cm and 4 cm combine to form a bubble of large radius R, under isothermal condition. Then R is approximately equal to **(MHT CET 2001)** (a) $(3^3 + 4^3)^{1/3}$ cm (b) $(3^3 + 4^3)^{1/2}$ cm (c) $(3^2 + 4^2)^{1/2}$ cm (d) $(3^3 + 4^2)^{1/3}$ cm **Answer: (c)**
67.	Meniscus of mercury in capillary is **(PMT MP 88)** (a) Concave (b) Convex (c) Plane (d) Cylindrical **Answer: (b)**
68.	Surface tension of liquid is independent of the (a) Temperature of the liquid (b) Area of the liquid surface (c) Nature of the liquid (d) Impurities present in the liquid **Answer: (b)**
69.	For a liquid which is rising in a capillary, he angle of contact is **(MHT-CET-2005)** (a) Obtuse (b) Acute (c) 180° (d) 90° **Answer: (b)**
70.	Two capillary tubes of the same material but of different radii are dipped in a liquid. The heights to which the liquid rises in the two tubes are 2.2 cm and 6.6 cm. the ratio of radii of the tubes will be **(MPPET 90)** (a) 1:9 (b) 1:3 (c) 9:1 (d) 3:1 **Answer: (d)**
71.	The dimension of surface tension are **(MHT-CET-2002)** (a) $[M LT^{-1}]$ (b) $[M L^2 T^{-2}]$ (c) $[M L^0 T^{-2}]$ (d) $[M L^{-1} T^{-2}]$ **Answer: (c)**
72.	Two soap bubbles have radii in the ratio 2:1. What is the ratio excess of pressure inside them? **(NCERT 90)** (a) 1:2 (b) 2:1 (c) 1:4 (d) 4:1

	Answer: (a)
73.	Water rises to a height of 2 cm in a capillary tube held vertically. When the tube is tilted 60° from the vertical, the length of the water column in the tube will be **(MHT-CET 99)** (a) 2 CM (b) 1 CM (c) 3 CM (d) 4 CM **Answer: (d)**
74.	Find the difference of air pressure between the inside and out side of a soap bubble of 5 mm diameter, if the surface tension is 1.6 N/m **(CPMT 92)** (a) 2560 N/m^2 (b) 3720 N/m^2 (c) 1208 N/m^2 (d) 10132 N/m^2 **Answer: (a)**
75.	For a water does not wet a glass rod, the angle of contact is **(MHT-CET-2006)** (a) Obtuse (b) Acute (c) 0° (d) 90° **Answer: (a)**

76.	The S.T. of soap solution is 25×10^{-3} N/m. the excess of pressure inside a soap bubble of diameter 1 cm is **(AIIMS 87)** (a) 10 Pa (b) 20 Pa (c) 5 Pa (d) None of these **Answer: (b)**
77.	When a liquid rises inside a capillary tube, the weight of the liquid in the capillary tube is supported (a) Entirely by atmospheric pressure (b) Entirely by the force due to surface tension (c) Partly by the force due to surface tension and partly by atmospheric pressure (d) Entirely due to the upward component of the reaction (R) to the surface tension **Answer: (d)**
78.	A liquid is kept in a glass beaker. Which molecules of the liquid have the highest potential energy? (a) Molecules at the bottom of the beaker (b) Molecules near the centre of the liquid (c) Molecules lying at half the depth of the liquid and touching the walls of the beaker (d) Molecules lying in the surface film **Answer: (d)**
79.	Work done in blowing a soap bubble of diameter 2cm, is (S.T. = 3×10^{-2} N/m) (a) 7.54×10^{-5} J (b) 7.54×10^{-6} J (c) 7.54×10^{-3} J (d) 7.54 J Answer: (a)
80.	Kerosene in the wick of lantern rises up because **(MNR 86)** (a) Of negligible viscosity (b) The diffusion of the oil through the wick (c) Of the surface tension of the oil (d) Wick attracts the kerosene

	Answer: (c)
81.	A soap bubble has radius 2 cm. the work done in bobbling the radius is (Surface tension is 30 dynes/cm). **(MNR-80)** (a) 9050 erg (b) 4525 erg (c) 8050 erg (d) 3525 erg **Answer: (a)**
82.	Two soap bubbles of radii 4 cm and 3 cm respectively coalesce under isothermal conditions to form a single bubble. What is the radius of the new single bubble? (a) 3 cm (b) 4 cm (c) 5 cm (d) 6 cm **Answer: (c)**
83.	A film of water is formed between two straight parallel wires, each of length 10 cm and separated by 4 mm. how much work should be done to increase their separation by 1 mm, while still maintaining their parallelism? ($T = 7.5 \times 10^{-2}$ N/m) (a) 0.5×10^{-5} J (b) 1.2×10^{-5} J (c) 1.5×10^{-5} J (d) 15×10^{-7} J **Answer: (c)**
84.	At the boiling point of water, its surface tension (a) Is infinite (b) Is zero (c) Is the same as that at room temperature (d) Is maximum **Answer: (b)**
85.	The surface tension of a liquid is 10^8 dyne/cm^2. it is equivalent to **(MHT-CET 1999)** (a) 10^7 N/m (b) 10^6 N/m (c) 10^5 N/m (d) 10^4 N/m **Answer: (a)**
86.	Nacl dissolved (added) in to water than it surface tension is **(MHT-CET-2008)** (a) Decreases (b) Increases (c) Remains same (d) All of these **Answer: (b)**
87.	The work done in splitting a drop of water of 1 mm radius into 10^6 droplets is **(PMT MP 88)** (a) 9.98×10^{-5} J (b) 8.95×10^{-5} J (c) 5.89×10^{-5} J (d) 5.98×10^{-5} J **Answer: (b)**
88.	Out of the following, which is not an example of capillary action **(MHT-CET-2006)** (a) Absorption of ink in blotting paper (b) Floating of wood on water surface (c) Rise of oil wick of a lamp (d) Ploughing of the field **Answer: (b)**
89.	The radius of a soap bubble is r. the surface tension of soap solution is T. keeping temperature constant, the radius of the soap bubble is doubled, the energy necessary for this will be **(CPMT 91)** (a) $24 \pi r^2 T$ (b) $8 \pi r^2 T$ (c) $12 \pi r^2 T$ (d) $16 \pi r^2 T$ **Answer: (a)**
90.	The surface of water in contact with glass wall is

	(a) Plane (b) Convex (c) Concave (d) Either convex or concave **Answer: (c)**
91.	Water rises to a height of 16.3 cm in a capillary tube of height 18 cm above the water level. If the tube is cut at a height of 12 cm **(C.P.M.T. 74)** (a) Water will come out in the form of fountain from the capillary tube (b) Water will stay at height of 12 cm in the capillary (c) The height of water in the capillary will be 10.3 cm (d) Water will flow down the sides of capillary tube **Answer: (b)**
92.	More liquid rises in a thin tube because of **(CPMT 87)** (a) Larger value of radius (b) Larger value of surface tension (c) Smaller value of S.T. (d) Smaller value of radius **Answer: (d)**
93.	A spherical liquid drop of radius R is divided into eight equal droplets. If surface tension is T, then the work done in this process will be **(CPMT 90)** (a) $2\pi R^2 T$ (b) $3\pi R^2 T$ (c) $4\pi R^2 T$ (d) $2\pi R T^2$ **Answer: (c)**
94.	Excess pressure inside a soap bubble is **(CPMT 92)** (a) Inversely proportional to its radius (b) Directly proportional to its radius (c) Directly proportional to square roots of its radius (d) Independent of its radius **Answer: (a)**
95.	When a liquid rises inside a capillary tube, the weight of the liquid in the tube is supported **(MHT-CET-200)** (a) By atmospheric pressure (b) Partly by atmospheric pressure and partly by surface tension (c) Entirely by the force due to surface tension (d) Partly by the force due to surface tension **Answer: (c)**
96.	The surface tension of a soap solution is 0.035 N/m. the energy needed to increase the radius of the bubble from 4 cm to 6 cm is **(MHT-CET-2007)** (a) 1.5×10^{-3} J (b) 1.5×10^{-2} J (c) 3×10^{-2} J (d) 1.5×10^{-4} J **Answer: (a)**
97.	Two spherical soap bubbles of a radii r_1 and r_2 in vacuum coalesce under isothermal conditions. The resulting bubble has the radius R such that **(MHT-CET-2001)** (a) $R = r_1 + r_2$ (b) $R = \frac{r_1 r_2}{r_1 + r_2}$ (c) $R^2 = r_1^2 + r_2^2$ (d) $R = \frac{r_1 + r_2}{r_2}$

	Answer: (c)
98.	When two capillary tube of different diameters are dipped vertically the rise of the liquid is **(MNR 87, NCERT 78)** (a) Same in both the tubes (b) More in tube of larger diameter. (c) Less in tube of smaller diameter (d) More in the tube of smaller diameter **Answer: (d)**
99.	The work done in blowing a bubble of radius R is W, then the work done in making a bubble of radius 2R from the same solution is, **(MHT-CET 99, 2005)** (a) $\frac{W}{2}$ (b) 2W (c) 4W (d) $2^{\frac{1}{3}}W$ **Answer: (c)**
100.	Two drops of a liquid are merged to from a single drop. In this process **(MHT CET 2000, 2005, PMT Delhi 82)** (a) Energy is released (b) Energy is absorbed (c) Energy is remains constant (d) First 'B' then 'C' **Answer: (a)**

CH 10

Mechanical properties of Fluids

(5 Hours, 4 Marks (1M-2Q, 2M-1Q)

Syllabus :

Pressure due to a fluid column; Pascal's law and its applications (hydraulic lift and hydraulic brakes). Effect of gravity on fluid pressure.
Viscosity, Stokes' law, terminal velocity, Reynold's number, streamline and turbulent flow. Critical velocity. Bernoulli's theorem and its applications.
Surface energy and surface tension, angle of contact, excess of pressure, application of surface tension ideas to drops, bubbles and capillary rise.

1. Introduction :

Fluid is a matter in a state which can flow. Liquids, gases, molten metal and tar are examples of fluids.
• Fluid mechanics is studied in two parts:
(i) Fluid statics - Study of the forces and pressures acting on stationary fluid. Pascal's law and Archimedes' principle and surface tension are discussed in fluid statics.
(ii) Fluid dynamics - Study of motion of fluid and properties related to it as a result of forces acting on fluid. Bernoulli's theorem and its applications and viscosity of fluid are discussed here. Fluid dynamics is studied in two sections: Hydrodynamics and Aerodynamics.

1. Pressure due to a fluid column; Pascal's law and its applications (hydraulic lift and hydraulic brakes). Effect of gravity on fluid pressure :

1. Define Pressure, Density, Pascal's Law. Explain various applications of Pascal's law ?

Pressure is the force acting on a surface per unit area in a direction perpendicular to it. It is a scalar quantity and its SI unit is N / m^2 named pascal (P_a) in honour of the French scientist Blasé Pascal. Its dimensional formula is $M^1 L^{-1} T^{-2}$. Thus,

Pressure, P (Pa) = $\frac{Force\ F}{Area\ A}$

A bigger unit of pressure is 'bar'. 1 bar = 10^5 Pa.
1 atmosphere pressure (atm) = 1.013 × 10^5 Pa or N /m^2 = 760 mm (76 cm) of Hg column.

Density :

Density is the ratio of mass to volume of an object. It is a scalar quantity and its S I unit is kg/m^3. Liquids are almost incompressible. Hence, the density of a liquid remains almost constant at a given temperature for small change in the value of pressure. Gases are compressible. Hence, the volume of gas decreases and density increases with increase of pressure.

Relative density / Specific density / Specific gravity:

"Relative density also known as specific density or specific gravity of a given substance is the ratio of its density to the density of water at 277 K (i.e., 4° C)."
It is a dimensionless quantity and hence does not have a unit. Also,
Relative (specific) density of an object = Mass of an Object / Mass of the same volume of water at 277 K

Pascal's Law

"A change in pressure applied to an enclosed (incompressible) fluid is transmitted undiminished to every point of the fluid and the walls of the containing vessel." This statement is known as Pascal's law.

Pascal's law is also given as "If the effect of gravitation is neglected, the pressure at every point in an incompressible liquid, in equilibrium, is the same."

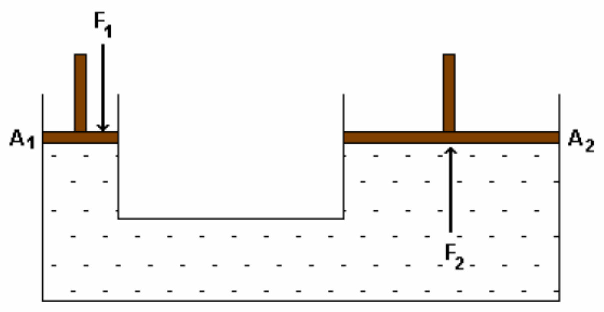
Applications of Pascal's Law:

(1) Hydraulic lift & Brakes :

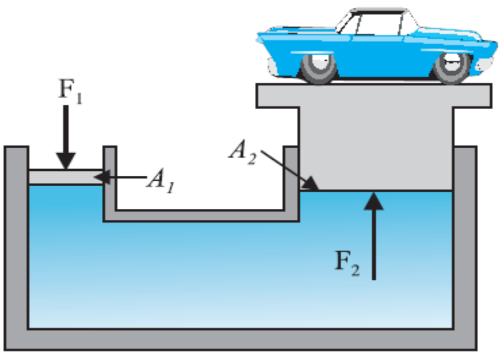
The figure shows the principle of a hydraulic lift used to raise heavy loads. This device has two vertical cylinders of different diameters connected by a horizontal tube. A liquid is filled in this vessel. Airtight pistons having cross-sectional areas A_1 and A_2 ($A_1 < A_2$) are fitted touching the liquid surface in both the cylinders. According to Pascal's law, in equilibrium, the pressure on liquid in both the arms is the same. Hence,

$$\frac{F_1}{A_1} = P_1 = P_2 = \frac{F_2}{A_2} \Rightarrow F_2 = F_1\left(\frac{A_2}{A_1}\right)$$

Thus, a large force, F_2, is generated using a small force, F_1, as the magnitude of F_2 is A_2/A_1 times the magnitude of F_1. Using Pascal's law, devices like hydraulic lift, hydraulic jack, hydraulic brake and hydraulic press are developed.



Principle of a hydraulic lift

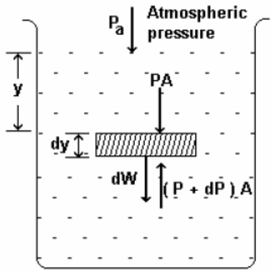


Hydraulic brakes in automobiles also work on the same principle. When we apply a little force on the pedal with our foot the master piston moves inside the master cylinder, and the pressure caused is transmitted through the brake oil to act on a piston of larger area. A large force acts on the piston and is pushed down expanding the brake shoes against brake lining. In this way a small force on the pedal produces a large retarding force on the wheel.

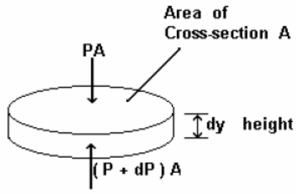
An important advantage of the system is that the pressure set up by pressing pedal is transmitted equally to all cylinders attached to the four wheels so that the braking effort is equal on all wheels.

(2) Pressure due to a fluid column:

For liquid of density ρ in a static equilibrium in a container, pressure at all points at the same depth (or in other words, at the same horizontal layer) is the same.



Pressure due to liquid column



Cylindrical volume of element

Consider an imaginary cylindrical volume element of height dy and cross-sectional area A at the depth y from the surface of liquid as shown in the above figure.

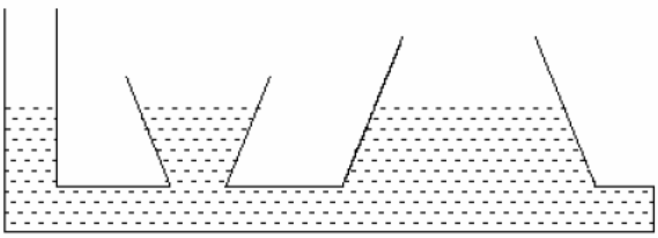
The weight of liquid in this volume element is dW = ρ g A dy
If P and P + dP are the pressures on the upper and lower faces of the element, then PA and (P + dP) A are the forces acting on them respectively. In equilibrium,
PA + dW = (P + dP) A
$\therefore$ PA + ρ g A dy = PA + A dP
$\therefore$ dP/dy = ρ g
This equation is valid for any fluid (liquid or gas). It shows that the pressure increases with increase in the depth. Here, ρ g is the weight density, i.e., weight per unit volume of the fluid. Its value for water is 9800 N / m3. Pressure P at the depth y = h can be obtained by integration as under

$$\int_{P_a}^{P} dP = \int_{0}^{h} \rho g \, dy$$

As ρ is independent of pressure and constant for liquid, the above integration gives P - Pa = ρ g h
$\therefore P = P_a + \rho g h$
This equation is valid only for incompressible fluid, i.e., liquid and gives the pressure at depth h in a liquid of density ρ.
Here, P ($= P_a + \rho g h$) is the absolute pressure whereas P - P_a (= ρ g h) is the **gauge pressure** also known as the **hydrostatic pressure**. In general,

$P_2 - P_1$ (= ρ g h) -------- (10.1)

The pressure at any point in a liquid does not depend on the shape or cross-sectional area of its container. This is known as hydrostatic paradox.
When liquid is filled in the containers of different shapes and sizes, joined at the bottom as shown in the figure, the height of liquid columns in all the containers is found to be the same.



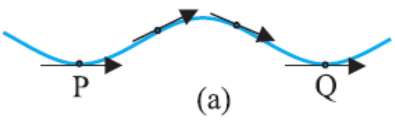
(3) Buoyancy and Archimedes' principal :
" When a body is partially or completely immersed in a liquid, the buoyant force acting on it is equal to the weight of the displaced liquid and it acts in the upward direction at the centre of gravity of the displaced liquid."
This statement was given by Archimedes and is known as Archimedes' principle. **Buoyant force = weight of the displaced liquid (or any fluid, i.e., liquid or gas) = decrease in the weight of the immersed body.**

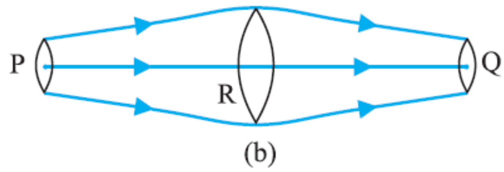
2. Streamline and turbulent flow. Critical velocity. Bernoulli's theorem and its applications :

2. Explain Streamlines, Tube of flow and Equation of continuity ?

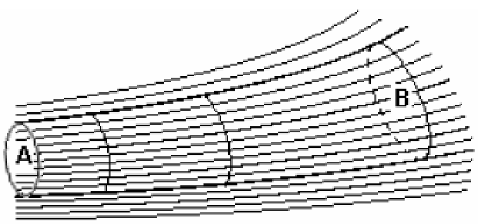
The path of motion of a fluid particle is called a line of flow. In a **steady flow**, velocity of each particle arriving at a point on this path remains constant with time. Hence, every particle reaching this point moves in the same direction with the same speed. However, when this particle moving on the flow line reaches a different point, its velocity may be different.

But this different velocity also remains constant with respect to time. The path so formed is called a streamline and such a flow is called **a streamline flow**. In unsteady flow, flow lines can be defined, but they are not streamlines as the velocity at a point on the flow line may not remain constant with time.
The path taken by a fluid particle under a steady flow is a **streamline**. It is defined as a curve whose tangent at any point is in the direction of the fluid velocity at that point.





Streamlines do not intersect each other, because if they do then two tangents can be drawn at the point of intersection and the particle may move in the direction of any tangent which is not possible.
Tube of flow:
The tubular region made up of a bundle of streamlines passing through the boundary of any surface is called a tube of flow. The tube of flow is surrounded by a wall made of streamlines. As the streamlines do not intersect, a particle of fluid cannot cross this wall. Hence the tube behaves somewhat like a pipe of the same shape.



Equation of continuity :

Consider planes perpendicular to the direction of fluid flow e.g., at three points P, R and Q in Fig.1 (b). The plane pieces are so chosen that their boundaries be determined by the same set of streamlines. This means that number of fluid particles crossing the surfaces as indicated at P, R and Q is the same. If area of cross-sections at these points are A_P, A_R and A_Q and speeds of fluid particles are v_P, v_R and v_Q, then mass of fluid Δm_P crossing at A_P in a small interval of time Δt is $\rho_P A_P v_P\ \Delta t$. Similarly mass of fluid Δm_R flowing or crossing at A_R in a small interval of time Δt is $\rho_R A_R v_R$ Ät and mass of fluid Δm_Q is $\rho_Q A_Q v_Q\ \Delta t$ crossing at A_Q. The mass of liquid flowing out equals the mass flowing in, holds in all cases. Therefore,

$\rho_P A_P v_P \Delta t = \rho_R A_R v_R \Delta t = \rho_Q A_Q v_Q \Delta t$ --------- (10.1)

For flow of incompressible fluids

$\rho_P = \rho_R = \rho_Q$

Equation (10.1) reduces to

$A_P v_P = A_R v_R = A_Q v_Q$ --------------- (10.2)

which is called the **equation of continuity** and it is a statement of conservation of mass in flow of incompressible fluids. In general **Av = constant** ---------- (10.3)

Av gives the volume flux or flow rate and remains constant throughout the pipe of flow.

Thus, at narrower portions where the streamlines are closely spaced, velocity increases and its vice versa. From (Fig 10.1b) it is clear that $A_R > A_Q$ or $v_R < v_Q$, the fluid is accelerated while passing from R to Q. This is associated with a change in pressure in fluid flow in horizontal pipes.

Steady flow is achieved at low flow speeds. Beyond a limiting value, called critical speed, this flow loses steadiness and becomes **turbulent**. One sees this when a fast flowing whirlpool-like regions called ‘white water rapids are formed.

Figure 10.2 displays streamlines for some typical flows. For example, Fig. 10.2(a) describes a laminar flow where the velocities at different points in the fluid may have different magnitudes but their directions are parallel. Figure 10.2 (b) gives a sketch of **turbulent flow**.

Fig. 10.8 (a) Some streamlines for fluid flow.(b) A jet of air striking a flat plate placed perpendicular to it. This is an example of turbulent flow.

3. State and derive Bernoulli’s theorem ? Explain its Applications ?

Bernoulli’s relation may be stated as follows: **As we move along a streamline the sum of the pressure (P), the kinetic energy per unit volume ($\rho v^2/2$) and the potential energy per unit volume (ρgh) remains a constant.**

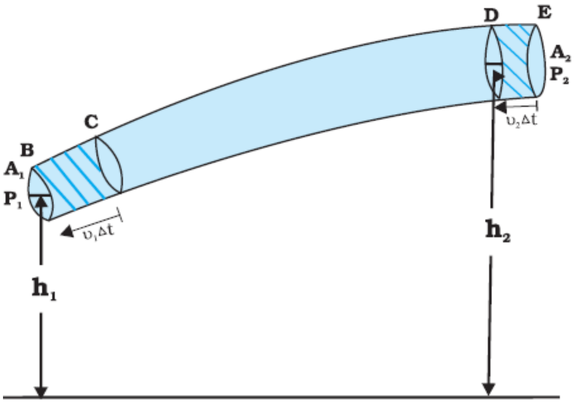


Fig. 10.9 The flow of an ideal fluid in a pipe of varying cross section. The fluid in a section of length v_1 Δt moves to the section of length v_2 Δt in time Δt.

Proof : Consider a fluid moving in a pipe of varying cross-sectional area. Let the pipe be at varying heights as shown in Fig. 10.9. We now suppose that an incompressible fluid is flowing through the pipe in a steady flow. Its velocity must change as a consequence of equation of continuity. A force is required to produce this acceleration, which is caused by the fluid surrounding it, the pressure must be different in different regions. Bernoulli's equation is a general expression that relates the pressure difference between two points in a pipe to both velocity changes (kinetic energy change) and elevation (height) changes (potential energy change). The Swiss Physicist Daniel Bernoulli developed this relationship in 1738.

Consider the flow at two regions 1 (i.e. BC) and 2 (i.e. DE). Consider the fluid initially lying between B and D. In an infinitesimal time interval Δt, this fluid would have moved. Suppose v_1 is the speed at B and v_2 at D, then fluid initially at B has moved a distance $v_1\Delta t$ to C ($v_1\Delta t$ is small enough to assume constant cross-section along BC). In the same interval Δt the fluid initially at D moves to E, a distance equal to $v_2\Delta t$. Pressures P_1 and P_2 act as shown on the plane faces of areas A_1 and A_2 binding the two regions. The work done on the fluid at left end (BC) is $W_1 = P_1A_1(v_1\Delta t) = P_1\Delta V$. Since the same volume ΔV passes through both the regions (from the equation of continuity) the work done by the fluid at the other end (DE) is $W_2 = P_2A_2(v_2\Delta t) = P_2\Delta V$ or, the work done on the fluid is $-P_2\Delta V$. So the total work done on the fluid is $W_1 - W_2 = (P_1 - P_2)\,\Delta V$

Part of this work goes into changing the kinetic energy of the fluid, and part goes into changing the gravitational potential energy. If the density of the fluid is ρ and $\Delta m = \rho A_1 v_1 \Delta t = \rho\Delta V$ is the mass passing through the pipe in time Δt, then change in gravitational potential energy is

$\Delta U = \rho g \Delta V\,(h_2 - h_1)$

The change in its kinetic energy is

$$\Delta K = \left(\frac{1}{2}\right) \rho\ \Delta V\,(v_2^2 - v_1^2)$$

We can employ the work – energy theorem (Chapter 6) to this volume of the fluid and this yields

$$(P_1 - P_2)\ \Delta V = \left(\frac{1}{2}\right) \rho\ \Delta V\,(v_2^2 - v_1^2) + \rho g \Delta V\,(h_2 - h_1)$$

We now divide each term by ΔV to obtain

$$(P_1 - P_2) = \left(\frac{1}{2}\right) \rho\ (v_2^2 - v_1^2) + \rho g\ (h_2 - h_1)$$

We can rearrange the above terms to obtain

$$P_1 + \left(\frac{1}{2}\right) \rho v_1^2 + \rho g h_1 = P_2 + \left(\frac{1}{2}\right) \rho v_2^2 + \rho g h_2$$ ---------- (

This is **Bernoulli's equation**. Since 1 and 2 refer to any two locations along the pipeline, we may write the expression in general as

$$P + \left(\frac{1}{2}\right) \rho v^2 + \rho g h = \text{constant}$$ --------- (

Thus, Bernoulli's relation may be stated as follows: As we move along a streamline the sum of the pressure (*P*), the kinetic energy per unit volume ($\rho v^2/2$) and the potential energy per unit volume ($\rho g h$) remains a constant.

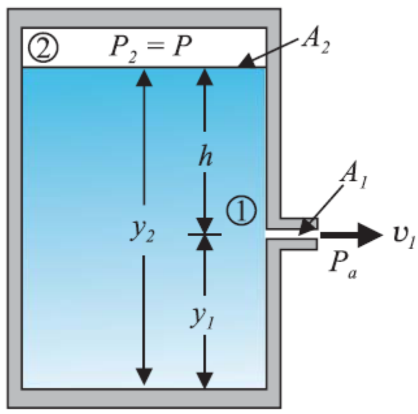
In practice, Bernoulli theorem has a large number of useful applications and can help explain a wide variety of phenomena for low viscosity incompressible fluids. Bernoulli's equation also does not hold for non-steady or turbulent flows, because in that situation velocity and pressure are constantly fluctuating in time. When a fluid is at rest i.e. its velocity is zero everywhere, Bernoulli's equation becomes :

$$P_1 + \rho g h_1 = P_2 + \rho g h_2$$

$$(P_1 - P_2) = \rho g\ (h_2 - h_1)$$ which is same as Eq. (10.1).

Applications of Bernoulli theorem :

(1) Speed of Efflux: Torricelli's Law :



The word efflux means fluid outflow. Torricelli discovered that the speed of efflux from an open tank is given by a formula identical to that of a freely falling body. Consider a tank containing a liquid of density ρ with a small hole in its side at a height y_1 from the bottom (see Fig. 10.10).

The air above the liquid, whose surface is at height y_2, is at pressure *P*. From the **equation of continuity** we have,

$$v_1 A_1 = v_2 A_2$$

$$v_2 = \frac{A_1}{A_2} v_1$$

If the cross sectional area of the tank A_2 is much larger than that of the hole ($A_2 >> A_1$), then we may take the fluid to be approximately at rest at the top, i.e. $v_2 = 0$. Now applying the Bernoulli equation at points 1 and 2 and noting that at the hole $P_1 = P_a$, the atmospheric pressure, we have from Bernoulli's relation,

$$P_1 + \left(\frac{1}{2}\right) \rho v_1^2 + \rho g h_1 = P_2 + \left(\frac{1}{2}\right) \rho v_2^2 + \rho g h_2$$

here, $h_1 = y_1$ and $h_2 = y_2$,
Taking $y_2 - y_1 = h$ we have

$$v_1 = \sqrt{2gh + \frac{2P - P_a}{\rho}}$$

When $P >> P_a$ and $2gh$ may be ignored, the speed of efflux is determined by the container pressure. Such a situation occurs in rocket propulsion. On the other hand if the tank is open to the atmosphere, then $P = P_a$ and

$$v_1 = \sqrt{2gh} \qquad \text{---------} \quad (\;)$$

This is the **speed of a freely falling body**. This Equation is known as **Torricelli's law**.

(2) Venturi-meter :

The Venturi-meter is a device to measure the flow speed of incompressible fluid. It consists of a tube with a broad diameter and a small constriction at the middle as shown in Fig. (10.11). A manometer in the form of a U-tube is also attached to it, with one arm at the broad neck point of the tube and the other at constriction as shown in Fig. (10.11). The manometer contains a liquid of density ρ_m. The speed v_1 of the liquid flowing through the tube at the broad neck area A is to be measured from equation of continuity Eq. (10.10) the speed at the constriction becomes
$v_2 = (A/a)\, v_1$.
Then using Bernoulli's equation, we get

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_1^2 (A/a)^2$$

so that

$$P_1 - P_2 = \frac{1}{2}\rho v_1^2 \left[\left(\frac{A}{a}\right)^2 - 1\right]$$

This pressure difference causes the fluid in the U tube connected at the narrow neck to rise in comparison to the other arm. The difference in height h measure the pressure difference.

$$P_1 - P_2 = \rho_m gh = \frac{1}{2}\rho v_1^2 \left[\left(\frac{A}{a}\right)^2 - 1\right]$$

So that the speed of fluid at wide neck is

$$v_1 = \sqrt{\left(\frac{2\rho_m gh}{\rho}\right)} \left(\left(\frac{A}{a}\right)^2 - 1\right)^{-1/2} \qquad \text{---------}$$

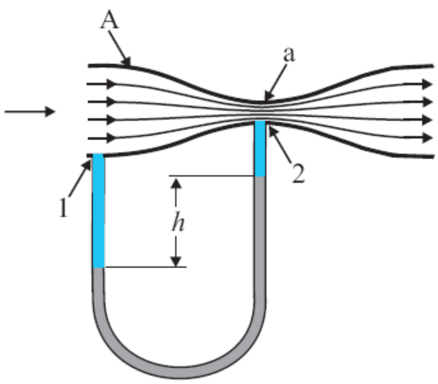


Fig. 10.11 A schematic diagram of Venturi-meter.

The principle behind this meter has many applications. The carburetor of automobile has a Venturi channel (nozzle) through which air flows with a large speed. The pressure is then lowered at the narrow neck and the petrol (gasoline) is sucked up in the chamber to provide the correct mixture of air to fuel necessary for combustion. Filter pumps or aspirators, Bunsen burner, atomisers and sprayers used for perfumes or to spray insecticides work on the same principle.

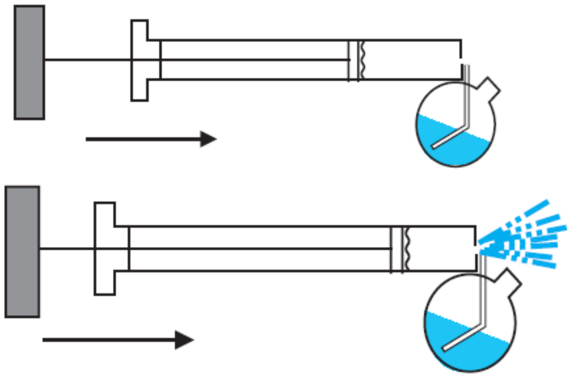


***Fig. :** The spray gun. Piston forces air at high speeds causing a lowering of pressure at the neck of the container.*

(3) Blood Flow and Heart Attack :

Bernoulli's principle helps in explaining blood flow in artery. The artery may get constricted due to the accumulation of plaque on its inner walls. In order to drive the blood through this constriction a greater demand is placed on the activity of the heart. The speed of the flow of the blood in this region is raised which lowers the pressure inside and the artery may collapse due to the external pressure. The heart exerts further pressure to open this artery and forces the blood through. As the blood rushes through the opening, the internal pressure once again drops due to same reasons leading to a repeat collapse. This may result in heart attack.

(4) Dynamic Lift :

Dynamic lift is the force that acts on a body, such as airplane wing, a hydrofoil or a spinning ball, **by virtue of its motion through a fluid**. In many games such as cricket, tennis, baseball, or golf, we notice that a spinning ball deviates from its parabolic trajectory as it moves through air. This deviation can be partly explained on the basis of Bernoulli's principle.

(i) **Ball moving without spin**: Fig. 10.13(a) shows the streamlines around a nonspinning ball moving relative to a fluid. From the symmetry of streamlines it is clear that the velocity of fluid (air) above and below the ball at corresponding points is the same resulting in zero pressure difference. The air therefore, exerts no upward or downward force on the ball.

(ii) **Ball moving with spin**: A ball which is spinning drags air along with it. If the surface is rough more air will be dragged.

Fig 10.13(b) shows the streamlines of air for a ball which is moving and spinning at the same time. The ball is moving forward and relative to it the air is moving backwards. Therefore, the velocity of air above the ball relative to it is larger and below it is smaller. The stream lines thus get crowded above and rarified below.

This difference in the velocities of air results in the pressure difference between the lower and upper faces and there is a net upward force on the ball. This **dynamic lift due to spinning** is called **Magnus effect**.

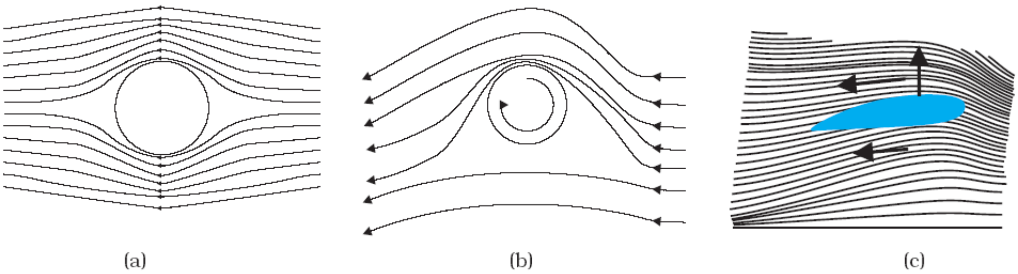


Fig 10.13 *(a) Fluid streaming past a static sphere. (b) Streamlines for a fluid around a sphere spinning clockwise. (c) Air flowing past an aerofoil.*

Aerofoil or lift on aircraft wing: Figure 10.13 (c) shows an aerofoil, which is a solid piece shaped to provide an upward dynamic lift when it moves horizontally through air. The cross-section of the wings of an aeroplane looks somewhat like the aerofoil shown in Fig. 10.13 (c) with streamlines around it. When the aerofoil moves against the wind, the orientation of the wing relative to flow direction causes the streamlines to crowd together above the wing more than those below it. The flow speed on top is higher than that below it. There is an upward force resulting in a dynamic lift of the wings and this balances the weight of the plane.

3. Viscosity, Stokes' law, terminal velocity, Reynold's number :

4. What is viscosity ? Explain co-efficient of viscosity with SI unit & dimension ?

Most of the fluids are not ideal ones and offer some resistance to motion. This resistance to fluid motion is like an internal friction analogous to friction when a solid moves on a surface. It is called **viscosity**. This force exists when there is relative motion between layers of the liquid.
The coefficient of viscosity (pronounced 'eta') for a fluid is defined as the ratio of shearing stress to the strain rate.

$$\eta = \frac{F/A}{v/l} = \frac{F\,l}{vA}$$

The SI unit of viscosity is poiseiulle (Pl). Its other units are N s m^{-2} or Pa s. The dimensions of viscosity are $[ML^{-1}T^{-1}]$. Generally thin liquids like water, alcohol etc. are less viscous than thick liquids like coal tar, blood, glycerin etc.
The relative viscosity (η/η_{water}) of blood remains constant between 0^oC and 37^oC. The viscosity of liquids decreases with temperature while it increases in the case of gases.

5. State & Explain Stokes' law ?

When a body falls through a fluid it drags the layer of the fluid in contact with it. A relative motion between the different layers of the fluid is set and as a result the body experiences a retarding force. Falling of a raindrop and swinging of a pendulum bob are some common examples of such motion. It is seen that the viscous force is proportional to the velocity of the object and is opposite to the direction of motion. The other quantities on which the force *F* depends on viscosity η of the fluid and radius *a* of the sphere. Sir George G. Stokes (1819-1903), an English scientist enunciated clearly the viscous drag force *F* as

$F = 6\pi\eta av$ -------- (1)

This is known as Stokes' law. We shall not derive Stokes' law.

6. Explain terminal velocity ?

The Stokes' law law is an interesting example of retarding force which is proportional to velocity. We can study its consequences on an object falling through a viscous medium. We consider a raindrop in air. It accelerates initially due to gravity. As the velocity increases, the retarding force also increases. Finally when viscous force plus buoyant force becomes equal to force due to gravity, the net force becomes zero and so does the acceleration. The sphere (raindrop) then descends with a constant velocity. Thus in equilibrium, this **terminal velocity** v_t is given by

$6\pi\,\eta a v_t = (4\pi/3)\, a^3\, (\rho-\sigma)g$

where ρ and σ are mass densities of sphere and the fluid respectively. We obtain

$$v_t = 2a^2\,(\rho-\sigma)g\,/\,(9\eta) \quad \text{-------------} \ (\ )$$

So the **terminal velocity** v_t depends on the square of the radius of the sphere and inversely on the viscosity of the medium.

7. Write a note on Reynold's number ?

When the rate of flow of a fluid is large, the flow no longer remain laminar, but becomes turbulent. In a turbulent flow the velocity of the fluids at any point in space varies rapidly and randomly with time. Some circular motions called eddies are also generated. An obstacle placed in the path of a fast moving fluid causes turbulence [Fig. 10.8 (b)]. The smoke rising from a burning stack of wood, oceanic currents are turbulent. Twinkling of stars is the result of atmospheric turbulence. The wakes in the water and in the air left by cars, aeroplanes and boats are also turbulent.

Osborne **Reynolds** (1842-1912) observed that turbulent flow is less likely for viscous fluid flowing at low rates. He defined a dimensionless number, whose value gives one an approximate idea whether the flow would be turbulent. This number is called the **Reynolds R_e.**

$R_e = \rho vd / \eta$ ------- (10.21)

where ρ is the density of the fluid flowing with a speed *v*, *d* stands for the dimension of the pipe, and η is the viscosity of the fluid. R_e is a dimensionless number and therefore, it remains same in any system of units. It is found that flow is streamline or laminar for R_e less than 1000. The flow is turbulent for $R_e > 2000$. The flow becomes unsteady for R_e between 1000 and 2000. The **critical value of R_e** (known as **critical Reynolds number**), at which turbulence sets, is found to be the same for the geometrically similar flows. For example when oil and water with their different densities and viscosities, flow in pipes of same shapes and sizes, turbulence sets in at almost the same value of R_e. Using this fact a small scale laboratory model can be set up to study the character of fluid flow. They are useful in designing of ships, submarines, racing cars and aeroplanes.

R_e can also be written as

$R_e = \rho v^2 / (\eta v/d) = \rho A v^2 / (\eta A v/d)$ ---------- (10.22)

= inertial force/force of viscosity.

Thus R_e represents the ratio of inertial force (force due to inertia i.e. mass of moving fluid or due to inertia of obstacle in its path) to viscous force.

Turbulence promotes mixing and increases the rates of transfer of mass, momentum and energy. The blades of a kitchen mixer induce turbulent flow and provide thick milk shakes as well as beat eggs into a uniform texture.

4. Surface energy and surface tension, angle of contact, excess of pressure, application of surface tension ideas to drops, bubbles and capillary rise :

8. What is surface tension ? How it can be explained based on Surface Energy concept ?

Surface tension is a force per unit length (or surface energy per unit area) acting in the plane of the interface between the plane of the liquid and any other substance; it also is the extra energy that the molecules at the interface have as compared to molecules in the interior.

An extra energy called **surface energy** is associated with surface of liquids, the creation of more surface (spreading of surface) keeping other things like volume fixed requires additional energy.

Expression for surface tension :
Consider a horizontal liquid film ending in bar free to slide over parallel guides Fig (10.17).

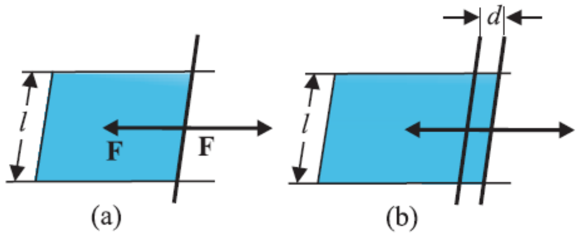


Fig : 10.17

Suppose that we move the bar by a small distance d as shown. Since the area of the surface increases, the system now has more energy, this means that some work has been done against an internal force. Let this internal force be **F**, the work done by the applied force is **F**.**d** = Fd. From conservation of energy, this is stored as additional energy in the film. If the surface energy of the film is S per unit area, the extra area is $2dl$. A film has two sides and the liquid in between, so there are two surfaces and the **extra energy** is

$S(2dl) = Fd$ ------ (10.23)

Or, $S = Fd/2dl = F/2l$ ----- (10.24)

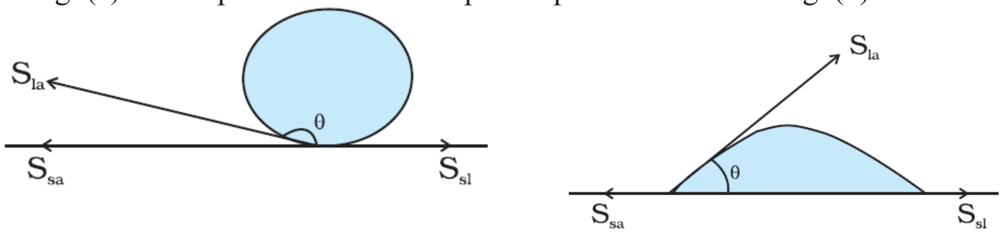
This quantity S is the magnitude of surface tension. It is equal to the surface energy per unit area of the liquid interface and is also equal to the force per unit length exerted by the fluid on the movable bar. Thus, more appropriately, the surface energy is the energy of the interface between two materials and depends on both of them.

At any point on the interface besides the boundary, we can draw a line and imagine equal and opposite surface tension forces S per unit length of the line acting perpendicular to the line, in the plane of the interface. The line is in equilibrium. To be more specific, imagine a line of atoms or molecules at the surface. The atoms to the left pull the line towards them; those to the right pull it towards them! This line of atoms is in equilibrium under tension. If the line really marks the end of the interface, as in Figure 10.16 (a) and (b) there is only the force S per unit length acting inwards.

The value of surface tension depends on temperature. Like viscosity, the surface tension of a liquid usually falls with temperature.

9. How surface tension is related with angle of contact ? Explain ?

The surface of liquid near the plane of contact, with another medium is in general curved. The angle between tangent to the liquid surface at the point of contact and solid surface inside the liquid is termed as angle of contact. It is denoted by θ. It is different at interfaces of different pairs of liquids and solids. The value of θ determines whether a liquid will spread on the surface of a solid or it will form droplets on it. For example, water forms droplets on lotus leaf as shown in Fig. (a) while spreads over a clean plastic plate as shown in Fig. (b).



(a) (b)

We consider the three interfacial tensions at all the three interfaces, liquid-air, solid-air and solid-liquid denoted by S_{la}, S_{sa} & S_{sl} respectively as given in Fig.

At the line of contact, the surface forces between the three media must be in equilibrium. From the Fig. (b) the following relation is easily derived.

$$S_{la} \cos \theta + S_{sl} = S_{sa}$$

The angle of contact is an obtuse angle if $S_{sl} > S_{la}$ as in the case of water-leaf interface while it is an acute angle if $S_{sl} < S_{la}$ as in the case of water-plastic interface. When θ is an obtuse angle then molecules of liquids are attracted strongly to themselves and weakly to those of solid, it costs a lot of energy to create a liquid-solid surface, and liquid then does not wet the solid. This is what happens with water on a waxy or oily surface, and with mercury on any surface. On the other hand, if the molecules of the liquid are strongly attracted to those of the solid, this will reduce S_{sl} and therefore, cos θ may increase or θ may decrease. In this case θ is an acute angle. This is what happens for water on glass or on plastic and for kerosene oil on virtually anything (it just spreads). Soaps, detergents and dying substances are wetting agents. When they are added the angle of contact becomes small so that these may penetrate well and become effective. Water proofing agents on the other hand are added to create a large angle of contact between the water and fibres.

10. Write a note on application of surface tension ideas to drops, bubbles, capillary rise & Detergents ?

(1) Drops and bubbles :

One consequence of surface tension is that free liquid drops and bubbles are spherical if effects of gravity can be neglected.

To answer the questions Why are drops and bubbles spherical? What keeps soap bubbles stable?, To answer these questions, a liquid air interface has energy, so for a given volume the surface with minimum energy is the one with the least area. The sphere has this property. Though it is out of the scope of this book, but you can check that a sphere is better than at least a cube in this respect! So, if gravity and other forces (e.g. air resistance) were ineffective, liquid drops would be spherical.

(a) Excess of pressure :

Another interesting consequence of surface tension is that the pressure inside a spherical drop Fig. 10.20(a) is more than the pressure outside. Suppose a spherical drop of radius r is in equilibrium. If its radius increase by Δr. The extra surface energy is

$[4\pi(r + \Delta r)^2 - 4\pi r^2]\, S_{la} = 8\pi r\, \Delta r\, S_{la}$ ---------

If the drop is in equilibrium this energy cost is balanced by the energy gain due to expansion under the pressure difference (Pi – Po) between the inside of the bubble and the outside. The work done is

$W = (P\mathrm{i} - P\mathrm{o})\, 4\pi r^2 \Delta r$ -------------- (10.28)

so that

$(P\mathrm{i} - P\mathrm{o}) = (2\, S_{la}/\, r)$ -------------- (10.29)

In general, for a liquid-gas interface, the convex side has a higher pressure than the concave side. For example, an air bubble in a liquid, would have higher pressure inside it.
See Fig 10.20 (b).

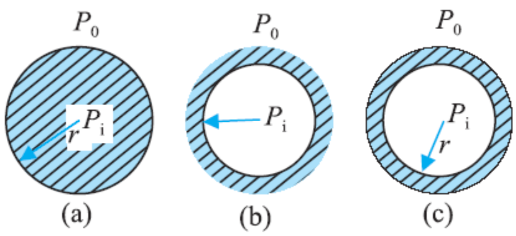


***Fig. 10.20** Drop, cavity and bubble of radius r.*

A bubble Fig 10.20 (c) differs from a drop and a cavity; in this it has two interfaces. Applying the above argument we have for a bubble

$(P\mathrm{i} - P\mathrm{o}) = (4\, S_{la}/\, r)$ --------- (10.30)

This is probably why you have to blow hard, but not too hard, to form a soap bubble. A little extra air pressure is needed inside!

(3) Capillary rise :

One consequence of the pressure difference across a curved liquid-air interface is the well known effect that water rises up in a narrow tube in spite of gravity. The word capilla means hair in latin; if the tube were hair thin, the rise would be very large. To see this, consider a vertical capillary tube of circular cross section (radius a) inserted into an open vessel of water (Fig. 10.21). The contact angle between water and glass is acute. Thus the surface of water in **the capillary is concave. This means that there is a pressure difference between the two sides of the top surface. This is given by**

$(Pi - Po) = (2S/r) = 2S/(a \sec\theta)$

$= (2S/a) \cos\theta$ (10.31)

Thus the pressure of the water inside the tube, just at the meniscus (air-water interface) is less than the atmospheric pressure. Consider the two points A and B in Fig. 10.21(a). They must be at the same pressure, namely

$P0 + h\,\rho\, g = Pi = PA$ (10.32)

where ρ is the density of water and h is called the capillary rise [Fig. 10.21(a)]. Using Eq. (10.31) and (10.32) we have

$h \rho g = (P_i - P_0) = (2S \cos \theta)/a$ ----------- (10.33)

$h = 2S/(\rho g a)$ ------- ( )

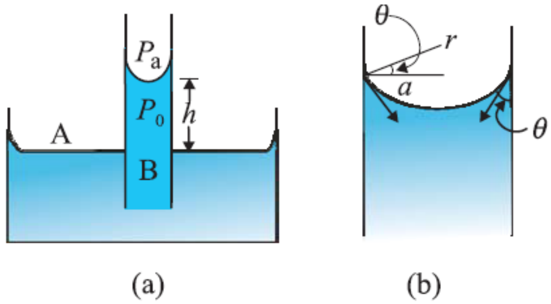


Fig. 10.21 *Capillary rise, (a) Schematic picture of a narrow tube immersed water. (b) Enlarged picture near interface.*

The Eqs. (10.28) and (10.29) make it clear that the capillary rise is due to surface tension. It is larger, for a smaller a. Typically it is of the order of a few cm for fine capillaries.

(3) Detergents and Surface Tension :

Washing with water does not remove grease stains. This is because water does not wet greasy dirt; i.e., there is very little area of contact between them. If water could wet grease, the flow of water could carry some grease away.

Something of this sort is achieved through detergents. The molecules of detergents are hairpin shaped, with one end attracted to water and the other to molecules of grease, oil or wax, thus tending to form water-oil interfaces. The result is shown in Fig. 10.22 as a sequence of figures.

In our language, we would say that addition of detergents, whose molecules attract at one end and say, oil on the other, reduces drastically the surface tension *S* (water-oil). It may even become energetically favourable to form such interfaces, i.e., globs of dirt surrounded by detergents and then by water. This kind of process using surface active detergents or surfactants is important not only for cleaning, but also in recovering oil, mineral ores etc.

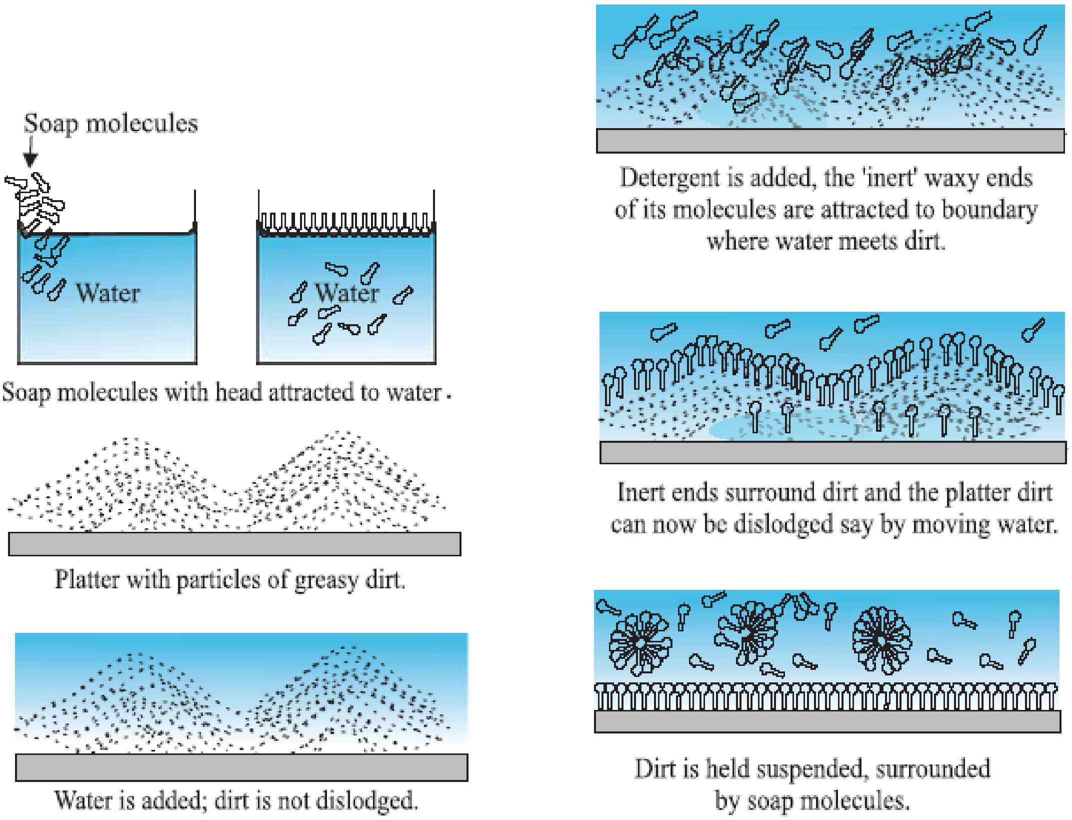


Fig. 10.22 *Detergent action in terms of what detergent molecules do.*

Physical Quantity	Symbol	Dimensions	Unit	Remarks
Pressure	P	$[M L^{-1} T^{-2}]$	pascal (Pa)	1 atm = 1.013×10^5 Pa, Scalar
Density	ρ	$[M L^{-3}]$	kg m^{-3}	Scalar
Specific Gravity		No	No	$\frac{\rho_{substance}}{\rho_{water}}$, Scalar
Co-efficient of viscosity	η	$[M L^{-1} T^{-1}]$	Pa s or poiseiulles (Pl)	Scalar
Reynold's Number	R_e	No	No	$R_e = \frac{\rho v d}{\eta}$ scalar
Surface Tension	S	$[M T^{-2}]$	N m^{-1}	Scalar

1 Marks Questions

1. What are fluids?
The materials that can flow are called fluids.

2. How are fluids different from solids?
Fluid has no definite shape of its own.

3. Define thrust of a liquid.
The total normal force exerted by a fluid on any surface in contact with it is called thrust of a liquid.

4. Define liquid pressure.
Liquid pressure is defined as the normal force acting per unit area.

5. Is pressure a scalar quantity?
Yes

6. Write the dimensions of pressure?
$ML^{-1} T^{-2}$

7 What is the SI unit of pressure?
Nm^{-2}

8. What is the atmosphere pressure at sea level in Pascal?
1.013×10^5 Pa

9. The blood pressure in human is greater at the feet then at brain Why?
The height of blood column is large at the feet than at the brain as a result blood pressure is human is grater at the feet.

10. Write the dimensions of density?
ML^{-3}

11. What is the SI unit of density?
Kg m^{-3}

12. Is density scalar or vector quantity?
Scalar quantity.

13. What is the density of water at 40c ?
1.0×10^3 Kg m^{-3}

14. Define relative density.
The relative density of a substance is the ratio of its density to the density of water at 4^0 C.

15. State Pascal's law.
Whenever external pressure is applied on any part of a fluid contained in a vessel it is transmitted undiminished and equally in all directions.

16. What is gauge pressure?
The gauge pressure is the difference of the actual pressure & the atmospheric pressure.

17. Write the expression for pressure exerted by a fluid .
P = r gh

18. Define torr.
A pressure equivalent of 1mm is called a torr.

19. Write the relation between torr & Pascal.
1 torr = 133Pa

20. Write the relation between bar & Pascal.
1 bar = 10^5Pa

21. Name the devices they work on the basis of Pascal's law.
Hydraulic lift, hydraulic brakes

22. What is fluid dynamics?
The study of the fluids in motion is known as fluid dynamics.

23. Define streamline flow.
The regular and orderly flow of a liquid is known as streamline flow.

24. Define turbulent flow.
The irregular & disorderly flow of a liquid is known as turbulent flow.

25. Write the equation of continuity.
av = constant

26. State Bernoulli's principle.
The sum of the pressure, KE per unit volume & the potential energy per unit volume remains a constant.

27. Write the Bernoulli's equation.
$P + \frac{v^2}{2} + \rho gh =$ constant.

28. What is venturimeter?
It is a device used to measure the flow speed of incompressible fluid.

29. On what principle venturimeter works?
Bernoulli's theorem.

30. Define dynamic lift.
Dynamic lift is the force that acts on a body by virtue of its motion through a fluid.

31. What is Magnus effect?
Dynamic lift due to spinning is called magnus effect.

32. Why two streamlines cannot cross each other?
It two streamlines cross each other than at the point of intersection the liquid should move simultaneously in two different direction this is not possible.

33. Define viscosity.
The property of a liquid by which it opposes the relative motion between its different layers is called viscosity. It is also called fluid friction.

34. When viscosity comes to exist?
When there is a relative motion between layers of the liquid.

35. Define coefficient of viscosity.
It is defined as the ratio of shearing stress to the strain rate.

36. What is the Si unit of viscosity?
Nsm-2 or Pa S

37. Write the dimensions of viscosity.
$ML^{-1}T^{-1}$

38. How viscosity of liquid varies with temperature?
Viscosity of liquid decreases with temperature.

39. How viscosity of gases varies with temperature?
Viscosity of gases increases with temperature.

40. State Stokes law.
The viscous force acting on an object moving in a fluid is directly proportional to the velocity of the object.

41. What is Reynolds's number?
Reynold's number is a pure number which determine the type of flow of a liquid through a pipe.

42. Define surface tension.
Surface tension is defined as the tangential force per unit length acting normally on either side of an imaginary line drawn on the surface of a liquid.

43. Define surface energy.
The extra potential energy of the molecules in the surface of a liquid is called surface energy.

44. How surface tension depends on temperature?
The surface tension of a liquid decreases with increases in temperature.

45. Write the expression to Measure surface tension.
S=F/2l

46. Define angle of contact.
The angle between the tangent to the liquid surface at the point of contact & the solid surface inside the liquid is called the angle of contact.

47. When does liquid wets the surface of solid?
If angle of contact is acute (q < 90^0).

48. When does liquid not wets the surface of solid?
If angle of contact is obtuse (q > 900).

49. Why drops & bubbles are spherical in shape?
Because of surface tension.

50. Mention the expression for capillary rise.
$h = \frac{2T\cos\theta}{r\rho g}$

51. What happens to the surface tension of water when soup is added to it?
Surface tension decreases.

Two marks Questions

1. Give two differences between liquids & gases.
a) A liquid has a definite size but not a definite shape A gas has neither a definite size nor a definite shape.
b) A liquid is nearly incompressible. A gas is highly compressible.

2. Distinguish between liquid thrust & pressure.
Liquid thrust is the total normal force exerted by a fluid on walls of container where as pressure is fluid thrust per unit area.

3. Name the SI unit of fluid pressure & write the dimensional formula.
SI unit is Nm-2 & Dimensions are $ML^{-1}T^{-2}$

4. How the pressure at a point in a liquid vary with a) Depth of the point b) density of the liquid
A) directly proportional B) directly proportional

5. What is the difference between atmospheric pressure & gauge pressure?
The pressure of the atmosphere at any point is known as atmosphere pressure & gauge pressure is the difference of the actual pressure & the atmospheric pressure.

6. Mention any two application of Pascal's law.
Hydraulic brake, Hydraulic lift, Hydraulic press.

7. State & explain Archimedes principle.
When a body is immersed completely or partially in a liquid the apparent loss of weight of the body is equal to the weight of the liquid displaced.

8. What is a force of buoyancy? What is its effect?
The resultant up thrust experienced by a body immersed completely or partially in a liquid is called buoyancy because of buoyancy weight of the body decreases.

9. Explain hydrostatic paradox.
The liquid pressure is the same at all points at the same horizontal level (same depth) the result is appreciated through the example of hydrostatic paradox when different shaped vessels are connected at the bottom by a horizontal pipe & on filling with water the level in the three vessels is the same through they hold different amounts of water.

10. Mention the types of flow of fluid.
Streamline flow & Turbulent flow.

11. Distinguish between stream line flow & turbulent flow.

Streamline flow	Turbulent flow
1. It is a regular & orderly flow of liquid.	1. It is irregular & disorderly flow of liquid.
2. In Stream line flow velocity of all the liquid particles is the same at a given point.	2. In Turbulent flow the velocity of all the liquid particles is different at a given point.
3. The motion of liquid particles is parallel to each other.	3. The motion of liquid particle is not parallel to each other.
4. Every liquid particles moves with a velocity less then the critical velocity.	4. Every liquid particles moves with a velocity grater than the critical velocity.

12. State & explain equation of continuity.
It states that the product of area of cross section & the speed of a liquid remains the same at all points of a tube of flow.
If 'a' is the area of cross section of the tube at a point & v is the velocity of liquid in that region then
$v \propto 1/a$ then $v\,a = \text{constant}$

13. What are the different types of energy possessed by a liquid in motion? Write their Expressions. (any two)
Potential energy = ρgh. Kinetic energy = $\frac{1}{2}\rho v^2$ & pressure energy = $\frac{F}{\rho}$

14. State & explain Bernoulli's theorem.

Statement : Along the streamline of an ideal fluid the sum of the potential energy kinetic energy & pressure energy per unit mass remains constant.

i.e $gh + \frac{v^2}{2} + \frac{F}{\rho} = \text{constant}$

For a liquid having horizontal flow h is constant then

$\frac{\vartheta}{\rho} + \frac{v^2}{2} = \text{constant.}$

Thus the velocity of flow at any point increases the pressure at that point decreases & vice versa.

15. Write a note on uplift of an aircraft .

Bernoulli's principle that the pressure of any fluid decreases with increase in its velocity is used in designing air craft wings.

The shape of the wings of an air craft is such that the speed of the air above the aircraft is greater than the speed below the wings by Bernoulli's theorem it follows that the pressure below the wing is greater than that above. As a result an upward force is produced which lifts the air craft.

16. Explain the working of an atomizer.

An atomizer works on Bernoulli's principle that the pressure of a fluid decreases with increases in its peed. It consists of a cylinder fitted with piston at one end & the other end terminates in a small constriction. The constriction is connected to a vessel through a narrow tube. The air in the cylinder is pushed using the piston. As the air passes though the constriction its speed is considerably increases & consequently pressure drops the liquid rises from the vessel & is sprayed with the expelled air.

17. Define co-efficient of viscosity & name its SI unit.

Coefficient of viscosity is defined the ratio of shearing stress to the strain rate i.e. $\eta = \frac{F/A}{V/l} = \frac{Fl}{VA}$

SI unit is poiseiulle or NSm^{-2} or Pa.s.

18. State & explain Stokes law.

Statement : the viscous force acting on an object moving in a fluid is directly proportional to the velocity of the object.

Stoke's showed that the viscous force F acting on a body moving in a fluid is directly proportional to its terminal velocity v i.e F a v or F = Kv

Where K is the constant of proportionality.

19. Write Stokes formula & explain the terms.

For a spherical solid object $F = 6\pi\eta rv$

Where $6\pi \rightarrow$ constant

$\eta \rightarrow$ coefficient of viscosity of liquid column

$r \rightarrow$ radius of the spherical object

$v \rightarrow$ terminal velocity

20. Define surface tension & write its SI unit.

Surface tension is defined as the tangential force per unit length acting normally on either side of an imaginary line drawn on the surface of a liquid.

If F is the force acting on a length of an imaginary line drawn on the surface of a liquid then $F \propto L$ or $F = TL$ $\therefore$ $T = \frac{F}{L}$ SI unit is Nm^{-1}

21. When does liquid wets the surface of solid? Give an example.
If the angle of contact is acute ie $\theta < 90°$ liquid wets surface Ex: water & glass.

22. When does liquid do not wets the surface of solid? Give an example.
If the angle of contact is obtuse ie q>900 liquid do not wets the surface Ex: Mercury & glass.

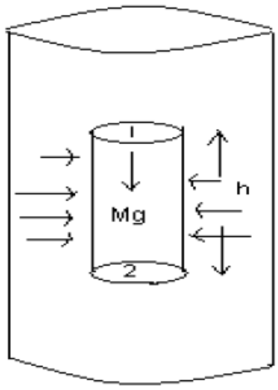
23. Write the expression for capillary rise of liquid & explain terms.
$h = \frac{2T\cos\theta}{r\rho g}$
$T \rightarrow$ surface tension of the liquid $r \rightarrow$ radius of capillary tube
$\rho \rightarrow$ density of liquid $g \rightarrow$ acceleration due to gravity
$\theta \rightarrow$ Angle of contact

24. Explain formations of drops & bubbles.
A liquid surface has a tendancy to have minimum surface area due to the property of surface tension. for a given volume, the surface area is minimum for a sphere. This is why small drops of liquid and bubbles attain spherical shape

25. Explain action of detergents.
When detergent is added to water it decreases the surface tension of water. Therefore when a dirty cloth is dipped in soapwater,it penetrates in to the interior parts of cloth and removes the dirt.

4 MARKS QUESTIONS

1. Derive an expression for pressure inside a liquid.
Consider a fluid at rest in a container as shown in the fig. the point 1 is at a height h above a point 2 & the pressures at point 1& 2 are P_1 & P_2 respectively consider a cylindrical element of fluid having area of base A & height h. As the fluid is at rest the resultant vertical forces should balances the weight of the element. The forces acing in the vertical direction are due to the fluid pressure at the top (P_1 A) ($\therefore P = F/A$) acting downward at the bottom (P_2 A) acting upward. If mg is weight of the fluid in the cylinder we have
$(P_2 - P_1) A = F$ but $F = mg$
$(P_2 – P_1) A = mg$ ……….. (1)
If ρ is the density of the fluid then mass of fluid is $m = \rho V = \rho h A$ ($V= hA$) Equation (1) becomes
$(P_2 - P_1) A = \rho h A g$
$P_2 - P_1 = \rho g h$ ……… (2)
If point 1 is shifted to the top of the fluid which is open to the atmosphere then $P_1 = P_a$ atmospheric pressure & replace P_2 by P then equation (2) becomes
$P - P_a = \rho gh$
Or $P = P_a + \rho gh$



2. Explain how Pascal's law is applied in a hydraulic lift?

For a confined static liquid pressure applied at any point in the liquid is transmitted equally & undiminished in all direction throughout the liquid.

At piston P, the force F_1 acts over the area A_1

$P_1 = F_1/A_1$ ---------

At piston Q the force F_2 acts over area A_2 $\therefore P_2 = F_2/A_2$

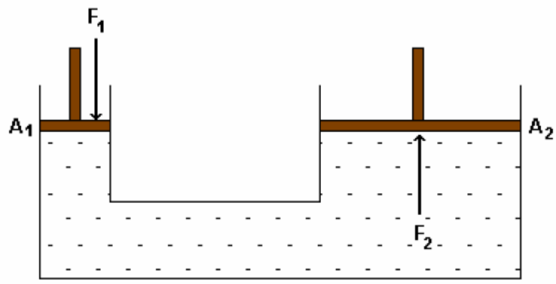
But according to Pascal's law the pressure P_1 is transmitted equally to piston Q

$\therefore P_1 = P_2 \quad \rightarrow \quad F_1/A_1 = F_2/A_2$

$F_2 = F_1 (A_2 / A_1)$

Since $A_2 > A_1$, $F_2 > F_1$

Thus a small force applied on the smaller piston appears as a large force on the larges piston.



Principle of a hydraulic lift

3. a) State Stokes law.

b) Show that terminal velocity of a sphere falling through a viscous medium is proportional to Square of its radius.

The viscous force acting on an object moving in a fluid is directly proportional to the velocity of the object. When a body falls through a fluid it drags the layer of the fluid in contact with it. A relative motion between the different layers of the fluid is set and as a result the body experiences a retarding force. Stokes an English scientist enunciated clearly the viscous drag force F as

$\mathbf{F = 6\pi\eta av}$

When a body falls through a fluid, initially it accelerates due to gravity. As the velocity increases the retarding force also increases. Finally when viscous force & buoyant force becomes equal to force due to gravity the net force becomes zero & acceleration also zero. Then the body moves with constant velocity called terminal velocity Vt is given by F = $6\pi\eta a v_t = \frac{4}{3}\pi a^3 (\rho - \sigma) g$.

When ρ & σ are densities of sphere & the fluid respectively then.

$$V_t = \frac{2a^2 (\rho - \sigma)g}{9\,\eta}$$

the terminal velocity V_t is directly proportional to square of the radius of the sphere & inversely on the viscosity of the medium.

4. Explain Bernoulli's principle.

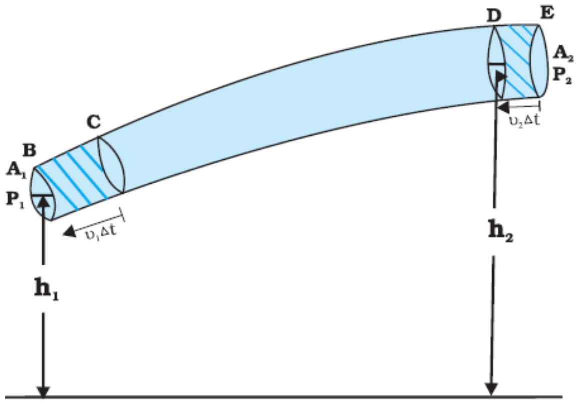
Bernoulli's theorem relates the speed of a fluid at a given point, the pressure at that point & the height of that point above a reference level. consider a liquid contained between cross sections A & B of the tube as shown in fig. The height of A & B are h_1 & h_2 respectively from a reference level.

Let the pressure at A & B be P_1 & P_2. The velocities at A and B be V_1 and V_2 and the density of the liquid is ρ. According to Bernoulli's theorem :

$$P_1 + \frac{1}{2}\rho V_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho V_2^2 + \rho g h_2$$

$$\text{i.e } P + \frac{1}{2}\rho V^2 + \rho g h = \text{constant.}$$

This is known as Bernoulli's equation. Thus for incompressible non viscous fluid in steady state flow, the sum of pressure energy, kinetic energy and potential energy per unit volume is constant.



5. Explain Torricelli's law (speed of efflux).

(1) Speed of Efflux: Torricelli's Law :

The word efflux means fluid outflow. Torricelli discovered that the speed of efflux from an open tank is given by a formula identical to that of a freely falling body. Consider a tank containing a liquid of density ρ with a small hole in its side at a height y_1 from the bottom (see Fig. 10.10). The air above the liquid, whose surface is at height y_2, is at pressure P. From the **equation of continuity** we have,

$$\upsilon_1 A_1 = \upsilon_2 A_2$$

$$\upsilon_2 = \frac{A_1}{A_2}\upsilon_1$$

If the cross sectional area of the tank A_2 is much larger than that of the hole ($A_2 >> A_1$), then we may take the fluid to be approximately at rest at the top, i.e. $v_2 = 0$. Now applying the Bernoulli equation at points 1 and 2 and noting that at the hole $P_1 = P_a$, the atmospheric pressure, we have from Bernoulli's relation,

$$P_1 + \left(\frac{1}{2}\right)\rho v_1^2 + \rho g h_1 = P_2 + \left(\frac{1}{2}\right)\rho v_2^2 + \rho g h_2$$

here, $h_1 = y_1$ and $h_2 = y_2$,
Taking $y_2 - y_1 = h$ we have

$$v_1 = \sqrt{2gh + \frac{2P - P_a}{\rho}}$$

When $P >> P_a$ and $2gh$ may be ignored, the speed of efflux is determined by the container pressure. Such a situation occurs in rocket propulsion. On the other hand if the tank is open to the atmosphere, then $P = P_a$ and

$$v_1 = \sqrt{2gh} \quad \text{---------} \quad (\,)$$

This is the speed of a freely falling body. This Equation is known as **Torricelli's law**.

6. Derive an expression to measure surface energy of a liquid.

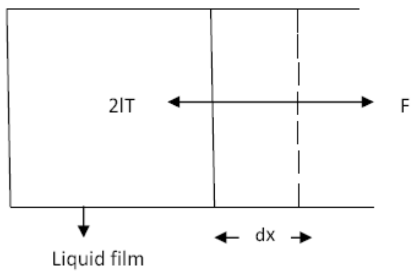
Consider a liquid film held by a U shaped wire and movable wire at one side. Let l be the length of the movable wire. Since the liquid film has two surfaces, the wire experiences a force 2lT. If the wire is moved to stretch the surface through a distance dx then

Work done = force × displacement.

= 2lT × dx

This is equal to the energy gained by the surface, 2l dx is the increase in the surface area of the film.

Thus surface energy = T × surface area



5 marks questions

1. State Bernoulli's principle. explain the Bernoulli's equation for the flow of an ideal fluid in stream line motion. Mention any two applications.

Statement: Along the stream line of an ideal fluid, the sum of the potential energy, kinetic energy & pressure energy per unit mass remains constant.

Explanation: It is same as Q.No 4 (4 mark question)

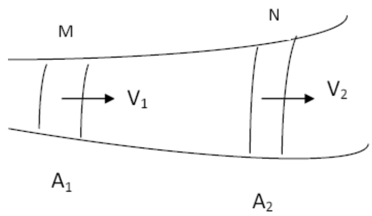
Application: Venturimeter, Atomisers and sprayers

2. Describe different types of flow of fluids. State and explain equation of continuity.

There are two type of flow of fluids (1) stream line (2) Turbulent flow.

In stream line flow velocity of all the liquid particles is the same at a given point & it is regular & orderly flow.

In turbulent flow the velocity of all the liquid particles is different at a given point & it is irregular & disorderly flow of liquid.
When a non viscous & in compressible liquid flow in streamline motion in a tube of non uniform cross section then the product of the area of cross section & velocity of liquid at any point remains constant.
If 'a' is the area of cross section of the tube at a point & V be the velocity of liquid then $V \propto 1/a$ or $V a$ = constant.
This is known as the equation of continuity.
Similarly let A_1 & A_2 be the areas of cross section at M & N respectively. Let V_1 & V_2 be the velocities in these sections then according to the equation of continuity $V_1A_1 = V_2A_2$



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TEXT BOOK EXERCISES

10.1 Explain why (a) The blood pressure in humans is greater at the feet than at the brain (b) Atmospheric pressure at a height of about 6 km decreases to nearly half of its value at the sea level, though the height of the atmosphere is more than 100 km

(c) Hydrostatic pressure is a scalar quantity even though pressure is force divided by area.

ANS:

(a) The pressure of a liquid is given by the relation:

$P = h\rho g$

Where,

P = Pressure

h = Height of the liquid column

ρ = Density of the liquid

g = Acceleration due to the gravity

It can be inferred that pressure is directly proportional to height. Hence, the blood pressure in human vessels depends on the height of the blood column in the body. The height of the blood column is more at the feet than it is at the brain. Hence, the blood pressure at the feet is more than it is at the brain.

(b) Density of air is the maximum near the sea level. Density of air decreases with increase in height from the surface. At a height of about 6 km, density decreases to nearly half of its value at the sea level. Atmospheric pressure is proportional to density. Hence, at a height of 6 km from the surface, it decreases to nearly half of its value at the sea level.

(c) When force is applied on a liquid, the pressure in the liquid is transmitted in all directions. Hence, hydrostatic pressure does not have a fixed direction and it is a scalar physical quantity.

10.2 Explain why (a) The angle of contact of mercury with glass is obtuse, while that of water with glass is acute. (b) Water on a clean glass surface tends to spread out while mercury on the same surface tends to form drops. (Put differently, water wets glass while mercury does not.)

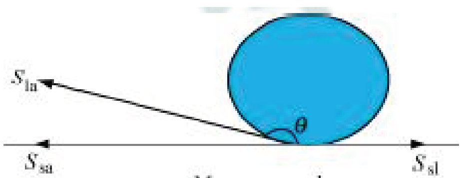
(c) Surface tension of a liquid is independent of the area of the surface

(d) Water with detergent disolved in it should have small angles of contact.

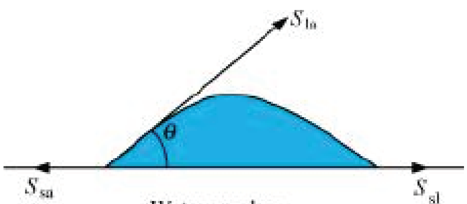
(e) A drop of liquid under no external forces is always spherical in shape

ANS:

(a) The angle between the tangent to the liquid surface at the point of contact and the surface inside the liquid is called the angle of contact (θ), as shown in the given figure.



Mercury on glass



Water on glass

S_{la}, S_{sa}, and S_{sl} are the respective interfacial tensions between the liquid-air, solidair, and solid-liquid interfaces. At the line of contact, the surface forces between the three media must be in equilibrium, i.e.,

$$\cos\theta = \frac{S_{sa} - S_{sl}}{S_{la}}$$

The angle of contact θ , is obtuse if $S_{sa} < S_{la}$ (as in the case of mercury on glass). This angle is acute if $S_{sl} < S_{la}$ (as in the case of water on glass).

(b) Mercury molecules (which make an obtuse angle with glass) have a strong force of attraction between themselves and a weak force of attraction toward solids. Hence, they tend to form drops. On the other hand, water molecules make acute angles with glass. They have a weak force of attraction between themselves and a strong force of attraction toward solids. Hence, they tend to spread out.

(c) Surface tension is the force acting per unit length at the interface between the plane of a liquid and any other surface. This force is independent of the area of the liquid surface. Hence, surface tension is also independent of the area of the liquid surface.

(d) Water with detergent dissolved in it has small angles of contact (θ). This is because for a small θ, there is a fast capillary rise of the detergent in the cloth. The capillary rise of a liquid is directly proportional to the cosine of the angle of contact (θ). If θ is small, then cos θ will be large and the rise of the detergent water in the cloth will be fast.

(e) A liquid tends to acquire the minimum surface area because of the presence of surface tension. The surface area of a sphere is the minimum for a given volume. Hence, under no external forces, liquid drops always take spherical shape.

10.3 Fill in the blanks using the word(s) from the list appended with each statement:
(a) Surface tension of liquids generally . . . with temperatures (increases / decreases)
(b) Viscosity of gases . .. with temperature, whereas viscosity of liquids . . . with temperature (increases / decreases)
(c) For solids with elastic modulus of rigidity, the shearing force is proportional to . . . , while for fluids it is proportional to . .. (shear strain / rate of shear strain)
(d) For a fluid in a steady flow, the increase in flow speed at a constriction follows (conservation of mass / Bernoulli's principle)
(e) For the model of a plane in a wind tunnel, turbulence occurs at a ... speed for turbulence for an actual plane (greater / smaller)
ANS:
(a) decreases
The surface tension of a liquid is inversely proportional to temperature.
(b) increases; decreases
Most fluids offer resistance to their motion. This is like internal mechanical friction, known as viscosity. Viscosity of gases increases with temperature, while viscosity of liquids decreases with temperature.
(c) Shear strain; Rate of shear strain
With reference to the elastic modulus of rigidity for solids, the shearing force is proportional to the shear strain. With reference to the elastic modulus of rigidity for fluids, the shearing force is proportional to the rate of shear strain.
(d) Conservation of mass/Bernoulli's principle
For a steady-flowing fluid, an increase in its flow speed at a constriction follows the conservation of mass/Bernoulli's principle.

(e) Greater
For the model of a plane in a wind tunnel, turbulence occurs at a greater speed than it does for an actual plane. This follows from Bernoulli's principle and different Reynolds' numbers are associated with the motions of the two planes.

10.4 Explain why (a) To keep a piece of paper horizontal, you should blow over, not under, it
(b) When we try to close a water tap with our fingers, fast jets of water gush through the openings between our fingers
(c) The size of the needle of a syringe controls flow rate better than the thumb pressure exerted by a doctor while administering an injection
(d) A fluid flowing out of a small hole in a vessel results in a backward thrust on the vessel
(e) A spinning cricket ball in air does not follow a parabolic trajectory
ANS:
(a) When air is blown under a paper, the velocity of air is greater under the paper than it is above it. As per Bernoulli's principle, atmospheric pressure reduces under the paper. This makes the paper fall. To keep a piece of paper horizontal, one should blow over it. This increases the velocity of air above the paper. As per Bernoulli's principle, atmospheric pressure reduces above the paper and the paper remains horizontal.
(b) According to the equation of continuity:
Area × Velocity = Constant
For a smaller opening, the velocity of flow of a fluid is greater than it is when the opening is bigger. When we try to close a tap of water with our fingers, fast jets of water gush through the openings between our fingers. This is because very small openings are left for the water to flow out of the pipe. Hence, area and velocity are inversely proportional to each other.
(c) The small opening of a syringe needle controls the velocity of the blood flowing out. This is because of the equation of continuity. At the constriction point of the syringe system, the flow rate suddenly increases to a high value for a constant thumb pressure applied.
(d) When a fluid flows out from a small hole in a vessel, the vessel receives a backward thrust. A fluid flowing out from a small hole has a large velocity according to the equation of continuity:
Area × Velocity = Constant
According to the law of conservation of momentum, the vessel attains a backward velocity because there are no external forces acting on the system.
(e) A spinning cricket ball has two simultaneous motions – rotatory and linear. These two types of motion oppose the effect of each other. This decreases the velocity of air flowing below the ball. Hence, the pressure on the upper side of the ball becomes lesser than that on the lower side. An upward force acts upon the ball.
Therefore, the ball takes a curved path. It does not follow a parabolic path.

10.5 A 50 kg girl wearing high heel shoes balances on a single heel. The heel is circular with a diameter 1.0 cm. What is the pressure exerted by the heel on the horizontal floor ?
ANS:
Mass of the girl, $m = 50$ kg
Diameter of the heel, $d = 1$ cm = 0.01 m
Radius of the heel, $r = d/2 = 0.005\ m$
Area of the heel = πr^2
$= \pi (0.005)^2 \quad = 7.85 \times 10^{-5}\ m^2$

Force exerted by the heel on the floor:
$F = mg \quad = 50 \times 9.8 \quad = 490$ N
Pressure exerted by the heel on the floor:
P = F/A
$= \frac{490}{7.85 \times 10^{-5}}$
$= 6.24 \times 10^6$ N m^{-2}
Therefore, the pressure exerted by the heel on the horizontal floor is 6.24×10^6 Nm^{-2}.

10.6 Toricelli's barometer used mercury. Pascal duplicated it using French wine of density 984 kg m^{-3}. Determine the height of the wine column for normal atmospheric pressure.
ANS:
10.5 m
Density of mercury, $\rho_1 = 13.6 \times 103$ kg/m^3
Height of the mercury column, $h_1 = 0.76$ m
Density of French wine, $\rho_2 = 984$ kg/m^3
Height of the French wine column = h_2
Acceleration due to gravity, g = 9.8 m/s^2
The pressure in both the columns is equal, i.e.,
Pressure in the mercury column = Pressure in the French wine column

$$\rho_1 h_1 g = \rho_2 h_2 g$$

$$h_2 = \frac{\rho_1 h_1}{\rho_2}$$

$$= \frac{13.6 \times 10^3 \times 0.76}{984}$$

= 10.5 m
Hence, the height of the French wine column for normal atmospheric pressure is 10.5 m.

10.7 A vertical off-shore structure is built to withstand a maximum stress of 109 Pa. Is the structure suitable for putting up on top of an oil well in the ocean ? Take the depth of the ocean to be roughly 3 km, and ignore ocean currents.
ANS:
Yes
The maximum allowable stress for the structure, $P = 109$ Pa
Depth of the ocean, $d = 3$ km $= 3 \times 10^3$ m
Density of water, $\rho = 103$ kg/m^3
Acceleration due to gravity, g = 9.8 m/s^2
The pressure exerted because of the sea water at depth, $d = \rho dg$
$= 3 \times 10^3 \times 10^3 \times 9.8$
$= 2.94 \times 10^7$ Pa
The maximum allowable stress for the structure (10^9 Pa) is greater than the pressure of the sea water (2.94×10^7 Pa). The pressure exerted by the ocean is less than the pressure that the structure can withstand. Hence, the structure is suitable for putting up on top of an oil well in the ocean.

10.8 A hydraulic automobile lift is designed to lift cars with a maximum mass of 3000 kg. The area of cross-section of the piston carrying the load is 425 cm^2. What maximum pressure would the smaller piston have to bear ?

ANS:

The maximum mass of a car that can be lifted, m = 3000 kg

Area of cross-section of the load-carrying piston, $A = 425\text{ cm}^2 = 425 \times 10^{-4}\text{ m}^2$

The maximum force exerted by the load, $F = mg$

$= 3000 \times 9.8$

$= 29400$ N

The maximum pressure exerted on the load-carrying piston,

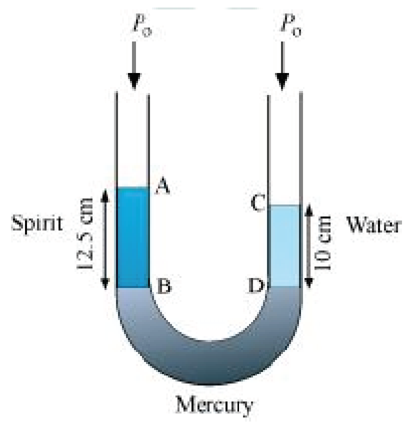
$= 6.917 \times 10^5$ Pa

Pressure is transmitted equally in all directions in a liquid. Therefore, the maximum pressure that the smaller piston would have to bear is 6.917×10^5 Pa.

10.9 A U-tube contains water and methylated spirit separated by mercury. The mercury columns in the two arms are in level with 10.0 cm of water in one arm and 12.5 cm of spirit in the other. What is the specific gravity of spirit ?

ANS:

The given system of water, mercury, and methylated spirit is shown as follows:



Height of the spirit column, h_1 = 12.5 cm = 0.125 m

Height of the water column, h_2 = 10 cm = 0.1 m

P_0 = Atmospheric pressure

ρ_1 = Density of spirit

ρ_2 = Density of water

Pressure at point B = $P_0 + h_1\rho_1 g$

Pressure at point D = $P_0 + h_2\rho_2 g$

Pressure at points B and D is the same.

$$P_0 + h_1\rho_1 g = h_2\rho_2 g$$

$$\frac{\rho_1}{\rho_2} = \frac{h_2}{h_1} \qquad = \frac{10}{12.5} = 0.8$$

Therefore, the specific gravity of spirit is 0.8.

10.10 In the previous problem, if 15.0 cm of water and spirit each are further poured into the respective arms of the tube, what is the difference in the levels of mercury in the two arms ? (Specific gravity of mercury = 13.6)

ANS:

Height of the water column, h_1 = 10 + 15 = 25 cm

Height of the spirit column, h_2 = 12.5 + 15 = 27.5 cm

Density of water, $\rho_1 = 1\text{ g cm}^{-3}$

Density of spirit, $\rho_2 = 0.8\text{ g cm}^{-3}$

Density of mercury = 13.6 g cm^{-3}

Let h be the difference between the levels of mercury in the two arms.
Pressure exerted by height h, of the mercury column:
$= h\rho g$
$= h \times 13.6g$ … (i)
Difference between the pressures exerted by water and spirit:
$= h_1\rho_1 g - h_1\rho_1 g$
= g(25 × 1 – 27.5 × 0.8)
= 3g … (ii)
Equating equations (i) and (ii), we get:
13.6 hg = 3g
h = 0.220588 ≈ 0.221 cm
Hence, the difference between the levels of mercury in the two arms is 0.221 cm.

10.11 Can Bernoulli's equation be used to describe the flow of water through a rapid in a river ? Explain.
ANS:
No
Bernoulli's equation cannot be used to describe the flow of water through a rapid in a river because of the turbulent flow of water. This principle can only be applied to a streamline flow.

10.12 Does it matter if one uses gauge instead of absolute pressures in applying Bernoulli's equation ? Explain.
ANS:
No
It does not matter if one uses gauge pressure instead of absolute pressure while applying Bernoulli's equation. The two points where Bernoulli's equation is applied should have significantly different atmospheric pressures.

10.13 Glycerine flows steadily through a horizontal tube of length 1.5 m and radius 1.0 cm. If the amount of glycerine collected per second at one end is 4.0 × 10–3 kg s^{-1}, what is the pressure difference between the two ends of the tube ? (Density of glycerine = 1.3×10^3 kg m^{-3} and viscosity of glycerine = 0.83 Pa s). [You may also like to check if the assumption of laminar flow in the tube is correct].
ANS:
9.8×10^2 Pa
Length of the horizontal tube, l = 1.5 m
Radius of the tube, r = 1 cm = 0.01 m
Diameter of the tube, $d = 2r$ = 0.02 m
Glycerine is flowing at a rate of 4.0×10^{-3} kg s^{-1}.
$M = 4.0 \times 10^{-3}$ kg s^{-1}
Density of glycerine, $\rho = 1.3 \times 10^3$ kg m^{-3}
Viscosity of glycerine, η = 0.83 Pa s
Volume of glycerine flowing per sec: V = M/ρ
$$= \frac{4.0 \times 10^{-3}}{1.3 \times 10^3}$$
$= 3.08 \times 10^{-6}$ m^3 s^{-1}

According to Poiseville's formula, we have the relation for the rate of flow:

$$V = \frac{\pi p r^4}{8\eta l}$$

Where, p is the pressure difference between the two ends of the tube

$$\therefore p = \frac{V 8\eta l}{\pi r^4} \qquad = \frac{3.08 \times 10^{-6} \times 8 \times 0.83 \times 1.5}{\pi \times (0.01)^4}$$

$= 9.8 \times 10^2$ Pa

Reynolds' number is given by the relation:

$$R = \frac{4\rho V}{\pi d\eta} \qquad = \frac{4 \times 1.3 \times 10^3 \times 3.08 \times 10^{-6}}{\pi \times (0.02) \times 0.83} = 0.3$$

Reynolds' number is about 0.3. Hence, the flow is laminar.

10.14 In a test experiment on a model aeroplane in a wind tunnel, the flow speeds on the upper and lower surfaces of the wing are 70 m s^{-1} and 63 m s-1 respectively. What is the lift on the wing if its area is 2.5 m^2 ? Take the density of air to be 1.3 kg m^{-3}.

ANS:

Speed of wind on the upper surface of the wing, $V_1 = 70$ m/s

Speed of wind on the lower surface of the wing, $V_2 = 63$ m/s

Area of the wing, $A = 2.5$ m^2

Density of air, $\rho = 1.3$ kg m^{-3}

According to Bernoulli's theorem, we have the relation:

$$P_1 + \frac{1}{2}\rho V_1^2 = P_2 + \frac{1}{2}\rho V_2^2$$

$$P_2 - P_1 = \frac{1}{2}\rho\left(V_1^2 - V_2^2\right)$$

Where,

P_1 = Pressure on the upper surface of the wing

P_2 = Pressure on the lower surface of the wing

The pressure difference between the upper and lower surfaces of the wing provides lift to the aeroplane.

Lift on the wing = $(P_2 - P_1)A$

$$= \frac{1}{2}\rho\left(V_1^2 - V_2^2\right)A$$

$$= \frac{1}{2}1.3\left((70)^2 - (63)^2\right) \times 2.5$$

= 1512.87

$= 1.51 \times 10^3$ N

Therefore, the lift on the wing of the aeroplane is 1.51×10^3 N.

10.15 Figures 10.23(a) and (b) refer to the steady flow of a (non-viscous) liquid. Which of the two figures is incorrect ? Why ?

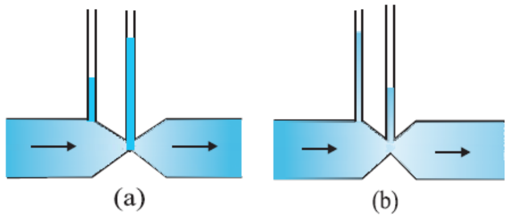
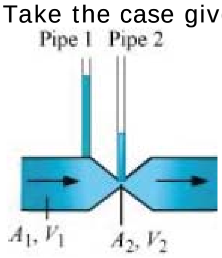


Fig. 10.23

ANS:
(a)
Take the case given in figure (b).



Where,
A_1 = Area of pipe1
A_2 = Area of pipe 2
V_1 = Speed of the fluid in pipe1
V_2 = Speed of the fluid in pipe 2
From the law of continuity, we have:

$$A_1V_1 = A_2V_2$$

When the area of cross-section in the middle of the venturimeter is small, the speed of the flow of liquid through this part is more. According to Bernoulli's principle, if speed is more, then pressure is less.
Pressure is directly proportional to height. Hence, the level of water in pipe 2 is less. Therefore, figure (a) is not possible.

10.16 The cylindrical tube of a spray pump has a cross-section of 8.0 cm^2 one end of which has 40 fine holes each of diameter 1.0 mm. If the liquid flow inside the tube is 1.5 m min^{-1}, what is the speed of ejection of the liquid through the holes ?

ANS:
Area of cross-section of the spray pump, $A_1 = 8\ \text{cm}^2 = 8 \times 10^{-4}\ \text{m}^2$
Number of holes, $n = 40$
Diameter of each hole, $d = 1\ \text{mm} = 1 \times 10^{-3}\ \text{m}$
Radius of each hole, $r = d/2 = 0.5 \times 10^{-3}\ \text{m}$
Area of cross-section of each hole, $a = \pi r2 = \pi\,(0.5 \times 10^{-3})^2\ \text{m}^2$
Total area of 40 holes, $A^2 = n \times a$
$= 40 \times \pi\,(0.5 \times 10^{-3})^2\ \text{m}^2$
$= 31.41 \times 10^{-6}\ \text{m}^2$
Speed of flow of liquid inside the tube, $V_1 = 1.5\ \text{m/min} = 0.025\ \text{m/s}$
Speed of ejection of liquid through the holes = V_2
According to the law of continuity, we have:

$$A_1V_1 = A_2V_2$$

$V_2 = \frac{A_1 V_1}{A_2}$ $= \frac{8 \times 10^{-4} \times 0.025}{31.61 \times 10^{-6}}$

= 0.633 m/s

Therefore, the speed of ejection of the liquid through the holes is 0.633 m/s.

10.17 A U-shaped wire is dipped in a soap solution, and removed. The thin soap film formed between the wire and the light slider supports a weight of 1.5×10^{-2} N (which includes the small weight of the slider). The length of the slider is 30 cm. What is the surface tension of the film ?

ANS:

The weight that the soap film supports, $W = 1.5 \times 10^{-2}$ N

Length of the slider, $l = 30$ cm = 0.3 m

A soap film has two free surfaces. ∴ Total length = $2l = 2 \times 0.3 = 0.6$ m

Surface tension

$S = \frac{\text{Force or Weight}}{2l}$ $= \frac{1.5 \times 10^{-2}}{0.6}$ $= 2.5 \times 10^{-2}$ N/m

Therefore, the surface tension of the film is 2.5×10^{-2} N m^{-1}.

10.18 Figure 10.24 (a) shows a thin liquid film supporting a small weight = 4.5×10^{-2} N. What is the weight supported by a film of the same liquid at the same temperature in Fig. (b) and (c) ? Explain your answer physically.

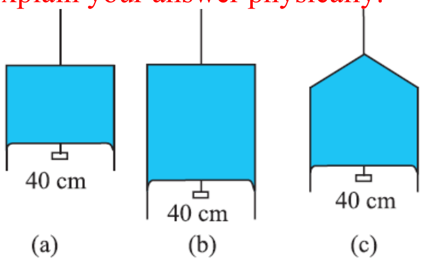


Figure 10.24

ANS:

Take case (a):

The length of the liquid film supported by the weight, $l = 40$ cm = 0.4 cm

The weight supported by the film, $W = 4.5 \times 10^{-2}$ N

A liquid film has two free surfaces.

∴ Surface tension = W/2l

$= \frac{4.5 \times 10^{-2}}{2 \times 0.4} = 5.625 \times 10^{-2}$ N m^{-1}

In all the three figures, the liquid is the same. Temperature is also the same for each case. Hence, the surface tension in figure (b) and figure (c) is the same as in figure (a), i.e., 5.625×10^{-2} N m^{-1}.

Since the length of the film in all the cases is 40 cm, the weight supported in each case is 4.5×10^{-2} N.

10.19 What is the pressure inside the drop of mercury of radius 3.00 mm at room temperature ? Surface tension of mercury at that temperature (20 °C) is 4.65×10^{-1} N m^{-1}. The atmospheric pressure is 1.01×10^{5} Pa. Also give the excess pressure inside the drop.

ANS:

1.01×10^5 Pa; 310 Pa
Radius of the mercury drop, $r = 3.00$ mm $= 3 \times 10^{-3}$ m
Surface tension of mercury, $S = 4.65 \times 10^{-1}$ N m^{-1}
Atmospheric pressure, $P_0 = 1.01 \times 10^5$ Pa
Total pressure inside the mercury drop
= Excess pressure inside mercury + Atmospheric pressure

$$= \frac{2S}{r} + P_0$$

$$= \frac{2 \times 4.65 \times 10^{-1}}{3 \times 10^{-3}} + 1.01 \times 10^5$$

$= 1.0131 \times 10^5$
$= 1.01 \times 10^5$ Pa
Excess pressure = 2S/r

$$= \frac{2 \times 4.65 \times 10^{-1}}{3 \times 10^{-3}} = 310 \text{ Pa}$$

10.20 What is the excess pressure inside a bubble of soap solution of radius 5.00 mm, given that the surface tension of soap solution at the temperature (20 °C) is 2.50×10^{-2} N m^{-1} ? If an air bubble of the same dimension were formed at depth of 40.0 cm inside a container containing the soap solution (of relative density 1.20), what would be the pressure inside the bubble ? (1 atmospheric pressure is 1.01×10^5 Pa).

ANS:
Excess pressure inside the soap bubble is 20 Pa;
Pressure inside the air bubble is 1.06×10^5 Pa
Soap bubble is of radius, $r = 5.00$ mm $= 5 \times 10^{-3}$ m
Surface tension of the soap solution, $S = 2.50 \times 10^{-2}$ Nm^{-1}
Relative density of the soap solution = 1.20 ∴ Density of the soap solution, $\rho = 1.2 \times 10^3$ kg/m^3
Air bubble formed at a depth, $h = 40$ cm $= 0.4$ m
Radius of the air bubble, $r = 5$ mm $= 5 \times 10^{-3}$ m
1 atmospheric pressure $= 1.01 \times 10^5$ Pa
Acceleration due to gravity, g = 9.8 m/s^2
Hence, the excess pressure inside the soap bubble is given by the relation:
P = 4S/r

$$= \frac{4 \times 2.5 \times 10^{-2}}{5 \times 10^{-3}} = 20 \text{ Pa}$$

Therefore, the excess pressure inside the soap bubble is 20 Pa.
The excess pressure inside the air bubble is given by the relation:
P' = 2S/r

$$= \frac{2 \times 2.5 \times 10^{-2}}{5 \times 10^{-3}} = 10 \text{ Pa}$$

Therefore, the excess pressure inside the air bubble is 10 Pa.
At a depth of 0.4 m, the total pressure inside the air bubble = Atmospheric pressure + $h\rho g + P'$

$= 1.01 \times 10^5 + 0.4 \times 1.2 \times 10^3 \times 9.8 + 10$

$= 1.057 \times 10^5$ Pa

$= 1.06 \times 10^5$ Pa

Therefore, the pressure inside the air bubble is 1.06×10^5 Pa

Additional Exercises

10.21 A tank with a square base of area 1.0 m2 is divided by a vertical partition in the middle. The bottom of the partition has a small-hinged door of area 20 cm2. The tank is filled with water in one compartment, and an acid (of relative density 1.7) in the other, both to a height of 4.0 m. compute the force necessary to keep the door close.

ANS:

Base area of the given tank, $A = 1.0\ m^2$

Area of the hinged door, $a = 20\ cm^2 = 20 \times 10^{-4}\ m^2$

Density of water, $\rho_1 = 10^3\ kg/m^3$

Density of acid, $\rho_2 = 1.7 \times 10^3\ kg/m^3$

Height of the water column, $h_1 = 4$ m

Height of the acid column, $h_2 = 4$ m

Acceleration due to gravity, g = 9.8

Pressure due to water is given as:

$P_1 = h_1 \rho_1 g$

$= 4 \times 10^3 \times 9.8$

$= 3.92 \times 10^4$ Pa

Pressure due to acid is given as:

$P_2 = h_2 \rho_2 g$

$= 4 \times 1.7 \times 10^3 \times 9.8$

$= 6.664 \times 10^4$ Pa

Pressure difference between the water and acid columns:

$\Delta P = P_2 - P_1$

$= 6.664 \times 10^4 - 3.92 \times 10^4$

$= 2.744 \times 10^4$ Pa

Hence, the force exerted on the door = $\Delta P \times a$

= 2.744 × 104 × 20 × 10_{-4} = 54.88 N

Therefore, the force necessary to keep the door closed is 54.88 N.

10.22 A manometer reads the pressure of a gas in an enclosure as shown in Fig. 10.25 (a) When a pump removes some of the gas, the manometer reads as in Fig. 10.25 (b) The liquid used in the manometers is mercury and the atmospheric pressure is 76 cm of mercury.

(a) Give the absolute and gauge pressure of the gas in the enclosure for cases (a) and (b), in units of cm of mercury.

(b) How would the levels change in case (b) if 13.6 cm of water (immiscible with mercury) are poured into the right limb of the manometer ? (Ignore the small change in the volume of the gas).

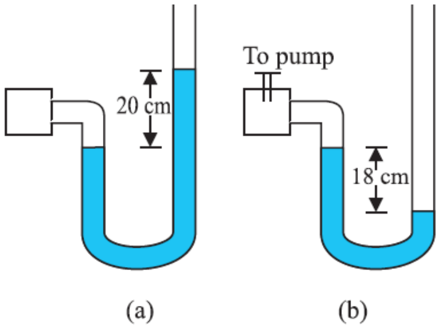


Fig. 10.25

ANS:

(a) 96 cm of Hg & 20 cm of Hg; 58 cm of Hg & –18 cm of Hg

(b) 19 cm

(a) *For figure* (a)

Atmospheric pressure, $P0$ = 76 cm of Hg

Difference between the levels of mercury in the two limbs gives gauge pressure

Hence, gauge pressure is 20 cm of Hg.

Absolute pressure = Atmospheric pressure + Gauge pressure

= 76 + 20 = 96 cm of Hg

For figure (b)

Difference between the levels of mercury in the two limbs = –18 cm

Hence, gauge pressure is –18 cm of Hg.

Absolute pressure = Atmospheric pressure + Gauge pressure

= 76 cm – 18 cm = 58 cm

(b) 13.6 cm of water is poured into the right limb of figure (b).

Relative density of mercury = 13.6

Hence, a column of 13.6 cm of water is equivalent to 1 cm of mercury.

Let h be the difference between the levels of mercury in the two limbs.

The pressure in the right limb is given as:

P_R = Atmospheric pressure + 1 cm of Hg

= 76 + 1 = 77 cm of Hg … (*i*)

The mercury column will rise in the left limb.

Hence, pressure in the left limb, $P_L = 58 + h$ ------- (2)

Equating equations (*i*) and (*ii*), we get:

$77 = 58 + h \quad \therefore h = 19$ cm

Hence, the difference between the levels of mercury in the two limbs will be 19 cm.

10.23 Two vessels have the same base area but different shapes. The first vessel takes twice the volume of water that the second vessel requires to fill upto a particular common height. Is the force exerted by the water on the base of the vessel the same in the two cases ? If so, why do the vessels filled with water to that same height give different readings on a weighing scale ?

ANS

Yes

Two vessels having the same base area have identical force and equal pressure acting on their common base area. Since the shapes of the two vessels are different, the force exerted on the sides of the vessels has non-zero vertical components.

When these vertical components are added, the total force on one vessel comes out to be greater than that on the other vessel. Hence, when these vessels are filled with water to the same height, they give different readings on a weighing scale.

10.24 During blood transfusion the needle is inserted in a vein where the gauge pressure is 2000 Pa. At what height must the blood container be placed so that blood may just enter the vein ? [Use the density of whole blood from Table 10.1].

ANS:
Gauge pressure, $P = 2000$ Pa
Density of whole blood, $\rho = 1.06 \times 10^3$ kg m^{-3}
Acceleration due to gravity, g = 9.8 m/s^2
Height of the blood container = h
Pressure of the blood container, $P = h\rho g$
$\therefore$ h = P/ρg

$$= \frac{2000}{1.06 \times 10^3 \times 9.8} = \quad = 0.1925 \text{ m}$$

The blood may enter the vein if the blood container is kept at a height greater than 0.1925 m, i.e., about 0.2 m.

10.25 In deriving Bernoulli's equation, we equated the work done on the fluid in the tube to its change in the potential and kinetic energy. (a) What is the largest average velocity of blood flow in an artery of diameter 2×10^{-3} m if the flow must remain laminar ? (b) Do the dissipative forces become more important as the fluid velocity increases ? Discuss qualitatively.

ANS:
(a) 1.966 m/s **(b)** Yes
(a) Diameter of the artery, d = 2 × 10–3 m
Viscosity of blood,
Density of blood, ρ = 1.06 × 103 kg/m3
Reynolds' number for laminar flow, $N_R = 2000$
The largest average velocity of blood is given as:

$$V_{\text{arg}} = \frac{N_R \eta}{\rho d}$$

$$= \frac{2000 \times 2.084 \times 10^{-3}}{1.06 \times 10^3 \times 2 \times 10^{-3}} = 1.966 \text{ m/s}$$

Therefore, the largest average velocity of blood is 1.966 m/s.
(b) As the fluid velocity increases, the dissipative forces become more important. This is because of the rise of turbulence. Turbulent flow causes dissipative loss in a fluid.

10.26 (a) What is the largest average velocity of blood flow in an artery of radius 2×10^{-3}m if the flow must remain lanimar? (b) What is the corresponding flow rate ? (Take viscosity of blood to be 2.084×10^{-3} Pa s).

ANS:
(a) Radius of the artery, $r = 2 \times 10^{-3}$ m
Diameter of the artery, $d = 2 \times 2 \times 10^{-3}$ m $= 4 \times 10^{-3}$m
Viscosity of blood, $\eta = 2.084 \times 10^{-3}$ Pa s

Density of blood, $\rho = 1.06 \times 10^3$ kg/m^3
Reynolds' number for laminar flow, $N_R = 2000$
The largest average velocity of blood is given by the relation:

$$V_{avg} = \frac{N_R \eta}{\rho d}$$

$$= \frac{2000 \times 2.084 \times 10^{-3}}{1.06 \times 10^3 \times 4 \times 10^{-3}} = 0.983 \text{ m/s}$$

Therefore, the largest average velocity of blood is 0.983 m/s.
(b) Flow rate is given by the relation:
$R = \pi r^2 V_{avg}$

$$= 3.14 \times \left(2 \times 10^{-3}\right)^2 \times 0.983$$

$$= 1.235 \times 10^{-5} \text{ m}^3 \text{s}^{-1}$$

Therefore, the corresponding flow rate

$$= 1.235 \times 10^{-5} \text{ m}^3 \text{s}^{-1}$$

10.27 A plane is in level flight at constant speed and each of its two wings has an area of 25 m^2. If the speed of the air is 180 km/h over the lower wing and 234 km/h over the upper wing surface, determine the plane's mass. (Take air density to be 1 kg m^{-3}).

ANS:
The area of the wings of the plane, $A = 2 \times 25 = 50$ m^2
Speed of air over the lower wing, $V_1 = 180$ km/h = 50 m/s
Speed of air over the upper wing, $V_2 = 234$ km/h = 65 m/s
Density of air, $\rho = 1$ kg m^{-3}
Pressure of air over the lower wing = P_1
Pressure of air over the upper wing= P_2

The upward force on the plane can be obtained using Bernoulli's equation as:

$$P_1 + \frac{1}{2}\rho V_1^2 = P_2 + \frac{1}{2}\rho V_2^2$$

$$P_1 - P_2 = \frac{1}{2}\rho\left(V_2^2 - V_1^2\right) \quad \ldots (i)$$

The upward force (F) on the plane can be calculated as:

$$(P_1 - P_2)A$$

$$= \frac{1}{2}\rho\left(V_2^2 - V_1^2\right)A \quad \text{Using equation } (i)$$

$$= \frac{1}{2} \times 1 \times \left((65)^2 - (50)^2\right) \times 50$$

$$= 43125 \text{ N}$$

Using Newton's force equation, we can obtain the mass (m) of the plane as:
F = mg

$$\therefore m = \frac{43125}{9.8} = 4400.51 \text{ kg} \sim 4400 \text{ kg}$$

Hence, the mass of the plane is about 4400 kg.

10.28 In Millikan's oil drop experiment, what is the terminal speed of an uncharged drop of radius 2.0×10^{-5} m and density 1.2×10^3 kg m–3. Take the viscosity of air at the temperature of the experiment to be 1.8×10^{-5} Pa s. How much is the viscous force on the drop at that speed ? Neglect buoyancy of the drop due to air.

ANS:

Terminal speed = 5.8 cm/s; Viscous force = 3.9×10^{-10} N

Radius of the given uncharged drop, $r = 2.0 \times 10^{-5}$ m

Density of the uncharged drop, $\rho = 1.2 \times 10^3$ kg m^{-3}

Viscosity of air, $\eta = 1.8 \times 10^{-5}$ Pa s

Density of air can be taken as zero in order to neglect buoyancy of air.

Acceleration due to gravity, g = 9.8 m/s^2

Terminal velocity (v) is given by the relation:

$$v = \frac{2r^2 \times (\rho - \rho_0) g}{9\eta}$$

$$= \frac{2 \times (2.0 \times 10^{-5})^2 (1.2 \times 10^3 - 0) \times 9.8}{9 \times 1.8 \times 10^{-5}} \qquad = 5.807 \times 10^{-2} \text{ m s}^{-1}$$

$$= 5.8 \text{ cm s}^{-1}$$

Hence, the terminal speed of the drop is 5.8 cm s^{-1}.

The viscous force on the drop is given by:

$$F = 6\pi\eta r v$$

$$\therefore F = 6 \times 3.14 \times 1.8 \times 10^{-5} \times 2.0 \times 10^{-5} \times 5.8 \times 10^{-2}$$

$$= 3.9 \times 10^{-10} \text{ N}$$

Hence, the viscous force on the drop is 3.9×10^{-10} N.

10.29 Mercury has an angle of contact equal to 140° with soda lime glass. A narrow tube of radius 1.00 mm made of this glass is dipped in a trough containing mercury. By what amount does the mercury dip down in the tube relative to the liquid surface outside ? Surface tension of mercury at the temperature of the experiment is 0.465 N m^{-1}. Density of mercury = 13.6×10^3 kg m^{-3}.

ANS:

Angle of contact between mercury and soda lime glass, $\theta = 140°$

Radius of the narrow tube, $r = 1$ mm $= 1 \times 10^{-3}$ m

Surface tension of mercury at the given temperature, $s = 0.465$ N m^{-1}

Density of mercury, $\rho = 13.6 \times 10^3$ kg/m^3

Dip in the height of mercury = h

Acceleration due to gravity, g = 9.8 m/s^2

Surface tension is related with the angle of contact and the dip in the height as:

$$s = \frac{h\rho g r}{2\cos\theta}$$

$$\therefore h = \frac{2s\cos\theta}{r\rho g}$$

$$= \frac{2 \times 0.465 \times \cos 140}{1 \times 10^{-3} \times 13.6 \times 10^{3} \times 9.8}$$
$$= -0.00534 \text{ m}$$
$$= -5.34 \text{ mm}$$

Here, the negative sign shows the decreasing level of mercury. Hence, the mercury level dips by 5.34 mm.

10.30 Two narrow bores of diameters 3.0 mm and 6.0 mm are joined together to form a U-tube open at both ends. If the U-tube contains water, what is the difference in its levels in the two limbs of the tube ? Surface tension of water at the temperature of the experiment is 7.3×10^{-2} N m^{-1}. Take the angle of contact to be zero and density of water to be 1.0×10^{3} kg m–3 ($g = 9.8$ m s^{-2}) .

ANS:

Diameter of the first bore, $d_1 = 3.0$ mm $= 3 \times 10^{-3}$ m

Hence, the radius of the first bore, $r_1 = \frac{d_1}{2} = 1.5 \times 10^{-3}$ m

Diameter of the second bore, $d_2 = 6.0$ mm

Hence, the radius of the second bore, $r_2 = \frac{d_2}{2} = 3 \times 10^{-3}$ m

Surface tension of water, $s = 7.3 \times 10^{-2}$ N m^{-1}

Angle of contact between the bore surface and water, $\theta = 0$

Density of water, $\rho = 1.0 \times 103$ kg/m^{-3}

Acceleration due to gravity, $g = 9.8$ m/s^2

Let $h1$ and $h2$ be the heights to which water rises in the first and second tubes respectively. These heights are given by the relations:

$$h_1 = \frac{2s\cos\theta}{r_1\rho g} \quad \dots (i)$$
$$h_2 = \frac{2s\cos\theta}{r_2\rho g} \quad \dots (ii)$$

The difference between the levels of water in the two limbs of the tube can be calculated as:

$$= \frac{2s\cos\theta}{r_1\rho g} - \frac{2s\cos\theta}{r_2\rho g}$$
$$= \frac{2s\cos\theta}{\rho g}\left[\frac{1}{r_1} - \frac{1}{r_2}\right] \qquad = \frac{2 \times 7.3 \times 10^{-2} \times 1}{1 \times 10^{3} \times 9.8}\left[\frac{1}{1.5 \times 10^{-3}} - \frac{1}{3 \times 10^{-3}}\right]$$
$$= 4.966 \times 10^{-3} \text{ m}$$
$$= 4.97 \text{ mm}$$

Hence, the difference between levels of water in the two bores is 4.97 mm.

CH 11

Thermal Properties of Matter

(10 Hours, 9 Marks (1M-1Q, 3M-1Q, 5M(NP)-1Q)

Syllabus :

Heat temperature, thermal expansion; thermal expansion of solids, liquids and gases, ideal gas laws, isothermal and adiabatic processes; anomalous expansion and its effect, specific heat capacity: Cp, Cv-calorimetry; change of state-specific latent heat capacity. Heat transfer-conduction, convection and radiation, Blackbody radiation kirchoff's Law, absorptive and emissive powers and greenhouse effect thermal conductivity, Newton's law of cooling, wein's displacement Law, stefan's law.

1. Heat, Temperature,

Temperature is a relative measure, or indication of hotness or coldness. A hot utensil is said to have a high temperature, and ice cube to have a low temperature. An object that has a higher temperature than another object is said to be hotter.

Heat is the form of energy transferred between two (or more) systems or a system and its surroundings by virtue of temperature difference. The SI unit of heat energy transferred is expressed in joule (J) while SI unit of temperature is kelvin (K), and °C is a commonly used unit of temperature. When an object is heated, many changes may take place.
Its temperature may rise, it may expand or change state.

A measure of temperature is obtained using a thermometer. The commonly used property is variation of the volume of a liquid with temperature. For example, a common thermometer (the liquid-in-glass type). Mercury and alcohol are the liquids used in most liquid-in-glass thermometers.
Thermometers are calibrated so that a numerical value may be assigned to a given temperature. For the definition of any standard scale, two fixed reference points are needed.
Since all substances change dimensions with temperature, an absolute reference for expansion is not available. However, the necessary fixed points may be correlated to physical phenomena that always occur at the same temperature. The ice point and the steam point of water are two convenient fixed points and are known as the freezing and boiling points.
These two points are the temperatures at which pure water freezes and boils under standard pressure. The two familiar temperature scales are the Fahrenheit temperature scale and the Celsius temperature scale. The ice and steam point have values 32 °F and 212 °F respectively, on the Fahrenheit scale and 0 °C and 100 °C on the Celsius scale. On the Fahrenheit scale, there are 180 equal intervals between two reference points, and on the celsius scale, there are 100.

A relationship for converting between the two scales may be obtained from an equation :

$$\frac{t_F - 32}{180} = \frac{t_C}{100}$$

Liquid-in-glass thermometers show different readings for temperatures other than the fixed points because of differing expansion properties. A thermometer that uses a gas, however, gives the same readings regardless of which gas is used.

Using Boyle's law, and Charles' law, we can write PV = constant and V/T = constant for a given quantity of gas, then PV/T should also be a constant. This relationship is known as ideal gas law. Then **ideal-gas equation can be written as** :

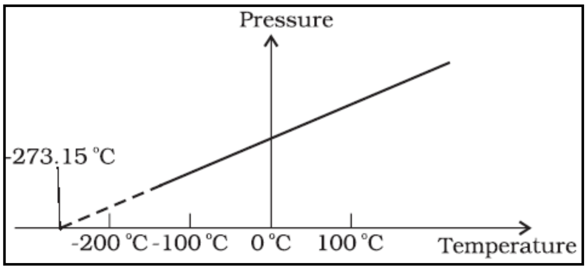
$$\frac{PV}{T} = \mu R \qquad \text{or } PV = \mu RT \qquad \text{---------} \quad (1)$$

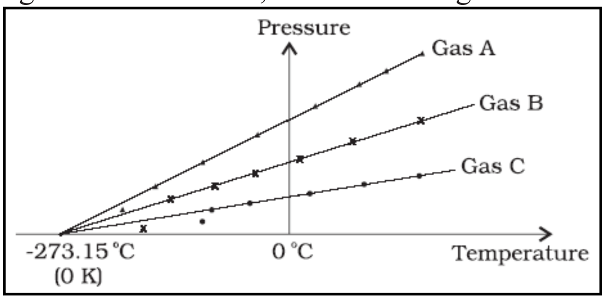
where, μ is the number of moles in the sample of gas and R is called universal gas constant:
$R = 8.31 \text{ J mol}^{-1} \text{ K}^{-1}$

From Eqn. (1) we have learnt that the pressure and volume are directly proportional to temperature : $PV \propto T$. This relationship allows a gas to be used to measure temperature in a constant volume gas thermometer.

Holding the volume of a gas constant, it gives $P \propto T$. Thus, with a constant-volume gas thermometer, temperature is read in terms of pressure.

A plot of pressure versus temperature gives a straight line in this case, as shown in Fig. 11.2.





However, measurements on real gases deviate from the values predicted by the ideal gas law at low temperature. But the relationship is linear over a large temperature range, and it looks as though the pressure might reach zero with decreasing temperature if the gas continued to be a gas. The absolute minimum temperature for an ideal gas, therefore, inferred by extrapolating the straight line to the axis, as in Fig. 11.3. This temperature is found to be – 273.15 °C and is designated as **absolute zero**.

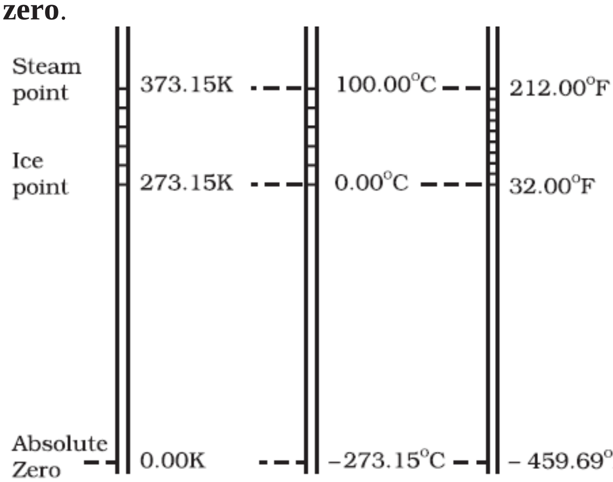


Fig. 11.4 Comparision of the Kelvin, Celsius and Fahrenheit temperature scales.

Absolute zero is the foundation of the Kelvin temperature scale or absolute scale temperature named after the British scientist Lord Kelvin. On this scale, – 273.15 °C is taken as the zero point, that is 0 K.
The size of the unit for Kelvin temperature is the same celsius degree, so temperature on these scales are related by
$T = t_C + 273.15$ --------- ( )

Types of thermometers
a) **Liquid thermometers:-** It works on the principal of change in volume of liquid with change in temperature.
b) **Gas thermometers :-** It works on the principal of change in the pressure with change in temperature.
c) **Platinum resistance thermometer :-** It works on the principal of change of resistance with change of temperature.
d) **Thermoelectric thermometer : -** It works on the principal of change of Thermo emf with change in temperature.
e) **Radiation pyrometers:-** It works on the principle of amount of radiation falls .
f) **Bimetallic strip thermometer: -** It works on the principle of linear expansion of solid with temperature.

2. Thermal expansion of solids, liquids and gases, ideal gas laws, isothermal and adiabatic processes; anomalous expansion and its effect :

A change in the temperature of a body causes change in its dimensions. The increase in the dimensions of a body due to the increase in its temperature is called thermal expansion. The expansion in length is called **linear expansion**. The expansion in area is called **area expansion**. The expansion in volume is called **volume expansion.**

(a) Linear expansion → $\Delta l/l = \alpha_1 \Delta T$ ------ (2) where α1 is known as the **coefficient of linear expansion** and is characteristic of the material of the rod.

(b) Area expansion → $\Delta A/A = 2\alpha_1 \Delta T$ ------ (3)

We find that copper expands about five times more than glass for the same rise in temperature. Normally, metals expand more and have relatively high values of α_1.

(c) Volume expansion → $\Delta V/V = 2\alpha_1 \Delta T$ ------ (4)

Here, we consider the fractional change in volume, $\Delta V/V$, of a substance for temperature change ΔT and define the **coefficient of volume expansion**, α_V as

$$\alpha_V = \left(\frac{\Delta V}{V}\right)\frac{1}{\Delta T}$$

Here αV is also a characteristic of the substance but is not strictly a constant. It depends in general on temperature. It is seen that α_V becomes constant only at a high temperature.

Water exhibits an anomalous behaviour; it contracts on heating between 0 °C and 4 °C. The volume of a given amount of water decreases as it is cooled from room temperature, until its temperature reaches 4 °C, [Fig. 11.7(a)]. Below 4 °C, the volume increases, and therefore the density decreases [Fig. 11.7(b)].
This means that water has a maximum density at 4 °C. This property has an important environmental effect: Bodies of water, such as lakes and ponds, freeze at the top first. As a lake cools toward 4 °C, water near the surface loses energy to the atmosphere, becomes denser, and sinks; the warmer, less dense water near the bottom rises. However, once the colder water on top reaches temperature below 4 °C, it becomes less dense and remains at the surface, where it freezes. If water did not have this property, lakes and ponds would freeze from the bottom up, which would destroy much of their animal and plant life.
Gases at ordinary temperature expand more than solids and liquids.

3. Specific heat capacity; Cp, Cv - calorimetry; change of state - latent heat capacity.

The quantity of heat required to warm a given substance depends on its mass, *m*, the change in temperature, ΔT and the nature of substance. The change in temperature of a substance, when a given quantity of heat is absorbed or rejected by it, is characterised by a quantity called the **heat capacity** of that substance. We define heat capacity, *S* of a substance as

$$S=\frac{\Delta Q}{\Delta T} \quad \text{---------} \quad (\,)$$

where ΔQ is the amount of heat supplied to the substance to change its temperature from T to $T + \Delta T$.

Every substance has a unique value for the amount of heat absorbed or rejected to change the temperature of unit mass of it by one unit. This quantity is referred to as the **specific heat capacity** of the substance.
If ΔQ stands for the amount of heat absorbed or rejected by a substance of mass *m* when it undergoes a temperature change ΔT, then the specific heat capacity, of that substance is given by

$$s=\frac{S}{m}=\frac{1}{m}\frac{\Delta Q}{\Delta T} \quad \text{-------} \quad (\,)$$

The **specific heat capacity** is the property of the substance which determines the change in the temperature of the substance (undergoing no phase change) when a given quantity of heat is absorbed (or rejected) by it. It is defined as the amount of heat per unit mass absorbed or rejected by the substance to change its temperature by one unit. It depends on the nature of the substance and its temperature.
The SI unit of specific heat capacity is J kg^{-1} K^{-1}.

If the amount of substance is specified in terms of moles ⌠, instead of mass *m* in kg, we can define heat capacity per mole of the substance by

$$C=\frac{S}{\mu}=\frac{1}{\mu}\frac{\Delta Q}{\Delta T}$$

where *C* is known as **molar specific heat capacity** of the substance. Like *S*, *C* also depends on the nature of the substance and its temperature. The SI unit of molar specific heat capacity is J mol^{-1} K^{-1}.

If the gas is held under constant pressure during the heat transfer, then it is called the **molar specific heat capacity at constant pressure** and is denoted by C_p. On the other hand, if the volume of the gas is maintained during the heat transfer, then the corresponding molar specific heat capacity is called **molar specific heat capacity at constant volume** and is denoted by C_v.

Calorimetry :

Calorimetry means measurement of heat. When a body at higher temperature is brought in contact with another body at lower temperature, the heat lost by the hot body is equal to the heat gained by the colder body, provided no heat is allowed to escape to the surroundings. A device in which heat measurement can be made is called a **calorimeter**. It consists a metallic vessel and stirrer of the same material like copper or alumiunium. The vessel is kept inside a wooden jacket which contains heat insulating materials like glass wool etc. The outer jacket acts as a heat shield and reduces the heat loss from the inner vessel. There is an opening in the outer jacket through which a mercury thermometer can be inserted into the calorimeter. The following example provides a method by which the specific heat capacity of a given solid can be determined by using the principle, heat gained is equal to the heat lost.

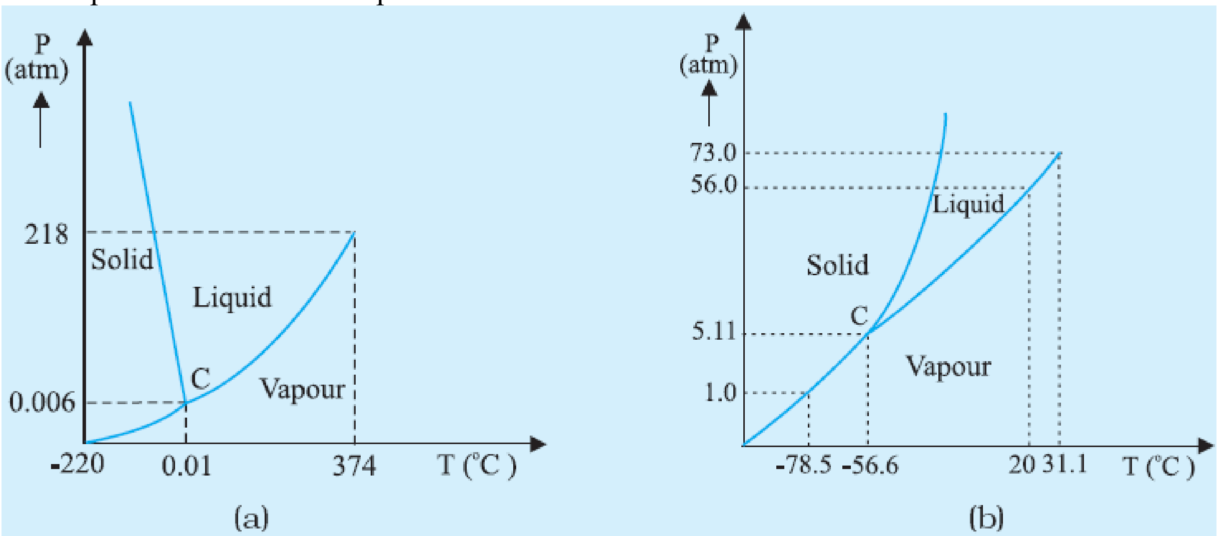
Change of state :

Matter normally exists in three states: solid, liquid, and gas. A transition from one of these states to another is called a change of state. Two common changes of states are solid to liquid and liquid to gas (and vice versa). These changes can occur when the exchange of heat takes place between the substance and its surroundings.
The change of state from solid to liquid is called **melting** and from liquid to solid is called **fusion**. It is observed that the temperature remains constant until the entire amount of the solid substance melts. That is, **both the solid and liquid states of the substance coexist in thermal equilibrium during the change of states from solid to liquid.** The temperature at which the solid and the liquid states of the substance in thermal equilibrium with each other is called its **melting point**. It is characteristic of the substance. It also depends on pressure. The melting point of a substance at standard atomspheric pressure is called its **normal melting point**.

To explains why cooking is difficult on hills : At high altitudes, atmospheric pressure is lower, reducing the boiling point of water as compared to that at sea level. On the other hand, boiling point is increased inside a pressure cooker by increasing the pressure. Hence cooking is faster. The boiling point of a substance at standard atmospheric pressure is called its **normal boiling point**.
However, all substances do not pass through the three states: solid-liquid-gas. There are certain substances which normally pass from the solid to the vapour state directly and vice versa. The change from solid state to vapour state without passing through the liquid state is called **sublimation**, and the substance is said to sublime. Dry ice (solid CO_2) sublimes, so also iodine. During the sublimation process both the solid and vapour states of a substance coexist in thermal equilibrium.

Triple Point :

The temperature of a substance remains constant during its change of state (phase change). A graph between the temperature T and the Pressure P of the substance is called a phase diagram or ***P* – *T* diagram**. The following figure shows the phase diagram of water and CO2. Such a phase diagram divides the $P - T$ plane into a solid-region, the vapour-region and the liquid-region. The regions are separated by the curves such as sublimation curve (BO), **fusion curve** (AO) and **vaporisation curve** (CO). The points on **sublimation curve** represent states in which solid and vapour phases coexist. The point on the sublimation curve BO represent states in which the solid and vapour phases co-exist. Points on the fusion curve AO represent states in which solid and liquid phase coexist. Points on the vapourisation curve CO represent states in which the liquid and vapour phases coexist. The temperature and pressure at which the fusion curve, the vaporisation curve and the sublimation curve meet and all the three phases of a substance coexist is called the **triple point** of the substance. For example the triple point of water is represented by the temperature 273.16 K and pressure 6.11×10^{-3} *Pa*.



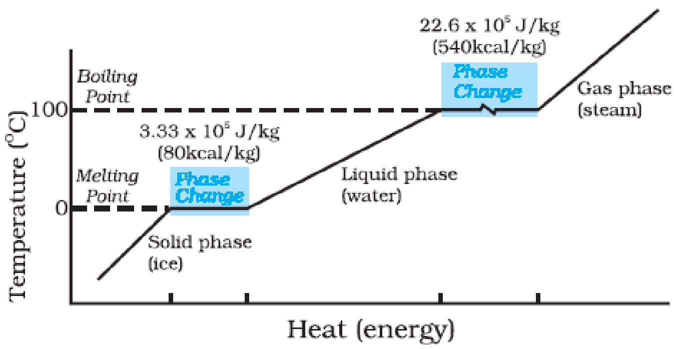
Pressure-temperature phase diagrams for (a) water and (b) CO2

Latent Heat :

We know that certain amount of heat energy is transferred between a substance and its surroundings when it undergoes a change of state. The amount of heat per unit mass transferred during change of state of the substance is called latent heat of the substance for the process.

The heat required during a change of state depends upon the heat of transformation and the mass of the substance undergoing a change of state. Thus, if mass m of a substance undergoes a change from one state to the other, then the quantity of heat required is given by $Q = m L$
or $L = Q/m$ ------------ (11.13)
where L is known as latent heat and is a characteristic of the substance. Its SI unit is J kg–1. The value of L also depends on the pressure. Its value is usually quoted at standard atmospheric pressure. The latent heat for a solid-liquid state change is called the **latent heat of fusion** (L_f), and that for a liquid-gas state change is called the **latent heat of vaporisation** (L_v). These are often referred to as the heat of fusion and the heat of vaporisation.

A plot of temperature versus heat energy for a quantity of water is shown in Fig. .



4. Heat transfer-conduction, convection and radiation, Blackbody radiation, Kirchoff's Law, absorptive and emissive powers and greenhouse effect, thermal conductivity, Newton's law of cooling, Wein's displacement Law, Stefan's law.

1. Heat transfer-conduction, convection and radiation :

There are three distinct modes of heat transfer : conduction, convection and radiation.

Conduction :

Conduction is the mechanism of transfer of heat between two adjacent parts of a body because of their temperature difference.
Gases are poor thermal conductors while liquids have conductivities intermediate between solids and gases. Heat conduction may be described quantitatively as the time rate of heat flow in a material for a given temperature difference.
In conduction, the rate of flow of heat (or heat current) H is proportional to the temperature difference ($T_C - T_D$) and the area of cross section A and is inversely proportional to the length L :

$$H = KA\frac{T_C - T_D}{L} \quad \text{--------} \quad (\ )$$

The constant of proportionality K is called the **thermal conductivity** of the material. The greater the value of K for a material, the more rapidly will it conduct heat. The SI unit of K is J S^{-1} m^{-1} K^{-1} or W m^{-1} K^{-1}. Thermal conductivity values vary slightly with temperature, but can be considered to be constant over a normal temperature range.

Convection :

Convection is a mode of heat transfer by actual motion of matter. It is possible only in fluids. Convection can be natural or forced. In natural convection, gravity plays an important part. When a fluid is heated from below, the hot part expands and, therefore, becomes less dense. Because of buoyancy, it rises and the upper colder part replaces it. This again gets heated, rises up and is replaced by the colder part of the fluid. The process goes on. This mode of heat transfer is evidently different from conduction. Convection involves bulk transport of different parts of the fluid. In forced convection, material is forced to move by a pump or by some other physical means. The common examples of forced convection systems are forced-air heating systems in home, the human circulatory system, and the cooling system of an automobile engine. In the human body, the heart acts as the pump that circulates blood through different parts of the body, transferring heat by forced convection and maintaining it at a uniform temperature.

Ex. 1 : Natural convection is responsible for many familiar phenomena. During the day, the ground heats up more quickly than large bodies of water do. This occurs both because the water has a greater specific heat and because mixing currents disperse the absorbed heat throughout the great volume of water. The air in contact with the warm ground is heated by conduction. It expands, becoming less dense than the surrounding cooler air. As a result, the warm air rises (air currents) and other air moves (winds) to fill the space-creating a sea breeze near a large body of water. Cooler air descends, and a thermal convection cycle is set up, which transfers heat away from the land. At night, the ground loses its heat more quickly, and the water surface is warmer than the land.

Ex. 2 : The other example of natural convection is the steady surface wind on the earth blowing in from north-east towards the equator, the so called trade wind. A reasonable explanation is as follows : the equatorial and polar regions of the earth receive unequal solar heat. Air at the earth's surface near the equator is hot while the air in the upper atmosphere of the poles is cool. In the absence of any other factor, a convection current would be set up, with the air at the equatorial surface rising and moving out towards the poles, descending and streaming in towards the equator. The rotation of the earth, however, modifies this convection current. Because of this, air close to the equator has an eastward speed of 1600 km/h, while it is zero close to the poles. As a result, the air descends not at the poles but at 30° N (North) latitude and returns to the equator. This is called **trade wind**.

Radiation :

Conduction and convection require some material as a transport medium. These modes of heat transfer cannot operate between bodies separated by a distance in vacuum. But the earth does receive heat from the sun across a huge distance and we quickly feel the warmth of the fire nearby even though air conducts poorly and before convection can set in. The third mechanism for heat transfer needs no medium; it is called radiation and the energy so radiated by electromagnetic waves is called radiant energy. In an electromagnetic wave electric and magnetic fields oscillate in space and time. Like any wave, electromagnetic waves can have different wavelengths and can travel in vacuum with the same speed, namely the speed of light i.e., 3×10^8 m s^{-1} .

The electromagnetic radiation emitted by a body by virtue of its temperature like the radiation by a red hot iron or light from a filament lamp is called thermal radiation. When this thermal radiation falls on other bodies, it is partly reflected and partly absorbed. The amount of heat that a body can absorb by radiation depends on the colour of the body.

Ex. 1 : We find that black bodies absorb and emit radiant energy better than bodies of lighter colours. This fact finds many applications in our daily life. We wear white or light coloured clothes in summer so that they absorb the least heat from the sun. However, during winter, we use dark coloured clothes which absorb heat from the sun and keep our body warm. The bottoms of the utensils for cooking food are blackened so that they absorb maximum heat from the fire and give it to the vegetables to be cooked.

Ex. 2 : Similarly, a Dewar flask or thermos bottle is a device to minimise heat transfer between the contents of the bottle and outside. It consists of a double-walled glass vessel with the inner and outer walls coated with silver. Radiation from the inner wall is reflected back into the contents of the bottle. The outer wall similarly reflects back any incoming radiation. The space between the walls is evacuated to reduce conduction and convection losses and the flask is supported on an insulator like cork. The device is, therefore, useful for preventing hot contents (like milk) from getting cold, or alternatively to store cold contents (like ice).

Newton's law of cooling :

It can be shown that a hot body loses heat to its surroundings in the form of heat radiation. The rate of loss of heat depends on the difference in temperature between the body and its surroundings. Newton was the first to study, in a systematic manner, the relation between the heat lost by a body in a given enclosure and its temperature.
According to Newton's law of cooling, the rate of loss of heat, – dQ/dt of the body is directly proportional to the difference of temperature $\Delta T = (T_2 - T_1)$ of the body and the surroundings. The law holds good only for small difference of temperature. Also, the loss of heat by radiation depends upon the nature of the surface of the body and the area of the exposed surface. We can write

$$-\frac{dQ}{dt}=k(T_2 \quad T_1) \qquad \text{----------} \quad (1)$$

where k is a positive constant depending upon the area and nature of the surface of the body. Suppose a body of mass m and specific heat capacity s is at temperature T_2. Let T_1 be the temperature of the surroundings. If the temperature falls by a small amount dT_2 in time dt, then the amount of heat lost is dQ = ms dT_2
∴ Rate of loss of heat is given by

$$\frac{dQ}{dt}=ms\frac{dT_2}{dt}$$

From Eqs. (11.15) and (11.16) we have

$$-m\,s\frac{dT_2}{dt}=k(T_2-T_1)$$

$$\frac{dT_2}{T_2-T_1}=-\frac{k}{ms}dt=-K\,dt$$

where $K = k/m\,s$
On integrating,
$\log_e (T_2 - T_1) = -K\,t + c$ ------------ (11.18)
or $T_2 = T_1 + C'$ e–Kt; where $C' = e^c$ ---------------- (11.19)
Equation (11.19) enables you to calculate the time of cooling of a body through a particular range of temperature.

For small temperature differences, the rate of cooling, due to conduction, convection, and radiation combined, is proportional to the difference in temperature. It is a valid approximation in the transfer of heat from a radiator to a room, the loss of heat through the wall of a room, or the cooling of a cup of tea on the table.

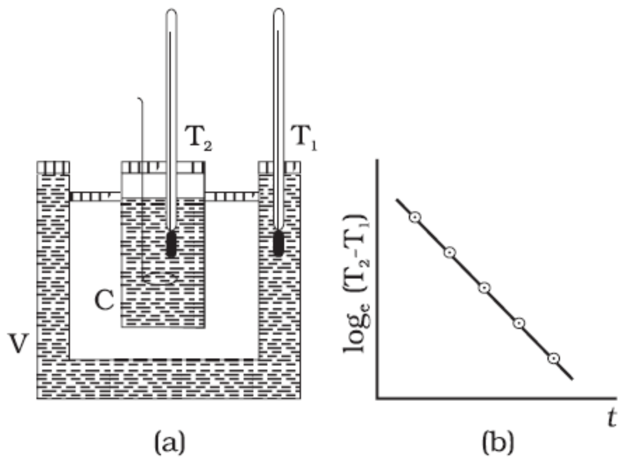


Fig. 11.19 *Verification of Newton's Law of cooling.*

Newton's law of cooling can be verified with the help of the experimental set-up shown in Fig. 11.19(a). The set-up consists of a double walled vessel (V) containing water in between the two walls. A copper calorimeter (C) containing hot water is placed inside the double walled vessel. Two thermometers through the corks are used to note the temperatures T_2 of water in calorimeter and T_1 of hot water in between the double walls respectively.

Temperature of hot water in the calorimeter is noted after equal intervals of time. A graph is plotted between loge (T_2–T_1) and time (t). The nature of the graph is observed to be a straight line having a negative slope as shown in Fig. 11.19(b). This is in support of Eq. (11.18).

Quantity	Symbol	Dimensions	Unit	Remark
Amount of substance	μ	[mol]	mol	
Celsius temperature	t_c	[K]	°C	
Kelvin absolute temperature	T	[K]	K	$t_c = T - 273.15$
Co-efficient of linear expansion	α_l	[K^{-1}]	K^{-1}	
Co-efficient of volume expansion	α_v	[K^{-1}]	K^{-1}	$\alpha_v = 3\,\alpha_l$
Heat supplied to a system	ΔQ	[$ML^2 T^{-2}$]	J	Q is not a state variable
Specific heat	s	[$L^2 T^{-2} K^{-1}$]	J kg^{-1} K^{-1}	
Thermal Conductivity	K	[$M LT^{-3}K^{-1}$]	J $s^{-1}K^{-1}$	$H = -KA \frac{dT}{dx}$

1) Write the differences between heat and temperature

Sl No	Heat	Temperature
1	It is a form of energy	It is a measure of degree of hotness of a body
2	Heat is responsible for temperature	It is one of the effects of heat
3	It is the sum of energies of all molecules	It is the average kinetic energy of the molecules
4	S I unit is joule	S I unit is Kelvin

2) A thermometer is kept in direct sun light what does it measure temperature of air or temperature of sun.
Temperature of air

3) What is the temperature of vacuum?
No.
If you have a perfect vacuum, it has no temperature. Temperature is a measure of how fast the molecules of the medium are moving, and with no molecules to measure, temperature is meaningless.

4) Why a thick glass tumbler does break when a hot liquid is poured in it?
The inside of the glass expands faster because the heat comes to it faster than the outside so it cracks under pressure.

5) A thin rod and a thick rod of the same material are allowed to expand by giving the same amount of heat which expands more?
Thin rod

6) What does the temperature scale on the thermometer measure?
It measures the average Kinetic Energy of a substance. In other words, it measures the average motion of the molecules in a substance.

7) Imagine you put a thermometer in your mouth. How does the heat flowing from your mouth affect the atoms in the thermometer?
Heat flows from warm places to cooler places. Thus, heat is conducted into the cooler thermometer. The heat causes the molecules in the thermometer to vibrate more quickly. The increased kinetic energy of the atoms causes them to spread out and rise up the tube. The end result is that you can read your body temperature.

Q.1 What is thermal energy ?

Ans: The total kinetic energy of all the molecules of the body is called thermal energy.

Q.2. Distinguish between heat and temperature,

Sl. No.	Heat	Temperature
1	Heat is a transfer of energy between two systems or a system and its surroundings by virtue of temperature difference.	Temperature is a measure degree of hotness or coldness of a body.
2	J (joule) is its SI unit.	K (kelvin) is its SI unit.
3	It is action	It is reaction

Q. 3 what property of material is used in designing thermometers.

Ans: Thermal expansion.

Q.4 Name the instrument commonly used to measure temperature.

Ans : Thermometer.

Q. 5. Which are the convenient temperature points of water are used normally while designing thermometer?

Ans : a) Ice point/ freezing point of water. b) steam point/ boiling point of water.

Q. 6. Write the relation between Fahrenheit and Celsius scales.

Ans: $t_F = 32 + \frac{9}{5}t_c$ OR $t_c = \frac{5}{9}(t_f - 32)$

Q.7.Write the value of ice point and steam point in Fahrenheit scale and Celsius scale

Details	Ice point	Steam point
Fahrenheit scale	$32^0 F$	$212^0 F$
Celsius scales	$0^0 C$	$100^0 C$

Q. 8. Write the relation between Celsius scale and kelvin scale of temperature.

Ans: T = (273.15 + t) Where T is temp. In kelvin scale and t is temp. in Celsius scale.

Q.9. State Boyle's law.

Ans: at constant temperature the pressure of a given mass of gas varies inversely as its volume.

$p \propto \frac{1}{v}$ at constant temperature

pv=constant

Q.10. state charle's law.

Ans: at constant pressure the volume of a given mass of a gas is directly proportional to its absolute temperature.

$V \propto T$ or $\frac{v}{T} = constant$

Q.11. Arrive at ideal gas law.

Ans: we know from Boyle's law pv = constant and from charle's law $\frac{v}{T} = constant$

Therefore $\frac{pv}{T} = constant$ = μR

Therefore pv = μRT where μ = number of moles of the gas

R = universal gas constant = 8.31 J/M/K

Q.12. what is thermal expansion?

Ans: The measure in the dimensions of a body due to increase in its temperature is called thermal expansion.

Q.13. Define coefficient of linear expansion.

Ans: $\alpha = \frac{(l_2 - l_1)}{l(t_2 - t_1)} = \frac{\Delta l}{l.\Delta T}$

It is the ratio of change in length to the original length.

Q.14. Define coefficient of volume expansion.

Ans: It is the ratio of change in volume to the original volume per degree rise in temperature

$\alpha_v = \frac{\Delta v}{v.\Delta T}$

Q.15. what is anomalous expansion of water?

Ans: Water contracts on heating between 0^0 C and 4^0 C. the volume of a given amount of water decreases as it is cooled from room temperature till 4^0 C, then volume increases and density decreases.

Q.16. what is the importance of anomalous expansion of water?

Ans: When water in lakes, ponds, rivers freezes at 4^0 C the density decreases and ice starts floating on water which helps as insulation for the further freeze. Water below ice layer remains as water and helps the animal and plant life inside it.

Q.17. Arrive at the relation between coefficient of linear and volume expansion.

Ans: Let α_l and α_v be the linear and volume expansion

We have $\alpha_l = \frac{\Delta l}{l.\Delta T}$; $\alpha_v = \frac{\Delta v}{v.\Delta T}$

$\Delta v = (l + \Delta l)^3 - l^3$

$= l^3 + \Delta l^3 + 3l^2\Delta l + 3l\Delta l^3 - l^3$

$= 3l^2\Delta l$ (neglecting power of Δl)

$= 3l^3 \frac{\Delta l}{l} = 3v.\alpha_l \Delta T$

$\frac{\Delta v}{v.\Delta T} = 3\ \alpha_l = \alpha_v$

Therefore $\alpha_v = 3\alpha_l$

Q.18. Define heat capacity of a substance.

Ans: it is defined as the quantity of heat required to raise the temperature of a given substance by unit degree Celsius.

$S = \frac{\Delta Q}{\Delta T}$

Q.19. Define specific heat capacity . write its unit.

Asn: It is the quantity of heat required to raise the temperature of unit mass of a substance by one unit.

$S = \frac{1}{m}\frac{\Delta Q}{\Delta T}$ unit = J/Kg/K

Q.20. Define molar specific heat capacity and write its unit.

Ans: It is the quantity of heat required to raise the temperature of one mole of substance by one kelvin.

$C = \frac{1}{\mu}\frac{\Delta Q}{\Delta T}$ unit = J/mol/K

Q.20. Define specific heat at constant pressure and constant volume.

Q.21. What is calorimetry? Write principle of calorimetry.

Ans: It means measurement of heat.

Principle- Heat lost by one body = Heat gained

Q.22. What is calorimeter?

Ans: A device in which heat measurement can be made.

Q.23. Define the terms melting point and boiling point.

Ans: the temperature at which the solid and the liquid states of the substance are in thermal equilibrium with each other.

Q.24. What is regelation?

Ans: the phenomenon of refreezing when a wire cuts the ice slab due to pressure and passes through it without splitting is called regelation.

Q.25. What is normal boiling point?

Ans: The boiling point of a substance at atmospheric pressure is called normal boiling point.

Q.26. How boiling point depends on pressure?

Ans: When pressure is increased boiling point increases.

Q.27. Why cooking is difficult on hills?

Ans: At high altitudes, atmospheric pressure is lower , which reduces the boiling point.

Q.28. What is the principle involved in pressure cooker?

Ans: Boiling point increases with increase in pressure. Hence food gets cooked faster.

Q.29. What is sublimation?

Ans: the change from solid state to vapour state without passing through a liquid state.

Q.30. What is latent heat?

Ans: The quantity of heat required to change the state of the substance without change in temperetaure.

Q = mL $\quad L = \frac{Q}{m}$ unit = J/Kg

Q.31. Define the terms , latent heat of fusion and latent heat of vaporisation.

Ans: Latent heat of fusion – Quantity of heat required to change the state from solid to liquid at constant temperature.

latent heat of vaporisation - Quantity of heat required to change the state from liquid to vapour at constant temperature.

Q.32. Steam burns are more serious than boiling water burns why?

Ans: Latent heat of vaporisation = 22.6×10^5 J/Kg

This heat gets released when steam changes its state from vapour to liquid. Hence steam burns are more serious than boiling water burns.

Q.33. Mention the different methods of heat transfer.

Ans: Conduction , convection and radiation.

Q.34. What is conduction?

Ans: It is the mechanism of transfer of heat between two adjacent parts of a body because of there temperature differences.

Q.35. State laws of thermal conductivity.

Ans: The rate of flow of heat at steady state in a conductor of length L is proportional to the temperature difference and area of cross section A and inversely proportional to the length L.

$H = KA\,\frac{T_c - T_o}{L}$ where K = coefficient of thermal conductivity

Q.36. Define K-coefficient of thermal conductivity.

Ans: It is defined as the quantity of heat flowing through a conductor of unit length and unit area of cross section in unit time to have a temperature difference of 1^0 C.

S.I unit = J/S/mol/K or W/mol/K

Q.37. Why a layer of earth or foam insulation on the cooling of concrete roofs is preferred?

Ans: Houses made of concrete roofs get very hot during summer , because thermal conductivity of concrete is high. So to keep the rooms cooler it is preferred.

Q.38. Why cooking pots have copper coating on the bottom?

Ans: copper is a good conductor of heat, promotes distribution od heat over the bottom of a pot for uniform cooking.

Q.39. What is convection?

Ans: The process of transmission of heat by the actual motion of matter.

Q.40. What is trade wind?

Ans: Due to the rotation of the earth air closed to the equator has an east ward speed of 1600 Km/hr, while it is zero closed to the poles. As a result , the air descends not at the poles but at 30^0N latitude and returns to the equator is called trade wind.

Q.41. What is radiation?

Ans: It is a process of transmission of heat without affecting the intervening medium.

Q.42. write the properties of thermal radiation.

Ans: 1) They travel in straight line.

2) They are electromagnetic in nature.

3) They travel with speed of light.

4) They undergo reflection, refraction and interference.

5) They obey inverse square law.

Q.43. Why black cloths are preferred in water?

Ans: Black absorbs more heat and keep our body warm.

Q.44. State Newton's law of cooling.

Ans: The rate of loss of heat of the body is directly proportional to the difference of temperature of the body and its surroundings.

$$-\frac{dQ}{dt} = K(T_2 - T_1) \quad \text{where K is constant.}$$

Q.45. Derive the expression $T_2 = T_1 + e^{-kt+C}$ using Newton's law of cooling.

Ans: Let us consider a body of mass m and specific heat capacity S at temperature T_2. Let T_1 is temperature of surroundigs.

If the fall in tempetarure is dT_2 in time dt , then the amount of heat lost

$dQ = msdT_2$

Rate of heat lost $\frac{dQ}{dt} = ms\frac{dT_2}{dt}$ (1)

From Newton's law $-\frac{dQ}{dt} = K(T_2 - T_1)$ (2)

From (1) and (2) $ms\frac{dT_2}{dt} = -K(T_2 - T_1)$

$$\frac{dT_2}{(T_2-T_1)} = \frac{-K}{ms}\,dt = -Kdt$$

$$log_e(T_2 - T_1) = -Kt + C$$

Therefore $T_2 - T_1 = e^{-kt+C}$

Therefore $T_2 = T_1 + e^{-kt+C}$

EXERCISES

11.1 The triple points of neon and carbon dioxide are 24.57 K and 216.55 K respectively. Express these temperatures on the Celsius and Fahrenheit scales.

11.2 Two absolute scales *A* and *B* have triple points of water defined to be 200 A and 350 B. What is the relation between T_A and T_B ?

11.3 The electrical resistance in ohms of a certain thermometer varies with temperature according to the approximate law :
$R = R_o [1 + \alpha (T - T_o)]$
The resistance is 101.6 Ω at the triple-point of water 273.16 K, and 165.5 Ω at the normal melting point of lead (600.5 K). What is the temperature when the resistance is 123.4 Ω ?

11.4 Answer the following :
(a) The triple-point of water is a standard fixed point in modern thermometry. Why ? What is wrong in taking the melting point of ice and the boiling point of water as standard fixed points (as was originally done in the Celsius scale) ?
(b) There were two fixed points in the original Celsius scale as mentioned above which were assigned the number 0 °C and 100 °C respectively. On the absolute scale, one of the fixed points is the triple-point of water, which on the Kelvin absolute scale is assigned the number 273.16 K. What is the other fixed point on this (Kelvin) scale ?
(c) The absolute temperature (Kelvin scale) *T* is related to the temperature t_c on the Celsius scale by $t_c = T - 273.15$
Why do we have 273.15 in this relation, and not 273.16 ?
(d) What is the temperature of the triple-point of water on an absolute scale whose unit interval size is equal to that of the Fahrenheit scale ?

11.5 Two ideal gas thermometers *A* and *B* use oxygen and hydrogen respectively. The following observations are made :

Temperature	Pressure thermometer A	Pressure thermometer B
Triple-point of water	1.250×10^5 Pa	0.200×10^5 Pa
Normal melting point of sulphur	1.797×10^5 Pa	0.287×10^5 Pa

(a) What is the absolute temperature of normal melting point of sulphur as read by thermometers *A* and *B* ?

(b) What do you think is the reason behind the slight difference in answers of thermometers *A* and *B* ? (The thermometers are not faulty). What further procedure is needed in the experiment to reduce the discrepancy between the two readings ?

11.6 A steel tape 1m long is correctly calibrated for a temperature of 27.0 °C. The length of a steel rod measured by this tape is found to be 63.0 cm on a hot day when the temperature is 45.0

°C. What is the actual length of the steel rod on that day ? What is the length of the same steel rod on a day when the temperature is 27.0 °C ? Coefficient of linear expansion of steel = 1.20 × 10–5 K^{-1} .

11.7 A large steel wheel is to be fitted on to a shaft of the same material. At 27 °C, the outer diameter of the shaft is 8.70 cm and the diameter of the central hole in the wheel is 8.69 cm. The shaft is cooled using 'dry ice'. At what temperature of the shaft does the wheel slip on the shaft? Assume coefficient of linear expansion of the steel to be constant over the required temperature range : $\alpha_{steel} = 1.20 \times 10^{-5} K^{-1}$.

11.8 A hole is drilled in a copper sheet. The diameter of the hole is 4.24 cm at 27.0 °C. What is the change in the diameter of the hole when the sheet is heated to 227 °C?
Coefficient of linear expansion of copper = $1.70 \times 10^{-5} K^{-1}$.

11.9 A brass wire 1.8 m long at 27 °C is held taut with little tension between two rigid supports. If the wire is cooled to a temperature of –39 °C, what is the tension developed in the wire, if its diameter is 2.0 mm ? Co-efficient of linear expansion of brass = $2.0 \times 10^{-5} K^{-1}$; Young's modulus of brass = 0.91×10^{11} Pa.

11.10 A brass rod of length 50 cm and diameter 3.0 mm is joined to a steel rod of the same length and diameter. What is the change in length of the combined rod at 250 °C, if the original lengths are at 40.0 °C? Is there a 'thermal stress' developed at the junction ? The ends of the rod are free to expand (Co-efficient of linear expansion of brass = $2.0 \times 10^{-5} K^{-1}$, steel = $1.2 \times 10^{-5} K^{-1}$).

11.11 The coefficient of volume expansion of glycerin is 49 × 10–5 K–1. What is the fractional change in its density for a 30 °C rise in temperature ?

11.12 A 10 kW drilling machine is used to drill a bore in a small aluminium block of mass 8.0 kg. How much is the rise in temperature of the block in 2.5 minutes, assuming 50% of power is used up in heating the machine itself or lost to the surroundings.
Specific heat of aluminium = 0.91 J $g^{-1} K^{-1}$.

11.13 A copper block of mass 2.5 kg is heated in a furnace to a temperature of 500 °C and then placed on a large ice block. What is the maximum amount of ice that can melt? (Specific heat of copper = 0.39 J g–1 K–1; heat of fusion of water = 335 J g–1).

11.14 In an experiment on the specific heat of a metal, a 0.20 kg block of the metal at 150 °C is dropped in a copper calorimeter (of water equivalent 0.025 kg) containing 150 cm3 of water at 27 °C. The final temperature is 40 °C. Compute the specific heat of the metal. If heat losses to the surroundings are not negligible, is your answer greater or smaller than the actual value for specific heat of the metal ?

11.15 Given below are observations on molar specific heats at room temperature of some common gases.

Gas	Molar specific heat (C_v) (cal mol^{-1} K^{-1})
Hydrogen	4.87
Nitrogen	4.97
Oxygen	5.02
Nitric oxide	4.99
Carbon monoxide	5.01
Chlorine	6.17

The measured molar specific heats of these gases are markedly different from those for monatomic gases. Typically, molar specific heat of a monatomic gas is 2.92 cal/mol K. Explain this difference. What can you infer from the somewhat larger (than the rest) value for chlorine ?

11.16 Answer the following questions based on the *P-T* phase diagram of carbon dioxide:
(a) At what temperature and pressure can the solid, liquid and vapour phases of CO_2 co-exist in equilibrium ?
(b) What is the effect of decrease of pressure on the fusion and boiling point of CO_2 ?
(c) What are the critical temperature and pressure for CO_2 ? What is their significance ?
(d) Is CO_2 solid, liquid or gas at (a) –70 °C under 1 atm, (b) –60 °C under 10 atm,
(c) 15 °C under 56 atm ?

11.17 Answer the following questions based on the *P – T* phase diagram of CO_2:
(a) CO_2 at 1 atm pressure and temperature – 60 °C is compressed isothermally. Does it go through a liquid phase ?
(b) What happens when CO_2 at 4 atm pressure is cooled from room temperature at constant pressure ?
(c) Describe qualitatively the changes in a given mass of solid CO_2 at 10 atm pressure and temperature –65 °C as it is heated up to room temperature at constant pressure.
(d) CO_2 is heated to a temperature 70 °C and compressed isothermally. What changes in its properties do you expect to observe ?

11.18 A child running a temperature of 101°F is given an antipyrin (i.e. a medicine that lowers fever) which causes an increase in the rate of evaporation of sweat from his body. If the fever is brought down to 98 °F in 20 min, what is the average rate of extra evaporation caused, by the drug. Assume the evaporation mechanism to be the only way by which heat is lost. The mass of the child is 30 kg. The specific heat of human body is approximately the same as that of water, and latent heat of evaporation of water at that temperature is about 580 cal g^{-1}.

11.19 A ‘thermacole’ icebox is a cheap and efficient method for storing small quantities of cooked food in summer in particular. A cubical icebox of side 30 cm has a thickness of 5.0 cm. If 4.0 kg of ice is put in the box, estimate the amount of ice remaining after 6 h. The outside temperature is 45 °C, and co-efficient of thermal conductivity of thermacole is 0.01 J s^{-1} m^{-1} K^{-1}. [Heat of fusion of water = 335×10^3 J kg^{-1}]

11.20 A brass boiler has a base area of 0.15 m2 and thickness 1.0 cm. It boils water at the rate of 6.0 kg/min when placed on a gas stove. Estimate the temperature of the part of the flame in contact with the boiler. Thermal conductivity of brass = 109 J s^{-1} m^{-1} K^{-1} ; Heat of vaporisation of water = 2256 × 10^3 J kg^{-1}.

11.21 Explain why :
(a) a body with large reflectivity is a poor emitter
(b) a brass tumbler feels much colder than a wooden tray on a chilly day
(c) an optical pyrometer (for measuring high temperatures) calibrated for an ideal black body radiation gives too low a value for the temperature of a red hot iron piece in the open, but gives a correct value for the temperature when the same piece is in the furnace
(d) the earth without its atmosphere would be inhospitably cold
(e) heating systems based on circulation of steam are more efficient in warming a building than those based on circulation of hot water

11.22 A body cools from 80 °C to 50 °C in 5 minutes. Calculate the time it takes to cool from 60 °C to 30 °C. The temperature of the surroundings is 20 °C.

RADIATION MCQ

1.	512 calories of radiation is incident on a body. It absorbs 220 calories the coefficient of emission is **(MHT-CET 2001)** (a) 0.43 (b) 0.57 (c) 1.7 (d) 0.34	Answer: (b)
2.	Heat energy received by the earth from the sun is due to **(CPMT 1994)** (a) Convection (b) Radiation (c) Reflection of light (d) Transmission of light	Answer: (b)
3.	Two sphere of same material have radii r_1 and r_2. they are heated to same temperature and kept in same enclosure at low temperature their rates of loss of heat are in the ratio **(JIPMER 98)** (a) $\frac{r_2^2}{r_1^2}$ (b) $\frac{r_1^2}{r_2^2}$ (c) $\frac{r_1}{r_2}$ (d) $\frac{r_2}{r_1}$	Answer: (b)
4.	Two stars A and B radiate maximum energy at 3600°A and 3600°A respectively. Then the ratio of absolute temperatures of A and B is **(P.M.T. MP 91)** (a) 256: 81 (b) 81: 256 (c) 3: 4 (d) 4: 3	Answer: (d)
5.	Emissivity of perfectly black body is **(MHT-CET 2003)** (a) 1 (b) 2 (c) 5 (d) 0	Answer: (a)
6.	Absorptive power of perfectly black body is **(PMT-89)** (a) Zero (b) Infinity (c) One (d) Constant	Answer: (c)

7.	Newton's law of cooling is also applicable to **(NCERT)** (a) Convection losses (b) Natural convection losses (c) Forced convection losses (d) None of the above	Answer: (c)
8.	Which of the following statement is wrong? **(NCERT 76)** (a) Rough surfaces are better radiators tan smooth surfaces (b) Highly polished mirror surfaces are very good radiators (c) Black surfaces are better absorbers then white ones (d) Black surfaces are better radiators then white ones	Answer: (b)
9.	The best ideal black body is **(CPMT-86)** (a) Lamp of charcoal heated to high temperature (b) Metal coated with a black dye (c) Glass surface coated with coalter (d) Hollow enclosure blackened inside and having a small hole	Answer: (d)
10.	The earth intercepts approximately one billionth of the power radiated by the sun. if the surface temperature of the sun were to drop by a factor of 2, the average radiant energy incident on earth per second would reduce by factor of **(MHT-CET 2001)** (a) 2 (b) 4 (c) 8 (d) 16	Answer: (d)
11.	A metal piece heated to T_1 °K. The temperature of the surrounding is T_2 °K. the heat in the surrounding due to radiation is proportional to **(NCERT 72)** (a) $(T_1^4 - T_2^4)$ (b) $(T_1 - T_2)^4$ (c) $(T_1^4 + T_2^4)$ (d) $(T_1^3 - T_2^3)$	Answer: (a)
12.	If the temperature of the sun is doubled. The rate of energy received on the earth will be increased by a factor of **(CBSE-93,94)** (a) 2 (b) 4 (c) 8 (d) 16	Answer: (d)
13.	A body in a room cools from 90°C to 80°C in 5 minutes. the time taken to cool from 70°C to 60°C is **(CPMT 92)** (a) Less than 5 minute (b) 5 minute (c) More than 5 minutes (d) Less or more than 5 minutes depending upon the nature of the body	Answer: (c)
14.	A sphere, a cube and a thin circular plate, all made of the same	Answer: [illegible]

	material and having the same mass are initially heated to a temperature of 3000°K, which of these will cool fastest? **(MNR 83)** (a) Sphere (b) Cube (c) Plate (d) None	
15.	The temperature of a piece of metal is raised from 27°C to 51.2°C. the rate at which metal radiates energy increases nearly **(MNR 87)** (a) 1.36 times (b) 2.36 times (c) 3.36 times (d) 4.36 times	Answer: (a)
16.	A cup of tea cools from 80°C to 60°C in one minute. The ambient temperature is 30°C. in cooling from 60°C to 50°C, it will take. **(MP-PMT-95, MHT-CET 2002)** (a) 50 sec (b) 90 sec (c) 60 sec (d) 30 sec	Answer: (a)
17.	Spectrum of a perfectly black body is **(MP PMT-89, MHT-CET 2005)** (a) Line spectrum (b) Band spectrum (c) Continuous spectrum (d) None of these	Answer: (c)
18.	An object is at the temperature of 400°C. at what temperature would it radiant energy twice as first? The temperature of surrounding may be assumed to be negligible **(PMT-MP 90)** (a) 200°C (b) 200°K (c) 800°C (d) 800°K	Answer: (d)
19.	A body at high temperature T °K radiates heat at rate proportional to **(MHT CET 2001)** (a) T^4 (b) T^{-4} (c) T (d) T^2	Answer: (a)
20.	Co-efficient of reflection, coefficient of absorption and coefficient transmission are related as **(MHT-CET 2005)** (a) $a + r + 1 = 1$ (b) $a + r + t \neq 1$ (c) $a + r = -t$ (d) $a \neq r + 1$	Answer: (a)

21.	The earth receives at its surface radiation from the sun at the rate of 1400 W/m^2. the distance of the centre of the sun from the surface of the earth is 1.5×10^{11} m and the radius of the sun is 7.0×10^8 m. treating sun as a black body, it follows from the above data that its surface temperature is, **(I.I.T 89)** (a) 5801 K (b) 10^6 K (c) 50.1 K (d) 2801°C	Answer: (a)
22.	The process of heat transfer in which heat is transferred with actual migration of medium particles is known as **(AFMC-94)** (a) Conduction (b) Convection (c) Radiation (d) Reflection	Answer: (b)
23.	A black body is at a temperature of 500K. it emits energy at a rate which is proportional to **(CBSE 97)** (a) 500 (b) $(500)^2$ (c) $(500)^3$ (d) $(500)^4$	Answer: (d)
24.	Which of the following is most repaid process **(NCERT 80)** (a) Conduction (b) Convection (c) Radiation (d) None of these	Answer: (c)
25.	The wavelength of maximum energy released during an atomic explosion was 2.93×10^{-10}m. the maximum temperature attained must be (Wien's constant = 2.93×10^{-3}mK) **(MHT-CET 2002)** (a) 5.86×10^7 K (b) 10^{-13} K (c) 10^{-7} K (d) 10^7 K	Answer: (d)

26.	If a = 0.72, r = 0.24, then the value of t is **(MHT-CET 2003)** (a) 0.02 (b) 0.04 (c) 0.4 (d) 0.2	Answer: (b)

27.	A body cools from 50°C to 46°C in 5 minutes and to 40°C in the next 10 minutes. The surrounding temperature is **(MHT-CET-1999)** (a) 30°C (b) 28°C (c) 36°C (d) 32°C	Answer: (a)
28.	According to Wien's law **(PMT MP 88)** (a) λ_m T = constant (b) $\frac{\lambda_m}{T} =$ constant (c) $\frac{T}{\lambda_m} =$ constant (d) T + λ_m = constant	Answer: (a)
29.	A black body is at 300 K. it emits energy at a rate which is proportional to **(AIMS-2002)** (a) (300) (b) $(300)^2$ (c) $(300)^4$ (d) $(300)^3$	Answer: (c)
30.	Which of the following will radiate heat to large extent? **(MNR-92)** (a) Rough surface (b) Polished surface (c) Black rough surface (d) Black polished surface	Answer: (c)
31.	The rate of radiation of black body at 0°C is E watt. The n the rate of radiation of this black body at 273°C will be **(MP-PMT 89)** (a) 16 E (b) 8 E (c) 4 E (d) E	Answer: (a)
32.	A black body radiates energy at the rate of E watt m^{-2} at a temperature of T Kelvin. When the temperature is reduced to T/2 Kelvin, the radiant energy will become **(MPPMT 92, CPMT 88, MNR 93)** (a) $\left(\frac{E}{2}\right)$ (b) $\left(\frac{E}{8}\right)$ (c) $\left(\frac{E}{16}\right)$ (d) None of the above	Answer: (c)
33.	Woolen clothes keep the body warm, because wool	Answer: (a)

	(EAMCET 78, AIMS 98) (a) Is a bad conductor (b) Increases the temperature of body (c) Decreases the temperature (d) All of these	
34.	The good absorber of heat are **(PMT 89)** (a) Non-emitter (b) Poor-emitter (c) Good-emitter (d) Highly polished	Answer: (c)
35.	The maximum energy in the thermal radiation from a hot source occurs at a wavelength of 11×10^{-5} cm. according to Wien's law, the temperature of the source (on Kelvin scale) will be n times the temperature of another source on (Kelvin scale) for which the wavelength at a maximum energy is 5.5×10^{-5} cm. The value of n is **(CPMT 91)** (a) 2 (b) 4 (c) 1/2 (d) 1	Answer: (a)
36.	Two sphere of the same material have radii 1 m and 4 m and temperature 4000 K and 2000 K respectively. the energy radiated per second by the first sphere is **(I.I.T 88)** (a) Greater then by the second (b) Less than that by the second (c) Equal in both cases (d) The information is incomplete to draw and conclusion	Answer: (c)
37.	A surface at temperature T_0 °K receives power P by radiation from a small sphere at temperature $T >> T_0$ and at a distance d. if both T and d are doubled, the power received by surface will becomes approximately **(NSEP 89)** (a) P (b) 2 P (c) 4 P (d) 16 P	Answer: (c)
38.	If temperature of a black body increases from 7°C to 287°C, then the rate of energy radiation becomes **(HARYANA PMT-2000)** (a) $\left(\frac{287}{7}\right)^4$ times (b) 16 times (c) 4 times (d) 2 times	Answer: (b)
39.	A bucket full of hot water is kept in a room and it cools from 75°C to	Answer: (b)

	70°C in t_1 minutes from 70°C to 65°C in t_2 minutes and from 65°C to 60°C in t_3 minutes; then **(NCERT 80, CBSE 95, MHT-CET 99)** (a) $t_1 - t_2 = t_3$ (b) $t_1 < t_2 < t_3$ (c) $t_1 > t_2 > t_3$ (d) $t_1 < t_2 > t_3$	
40.	The velocity with which thermal radiation travels in vacuum is **(EAMCET-82)** (a) 3×10^6 m/s (b) 3×10^7 m/s (c) 3×10^8 m/s (d) 3×10^{-16} m/s	Answer: (c)
41.	In which process the rate of transfer of heat is maximum? **(CMEE 94)** (a) Conduction (b) Convection (c) Radiation (d) In all these heat is transferred with the same velocity	Answer: (c)
42.	A body cools from 60°C to 50°C in 10 minutes. If the room temperature is 25°C and assuming Newton's law of cooling to hold good, the temperature of the body at the end of the next 10 minutes will be **(MP-PMT 98)** (a) 38.5°C (b) 40°C (c) 42.85°C (d) 45°C	Answer: (c)
43.	Two spheres made of same material have radii in the ratio 2:1. if both the spheres are at same temperature, then what is the ratio of heat radiation energy emitted per second by them? **(MHT-CET 2004)** (a) 1: 4 (b) 4: 1 (c) 3: 4 (d) 4: 3	Answer: (b)
44.	Solar radiation emitted by sun resembles that emitted by a black body at a temperature of 6000 K. maximum intensity is emitted at a wavelength of about 4800 A°. if the sun were cooled down from 6000 K to 3000 K, then the peak intensity would occur at a wavelength of **(PMT MP 91)** (a) 4800 A° (b) 9600 A° (c) 2400 A° (d) 19200 A°	Answer: (b)
45.	A perfectly black body emits radiation at temperature T_1 K. if it sis to radiate 16 times this power, its temperature T_2. will be **(BHU 94)** (a) $T_2 = 16\ T_1$ (b) $T_2 = 8\ T_1$	Answer: (d)

	(c) $T_1 = 4\ T_1$ (d) $T_2 = 2\ T_1$	
46.	Unit of Stefan's constant is given by **(MHT-CET 2006)** (a) $W/\ m\ K^2$ (b) $W/\ m^2\ K^2$ (c) $W^2/\ m^2\ K^4$ (d) $W/\ mK$	Answer: (b)
47.	A black body at hot temperature at 227°C radiates heat at a rate of 5 cal/cm^2 s. at a temperature of 727°C the rate of heat radiated per unit area will be **(MHT-CET 2002)** (a) 50 cal/cm^2 s (b) 250 cal/cm^2 s (c) 80 cal/cm^2 s (d) 100 cal/cm^2 s	Answer: (c)
48.	According to Kirchhoff's law of radiation which of the following relation is correct **(MHT CET 2006)** (a) $E_b = E_a$ (b) $E_b = \frac{E}{a}$ (c) $E_b = E^2 a$ (d) $E_b = E - a$	Answer: (b)
49.	A person with dark skin as compared to a person with white skin will experience **(CPMT-88)** (a) Less heat and less cold (b) More heat and more cold (c) More heat and less cold (d) Less heat and more cold.	Answer: (b)
50.	For which of the following process, the thermal conduction is maximum? **(AFMC PUNE-2002)** (a) Combustion (b) Radiation (c) Convection (d) Conduction	Answer: (b)

51.	The flow of heat from a hot body to a cold body is an example of **(MHT-CET-2001)** (a) Isothermal process (b) Reversible process (c) Adiabatic process	Answer: (d)

	(d) Irreversible process	
52.	Three black bodies A, B and C in the form of cubes of sides in the ratio of 3: 4: 5 are kept at the same high temperature. The ratio of the quantity of heat lost per second by A, B and C will be (a) 27: 64: 125 (b) 5: 4: 3 (c) 9: 16: 25 (d) 25: 16: 9	Answer: (c)
53.	A sphere, a cube and a thin circular plate all made of the same material and having the same mass are initially heated to a temperature of 300°C. which one of these cools faster? (a) Circular plate (b) Sphere (c) Cube (d) All will cool at the same rate	Answer: (a)
54.	The emissive of a perfectly black body is (a) 0 (b) 0.5 (c) 1 (d) 0.75	Answer: (c)
55.	If the wavelengths of maximum intensity of radiation emitted by the moon and the sun are 10^{-4} m and 0.5×10^{-6}m respectively, then the ratio of their temperature is (a) 200 (b) 100 (c) $\frac{1}{200}$ (d) $\frac{1}{100}$	Answer: (c)
56.	The coefficient of transmission of a perfectly black body is (a) Zero (b) One (c) 0.5 (d) 0.75	Answer: (a)
57.	When a black body is heated, it emits heat radiations of (a) Infrared wavelengths (b) Ultraviolet wavelengths (c) All wavelengths (d) A particular wavelengths	Answer: (c)
58.	An ideal black body is represented by (a) A metal coated with a black dye (b) A glass surface coated with coal tar (c) A hollow enclosure blackened from inside and having a small hole (d) A lump of charcoal heated to a high temperature	Answer: (c)

59.	The temperature of a furnace is 2327°C and the intensity is maximum in its radiation spectrum at 1200 $\mathring{A}$. If the intensity in the spectrum of a star is maximum at 4800 $\mathring{A}$, then the surface temperature of the star is (a) 6500 K (b) 6000 K (c) 4800 K (d) 7500 K	Answer: (a)
60.	The coefficient of transmission and coefficient of reflection for a given body at a given temperature are 0.33 and 0.64 respectively. Then the coefficient of emission of the same body at the same temperature will be (a) 0.3 (b) 0.003 (c) 0.03 (d) 0.33	Answer: (c)
61.	The correct equation out of the following is **(MHT-CET-2006)** (a) $E.E_b = a$ (b) $\frac{E}{E_b} = a$ (c) $\frac{E_b}{E} = a$ (d) $E.E_b = \frac{1}{a}$	Answer: (b)
62.	The maximum wavelength of radiation emitted by a black body at 1227°C is λm. what is its maximum wavelength at 2227°C? (a) $\frac{\lambda m}{2}$ (b) $\frac{\lambda m}{3}$ (c) $\frac{3}{5}\lambda m$ (d) $\frac{9}{25}\lambda m$	Answer: (c)
63.	According to Prevost's theory of heat exchange, the heat exchange shops at **(MHT-CET-2007)** (a) 0°C (b) – 5°C (c) – 273°C (d) – 273 K	Answer: (c)
64.	The rate of cooling of a body is 0.5°C/minute, when the body is 50°C above the surrounding. What is the rate of cooling if the body is 30°C above the surrounding?	Answer: (b)

	(a) 24°C/hour (b) 18°C/hour (c) 30°C/hour (d) 12°C/hour	
65.	Coefficient of transmission and coefficient of reflection for a given body are 0.22 and 0.74 respectively. Then, at a given temperature, the coefficient of emission for the body is **(MHT-CET-2005)** (a) 0.4 (b) 0.04 (c) 0.96 (d) 0.22	Answer: (b)
66.	If a black body is heated from 27°C to 927°C, then the ratio of the radiation emitted by the body at the two temperatures will be (a) 1: 4 (b) 1: 16 (c) 1: 256 (d) 1: 64	Answer: (c)
67.	A body cools from 100°C to 70°C in 8 minutes. If the room temperature is 15°C and assuming Newton's law of cooling holds good, then time required for the body to cool from 70°C to 40°C is **(MHT-CET-2005)** (a) 14 s (b) 10 s (c) 8 s (d) 5 s	Answer: (a)
68.	A body radiates heat at the rate of 4 cal/m^2/s, when its temperature is 227°C. what is the heat radiated by the same body when its temperature is 727°C? (a) 8 cal/m^2/s (b) 16 cal/m^2/s (c) 64 cal/m^2/s (d) 32 cal/m^2/s	Answer: (c)
69.	Two thermometers A and B exposed to sunlight. The value of A is painted black but that of B is not painted. The correct statement regarding this case is **(MHT-CET-1999)** (a) Temperature of B will rise faster (b) Temperature of A will remain more than B (c) Both of A and B show equal rise from the beginning (d) Temperature of A will rise faster than B but the final temperature will be same in both	Answer: (d)
70.	Two spheres made of same material have radii in the ratio 2: 1. if both the spheres are at same temperature, then what is the ratio of heat radiation energy emitted per second by them? **(MHT-CET-2004)** (a) 1: 4 (b) 4: 1 (c) 3: 4	Answer: (b)

	(d) 4: 3	
71.	The temperature of a black body is gradually increased. The colour of the body will change from (a) White-green-red (b) Red-yellow-blue (c) Red-violet-yellow (d) Yellow-green-red	Answer: (b)
72.	Radiation emitted by a surface is directly proportional to **(MHT-CET-2000)** (a) Third power of its temperature (b) Equal to its temperature (c) Twice power of its temperature (d) Fourth power of its temperature	Answer: (d)
73.	The coefficient of absorption of perfectly black body is **(MHT-CET-2004)** (a) 1 (b) 0 (c) 0.75 (d) None of these	Answer: (a)
74.	Two bodies A and B at temperatures T_1 K and T_2 K respectively have the same dimensions. Their emissivities are in the ratio of 1: 3. if they radiate the same amount of heat per unit time, then the relation between their temperatures is given by (a) $\frac{T_1}{T_2} = \frac{1}{3}$ (b) $\frac{T_1}{T_2} = \frac{81}{1}$ (c) $\frac{T_1}{T_2} = 3^{1/4} : 1$ (d) $\frac{T_1}{T_2} = 9^{1/4} : 1$	Answer: (c)
75.	A body radiates heat at the ate of 5 cal/m^2-s when its temperature is 227°C. the heat radiated by the same body when its temperature is 727°C is **(MHT-CET-2007)** (a) 10 cal/m^2-s (b) 20 cal/m^2-s (c) 40 cal/m^2-s (d) 80 cal/m^2-s	Answer: (d)

76.	Which one of the following constants is not related to radiation? (a) Solar constant	Answer: (c)

	(b) Stefan's constant (c) Boltzmann's constant (d) Wien's constant	
77.	Rate of cooling of body is 0.5°C/min, when the system is 50°C above the surroundings. When a system is 30°C above the surroundings, the rate of cooling will be **(MHT-CET-2004)** (a) 0.3°C/min (b) 0.6°C/min (c) 0.7°C/min (d) 0.4°C/min	Answer: (a)
78.	The sun emits light with a maximum wavelength of 510 nm while another star X emits light of maximum wavelength of 350 nm. What is the ratio of the surface temperatures of the sun and the star? (a) 0.35 (b) 1.5 (c) 1.1 (d) 0.69	Answer: (d)
79.	A sphere cube and a thin circular plate all made of the same material and having the same mass are initially heated to a temperature of 300°C. which of these cools fastest? **(MHT-CET-2000)** (a) Sphere (b) Cube (c) Plate (d) None of these	Answer: (c)
80.	An extremely hot star would appear to be (a) Yellow (b) Orange (c) Blue (d) Red	Answer: (c)
81.	A body at higher temperature T°K radiates heat at a rate which is proportional to **(MHT-CET-2001)** (a) T (b) T^2 (c) T^{-4} (d) T^4	Answer: (d)
82.	A black body emissive power is 81 J/m^2 sec it is at 300 K another one ordinary body emissivity is 0.8 it is at 500 K then what is the emissive power of ordinary body? **(MHT-CET-2008)** (a) 200 J/m^2 sec (b) 300 J/m^2 sec (c) 400 J/m^2 sec (d) 500 J/m^2 sec	Answer: (d)
83.	Which one of the following constants is related to radiation? (a) Gravitational constant	Answer: (b)

	(b) Planck's constant (c) Critical constant (d) Boltzmann's constant	
84.	If a black body is heated from 27°C to 927°C. then the ratio of radiation emitted will be **(MHT-CET-2005)** (a) 1: 4 (b) 1: 16 (c) 1: 8 (d) 1: 256	Answer: (d)
85.	You are given a sphere, a cube and a thin circular plate, all made from copper and all having the same mass. If they are heated to a temperature of 200°C, then which are of them will cool fasted? (a) Sphere (b) Thin circular plate (c) Cube (d) All at the same rate	Answer: (b)
86.	For a given body at a particular temperature, its coefficients of reflection and transmission are 0.63 and 0.21 respectively. Then its coefficient of emission at the same temperature will be (a) 0.16 (b) 0.32 (c) 0.84 (d) 0.36	Answer: (a)
87.	The unit of Stefa's constant is **(MHT-CET-2006)** (a) watt/m^2 K^4 (b) watt/m^3 K (c) watt/m^2 K (d) watt/m^3 K^4	Answer: (a)
88.	A body of surface area 25×10^{-3} m^2 emits 0.4 Kcal of heat in 20 s at 50°C. what is the emissive power of the body at that temperature? (a) 0.5 Kcal/m^2/S (b) 0.6 Kcal/m^2/S (c) 0.7 Kcal/m^2/S (d) 0.8 Kcal/m^2/S	Answer: (d)
89.	The wavelength of maximum energy release during an atomic explosion was 2.93×10^{-10} m. the maximum temperature attained must be (Wein's constant = 2.93×10^{-3} mK) **(MHT-CET-2002)** (a) 5.86×10^7 K (b) 10^{-13} k (c) 10^{-7} K (d) 10^7 K	Answer: (d)
90.	The solar constant for the earth is about 1.8 J/m^2/S. what is the solar constant for a black body situated on a planet which is situated at a distance of 0.3 times the distance of the earth from the sun? (a) 9 J/m^2/S	Answer: (d)

	(b) 12 $J/m^2/S$ (c) 15 $J/m^2/S$ (d) 20 $J/m^2/S$	
91.	Emissivity of perfectly black body is **(MHT-CET-2003)** (a) 1 (b) 2 (c) 5 (d) 0	Answer: (a)
92.	The velocity of thermal radiation (V) is related to the velocity of light (C) as (a) V > C (b) V < C (c) $V = \frac{C}{2}$ (d) V = C	Answer: (d)
93.	If a = 0.72, r = 0.24, then value of t is **(MHT-CET-2003)** (a) 0.02 (b) 0.04 (c) 0.4 (d) 0.2	Answer: (b)
94.	Wien's constant units is **(MHT-CET-2008)** (a) m.k (b) kg (c) m.m (d) mg.	Answer: (a)
95.	A liquid takes 5 minutes to cool from 80°C to 50°C. how much time it will take to cool from 60°C to 30°C, if the temperature of the surrounding is 20°C? (a) 5 minutes (b) 7 minutes (c) 9 minutes (d) 12 minutes	Answer: (c)

CH 12

Thermodynamics

(8 Hours, 6 Marks = 1Q-1M, 1Q-5M(T))

Syllabus :

Thermal equilibrium and definition of temperature (zeroth law of thermodynamics). Heat, work and internal energy. First law of thermodynamics. Second law of thermodynamics: reversible and irreversible processes. Heat engines and refrigerators.

1. Thermal equilibrium and definition of temperature (zeroth law of thermodynamics) :

Thermodynamics is the branch of physics that deals with the concepts of heat and temperature and the inter-conversion of heat and other forms of energy. Thermodynamics is a macroscopic science. It deals with bulk systems and does not go into the molecular constitution of matter.

The state of a system is an equilibrium state if the macroscopic variables that characterise the system do not change in time. For example, a gas inside a closed rigid container, completely insulated from its surroundings, with fixed values of pressure, volume, temperature, mass and composition that do not change with time, is in a state of **thermodynamic equilibrium**.

In thermal equilibrium, the temperatures of the two systems are equal.

Zeroth Law of Thermodynamics, which states that **'two systems in thermal equilibrium with a third system separately are in thermal equilibrium with each other'**. R.H. Fowler formulated this law in 1931 long after the first and second Laws of thermodynamics were stated and so numbered.

The Zeroth Law clearly suggests that when two systems *A* and *B*, are in thermal equilibrium, there must be a physical quantity that has the same value for both. This thermodynamic variable whose value is equal for two systems in thermal equilibrium is called temperature (T).
Thus, if *A* and *B* are separately in equilibrium with *C*, $T_A = T_C$ and $T_B = T_C$. This implies that $T_A = T_B$ i.e. the systems *A* and *B* are also in thermal equilibrium. We have arrived at the concept of temperature formally via the Zeroth Law.

2. Heat, work and internal energy. First law of thermodynamics :

Internal energy is thus, the sum of molecular kinetic and potential energies in the frame of reference relative to which the centre of mass of the system is at rest. Thus, it includes only the (disordered) energy associated with the random motion of molecules of the system. We denote the internal energy of a system by U.

The notion of heat should be carefully distinguished from the notion of internal energy. Heat is certainly energy, but it is the energy in transit. This is not just a play of words. The distinction is of basic significance. The state of a thermodynamic system is characterized by its internal energy, not heat. A statement like **'a gas in a given state has a certain amount of heat'** is as meaningless as the statement that **'a gas in a given state has a certain amount of work'**. In contrast, **'a gas in a given state has a certain amount of internal energy'** is a perfectly meaningful statement. Similarly, the statements **'a certain amount of heat is supplied to the system' or 'a certain amount of work was done by the system'** are perfectly meaningful.

To summarise, heat and work in thermodynamics are not state variables. They are modes of energy transfer to a system resulting in change in its internal energy, which, as already mentioned, is a state variable.

First law of thermodynamics :

The internal energy U of a system can change through two modes of energy transfer : heat and work. Let ΔQ = Heat supplied *to* the system *by* the surroundings ΔW = Work done *by* the system *on* the surroundings

ΔU = Change in internal energy of the system

The general principle of conservation of energy then implies that

$\Delta Q = \Delta U + \Delta W$ ----------- (12.1)

i.e. the energy (ΔQ) supplied to the system goes in partly to increase the internal energy of the system (ΔU) and the rest in work on the environment (ΔW). Equation (12.1) is known as the **First Law of Thermodynamics**.

It is simply the general law of conservation of energy applied to any system in which the energy transfer from or to the surroundings is taken into account.

Let us put Eq. (12.1) in the alternative form $\Delta Q - \Delta W = \Delta U$ ---------- (12.2)

Now, the system may go from an initial state to the final state in a number of ways. For example, to change the state of a gas from ($P1$, $V1$) to ($P2$, $V2$), we can first change the volume of the gas from $V1$ to $V2$, keeping its pressure constant i.e. we can first go the state ($P1$, $V2$) and then change the pressure of the gas from $P1$ to $P2$, keeping volume constant, to take the gas to ($P2$, $V2$). Alternatively, we can first keep the volume constant and then keep the pressure constant. Since U is a state variable, ΔU depends only on the initial and final states and not on the path taken by the gas to go from one to the other. However, ΔQ and ΔW will, in general, depend on the path taken to go from the initial to final states. From the First Law of Thermodynamics, Eq. (12.2), it is clear that the combination $\Delta Q - \Delta W$, is however, path independent. This shows that if a system is taken through a process in which $\Delta U = 0$ (for example, isothermal expansion of an ideal gas,

$\Delta Q = \Delta W$

i.e., heat supplied to the system is used up entirely by the system in doing work on the environment.

If the system is a gas in a cylinder with a movable piston, the gas in moving the piston does work. Since force is pressure times area, and area times displacement is volume, work done by the system against a constant pressure P is

$\Delta W = P\,\Delta V$

where ΔV is the change in volume of the gas.

Thus, for this case, Eq. (12.1) gives

$\Delta Q = \Delta U + P\,\Delta V$ ---------- (12.3)

The First Law of Thermodynamics is the principle of conservation of energy.

Ex : As an application of Eq. (12.3), consider the change in internal energy for 1 g of water when we go from its liquid to vapour phase. The measured latent heat of water is 2256 J/g. i.e., for 1 g of water ΔQ = 2256 J. At atmospheric pressure, 1 g of water has a volume 1 cm3 in liquid phase and 1671 cm3 in vapour phase.
Therefore,
$\Delta W = P (V_g - V_l) = 1.013 \times 105 \times (1670) \times 10{-}6 = 169.2$ J
Equation (12.3) then gives
$\Delta U = 2256 - 169.2 = 2086.8$ J
We see that most of the heat goes to increase the internal energy of water in transition from the liquid to the vapour phase.

A process in which the temperature of the system is kept fixed throughout is called an **isothermal process**.
In **isobaric processes** the pressure is constant while in **isochoric processes** the volume is constant.
Finally, if the system is insulated from the surroundings and no heat flows between the system and the surroundings, the process is **adiabatic**.

Type of processes	Feature
Isothermal	Temperature constant
Isobaric	Pressure constant
Isochoric	Volume constant
Adiabatic	No heat flow between the system and the surroundings ($\Delta Q = 0$)

3. Second law of thermodynamics: reversible and irreversible processes. Heat engines and refrigerators :

Second law of thermodynamics :

The Second Law of Thermodynamics gives a fundamental limitation to the efficiency of a heat engine and the co-efficient of performance of a refrigerator. In simple terms, it says that efficiency of a heat engine can never be unity.
The following two statements, one due to Kelvin and Planck denying the possibility of a perfect heat engine, and another due to Clausius denying the possibility of a perfect refrigerator or heat pump, are a concise summary of these observations.
Kelvin-Planck statement
No process is possible whose sole result is the absorption of heat from a reservoir and the complete conversion of the heat into work.
Clausius statement
No process is possible whose sole result is the transfer of heat from a colder object to a hotter object. It can be proved that the two statements above are completely equivalent.

Reversible and irreversible processes :

A thermodynamic process (state $i \rightarrow$ state f) is reversible if the process can be turned back such that both the system and the surroundings return to their original states, with no other change anywhere else in the universe.
A reversible process is an idealised notion. A process is reversible only if it is quasi-static (system in equilibrium with the surroundings at every stage) and there are no dissipative effects. For example, a quasi-static isothermal expansion of an ideal gas in a cylinder fitted with a frictionless movable piston is a reversible process.

The spontaneous processes of nature are irreversible. The free expansion of a gas is irreversible. The combustion reaction of a mixture of petrol and air ignited by a spark cannot be reversed. Cooking gas leaking from a gas cylinder in the kitchen diffuses to the entire room. The diffusion process will not spontaneously reverse and bring the gas back to the cylinder.

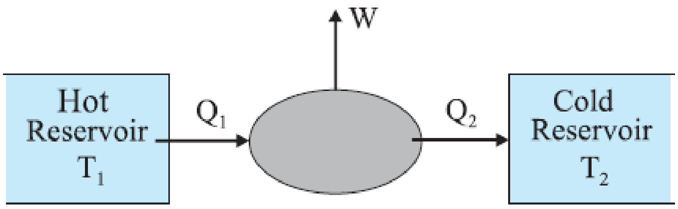
A reversible process would amount to conversion of heat entirely into work, violating the Second Law of Thermodynamics. Irreversibility is a rule rather an exception in nature.

Irreversibility arises mainly from two causes: one, many processes (like a free expansion, or an explosive chemical reaction) take the system to non-equilibrium states; two, most processes involve friction, viscosity and other dissipative effects (e.g., a moving body coming to a stop and losing its mechanical energy as heat to the floor and the body; a rotating blade in a liquid coming to a stop due to viscosity and losing its mechanical energy with corresponding gain in the internal energy of the liquid). Since dissipative effects are present everywhere and can be minimised but not fully eliminated, most processes that we deal with are irreversible.

Heat engines and refrigerators :

Heat engine :

Heat engine is a device by which a system is made to undergo a cyclic process that results in conversion of heat to work.
(1) It consists of a **working substance**–the system. For example, a mixture of fuel vapour and air in a gasoline or diesel engine or steam in a steam engine are the working substances.
(2) The working substance goes through a cycle consisting of several processes. In some of these processes, it absorbs a total amount of heat $Q1$ from an external reservoir at some high temperature $T1$.
(3) In some other processes of the cycle, the working substance releases a total amount of heat $Q2$ to an external reservoir at some lower temperature $T2$.
(4) The work done (W) by the system in a cycle is transferred to the environment via some arrangement (e.g. the working substance may be in a cylinder with a moving piston that transfers mechanical energy to the wheels of a vehicle via a shaft).
The basic features of a heat engine are schematically represented in Fig. 12.9.



The cycle is repeated again and again to get useful work for some purpose. The discipline of thermodynamics has its roots in the study of heat engines. A basic question relates to the efficiency of a heat engine. The efficiency (η) of a heat engine is defined by $\eta = W/Q_1$
where Q_1 is the heat input i.e., the heat absorbed by the system in one complete cycle
and W is the work done on the environment in a cycle. In a cycle, a certain amount of heat ($Q2$) may also be rejected to the environment. Then, according to the First Law of Thermodynamics, over one complete cycle,

$W = Q_1 - Q_2$ ------------- (12.19)

i.e.,

$\eta = 1 - \frac{Q_2}{Q_1}$ ---------- (12.20)

For $Q_2 = 0$, $\eta = 1$, i.e., the engine will have 100% efficiency in converting heat into work.

Note that the First Law of Thermodynamics i.e., the energy conservation law does not rule out such an engine. But experience shows that such an ideal engine with η = 1 is never possible, even if we can eliminate various kinds of losses associated with actual heat engines. It turns out that there is a fundamental limit on the efficiency of a heat engine set by an independent principle of nature, called the Second Law of Thermodynamics.

The mechanism of conversion of heat into work varies for different heat engines. Basically, there are two ways : the system (say a gas or a mixture of gases) is heated by an external furnace, as in a steam engine; or it is heated internally by an exothermic chemical reaction as in an internal combustion engine.

Refrigerators :

A refrigerator is the reverse of a heat engine. Here the working substance extracts heat Q_2 from the cold reservoir at temperature T_2, some external work W is done on it and heat Q_1 is released to the hot reservoir at temperature T_1 (Fig. 12.10).

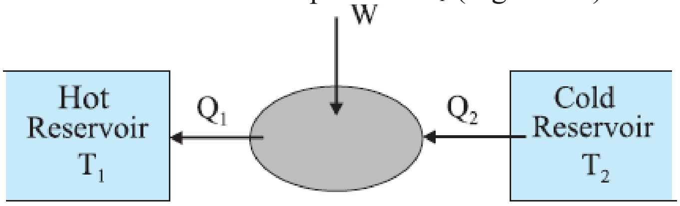


Fig. : *Schematic representation of a refrigerator or a heat pump, the reverse of a heat engine.*

A heat pump is the same as a refrigerator. What term we use depends on the purpose of the device. If the purpose is to cool a portion of space, like the inside of a chamber, and higher temperature reservoir is surrounding, we call
the device a refrigerator; if the idea is to pump heat into a portion of space (the room in a building when the outside environment is cold), the device is called a heat pump.

In a refrigerator the working substance (usually, in gaseous form) goes through the following steps : (a) sudden expansion of the gas from high to low pressure which cools it and converts it into a vapour-liquid mixture, (b) absorption by the cold fluid of heat from the region to be cooled converting it into vapour, (c) heating up of the vapour due to external work done on the system, and (d) release of heat by the vapour to the surroundings, bringing it to the initial state and completing the cycle.

The coefficient of performance (α) of a refrigerator is given by $\alpha = \frac{Q_2}{W}$

where Q_2 is the heat extracted from the cold reservoir and W is the work done on the system–the refrigerant. (α for heat pump is defined as Q_1/W) Note that while η by definition can never exceed 1, α can be greater than 1.

By energy conservation, the heat released to the hot reservoir is

$Q_1 = W + Q_2$

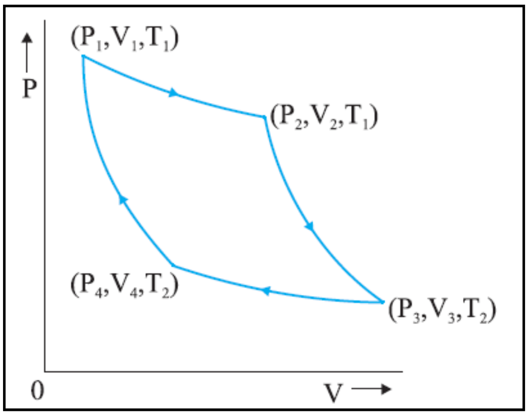
$$\alpha = \frac{Q_2}{Q_1 - Q_2} \quad \text{---------} \ (\ )$$

In a heat engine, heat cannot be fully converted to work; likewise a refrigerator cannot work without some external work done on the system, i.e., the coefficient of performance(α) cannot be infinite.

Carnot Engine :

Carnot Engine is an ideal, reversible heat engine operating between two temperatures T_1 (source) and T_2 (sink). The Carnot cycle consists of two isothermal processes connected by two adiabatic processes.

The working substance of the Carnot engine to be an ideal gas. Such an engine must have the following sequence of steps constituting one cycle, called the Carnot cycle.



Carnot cycle consists of two adiabatic and two isothermal processes, all are reversible. To describe the Carnot cycle, assume the working substance or ideal gas contained in a cylinder fitted with a movable piston.

* The cylinder walls and piston are non-conducting.

Step $_{1\to2}$ **:** Isothermal expansion of the gas taking its state from (P_1, V_1, T_1) to (P_2, V_2, T_1).

Thus the work done ($W_{1\to2}$) by the gas on the environment is

$$W_{1\to2} = Q_1 = \mu R T_1 \ln\frac{V_2}{V_1} \quad \text{---------} \ (1)$$

Step $_{2\to3}$**:** Adaibatic expansion of the gas from (P_2, V_2, T_1) to (P_3, V_3, T_2).

Thus the work done ($w_{2\to3}$) by the gas is

$$w_{2\to3} = \frac{\mu R(T_1 - T_2)}{\gamma - 1} \quad \text{---------} \ (2)$$

Step $_{3\to4}$ **:** Isothermal compression of the gas taking its state from (P_3, V_3, T_2) to (P_4, V_4, T_2).

Heat released (Q_2) by the gas to the reservior at temperature T_2.

Thus the work done ($w_{3\to4}$) on the gas by the environment is

$$w_{3\to4} = Q_2 = \mu RT_2 \ln\frac{V_3}{V_4} \quad \text{---------} (3)$$

Step $_{4\to1}$ **:** Adaibatic compression of the gas from (P_4, V_4, T_2) to (P_1, V_1, T_1).

Thus the work done ($w_{4\to1}$) on the gas is

$$w_{4\to1} = \frac{\mu R(T_1 - T_2)}{\gamma - 1} \quad \text{---------} (4)$$

From equation (1) to (4) total work done by the gas in one complete cycle

$$W = W_{1\to2} + W_{2\to3} - W_{3\to4} - W_{4\to1}$$

$$W = \mu RT_1 \ln\frac{V_2}{V_1} - \mu RT_2 \ln\frac{V_3}{V_4}$$

The efficiency of the carnot engine is

$$\eta = \frac{W}{Q_1} = 1 - \frac{Q_2}{Q_1}$$

$$\eta = 1 - \frac{T_2}{T_1}\left(\frac{\ln\frac{V_3}{V_4}}{\ln\frac{V_2}{V_1}}\right) \quad \text{------------}(5)$$

Step $_{2\to3}$: is an adiabatic process

Then $T_1\, V_2^{\gamma-1} = T_2\, V_3^{\gamma-1}$

$$\text{i.e.}\ \frac{V_2}{V_3} = \left(\frac{T_2}{T_1}\right)^{1/(\gamma-1)} \quad \text{------------}(6)$$

Similarly in Step $_{4\to1}$ is an adiabatic process

Then $T_2\, V_4^{\gamma-1} = T_1\, V_1^{\gamma-1}$

$$\text{i.e.}\ \frac{V_1}{V_4} = \left(\frac{T_2}{T_1}\right)^{1/(\gamma-1)} \quad \text{------------}(7)$$

From equations (6) and (7) we get

$$\frac{V_3}{V_4} = \frac{V_2}{V_1} \qquad \text{------------(8)}$$

From equations (8) and (5) we get

$$\eta = 1 - \frac{T_2}{T_1} \qquad \text{------------(9)}$$

is the expression for efficiency of Carnot engine.

The efficiency of a Carnot engine is given by $\eta = 1 - \frac{T_2}{T_1}$

Carnot's theorem :

(a) working between two given temperatures $T1$ and $T2$ of the hot and cold reservoirs respectively, no engine can have efficiency more than that of the Carnot engine and

(b) the efficiency of the Carnot engine is independent of the nature of the working substance.

ONE MARK QUESTIONS

1. What is Thermodynamics?

The branch of physics that deals with the concept of heat and temperature and inter-conversion of heat and other forms of energy.

2. Mention the Macroscopic variables to specify the thermodynamics.

Pressure, Volume, Temperature, mass and composition.

3. How does thermodynamics differ from Mechanics?

Mechanics deals with motion of particles under the action of forces, while Thermodynamics concerned with internal Microscopic state of the body.

4. What is thermodynamic equilibrium?

The system is said to be in the Thermodynamic equilibrium when the macroscopic variables do not change with time.

5. Give the meaning of the term 'adiabatic wall'.

It is an insulating wall (can be movable) that does not allow flow of energy (heat) from one system to another.

6. Give the meaning of 'diathermic wall'.

It is a conducting wall that allows energy flow (heat) from one system to another.

7. Name the scientist who formulated. Zeroth law of thermodynamics.

R.H. Fowler in 1931.

8. Write the significance of Zero'th law of thermodynamics?

It signifies the concept of temperature.

9. Mention the factor on which internal energy depends on.

It depends on 'state of system'

10. Mention the modes of changing internal energy.

Heat and work.

11. What are thermodynamic state variables?

Variables like Pressure, Temperature, internal Energy which determines the Thermodynamic state are called Thermodynamics state variables.

12. Mention the modes of energy transfer to a system.

Heat and work.

13. Define Internal energy.

Internal energy is the sum of molecular kinetic and potential energies in the frame of reference relative to which the centre of mass of the system is at rest.

14. Mention the principle of first law of thermodynamics
Law of conservation of energy.

15. Define specific heat capacity of water.
Specific heat capacity of water is equal to the amount of heat required to raise the temperature of 1kg of water by 1K.

16. Write the value of specific heat capacity of water.
4186 Jkg-1K-1

17. Define one calorie.
One calorie is defined to be the amount of heat required to raise the temperature of 1g of water from 14.5^0C to 15.5^0C.

18. What is equation of state?
The relation connecting between state variables is called equation of state.

19. Give the equation of state of ideal gas.
PV = RT for moles of ideal gas.

20. What is isothermal curve?
The pressure- volume curve for fixed temperature is called isothermal curve.

21. Mention the kinds of thermodynamic state variables.
Extensive variables and intensive variable.

22. Give an example of extensive state variable.
Volume, Mass, Internal Energy etc.

23. Give an example of intensive state variable.
Pressure, Temperature, Density etc.

24. What is quasi-static process?
The process in the system the variables (PTV) changes vary slowly so that the system remain in thermal and mechanical equilibrium with its surrounding throughout is called quasi-static process.

25. What is isothermal process?
A process in which the temperature of the system is kept constant throughout is called isothermal process. (i.e. $\Delta T = 0$)

26. What is isobaric process?
A process in which the pressure is kept constant is called isobaric process. (i.e. $\Delta P=0$)

27. What is isochoric process?

A process in which the volume is kept fixed is called isochoric process (i.e. $\Delta V=0$)

28. What is adiabatic process?
A process in which no heat flows between system and its surrounding is called adiabatic process. (i.e. $\Delta Q = 0$)

29. When does gas absorb heat and does work during isothermal process?
during isothermal expansion.

30. When does gas release heat and work is done by the surrounding?
during isothermal compression.

31. Mention the condition for work done by the gas in an adiabatic process.
during adiabatic expansion.

32. Mention the condition for work done on the gas in an adiabatic process.
during adiabatic compression.

33. What is the work done in isochoric process?
Zero Joule (because heat absorbed by the gas is entirely utilized to change its internal energy only).

34. Mention the expression for work done in isobaric process.
$W = P(V_2 - V_1)$

35. What is the work done in cyclic process?
Total work done is equal to total heat absorbed.

36. What is heat engine?
Heat engine is a device by which a system is made to undergo a cyclic process that results in conversion of heat into work.

37. Define the efficiency of a heat engine.
It is defined as the ratio of useful work done (W) to the heat input (Q_1)

38. Mention the expression for coefficient of performance of refrigerator
$\alpha = Q_2/W$

39. Mention the expression for amount of heat released to the heat reservoir.
$Q_1 = W + Q_2$

40. What is Heat pump?
A device used to increase the temperature of a system is known as heat pump.

41. Mention the thermodynamic process in which highest efficiency is possible.

Reversible process.

42. Mention the thermodynamic process in which lowest efficiency is possible.
Irreversible process.

43. What is Carnot engine?
A reversible heat engine operating between two temperatures is called a Carnot engine.

44. Who designed the Carnot engine?
N.L. Sodi Carnot - a French Engineer.

45. What is Carnot cycle?
The sequence of steps constituting one cycle of operation of Carnot Engine is called Carnot cycle.

46. Name the working substance used in Carnot cycle.
Ideal Gas.

TWO MARK QUESTIONS :

1. State and explain Zero'th law of thermodynamics.
Statement: "two systems in thermal equilibrium with a third system separately are in thermal equilibrium with each other".
Explanation : If A and B are two systems are equilibrium with system C then $T_A = T_C$ and $T_B = T_C$.
This implies that $T_A = T_B$ i.e. the system A and B are also in thermal equilibrium. Thus we arrived the concept of temperature formally via the Zero'th Law.

2. Justify the statement "A gas in a given state has a certain amount of work".
No, "A gas in a given state has a certain amount of internal energy" is a meaningful statement.

3. Justify the statement "A gas in a given state has a certain amount of heat"
No, "A certain amount of heat is supplied to the system or a certain amount of work was done by the system" is a meaningful statement.

4. State and explain First law of thermodynamics.
"When an amount of heat energy is supplied to the system, part of it increases internal energy of the system and remaining is utilized by the system to do work".
Explanation: If ΔQ is heat supplied to the system by the surrounding, Δu is change in internal energy of the system and Dw is work done by the system on the surrounding.
Then according to first law of thermodynamics $\Delta Q = \Delta u + \Delta w$
* It is the principle of the conservation of energy.

5. Explain isothermal process by applying first law of thermodynamics.
For isothermal process temperature remains constant i.e. $\Delta u = 0$.

Therefore according to first law of thermodynamics $\Delta Q = \Delta w$ Heat supplied to the system is used up entirely by the system in doing work on environment.

6. Explain isobaric process by applying first law of thermodynamics.

For isobaric process pressure remains constant. $\Delta W = P\,\Delta V$ i.e. work done by the system. Then we write first law of thermodynamics as $\Delta Q = \Delta u + P\,\Delta V$.

7. Write the equation of state for ideal gas. Explain the terms.

For an ideal gas, the equation of state is $PV = \mu RT$., where P is pressure, V is volume, R is universal gas constant and T is absolute temperature and μ is number of moles.

8. Show that molar specific heat of solids, C = 3R.

Consider a solid of N atoms, each vibrating about its mean position. An oscillator in one dimension has average energy of 2 x ½ kBT = kBT. For a mole of a solid.

The total energy, $U = 3k_BT \times N_A = 3\,RT$

At constant pressure, $\Delta Q = \Delta u + P\Delta V \cong \Delta U$

Since for a solid D V is negligible. Therefore, molar specific heat.

$$C = \frac{\Delta Q}{\Delta T} = \frac{\Delta U}{\Delta T} = 3R \qquad C = 3R$$

9. Mention the two ways of Mechanism of conversion of heat into work.

*The system is heated by an external furnace as in steam engine.

*The system is heated internally by an exothermic chemical reaction as in internal combustion engine.

10. State both forms of second law of thermodynamics.

Kelvin-Planck statement: “No process is possible whose sole result is the absorption of heat from a reservoir and the complete conversion of the heat into work”

Clausius statement: “No process is possible whose sole result is the transfer of heat from a colder object to a hotter object”.

* It can be proved that the two statements above are completely equivalent.

11. According to II law of thermodynamic, what are the limitations of efficiency and co-efficient of performance.

According the II Law,

*efficiency never be unity or never exceed unity.

*Coefficient of performance never be infinite.

12. What is reversible process? Give an example.

A process is reversible only if it is quasi-static and non dissipative.

Eg. A quasi-static isothermal expansion of an ideal gas in a cylinder fitted with a frictionless movable piston is reversible process.

13. What is irreversible process? Give an example.

A process which takes the system to the non equilibrium state is called irreversible process.

*It can't be retraced in the reverse direction.
Eg. The Free expansion of gas. The combustion reaction of mixture of petrol and air, ignited by a spark.

14. Mention the causes of irreversibility.
Friction, viscosity and other dissipative effect.

QUESTIONS CARRIES 4 OR 5 MARKS.
1. Using first law of thermodynamics, arrive at the relation, Cp-Cv = R.
According to 1st law of thermodynamics $\Delta Q = \Delta u + P\,\Delta V$,
consider one mole of ideal gas, if ΔQ is the amount of heat absorbed at constant volume then $\Delta V = 0$.
and molar specific heat of gas at constant volume becomes

$$Cv = \left(\frac{\Delta Q}{\Delta T}\right)_v = \left(\frac{\Delta U}{\Delta T}\right)_v = \left(\frac{\Delta U}{\Delta T}\right) \text{ --------------- } 1$$

Where the subscript v is dropped in the last step, since U of an ideal gas depends only on temperature.
If ΔQ is the amount of heat absorbed at constant pressure then

$$C_p = \left(\frac{\Delta Q}{\Delta T}\right)_p = \left(\frac{\Delta U}{\Delta T}\right)_p + P\left(\frac{\Delta V}{\Delta T}\right)_p \quad \text{or}$$

$$C_p = \left(\frac{\Delta Q}{\Delta T}\right) = \left(\frac{\Delta U}{\Delta T}\right) + P\left(\frac{\Delta V}{\Delta T}\right) \text{---------------} 2$$

Since U of an ideal gas depend only on T.
For one mole of ideal gas PV = RT which gives

$$P\left(\frac{\Delta V}{\Delta T}\right)_p = R \text{ --------------- } 3$$

substituting equations 1 and 3 in equation 2, we get

$$C_p = C_v + R$$
$$\Rightarrow C_p - C_v = R$$

[This relation is called Mayer's relation. Where R is universal gas constant and C**p** and C**v** are molar specific heat capacities of an ideal gas at constant pressure and constant volume respectively.]

2. What is isothermal process? Obtain the equation of isothermal process
OR
What is isothermal process? Obtain the expression for work done by isothermal process.
A thermodynamic process which takes place at a constant temperature is called an isothermal process.
For an ideal gas in isothermal process, the equation of state for mole of gas is PV = constant => PV = μRT
The gas under goes isothermal expansion from (P_1V_1) to (P_2V_2), at any intermediate stage with pressure P, and volume changes from V to V + ΔV
Then work done $\Delta W = P\,\Delta V$.
Taking ($\Delta V \to 0$) and summing the quantity ΔW over the entire process.

$$W = \int_{V_1}^{V_2} PdV = \mu RT\int_{V_1}^{V_2}\frac{dV}{V} = \mu RT\ln\frac{V_2}{V_1} \qquad W = \mu RT\ln\frac{V_2}{V_1}$$

This is the expression for work done during isothermal expansion.

3. What is adiabatic process? Obtain the expression for work done in adiabatic process.

A thermodynamic process during which no heat enters or leaves the system is known as an adiabatic process.

Consider 'μ' moles of ideal gas undergoing a adiabatic expansion from V_1 to V_2. For adiabatic process PV = constant, where ' g ' is the ratio of specific heats at constant pressure and at constant volume. i.e. $\gamma = \frac{C_P}{C_V}$.

If gas under goes change in its state adiabatically from (P_1, V_1) to (P_2, V_2) Then = $(P_1V_1)^{\gamma}$ = $(P_2V_2)^{\gamma}$

The work done in adiabatic change of an ideal gas from the sate (P_1, V_1, T_1) to the state (P_2, V_2,T_2) is

$$W = \int_{V_1}^{V_2} PdV = \text{constant } x \int_{V_1}^{V_2}\frac{dV}{V^{\gamma}} = \text{constant } x \left.\frac{V^{-\gamma+1}}{1-\gamma}\right|_{V1}^{V2}$$

$$= \frac{\text{constant}}{(1-\gamma)} x \left[\frac{1}{V_2^{\gamma-1}} - \frac{1}{V_1^{\gamma-1}}\right]$$

From equation (1) the constant is $(P_1V_1)^{\gamma}$ or $(P_2V_2)^{\gamma}$ then

$$W = \frac{\text{constant}}{(1-\gamma)} x \left[\frac{1}{V_2^{\gamma-1}} - \frac{1}{V_1^{\gamma-1}}\right]$$

$$W = \frac{1}{(1-\gamma)} x \left[\frac{P_2V_2^{\gamma}}{V_2^{\gamma-1}} - \frac{P_1V_1^{\gamma}}{V_1^{\gamma-1}}\right]$$

$$\text{or} \quad W = \frac{1}{(1-\gamma)}(P_2V_2 - P_1V_1) = \frac{\mu R(T_1 - T_2)}{\gamma - 1}$$

$$W = \frac{\mu R(T_1 - T_2)}{\gamma - 1}$$ is the expression for work done by the gas in adiabatic process.

4. Explain the features of a heat engine with schematic diagram.

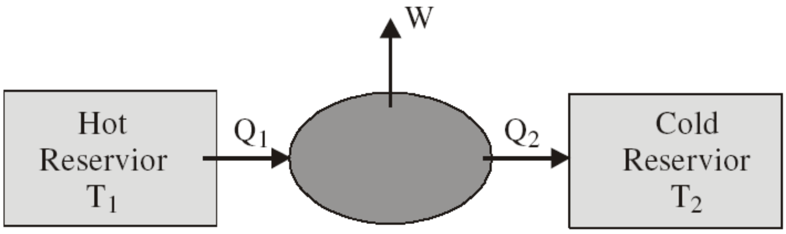
Heat engine is a device by which a system is made to undergo a cyclic process that results in conversion of heat to work.

* It consists of a working substance-the system. Eg. a mixture of fuel vapour and air in a gasoline.

* The working substance goes through a cycle consisting of several processes. In some of these processes, it absorbs a total amount of heat Q_1 from an external reservoir at some high temperature T_1.

* In some other processes of the cycle, the working substance releases a total amount of heat Q_2 to an external reservoir at some lower temperature T_2.

* The work done (W) by the system in a cycle is transferred to the environment via some arrangement like cylinder with piston system and is as shown in fig.



The cycle is repeated again and again to get useful work for some purpose.

5. Distinguish between isothermal and adiabatic process.

Isothermal	Adiabatic
1. Temperature remains constant	1. Heat energy exchange is 0.
2. It is a slow process	2. It is sudden process.
3. It is represented by equation PV =Const.	3. It is represented by equation PV^{γ} = constant.
4. Slope of PV graph is comparatively small.	4. Slope of PV graph is comparatively large(γ time)
5. Work done $W = \mu RT \ln\frac{V_2}{V_1}$	5. $W = \frac{\mu R(T_1 - T_2)}{\gamma - 1}$

6. Describe the Carnot cycle of operation using PV diagram.

Carnot cycle consists of two adiabatic and two isothermal processes, all are reversible. To describe the carnot cycle, assume the working substance or ideal gas contained in a cylinder fitted with a movable piston.

* The cylinder walls and piston are non-conducting.

Step $_{1\to2}$ **:** Isothermal expansion of the gas taking its state from (P_1, V_1, T_1) to (P_2, V_2, T_1).

Thus the work done ($W_{1\to2}$) by the gas on the environment is

$$W_{1\to2} = Q_1 = \mu RT_1 \ln\frac{V_2}{V_1} \quad \text{---------} \ (1)$$

Step $_{2\to3}$**:** Adaibatic expansion of the gas from (P_2, V_2, T_1) to (P_3, V_3, T_2).

Thus the work done ($w_{2\to3}$) by the gas is

$$w_{2\to3} = \frac{\mu R(T_1 - T_2)}{\gamma - 1} \quad \text{---------} \ (2)$$

Step $_{3\to4}$ **:** Isothermal compression of the gas taking its state from (P_3, V_3, T_2) to (P_4, V_4, T_2).

Heat released (Q_2) by the gas to the reservior at temperature T_2.

Thus the work done $(w_{3\to4})$ on the gas by the environment is

$$w_{3\to4} = Q_2 = \mu RT_2 \ln\frac{V_3}{V_4} \quad \text{---------} (3)$$

Step $_{4\to1}$ **:** Adaibatic compression of the gas from (P_4, V_4, T_2) to (P_1, V_1, T_1).

Thus the work done $(w_{4\to1})$ on the gas is

$$w_{4\to1} = \frac{\mu R(T_1 - T_2)}{\gamma - 1} \quad \text{---------} (4)$$

From equation (1) to (4) total work done by the gas in one complete cycle

$$W = W_{1\to2} + W_{2\to3} - W_{3\to4} - W_{4\to1}$$

$$W = \mu RT_1 \ln\frac{V_2}{V_1} - \mu RT_2 \ln\frac{V_3}{V_4}$$

The efficiency of the carnot engine is

$$\eta = \frac{W}{Q_1} = 1 - \frac{Q_2}{Q_1}$$

$$\eta = 1 - \frac{T_2}{T_1}\left(\frac{\ln\frac{V_3}{V_4}}{\ln\frac{V_2}{V_1}}\right) \quad \text{------------}(5)$$

Step $_{2\to3}$: is an adiabatic process

$$\text{Then } T_1 V_2^{\gamma-1} = T_2 V_3^{\gamma-1}$$

$$\text{i.e.} \frac{V_2}{V_3} = \left(\frac{T_2}{T_1}\right)^{1/(\gamma-1)} \quad \text{------------}(6)$$

Similarly in Step $_{4\to1}$ is an adiabatic process

$$\text{Then } T_2 V_4^{\gamma-1} = T_1 V_1^{\gamma-1}$$

$$\text{i.e.} \frac{V_1}{V_4} = \left(\frac{T_2}{T_1}\right)^{1/(\gamma-1)} \quad \text{------------}(7)$$

From equations (6) and (7) we get

$$\frac{V_3}{V_4} = \frac{V_2}{V_1} \qquad \text{------------(8)}$$

From equations (8) and (5) we get

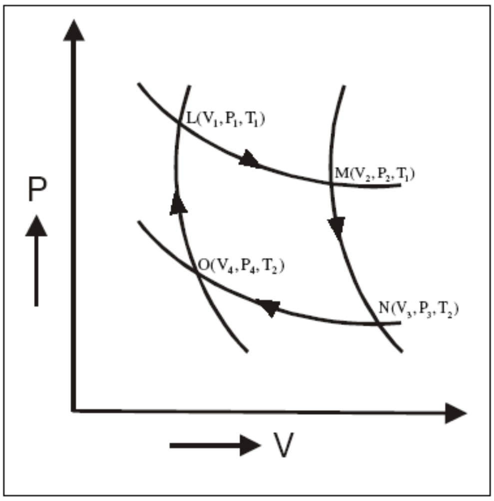
$$\eta = 1 - \frac{T_2}{T_1} \qquad \text{------------(9)}$$

is the expression for efficiency of Carnot engine.

Note : (i) The interesting aspect of h of Carnot engine is that it is independent of the nature of the working substance. But Carnot used an ideal gas operation which is not strictly followed by real gases or fuels.
(ii) Theoretically h can be 100%.
(iii) The efficiency of Carnot's ideal engine depends only on the temperatures of the source and the sink.
(iv) Efficiency of any reversible engine working between same two temperatures is same.

7. Discuss the Carnot cycle of operation and deduce the expression for efficiency.

Discuss the Carnot cycle of operation and deduce the expression for efficiency. Carnot engine is works on the principle of Carnot cycle made up of four stages are represented as shown in (PV) diagram.

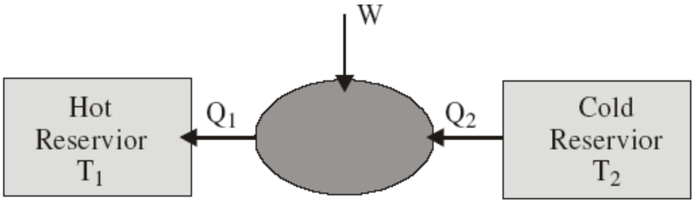


1) Isothermal expansion (LM) : The gas is allowed to expand isothermally.
During the expansion gas absorb an amount of heat Q_1 from the source at T_1,
(2) Adiabatic expansion (MN) : The gas is allowed to expand adiabatically till the temperature of gas falls to T_2, the temperature of the sink,
(3) Isothermal compression (NO) : The gas is compressed slowly at constant temperature T_2. During this process a certain amount of heat Q_2 is rejected into the sink,
(4) Adiabatic compression (OL): The gas is further compressed adiabatically till it returns to its initial state L.

The cycle of operations, LM, MN, NO and OL is called the Carnot cycle. It is represented by the closed loop LMNOL.

8. What is refrigerator ? Explain the working of refrigerator with schematic diagram.

A refrigerator is the reverse of a heat engine. Here the working substance extracts heat Q_2 from the cold reservoir at temperature T_2, some external work W is done on it and heat Q_1 is released to the hot reservoir at temperature T_1.



In a refrigerator the working substance (usually, in gaseous form) goes through the following steps:
(a) Sudden expansion of the gas from high to low pressure which cools it and converts it into a vapour-liquid mixture,
(b) Absorption by the cold fluid of heat from the region to be cooled converting it into vapour.
(c) Heating up of the vapour due to external work done on the system, and
(d) Release of heat by the vapour to the surroundings, bringing it to the initial state and completing the cycle.
The coefficient of performance (a) of a refrigerator is given by $\alpha = \frac{Q_2}{W}$

PROBLEMS :

1. A cylinder with a movable piston contains 3 moles of hydrogen at constant temperature and pressure. The walls of a cylinder are made up of a heat insulator, and the piston is insulated by having a pile of sand on it. By what factor does the pressure of a gas increases if the gas is compressed to half its original volume?

Ans :

Since the process is adiabatic

$$P_1V_1^{\gamma} = P_2V_2^{\gamma}$$

$$\frac{V_1}{V_2} = \frac{2}{1}, \quad \gamma = 7/5 \text{ for hydrogen}$$

∴ Factor by which the pressure of the gas increases

$$\Rightarrow \frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^{\gamma} \qquad \Rightarrow \frac{P_2}{P_1} = (2)^{7/5}$$

2. In changing the state of a gas adiabatically from an equilibrium state A to another equilibrium state B, an amount of work equal to 22.3 J is done on the system. If the gs is taken from state A to B via a process in which the net heat absorbed by the system is 9.35 cal., how much is the network done by the system in the later case? (Take 1 cal = 4.19J)

Since, the system is going from A to B and then back to A, it is undergoing a cyclic change. Now in cyclic change there is no change in internal energy $\Delta U = 0$.
From first law of thermodynamics
$\Delta Q = \Delta U + \Delta W = \Delta W$ since $\Delta U = 0$
The amount of heat absorbed by the system
$\Delta Q = 9.35 \text{ cal} - 22.3 \text{ J}$
$= (9.35 \times 4.19 - 22.3) \text{ J}$
$= 16.87 \text{ J}$

∴ The net work done by the system = ΔW 16.87 J

3. An electric heater supplies heat to a system at the rate of 10 W. If the system performs work at a rate of 75 joules per second. At what rate is the internal energy increasing ?

Given : heat supplied per second, $\Delta Q = 100$ J
Work done by the system per second = $\Delta W = 75$J
Increase in internal energy per second = $\Delta U = ?$
From the first law of thermodynamics
$\Delta Q = \Delta U + \Delta W = \Delta W \qquad \Delta U = \Delta Q - \Delta W$
$\Delta U = 100 \text{ J} - 75 \text{ J} \qquad \Delta U = 25$ J per second.
or $\Delta U = 25$ W

4. A steam engine delivers 5.4 x 10^8J of work per minute and services 3.6 x 10^9J of heat per minute from the boiler. What is the efficiency of the engine? How much heat is wasted per minute?

Given : Q_1 = heat absorbed from the boiler per minute = 3.6 x 10^9 J
W = work done per minute by the steam engine = 5.4 x 10^8 J
Q_2 = heat wasted / rejected per minute = ?
= percentage efficiency of the heat engine = ?
We have = $\eta\% = \frac{W}{Q_1} \times 100$

$$= \frac{5.4\text{x}10^8 \text{J}}{3.6\text{x}10^9 \text{J}} \text{ x } 100 \qquad = \frac{3}{20} \text{ x } 100 = 15\%$$

Also heat absorbed $Q_1 = W + Q_2$
heat wasted $Q_2 = Q_1 - W$ = 36 x 10^8 - 5.4 x 10^8
= 30.6 x 10^8 J/min. = 3.06 x 10^9 J/m
= 3.1 x 10^9 J/min.

5. A perfect Carnot engine utilizes an ideal gas. The source temperature is 500 K and sink temperature is 375K. If the engine takes 600K cal per cycle from the source, compute:
(a) the efficiency of the engine.
(b) Work done per cycle.
(c) Heat rejected to the sink per cycle.

Given : T1 = 500K
T2 = 375K
Q1 = Heat absorbed per cycle = 600K cal.

We have $\eta = 1 - \frac{T_2}{T_1} \qquad \eta = \frac{T_1 - T_2}{T_1} = \frac{500\text{-}375}{500} = \frac{125}{500} = 0.25$

$\eta\% = 0.25 \text{ x } 100 = 25\%$
Suppose W = work done per cycle
from the relation, $\eta = W/Q_1$, we get
$W = \eta Q_1 = 0.25 \text{ x } 600$ K cal.

= 150K cal

= 150 x 10^3 x 4.2 J

= 6.3 x 10^5 J.

Suppose Q_2 = heat rejected to the sink.
Then $W = Q_1 - Q_2$, we get
$Q_2 = Q_1 - W = 600 - 150 = 450$ K cal.

6. A refrigerator has to transfer an average of 263 J of heat per second from temperature - 10^0C to 25^0C. Calculate the average power consumed assuming ideal reversible cycle and no other losses.

$T_1 = 25 + 273 = 298$ K $\quad T_2 = -10 + 273 = 263$ K $\quad Q_2 = 263$ JS^{-1}

We have $\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$ $\quad Q_1 = \frac{T_1}{T_2} \times Q_2 = \frac{298}{263} \times 263$

$Q_1 = 298$ Js^{-1}
Average power consumed = $Q_1 - Q_2$
= (298 - 263) Js^{-1}
= 35 W

7. What is the coefficient of performance (b) or a Carnot refrigerator working between 30^0C and 0^0C?

Given : $T_2 = 0^0C = 273$ K
$T_1 = 30^0C = 273 + 30 = 303$ K
b = ?
We have

$$\beta = \frac{T_2}{T_1 - T_2} \qquad \beta = \frac{273}{303-273} = \frac{273}{30} = 9.1$$

8. A certain volume of dry air at NTP is allowed to expand 4 times of its original volume under (a) isothermal conditions (b) adiabatic conditions. Calculate the final pressure and temperature in each case g = 1.4.

Given : $V_1 = V$
$V_2 = 4V$
$P_1 = 76$ cm of Hg.
P_2 = ?
g = 1.4
$T_1 = 273$ K
T_2 = ?
For isothermal expansion $P_1 V_1 = P_2 V_2$
$P_2 = P_1 (V_1/V_2) = 72/4 = 19$ cm of Hg.
As the process is isothermal, therefore the final temperature will be the same as the initial temperature.
i.e. $T_2 = 273$K
For Adiabatic expansion:

$$P_1 V_1^{\gamma} = P_2 V_2^{\gamma}$$

$P_2 = P_1\left(\frac{V_1}{V_2}\right)^{\gamma}$ = $P_2 = 76\left(\frac{1}{4}\right)^{1.4}$ = 76 x $(0.25)^{1.4}$ = 10.91 cm of Hg.

Also :

$$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$$

$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1}$ $T_2 = 273\left(\frac{1}{4}\right)^{1.4-1}$ $= \frac{273}{(4)^{0.4}}$ = 156.8 K

9. In a refrigerator, heat from inside at 277 K is transferred to a room at 300K. How many joules of heat will be delivered to the room for each joule of electric energy consumed ideally?

T_2 = 277K, T_1 = 300 K, Q_1 =?, W = 1 J

We have $\beta = \frac{T_2}{T_1 - T_2}$

$$\beta = \frac{277}{300-277} = \frac{277}{23} = 12.04$$

Also $\beta = \frac{Q_2}{W} = \frac{Q_2}{1}$

We get $\beta = Q_2$

Q_2 = 12.04 J

Now $W = Q_1 - Q_2$

or $Q_1 = W + Q_2$

i.e. Total heat delivered to the room,

Q_1 = 1 + 12.04 = 13.04 J

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TEXTBOOK EXERCISES

12.1 A geyser heats water flowing at the rate of 3.0 litres per minute from 27 °C to 77 °C. If the geyser operates on a gas burner, what is the rate of consumption of the fuel if its heat of combustion is 4.0×10^4 J/g ?

ANS :

Water is flowing at a rate of 3.0 litre/min.
The geyser heats the water, raising the temperature from 27°C to 77°C.
Initial temperature, T_1 = 27°C
Final temperature, T_2 = 77°C ∴ Rise in temperature, $\Delta T = T_2 - T_1$
= 77 – 27= 50°C
Heat of combustion = 4 × 104 J/g
Specific heat of water, $c = 4.2$ J g^{-1} °C^{-1}
Mass of flowing water, m = 3.0 litre/min = 3000 g/min
Total heat used, $\Delta Q = mc\,\Delta T$
= 3000 × 4.2 × 50
= 6.3×10^5 J/min
∴ Rate of consumption = $(6.3 \times 10^5) / 4\times 10^4$ = 15.75 g/min

12.2 What amount of heat must be supplied to 2.0×10^{-2} kg of nitrogen (at room temperature) to raise its temperature by 45 °C at constant pressure ? (Molecular mass of N_2 = 28; R = 8.3 J mol^{-1} K^{-1}.)

ANS:

Mass of nitrogen, $m = 2.0 \times 10^{-2}$ kg = 20 g
Rise in temperature, ΔT = 45°C
Molecular mass of N_2, M = 28
Universal gas constant, R = 8.3 J mol^{-1} K^{-1}
Number of moles, n = m/M

$$= \frac{2.0 \times 10^{-2} \times 10^{3}}{28} = 0.714$$

Molar specific heat at constant pressure for nitrogen, C_P = (7/2)R = (7/2) × 8.3 = 29.05 J mol^{-1} K^{-1}
The total amount of heat to be supplied is given by the relation:
$\Delta Q = nC_P\,\Delta T$
= 0.714 × 29.05 × 45
= 933.38 J
Therefore, the amount of heat to be supplied is 933.38 J.

12.3 Explain why
(a) Two bodies at different temperatures T_1 and T_2 if brought in thermal contact do not necessarily settle to the mean temperature $(T_1 + T_2)/2$.
(b) The coolant in a chemical or a nuclear plant (i.e., the liquid used to prevent the different parts of a plant from getting too hot) should have high specific heat.
(c) Air pressure in a car tyre increases during driving.
(d) The climate of a harbour town is more temperate than that of a town in a desert at the same latitude.

ANS:
(a) When two bodies at different temperatures T_1 and T_2 are brought in thermal contact, heat flows from the body at the higher temperature to the body at the lower temperature till equilibrium is achieved, i.e., the temperatures of both the bodies become equal. The equilibrium temperature is equal to the mean temperature $(T_1 + T_2)/2$ only when the thermal capacities of both the bodies are equal.
(b) The coolant in a chemical or nuclear plant should have a high specific heat. This is because higher the specific heat of the coolant, higher is its heat-absorbing capacity and vice versa. Hence, a liquid having a high specific heat is the best coolant to be used in a nuclear or chemical plant. This would prevent different parts of the plant from getting too hot.
(c) When a car is in motion, the air temperature inside the car increases because of the motion of the air molecules. According to Charles' law, temperature is directly proportional to pressure. Hence, if the temperature inside a tyre increases, then the air pressure in it will also increase.
(d) A harbour town has a more temperate climate (i.e., without the extremes of heat or cold) than a town located in a desert at the same latitude. This is because the relative humidity in a harbour town is more than it is in a desert town.

12.4 A cylinder with a movable piston contains 3 moles of hydrogen at standard temperature and pressure. The walls of the cylinder are made of a heat insulator, and the piston is insulated by having a pile of sand on it. By what factor does the pressure of the gas increase if the gas is compressed to half its original volume ?
ANS:
The cylinder is completely insulated from its surroundings. As a result, no heat is exchanged between the system (cylinder) and its surroundings. Thus, the process is adiabatic.
Initial pressure inside the cylinder = P_1
Final pressure inside the cylinder = P_2
Initial volume inside the cylinder = V_1
Final volume inside the cylinder = V_2
Ratio of specific heats, $\gamma = 1.4$
For an adiabatic process, we have:

$$P_1V_1^{\gamma} = P_2V_2^{\gamma}$$

The final volume is compressed to half of its initial volume.
$\therefore$ $V_2 = V_1/2$

$$P_1(V_1)^{\gamma} = P_2\left(\frac{V_1}{2}\right)^{\gamma}$$

$$\frac{P_2}{P_1} = \frac{(V_1)^{\gamma}}{\left(\frac{V_1}{2}\right)^{\gamma}} = (2)^{\gamma} = (2)^{1.4} = 2.639$$

Hence, the pressure increases by a factor of 2.639.

12.5 In changing the state of a gas adiabatically from an equilibrium state *A* to another equilibrium state *B*, an amount of work equal to 22.3 J is done on the system. If the gas is taken from state *A* to *B* via a process in which the net heat absorbed by the system is 9.35 cal, how much is the net work done by the system in the latter case ?

(Take 1 cal = 4.19 J)
ANS:
The work done (*W*) on the system while the gas changes from state *A* to state *B* is 22.3 J.
This is an adiabatic process. Hence, change in heat is zero. ∴ $\Delta Q = 0$
$\Delta W = -22.3$ J (Since the work is done on the system)
From the first law of thermodynamics, we have:
$\Delta Q = \Delta U + \Delta W$
Where,
ΔU = Change in the internal energy of the gas ∴ $\Delta U = \Delta Q - \Delta W = -(-22.3 \text{ J})$
$\Delta U = +22.3$ J
When the gas goes from state *A* to state *B* via a process, the net heat absorbed by the system is:
$\Delta Q = 9.35 \text{ cal} = 9.35 \times 4.19 = 39.1765$ J
Heat absorbed, $\Delta Q = \Delta U + \Delta Q$ ∴$\Delta W = \Delta Q - \Delta U$
$= 39.1765 - 22.3$
$= 16.8765$ J
Therefore, 16.88 J of work is done by the system.

12.6 Two cylinders *A* and *B* of equal capacity are connected to each other via a stopcock. *A* contains a gas at standard temperature and pressure. *B* is completely evacuated. The entire system is thermally insulated. The stopcock is suddenly opened. Answer the following :
(a) What is the final pressure of the gas in *A* and *B* ?
(b) What is the change in internal energy of the gas ?
(c) What is the change in the temperature of the gas ?
(d) Do the intermediate states of the system (before settling to the final equilibrium state) lie on its *P-V-T* surface ?
ANS:
(a) 0.5 atm
(b) Zero
(c) Zero
(d) No
Explanation:
(a) The volume available to the gas is doubled as soon as the stopcock between cylinders *A* and *B* is opened. Since volume is inversely proportional to pressure, the pressure will decrease to one-half of the original value. Since the initial pressure of the gas is 1 atm, the pressure in each cylinder will be 0.5 atm.
(b) The internal energy of the gas can change only when work is done by or on the gas. Since in this case no work is done by or on the gas, the internal energy of the gas will not change.
(c) Since no work is being done by the gas during the expansion of the gas, the temperature of the gas will not change at all.
(d) The given process is a case of free expansion. It is rapid and cannot be controlled. The intermediate states do not satisfy the gas equation and since they are in non-equilibrium states, they do not lie on the *P-V-T* surface of the system.

12.7 A steam engine delivers 5.4×10^8J of work per minute and services 3.6×10^9J of heat per minute from its boiler. What is the efficiency of the engine? How much heat is wasted per minute?

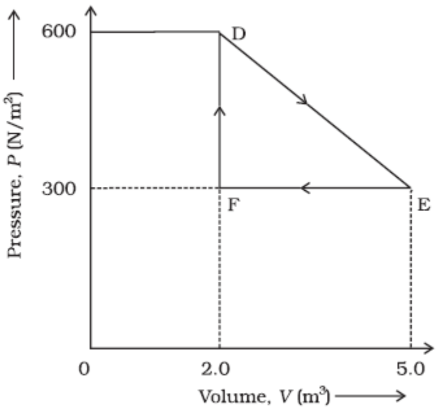
ANS:
Work done by the steam engine per minute, $W = 5.4 \times 10^8$ J
Heat supplied from the boiler, $H = 3.6 \times 10^9$ J
Efficiency of the engine = Output Energy/Input Energy

$$\therefore \eta = \frac{W}{H} = \frac{5.4 \times 10^8}{3.6 \times 10^9} = 0.15$$

Hence, the percentage efficiency of the engine is 15 %.
Amount of heat wasted = $3.6 \times 10^9 - 5.4 \times 10^8$
$= 30.6 \times 10^8 = 3.06 \times 10^9$ J
Therefore, the amount of heat wasted per minute is 3.06×10^9 J.

12.8 An electric heater supplies heat to a system at a rate of 100W. If system performs work at a rate of 75 joules per second. At what rate is the internal energy increasing?
ANS:
Heat is supplied to the system at a rate of 100 W. ∴Heat supplied, $Q = 100$ J/s
The system performs at a rate of 75 J/s. ∴Work done, $W = 75$ J/s
From the first law of thermodynamics, we have:
$Q = U + W$
Where,
U = Internal energy ∴ $U = Q - W$
$= 100 - 75$
$= 25$ J/s
$= 25$ W
Therefore, the internal energy of the given electric heater increases at a rate of 25 W.

12.9 A thermodynamic system is taken from an original state to an intermediate state by the linear process shown in Fig. (12.13)



Its volume is then reduced to the original value from E to F by an isobaric process. Calculate the total work done by the gas from D to E to F
ANS:
Total work done by the gas from D to E to F = Area of ΔDEF
Area of ΔDEF = $\frac{1}{2}\ DE\ \times EF$
Where,

DF = Change in pressure
= 600 N/m2 – 300 N/m2
= 300 N/m2
FE = Change in volume
= 5.0 m3 – 2.0 m3
= 3.0 m3
Area of ΔDEF = $= \frac{1}{2}\ DE\ \times EF = = \frac{1}{2}\ \times 300\ \times 3 = 450$ J
Therefore, the total work done by the gas from D to E to F is 450 J.

12.10 A refrigerator is to maintain eatables kept inside at 90C. If room temperature is 360C, calculate the coefficient of performance.
ANS:
Temperature inside the refrigerator, $T1$ = 9°C = 282 K
Room temperature, $T2$ = 36°C = 309 K
Coefficient of performance = $T_1/(T_2 - T_1)$

$$= \frac{282}{309 - 282} = 10.44$$

Therefore, the coefficient of performance of the given refrigerator is 10.44.

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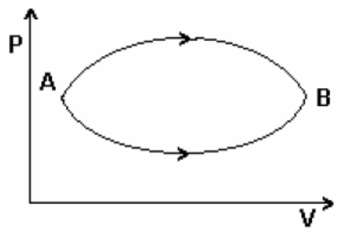
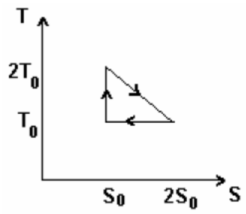
OBJECTIVE QUESTIONS

1) Which of the following is incorrect regarding the first law of thermodynamics ?
(a) It is the restatement of the principle of conservation of energy.
(b) It is not applicable to any cyclic process.
(c) It introduces the concept of entropy.
(d) It introduces the concept of the internal energy. [AIEEE 2005]

2) A system goes from A to B via two processes I and II as shown in figure. If ΔU_1 and ΔU_2 are the changes in internal energies in the processes I and II respectively, then
(a) relation between ΔU_1 and ΔU_2 cannot be determined
(b) $\Delta U_1 = \Delta U_2$ (c) $\Delta U_2 < \Delta U_1$ (d) $\Delta U_2 > \Delta U_1$
[AIEEE 2005]



3) The temperature – entropy diagram of a reversible engine cycle is given in the figure. Its efficiency is
(a) 1 / 4 (b) 1 / 2
(c) 2 / 3 (d) 1 / 3
[AIEEE 2005]



4) A gaseous mixture consists of 16 g of Helium and 16 g of oxygen. The ratio Cp / Cv of the mixture is
(a) 1.62 (b) 1.59 (c) 1.54 (d) 1.4 [AIEEE 2005]

5) Which of the following statements is correct for any thermodynamic system ?
(a) The internal energy changes in all processes.
(b) Internal energy and entropy are state functions.
(c) The change in entropy can never be zero.
(d) The work done in an adiabatic process is always zero. [AIEEE 2004]

6) One mole of ideal monatomic gas ($\gamma = 5/3$) is mixed with one mole of diatomic gas ($\gamma = 7/5$). What is γ for the mixture ?
(a) 3 / 2 (b) 23 / 15 (c) 35 / 23 (d) 4 / 3 [AIEEE 2004]

7) Two thermally insulated vessels 1 and 2 are filled with air at temperature (T_1, T_2), volume (V_1, V_2) and pressure (P_1, P_2) respectively. If the valve joining the two vessels is opened, the temperature inside he vessel at equilibrium will be

(a) $T_1 + T_2$ (b) $\frac{T_1 + T_2}{2}$ (c) $\frac{T_1 T_2 (P_1 V_1 + P_2 V_2)}{P_1 V_1 T_2 + P_2 V_2 T_1}$ (d) $\frac{T_1 T_2 (P_1 V_1 + P_2 V_2)}{P_1 V_1 T_1 + P_2 V_2 T2}$

[AIEEE 2004]

8) "Heat cannot by itself flow from a body at lower temperature to a body at higher temperature" is a statement as a consequence of
(a) conservation of mass (b) conservation of momentum
(c) first law of thermodynamics (d) second law of thermodynamics [AIEEE 2003]

9) During an adiabatic process, the pressure of a gas is found to be proportional to the cube of its absolute temperature. The ratio Cp / Cv for the gas is
(a) 2 (b) 3 / 2 (c) 4 / 3 (d) 5 / 3 [AIEEE 2003]

10) A Carnot engine takes 3 × 106 cal. of heat from a reservoir at 627° C and gives it to a sink at 27° C. The work done by the engine is
(a) zero (b) 4.2 × 106 J (c) 8.4 × 106 J (d) 16.8 × 106 J
[AIEEE 2003]

11) Which of the following parameters does not characterize the thermodynamic state of matter ?
(a) work (b) volume (c) pressure (d) temperature [AIEEE 2003]

12) For an isothermal expansion of a perfect gas, the value of ΔP / P is equal to
(a) - ΔV / V (b) γ ΔV / V (c) - γ ΔV / V (d) - γ 2ΔV / V [AIEEE 2002]
13) The translational kinetic energy of gas molecules at temperature T for one mole of a gas is
(a) (3 / 2) RT (b) (9 / 2) RT (c) (1 / 3) RT (d) (5 / 2) RT [AIEEE 2002]

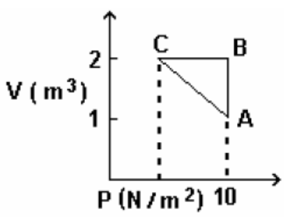
14) A gas at 300 K, enclosed in a container, is placed in a fast moving train. When the train is in motion, the temperature of the gas
(a) rises above 300 K (b) falls below 300 K
(c) remains unchanged (d) becomes unsteady [AIEEE 2002]

15) An ideal gas expands isothermally from volume V1 to V2 and then compressed to original volume V1 adiabatically. Initial pressure is P1 and final pressure is P3. The total work done is W. Then
(a) P3 > P1, W > 0 (b) P3 < P1, W < 0
(c) P3 > P1, W < 0 (d) P3 = P1, W = 0 [IIT 2004]

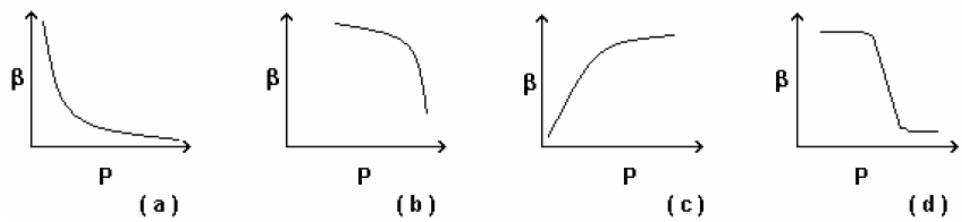
16) Two rods, one of aluminium and the other made of steel, having initial length *l*1 and *l*2 are connected together to form a single rod of length *l*1 + *l*2. The coefficients of linear expansion for aluminium and steel are α a and α s respectively. If the lengthy of each rod increases by the same amount when their temperatures are raised by t° C, then the ratio $l_1 / (l_1 + l_2)$ is equal to

(a) $\frac{\alpha_s}{\alpha_a}$ (b) $\frac{\alpha_a}{\alpha_s}$ (c) $\frac{\alpha_s}{\alpha_a + \alpha_s}$ (d) $\frac{\alpha_a}{\alpha_a + \alpha_s}$ **[IIT 2003]**

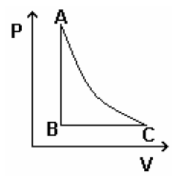
17) An ideal gas is taken through the cycle A → B → C → A, as shown in the figure. If the net heat supplied to the gas in the cycle is 5 J, the work done by the gas in the process C → A is
(a) - 5 J (b) - 10 J (c) - 15 J (d) - 20 J
[IIT 2002]



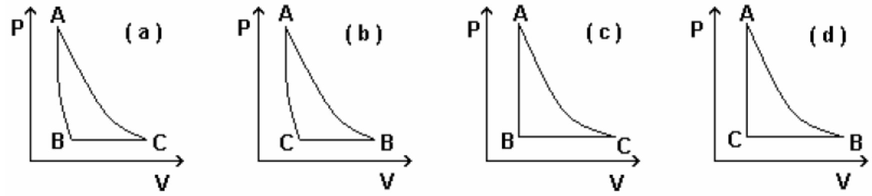
18) Which of the following graphs correctly represents the variation of β = - (dV/dP) / V with P for an ideal gas at constant temperature ?



[IIT 2002]

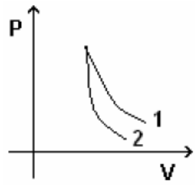


19) The PT diagram for an ideal gas is shown in the figure, where AC is an adiabatic process. Which of the following is the corresponding PV diagram ? [IIT 2003]



20) In a given process of an ideal gas, $dW = 0$ and $dQ < 0$. Then for the gas
(a) the temperature will decrease (b) the volume will increase
(c) the pressure will remain constant (d) the temperature will increase [IIT 2001]

21) P - V plots for two gases during adiabatic processes are shown in the figure. Plots 1 and 2 should correspond respectively to:



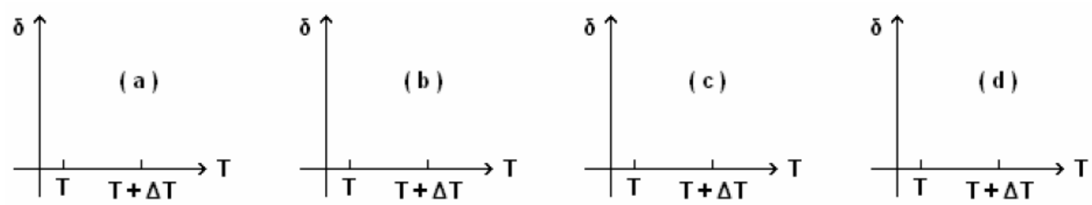
(a) He and O_2 (b) O_2 and He
(c) He and Ar (d) O_2 and N_2
[IIT 2001]

22) A monatomic ideal gas, initially at temperature T_1, is enclosed in a cylinder fitted with a frictionless piston, The gas is allowed to expand adiabatically to a temperature T_2 by releasing the piston suddenly. If L_1 and L_2 are the lengths of the gas column before and after expansion respectively, then T_1 / T_2 is given by

(a) $\left(\frac{L_1}{L_2}\right)^{\frac{2}{3}}$ (b) $\frac{L_1}{L_2}$ (c) $\frac{L_2}{L_1}$ (d) $\left(\frac{L_2}{L_1}\right)^{\frac{2}{3}}$ [IIT 2000]

23) Starting with the same initial conditions, an ideal gas expands from volume V_1 to V_2 in three different ways. The work done by the gas is W_1 if the process is purely isothermal, W_2 if purely isobaric and W_3 if purely adiabatic. Then
(a) $W_2 > W_1 > W_3$ (b) $W_2 > W_3 > W_1$
(c) $W_1 > W_2 > W_3$ (d) $W_1 > W_3 > W_2$
[IIT 2000]

24) An ideal gas is initially at temperature T and volume V. Its volume is increased by ΔV due to an increase in temperature ΔT, pressure remaining constant. The quantity $\delta = \Delta V / V\Delta T$ varies with temperature as [IIT 2000]



25) A gas mixture consists of 2 moles of oxygen and 4 moles of argon at temperature T. Neglecting all vibrational modes, the total internal energy of the system is
(a) 4 RT (b) 15 RT (c) 9 RT (d) 11 RT [IIT 1999]

26) Let v , v_{rms} and v_p respectively denote the mean speed, root mean square speed and most probable speed of the molecules in an ideal monatomic gas at absolute temperature T. The mass of a molecule is m. Then
(a) no molecule can have energy greater than $\sqrt{2}\ v_{rms}$
(b) no molecule can have speed less than $v_p / \sqrt{2}$
(c) $v_p < v < v_{rms}$
(d) the average kinetic energy of a molecule is (3/4) m V_P^2 **[IIT 1998]**

27) A vessel contains a mixture of one mole of oxygen and two moles of nitrogen at 300 K. The ratio of the average rotational kinetic energy per O2 to per N2 molecule is
(a) 1 : 1 (b) 1 : 2 (c) 2 : 1
(d) depends on the moment of inertia of the two molecules [IIT 1998]

28) Two cylinders A and B fitted with pistons contain equal amounts of an ideal diatomic gas at 300 K. The piston of A is free to move, while that of B is held fixed. The same amount of heat is given to the gas in each cylinder. If the rise in temperature of the gas in A is 30 K, then the rise in temperature of the gas in B is
(a) 30 K (b) 18 K (c) 50 K (d) 42 K [IIT 1998]

29) Two identical containers A and B with frictionless pistons contain the same ideal gas at the same temperature and the same volume V. The mass of the gas in A is mA and that in B is mB. The gas in each cylinder s now allowed to expand isothermally to the same final volume 2V. The changes in the pressure in A and B are found to be ΔP and 1.5 ΔP respectively. Then
(a) 4mA = 9mB (b) 2mA = 3mB (c) 3mA = 2mB (d) 9mA = 4mB [IIT 1998]

30) During the melting of a slab of ice at 273 K at atmospheric pressure
(a) positive work is done by he ice-water system on the atmosphere
(b) positive work is done by he ice-water system by the atmosphere
(c) the internal energy of the ice-water system increases
(d) the internal energy of the ice-water system decreases [IIT 1998]

31) A given quantity of an ideal gas is at pressure P and absolute temperature T. The isothermal bulk modulus of the gas is
(a) (3/2)P (b) P (c) (3/2) P (d) 2P [IIT 1998]

32) The average translational kinetic energy of O2 (molar mass 32) molecules at a particular temperature is 0.048 eV. The translational kinetic energy of N_2 (molar mass 28) in eV at the same temperature is
(a) 0.0015 (b) 0.003 (c) 0.048 (d) 0.768 [IIT 1997]

33) A vessel contains 1 mole of O_2 gas (molar mass 32) at a temperature T. The pressure of the gas is P. An identical vessel containing one mole of the gas (molar mass 4) at a temperature 2T has a pressure of
(a) P / 8 (b) P (c) 2P (d) 8P [IIT 1997]

34) The temperature of an ideal gas is increased from 120 K to 480 K. If at 120 K the root mean square velocity of the gas molecules is V, then at 480 K it becomes
(a) 4V (b) 2V (c) V / 2 (d) V / 4 [IIT 1996]

35) An ideal gas is taken from the state A (pressure P, volume V) to state B (pressure P / 2, volume 2V) along a straight line path in the P-V diagram. Select the correct statement (s) from the following :
(a) The work done by the gas in the process A to B exceeds that work that would be done by it f the system were taken from A to B along an isotherm.
(b) In the T-V diagram, the path AB becomes a part of a parabola.
(c) In the P-T diagram, the path AB becomes a part of a hyperbola.
(d) In going from A to B, the temperature T of the gas first increases to a maximum value and then decreases. [IIT 1993]

36) Three closed vessels A, B and C are at the same temperature T and contain gases which obey the Maxwellian distribution law of velocities. Vessel A contains only O_2, B only N_2 and C a mixture of equal quantities of O_2 and N_2. If the average speed of the O_2 molecules in vessel A is V_1, that of the N_2 molecules in vessel B is V_2, the average speed of the O_2 molecules in vessel C is
(a) $(V_1 + V_2) / 2$ (b) V_1 (c) $(V_1 V_2)^{1/2}$ (d) $\sqrt{(3kT / M)}$ [IIT 1992]

37) When an ideal diatomic gas is heated at constant pressure, the fraction of the heat energy supplied which increases the internal energy of the gas is
(a) 2 / 5 (b) 3 / 5 (c) 3 / 7 (d) 5 / 7 [IIT 1990]

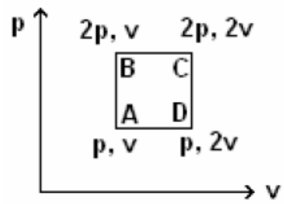
38) For an ideal gas:
(a) the change in internal energy in a constant reassure process from temperature T_1 to T_2 is equal to $nC_v(T_2 - T_1)$, where C_v is the molar specific heat at constant volume and n the number of moles of the gas
(b) the change in internal energy of the gas and the work done by the gas are equal in magnitude in an adiabatic process
(c) the internal energy does not change in an isothermal process
(d) no heat is added or removed in an adiabatic process [IIT 1989]

39) If one mole of a monatomic gas ($\gamma = 5 / 3$) is mixed with one mole of a diatomic gas ($\gamma = 7 / 5$), the value of γ for the mixture is
(a) 1.40 (b) 1.50 (c) 1.53 (d) 3.07 [IIT 1988]

40) 70 calories of heat are required to raise the temperature of 2 moles of an ideal gas at constant pressure from 30° C to 35° C. he amount of heat required (in calories) to raise the temperature of the same gas through the same range (30° C to 35° C) at constant volume is
(a) 30 (b) 50 (c) 70 (d) 90 [IIT 1985]

41) At room temperature, the r.m.s. speed of the molecules of a certain diatomic gas is found to be 1930 m / s. The gas is
(a) H_2 (b) F_2 (c) O_2 (d) Cl_2 [IIT 1984]

42) An ideal monatomic gas is taken round the cycle ABCDA as shown in the p-v diagram. The work done during the cycle is
(a) pv (b) 2pv (c) (1/2) pv (d) zero [IIT 1983]



Answers

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
b,c	b	d	a	b	a	c	d	b	c	a	a	a	c	c	c	a	a	none	a	b

22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
d	a	c	d	c,d	a	d	c	b,c	b	c	c	b	a,b	d	d	a,b,c,d	c	b

41	42
a	a

CH 13

Kinetic Theory

(5 Hours, 4 Marks = 1Q-1M, 1Q-3M)

Syllabus :

Equation of state of a perfect gas, work done in compressing a gas. Kinetic theory of gases-assumptions, concept of pressure. Kinetic energy and temperature: rms speed of gas molecules; degrees of freedom, law of equipartition of energy (statement only) and application to specific heats of gases; concept of mean free path, Avogadro's number.

1. Equation of state of a perfect gas, work done in compressing a gas :

We know that molecules (made up of one or more atoms) constitute matter. Electron microscopes and scanning tunnelling microscopes enable us to even see them. The size of an atom is about an angstrom (10^{-10} m).

In solids, which are tightly packed, atoms are spaced about a few angstroms (2 Å) apart. In liquids the separation between atoms is also about the same. In liquids the atoms are not as rigidly fixed as in solids, and can move around. This enables a liquid to flow. In gases the interatomic distances are in tens of angstroms. The average distance a molecule can travel without colliding is called the **mean free path**. The mean free path, in gases, is of the order of thousands of angstroms. The atoms are much freer in gases and can travel long distances without colliding. If they are not enclosed, gases disperse away. In solids and liquids the closeness makes the interatomic force important. The force has a long range attraction and a short range repulsion. The atoms attract when they are at a few angstroms but repel when they come closer. The static appearance of a gas is misleading. The gas is full of activity and the equilibrium is a dynamic one. In dynamic equilibrium, molecules collide and change their speeds during the collision. Only the average properties are constant.

The ideal gas equation connecting pressure (P), volume (V) and absolute temperature (T) is

$PV = \mu\, RT = k_B\, NT$

where μ is the number of moles and N is the number of molecules. R and k_B are universal constants.

$R = 8.314$ J mol^{-1} K^{-1}, $\quad k_B = R/N_A$

$N = 1.38 \times 10^{-23}$ J K^{-1}

Real gases satisfy the ideal gas equation only approximately, more so at low pressures and high temperatures.

2. Kinetic theory of gases-assumptions, concept of pressure. :

3. Kinetic energy and temperature: rms speed of gas molecules; degrees of freedom,

4. Law of equipartition of energy (statement only) and application to specific heats of gases; concept of mean free path, Avogadro's number.

ONE MARK OUESTION:

1. Name any one scientist who explained the behavior of gases considering it to be made up of tiny particles.

Newton, Boyle.

2. Based on which idea kinetic theory of gases explain the behavior of gases?

Gases consist of rapidly moving atoms or molecules

3. Mention any one measurable property of gas.

Viscosity, conduction, diffusion, pressure.

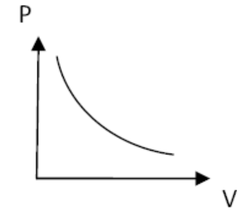
4. Name any one scientist who suggested that matter consists of indivisible constituents.

Kanada, Damocritus.

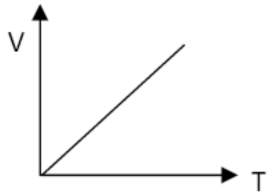
5. What is an ideal gas or a perfect gas?

Is one which obeys Boyle's law and Charl's law at all values of temperature and pressure.

6. Draw P-V curve or diagram for Boyle's law.



7. Draw P-V curve for Charle's law.



8. What is the value of universal gas constant 'R'?

8.314 J mol^{-1} K^{-1}

9. On what factor does the internal energy of a gas depend on?

Temperature.

10. On what factors does internal energy of the gas doesn't depend?

Pressure, volume.

11. Define degree's of freedom.

Number of co-ordinates required to specify the position of a molecule.

12. How many degrees of freedom a mono atomic gas molecule possess?
Three translational degrees of freedom.

13. How many degrees of freedom a di-atomic gas molecule possess?
Three translational degrees of freedom and two rotational degrees of freedom.

14. State the law of equipartition of energy.
It states that for any system in thermal equilibrium, the total energy is equally distributed among its various degrees of freedom and the energy associated with each molecule per degree of freedom is (1/2) $K_B T$.

15. What is the value of Boltzmann's constant?
1.38×10^{-23} JK^{-1}

16. What is the value of ratio of specific heats for a mono atomic gas molecule?
5/3

17. What is the value of ratio of specific heats of di-atomic gas molecule?
7/5

18. Write the general formula for ratio of specific heats for poly atomic gases.
$\gamma = \frac{(4+f)}{(3+f)}$

19. Write the equation connecting Cp,Cv & R.
$(C_P - C_V) = R$

20. Define mean free path.
The average distance travelled by a gas molecule between two successive collisions .

21. Write the equation for pressure of an ideal gas.
$P = (1/3)\, nm\bar{v}^{\,2}$

22. Write the order of mean free path in gases .
1000 Å

TWO MARK QUESTION:

23. Show that specific heat of solids C=3R.
For a mole of solid $U = 3K_B T N_A = 3RT$
At constant pressure $\Delta Q = \Delta U$
$\therefore \quad C = \frac{\Delta Q}{\Delta T} = \frac{\Delta U}{\Delta T} = 3R$

24. What are the important characteristics of an ideal gas?
The molecules of a gas are point masses, inter molecular force among molecules is zero.

25. State and explain Boyle's law.
At constant temperature T, the volume V of a given mass of gas is inversely proportional to its

pressure P i.e $V \propto 1/P$ or PV = K.

26. State and explain Charle's law.
At constant pressure P, the volume V of a given mass of gas is directly proportional to its absolute temperature T i.e V/T = constant or V = KT.

27. Write the perfect gas equation and explain the terms.
PV = μRT where P is the pressure, V volume, μ number of moles, R gas constant T temperature.

28. State and explain Dalton's law of partial pressures.
The total pressure of a mixture of ideal gases is the sum of the partial pressures.
Consider a mixture of two ideal gases μ_1 number of moles of gas 1, μ_2 number of moles of gas 2
At a temperature T, pressure P

$$PV=(\mu_1+\mu_2)RT$$

$$P=\mu_1\frac{RT}{V}+\mu_2\frac{RT}{V}$$

$P = P_1 + P_2$ where P_1 is pressure of gas 1 and P_2 is pressure of gas 2

29. Write the equation for mean free path and explain the terms.
$I = \frac{1}{\sqrt{2}\pi n d^2}$ where n is number of moles, d diameter of the molecule.

30. Explain the term free mean free path.
During the motion of molecules of a gas, the molecule suffers number of collisions with other gas molecules. During two successive collisions, a molecule of a gas moves in a straight line with constant velocity and the average distance travelled by a molecule during all collisions is known as mean free path.

FOUR MARK QUESTION:

31. Mention the salient features of kinetic theory of gases developed by Maxwell and Boltzmann.
(1) A gas consist of large number of tiny particles called molecules which are perfectly rigid and elastic.
(2) At ordinary temperature and pressure, the size of the molecules is negligible compared with the average distance between the molecules.
(3) The molecules have velocities ranging from 0 to 7 ,so that average velocity of random motion is zero.
(4) The collision between the two molecules is perfectly elastic that is the kinetic energy remains conserved in the collision .
(5) There is no inter molecular force of attraction between the molecules of the gas .
(6) Between two successive collisions a molecule traverses straight line path with constant speed called mean free path of the molecule.

32. Derive the equation $P = \frac{\rho RT}{M_0}$

Perfect gas equation is PV= μRT where R = $N_A K_B$ were N_A is number of moles in the sample, K_B is Boltzmann constant.

Choosing Kelvin's we have $\mu = M/M_0 = N/N_A$ where M is mass of the gas containing N molecules, M_0 is molar mass

$$PV=\mu RT=\frac{M}{M_0}RT \text{ or } P=\frac{M}{V}\cdot\frac{RT}{M_0}$$

$$\text{Or } p=\frac{\rho}{M_0}RT \quad \text{where } \frac{M}{V}=\rho$$

33. Deduce the expression PV = RT.

From Boyle's law $V \propto 1/P$ or $V = K/P$ ……………(1) at constant temperature
From Charle's law $V \propto T$ at constant pressure
OR V = KT ……………..(2)
From equation (1) and (2) $V \propto T/P$ or PV/T = constant
For one mole of gas, the constant has same value for all gases and is called universal gas constant R
∴ PV/T = R or PV = RT
For μ number of moles PV = RT.

34. Determine the specific heat capacity of a mono atomic gas molecule.

A mono atomic gas molecule has three translational degrees of freedom. The average energy of a molecule at temperature T is (3/2) K_BT
Total internal energy U = $(3/2)K_B T N_A$ = (3/2) RT

The molar specific heat at constant volume $C_V = \frac{dU}{dT} = \frac{3}{2}R$

We have $C_P = C_V + R = \frac{3}{2}R + R = \frac{5}{2}R$

∴ ratio of specific heats $\gamma = \frac{C_P}{C_V} = \frac{5/2}{3/2} = \frac{5}{3}$

35. Determine the specific heat capacity of a di-atomic gas molecule treated as a rigid rotator.

Di-atomic gas molecule has three translational degrees of freedom and two rotational degrees of freedom, totally five. Average energy U = $(5/2)K_B T N_A$ = (5/2) RT

The molar specific heat at constant volume $C_V = \frac{dU}{dT} = \frac{5}{2}R$

We have $C_P = C_V + R = \frac{5}{2}R + R = \frac{7}{2}R$

∴ ratio of specific heats $\gamma = \frac{C_P}{C_V} = \frac{7/2}{5/2} = \frac{7}{5}$

Note : If the di-atomic molecule is not a rigid rotator, it has a additional vibration mode.

$$U=\left[\frac{5}{2}k_BT + k_BT\right]N_A = \frac{7}{2}RT$$

$$C_v=\frac{dU}{dT}=\frac{7}{2}R$$

$$C_p = C_v + R$$

$$\therefore \gamma = \frac{C_p}{C_v} = \frac{9/2\ R}{7/2\ R} = \frac{9}{7}$$

36. Deduce the equation $\gamma = \frac{(4+f)}{(3+f)}$

For poly atomic gas molecule, there are three translational three rotational and 'f' vibrational degrees of freedom.

The average internal energy U = $[\frac{3}{2}k_B\text{T} + \frac{3}{2}k_B T + \text{f X } k_B\text{T}]\ N_A$

$=(3+\text{f})k_B T N_A = (3+\text{f})\text{RT}$

$C_v = \frac{dU}{dT} = (3+\text{f})\text{R}$

$C_p = C_v + \text{R}$

$=(3+\text{f})\text{R}+\text{R} = (4+\text{f})\text{R}$

$\gamma = \frac{C_p}{C_v} = \frac{(4+f)}{(3+f)}$

37. Apply law of equipartition of energy to a monoatomic gas molecule.

A monoatomic gas molecule is free to move in space and has 3 translational degrees of freedom . Each translational degree of freedom contributes for the total energy and = $\frac{1}{2}$ K_BT.

The kinetic energy of a monoatomic molecule is

$E_T = \frac{1}{2}\text{m}V_x{}^2 + \frac{1}{2}\text{m}v_y{}^2 + \frac{1}{2}\text{m}V_z{}^2$

For a gas at temperature T, average value of energy is denoted by $<E_T>$

$<E_T> = \frac{1}{2}\text{m}V_x{}^2 + \frac{1}{2}\text{m}V_y{}^2 + \frac{1}{2}\text{m}V_z{}^2 = \frac{3}{2}k_B\text{T}$

Since there is no preferred direction

$<\frac{1}{2}\text{m}V_x{}^2> = <\frac{1}{2}\text{m}V_y{}^2> = <\frac{1}{2}\text{m}V_z{}^2> = \frac{1}{2}k_B\text{T}$.

38. Apply the law of equipartition of energy to a di-atomic gas molecule treating it as a rigid rotator.

Diatomic molecule has three translational degrees of in addition to that ,it can also rotate about its center of mass i.e, two independent rotational degrees of freedom . Since the diatomic molecule is treated as a rigid rotator , the molecule does not vibrate. Each degree of freedom contributes to the total energy consisting of translational energy Et and rotational energy Er .

$E_t + E_r = \frac{1}{2}\text{m}V_x^2 + \frac{1}{2}\text{m}V_y^2 + \frac{1}{2}\text{m}V_z^2 + \frac{1}{2}I_1\omega_1{}^2 + \frac{1}{2}I_2\omega_2{}^2$

where ω_1 and ω_2 are angular speeds about the axes 1 and 2, I_1 and I_2 are corresponding moment of inertia.

FIVE MARK QUESTION:

39.Mention any five assumptions of kinetic theory of gases.

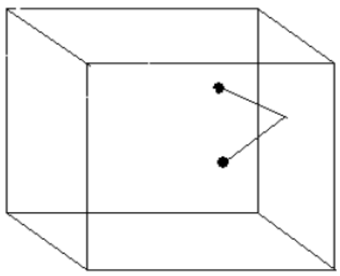
The following are the assumptions :

a) A gas consists of large number of tiny particles called molecules which are perfectly rigid and

elastic.
b) The molecules are in a state of continuous motion moving in all directions with all possible velocities.
c) At ordinary temperature and pressure, the size of the molecule is negligible compared with the average distance between the molecules.
d) The molecules have velocities ranging from 0 to ∞ ,so that average velocity of random motion is zero.
e) The collision between two molecules is perfectly elastic that is kinetic energy remains conserved in he collision.
f) Between two successive collisions, molecules traverse a straight line path with constant speed called free path of the molecule.
g) There is no inter molecular force of attraction between the molecules of the gas.
h) The collisions are almost instantaneous and the molecules obey Newton's laws of motion.

40. Derive an expression for pressure of an ideal gas.

Consider a gas enclosed in a cube of side l . Let the axes be parallel to the sides of the cube. A molecule with velocity (V_x, V_y, V_z) hits the planar wall parallel to y z plane of area l^2. Since the collision is elastic, the molecule rebounds with same velocity, its y and z components do not change in the collision, but the x component reverses its sign. Velocity after collision is given by ($-V_x$, V_y, V_z).
Change in momentum of the molecule is given by $-mV_x -(+mV_x) =-2mV_x$.
By the principle of conservation of momentum, the momentum imparted to the wall = 2m V_x. In a small time, interval Δt a molecule with a velocity V_x will hit the wall if it is in a distance V_x Δt from the wall.
That is all molecules within the volume AV_x Δt only can hit the wall in time Δt. On an average half the molecules move towards the wall and the other half away from the wall.



No of molecules with velocity (IJQIK,IL) hitting the wall in a time Δt is

41. From kinetic theory of gases explain the kinetic interpretation of temperature.

42. Define mean free path and hence derive the expression

CH 14

Oscillations

(8 Hours, 7 Marks (2M-1Q, 5M(T)-1Q)

Syllabus :

Periodic motion - period, frequency, displacement as a function of time. Periodic functions. Simple harmonic motion (S.H.M) and its equation; phase; oscillations of a spring–restoring force and force constant; energy in S.H.M. Kinetic and potential energies; simple pendulum–derivation of expression for its time period; free and forced and damped oscillations (qualitative ideas only), resonance.

14.1. Periodic motion - period, frequency, displacement as a function of time.

1. What is periodic & Oscillatory motion ?

Any motion that repeats itself at regular intervals of time is called **periodic motion.**

If the body is given a small displacement from the position, a force comes into play which tries to bring the body back to the equilibrium point, giving rise to **oscillations** or **vibrations**. During the oscillations the body moves to and fro from its mean position.

Every oscillatory motion is periodic, but every periodic motion need not be oscillatory. Circular motion is a periodic motion, but it is not oscillatory.

There is no significant difference between oscillations and vibrations. It seems that when the frequency is small, we call it oscillation (like the oscillation of a branch of a tree), while when the frequency is high, we call it vibration (like the vibration of a string of a musical instrument).
Simple harmonic motion is the simplest form of oscillatory motion. This motion arises when the force on the oscillating body is directly proportional to its displacement from the mean position, which is also the equilibrium position. Further, at any point in its oscillation, this force is directed towards the mean position.

2. Explain the properties of a periodic wave in terms of (a) period, (b) frequency, (c) displacement as a function of time?

(a) Any motion that repeats itself at regular intervals of time is called **periodic motion**. **The smallest interval of time after which the motion is repeated is called its period** denoted by T and its SI unit is second.
(b) The reciprocal of T gives the number of repetitions that occur per unit time. This quantity is called the **frequency of the periodic motion**. It is represented by the symbol υ . The relation between v and T is $v = 1/T$. The unit of υ is thus s^{-1} or Hz.
(c) We know that displacement of a particle as the change in its position vector. It refers to change with time of any physical property under consideration. For example, in case of rectilinear motion of a steel ball on a surface, the distance from the starting point as a function of time is its position displacement.
For oscillatory motion, the angular displacement θ is the angle from mean position.

The displacement can be represented by a mathematical function of time. A periodic function can be represented by

f(t) = A cos ωt. ------- **(1)**
If the function f(t) is periodic, then its period is **T = 2π/ω** -------- **(2)**

3. Show that any periodic function can be expressed as a superposition of sine and cosine functions of different time periods with suitable coefficients.

A periodic function can be represented by **f(t) = A cos ωt.** ------- **(1)**
If the function f(t) is periodic, then its period is **T = 2π/ω** -------- **(2)**
Thus, the function $f(t)$ is periodic with period T, $f(t) = f(t+T)$
The same result is obviously correct if we consider a sine function, $f(t) = A \sin \omega t$. Further, a linear combination of sine and cosine functions like, $f(t) = A \sin \omega t + B \cos \omega t$ --------- (3)
is also a periodic function with the same period T. Taking, $A = D \cos \phi$ *and* $B = D \sin \phi$ Eq. (3) can be written as,
$f(t) = D \sin(\omega t + \phi)$, ---------- (4)

Here D and ϕ are constant given by
$D = \sqrt{A^2 + B^2}$ and $\tan^{-1} = \frac{B}{A}$
According to B.J. Fourier, **Any periodic function can be expressed as a superposition of sine and cosine functions of different time periods with suitable coefficients**.

14.2. Periodic functions. Simple harmonic motion (S.H.M) and its equation; phase; oscillations of a spring–restoring force and force constant;

4. What is simple harmonic motion ? Obtain an expression for it ?

In simple harmonic motion, the position of the particle with reference to the origin gives its displacement at any instant of time. For such a motion the displacement $x(t)$ of the particle from a certain chosen origin is found to vary with time as,
$x(t) = A \cos(\omega t + \phi)$ -------- (5)
in which A, ω, and ϕ are constants.

$$\underset{\text{Displacement}}{\underset{\uparrow}{x(t)}} = \underset{\text{Amplitude}}{\underset{\uparrow}{A}} \cos(\overbrace{\underset{\text{Angular frequency}}{\underset{\uparrow}{\omega t}} + \underset{\text{Phase constant}}{\underset{\uparrow}{\phi}}}^{\text{Phase}})$$

The motion represented by Eq. (5) is called **simple harmonic motion** (SHM); a term that means the periodic motion is a sinusoidal function of time.

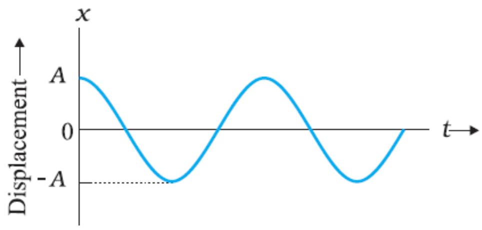


Fig. 1 : Sinusoidal motion.

(1) The quantity A is called the **amplitude** of the motion. It is a positive constant which represents the magnitude of the maximum displacement of the particle in either direction. The cosine function in Eq. (5) varies between the limits ±1, so the displacement $x(t)$ varies between the limits $\pm A$.

(2) The time varying quantity, $(\omega t + \phi)$, in Eq. (5) is called the **phase** of the motion. It describes the state of motion at a given time. The constant $\phi$is called the **phase constant** (or **phase angle**). The value of ϕ depends on the displacement and velocity of the particle at $t = 0$. It can be seen that the phase constant signifies the initial conditions.

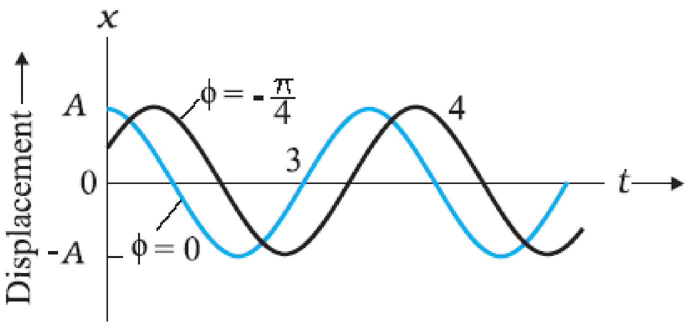


Fig.2: A plot for $\phi = 0$ and $\phi = -\pi/4$.

(3) If $\phi = 0$, the Eqn., (5) reduces to $x(t) = A \cos(\omega t)$ -------- *(6)*

5. Show that Simple harmonic motion is the projection of uniform circular motion on a diameter of the circle in which the latter motion takes place.

As shown in Fig, it can be shown that the motion of a **reference particle** P executing a uniform circular motion with (constant) angular speed ω in a **reference circle**. The radius A of the circle is the magnitude of the particle's position vector. At any time t, the angular position of the particle is $\omega t + \varphi$, where φ is its angular position at $t = 0$. The projection of particle P on the x-axis is a point P′ , which we can take as a second particle. The projection of the position vector of particle P on the x-axis gives the location $x(t)$ of P′ . Thus we have, $x(t) = A \cos(\omega t + \varphi)$ which is the same as Eq. (5). This shows that if the reference particle P moves in a uniform circular motion, its projection particle P' executes a simple harmonic motion along a diameter of the circle.

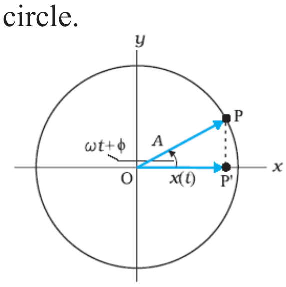


Fig.3: *The motion of a reference particle* P *executing a uniform circular motion with (constant) angular speed ω in a reference circle of radius A.*

6. Obtain an expression for velocity & acceleration of a simple harmonic wave ?

In SHM, the magnitude of velocity, **v**, with which the reference particle P (Fig.3) is moving in a circle is related to its angular speed, ω, as $v = \omega A$, where A is the radius of the circle described by the particle P. The magnitude of the velocity vector **v** of the projection particle is ωA ; its projection on the x-axis at any time t, as shown in Fig. 2, is

$v(t) = -\omega A \sin(\omega t + \phi)$ ------- (6)

The negative sign appears because the velocity component of P is directed towards the left, in the negative direction of x. Equation (6) expresses the instantaneous velocity of the particle P′(projection of P). Therefore, **it expresses the instantaneous velocity of a particle executing SHM.** Equation (7) can also be obtained by differentiating Eq. (6) with respect to time as,

$$v(t) = \frac{d}{dt}x(t) \quad \text{---------} \quad (7)$$

Expression for Acceleration :

When a particle executing a uniform circular motion is subjected to a radial acceleration **a** directed towards the centre. Figure 3 shows such a radial acceleration, **a**, of the reference particle P executing uniform circular motion. The magnitude of the radial acceleration of P is $\omega^2 A$. Its projection on the x-axis at any time t is,

$a(t) = -\omega^2 A \cos(\omega t + \varphi)$

$= -\omega^2 x(t) \quad \text{----------} \quad (8)$

which is the acceleration of the particle P′ (the projection of particle P). Equation (8), therefore, represents the instantaneous acceleration of the particle P′, which is executing **SHM**. Thus Eqn. (8) **expresses the acceleration of a particle executing SHM**. It is an important result for *SHM*. It shows that in **SHM**, **the acceleration is proportional to the displacement and is always directed towards the mean position**.

Eq. (8) can also be obtained by differentiating Eq. (7) with respect to time as,

$$a(t) = \frac{d}{dt}v(t) \quad \text{---------} \quad (9)$$

7. Explain SHM in terms of the concept of Force Law ?

Using the concept of Newton's second law and Acceleration of SH wave, we can write,

$F(t) = ma$

$= -m\omega^2 x(t)$

or $F(t) = -k\,x(t) \quad \text{-------} \quad (10)$

where $k = m\omega^2 \quad \text{----------} \quad (11)$

or

$$\boldsymbol{\omega} = \sqrt{\frac{k}{m}} \quad \text{---------} \quad (12)$$

Equation (10) gives the force acting on the particle. It is proportional to the displacement and directed in an opposite direction. Therefore, it is a restoring force.

It can be noted that unlike the centripetal force for uniform circular motion that is constant in magnitude, the restoring force for SHM is time dependent. The force law expressed by Eq. (10) can be taken as an alternative definition of simple harmonic motion. It states : **Simple harmonic motion is the motion executed by a particle subject to a force, which is proportional to the displacement of the particle and is directed towards the mean position.**

14.3. Energy in S.H.M. Kinetic and potential energies; simple pendulum– derivation of expression for its time period;

8. Obtain an expression for total energy of SH wave.

A particle executing simple harmonic motion has kinetic and potential energies, both varying between the limits, zero and maximum.

We know that the velocity of a particle executing SHM, is a periodic function of time. It is zero at the extreme positions of displacement. Therefore, the kinetic energy (*K*) of such a particle, which is defined as

$k = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2 A^2 \sin^2(\omega t + \varphi) = \frac{1}{2}kA^2 \sin^2(\omega t + \varphi)$ -------- (13)

is also a periodic function of time, being zero when the displacement is maximum and maximum when the particle is at the mean position. Note, since the sign of *v* is immaterial in *K*, the period of *K* is *T*/2.

Also we know that the potential energy associated with the conservative forces is $U = \frac{1}{2}kx^2$.

Hence the potential energy of a particle executing simple harmonic motion is, $U(x) = \frac{1}{2}kx^2$

$= \frac{1}{2}kA^2 \cos^2(\omega t + \varphi)$ -------- (14)

Thus the potential energy of a particle executing simple harmonic motion is also periodic, with period *T*/2, being zero at the mean position and maximum at the extreme displacements.

The total energy of the system can be calculated using Eqns. (13) & (14) as ***E* = *U* + *K***

$$= \frac{1}{2}k\,A^2\cos^2(\omega t + \phi) + \frac{1}{2}k\,A^2\sin^2(\omega t + \phi)$$

$$= \frac{1}{2}k\,A^2\left[\cos^2(\omega t + \phi) + \sin^2(\omega t + \phi)\right]$$

The quantity within the square brackets above is unity and we have, $\mathbf{E = \frac{1}{2}kA^2}$ ------- (15)

The total mechanical energy of a harmonic oscillator is thus independent of time as expected for motion under any conservative force. The time and displacement dependence of the potential and kinetic energies of a linear simple harmonic oscillator are shown in Fig. 4.

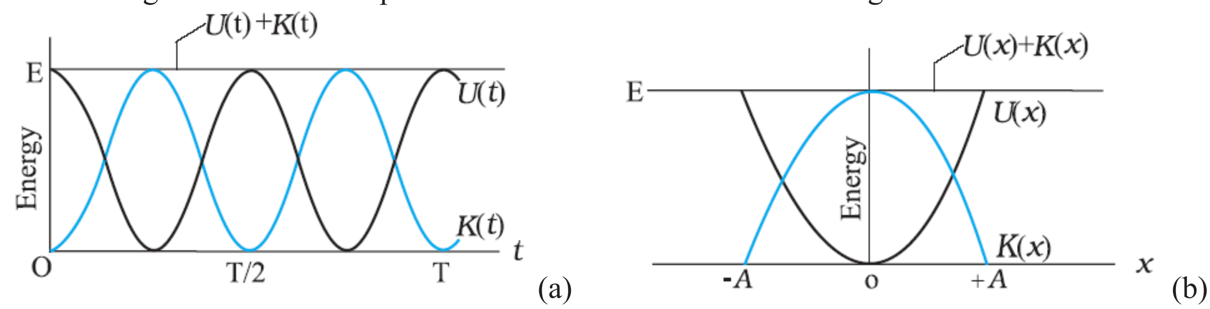


Fig.4: (a) Potential energy U(t), kinetic energy K(t)and the total energy E as functions of time t for a linear harmonic oscillator. All energies are positive and the potential and kinetic energies peak twice in every period of the oscillator. (b) Potential energy U(x), kinetic energy K(x) and the total energy E as functions of position x for a linear harmonic oscillator with amplitude A. For x = 0, the energy is all kinetic and for x = ± A it is all potential.

It is observed that in a linear harmonic oscillator, all energies are positive and peak twice during every period. For $x = 0$, the energy is all kinetic and for $x = \pm A$ it is all potential. In between these extreme positions, the potential energy increases at the expense of kinetic energy. This behaviour of a linear harmonic oscillator suggests that it possesses an element of springiness and an element of inertia. The former stores its potential energy and the latter stores its kinetic energy.

9. What is simple pendulum ? Derivation of expression for its time period ?

A **simple pendulum** consists of a particle of mass m (called the bob of the pendulum) suspended from one end of an unstretchable, massless string of length L fixed at the other end as shown in Fig. 5 (a). The bob is free to swing to and fro in the plane of the page, to the left and right of a vertical line through the pivot point.

The forces acting on the bob are the force **T**, tension in the string and the gravitational force $\mathbf{F_g}$ (= m **g**), as shown in Fig. 6 (b). The string makes an angle θ with the vertical. We resolve the force $\mathbf{F_g}$ into a radial component F_g cos θ and a tangential component F_g sin θ. The radial component is cancelled by the tension, since there is no motion along the length of the string. The tangential component produces a restoring torque about the pendulum's pivot point. This

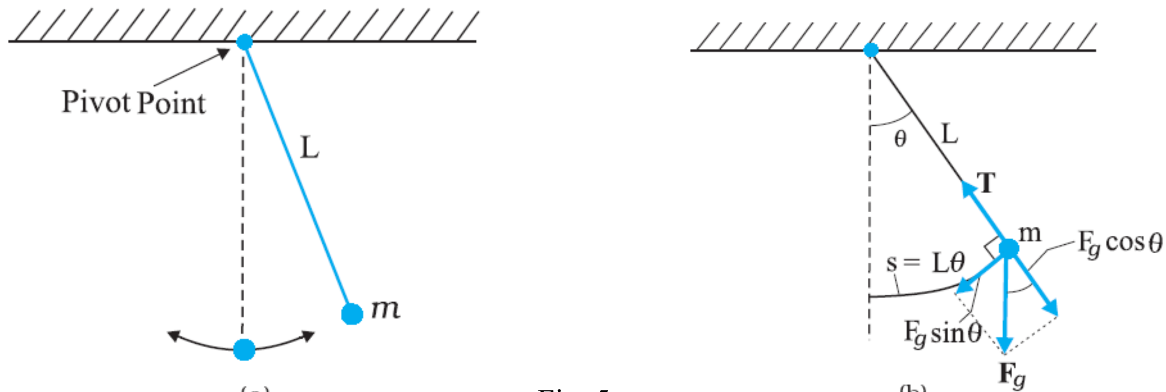


Fig. 5.

torque always acts opposite to the displacement of the bob so as to bring it back towards its central location. The central location is called the **equilibrium position** (θ = 0), because at this position the pendulum would be at rest if it were not swinging.

The restoring torque τ is given by,

$\tau = -L\,(F_g \sin\theta)$ --------- (16)

where the negative sign indicates that the torque acts to reduce θ, and L is the length of the moment arm of the force Fg sin θ about the pivot point. For rotational motion we have,

$\tau = I\,\alpha$ --------- (17)

where I is the pendulum's rotational inertia about the pivot point and α is its angular acceleration about that point. From Eqns. (16) and (17) we have,

$-L\,(F_g \sin\theta) = I\,\alpha$

Substituting the magnitude of F_g, i.e. mg, we have,

$-L\,m\,g \sin\theta = I\,\alpha$

or, $\alpha = -\frac{mgL}{I}\sin\theta$ -------- (18)

For small value of θ, sin θ = θ, and hence, Eqn. (18) becomes, $\alpha = -\frac{mgL}{I}\theta$ ------- (19)

Equation (19) is the angular analogue of Eq. (8) and tells us that the angular acceleration of the pendulum is proportional to the angular displacement θ but opposite in sign. Thus as the pendulum moves to the right, its pull to the left increases until it stops and begins to return to the left. Similarly, when it moves towards left, its acceleration to the right tends to return it to the right and so on, as it swings to and fro in SHM. Thus the motion of a **simple pendulum swinging through small angles is approximately SHM**.

Comparing Eq. (19) with Eq. (8), we see that the angular frequency of the pendulum is,

$\boldsymbol{\omega = \sqrt{\frac{mgL}{I}}}$ and the period of the pendulum, *T*, is given by, $\boldsymbol{T = \sqrt{\frac{I}{mgL}}}$ **------** (20)

All the mass of a simple pendulum is centered in the mass *m* of the bob, which is at a radius of *L* from the pivot point. Therefore, for this system, we can write $I = m\ L^2$ and substituting this in Eqn. (20) we get,

$\boldsymbol{T = \sqrt{\frac{L}{g}}}$ ---------- (21)

Equation (21) represents a simple expression for the time period of a simple pendulum.

Problem :

(1) What is the length of a simple pendulum, which ticks seconds ?

The time period of a simple pendulum is given by, $\boldsymbol{T = \sqrt{\frac{L}{g}}}$

then $L = T^2.g$

14.4. Free and forced and damped oscillations (qualitative ideas only), resonance :

10. Write a note on Free and Forced and Damped Oscillations ?

(a) Free Oscillations :

A person swinging in a swing without anyone pushing it or a simple pendulum, displaced and released, are examples of free oscillations. In both the cases, the amplitude of swing will gradually decrease and the system would, ultimately, come to a halt. Because of the ever-present dissipative forces, the free oscillations cannot be sustained in practice. They get damped.

(b) Forced Oscillations :

If we apply external force on an oscillator, we find that not only the oscillations can now be maintained but the amplitude can also be increased. Under this condition the swing has **forced,** or **driven, oscillations**. In case of a system executing driven oscillations under the action of a harmonic force, two angular frequencies are important : (1) **the natural** angular frequency ω of the system, which is the angular frequency at which it will oscillate if it were displaced from equilibrium position and then left to oscillate freely, and (2) the angular frequency ω_d of the external force causing the driven oscillations.

In forced oscillations, the steady state motion of the particle (after the force oscillations die out) is simple harmonic motion whose frequency is the frequency of the driving frequency ω_d , not the natural frequency ω of the particle. Under forced oscillation, the phase of harmonic motion of the particle differs from the phase of the driving force.

(c) Damped Oscillations :

If the motion of a simple pendulum swinging in air, dies out eventually due to the air drag and the friction at the support oppose the motion of the pendulum and dissipate its energy gradually, then the pendulum is said to execute **damped oscillations**.
In general, the mechanical energy in a real oscillating system decreases during oscillations because external forces, such as drag, inhibit the oscillations and transfer mechanical energy to thermal energy. The real oscillator and its motion are then said to be *damped*. If the *damping force* is given by $F_d = -bv$, where v is the velocity of the oscillator and b is a *damping constant*, then the displacement of the oscillator is given by,

$$x(t) = A e^{-bt/2m} \cos(\omega' t + \phi)$$

where ω′, the angular frequency of the damped oscillator, is given by

$$\sqrt{\frac{k}{m} = \frac{b^2}{4m^2}}$$

If the damping constant is small then ω′=ω, where ω is the angular frequency of the undamped oscillator. The mechanical energy E of the damped oscillator is given by

$$E(t)\quad \frac{1}{2}kA^2e^{\ bt/m}$$

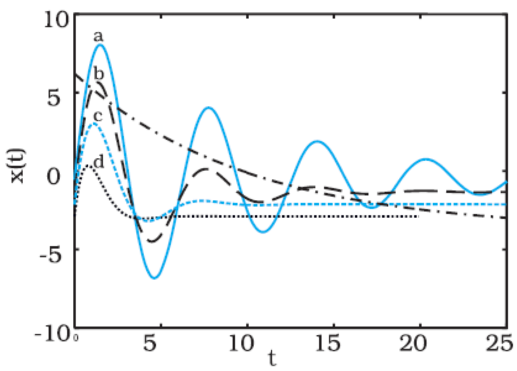


Fig.6: Displacement as a function of time in damped harmonic oscillations. Damping goes on increasing successively from curve a *to* d.

In the ideal case of zero damping, the amplitude of simple harmonic motion at resonance is infinite. This is no problem since all real systems have some damping, however, small.

11. What is resonance in forced oscillations ? Give the condition for resonance ?

If an external force with driving angular frequency ω_d acts on an oscillating system with natural angular frequency ω, the system oscillates with angular frequency ω_d. The amplitude of oscillations is the greatest when $\boldsymbol{\omega_d = \omega}$ a condition called *resonance*.
Thus the phenomenon of increase in amplitude when the driving force is close to the natural frequency of the oscillator is called **resonance**.

Examples :
All mechanical structures have one or more natural frequencies, and if a structure is subjected to a strong external periodic driving force that matches one of these frequencies, the resulting oscillations of the structure may rupture it.
(1) The Tacoma Narrows Bridge at Puget Sound, Washington, USA was opened on July 1, 1940. Four months later winds produced a pulsating resultant force in resonance with the natural frequency of the structure. This caused a steady increase in the amplitude of oscillations until the bridge collapsed. It is for the same reason the marching soldiers break steps while crossing a bridge.
(2) Aircraft designers make sure that none of the natural frequencies at which a wing can oscillate match the frequency of the engines in flight. Earthquakes cause vast devastation.

(3) It is interesting to note that sometimes, in an earthquake, short and tall structures remain unaffected while the medium height structures fall down. This happens because the natural frequencies of the short structures happen to be higher and those of taller structures lower than the frequency of the seismic waves.

One mark questions

1. Define frequency of periodic motion.
The number of repetitions that occur per unit time (second) is called the frequency of the periodic motion.

2. Give an example for a non-simple harmonic periodic motion.
Rotation of earth about its axis, Revolution of earth around the sun, etc..

3. What is the SI unit of frequency?
hertz (Hz).

4. Give the relation between period and frequency of periodic motion.
$\boldsymbol{\vartheta = \frac{1}{T}}$

5. What is the mean position(or equilibrium position) of an oscillating body?
It is a position of the particle where the net force acting on it is zero.

6. Define the phase of particle in oscillatory motion.
Phase of a vibrating particle is defined as state of vibration regarding position and direction of motion at that instant of motion.

7. What is the net external force acting on the body at its equilibrium position?
Zero.

8. Where will be the force acting on a particle executing SHM maximum?
At the extreme positions.

9. Define amplitude of simple harmonic motion.
The magnitude of the maximum displacement of the particle in either direction of SHM is called the amplitude.

10. What is the SI unit of angular frequency?
Radian per second (rad/s).

11. Give the relation between angular frequency and frequency.
$\omega = = 2\pi\vartheta$, where is ϑ frequency and ω is angular frequency.

12. Mention the relation between angular frequency and period.

$$\boldsymbol{\omega = \frac{2\pi}{T}}$$

13. Write the relation between m, ω and k, where the terms have usual meaning.
Angular frequency $\omega = \sqrt{\frac{k}{m}}$

14. Give the expression of the force law (Hooke's law) for a particle executing SHM.
F(t) = – k x(t).

15. Mention the expression for velocity of a particle executing SHM.
Velocity : $v(t) = -\omega A \sin(\omega t + \phi)$.

16. What is the phase difference between velocity and displacement of a particle executing SHM?
$\frac{\pi}{2}$ or 90°

17. What is the phase difference between acceleration and displacement of a particle executing SHM?
π or 180°.

18. Write the relation between velocity amplitude 'v', the displacement amplitude 'A' and the angular frequency 'ϑ' of SHM.
$v = \omega A$.

19. Give the expression for acceleration of a particle executing SHM.
Acceleration : $a(t) = -\omega^2 A \cos(\omega t + \phi) = -\omega^2 x(t)$

20. Give the expression for kinetic energy of the particle executing SHM at mean position.
Kinetic energy at mean position(maximum value) = $K_{max} = \frac{1}{2} m\omega^2 A^2$ or $K_{max} = \frac{1}{2} k A^2$

21. What is the minimum value of kinetic energy of a particle executing SHM?
Zero (at extreme positions of displacement).

22. Does the total mechanical energy of a harmonic oscillator depend on time?
No, the total mechanical energy of a harmonic oscillator is *independent of time*.

23. When will the motion of a simple pendulum be simple harmonic?
The motion of a simple pendulum swinging through small angles is approximately SHM.

24. The time period of simple pendulum is T, What is the time period when mass of the bob is doubled?
Time period = T, since the time period of simple pendulum is independent of mass of the bob.

25. How does the time period of a simple pendulum vary when it is taken from equator to poles?
The time period decreases as the value of g is more at the poles than that at equator.

26. What happens to the time period of a simple pendulum when it is taken from earth to the moon?
The time period increases as the value of g is less on the moon than that on earth surface.

27. How does the time period of simple pendulum vary with its length?
Time period of simple pendulum is directly proportional to the square root of its length.

28. How is the time period of the pendulum affected when it is taken to hills or in to mines?
The time period increases as the value of g is less on hills or in mines than that at surface.

29. What is the cause for damped oscillations?
Air drag, viscous force and friction at support oppose the oscillations are main causes for damping.

30. What happens to the mechanical energy of the particle executing damped oscillations?
Mechanical energy of the particle decreases.

31. Whether the amplitude increase or decrease or remains same in damped scillations?
Amplitude decreases gradually in damped oscillations.

32. What is resonance?
The phenomenon of increase in amplitude when the driving frequency (applied) is close to the natural frequency of the oscillator is called resonance.

33. What is the frequency of oscillation of a simple pendulum mounted in a cabin that is freely falling under gravity?
Frequency is zero since the gravity disappears for free fall.

34. What is the condition for resonance?
$\omega_d = \omega$ where ω is natural angular frequency and ω_d is driven angular frequency.

Two marks questions

1. What is periodic motion? Give an example.
The motion that repeats itself at regular intervals of time is called periodic motion.
E.g.: The motion of leaves and branches of a tree in breeze, orbital motion of planets in the solar system, oscillations of loaded spring, movement of the pendulum of a clock.

2. Define period of periodic motion.
The smallest interval of time after which the motion is repeated is called time period. SI unit of period is second (s).

3. On an average a human heart is found to beat 75 times in a minute. Calculate its frequency and period.
The beat frequency of heart = 75/(1 min) = 75/(60 s) = 1.25 s^{-1} = 1.25 Hz

The time period T= $1/(1.25\ s^{-1}) = 0.8$ s

4. A particle takes 32s to make 20 oscillations. Calculate time period and frequency.

ANS :

Time period =

$$\frac{\text{Total time}}{\text{Number of oscillations}} = \frac{32\ s}{20} = 1.6\ s$$

Frequency = 1/Time Period = 1/1.6 = 0.625 Hz.

5. Which of the following functions of time represent (a) periodic and(b) non periodic. Also give the period for each. (a) sin3ωt (b) sinωt+ cosωt (c) sinωt+ cos2ωt + sin4ωt (d) 3 cos(π/4 – 2ωt) (e) $\sin^2\omega t$ (f) $\sin^3\omega t$ (g) $e^{-\omega t}$ (h) log (ωt)

ANS :

(a) sin3ωt is a periodic function. Period of the function is 2π/3ω

(b) sinωt+ cos ωtis a periodic function.

sinωt+ cos ωt = $\sqrt{2}\sin(\omega t+\pi/4)$

Now$\sqrt{2}\sin(\omega t+\pi/4) = \sqrt{2}\sin(\omega t+\pi/4+2\pi) = \sqrt{2}\sin[\omega(t+2\pi/\omega)+\pi/4]$

∴ Period of the function is 2π/ω.

(c) sinωt+ cos 2ωt + sin 4ωtis a periodic function.

sinωthas a period of 2π/ω, cos2ωt has a period of 2π/2ωand

sin 4ωthas a period of 2π/4ω.

∴ Period of the function is 2π/ω.

(d) 3 cos (π/4 – 2ωt)is a periodic function. Period of the function is 2π/2ω= π/ω

(e) $\sin^2\omega t$= ½ – (½ cos 2ωt) is a periodic function. Period of the function is 2π/2ω = π/ω

(f) $\sin^3\omega t$ is a periodic function. Period of the function is 2π/ω

(g) $e^{-\omega t}$ is a non-periodic function.

$e^{-\omega t}$decreases monotonically with increasing time and tends to zero as t → ∞ and never repeats its value.

(h) log (ωt) is a non-periodic function.

log(ωt) increases monotonically with time t. Itnever repeats its value.

As t → ∞,log(ωt) diverges to ∞. ∴ It cannot represent any kind of physicaldisplacement.

6. Give the expression for displacement of a particle executing simple harmonic motion ?

ANS :

$x(t) = A\cos(\omega t + \phi)$,

Where $x(t)$ is displacement as a function of time,

A is Amplitude, ω is angular frequency,

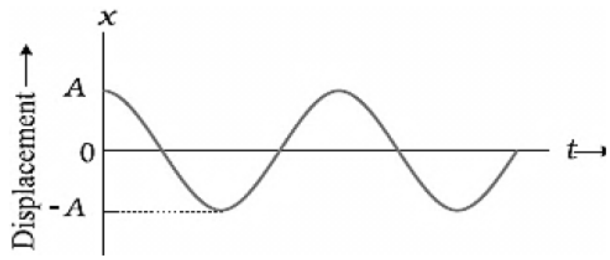
f is phase constant and (ωt+ ϕ) is phase.

7. Plot the graph of $x(t)$ as a function of time for the motion represented by the equation $x(t) = A\cos(\omega t+\phi)$, Where A, ω and ϕ are constants.

ANS :

Displacement : $x(t) = A\cos(\omega t+\phi)$,

The graph of x as a function of time for the SHM is shown in the adjacent figure.



8. The displacement of a particle executing SHM is given by $x(t) = A\cos(\omega t + \phi)$. Write the expression for velocity and acceleration.

ANS :

Displacement : $x(t) = A\cos(\omega t+ \phi)$,

Velocity: $v(t) = – A\omega\sin (\omega t+ \phi)$ and acceleration: $a(t) = – A\omega^2\cos(\omega t+ \phi)$

9. Does the function 'sin ωt – cos ωt represent SHM? Find its (i) period and (ii) phase angle.

ANS :

The function $\sin\omega t–\cos\omega t$ represents SHM.

Because $\sin \omega t – \cos \omega t = \sin \omega t – \sin (\pi/2 – \omega t) = 2 \cos (\pi/4) \sin (\omega t – \pi/4) = \sqrt{2} \sin (\omega t –\pi/4)$

This function represents a simple harmonic motion having a period $T = 2\pi/\omega$ and a phase angle $(–\pi/4)$ or $(7\pi/4)$.

10. Mention the expression for kinetic energy of a particle executing SHM. Explain the terms.

ANS :

Kinetic energy: $K = \frac{1}{2}m\omega^2A^2\sin^2(\omega t+\phi)$ or $K = \frac{1}{2}k\,A^2\sin^2(\omega t+\phi)$

Where m is mass of the particle executing SHM, ω is angular frequency, A is Amplitude, ϕ is phase constant, (ωt + ϕ) is phase and k is force constant.

11. Where is the kinetic energy of a particle executing SHM (i) minimum and (ii) maximum ?

ANS :

(i) Minimum at extreme positions (ii) maximum at mean position.

12. Give the expression for the time period of a simple harmonic oscillator (spring system).

ANS :

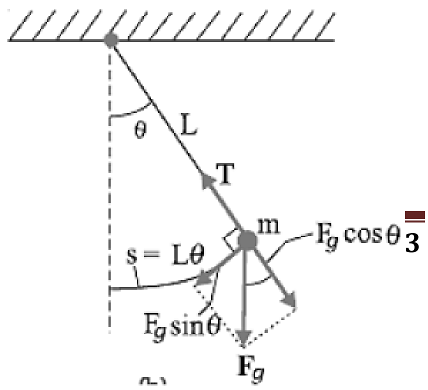
The time period of a simple harmonic oscillator : $T = 2p\sqrt{\frac{m}{k}}$

Where m is the mass of the load attached and k is the force constant.

13. Draw the free-body diagram of the simple pendulum showing the forces acting on the bob.

ANS :

Free-body diagram of the simple pendulum is shown in the adjacent diagram.



Where m is the mass of the bob of the pendulum,
L is the length of the pendulum, T is the tension in the string, F_g (= mg) is the gravitational force,
$F_g \cos\theta$ radial component of gravitational force and
$F_g \sin\theta$ is the tangential component of gravitational force.

14. Mention the expression for potential energy of a particle executing SHM. Explain the terms.

ANS :

Potential energy $U = \frac{1}{2}kx^2 = \frac{1}{2}\, m\omega^2 A^2 \cos^2(\omega t + \phi)$ or $U = \frac{1}{2}\, k\, A^2 \cos^2(\omega t + \phi)$

Where m is mass of the particle executing SHM, ω is angular frequency, A is Amplitude,
ϕ is phase constant, (ωt + ϕ) is phase and k is force constant.

15. Where is the potential energy of a particle executing SHM (i) minimum and (ii) maximum ?

ANS :

(i) Minimum at mean position (ii) maximum at extreme positions.

16. Give the expression for total mechanical energy of a particle executing SHM.

ANS :

Total mechanical energy : $E = K + U = \frac{1}{2}\, m\omega^2 A^2 \sin^2(\omega t + \phi) + \frac{1}{2}\, m\omega^2 A^2 \cos^2(\omega t + \phi)$

$$= \frac{1}{2}kA^2$$

Where A is Amplitude and k is force constant.

17. Mention the expression for time period of simple pendulum. Explain the terms.

ANS :

Time period of oscillation of the pendulum: $T = 2\pi\sqrt{\frac{L}{g}}$

Where L is the length of the pendulum and g is acceleration due to gravity.

18. What is the length of a simple pendulum, which ticks seconds ?

ANS :

Time period of oscillation of the given pendulum : T = 2 s

Time period : $T = 2p\sqrt{\frac{L}{g}}$

$$L = \left(\frac{g\,T^2}{4\,\pi^2}\right) = \left(\frac{9.8 \times 2^2}{4 \times 3.142^2}\right) = 0.9927\,m \approx 1\,m$$

19. Write the expression for time period of oscillations of loaded spring. Explain the terms.

ANS :

Time period of oscillation of loaded spring: $T = 2\pi\sqrt{\frac{m}{k}}$,

Where m is the mass of the load attached and k is the spring constant.

20. What are free oscillations? Give an example.

ANS :
The oscillations made by a body (particle) when it is left free itself, it oscillates with a frequency of its natural frequency are called free oscillations.
E.g.: The oscillations of a pendulum, the oscillations of loaded spring, oscillations of the prongs of a tuning fork, etc..

21. What are damped oscillations? Give an example.
ANS :
The oscillations of a simple pendulum (or any other oscillating particle) are opposed by air drag and friction at its support. As a result pendulum makes oscillations with decreasing amplitude. Such oscillations are called damped oscillations.
E.g.: The oscillations of a pendulum, the oscillations of loaded spring, oscillations of the prongs of a tuning fork, etc..

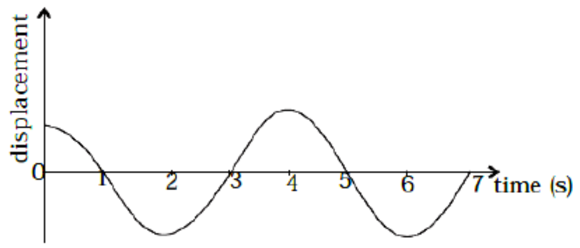
22. What are forced or driven oscillations? Give an example.
ANS :
The oscillations in which the a of external agency (force) are called E.g.: When the stem of vibrating tuning fork pressed on a table, table executes forced oscillations with a frequency equal to frequency of tuning fork.

23. What are the two basic characteristics of a simple harmonic motion?
ANS :
The two basic characteristics of a simple harmonic motion :
(i) Restoring force is proportional to the displacement of the particle from mean position.
(ii) Restoring force is directed towards the mean position.

24. What is the ratio of maximum acceleration to the maximum velocity of a simple harmonic oscillator?
ANS :
Maximum acceleration: am = $A\omega^2$ -------- (1)
Maximum velocity: $v_m = A\omega$ ...………..(2)
$\rightarrow$ Required ratio, (1) ÷ (2) gives $a_m/v_m = \omega$, is the angular frequency.

25. What is the ratio between the distance travelled by the oscillator in one time period and amplitude?
ANS :
Ratio between the distance travelled by the oscillator in one time period and amplitude: Distance travelled in by the oscillator in one time period is 4A, where A is the Amplitude. Required ratio = 4A/A = 4

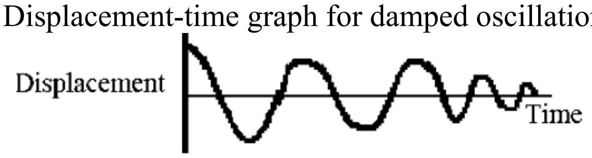
26. The displacement-time curve for a particle executing S.H.M. is given. (i) What is the time period of S.H.M? (ii) What is the phase of the particle at t = 2s?



ANS :
(i) Time period of S.H.M is 4s
(ii)The phase of the particle at t = 2s is π.

27. Give the expression for damping force. Explain the terms.
ANS :
Damping force : Fd = – b v
Where b is damping constant (always positive) and v is velocity.
Note : The negative sign indicates that the force is opposite to the velocity at every moment.

28. Draw the displacement time graph for damped oscillations.
ANS :
Displacement-time graph for damped oscillations is as shown in the adjacent figure.



29. Write the expression for angular frequency of damped oscillator. Explain the terms.
ANS :
Angular frequency of damped oscillator : $\boldsymbol{\omega}' = \sqrt{\frac{\boldsymbol{k}}{\boldsymbol{m}} - \frac{\boldsymbol{b}^2}{4\boldsymbol{m}^2}}$
Where b is damping constant, m is the mass and k is the force constant.

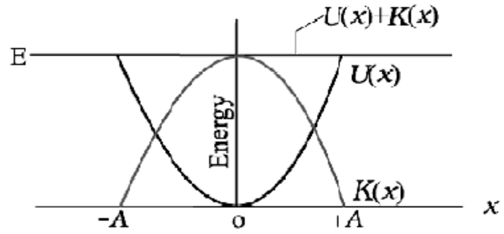
30. Give the expression for total mechanical energy of the damped oscillator. Explain the terms.
ANS :
The total mechanical energy of the damped oscillator : $E = \frac{1}{2} kA^2 e^{-bt/m}$
Where A is Amplitude, k is force constant, m is the mass and b is damping constant.

31. A man with a wristwatch on his hand falls from the top of a tower. Does the watch give correct time during the free fall? And why?
ANS :
Yes, the clock gives correct time.
The motion in the wristwatch depends on spring action and has nothing to do with acceleration due to gravity.
i.e., The motion of spring in the watch is not affected by acceleration due to gravity.

32. Give the graphical representation of the variation of potential energy, kinetic energy and the total energy as functions of position *x* for a linear harmonic oscillator with amplitude A

ANS :

The graphical representation of the variation of potential energy U(x), kinetic energy K(x) and the total energy E as functions of position x for a linear harmonic oscillator with amplitude A is as shown in the adjacent diagram.



Four/Five Marks Questions :

1. State force law definition of simple harmonic motion and hence obtain the expression for angular frequency.

Simple harmonic motion is the motion executed by a particle subject to a force, which is proportional to the displacement of the particle and is directed towards the mean position.

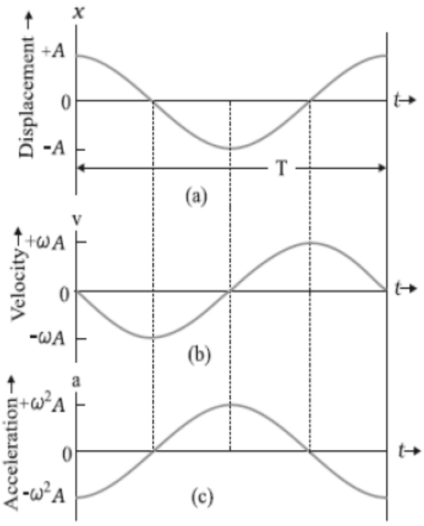
Consider a particle of mass m executing SHM with an angular frequency ω.

From Newton's second law: $F(t) = ma = -m\omega^2 x(t)$……(1) since acceleration $a = -\omega^2 x(t)$

The restoring force Facting on the particle $F(t) = -k\,x(t)$……..(2) , k is force constant.

From (1) and (2), $k = m\omega^2$ $\therefore\ \omega = \sqrt{\frac{k}{m}}$

2. Plot the velocity-time graph and acceleration-time graph of a particle executing SHM whose displacement is given by *x*(t) = A cos(ωt). Also give the expressions for the velocity and acceleration.



The diagram (a) Represents Displacement

The diagram (b) Represents velocity

The diagram (c) Represents acceleration

Displacement : $x(t) = A\cos(\omega t)$

Velocity : $v(t) = -A\omega\sin(\omega t)$

Acceleration : $a(t) = -A\omega^2\cos(\omega t)$

Where A is Amplitude, ω is a angular frequency, and (ωt) is phase.

3. Arrive at the expression for time period of oscillation of a mass attached to a spring ?

Derivation of expression for time period of oscillation of a mass attached to a spring:

Consider the small oscillations of a block of mass m fixed to a spring, which in turn is fixed to a support.

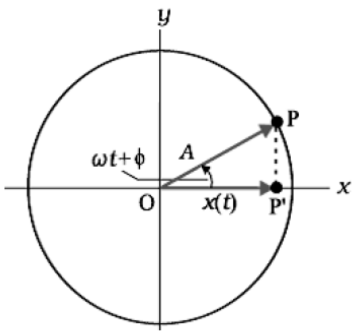
From Newton's second law, F(t) = ma , but acceleration a = $-\omega^2 x(t)$
i.e., F(t) = $-m\omega^2 x(t)$……..…..…(1)
The magnitude of the restoring force acting on the mass is proportional to the deformation or the displacement and acts in opposite direction
i.e., F(t) = – k $x(t)$……………..(2)

From (1) and (2), $k = m\omega^2$ or $\omega = \sqrt{\frac{k}{m}}$

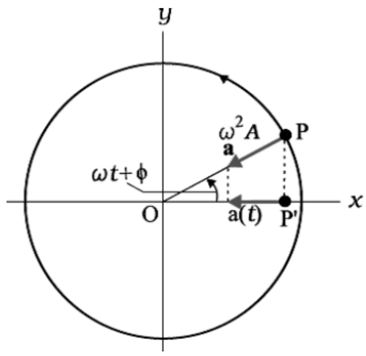
Time period of oscillation: $T = 2\pi/\omega = 2\pi\sqrt{\frac{m}{k}}$

4. Explain simple harmonic motion with reference to uniform circular motion with the help of a diagram.

The motion of a reference particle P executing a uniform circular motion with constant angular speed ω in a reference circle is shown in the adjacent figure.
The radius A of the circle is the magnitude of the particle's position vector.
At any time t, the angular position of the particle is (ωt + ϕ)
where f is its angular position at t = 0
The projection of particle P on the x-axis is a point P', which we can take as a second particle. The projection of the position vector of particle P on the x-axis gives the location $x(t)$ of P'.
Thus we have, $x(t) = A\cos(\omega t + \phi)$.
This shows that if the reference particle P moves in a uniform circular motion, its projection particle P' executes a simple harmonic motion along a diameter of the circle.
Thus Simple harmonic motion is the projection of uniform circular motion on a diameter of the
circle in which the latter motion takes place.



5. Show that in simple harmonic motion, the acceleration is directly proportional to its displacement at the given instant



Consider the a particle executing SHM,
Let the displacement: $x(t) = A\cos(\omega t + \phi)$.
Differentiating displacement w.r.t. 't'
Velocity: v(t) = d/dt (xt)] $= -A\omega\sin(\omega t + \phi)$
Differentiating velocity w.r.t. 't'
Acceleration: a(t) = d/dt [v(t)]
$= -A\omega^2\cos(\omega t + \phi) = -\omega^2 x(t)$
=> Acceleration: $a(t) \propto x(t)$
∴ In SHM, the acceleration is proportional to the displacement at any instant of time.

6. Derive the expression for the kinetic energy and potential energy ?

Derivation of expression for the kinetic energy of a harmonic oscillator:

Kinetic energy: $K = \frac{1}{2}mv^2$ =

Kinetic energy: $K=\frac{1}{2}mv^2$ where v = velocity= – $A\omega\sin(\omega t+\phi)$, m is mass of the particle

i.e., $K=\frac{1}{2}m\omega^2A^2\sin^2(\omega t+\phi)=\frac{1}{2}kA^2\sin^2(\omega t+\phi)$, where $k=m\omega^2$

Derivation of expression for potential energy of a harmonic oscillator:

Potential energy: $U=\frac{1}{2}kx^2$ where $x = A\cos(\omega t+\phi)$ and k is force constant

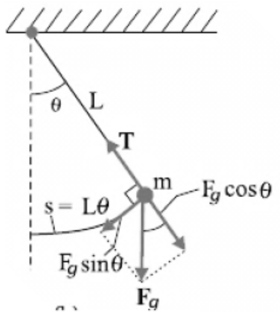
i.e., $U=\frac{1}{2}kA^2\cos^2(\omega t+\phi)=\frac{1}{2}m\omega^2A^2\cos^2(\omega t+\phi)$

Where ω is angular frequency, A is Amplitude,

ω is angular frequency, ϕ is phase constant and $(\omega t+\phi)$ is phase.

7. Arrive at an expression for the time period of simple pendulum.

Derivation of the expression for the time period of simple pendulum:



Let m be the mass of the bob of the simple pendulum executing oscillations about the mean position as shown in the diagram.

L be the length of the simple pendulum,

T be the tension in the string,

F_g (= mg) be the gravitational force acting vertically down,

$F_g\cos\theta$ be the radial component of gravitational force and

$F_g\sin\theta$ be the tangential component of gravitational force.

Torque t = –L ($m_g\sin\theta$) and from rotational motion, t= I α

=> $I\alpha = -mg\,L\sin\theta$, where_is the angular acceleration and

I is the pendulum's rotational inertia of the system about the pivot point.

$\Rightarrow \alpha=-\left(\frac{mgL}{I}\right)\sin\theta$ and for small value of θ, $\sin\theta\approx\theta$

$\therefore \alpha=-\left(\frac{mgL}{I}\right)\theta$ comparing this equation with $a=-\omega^2x$ as they are similar.

We have, $\omega^2=\frac{mgL}{I}\Rightarrow\omega=\sqrt{\frac{mgL}{I}}$

Time period: $T=\frac{2\pi}{\omega}=2\pi\sqrt{\frac{I}{mgL}}$

The string of simple pendulum is mass less. So moment of inertia of the bob is $I = mL^2$.

$\therefore$ Time period of the simple pendulum is $T = 2\pi\sqrt{\frac{L}{g}}$

8. Discuss the effect of damping force on a system (a mass attached to a spring in a viscous medium and obtain an expression for displacement

Let the damping force: $F_d = -b\,v$ .... (1)

Where b is a positive constant called damping constant and v is velocity of the particle. The negative sign indicates that the force is opposite to the velocity at every moment. When the mass mis attached to the spring and released, the spring will elongate a little and the mass will settle at some height. This position is the equilibrium position of the mass. If the mass is pulled down or pushed up a little, the restoring force on the block due to the spring is $F_S = -kx$, where x is the displacement of the mass from its equilibrium position. Thus the total force acting on the mass at any time t is

$F = -k\,x - b\,v$……….……..….(2)

If a(t) is the acceleration of the mass at time t, then by

Newton's second law of motion F = ma (t)

(2) = $ma(t) = -kx(t) - bv(t)$ ……………......(3)

Using $a(t) = \frac{d^2x}{dt^2}$ and $v(t) = \frac{dx}{dt}$ in (3)

We get, $m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$ .... (4)

The solution of eqn. (4) describes the motion of the block under the influence of a damping force which is proportional to velocity.

The solution of the eqn.(4) is of the form,

$x(t) = A\,e^{-bt/2m}\cos(\omega' t + \phi)$..............(5)

where $Ae^{-bt/2m}$ is the amplitude and ω' is the angular frequency of the damped oscillator.

Four/Five Marks Problems

1. A particle executing S.H.M. has a maximum speed of 30 cm/s and a maximum acceleration of 60 cm/s2. Calculate the period of the oscillation.

Given, maximum velocity: $v_m = A\omega = 30$ cm/s = 0.3m/s …………(1)

Maximum acceleration: $a_m = A\,\omega^2 = 60$ cm/s^2 = 0.6m/s^2 ...………..(2)

(2) ÷ (1) => ω = 2 or 2π/T = 2 => T = π second.

2. A particle oscillates with SHM according to the equation $x = 5\cos(2\pi t + \pi/4)$ metre. At t = 2.5 s, calculate the (a) displacement, (b) speed and (c)

A body oscillates with SHM according to the equation : $x = 5\cos(2\pi t + \pi/4)$ metre.

At t = 2.5 s,

(a) Displacement = 5 cos [(2π) ×2.5 s + π/4]

= 5 cos [(5π + π/4)] =–5 cos(π/4) = –5 × 0.7071m = –3.536 m

(b) Differentiating $x = 5\cos(2\pi t + \pi/4)$ w.r.t. 't',

Velocity: v = dx/dt

= –5 [sin(2πt + π/4)] × (2π) = –10πsin(2πt+ π/4)

At t = 2.5 s,

Velocity = –10_sin[2π × (2.5) + π/4)] = –10πsin(5π + π/4) = –10 πsin(π + π/4)= 10 πsin(π/4)

= 10(3.142) (0.7071) = 22.22 m/s

(c) Differentiating velocity v = –10π sin(2πt+ π/4) w.r.t. 't',

Acceleration: a = dv/dt

= –10π [cos(2πt + π/4)] ×(2π) = – 20π^{2}sin(2πt + π/4)

At t = 2.5 s,

Acceleration: a = – 20π^{2}cos[2π × (2.5) + π/4] = –20π^{2}cos(5π + π/4) = 20(3.142)^{2}cos(π/4)

= 139.6 m/s^2

3. A 5 kg collar is attached to a spring of spring constant 500 N a horizontal rod. The collar is displaced from its equilibrium position. Calculate (a) the period of oscillation, (b) the maximum speed and (c) maximum acceleration of the collar.

Given mass m = 5 kg, Spring constant k= 500 Nm^{-1} and amplitude A= 10.0 cm= 0.01m

(a) The period of oscillation as given by $T = 2\pi\sqrt{\frac{m}{k}} = 2(3.142)\sqrt{\frac{5}{500}} = \frac{6.284}{10} = 0.6284$ s

(b) The maximum speed: $v_m = A\omega = A\sqrt{\frac{k}{m}} = 0.1\times\sqrt{\frac{500}{5}} = 1$ m/s

(c) The maximum acceleration: $a_{max} = -\omega^2 A = \frac{k}{m}\times A = \frac{500}{5}\times 0.1 = 10\ m/s^2$

4. A spring balance has a scale that reads from 0 to 50 kg. The length of the scale is 20cm. A body suspended from this balance, when displaced and released, oscillates with a period of 0.6 s. What is the weight of the body.

The spring constant: $k = \frac{F}{l} = \frac{mg}{l} = \frac{50\times 9.8}{0.2} = 2450$ N/m

Time period: $T = 2\pi\sqrt{\frac{m}{k}} \Rightarrow 0.6 = 2\pi\sqrt{\frac{m}{2450}} \Rightarrow m = 22.34$ kg

Weight of the body is W= m g = 22.34 × 9.8 = 218.9 N

5. A spring having a spring constant horizontal table as shown in the figure. A mass of 3 kg is attached to the free end of the spring. The mass is then pulled sideways to a distance of 2.0 cm and released. Determine (i) the frequency of oscillations,
(ii) maximum acceleration of the mass and (iii) the maximum speed of the mass.

Given spring constant k =1200 Nm^{-1}, mass m= 3 kg and amplitude A =2.0 cm = 0.02m

(i)The frequency of oscillations: $\upsilon = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

$= \frac{1}{6.284}\sqrt{\frac{1200}{3}} = \frac{1}{6.284}\sqrt{400} = \frac{20}{6.284} = 3.183\,Hz$

(ii) The maximum acceleration of the mass: $a_m = \omega^2 A = \frac{k}{m}A = \frac{1200}{3}\times 0.02 = 8.0\ m/s^2$

(iii) The maximum speed of the mass: $v_m = A\omega = A\sqrt{\frac{k}{m}} = 0.02\times\sqrt{\frac{1200}{3}} = 0.04$ m/s

6. A particle executes SHM along the x the equation: $x(t) = 2.5\cos(4\pi t + \pi/6)$.
Determine the amplitude, frequency ?

Displacement: $x(t) = 2.5\cos(4\pi t + \pi/6)$.

Comparing it with, $x(t) = A\cos(\pi\, t + f)$

Amplitude A = 2.5m,

Angular frequency: $\omega = 2\pi/T = 4\pi$ => Time period T = 0.5 s

Frequency = 1/T = 1/0.5 = 2 Hz and Phase constant $\phi = \pi/6$

7. The acceleration due to gravity a simple pendulum on the surface of moon if its time period on the surface.

The time period of oscillation of simple pendulum: $T = 2\pi\sqrt{\frac{L}{g}}$

On earth surface: $T1 = 1.5 = 2\pi\sqrt{\frac{L}{9.8}}$ ………………. (1)

On moon surface: $T2 = 2\pi\sqrt{\frac{L}{1.7}}$ ……………. (2)

$$(2)\div(1) \Rightarrow \frac{T_2}{1.5} = \sqrt{\frac{L}{1.7}} \times \sqrt{\frac{9.8}{L}} = \sqrt{\frac{9.8}{1.7}} \Rightarrow T_2 = 1.5 \times 2.401 = 3.601\,s$$

8. A particle describes SHM with and velocity of the particle when the displacement is (a) 5cm, (b) 3 cm

Amplitude A = 5 cm = 0.05 m and time period T = 0.2 s.
Angular frequency: $\omega = 2\pi/T = 2\,(3.142) / 0.2 = 31.42$ rad/s

(a) Displacement x = 5 cm = 0.05 m
Here displacement = amplitude, so to find acceleration and velocity at extreme positions. At extreme positions the acceleration is maximum and the velocity is minimum.
∴ Maximum acceleration = $a_{max} = A\,\omega^2 = 0.05 \times (31.42)^2 = 49.36$ m/s^2
∴ Minimum velocity = $v_{min} = 0$
(b) Displacement x= 3 cm= 0.03 m,
Displacement: $x(t) = 0.05\cos(\omega t)$
But given $x(t)$= 0.03m => 0.03 =0.05cos(ωt)
∴ cos(ωt) = 3/5 and hence sin(ωt)= 4/5 = 0.8
The velocity of the at x= 3 cm is v = Aωsin(ωt) = 0.05 × (31.42) × 0.8 = 1.257 m/s
(c) Displacement x=0 cm, which is the mean position.
At mean position acceleration is minimum and velocity is maximum.
∴ Minimum acceleration = $a_{min} = 0$
∴ Maximum velocity = $v_{max} = A\omega = 0.05 \times 31.42 = 1.571$ m/s

9. A block of mass is 1 kg is fastened to a spring. The spring has a . The block is pulled to a distance frictionless surface from rest at block when it is 5 cm away from the mean position.

Mass m = 1 kg, spring constant k = 50 Nm^{-1}, Amplitude A =10cm = 0.1m and x = 5 cm = 0.05 m.

Angular frequency: $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{50}{1}} = 7.071$ rad/s

Displacement: $x(t) = 0.1\cos(7.071t)$

But given $x(t)$ = 5 cm = 0.05m ⇒ 0.05 = = 0.1 cos (7.071t)

∴cos (7.071t) = 0.5 = ½ and hence $\sin(7.071t) = \frac{\sqrt{3}}{2} = 0.866$

The velocity of the at $x(t)$ = 5 cm is v=Aωsin (ωt) = 0.1 × (7.071) × 0.866 = 0.6123 m/s

Kinetic energy: $K = \frac{1}{2}mv^2 = \frac{1}{2}(1)(0.6123)^2 = 0.1874$ J

Potential energy: $U = \frac{1}{2}k\,x^2 = \frac{1}{2}(50)(0.05)^2 = 0.0625$ J

Total energy of the block: E= K+U = 0.1874+0.0625 = 0.2499 J

10. The piston in the cylinder head of a locomotive has a stroke (twice the amplitude) of 1.0 m. If the piston moves with simple harmonic motion with an angular frequencyof 200 rad/min, what is its maximum speed and maximum acceleration

Amplitude : A = 1.0 m^2 = 0.5 m, angular frequency:_ = 200 rad/min = 200rad/60s=10/3 rad/s,
The maximum speed: $v_m = A\omega = 0.5 \times 10/3 = 1.667$ m/s
The maximum acceleration: $a_m = A\,\omega^2 = 0.5 \times (10/3)^2 = 5.556$ m/s^2

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NCERT Textbook Problems

Question 14.1:

Which of the following examples represent periodic motion?

(a) A swimmer completing one (return) trip from one bank of a river to the other and back.

(b) A freely suspended bar magnet displaced from its N-S direction and released.

(c) A hydrogen molecule rotating about its center of mass.

(d) An arrow released from a bow.

Answer

(b) and **(c)**

(a) The swimmer's motion is not periodic. The motion of the swimmer between the banks of a river is back and forth. However, it does not have a definite period. This is because the time taken by the swimmer during his back and forth journey may not be the same.

(b) The motion of a freely-suspended magnet, if displaced from its N-S direction and released, is periodic. This is because the magnet oscillates about its position with a definite period of time.

(c) When a hydrogen molecule rotates about its centre of mass, it comes to the same position again and again after an equal interval of time. Such motion is periodic.

(d) An arrow released from a bow moves only in the forward direction. It does not come backward. Hence, this motion is not a periodic.

Question 14.2:

Which of the following examples represent (nearly) simple harmonic motion and which represent periodic but not simple harmonic motion?

(a) the rotation of earth about its axis.

(b) motion of an oscillating mercury column in a U-tube.

(c) motion of a ball bearing inside a smooth curved bowl, when released from a point slightly above the lower most point.

(d) general vibrations of a polyatomic molecule about its equilibrium position.

Answer

(b) and **(c)** are SHMs

(a) and **(d)** are periodic, but not SHMs

(a) During its rotation about its axis, earth comes to the same position again and again in equal intervals of time. Hence, it is a periodic motion. However, this motion is not simple harmonic. This is because earth does not have a to and fro motion about its axis.

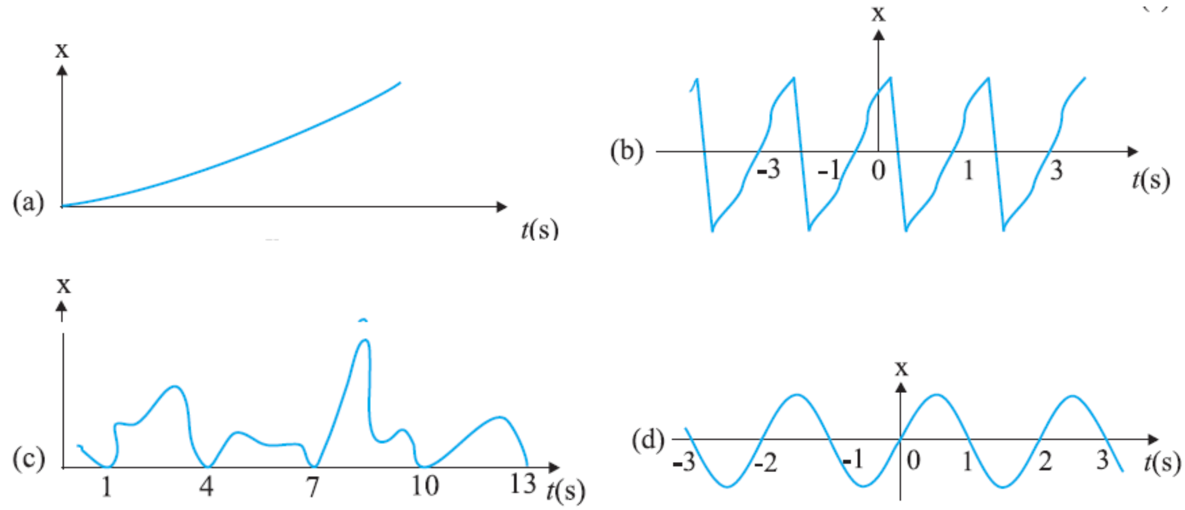
(b) An oscillating mercury column in a U-tube is simple harmonic. This is because the mercury moves to and fro on the same path, about the fixed position, with a certain period of time.

(c) The ball moves to and fro about the lowermost point of the bowl when released. Also, the ball comes back to its initial position in the same period of time, again and again. Hence, its motion is periodic as well as simple harmonic.

(d) A polyatomic molecule has many natural frequencies of oscillation. Its vibration is the superposition of individual simple harmonic motions of a number of different molecules. Hence, it is not simple harmonic, but periodic.

Question 14.3:

Following Figure depicts four *x*-*t* plots for linear motion of a particle. Which of the plots represent periodic motion? What is the period of motion (in case of periodic motion)?



Answer

(b) and **(d)** are periodic

(a) It is not a periodic motion. This represents a unidirectional, linear uniform motion. There is no repetition of motion in this case.

(b) In this case, the motion of the particle repeats itself after 2 s. Hence, it is a periodic motion, having a period of 2 s.

(c) It is not a periodic motion. This is because the particle repeats the motion in one position only. For a periodic motion, the entire motion of the particle must be repeated in equal intervals of time.

(d) In this case, the motion of the particle repeats itself after 2 s. Hence, it is a periodic motion, having a period of 2 s.

14.4 Which of the following functions of time represent (a) simple harmonic, (b) periodic but not simple harmonic, and (c) non-periodic motion? Give period for each case of periodic motion (ω is any positive constant) :

(a) $\sin \omega t - \cos \omega t$

(b) $\sin^3 \omega t$

(c) $3 \cos (\pi/4 - 2 \omega t)$

(d) $\cos \omega t + \cos 3 \omega t + \cos 5 \omega t$

(e) $\exp (-\omega^2 t^2)$

(f) $1 + \omega t + \omega^2 t^2$

Answer :

(a) SHM

The given function is:

$$\sin\omega t - \cos\omega t$$

$$= \sqrt{2}\left[\frac{1}{\sqrt{2}}\sin\omega t - \frac{1}{\sqrt{2}}\cos\omega t\right]$$

$$= \sqrt{2}\left[\sin\omega t \times \cos\frac{\pi}{4} - \cos\omega t \times \sin\frac{\pi}{4}\right]$$

$$= \sqrt{2}\sin\left(\omega t - \frac{\pi}{4}\right)$$

This function represents SHM as it can be written in the form: a sin ($\omega t + \phi$)
Its period is: $2\pi/\omega$

(b) Periodic, but not SHM
The given function is: $\sin^3\omega t = \frac{1}{2}[3\sin\omega t + \sin 3\omega t]$
The terms sinωt and sinωt individually represent simple harmonic motion (SHM). However, the superposition of two SHM is periodic and not simple harmonic.

(c) SHM
The given function is:

$$3\cos\left[\frac{\pi}{4} - 2\omega t\right]$$

$$= 3\cos\left[2\omega t - \frac{\pi}{4}\right]$$

This function represents simple harmonic motion because it can be written in the form:
a cos [$\omega t + \phi$]
Its period is: $2\pi/2\omega = \pi/\omega$

(d) Periodic, but not SHM
The given function is cos ωt + cos 3 ωt + cos 5 ωt. Each individual cosine function represents SHM. However, the superposition of three simple harmonic motions is periodic, but not simple harmonic.

(e) Non-periodic motion
The given function exp ($-\omega^2 t^2$) is an exponential function. Exponential functions do not repeat themselves. Therefore, it is a non-periodic motion.

(f) The given function $1 + \omega t + \omega^2 t^2$ is non-periodic.

Question 14.5:
A particle is in linear simple harmonic motion between two points, A and B, 10 cm apart. Take the direction from A to B as the positive direction and give the signs of velocity, acceleration and force on the particle when it is
(a) at the end A,

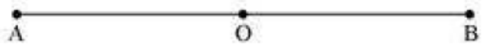
(b) at the end B,
(c) at the mid-point of AB going towards A,
(d) at 2 cm away from B going towards A,
(e) at 3 cm away from A going towards B, and
(f) at 4 cm away from B going towards A.

Answer

(a) Zero, Positive, Positive
(b) Zero, Negative, Negative
(c) Negative, Zero, Zero
(d) Negative, Negative, Negative
(e) Zero, Positive, Positive
(f) Negative, Negative, Negative

Explanation:

The given situation is shown in the following figure. Points A and B are the two end points, with AB = 10 cm. O is the midpoint of the path.

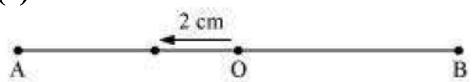


A particle is in linear simple harmonic motion between the end points

(a) At the extreme point A, the particle is at rest momentarily. Hence, its velocity is zero at this point.
Its acceleration is positive as it is directed along AO.
Force is also positive in this case as the particle is directed rightward.

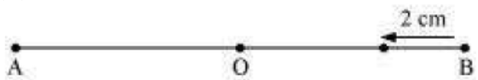
(b) At the extreme point B, the particle is at rest momentarily. Hence, its velocity is zero at this point.
Its acceleration is negative as it is directed along B.
Force is also negative in this case as the particle is directed leftward.

(c)



The particle is executing a simple harmonic motion. O is the mean position of the particle. Its velocity at the mean position O is the maximum. The value for velocity is negative as the particle is directed leftward. The acceleration and force of a particle executing SHM is zero at the mean position.

(d)

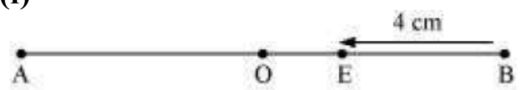


The particle is moving toward point O from the end B. This direction of motion is opposite to the conventional positive direction, which is from A to B. Hence, the particle's velocity and acceleration, and the force on it are all negative.

(e)

The particle is moving toward point O from the end A. This direction of motion is from A to B, which is the conventional positive direction. Hence, the values for velocity, acceleration, and force are all positive.

(f)



This case is similar to the one given in **(d)**.

Question 14.6:

Which of the following relationships between the acceleration a and the displacement x of a particle involve simple harmonic motion?

(a) $a = 0.7x$

(b) $a = -200x^2$

(c) $a = -10x$

(d) $a = 100x^3$

Answer

(c) A motion represents simple harmonic motion if it is governed by the force law:

$F = -kx$

$ma = -k$

$\therefore\ a = -\frac{k}{m}x$

Where,

F is the force

m is the mass (a constant for a body)

x is the displacement

a is the acceleration

k is a constant

Among the given equations, only equation $a = -10\,x$ is written in the above form with k/m =10. Hence, this relation represents SHM.

Question 14.7:

The motion of a particle executing simple harmonic motion is described by the displacement function, $x(t) = A\cos(\omega t + \varphi)$.

If the initial ($t = 0$) position of the particle is 1 cm and its initial velocity is ω cm/s, what are its amplitude and initial phase angle? The angular frequency of the particle is π s–1. If instead of the cosine function, we choose the sine function to describe the SHM: $x = B\sin(\omega t + \alpha)$, what are the amplitude and initial phase of the particle with the above initial conditions.

Answer

Initially, at $t = 0$:

Displacement, $x = 1$ cm

Initial velocity, $v = \omega$ cm/sec.

Angular frequency, $\omega = \pi$ rad/s^{-1}

It is given that:

$$x(t) = A\cos(\omega t + \phi)$$

$$1 = A\cos(\omega \times 0 + \phi) = A\cos\phi$$

$$A\cos\phi = 1 \qquad (i)$$

$$\text{Velocity}, v = \frac{dx}{dt}$$

$$\omega = -A\omega\sin(\omega t + \phi)$$
$$1 = -A\sin(\omega \times 0 + \phi) = -A\text{Sin}\phi$$
$$A\sin\phi = -1 \qquad (ii)$$

Squaring and adding equations (*i*) and (*ii*), we get:

$$A^2(\sin^2\phi + \cos^2\phi) = 1 + 1$$
$$A^2 = 2$$
$$\therefore A = \sqrt{2}\ \text{cm}$$

Dividing equation (*ii*) by equation (*i*), we get: $\tan\phi = -1$.

$$\therefore \phi = \frac{3\pi}{4}, \frac{7\pi}{4}, \ldots\ldots.$$

SHM is given as:

$$x = B\sin(\omega t + \alpha)$$

Putting the given values in this equation, we get:

$$1 = B\sin[\omega \times 0 + \alpha]$$
$$B\sin\alpha = 1 \qquad (iii)$$

Velocity, $v = \omega B\cos(\omega t + \alpha)$

Substituting the given values, we get:

$$\pi = \pi B\ \sin\alpha$$
$$B\sin\alpha = 1 \qquad (iv)$$

Squaring and adding equations (*iii*) and (*iv*), we get:

$$B^2[\sin^2\alpha + \cos^2\alpha] = 1 + 1$$
$$B^2 = 2$$
$$\therefore B = \sqrt{2}\ \text{cm}$$

Dividing equation (*iii*) by equation (*iv*), we get:

$$\frac{B\sin\alpha}{B\cos\alpha} = \frac{1}{1}$$

$$\tan\alpha = 1 = \tan\frac{\pi}{4}$$

$$\therefore \alpha = \frac{\pi}{4}, \frac{5\pi}{4}, \ldots\ldots\ldots$$

Question 14.8:

A spring balance has a scale that reads from 0 to 50 kg. The length of the scale is 20 cm. A body suspended from this balance, when displaced and released, oscillates with a period of 0.6 s. What is the weight of the body?

Answer

Maximum mass that the scale can read, $M = 50$ kg

Maximum displacement of the spring = Length of the scale, $l = 20$ cm = 0.2 m

Time period, $T = 0.6$ s

Maximum force exerted on the spring, $F = Mg$

Where,

g = acceleration due to gravity = 9.8 m/s^2

$F = 50 \times 9.8 = 490$

$\therefore$Spring constant, $k = \frac{F}{l} = \frac{490}{0.2} = 2450 \text{ N m}^{-1}$

Mass m, is suspended from the balance.

Time period, $T = 2\pi\sqrt{\frac{m}{k}}$

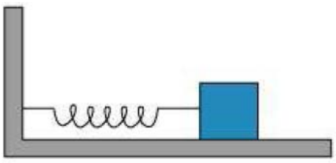
$$\therefore m = \left(\frac{T}{2\pi}\right)^2 \times k = \left(\frac{0.6}{2 \times 3.14}\right)^2 \times 2450 = 22.36 \text{ kg}$$

$\therefore$Weight of the body = $mg = 22.36 \times 9.8 = 219.167$ N

Hence, the weight of the body is about 219 N.

Question 14.9:

A spring having with a spring constant 1200 N m^{-1} is mounted on a horizontal table as shown in Fig. A mass of 3 kg is attached to the free end of the spring. The mass is then pulled sideways to a distance of 2.0 cm and released.



Determine (i) the frequency of oscillations, (ii) maximum acceleration of the mass, and (iii) the maximum speed of the mass.

Answer

Spring constant, $k = 1200$ N m^{-1}

Mass, $m = 3$ kg

Displacement, $A = 2.0$ cm = 0.02 cm

(i) Frequency of oscillation v, is given by the relation:

$$v = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Where, T is the time period

$$\therefore v = \frac{1}{2 \times 3.14}\sqrt{\frac{1200}{3}} = 3.18 \text{ m/s}$$

Hence, the frequency of oscillations is 3.18 m/s.

(ii) Maximum acceleration (a) is given by the relation: $a = \omega^2 A$

Where, ω = Angular frequency = $\sqrt{(k/m)}$

A = Maximum displacement

$$\therefore a = \frac{k}{m}A = \frac{1200 \times 0.02}{3} = 8 \text{ m s}^{-2}$$

Hence, the maximum acceleration of the mass is 8.0 m/s^2.

(iii) Maximum velocity, $v_{max} = A\omega$

$$= A\sqrt{\frac{k}{m}} = 0.02 \times \sqrt{\frac{1200}{3}} = 0.4 \text{ m/s}$$

Hence, the maximum velocity of the mass is 0.4 m/s.

Question 14.10:

In Exercise 14.9, let us take the position of mass when the spring is unstreched as x =0, and the direction from left to right as the positive direction of x-axis. Give x as afunction of time t for the oscillating mass if at the moment we start the stopwatch (t =0), the mass is

(a) at the mean position,

(b) at the maximum stretched position, and

(c) at the maximum compressed position.

In what way do these functions for SHM differ from each other, in frequency, in amplitude or the initial phase?

Answer

(a) x = 2sin 20t

(b) x = 2cos 20t

(c) x = –2cos 20t

The functions have the same frequency and amplitude, but different initial phases.

Distance travelled by the mass sideways, A = 2.0 cm

Force constant of the spring, k = 1200 N m^{-1}

Mass, m = 3 kg

Angular frequency of oscillation:

$$\omega = \sqrt{\frac{k}{m}} \qquad = \sqrt{\frac{1200}{3}} = \sqrt{400} \qquad = 20 \text{ rad s}^{-1}$$

(a) When the mass is at the mean position, initial phase is 0.

Displacement, $x = A\sin\omega t$ = 2sin 20t

(b) At the maximum stretched position, the mass is toward the extreme right. Hence, the initial phase is $\pi/2$

Displacement,

$$x = A\sin\left(\omega t + \frac{\pi}{2}\right) \quad = 2\sin\left(20t + \frac{\pi}{2}\right) \quad = 2\cos 20t$$

(c) At the maximum compressed position, the mass is toward the extreme left. Hence, the initial phase is 3π/2.

Displacement,

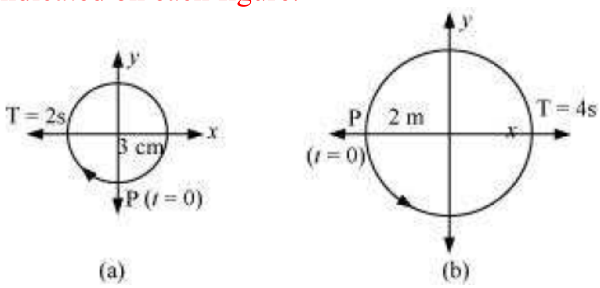
$$x = A\sin\left(\omega t + \frac{3\pi}{2}\right)$$

$$= 2\sin\left(20t + \frac{3\pi}{2}\right) = -2\cos 20t$$

The functions have the same frequency (20/2π Hertz) and amplitude (2 cm), but different initial phases (0, π/2. 3π/2 ....).

Question 14.11:

Figures 14.29 correspond to two circular motions. The radius of the circle, the period of revolution, the initial position, and the sense of revolution (i.e. clockwise or anticlockwise) are indicated on each figure.



Obtain the corresponding simple harmonic motions of the *x*-projection of the radius vector of the revolving particle P, in each case.

Answer

(a) Time period, $T = 2$ s

Amplitude, $A = 3$ cm

At time, $t = 0$, the radius vector OP makes an angle π/2 with the positive *x*-axis, i.e., phase angle $\phi = +\pi/2$.

Therefore, the equation of simple harmonic motion for the *x*-projection of OP, at time *t*, is given by the displacement equation:

$$x = A\cos\left[\frac{2\pi t}{T} + \phi\right]$$

$$= 3\cos\left(\frac{2\pi t}{2} + \frac{\pi}{2}\right) = -3\sin\left(\frac{2\pi t}{2}\right)$$

$$\therefore x = -3\sin \pi t \text{ cm}$$

(b) Time period, $T = 4$ s

Amplitude, $a = 2$ m

At time $t = 0$, OP makes an angle π with the *x*-axis, in the anticlockwise direction.

Hence, phase angle, $\Phi = +\pi$

Therefore, the equation of simple harmonic motion for the x-projection of OP, at time t, is given as:

$$x = a\cos\left(\frac{2\pi t}{T} + \phi\right) = 2\cos\left(\frac{2\pi t}{4} + \pi\right)$$

$$\therefore x = -2\cos\left(\frac{\pi}{2}t\right) \text{ m}$$

Question 14.12:

Plot the corresponding reference circle for each of the following simple harmonic motions. Indicate the initial ($t = 0$) position of the particle, the radius of the circle, and the angular speed of the rotating particle. For simplicity, the sense of rotation may be fixed to be anticlockwise in every case: (x is in cm and t is in s).

(a) $x = -2 \sin (3t + \pi/3)$

(b) $x = \cos (\pi/6 - t)$

(c) $x = 3 \sin (2\pi t + \pi/4)$

(d) $x = 2 \cos \pi t$

Answer

(a)

$$x = -2\sin\left(3t + \frac{\pi}{3}\right) = +2\cos\left(3t + \frac{\pi}{3} + \frac{\pi}{2}\right) = 2\cos\left(3t + \frac{5\pi}{6}\right)$$

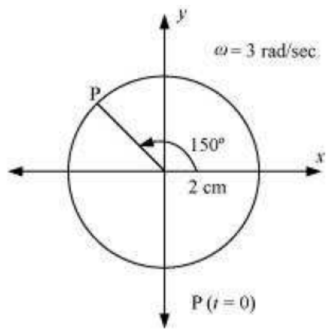
If this equation is compared with the standard SHM equation then

$x = A\cos\left(\frac{2\pi}{T}t + \phi\right)$, we get: Amplitude A = 2 cm

Phase angle, $\phi = 5\pi/6 = 150°$

Angular velocity, $\omega = 2\pi/T = 3$ rad/sec.

The motion of the particle can be plotted as shown in the following figure.



(b)

$$x = \cos\left(\frac{\pi}{6} - t\right) = \cos\left(t - \frac{\pi}{6}\right)$$

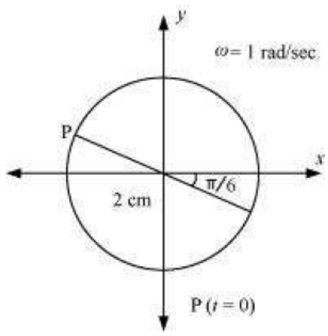
If this equation is compared with the standard SHM equation

$x = A\cos\left(\frac{2\pi}{T}t + \phi\right)$, then we get : Amplitude A = 1 cm

Phase angle, $\phi = -\pi/6 = -30°$

Angular velocity, $\omega = 2\pi/T = 1$ rad/sec.

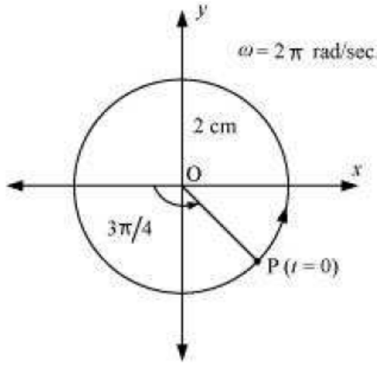
The motion of the particle can be plotted as shown in the following figure.



(c)

$$x = 3\sin\left(2\pi t + \frac{\pi}{4}\right)$$

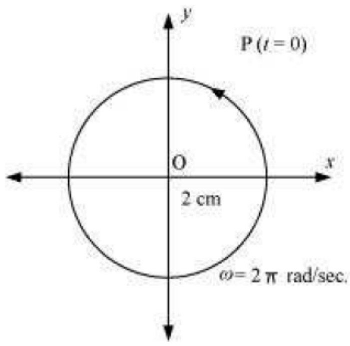
$$= -3\cos\left[\left(2\pi t + \frac{\pi}{4}\right) + \frac{\pi}{2}\right] = -3\cos\left(2\pi t + \frac{3\pi}{4}\right)$$



If this equation is compared with the standard SHM equation

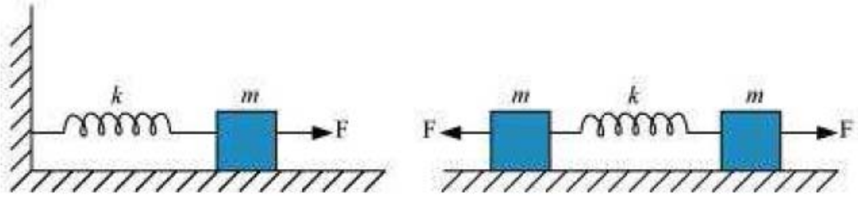
$$x = A\cos\left(\frac{2\pi}{T}t + \phi\right)$$

then, we get:
Amplitude, $A = 3$ cm
Phase angle, $\phi = 3\pi/4 = 135°$
Angular velocity, $\omega = 2\pi/T = 2\pi$ rad/sec.
The motion of the particle can be plotted as shown in the following figure.



(d) $x = 2 \cos \pi t$
If this equation is compared with the standard SHM equation $x = A\cos\left(\frac{2\pi}{T}t + \phi\right)$, then we get:
Amplitude, $A = 2$ cm
Phase angle, $\Phi = 0$
Angular velocity, $\omega = \pi$ rad/s
The motion of the particle can be plotted as shown in the following figure.

Question 14.13:
Figure 14.30 (a) shows a spring of force constant k clamped rigidly at one end and a mass m attached to its free end. A force F applied at the free end stretches the spring. Figure 14.30 (b) shows the same spring with both ends free and attached to a mass m at either end. Each end of the spring in Fig. 14.30(b) is stretched by the same force F.



(a) What is the maximum extension of the spring in the two cases?
(b) If the mass in Fig. (a) and the two masses in Fig. (b) are released, what is the period of oscillation in each case?
Answer
(a) For the one block system:
When a force F, is applied to the free end of the spring, an extension l, is produced. For the maximum extension, it can be written as:
$F = kl$
Where, k is the spring constant
Hence, the maximum extension produced in the spring, $l = F/k$
For the two block system:
The displacement (x) produced in this case is: $x = l/2$
Net force, $F = +2\ kx = 2k\ (l/2)$ $\therefore l = F/k$
(b) For the one block system:

For mass (m) of the block, force is written as:

$$F = ma = m\frac{d^2x}{dt^2}$$

Where, x is the displacement of the block in time t

$$\therefore m\frac{d^2x}{dt^2} = -kx$$

It is negative because the direction of elastic force is opposite to the direction of displacement.

$$\frac{d^2x}{dt^2} = -\left(\frac{k}{m}\right)x = -\omega^2 x \qquad \text{Where, } \omega^2 = \frac{k}{m}$$

$$\omega = \sqrt{\frac{k}{m}}$$ Where, ω is angular frequency of the oscillation,

∴Time period of the oscillation,

$$T = \frac{2\pi}{\omega} \qquad = \frac{2\pi}{\sqrt{\frac{k}{m}}} = 2\pi\sqrt{\frac{m}{k}}$$

For the two block system:

$$F = m\frac{d^2x}{dt^2} \qquad or \qquad m\frac{d^2x}{dt^2} = -2kx$$

It is negative because the direction of elastic force is opposite to the direction of displacement.

$$\frac{d^2x}{dt^2} = -\left[\frac{2k}{m}\right]x = -\omega^2 x$$

Where, Angular frequency, $\omega = \sqrt{\frac{2k}{m}}$

∴Time period,

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{2k}}$$

Question 14.14:

The piston in the cylinder head of a locomotive has a stroke (twice the amplitude) of 1.0 m. If the piston moves with simple harmonic motion with an angular frequency of 200 rad/min, what is its maximum speed?

Answer

Angular frequency of the piston, ω = 200 rad/ min.

Stroke = 1.0 m

Amplitude, A = 1.0/2 = 0.5 m

The maximum speed ($v$max) of the piston is give by the relation:

$$v_{\max} = A\omega$$
$$= 200 \times 0.5 = 100 \text{ m/min}$$

Question 14.15:
The acceleration due to gravity on the surface of moon is 1.7 ms^{-2}. What is the time period of a simple pendulum on the surface of moon if its time period on the surface of earth is 3.5 s? (*g* on the surface of earth is 9.8 ms^{-2})
Answer
Acceleration due to gravity on the surface of moon, = 1.7 m s^{-2}
Acceleration due to gravity on the surface of earth, g = 9.8 m s^{-2}
Time period of a simple pendulum on earth, T = 3.5 s

$$T = 2\pi\sqrt{\frac{l}{g}}$$

Where,
l is the length of the pendulum

$$\therefore l = \frac{T^2}{(2\pi)^2} \times g \qquad = \frac{(3.5)^2}{4 \times (3.14)^2} \times 9.8 \text{ m}$$

The length of the pendulum remains constant.
On moon's surface, time period,

$$T' = 2\pi\sqrt{\frac{l}{g'}} \qquad = 2\pi\sqrt{\frac{\frac{(3.5)^2}{4 \times (3.14)^2} \times 9.8}{1.7}} = 8.4 \text{ s}$$

Hence, the time period of the simple pendulum on the surface of moon is 8.4 s.

Question 14.16:
Answer the following questions:
(a) Time period of a particle in SHM depends on the force constant k and mass m of the particle: $T = 2\pi\sqrt{(m/k)}$
A simple pendulum executes SHM approximately. Why then is the time period of a pendulum independent of the mass of the pendulum?
(b) The motion of a simple pendulum is approximately simple harmonic for small angle oscillations. For larger angles of oscillation, a more involved analysis shows that T is greater than $2\pi\sqrt{(l/g)}$. Think of a qualitative argument to appreciate this result.
(c) A man with a wristwatch on his hand falls from the top of a tower. Does the watch give correct time during the free fall?
(d) What is the frequency of oscillation of a simple pendulum mounted in a cabin that is freely falling under gravity?
Answer
(a) The time period of a simple pendulum, $T = 2\pi\sqrt{(m/k)}$
For a simple pendulum, k is expressed in terms of mass m, as:
$k \propto m$ *or* m/k = Constant
Hence, the time period T, of a simple pendulum is independent of the mass of the bob.
(b) In the case of a simple pendulum, the restoring force acting on the bob of the pendulum is given as:

$F = -mg\sin\theta$
Where,
F = Restoring force
m = Mass of the bob
g = Acceleration due to gravity
θ = Angle of displacement
For small θ, $\sin\theta \approx \theta$
For large θ, $\sin\theta$ is greater than θ.
This decreases the effective value of g.
Hence, the time period increases as: $T = 2\pi\sqrt{(l/g)}$
Where, l is the length of the simple pendulum

(c) The time shown by the wristwatch of a man falling from the top of a tower is not affected by the fall. Since a wristwatch does not work on the principle of a simple pendulum, it is not affected by the acceleration due to gravity during free fall. Its working depends on spring action.

(d) When a simple pendulum mounted in a cabin falls freely under gravity, its acceleration is zero. Hence the frequency of oscillation of this simple pendulum is zero.

Question 14.17:
A simple pendulum of length l and having a bob of mass M is suspended in a car. The car is moving on a circular track of radius R with a uniform speed v. If the pendulum makes small oscillations in a radial direction about its equilibrium position, what will be its time period?
Answer
The bob of the simple pendulum will experience the acceleration due to gravity and the centripetal acceleration provided by the circular motion of the car.
Acceleration due to gravity = g
Centripetal acceleration = v^2/R
Where, v is the uniform speed of the car, R is the radius of the track
Effective acceleration ($a$eff) is given as:

$$a_{\text{eff}} = \sqrt{g^2 + \left(\frac{v^2}{R}\right)^2}$$

Time period,

$$T = 2\pi\sqrt{\frac{l}{a_{\text{eff}}}}$$

Where, l is the length of the pendulum
∴Time period, T =

$$= 2\pi\sqrt{\frac{l}{\sqrt{g^2 + \frac{v^4}{R^2}}}}$$

Question 14.18:
Cylindrical piece of cork of density of base area A and height h floats in a liquid of density ρ_i. The cork is depressed slightly and then released. Show that the cork oscillates up and down simple harmonically with a period

$T = 2\pi\sqrt{\frac{h\rho}{\rho_l g}}$ where ρ is the density of cork. (Ignore damping due to viscosity of the liquid).

Answer
Base area of the cork = A
Height of the cork = h
Density of the liquid = ρ_i
Density of the cork = ρ
In equilibrium:
Weight of the cork = Weight of the liquid displaced by the floating cork
Let the cork be depressed slightly by x. As a result, some extra water of a certain volume is displaced. Hence, an extra up-thrust acts upward and provides the restoring force to the cork.
Up-thrust = Restoring force, F = Weight of the extra water displaced
$F = -(\text{Volume} \times \text{Density} \times g)$
Volume = Area × Distance through which the cork is depressed
Volume = Ax
$\therefore F = -A\,x\,\rho_i\,g \ldots (i)$
According to the force law:
$F = kx \quad or \quad k = F/x$
Where, k is a constant
$k = F/x \; = -A\,\rho_i\,g \ldots (ii)$
The time period of the oscillations of the cork:
$T = 2\pi\sqrt{(m/k)}$ ------ *(iii)*
Where,
m = Mass of the cork
= Volume of the cork × Density
= Base area of the cork × Height of the cork × Density of the cork
= $Ah\rho$
Hence, the expression for the time period becomes:

$$T = 2\pi\sqrt{\frac{Ah\rho}{A\rho_l g}} = 2\pi\sqrt{\frac{h\rho}{\rho_l g}}$$

Question 14.19:
One end of a U-tube containing mercury is connected to a suction pump and the other end to atmosphere. A small pressure difference is maintained between the two columns. Show that, when the suction pump is removed, the column of mercury in the U-tube executes simple harmonic motion.
Answer
Area of cross-section of the U-tube = A
Density of the mercury column = ρ
Acceleration due to gravity = g
Restoring force, F = Weight of the mercury column of a certain height
$F = -(\text{Volume} \times \text{Density} \times g)$
$F = -(A \times 2h \times \rho \times g) = -2A\rho gh = -k \times$ Displacement in one of the arms (h)
Where, $2h$ is the height of the mercury column in the two arms

k is a constant, given by k = - (F/h) = 2Aρg
Time period,

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{2A\rho g}}$$

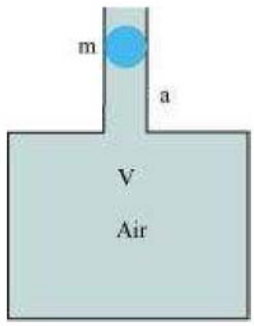
Where,
m is the mass of the mercury column
Let *l* be the length of the total mercury in the U-tube.
Mass of mercury, *m* = Volume of mercury × Density of mercury
= *Alρ*

$$\therefore T = 2\pi\sqrt{\frac{Al\rho}{2A\rho g}} = 2\pi\sqrt{\frac{l}{2g}}$$

Hence, the mercury column executes simple harmonic motion with time period

$$2\pi\sqrt{\frac{l}{2g}}.$$

Question 14.20:
An air chamber of volume *V* has a neck area of cross section *a* into which a ball of mass *m* just fits and can move up and down without any friction (Fig,). Show that when the ball is pressed down a little and released, it executes SHM. Obtain an expression for the time period of oscillations assuming pressure-volume variations of air to be isothermal [see Fig.



Answer
Volume of the air chamber = *V*
Area of cross-section of the neck = *a*
Mass of the ball = *m*
The pressure inside the chamber is equal to the atmospheric pressure.
Let the ball be depressed by *x* units. As a result of this depression, there would be a decrease in the volume and an increase in the pressure inside the chamber.
Decrease in the volume of the air chamber, R*V* = *ax*
Volumetric strain = (Change in volume /Original volume)

$$\Rightarrow \frac{\Delta V}{V} = \frac{ax}{V} \qquad B = \frac{\text{Stress}}{\text{Strain}} = \frac{-p}{\frac{ax}{V}}$$

Bulk Modulus of air,
In this case, stress is the increase in pressure. The negative sign indicates that pressure increases with a decrease in volume.

$$p = \frac{-Bax}{V}$$

The restoring force acting on the ball, $F = p \times a$

$$= \frac{-Bax}{V} \cdot a \qquad = \frac{-Ba^2 x}{V} \qquad \dots (i)$$

In simple harmonic motion, the equation for restoring force is:

$F = -kx$ … (ii)
Where, k is the spring constant
Comparing equations (i) and (ii), we get:

$$k = \frac{Ba^2}{V}$$

Time period,

$$T = 2\pi\sqrt{\frac{m}{k}} \qquad = 2\pi\sqrt{\frac{Vm}{Ba^2}}$$

Question 14.21:

You are riding in an automobile of mass 3000 kg. Assuming that you are examining the oscillation characteristics of its suspension system. The suspension sags 15 cm when the entire automobile is placed on it. Also, the amplitude of oscillation decreases by 50% during one complete oscillation. Estimate the values of (a) the spring constant k and (b) the damping constant b for the spring and shock absorber system of one wheel, assuming that each wheel supports 750 kg.

Answer

(a) Mass of the automobile, m = 3000 kg
Displacement in the suspension system, x = 15 cm = 0.15 m
There are 4 springs in parallel to the support of the mass of the automobile.
The equation for the restoring force for the system:
$F = -4kx = mg$
Where, k is the spring constant of the suspension system
Time period,

$$T = 2\pi\sqrt{\frac{m}{4k}}$$

And

$$k = \frac{mg}{4x} = \frac{3000 \times 10}{4 \times 0.15} = 5000 = 5 \times 10^4 \text{ N/m}$$

Spring constant, $k = 5 \times 10^4$ N/m

(b) Each wheel supports a mass, M = 3000/4 = 750 kg
For damping factor b, the equation for displacement is written as:

$$x = x_0 e^{-bt/2M}$$

The amplitude of oscillation decreases by 50%.

$$\therefore \; x = \frac{x_0}{2} \quad \Rightarrow \quad \frac{x_0}{2} = x_0 e^{-bt/2M}$$

$$\log_e 2 = \frac{bt}{2M}$$

$$\therefore b = \frac{2M \log_e 2}{t}$$

Where,

Time period,

$$t = 2\pi\sqrt{\frac{m}{4k}} = 2\pi\sqrt{\frac{3000}{4 \times 5 \times 10^4}} = 0.7691 \text{ s}$$

$$\therefore b = \frac{2 \times 750 \times 0.693}{0.7691} = 1351.58 \text{ kg/s}$$

Therefore, the damping constant of the spring is 1351.58 kg/s.

Question 14.22:

Show that for a particle in linear SHM the average kinetic energy over a period of oscillation equals the average potential energy over the same period.

Answer

The equation of displacement of a particle executing SHM at an instant *t* is given as:

x = A sin ωt

Where,

A = Amplitude of oscillation

ω = Angular frequency = √(k/m)

The velocity of the particle is:

$$v = \frac{dx}{dt} = A\omega\cos\omega t$$

The kinetic energy of the particle is:

$$E_k = \frac{1}{2}Mv^2 = \frac{1}{2}MA^2\omega^2\cos^2\omega t$$

The potential energy of the particle is:

$$E_p = \frac{1}{2}kx^2 = \frac{1}{2}M\omega^2A^2\sin^2\omega t$$

For time period *T*, the average kinetic energy over a single cycle is given as:

$$(E_k)_{\text{Avg}} = \frac{1}{T}\int_0^T E_k\, dt$$

$$= \frac{1}{T}\int_0^T \frac{1}{2}MA^2\omega^2\cos^2\omega t\, dt$$

$$= \frac{1}{2T}MA^2\omega^2\int_0^T \frac{(1+\cos 2\omega t)}{2}dt$$

$$= \frac{1}{4T}MA^2\omega^2\left[t + \frac{\sin 2\omega t}{2\omega}\right]_0^T = \frac{1}{4T}MA^2\omega^2(T)$$

$$= \frac{1}{4}MA^2\omega^2 \qquad \dots (i)$$

And, average potential energy over one cycle is given as:

$$(E_p)_{\text{Avg}} = \frac{1}{T}\int_0^T E_p\, dt$$

$$= \frac{1}{T}\int_0^T \frac{1}{2} M\omega^2 A^2 \sin^2 \omega t \, dt$$

$$= \frac{1}{2T} M\omega^2 A^2 \int_0^T \frac{(1-\cos 2\omega t)}{2} \, dt$$

$$= \frac{1}{4T} M\omega^2 A^2 \left[t - \frac{\sin 2\omega t}{2\omega} \right]_0^T \qquad = \frac{1}{4T} M\omega^2 A^2 (T)$$

$$= \frac{M\omega^2 A^2}{4} \qquad \ldots (ii)$$

It can be inferred from equations (*i*) and (*ii*) that the average kinetic energy for a given time period is equal to the average potential energy for the same time period.

Question 14.23:

A circular disc of mass 10 kg is suspended by a wire attached to its centre. The wire is twisted by rotating the disc and released. The period of torsional oscillations is found to be 1.5 s. The radius of the disc is 15 cm. Determine the torsional spring constant of the wire. (Torsional spring constant α is defined by the relation $J = -\alpha\, \theta$, where J is the restoring couple and θ the angle of twist).

Answer

Mass of the circular disc, m = 10 kg

Radius of the disc, r = 15 cm = 0.15 m

The torsional oscillations of the disc has a time period, T = 1.5 s

The moment of inertia of the disc is:

$I = \frac{1}{2} mr^2 \quad = 1/2\,(10)\,(0.15)^2 \quad = 0.1125$ kg m^2

Time period, $T = 2\pi\sqrt{(I/\alpha)}$

α is the torsional constant.

$$\alpha = \frac{4\pi^2 I}{T^2} \qquad = \frac{4 \times (\pi)^2 \times 0.1125}{(1.5)^2}$$

= 1.972 Nm/rad

Hence, the torsional spring constant of the wire is 1.972 Nm rad^{-1}.

Question 14.24:

A body describes simple harmonic motion with amplitude of 5 cm and a period of 0.2 s.

Find the acceleration and velocity of the body when the displacement is (a) 5 cm, (b) 3 cm, (c) 0 cm.

Answer

Amplitude, A = 5 cm = 0.05 m

Time period, T = 0.2 s

(a) For displacement, x = 5 cm = 0.05 m

Acceleration is given by:

$a = -\omega^2 x$

$= -\left(\frac{2\pi}{T}\right)^2 x$

$= -\left(\frac{2\pi}{0.2}\right)^2 \times 0.05$

$= -5\pi^2 \text{ m/s}^2$

Velocity is given by:

$v = \omega\sqrt{A^2 - x^2}$ $= \frac{2\pi}{\mathrm{T}}\sqrt{(0.05)^2 - (0.05)^2}$ $= 0$

When the displacement of the body is 5 cm, its acceleration is $-5\pi^2$ m/s^2 and velocity is 0.

(b) For displacement, $x = 3$ cm $= 0.03$ m

Acceleration is given by:

$a = -\omega^2 x$

$= -\left(\frac{2\pi}{T}\right)^2 x$

$= -\left(\frac{2\pi}{0.2}\right)^2 0.03$

$= -3\pi^2 \text{ m/s}^2$

Velocity is given by:

$v = \omega\sqrt{A^2 - x^2}$ $= \frac{2\pi}{T}\sqrt{A^2 - x^2}$ $= \frac{2\pi}{T}\sqrt{(0.05)^2 - (0.03)^2}$

$= \frac{2\pi}{0.2} \times 0.04$

$= 0.4\,\pi$ m/s

When the displacement of the body is 3 cm, its acceleration is -3π m/s2 and velocity is 0.4π m/s.

(c) For displacement, $x = 0$

Acceleration is given by:

$a = -\omega^2 x = 0$

Velocity is given by:

$v = \omega\sqrt{A^2 - x^2}$

$= \frac{2\pi}{T}\sqrt{A^2 - x^2}$

$= \frac{2\pi}{0.2}\sqrt{(0.05)^2 - 0}$ $= 0.5\pi \text{ m/s}$

When the displacement of the body is 0, its acceleration is 0 and velocity is $0.5\,\pi$ m/s.

Question 14.25:

A mass attached to a spring is free to oscillate, with angular velocity ω, in a horizontal plane without friction or damping. It is pulled to a distance *x*0 and pushed towards the centre with a velocity *v*0 at time *t* = 0. Determine the amplitude of the resulting oscillations in terms of the parameters ω, *x*0 and *v*0. [Hint: Start with the equation *x* = *a* cos (*ωt*+*θ*) and note that the initial velocity is negative.]

Answer

The displacement equation for an oscillating mass is given by: x = A cos (ωt + ϕ)

Where,

A is the amplitude

x is the displacement

θ is the phase constant

Velocity,

$$v = \frac{dx}{dt} = -A\omega \sin(\omega t + \theta)$$

At $t = 0$, $x = x_0$

$x_0 = A\cos\theta = x_0$ … (i)

And, $\frac{dx}{dt} = -v_0 = A\omega \sin\theta$

$A \sin\theta = \frac{v_0}{\omega}$ … (*ii*)

Squaring and adding equations (*i*) and (*ii*), we get:

$$A^2(\cos^2\theta + \sin^2\theta) = x_0^2 + \left(\frac{v_0^2}{\omega^2}\right)$$

$$\therefore A = \sqrt{x_0^2 + \left(\frac{v_0}{\omega}\right)^2}$$

Hence, the amplitude of the resulting oscillation is

$$\sqrt{x_0^2 + \left(\frac{v_0}{\omega}\right)^2}.$$

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PRACTICE QUESTIONS

1) An oscillator having a mass of 0.2 kg is executing simple harmonic oscillations. After 100 oscillations the amplitude becomes 1 / 7th of the original value. Find damping coefficient of this motion, if its periodic time is 2 seconds. [March, 2003]
(Ans: 3.89 dyne-sec /cm)

2) An oscillator having a mass of 100 gm is executing damped oscillations. After 1500 oscillations, the amplitude reduces to half of its original value. Find the damping coefficient if its time period is 2.22 seconds. [March, 2001]
(Ans: 0.042 dyne-sec /cm)

3) For damped oscillations, find the time for decrease of the amplitude to A / 32. Mass of oscillator = 200 gm and resistive force coefficient = 0.1 dyne-sec /cm [March, 2001]
(Ans: 3.85 hour)

4) In a simple harmonic motion, the velocities of a particle are 10 cm /s and 24 cm /s when its displacements are 12 cm and 5 cm respectively. Calculate its periodic time and amplitude. [October, 1998]
(Ans: 3.14 sec, 13 cm)

5) Find the resistive force on a body when its velocity is 7200 km/hr in a medium of damping constant 10 –3 N-s /m. [March, 1998]
(Ans: 2 N)

6) A body of mass 'm' is suspended at the end of a spring. The periodic time of the body is 4 seconds. If the mass is increased by 4 kg, the increase in periodic time is found to be 2 seconds. Calculate the mass 'm'. [October, 1997; October, 1991]
(Ans: 3.2 kg)

7) Angular frequency of S. H. oscillator is 2 rad /s. Initial displacement is 5 cm and initial velocity is 10 cm /s. Give equation for this S. H. M. [March, 1997]
[Ans : $5\sqrt{2}\sin(2t + \pi / 4)$ cm]

8) Find frequency of necessary external oscillating force to produce resonance in the seconds pendulum. [March, 1996]
(Ans: 0.5 Hz)

9) For a S. H. O., mass, length of path of oscillation, frequency and initial phase are 0.5 kg, 10 cm, 60 oscillations /minute and π / 3 radian respectively. Write an equation of displacement at time t. Find its force constant and mechanical energy. [October, 1995]
(Ans: y = 0.1 sin (2 πt + π / 3) m, 19.72 N /m, 0.025 joule)

10) A particle executes S. H. M. on a line 4 cm long. Its velocity when passing through the centre of the line is 12 cm /s. Find the period. [March, 1995]
(Ans: 1.05 seconds)

11) When and where will the potential and kinetic energy of an oscillator become equal starting its motion from mean position along Y-axis and having periodic time 4 seconds and amplitude 10 cm. [October, 1994]
(Ans: (i) after 0.5 second at the earliest, (ii) at 7.07 cm on either side of the mean position)

12) A particle of 10 gm executes S. H. M. of periodic time 6 seconds. One second after it has passed through its mean position, its velocity is found to be 6 cm /s. Calculate its mechanical energy. [March, 1994]
(Ans: 720 erg)

13) In a S. H. M., the value of maximum velocity is 10 m /s and the value of maximum acceleration is 20 π m /s 2. Find the periodic time. [March, 1993]
(Ans: 1 second)

14) The acceleration of a particle executing S. H. M. is - 20 π 2 cm / s 2. Its time period is 0.4 second and its amplitude is 20 mm. Calculate the velocity and displacement.
(Ans: 28.8 cm /s, 0.8 cm) [October, 1992]

15) A simple harmonic oscillator starts motion from the mean position. Its periodic time is 8 sec. Find the time at the end of which its kinetic energy will become equal to half of its mechanical energy. [March, 1992]
(Ans: earliest after 1 second from the start of motion)

16) A glass tube of U-shape is partly filled with mercury of density d. Height of mercury column in each limb is L. If free surface of mercury in one of the limbs is made to oscillate by giving displacement y, prove that the oscillations are simple harmonic.
[October, 1990]

17) The acceleration of a particle is 150 m/s 2 when its displacement in a S. H. M. is 15 mm.

Calculate its periodic time and frequency. [October, 1989]
(Ans: 0.063 s, 16 Hz)

18) A particle executing a S. H. M. has a maximum displacement of 4 cm and its acceleration at a distance of 1 cm from its mean position is 3 cm /s 2. What will be its velocity when it is at a distance of 2 cm from its mean position ? [March, 1989]
(Ans: 6 cm /s)

19) The equation of velocity of a simple harmonic oscillator is v = 6 π cos (πt + π/6). Find the equations for its displacement and acceleration. [March, 1988]
[Ans: y = 6 sin (πt + π/6), a = - 6 π 2 sin (πt + π/6)]

20) The acceleration of the particle executing S. H. M. is π 2 / 200 m /s 2, when its displacement is 0.02 m. Determine its period. [October, 1986]
(Ans: 4 seconds)

21) For a simple harmonic oscillator, the value of amplitude, mass and angular frequency are respectively 10 cm, 400 gm and 6 Hz. Find the force constant of the spring.
(Ans: 568 N / m) (Note: It should be frequency and not angular frequency.) [May, 1986]

22) When displacements of an oscillator performing S. H. M. are y 1 and y 2, its velocities are v 1 and v 2 respectively. Prove that its amplitude and periodic time are given by

$$A = \left[\frac{v_1^2 y_2^2 - v_2^2 y_1^2}{v_1^2 - v_2^2}\right]^{\frac{1}{2}} \quad \text{and} \quad T = 2\pi\left[\frac{y_2^2 - y_1^2}{v_1^2 - v_2^2}\right]^{\frac{1}{2}}$$

23) The displacement in cm of an oscillator performing S. H. M. at instant 't' is given by y = 5 sin (10 πt + π / 6). Find (i) amplitude, (ii) periodic time, (iii) initial phase, (iv) displacement after 0.2 second, (v) velocity at 0.2 second and (vi) acceleration after 0.2 second.
(Ans: (i) 5 cm, (ii) 0.2 s, (iii) π / 6, (iv) 2.5 cm, (v) 136 cm / s, (vi) 24.65 m / s^2)

24) The maximum velocity of an oscillator performing S. H. M. is 3 20 cm / s. What will be its velocity when it is midway between mid-position and positive end.
(Ans: 30 cm / s)

25) Find initial phase and amplitude for simple harmonic motion represented by an equation

y = 3 cos ωt + 4 sin ωt.
(Ans: 36 0 52, 5 units)

26) A wooden rod of mass M and cross section A floats vertically in the liquid of density d. Its centre of mass is inside the liquid surface. If it is slightly pressed down and released, prove that it performs S. H. M. Find its periodic time.
[Ans: T = 2 π (M / Adg) 1 / 2]

27) A S. H. O. oscillates with periodic time of 6 seconds. Find the time elapsed when its phase changes from π / 6 to 5 π / 6.
(Ans: 2 seconds)

28) If a block of mass 100 gm is suspended at the end of an elastic spring then its length increases by 9.8 cm. From this equilibrium condition, it is displaced upwards by 10 cm at time t = 0 and released. Obtain the displacement equation of its S. H. M.
[Ans: y = 0.1 sin (10t + π /2)]

29) A block of mass 1 Kg is suspended to a spring having spring constant 100 N /m. It performs damped oscillations in a medium having damping coefficient 12 Kg /s. Find its angular frequency of oscillations.
(Ans: 8.0 rad /s)

30) One S. H. O. with mass 0.5 kg performs damped S. H. M. in a medium having damping coefficient 75 g /s. Find out the time taken for the amplitude to reduce by 25 % of initial amplitude. Also find the time at the end of which the mechanical energy becomes one fourth its initial value.
(Ans: 3.84 s, 9.24 s)

31) For a damped S. H. O., m = 2 kg, k = 10 N / m. If its amplitude becomes 3 /4 th of its initial value at the end of 4 oscillations, find the damping coefficient for this S. H. O. Consider b / 2m << k / m.
(Ans: 0.102 kg / s)

32) Derive differential equation of damped oscillations from its solution.

OSCILLATIONS MCQ

1. The length of second's pendulum on the surface of earth is 1 m. the length of same pendulum on the surface of moon, where acceleration due to gravity is $(1/6)^{th}$ of the g on the surface of earth is **(NCERT 71)**
(a) 36 m (b) 1 m
(c) $\frac{1}{36}$ (d) $\frac{1}{6}m$

2. A mass M is suspended from a light spring. If the additional mass m is added, it displaces the spring by a distance x. now the combined mass will oscillate on the spring with time period equals to **(CPMT 89)**
(a) $T = 2\pi\sqrt{\frac{mg}{x(M+m)}}$ (b) $T = 2\pi\sqrt{\frac{x(M+m)}{mg}}$
(c) $T = \frac{\pi}{2}\sqrt{\frac{mg}{x(M+m)}}$ (d) $T = \frac{\pi}{2}\sqrt{\frac{(M+m)}{mgx}}$

3. The displacement of particle performing simple harmonic motion is given by, x □ 8 sin □t □ 6 cos □t, where distance is in cm and time is in second. The amplitude of motion is **(MHT-CET-2005)**
(a) 10 cm (b) 14 cm
(c) 2 cm (d) 3.5 cm

4. A simple pendulum is set up in a trolley which moves to the right with an acceleration a on a horizontal plane. Then the thread of the pendulum in the mean position makes an angle □ with the vertical **(CPMT 83)**
(a) $\tan^{-1}\left(\frac{a}{g}\right)$ in the forward direction
(b) $\tan^{-1}\left(\frac{a}{g}\right)$ in the backward direction
(c) $\tan^{-1}\left(\frac{g}{a}\right)$ in the backward direction
(d) $\tan^{-1}\left(\frac{g}{a}\right)$ in the forward direction

5. The angular velocity and the amplitude of a simple pendulum is '□' and 'a' respectively. At a displacement x from the mean position its kinetic energy is T and potential energy is V, then the ratio of T to V is
(CBSE 91)
(a) $\frac{x^2\omega^2}{a^2 - x^2\omega^2}$ (b) $\frac{x^2}{(a^2 - x^2)}$

(c) $\frac{a^2 - x^2\omega^2}{x^2\omega^2}$ (d) $\frac{a^2 - x^2}{x^2}$

6. A particle executes S.H.M. of amplitude A. at what distance from mean position its kinetic energy is equal to its potential energy? **(MHT-CET 99)**
(a) 0.51 A (b) 0.61 A
(c) 0.71 A (d) 0.81 A

7. A simple pendulum of length l and mass (bob) m is suspended vertically. The string makes an angle □ with the vertical. The restoring force acting on the pendulum, is **(MHT-CET-2005)**
(a) mg tan □ (b) mg sin □
(c) □ mg sin □ (d) □ mg cos □

8. The mass and diameter of a planet are twice those of earth. the period of oscillation of pendulum on this planet will be (if it is a second's pendulum on earth)
(IIT 73)
(a) $\frac{1}{\sqrt{2}}$Second (b) $2\times\sqrt{2}$ Second
(c) 2 second (d) $\frac{1}{2}$Second

9. A second's pendulum is placed in space laboratory orbiting around the earth at a height 3R from earth's surface where R is earth's radius. The time period of the pendulum will be **(CPMT 89)**
(a) Zero (b) $2\sqrt{3}$s
(c) 4 s (d) Infinite

10. The pendulum is acts as second pendulum on earth. Its time on a planet, whose mass and diameter are twice that of earth, is **(MHT-CET-2005)**
(a) $\sqrt{2}$s (b) 2 s
(c) $2\sqrt{2}$s (d) 1/$\sqrt{2}$s

11. A particle of mass m is hanging vertically by an ideal spring of force constant K. if the mass is made to oscillate vertically, its total energy is **(CPMT 78)**
(a) Maximum at extreme position
(b) Maximum at mean position
(c) Minimum at mean position
(d) Same at all positions

12. At a place where g □ 980 cm/sec^2. the length of seconds pendulum is about
(a) 50 cm (b) 100 cm
(c) 2 cm (d) 2 m

13. The maximum velocity for particle in SHM is 0.16 m/s and maximum acceleration is

0.64 m/s^2. The amplitude is **(MHT-CET-2004)**
(a) 4 □ 10$^{□2}$ m (b) 4 □ 10$^{□1}$ m
(c) 4 □ 10 m (d) 4 □ 10^0 m

14. A particle is vibrating in S.H.M. with an amplitude of 4 cm. at what displacement from the equilibrium position is its energy half potential and half kinetic?**(NCERT 84)**
(a) 2.5 cm (b) $\sqrt{2}$ cm
(c) 3 cm (d) 2 cm

15. The time period of a spring pendulum is **(CPMT 71)**
(a) $T = 2\pi\sqrt{\frac{m}{k}}$ (b) $T = 2\pi\sqrt{\frac{2k}{m}}$
(c) $T = 2\pi\sqrt{\frac{k}{m}}$ (d) $T = \pi\sqrt{\frac{2m}{k}}$

16. The equation of displacement of particle performing SHM is X = 0.25 sin (200 t). The maximum velocity is **(MHT-CET-2004)**
(a) 100 m/s (b) 200 m/s
(c) 50 m/s (d) 150 m/s

17. A pendulum suspended from the roof of a train has a period T (When the train is at rest). When the train is accelerating with a uniform acceleration 'a', the time period of the pendulum will **(NCERT 80)**
(a) Increase (b) Decrease
(c) Remain unaffected (d) Become infinite

18. A particle executing a vibratory motion while passing through the mean position has **(CPMT 92)**
(a) Maximum P.E. and minimum K.E.
(b) Maximum K.E. and minimum P.E.
(c) P.E. and K.E. both maximum
(d) P.E. and K.E. both minimum

19. The frequency of wave is 0.002 Hz. Its time period is **(MHT-CET-2004)**
(a) 100 s (b) 500 s
(c) 5000 s (d) 50 s

20. A simple pendulum has a period T. it is taken inside a lift moving up with uniform acceleration g/3. now its time period will be **(NCERT 90)**
(a) $\sqrt{2}\,T$ (b) $\frac{2T}{\sqrt{3}}$
(c) $\frac{\sqrt{3}}{2}T$ (d) $\frac{3T}{\sqrt{2}}$

21. For a magnet of time period T magnetic moment is M, if the magnetic moment becomes

one fourth of the initial value, then the time period of oscillation becomes.
(MHT CET 2006)
(a) Half of initial value
(b) One fourth of initial value
(c) Double of initial value
(d) Four time initial value

22. The value of displacement of particle performing SHM, when kinetic energy is (3/4)th of its total energy is **(MHT-CET-2004)**
(a) $x = \pm\frac{A}{2}$ (b) $x = \pm\frac{\sqrt{3}A}{2}$
(c) $x = \pm\frac{A}{4}$ (d) $x = \pm\frac{A}{\sqrt{2}}$

23. The shape of l □ T graph of simple pendulum is,
(CPMT-92)
(a) Curve (b) Parabola
(c) Straight line (d) Hyperbola

24. A simple pendulum is suspended from the roof of a trolley which moves in a horizontal direction with an acceleration 'a' then the time period is given
T □ 2□ $\sqrt{\frac{l}{g}}$, where g is equal to **(CBSE 91)**
(a) □ ag (b) 3 □ a
(c) g □ a (d) $\sqrt{g^2 + a^2}$

25. Two equal negative charges □q are fixed at point (0, a) and (0, □a) on the Y-axis A positive charge q is released from rest at point (2a, 0) on the X-axis. The charge Q will **(IIT 83)**
(a) Execute simple harmonic motion about the origin
(b) Move to the origin and remained at rest
(c) Move to infinity
(d) Execute oscillatory motion but not simple harmonic motion

Answers to Oscillation, Paper 1
1. Answer: (d)
2. Answer: (b)
3. Answer: (a)
4. Answer: (b)
5. Answer: (d)
6. Answer: (c)
7. Answer: (c)
8. Answer: (b)
9. Answer: (d)
10. Answer: (c)

11. Answer: (d)
12. Answer: (b)
13. Answer: (a)
14. Answer: (d)
15. Answer: (a)
17. Answer: (b)
18. Answer: (b)
19. Answer: (b)
20. Answer: (c)
21. Answer: (c)
22. Answer: (a)
23. Answer: (b)
24. Answer: (d)
25. Answer: (d)

1. The kinetic energy of a particle executing S.H.M. is 16 J when it is at its mean position. If the mass of the particle is 0.32 kg, then what is the maximum velocity of the particle? **(MHT-CET-2004)**
(a) 5 m/s (b) 15 m/s
(c) 10 m/s (d) 20 m/s

2. If a hole is bored along the diameter of the earth and a stone is dropped into the hole **(CPMT 84)**
(a) The stone reaches the centre of the earth and stops there
(b) The stone reaches the other side of the earth and stops there
(c) The stone executes simple harmonic motion about the centre of the earth
(d) The stone reaches the other side of the earth and escapes into space

3. A simple pendulum of length L and mass m is oscillating in a plane about a vertical line between angular limits -□ and□□. For an angular displacement □□□□□□□□ The tension in the string and the velocity of the bob are T and □ respectively. The following relation holds good under the above conditions **(IIT 86)**
(a) T cos□ □ mg
(b) T □ m g cos □ □ $\frac{mu^2}{L}$
(c) T □ mg cos □
(d) Tangential acceleration □a_T□ □ g sin □

4. When a particle performing uniform circular motion of radius 10 cm undergoes the SHM, what will be its amplitude? **(MHT-CET-2004)**
(a) 10 cm (b) 5 cm
(c) 2.5 cm (d) 20 cm

5. The work done by the tension in the string of a pendulum during one complete vibration is equal to
(NCERT 83)
(a) Potential energy of pendulum
(b) Total energy of pendulum
(c) Kinetic energy of pendulum
(d) Zero

6. A particle is executing simple harmonic motion with an amplitude a. when its kinetic energy is equal to its potential energy its distance from the mean position
(CPMT 90, PMT MP 87)
(a) $\frac{a}{2}$ (b) $\frac{a}{\sqrt{2}}$
(c) $\frac{a}{\sqrt{3}}$ (d) $\frac{a}{3}$

7. A magnet of magnetic M oscillates in magnetic field B with time period 2 sec. If now the magnet is cut into two half pieces parallel to the axis, then what is new time period if only one part oscillate in field? **(MHT-CET-2004)**
(a) 2 s (b) $2\sqrt{2}$ s
(c) $\frac{1}{\sqrt{2}}$s (d) 2.4 s

8. Time period of simple pendulum of length l and a place where acceleration due to gravity is g is T. what is the period of a simple pendulum of the same length at a place where the acceleration due to gravity is 1.029 is,
(CPMT 82)
(a) T (b) 1.02 T
(c) 0.99 T (d) 1.01 T

9. The potential energy of a particle with displacement X is U (X). the motion is simple harmonic, when
(K is a positive constant) **(CPMT 82)**
(a) $U = \frac{KX^2}{2}$ (b) U □ KX^2
(c) U □ K (d) U □ KX

10. A particle is subjected to two S.H.M.s x_1 □ A_1 sin □t and x_2 □ A_2 sin $\left(\omega t+\frac{\pi}{4}\right)$ The resultant S.H.M. will have an amplitude of **(IIT 96)**
(a) $\frac{A_1+A_2}{2}$ (b) $\sqrt{A_1^2+A_2^2}$

(c) $\sqrt{A_1^2 A_2^2 + \sqrt{2} A_1 A_2}$ (d) $A_1 \square A_2$

11. If velocity of a body is half the maximum velocity. Then what is the distance from the mean position?
(MHT CET 2002, 2003)
(a) 2a (b) $\frac{\sqrt{3}}{2}a$
(c) a (d) $\frac{a}{2}$

12. If the length of the simple pendulum is increased by 44%, then what is the change in time period of pendulum? **(MHT-CET-2004)**
(a) 22% (b) 20%
(c) 33% (d) 44%

13. A linear harmonic oscillator of force constant $2 \square 10^6$ N/m and amplitude 0.01 m has a total mechanical energy of 100 J. it's maximum potential energy is
(IIT 89)
(a) 100 J (b) 200 J
(c) 150 J (d) 0

14. The period of oscillation of a simple pendulum of constant length at earths surface is T, it period inside a mine is **(CPMT 73)**
(a) Greater than T.
(b) Less than T.
(c) Equal to T.
(d) Cannot be compared

15. A spring having a spring constant k is loaded with a mass m. the spring is cut into two equal parts and one of these is loaded again with the same mass. The new spring constant is **(NCERT 90)**
(a) $\frac{k}{2}$ (b) k
(c) 2k (d) k^2

16. Dimensions of force constant are **(MHT CET 2003)**
(a) $[M^1 L^0 T^{\square 2}]$ (b) $[M^{\square 1} L^0 T^{\square 2}]$
(c) $[M^1 L^1 T^{\square 2}]$ (d) $[M^{\square 1} L^1 T^{\square 2}]$

17. Spring is pulled down by 2 cm. What is amplitude of motion? **(MHT-CET-2003)**
(a) 0 cm (b) 6 cm
(c) 2 cm (d) 4 cm

18. The particle is performing S.H.M. along a straight line with amplitude 'a'. the potential energy is maximum when the displacement is **(CPMT 82)**

(a) A (b) $\frac{a}{2}$

(c) 0 (d) $\frac{a}{4}$

19. The period of a simple pendulum is doubled when **(CPMT 74)**
(a) Its length is doubled
(b) The mass of the bob is doubled
(c) Its length is made four times
(d) The mass of the bob and the length of the pendulum are doubled.

20. Particle moves from extreme position to mean position, its **(MHT-CET-2003)**
(a) Kinetic energy increases, potential increases decreases
(b) Kinetic energy decreases, potential increases
(c) Both remains constant
(d) Potential energy becomes zero and kinetic energy remains constant

21. If a particle is moving in a circle, with a uniform speed, then its motion is, **(CPMT 78)**
(a) Oscillatory (b) Periodic
(c) Non-periodic (d) Simple harmonic

22. A simple pendulum performs simple harmonic motion about x □ 0 with an amplitude A and time period T. the speed of the pendulum at $x = \frac{A}{2}$ will be **(PMT-MP 87)**

(a) $\frac{\pi A\sqrt{3}}{T}$ (b) $\frac{\pi A}{T}$

(c) $\frac{\pi A\sqrt{3}}{2T}$ (d) $\frac{3\pi^2 A}{T}$

23. If velocity of body is half the maximum velocity. Then what is the distance from the mean position? **(MHT-CET-2003)**

(a) 2 A (b) $\frac{\sqrt{3}}{2} x A$

(c) A (d) $\frac{A}{2}$

24. The necessary and sufficient condition for S.H.M. is **(NCERT 74)**
(a) Constant period
(b) Constant acceleration
(c) Proportionality between restoring force and displacement from equilibrium position in opposite direction
(d) None of the above

25. The motion of a particle executing simple harmonic motion is given by X □ 0.01 sin

$100\pi(t + 0.05)$, where X is in metres ant t in second. The time period in second is **(CPMT 90)**
(a) 0.001 (b) 0.02
(c) 0.1 (d) 0.2

Answers to Oscillation, Paper 2

1. Answer: (c)
2. Answer: (c)
3. Answer: (b)
4. Answer: (a)
5. Answer: (d)
6. Answer: (b)
7. Answer: (a)
8. Answer: (c)
9. Answer: (b)
10. Answer: (c)
11. Answer: (b)
12. Answer: (b)
13. Answer: (a)
14. Answer: (a)
15. Answer: (c)
16. Answer: (a)
17. Answer: (c)
18. Answer: (a)
19. Answer: (c)
20. Answer: (a)
21. Answer: (b)
22. Answer: (c)
23. Answer: (b)
24. Answer: (c)
25. Answer: (b)

1. Dimensions of force constant are **(MHT-CET-2003)**
(a) $M^{1}L^{0}T^{-2}$ (b) $M^{-1}L^{0}T^{-2}$
(c) $M^{0}L^{1}T^{-2}$ (d) MLT^{-2}

2. A bar magnet is in oscillatory motion its frequency is ‘n’ magnetic field induction is 0.4×10^{-5}T when the frequency is doubled due to increase the magnetic field induction. The what is increasing value to induction?
(a) 1.2×10^{-3} B (b) 1.2×10^{-4}
(c) 1.2×10^{-5} B (d) 1.2×10^{-6}

3. A particle moves such that its acceleration a is given by $a = -bx$, where x is the displacement from equilibrium position and b is a constant. The period of oscillation is **(CPMT 91, NCERT 84)**

(a) $\sqrt{\frac{2\pi}{b}}$ (b) $\frac{2\pi}{\sqrt{b}}$

(c) $\frac{2\pi}{b}$ (d) $\sqrt{\frac{2\pi}{b}}$

4. A force of 6.4N stretches a vertical spring by 0.1 m. the mass that must be suspended from the spring so that it oscillates with a period of (□/4) sec is **(Roorkee 88)**

(a) $\frac{\pi}{4}$kg (b) $\frac{1}{\pi}$kg

(c) 1 kg (d) 10 kg

5. Total energy of a particle executing S.H.M. is proportional to **(MHT CET 2002, CPMT 74)**

(a) Square of the amplitude of the motion
(b) Frequency of oscillation
(c) Velocity in equilibrium position
(d) Displacement form equilibrium position

6. The length of a pendulum is halved. Its energy will **(MH-CET-2002)**

(a) Decreased to half
(b) Increased to 2 times
(c) Decreased to one fourth
(d) Increased to 4 times

7. In simple harmonic motion which statement is wrong.
(MHT-CET-2008)

(a) A body in S.H.M. its velocity maximum at mean position
(b) A body in S.H.M. its K.E. less at extreme position
(c) A body in S.H.M. its acceleration more at extreme position its directions away from mean position
(d) A body in S.H.M its acceleration less at mean position

8. A simple pendulum is made of a body which is a hollow sphere containing mercury suspended by means of a wire. If a little mercury is drained off, the period of pendulum will **(NCERT 72)**

(a) Remain unchanged(b) Increase

(c) Decrease (d) Become erratic

9. Time period of pendulum is 6.28 sec and amplitude of oscillation is 3 cm. Maximum acceleration of pendulum is **(MHT-CET-2002)**

(a) 8 cm/s^2 (b) 0.3 cm/s^2

(c) 3 cm/s^2 (d) 58.2 cm/s^2

10. A body in simple harmonic motion. Its time period is 24 sec. at mean position to 4 sec after its velocity is □ m/sec. then find its path length. **(MHT-CET-2008)**

(a) 48 m (b) 58 m

(c) 68 m (d) 78 m

11. If the length of second's pendulum is increased by 2%, how may seconds it will lose per day? **(CPMT 92)**
(a) 3427 sec (b) 3727 sec
(c) 3927 sec (d) 864 sec

12. A mass of 1 kg attached to the bottom of a spring ahs a certain frequency of vibration. The following mass has to the added to it in order to reduce the frequency by half (Roorkee 88)
(a) 1 kg (b) 3 kg
(c) 2 kg (d) 4 kg

13. In S.H.M. path length is 4 cm and maximum acceleration is $2\square^2 cm/s^2$. Time period of motion is **(MHT-CET-2002)**
(a) 2 s (b) 4 s
(c) $\sqrt{2}$ s (d) 1/2 s

14. A magnet, when suspended in an external magnetic field, has period of oscillation of 4s. when it is cut length wise, and suspended in the same magnetic field, the period of vibration will be **(MHT-CET-2007)**
(a) $2\sqrt{2}$ s (b) 2 s
(c) $4\sqrt{2}$ s (d) 8 s

15. A body executes S.H.M. with an amplitude A. At what displacement, from the mean position, the kinetic energy of the body is one fourth of its total energy
(CBSE 90)
(a) $\frac{A}{4}$ (b) $\frac{A}{2}$
(c) $\frac{\sqrt{3}}{2}A$ (d) $\sqrt{3}A$

16. M kg weight is suspended from a weightless spring and it has time period T. if now 4M kg weight is suspended from the same spring, the new time period will be
(CPMT 79)
(a) T (b) 2T
(c) $\frac{T}{2}$ (d) 4T

17. The acceleration of particle executing S.H.M. when it is at mean position is **(MHT CET 2002)**
(a) Infinite (b) Varies
(c) Maximum (d) Zero

18. A.S.H.M. is represented by

x □ $5\sqrt{2}$ (sin 2 □t □ cos^2 □ t). Then amplitude of the S.H.M. is **(MHT CET 2004)**
(a) 10 cm (b) 20 cm
(c) 5 cm (d) 50 cm

19. W denotes to the total energy of a particle in linear S.H.M. At a point, equidistant from the mean position and extremity of the path of the particle **(MH-CET 2001)**
(a) K.E. of the particle will be w/2 and P.E. will also be w/2
(b) K.E. of the particle will be w/4 and P.E will be w/4
(c) K.E. of the particle will be 3w/4 and P.E. will be w/4
(d) K.E. of the particle will be w/8 and P.E. will be 7w/8

20. In SHM, graph of which of the following is a straight line? **(MHT-CET2007)**
(a) T.E. against displacement
(b) P.E. against displacement
(c) Acceleration against time
(d) Velocity against displacement

21. The potential energy of a particle in S.H.M. at a distance x from the equilibrium position is
(MH-CET 99)
(a) $\frac{1}{2}m\omega^2x^2$ (b) $\frac{1}{2}m\omega^2a^2$
(c) $\frac{1}{2}m\omega^2\left(a^2-x^2\right)$ (d) Zero

22. The velocity of a particle performing simple harmonic motion, when it passes through its mean position is
(MHT CET 2002)
(a) Infinite (b) Zero
(c) Minimum (d) Maximum

23. The time period of a bar magnet in uniform magnetic field $\vec{B}$ is T. It is cut into two halves, by cutting it parallel to its length then the time period of each part in same field is **(MHT-CET-2001)**
(a) $\sqrt{2}T$ (b) T
(c) 2T (d) None of these

24. The period of oscillation of a mass M, having from a spring of force constant k is T. When additional mass m is attached to the spring, the period of oscillation becomes 5T/4. m/M = **(MHT-CET-2007)**
(a) 9:16 (b) 25:16
(c) 25:9 (d) 19:9

25. The pendulum energy of a particle executing S.H.M. at a distance x from a equilibrium position is proportional to **(Roorkee 92)**

(a) $\sqrt{x}$ (b) x
(c) x^2 (d) x^3

Answers to Oscillation, Paper 3 (Go to top)

1. Answer: (a)
2. Answer: (c)
3. Answer: (b)
4. Answer: (c)
5. Answer: (a)
6. Answer: (b)
7. Answer: (c)
8. Answer: (b)
9. Answer: (c)
10. Answer: (a)
11. Answer: (d)
12. Answer: (d)
13. Answer: (a)
14. Answer: (b)
15. Answer: (c)
16. Answer: (b)
17. Answer: (d)
18. Answer: (a)
19. Answer: (c)
20. Answer: (a)
21. Answer: (a)
22. Answer: (d)
23. Answer: (b)
24. Answer: (d)
25. Answer: (c)

1. The period of simple pendulum is doubled when
(CPMT 74)
(a) Its length is doubled
(b) Its length is halved
(c) The length is made four times
(d) Mass of the bob is doubled

2. If the length of a simple pendulum is doubled keeping its amplitude constant its energy will be
(MHT CET 2001)
(a) Unchanged (b) Doubled
(c) Four times (d) Halved

3. If an simple pendulum oscillates with an amplitude of 50 mm and time period of 2s, then its maximum velocity is **(MHT-CET-2000)**

(a) 0.10 m/s (b) 0.16 m/s
(c) 0.25 m/s (d) 0.5 m/s

4. Starting from the extreme position, the time taken by an ideal simple pendulum to travel a distance of half of the amplitude is **(MHT-CET-2007)**
(a) T/6 (b) T/12
(c) T/13 (d) T/4

5. If a simple harmonic oscillator has got a displacement of 0.02 m and acceleration equal to 2.0 ms$^{□2}$ at any time, the angular frequency of the oscillator is equal to
(CBSE 92)
(a) 10 rad s$^{□1}$ (b) 0.1 rad s$^{□1}$
(c) 100 rad s$^{□1}$ (d) 1 rad s$^{□1}$

6. A body is executing S.H.M. when the displacements from the mean position are 4 cm and 5 cm, the corresponding velocities of the body are 10 cm/s and 8 cm/s respectively. The time period of oscillation is
(MHT CET 2001, CBSE 91)
(a) 5 second (b) 3.14 second
(c) 2 second (d) 6.28 second

7. The period of thin magnet is 4 sec. if it is divided into two equal halves then the time period of each part will be **(MHT CET 2004)**
(a) 4 sec (b) 1 sec
(c) 2 sec (d) 8 sec

8. The force constant of a wire is K and that of another wire of the same material is 2K. when both the wires are stretched, then work done is **(MHT-CET-2000)**
(a) W_2 □ 0.5 W_1 (b) W_2 □ W_1
(c) W_2 □ 2W_2 (d) W_2 □ 2W_1^2

9. The acceleration due to gravity changes from 9.8 m/s^2 to 9.5 m/s^2. To keep the period of pendulum constant, its length must changes by **(MHT-CET-2006)**
(a) 3 m (b) 0.3 m
(c) 0.3 cm (d) 3 cm

10. A simple harmonic oscillator has an amplitude a and time period T. the time required to travel from x □ $\frac{a}{2}$ is **(CBSE 92)**
(a) $\frac{T}{6}$ (b) $\frac{T}{4}$
(c) $\frac{T}{3}$ (d) $\frac{T}{2}$

11. Two springs of constants k_1 and k_2 equal maximum velocities, when executing simple harmonic motion. The ratio of their amplitudes (masses are equal) will be

(MHT-CET-2000)

(a) $\frac{k_1}{k_2}$ (b) $\frac{k_2}{k_1}$

(c) $\left(\frac{k_1}{k_2}\right)^{1/2}$ (d) $\left(\frac{k_2}{k_1}\right)^{1/2}$

12. A load of mass 100 gm increases the length of wire by 10 cm. If the system is kept in oscillation, its time period is **(MHT-CET-2006)**

(a) 0.314 s (b) 3.14 s

(c) 0.628 s (d) 6.28 s

13. A spring has a force constant K and a mass m is suspended from it. The spring is cut into half and the same a mss is suspended from one of the halves. If the frequency of oscillation in the first case is □, then the frequency in the second case will be **(CPMT 86)**

(a) 2 □ (b) □

(c) $\frac{\alpha}{2}$ (d) $\alpha\sqrt{2}$

14. A particle executes S.H.M. with amplitude 0.5 cm and frequency 100 Hz. The maximum speed of particle will be **(CPMT 84)**

(a) □ m/s (b) 5 □ □ 10□5 m/s

(c) 0.5 m/s (d) 100 □ m/s

15. A particle executes simple pendulum harmonic motion of amplitude A. at what distance from the mean position is its kinetic energy to its potential energy? **(MHT-CET-1999)**

(a) 0.51 A (b) 0.61 A

(c) 0.71 A (d) 0.81 A

16. A particle of mass m is executing SHM about its mean position. The total energy of the particle at given instant is

(a) $\frac{\pi^2 mA^2}{T^2}$ (b) $\frac{4\pi^2 mA^2}{T^2}$

(c) $\frac{2\pi^2 mA^2}{T^2}$ (d) $\frac{8\pi^2 mA^2}{T^2}$

17. Two bodies M and N of equal masses are suspended from two separate mass less springs of spring constants k_1 and k_2 respectively. If the two bodies oscillate vertically such that their maximum velocities are equal, the ratio of the amplitudes of M to that of N is **(IIT 88)**

(a) $\frac{k_1}{k_2}$ (b) $\frac{\sqrt{k_1}}{k_2}$

(c) $\frac{k_2}{k_1}$ (d) $\sqrt{\frac{k_2}{k_1}}$

18. A simple harmonic motion having an amplitude A and time period T is represented by the equation
y □ 5 sin □ (t □ 4)m then the values of
(A in m) and (T in sec) are **(MNR 91)**
(a) A □ 5, T □ 2 (b) A □ 10, T □ 1
(c) A □ 5, T □ 1 (d) A □ 10, T □ 2

19. If the length of simple pendulum is increased by 44% then what is the change in the time period of the pendulum? **(MHT-CET 2004)**
(a) 22 % (b) 20 %
(c) 33 % (d) 44 %

20. The maximum velocity and maximum acceleration of a body moving in a simple harmonic oscillator are 2m/s and $4 m/s^2$ the angular velocity is **(MHT-CET-1999)**
(a) 1 rad/s (b) 2 rad/s
(c) 4 rad/s (d) 5 rad/s

21. If a bar magnet of magnetic moment M is kept in a uniform magnetic field B, its time period of oscillation is T. The another magnet of same length and breadth is kept in a same magnetic field. If magnetic moment of new magnet is M/4, then its oscillation time period is
(MHT-CET-2006)
(a) T (b) 2T
(c) T/2 (d) T/4

22. For a particle executing simple harmonic motion, the kinetic energy K is given by K □ $K_0 \cos^2$ □t. the maximum value of potential energy is **(CPMT 81)**
(a) K_0 (b) Zero
(c) $\frac{K_0}{2}$ (d) Not obtainable

23. A body of mass 5 gm is moving at the centre with amplitude of 10 cm. its maximum velocity is 100 cm/s. its velocity will be 50 cm/s at a distance (in cm)
(CPMT 76)
(a) $5\sqrt{2}$ (b) $5\sqrt{3}$
(c) 10 (d) $10\sqrt{10}$

24. The unit of force constant is **(MHT-CET 99)**
(a) Nm (b) N/m
(c) N/kg (d) Nkg

25. The maximum velocity of a body in S.H.M. is 0.25 m/s and maximum acceleration is

0.75 m/s^2, the period of S.H.M. is

(a) $\left(\frac{\pi}{3}\right)$ second (b) $\left(\frac{\pi}{2}\right)$ second

(c) $\left(\frac{2\pi}{3}\right)$ second (d) □ second

Answers to Oscillation, Paper 4

1. Answer: (c)
2. Answer: (d)
3. Answer: (b)
4. Answer: (a)
5. Answer: (a)
6. Answer: (b)
7. Answer: (c)
8. Answer: (c)
9. Answer: (d)
10. Answer: (a)
11. Answer: (d)
12. Answer: (c)
13. Answer: (d)
14. Answer: (a)
15. Answer: (c)
16. Answer: (b)
17. Answer: (d)
18. Answer: (a)
19. Answer: (b)
20. Answer: (b)
21. Answer: (b)
22. Answer: (a)
23. Answer: (b)
24. Answer: (b)
25. Answer: (c)

CH 15
Waves
(10 Hours, 10 Marks (5MT-1Q, 5M NP-1Q)

Syllabus :
Wave motion. Transverse and longitudinal waves, speed of wave motion. Displacement relation for a progressive wave. Principle of superposition of waves, reflection of waves, standing waves in strings and organ pipes, fundamental mode and harmonics, Beats, Doppler effect.

1. Wave motion. Transverse and longitudinal waves :

1. Explain wave motion and types of waves ?

The disturbances produced in air or vacuum which move without the actual physical transfer or flow of matter as a whole, are called **waves**. Not all waves require a medium for their propagation. Electro-magnetic waves can travel through vacuum.

The waves we come across are mainly of three types: (a) mechanical waves, (b) electromagnetic waves and (c) matter waves. Mechanical waves are most familiar because we encounter them constantly; common examples include water waves, sound waves, seismic waves, etc.
All these waves have certain central features : They are governed by Newton's laws, and can exist only within a material medium, such as water, air, and rock. The common examples of electromagnetic waves are visible and ultraviolet light, radio waves, microwaves, x-rays etc. All electromagnetic waves travel through vacuum at the same speed *c*.

2. Explain Transverse and longitudinal waves with examples ?
Mechanical waves can be transverse or longitudinal depending on the relationship between the directions of disturbance or displacement in the medium and that of the propagation of wave.

In transverse waves, the constituents of the medium oscillate perpendicular to the direction of wave propagation and in longitudinal waves they oscillate along the direction of wave propagation.

Transverse waves are waves in which the particles of the medium oscillate perpendicular to the direction of wave propagation.

Longitudinal waves are waves in which the particles of the medium oscillate along the direction of wave propagation.

Progressive wave is a wave that moves from one point of medium to another.

It is found that generally transverse and longitudinal waves travel with different speeds in the same medium.

Example 1 : Given below are some examples of wave motion. State in each case if the wave motion is transverse, longitudinal or a combination of both:
(a) Motion of a kink in a longitudinal spring produced by displacing one end of the spring sideways.
(b) Waves produced in a cylinder containing a liquid by moving its piston back and forth.
(c) Waves produced by a motorboat sailing in water.
(d) Ultrasonic waves in air produced by a vibrating quartz crystal.

Answer

(a) Transverse and longitudinal, (b) Longitudinal, (c) Transverse and longitudinal,
(d) Longitudinal

2. Displacement relation for a progressive wave.

3. Explain Displacement relation for a progressive wave ?

Let $y(x, t)$ denote the transverse displacement of the element at position x at time t. As the wave sweeps through succeeding elements of the string, the elements oscillate parallel to the y-axis. At any time t, the displacement y of the element located at position x is given by

$y(x, t) = a \sin(kx - \omega t + \varphi)$ ------- (1)

One can as well choose a cosine function or a linear combination of sine and cosine functions such as, $y(x, t) = A \sin(kx - \omega t) + B \cos(kx - \omega t)$, -------- (2)

then in Eq. (1),

$$a = \sqrt{A^2 + B^2} \text{ and } \phi = \tan^{-1}\left(\frac{B}{A}\right) \quad \text{----------} \quad (3)$$

The function represented in Eq. (1) is periodic in position coordinate x and time t. It represents a transverse wave moving along the x-axis. At any time t, it gives the displacement of the elements of the string as a function of their position.

On the other hand a function, $y(x, t) = a \sin(kx + \omega t + \varphi)$, (4) represents a wave travelling in the negative direction of x-axis.

The set of four parameters a, φ, k, and ω in Eq. (1) completely describe a harmonic wave.

$$\overbrace{y(x,t)}^{\text{Displacement}} = \overbrace{a}^{\text{Amplitude}} \sin\big(\overbrace{\underset{\substack{\uparrow\\ \text{Angular}\\ \text{Wave}\\ \text{Number}}}{kx} - \underset{\substack{\uparrow\\ \text{Angular}\\ \text{Frequency}}}{\omega t} + \underset{\substack{\uparrow\\ \text{Initial}\\ \text{Phase}\\ \text{Angle}}}{\phi}}^{\text{Phase}}\big)$$

The graphs represent plots of Eqn. (1) for five different values of time t as the wave travels in positive direction of x-axis. A point of maximum positive displacement in a wave, shown by the arrow, is called ***crest***, and a point of maximum negative displacement is called ***trough***. The progress of the wave is indicated by the progress of the short arrow pointing to a crest of the wave towards the right.

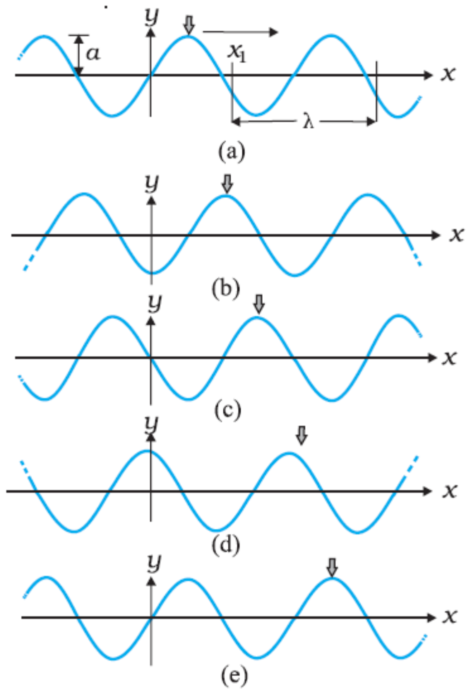


Fig. 1 : Plots of Eq. (1) for a wave travelling in the positive direction of an x-axis at five different values of time t.

(a) Amplitude :

The **amplitude** a of a wave such as that the magnitude of the maximum displacement of the elements from their equilibrium positions as the wave passes through them. Since a is a magnitude, it is a positive quantity, even if the displacement is negative.

(b) Phase :

The **phase** of the wave is the argument $(kx - \omega t + \varphi)$ of the oscillatory term $\sin(kx - \omega t + \varphi)$ in Eq. (15.2). It describes the state of motion as the wave sweeps through a string element at a particular position x. It changes linearly with time t. The sine function also changes with time, oscillating between +1 and –1. Its extreme positive value +1 corresponds to a peak of the wave moving through the element; then the value of y at position x is a. Its extreme negative value –1 corresponds to a valley of the wave moving through the element, then the value of y at position x is $-a$. Thus, the sine function and the time dependent phase of a wave correspond to the oscillation of a string element, and the amplitude of the wave determines the extremes of the element's displacement. The constant φ is called the **initial phase angle**. The value of φ is determined by the initial $(t = 0)$ displacement and velocity of the element (say, at $x = 0$).

(c) Wavelength :

The **wavelength** λ of a wave is the distance (parallel to the direction of wave propagation) between the consecutive repetitions of the shape of the wave. It is the minimum distance between two consecutive troughs or crests or two consecutive points in the same phase of wave motion.

(d) Angular Wave Number :

For $t = 0$ and $\varphi = 0$. At this time Eq. (1) reduces to $y(x, 0) = a \sin kx$

Since λ is defined as the least distance between points with the same phase, $k = 2\pi/\lambda$ *where k* is called the **propagation constant** or the **angular wave number** ; its SI unit is radian per metre or rad m^{-1}.

(e) Period :

The **period** of oscillation T of a wave is defined as the time any string element takes to move through one complete oscillation.

(f) Angular Frequency :

The least value of ωT is 2π, or $\omega = 2\pi/T$
ω is called the **angular frequency** of the wave, its SI unit is rad s^{-1}.

(g) Frequency :

The **frequency** v of a wave is defined as $1/T$ and is related to the angular frequency ω by
$\upsilon = 1/T = \omega/2\pi$
It is the number of oscillations per unit time made by a string element as the wave passes through it. It is usually measured in hertz.

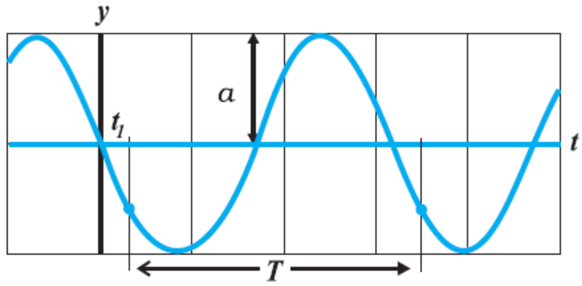


Fig. 2 : *A graph of the displacement of the string element at x = 0 as a function of time.*

3. Speed of wave motion :

(a) Speed of a transverse wave :

Consider the propagation of a travelling wave represented by Eq. (1) along a string. The wave is travelling in the positive direction of x. We find that an element of string at a particular position x moves up and down as a function of time but the waveform advances to the right. The displacement of various elements of the string at two different instants of time t differing by a small time interval Δt is depicted in Fig. 3 (the phase angle φ has been taken to be zero). It is observed that during this interval of time the entire wave pattern moves by a distance Δx in the positive direction of x. Thus the wave is travelling to the right, in the positive direction of x. The ratio $\Delta x/\Delta t$ is the wave speed v.

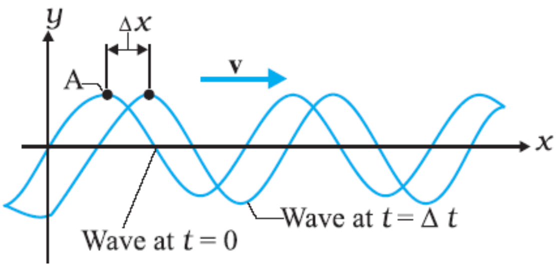


Fig. 3: *The plots of Eq.(1) at two instants of time differing by an interval Δt, at t = 0 and then at t = Δt. As the wave moves to the right at velocity* **v**, *the entire curve shifts a distance Δx during Δt. The point A rides the waveform but the string element moves only up and down.*

As the wave moves (see Fig.3), each point of the moving waveform represents a particular phase of the wave and retains its displacement y. If a point like A on the waveform retains its displacement as it moves, it follows from Eq. (1) that this is possible only when the argument is constant. It, therefore, follows that :
$kx - \omega t$ = constant ------- (4)

[$y(x, t) = a \sin(kx - \omega t + \varphi)$ ------- (1)]

To find the wave speed v, let us differentiate Eq. (4) with respect to time ;

$\frac{d}{dt} kx - \omega t = 0$ ------- (5) or $k \frac{d}{dt} x - \omega = 0$

$\frac{dx}{dt} = \frac{\omega}{k} = v$ --------- (6)
Relating ω to T and k to λ, we get,
$\mathbf{v = \frac{\omega}{k} = \frac{2\pi\omega}{2\pi k} = \lambda\upsilon = \frac{\lambda}{T}}$ **--------- (7)**

Equation (7) is a general relation valid for all progressive waves. It merely states that the wave moves a distance of one wavelength in one period of oscillation. The speed of a wave is related to its wavelength and frequency by the Eq. (7), **but it is determined by the properties of the medium.** For mechanical wave is to travel in a medium like air, water, steel, or a stretched string, it must cause the particles of that medium to oscillate as it passes through it. For this to happen, the medium must possess mass and elasticity. Therefore **the inertial properties like linear mass density** (or mass per unit length, in case of linear systems like a stretched string) and **the elastic properties** (restoring force set-up in the medium) determine how fast the wave can travel in the medium.

Speed of a Transverse Wave on Stretched String :

The speed of transverse waves on a string is found to be (i) directly proportional to **the elastic properties** (restoring force set-up in the medium) and (ii) inversely proportional to **the inertial properties like linear mass density of the medium.**
For waves on a stretched string, the restoring force is provided by tension T of the string and the inertial property is the linear mass density μ, which is the mass m of the string divided by its length L.

Using dimensional analysis, we can calculate the dimension of velocity standing waves.
The dimension of v is $[LT^{-1}]$, the dimension of T is $[MLT^{-2}]$ and the dimension of μ is $[ML^{-1}]$.

$\frac{[MLT^{-2}]}{[ML^{-1}]} = [L^2T^{-2}] = [LT^{-1}]^2$ => Square of dimension of speed v.

Based on this analysis we can write relation as speed $v = C\sqrt{\frac{T}{\mu}}$, where C is a constant and for approximation, its value can be taken as unity.
Thus the speed of transverse waves on a stretched string can be approximated as

$\mathbf{v} = \sqrt{\frac{T}{\mu}}$ ------- (8)

The speed of a wave along a stretched ideal string depends only on the tension (T) and the linear mass density (μ) of the string and does not depend on the frequency of the wave.
The **frequency** of the wave is determined by the source that generates the wave. The **wavelength** is then fixed by Eq. (7) in the form, $\lambda = \frac{\vartheta}{v}$ ------ (9)

(b) Speed of longitudinal waves :
In a longitudinal wave the constituents of the medium oscillate forward and backward in the direction of propagation of the wave. We know that the sound waves travel in the form of compressions and rarefactions of small volume elements of air. The property that determines the extent to which the volume of an element of a medium changes when the pressure on it changes, is the **bulk modulus** *B*, defined as,
$B = -\frac{\Delta P}{\Delta V/V}$ --------- (10)

Using dimensional analysis, the dimension of Pressure is [ ] and the dimension of Density is $[ML^{-3}]$. Then dimension of B becomes $[ML^{-1}T^{-2}]$.
Thus, the dimension of the ratio *B*/ ρ is,
$$\frac{\left[M\,L^{-1}\,T^{-2}\right]}{\left[M\,L^{-3}\right]} = \left[L^{2}\;\;T^{-2}\right]$$
=> Square of dimension of speed v --------- (11)
Therefore, on the basis of dimensional analysis the most appropriate expression for the speed of longitudinal waves in a medium is
$v = C\sqrt{\frac{B}{\rho}}$ --------- (12)
where *C* is a dimensionless constant and can be shown to be unity. Thus the speed of longitudinal waves in a medium is given by,
$v = \sqrt{\frac{B}{\rho}}$ --------- (13)
The speed of propagation of a longitudinal wave in a fluid therefore depends only on the bulk modulus and the density of the medium.

It can be shown that the speed of a longitudinal wave in the solid bar where the relevant modulus of elasticity is the Young's modulus of material of bar (Y), is given by,
$v = \sqrt{\frac{Y}{\rho}}$ --------- (14)
It may be noted that although the densities of liquids and solids are much higher than those of the gases, the speed of sound in them is higher. It is because liquids and solids are less compressible than gases, i.e. have much greater bulk modulus.

The speed of a longitudinal wave in an ideal gas is given by,
$v = \sqrt{\frac{P}{\rho}}$ --------- (15)
This relation was first given by Newton and is known as Newton's formula.

According to Newton's formula if we calculate for the speed of sound in air, we get the value which is about 15% smaller than speed of sound in air obtained experimentally (331 ms^{-1}). This mistake is corrected by Laplace as Follows :

If we examine the basic assumption made by Newton that the pressure variations in a medium during propagation of sound are isothermal, we find that this is not correct. It was pointed out by Laplace that the pressure variations in the propagation of sound waves are so fast that there is little time for the heat flow to maintain constant temperature.
These variations, therefore, are adiabatic and not isothermal. For adiabatic processes the ideal gas satisfies the relation, PV^{γ} = constant and the Bulk modulus $B_{ad} = \gamma P$, where γ is the ratio of two specific heats, C_p/C_V. The speed of sound is, therefore, given by,

$$v = \sqrt{\frac{\gamma P}{\rho}} \quad \text{---------} \quad (16)$$

This modification of Newton's formula is referred to as the **Laplace correction**.
For air γ = 7/5. Now using Eq. (16) to estimate the speed of sound in air at STP, we get a value 331.3 m s^{-1}, which agrees with the measured speed.

4. Principle of superposition of waves, reflection of waves, standing waves in strings and organ pipes, fundamental mode and harmonics,

4. Explain principle of superposition of waves. Obtain an expression for it in case of stretched string ?

Let us consider that two waves are travelling simultaneously along the same stretched string in opposite directions.
The sequence of pictures shown in Fig. 15.9 depicts the state of displacement of various elements of the string at different time instant. Each picture depicts the resultant waveform in the string at a given instant of time. It is observed that the **net displacement of any element of the string at a given time is the algebraic sum of the displacements due to each wave**. This way of addition of individual waveforms to determine the net waveform is called the **principle of superposition**. To put this rule in a mathematical form, let $y1(x, t)$ and $y2(x, t)$ be the displacements that any element of the string would experience if each wave travelled alone.
The displacement $y\,(x,t)$ of an element of the string when the waves overlap is then given by,

$$y\,(x, t) = y1(x, t) + y2(x, t) \quad \text{------------} \quad (17)$$

The principle of superposition can also be expressed by stating that **overlapping waves algebraically add to produce a resultant wave (or a net wave)**. The principle implies that the overlapping waves do not, in any way, alter the travel of each other.

Let a wave travelling along a stretched string be given by,

$$y1(x, t) = a \sin (kx - \omega t) \quad \text{--------} \quad (18)$$

and another wave, shifted from the first by a phase φ,

$$y^2(x, t) = a \sin (kx - \omega t + \varphi) \quad \text{--------} \quad (19)$$

Both the waves have the same angular frequency, same angular wave number k (same wavelength) and the same amplitude a.

They travel in the positive direction of x-axis, with the same speed. Their phases at a given distance and time differ by a constant angle φ. These waves are said to be out of phase by φ or have a phase difference φ.

Now, applying the superposition principle, the resultant wave is the algebraic sum of the two constituent waves and has displacement

$$y(x, t) = a \sin(kx - \omega t) + a \sin(kx - \omega t + \varphi) \quad \text{--------} \quad (20)$$

using trigonometric relations, we can write :

$$y(x, t) = [2a \cos \frac{1}{2}\phi] \sin(kx - \omega t + \frac{1}{2}\phi) \quad \text{--------} \quad (21)$$

Equation (8) shows that the resultant wave is also a sinusoidal wave, travelling in the positive direction of x-axis.

The resultant wave differs from the constituent waves in two respects: (1) its phase angle is (½)φ and (2) its amplitude is the quantity in brackets in Eqn. (21).

5. Write a note on reflection of waves

If the boundary of interface between two media is not completely rigid or is an interface between two different elastic media, the effect of boundary conditions on an incident pulse or a wave is somewhat complicated. A part of the wave is reflected and a part is transmitted into the second medium. If a wave is incident obliquely on the boundary between two different media the transmitted wave is called the **refracted wave**. The incident and refracted waves obey Snell's law of refraction, and the incident and reflected waves obey the usual laws of reflection.

The reflection of waves at a boundary or interface between two media as follows:

A travelling wave, at a rigid boundary or a closed end, is reflected with a phase reversal but the reflection at an open boundary takes place without any phase change. To express the above statement mathematically, let the incident wave be represented by

$y_i(x, t) = a \sin(kx - \omega t)$,

then, for reflection at a rigid boundary the reflected wave is represented by,

$y_r(x, t) = a \sin(kx + \omega t + \pi)$.

$= - a \sin(kx + \omega t)$ -------- (22)

For reflection at an open boundary, the reflected wave is represented by

$y_r(x, t) = a \sin(kx + \omega t)$ -------- (23)

6. Obtain an expression for standing waves in strings and organ pipes ?

(a) Standing waves in strings :

Consider a system which is bounded at both the ends such as a stretched string fixed at the ends or an air column of finite length. In such a system suppose that we send a continuous sinusoidal wave of a certain frequency, say, toward the right. When the wave reaches the right end, it gets reflected and begins to travel back. The left-going wave then overlaps the wave, travelling to the right. When the left-going wave reaches the left end, it gets reflected again and the newly reflected wave begins to travel to the right, overlapping the left-going wave. This process will continue and, therefore, very soon we have many overlapping waves, which interfere with one another. In such a system, at any point x and at any time t, there are always two waves, one moving to the left and another to the right. We, therefore, have

$y_1(x, t) = a \sin(kx - \omega t)$ (wave travelling in the positive direction of x-axis)
and $y_2(x, t) = a \sin(kx + \omega t)$ (wave travelling in the negative direction of x-axis).
The principle of superposition gives, for the combined wave
$y(x, t) = y_1(x, t) + y_2(x, t)$
$= a \sin(kx - \omega t) + a \sin(kx + \omega t)$
$= \mathbf{(2a \sin kx) \cos \omega t}$ ----------- *(23)*

The wave represented by Eq. (23) does not describe a travelling wave, as the waveform or the disturbance does not move to either side. Here, the quantity $2a \sin kx$ within the brackets is the amplitude of oscillation of the element of the string located at the position x.

In a travelling wave, in contrast, the amplitude of the wave is the same for all elements. Equation (23), therefore, represents a **standing wave**, a wave in which the waveform does not move. The formation of such waves is illustrated in Fig. 15.4.

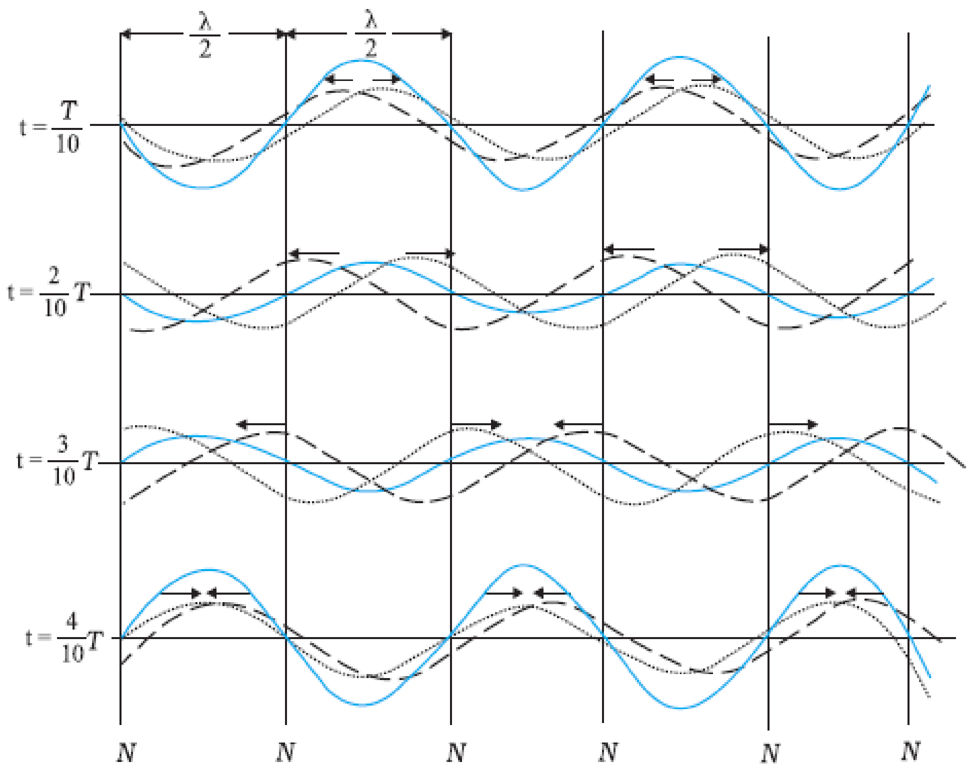


Fig. 15.4 : *The formation of a standing wave in a stretched string.*

It is seen that the points of maximum or minimum amplitude stay at one position. The amplitude is zero for values of kx that give $\sin kx = 0$. Those values are given by $kx = n\pi$, for $n = 0, 1, 2, 3,$ ... Substituting $k = 2\pi/\lambda$ in this equation, we get

$x = n\frac{\lambda}{2}$, for $n = 0, 1, 2, 3, ...$ ----------- (24)

The positions of zero amplitude are called **nodes**. Note that a distance of $\lambda/2$ or half a wavelength separates two consecutive nodes.

The amplitude has a maximum value of $2a$, which occurs for the values of kx that give $|\sin kx| = 1$. Those values are $kx = (n + ½)\pi$ for $n = 0, 1, 2, 3, ...$

Substituting $k = 2\pi/\lambda$ in this equation, we get $x = (n + ½)\frac{\lambda}{2}$ for $n = 0, 1, 2, 3, ...$ ----- (25)

as the positions of maximum amplitude. These are called the **antinodes**. The antinodes are separated by $\lambda/2$ and are located half way between pairs of nodes.

For a stretched string of length L, fixed at both ends, the two ends of the string have to be nodes. If one of the ends is chosen as position $x = 0$, then the other end is $x = L$. In order that this end is a node; the length L must satisfy the condition $L = n\frac{\lambda}{2}$, for $n = 1, 2, 3, ...$ -------- (26)

This condition shows that standing waves on a string of length L have restricted wavelength given by $\lambda = 2\frac{L}{n}$, for $n = 1, 2, 3, \ldots$ etc. --------- (27)

The frequencies corresponding to these wavelengths follow from Eq. (7) as

$v = n\frac{\vartheta}{2L}$, for $n = 1, 2, 3, \ldots$ etc. ---------- (28)

where v is the speed of travelling waves on the string. The set of frequencies given by Eq. (28) are called the natural frequencies or **modes** of oscillation of the system. This equation tells us that the natural frequencies of a string are integral multiples of the lowest frequency $v = \frac{\vartheta}{2L}$, which corresponds to $n = 1$. The oscillation mode with that lowest frequency is called the **fundamental mode** or the **first harmonic**. The **second harmonic** is the oscillation mode with $n = 2$. The **third harmonic** corresponds to $n = 3$ and so on. The frequencies associated with these modes are often labelled as v_1, v_2, v_3 and so on. The collection of all possible modes is called the **harmonic series** and n is called the harmonic number.

Some of the harmonics of a stretched string fixed at both the ends are shown in Fig. 15.5. According to the principle of superposition, a stretched string tied at both ends can vibrate simultaneously in more than one modes. Which mode is strongly excited depends on where the string is plucked or bowed. Musical instruments like sitar and violin are designed on this principle.

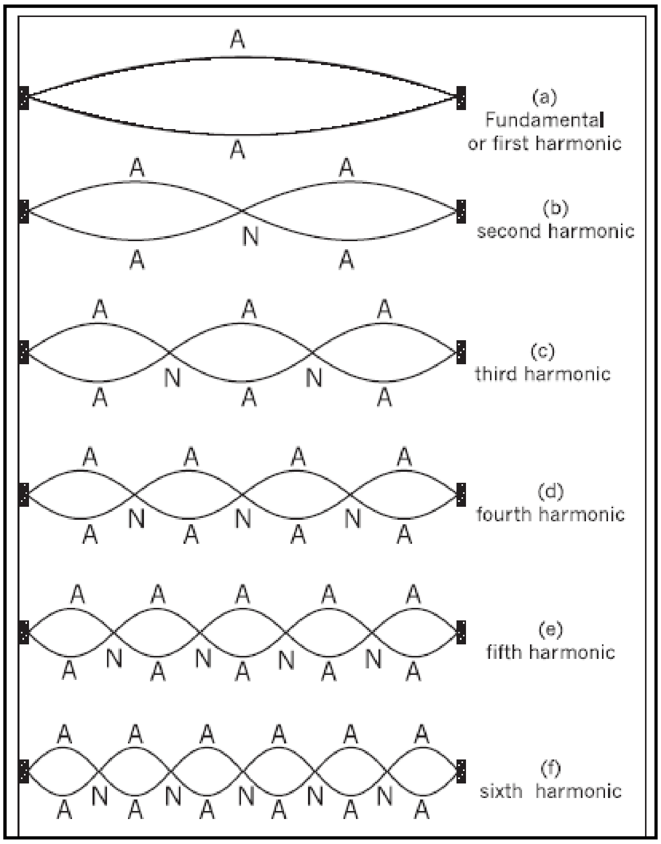


Fig. 15.5 : *Stationary waves in a stretched string fixed at both ends. Various modes of vibration are shown.*

(b) Standing waves in organ pipes :

Now we shall study the modes of vibration of a system closed at one end, with the other end being free. Air columns such as glass tubes partially filled with water provide examples of such systems. In these, the length of the air column can be adjusted by changing the water level in the tube. In such systems, the end of the air column in touch with the water suffers no displacement as the reflected and incident waves are exactly out of phase. For this reason the pressure changes here are the largest, since when the compressional part is reflected the pressure increase is doubled, and when the rarefaction is reflected the decrease in pressure is doubled. On the other hand, at the open end, there is maximum displacement and minimum pressure change. The two waves travelling in opposite directions are in phase here, so there are no pressure changes.
Now if the length of the air column is L, then the open end, $x = L$, is an **antinode** and therefore, it follows from Eq. (15.25) that

$L = (n + ½)\frac{\lambda}{2}$ for n = 0, 1, 2, 3, ...

The modes, which satisfy the condition

$$\boldsymbol{\lambda} = \frac{2L}{(n+\frac{1}{2})} \text{ for n = 0, 1,2,3,....... -------- (15.26)}$$

are sustained in such an air column. The corresponding frequencies of various modes of such an air column are given by,

$$\upsilon = \left(n + \frac{1}{2}\right)\frac{\vartheta}{2L}, \text{ for n=0, 1, 2, 3,..... ------- (15.27)}$$

Some of the normal modes in an air column with the open end are shown in Fig. 15.6. The fundamental frequency is $\frac{v}{4L}$ and the higher frequencies are **odd harmonics** of the fundamental frequency, i.e. 3 $\frac{v}{4L}$, 5 $\frac{v}{4L}$ $, etc.$

In the case of a pipe open at both ends, there will be antinodes at both ends, and **all harmonics** will be generated.
Normal modes of a circular membrane rigidly clamped to the circumference as in a tabla are determined by the boundary condition that no point on the circumference of the membrane vibrates. Estimation of the frequencies of normal modes of this system is more complex.

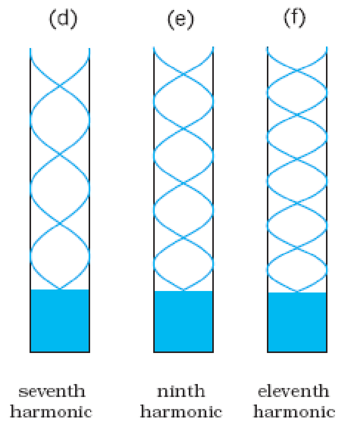


Fig. 15.6 Some of the normal modes of vibration of an air column open at one end.

7. Obtain an expression for fundamental mode and harmonics ?

Refer Q. 6

4. Beats, Doppler effect :

8. What do you meant by Beats ? How they are formed ?

The phenomenon of **wavering of sound intensity** when two waves of nearly same frequencies and amplitudes travelling in the same direction, are superimposed on each other is called **beats**. Beats is an interesting phenomenon arising from interference of waves.

When two waves having slightly different frequencies are superposed on each other, the time dependent variations of the displacements due to two sound waves at a particular location be
$s_1 = a \cos \omega_1 t$ and $s_2 = a \cos \omega_2 t$ --------- *(1)*
where $\omega_1 > \omega_2$. We have assumed, for simplicity, that the waves have same amplitude and phase. According to the superposition principle, the resultant displacement is

$$s = s_1 + s_2 = a(\cos \omega_1 t + \cos \omega_2 t)$$

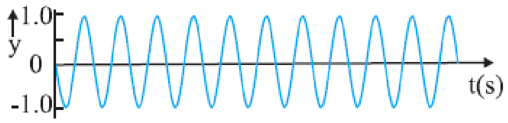
$$= 2\,a \cos\frac{(\omega_1 - \omega_2)t}{2} \cos\frac{(\omega_1 + \omega_2)t}{2} \quad \text{---------} \quad (2)$$

If we write $\omega_b = \dfrac{(\omega_1 - \omega_2)}{2}$ and $\omega_a = \dfrac{(\omega_1 + \omega_2)}{2}$ then Eq. (2) can be written as

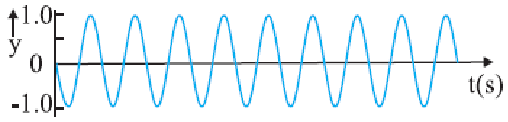
$s = [\, 2\,a \cos \omega_b t \,] \cos \omega_a t$ --------- *(3)*

If $|\omega_1 - \omega_2| << \omega_1, \omega_2, \omega_a >> \omega_b$, then in Eq. (3) the main time dependence arises from cosine function whose angular frequency is ω_a. The quantity in the brackets can be regarded as the amplitude of this function (which is not a constant but, has a small variation of angular frequency ω_b). It becomes maximum whenever $\cos \omega_b t$ has the value +1 or –1, which happens twice in each repetition of cosine function.
In other words, the intensity of the resultant wave waxes and wanes with a frequency which is $\omega_{beat} = 2\,\omega_b = \omega_1 - \omega_2$.
Now using the relation, $\omega = 2\pi\nu$, the beat frequency, ν_{beat}, is given by
$\nu_{beat} = \nu_1 - \nu_2$ --------- (4)

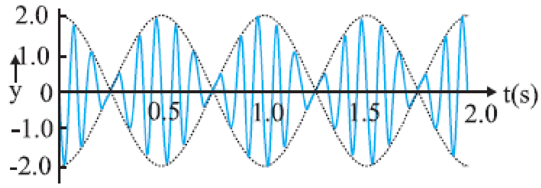
Thus we hear a waxing and waning of sound with a frequency equal to the difference between the frequencies of the superposing waves. The time-displacement graphs of two waves of frequency 11 Hz and 9 Hz is shown in Figs 7. (a) and (b). The result of their 'superposition' giving rise to beats of frequency 2 Hz. is shown in Fig. (c).



(a)



(b)



(c)

9. Explain Doppler effect ? Analyse doppler effects when (1) observer is stationary but the source is moving, (2) observer is moving but the source is stationary, and (3) both the observer and the source are moving.

When we approach a stationary source of sound with high speed, the pitch of the sound heard appears to be higher than that of the source. As the observer recedes away from the source, the observed **pitch** (or frequency) becomes lower than that of the source. This motion-related frequency change is called **Doppler effect**. The Austrian physicist Johann Christian Doppler first proposed the effect in 1842. Buys Ballot in Holland tested it experimentally in 1845. Doppler effect is a **wave phenomenon**, it holds not only for sound waves but also for electromagnetic waves.

We shall analyse changes in frequency under three different situations:

(1) Observer is stationary but the source is moving :
Let us choose the convention to take the direction from the observer to the source as the positive direction of velocity. Consider a source S moving with velocity v_s and an observer who is stationary in a frame in which the medium is also at rest. Let the speed of a wave of angular frequency ω and period T_0, both measured by an observer at rest with respect to the medium, be v. We assume that the observer has a detector that counts every time a wave crest reaches it. As shown in Fig. 5, at time $t = 0$ the source is at point S_1, located at a distance L from the observer, and emits a crest. This reaches the observer at time $t_1 = L/v$. At time $t = T_0$ the source has moved a distance $v_s T_0$ and is at point S_2, located at a distance $(L + v_s T_0)$ from the observer. At S_2, the source emits a second crest. This reaches the observer at

$$t_2 = T_0 + \frac{(L + v_s T_0)}{v}$$

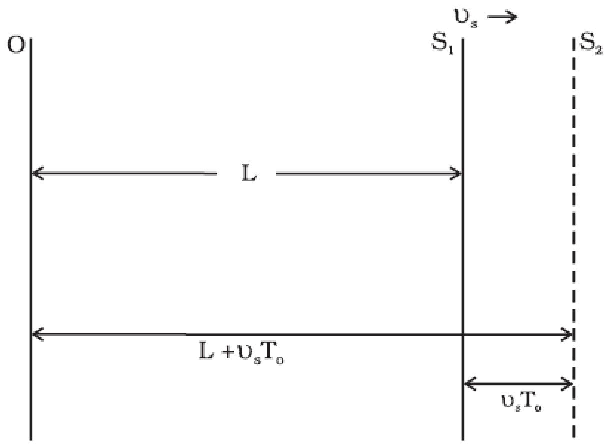


Fig. 5: A source moving with velocity v_s emits a wave crest at the point S_1. It emits the next wave crest at S_2 after moving a distance $v_s T_0$.

At time $n\,T_0$, the source emits its $(n+1)^{th}$ crest and this reaches the observer at time

$$t_{n+1} = nT_0 + \frac{(L + nv_sT_0)}{v}$$

Hence, in a time interval

$$\left[nT_0 + \frac{(L+nv_sT_0)}{v} - \frac{L}{v}\right]$$

the observer's detector counts n crests and the observer records the period of the wave as T given by

$$T = \left[nT_0 + \frac{(L+nv_sT_0)}{v} - \frac{L}{v}\right] / n$$

$$= T_0 + \frac{v_s T_0}{v}$$

$$= T_0 \left(1 + \frac{v_s}{v}\right) \quad \text{---------} \quad (1)$$

Equation (1) may be rewritten in terms of the frequency ν_o that would be measured if the source and observer were stationary, and the frequency ν observed when the source is moving, as

$$\nu = \nu_0 \left(1 + \frac{v_s}{v}\right)^{-1} \quad \text{---------} \quad (2)$$

If v_s is small compared with the wave speed v, taking binomial expansion to terms in first order in v_s/v and neglecting higher power, Eq. (2) may be approximated, giving

$$\nu = \nu_0 \left(1 - \frac{v_s}{v}\right) \quad \text{---------} \quad (3)$$

For a source approaching the observer, we replace v_s by $-v_s$ to get

$$\nu = \nu_0 \left(1 + \frac{v_s}{v}\right) \quad \text{----------} \quad (4)$$

The observer thus measures a lower frequency when the source recedes from him than he does when it is at rest. He measures a higher frequency when the source approaches him.

(2) Observer is moving but the source is stationary :

In this case we have to consider a reference frame of the moving observer. In this reference frame the source and medium are approaching at speed v_o and the speed with which the wave approaches is $v_o + v$.

Here the time interval between the arrival of the first and the $(n+1)^{th}$ crests is

$$t_{n+1} - t_1 = n\ T_0 - \frac{n\upsilon_0 T_0}{\upsilon_0 + \upsilon}$$

The observer thus, measures the period of the wave to be

$$= T_0\left(1 - \frac{\upsilon_0}{\upsilon_0 + \upsilon}\right)$$

$$= T_0\ \left(1 + \frac{\upsilon_0}{\upsilon}\right)^{-1}$$

giving

$$\upsilon = \upsilon_0\left(1 + \frac{\upsilon_0}{\upsilon}\right) \quad \text{----------} \quad (5)$$

If $\frac{\upsilon_0}{\upsilon}$ is small, the Doppler shift is almost same whether it is the observer or the source moving since Eq. (5) and the approximate relation Eq. (3) are the same.

(3) Both the observer and the source are moving :

Let us take the direction from the observer to the source as the positive direction. Let the source and the observer be moving with velocities v_s and v_o respectively as shown in Fig.8. Suppose at time $t = 0$, the observer is at O_1 and the source is at S_1, O_1 being to the left of S_1. The source emits a wave of velocity v, of frequency ν and period $T0$ all measured by an observer at rest with respect to the medium. Let L be the distance between O_1 and S_1 at $t = 0$, when the source emits the first crest. Now, since the observer is moving, the velocity of the wave relative to the observer is $v+v_0$. Therefore the first crest reaches the observer at time $t_1 = L/(v+v_0)$. At time $t = T_0$, both the observer and the source have moved to their new positions O_2 and S_2 respectively. The new distance between the observer and the source, $O_2\ S_2$, would be $L+(v_s–v_0)\ T_0]$. At S_2, the source emits a second crest. This reaches the observer at time.

$$t_2 = T_o + [L + (\upsilon_s - \upsilon_o)T_o)]\ /(\upsilon + \upsilon_o)$$

At time nT_o the source emits its $(n+1)^{th}$ crest and this reaches the observer at time

$$t_{n+1} = nT_o + [L + n\,(\upsilon_s - \upsilon_o)T_o)]\ /(\upsilon + \upsilon_o)$$

Hence, in a time interval $t_n+1\ \ –t_1$, i.e.,

$$nT_o + [L + n\,(\upsilon_s - \upsilon_o)T_o)]\ /(\upsilon + \upsilon_o) - L\,/(\upsilon + \upsilon_o),$$

the observer counts n crests and the observer records the period of the wave as equal to T given by

$$T = T_0\left(1 + \frac{\upsilon_s - \upsilon_o}{\upsilon + \upsilon_0}\right) = T_0\left(\frac{\upsilon + \upsilon_s}{\upsilon + \upsilon_0}\right)$$

The frequency ν observed by the observer is given by

$$\nu = \nu_0\left(\frac{\upsilon + \upsilon_0}{\upsilon + \upsilon_s}\right)$$

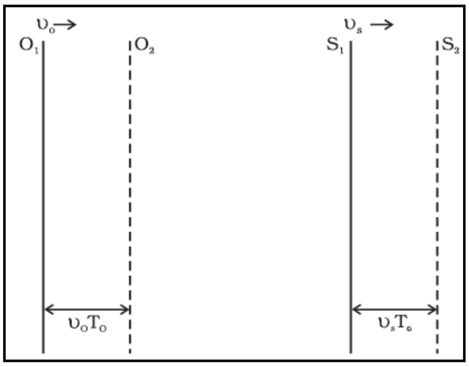


Fig. 8 : Both Observer and the source are moving

Consider a passenger sitting in a train moving on a straight track. Suppose she hears a whistle sounded by the driver of the train. What frequency will she measure or hear? Here both the observer and the source are moving with the same velocity, so there will be no shift in frequency and the passanger will note the natural frequency. But an observer outside who is stationary with respect to the track will note a higher frequency if the train is approaching him and a lower frequency when it recedes from him.

We have defined the direction from the observer to the source as the positive direction. Therefore, if the observer is moving towards the source, v_0 has a positive (numerical) value whereas if O is moving away from S, v_0 has a negative value. On the other hand, if S is moving away from O, v_s has a positive value whereas if it is moving towards O, v_s has a negative value. The sound emitted by the source travels in all directions. It is that part of sound coming towards the observer which the observer receives and detects. Therefore the relative velocity of sound with respect to the observer is $v + v_0$ in all cases.

If there is no medium present, the Doppler shifts are same irrespective of whether the source moves or the observer moves, since there is no way of distinction between the two situations.

POINTS TO PONDER

1. A wave is not motion of matter as a whole in a medium. A wind is different from the sound wave in air. The former involves motion of air from one place to the other. The latter involves compressions and rarefactions of layers of air.
2. In a wave, energy and *not the matter* is transferred from one point to the other.
3. Energy transfer takes place because of the coupling through elastic forces between neighbouring oscillating parts of the medium.
4. Transverse waves can propagate only in medium with shear modulus of elasticity, Longitudinal waves need bulk modulus of elasticity and are therefore, possible in all media, solids, liquids and gases.
5. In a harmonic progressive wave of a given frequency all particles have the same amplitude but different phases at a given instant of time. In a stationary wave, all particles between two nodes have the same phase at a given instant but have different amplitudes.
6. Relative to an observer at rest in a medium the speed of a mechanical wave in that medium (v) depends only on elastic and other properties (such as mass density) of the medium. It does not depend on the velocity of the source.
7. For an observer moving with velocity v_o relative to the medium, the speed of a wave is obviously different from v and is given by $v \pm v_o$.

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One Mark questions.

1. What is a wave?

A wave is a sort of disturbance which is transmitted in a medium without the bulk movement of particles of the medium.

Or

A wave is a sort of disturbance when a group of particles of the medium are disturbed, the pattern of disturbance that travels through the medium due to the periodic motion of the particles of the medium about their equilibrium position, with the transfer of energy and momentum and without the transfer of matter(particles) is called a wave.

2. What is a progressive wave?

A wave (disturbance) that travels continuously from one point of medium to another is called a progressive wave.

3. Does, all the waves requires a material medium for their propogation?

No

4. Does, a wave carry energy?

Yes

5. Name the properties of a medium which are responsible for the propogation of a mechanical wave?

Elastic and Inertial properties of medium

6. What are matter waves?

The wave associated with moving material particles are called matter waves.

7. Name the kind of wave that are employed in the working of an electron microscope?

Matter waves associated with electrons.

8. Define amplitude of a wave.

Amplitude is the maximum displacement of the particle on either side of the equilibrium position during wave propogation.

9. Define period of a wave.

It is the time taken by a wave to move through a distance of wavelength during wave propagation.

10. Define frequency of a wave.

It is the number of waves crossing a given cross section per second during wave propagation.

11. Define wavelength of a wave.

The distance between two consecutive particles of medium which are in the same state of vibration (phase) is called as wavelength.

12. Define wave velocity.

Wave velocity is defined as the distance traveled by the wave in one second.

13. Define phase of a vibrating particle?
The phase of a vibrating particle at a given instant of time is the state of vibration of a particle at that instant of time with reference to its equilibrium position.

14. Define propogation constant (or) angular wave number.
It is the number of waves that can be accommodated per unit length.

15. How is propogation constant related to wavelength of a wave?
$k = 2\pi/\lambda$

16. Name the factors which determine the speed of a propogation of an electromagnetic wave?
Permitivity and permeability of the medium

17. Name the quantity associated with a wave that remains unchanged when a wave travel from one medium to another?
Frequency of the wave

18. Name the quantities associated with a wave, that changes when a wave travels from one medium to another.
Wavelength and velocity of the wave.

19. What is sound?
Sound is a form of energy that produces a sensation of hearing.

20. How is sound produced?
Vibrating bodies surrounded by a material medium produces sound.

21. Why do we see the flash of lightening before we hear the thunder?
Because speed of light is much greater that the speed of sound.

22. What is a stationary wave?
When two progressive waves of equal amplitude, frequency and speed traveling in a medium along the same line but in opposite direction superimpose, the resulting waveform appears to be stationary pattern, such a wave is called stationary wave.

23. How much energy is transported by a stationary wave?
zero

24. What is a node?
Nodes are certain location (position) in a stationary wave, where the particles of the medium are completely at rest (zero displacement)

25. What is an antinode?
Antinodes are certain location (position) in a stationary wave, where the particle of

the medium vibrate with maximum displacement.

26. What is a segment (or) loop in a stationary wave?
The wave form (or) region between two consecutive node in a stationary wave is called a loop (or) segment.

27. What is the length of a loop in a stationary wave in terms of wavelength?
$\lambda/2$ (or) half of the wavelength

28. How much is the distance between a node and its neighbouring antinode?
$\lambda/4$

29. How much is the distance between a node and its neighbouring node.
$\lambda/2$

30. What happens to a wave, if it meets a rigid boundary?
The waves gets reflected (assuming that there is no absorption of energy by the boundary)

31. What happens to a wave, if it meets a boundary which is not completely rigid?
A part of the incident wave gets reflected and part of incident wave gets transmitted into the other medium (assuming that there is no absorption of energy by the boundary)

32. What is the phase angle between the incident wave and the wave reflected at a rigid boundary?
π radian (or) 180 degree

33. What is the phase angle between the incident wave and the wave reflected at a open boundary?
No phase change (or) zero.

34. Give the relation between phase difference and path difference.
Phase difference = $\left(\frac{2\pi}{\lambda}\right)$ path difference. Where l is the wavelength of the wave.

35. What are normal modes of oscillation in a stationary wave?
In a stationary wave, the possible frequencies of oscillation of the system is characterized by a set of natural frequencies called as normal modes of oscillation.

36. What is the meaning of the fundamental mode (or) first harmonic of oscillation in a stationary wave?
In a stationary wave, the oscillation of the system with lowest possible natural frequency is called as fundamental frequency (or) first harmonic.

37. What are harmonics in a stationary wave?
For a vibrating system the frequencies which are integral multiples of fundamental frequency are called harmonics.

38. What are overtones in a stationary wave?
For a vibrating system, frequencies greater than fundamental frequencies are called overtones.

39. What is resonance?
In case of forced vibration, when the frequency of the external agent causing vibration (applied force) becomes equal to natural frequency of the vibrating body. The body vibrates with maximum amplitude. This phenomena is called "Resonance".

40. What are beats?
The periodic waxing (increase (or) rise) and waning (decrease (or) fall) in the intensity of sound due to superposition of two sound waves of nearly same frequencies traveling in same direction are called beats.

41. What is beat period?
The time interval between two consecutive waxing (or) waning is called as beat period.

42. What is Doppler effect?
The apparent change in the frequency (pitch) of sound heard by the listener due to relative motion between the source producing the sound and the listener is called as Doppler effect.

43. Which harmonics are absent in a closed organ pipe?
Even harmonics.

44. Give the formula for speed of transverse wave on a stretched string.
$v = \sqrt{\frac{T}{\mu}}$ where T is the tension in the string and μ is the linear mass density

45. What is the increase in the speed of sound in air when the temperature of the air rises by 1^oC?
The speed of sound increases approximately by 0.61 ms^{-1} per degree centigrade rise in temperature.

46. Why a transverse mechanical wave cannot travel in gases?
Shear modulus of elasticity is absent in gaseous medium, which is necessary for the propogation of transverse wave.

47. How does the velocity of sound in air vary with temperature?
v a $\sqrt{T}$ i.e. velocity of sound in air is directly proportional to square root of its absolute temperature.

48. How does the velocity of sound in air vary with pressure?
Velocity of sound in air is independent of pressure provided temperature remains constant.

49. Give the dimensional formula for propogation constant.
$[L^{-1}]$

50. The fundamental frequency of a closed pipe is 80Hz. What is the frequency of first overtone.
Frequency of I overtone in closed pipe = 3 (fundamental frequency) = 3 x 80 = 240 Hz

51. Calculate the wavelength of a wave whose angular wave number is 10p radian m^{-1}?
$\lambda = 2\pi/k = 2\pi/10\pi = 0.2$ m

52. The distance between a node & an next antinode in a stationary wave pattern is 0.08m. What is the wavelength of the wave?
The distance between node & an next antinode is $\lambda/4$, $\therefore \lambda/4 = 0.8$, $\lambda = 0.32$ m

53. How is the frequency of an air column in an open pipe related with the temperature of air?
Frequency of air column in an open pipe is directly proportional to square root of its absolute temperature ($v \ \alpha \ \sqrt{T}$)

54. A sound wave has a velocity of 330 ms-1 at one atmospheric pressure. What will be its velocity at 4 atmospheric pressure?
330 ms^{-1}

55. What happens to the frequency of the wave when it travels from water to air?
Frequency remains the same

56. Give the relation between time period and frequency of a wave.
Time period = 1/(*frequency)*

57. Is Doppler effect observed for sound waves only?
No.

58. What is the distance between two consecutive antinodes in a stationary wave of wavelength 2m?
1 m

59. How does speed of a transverse wave on a stretched string vary with its tension?
Speed of a transverse wave on a stretched string is directly proportional to square of its tension. i.e. ($v \ \alpha \ \sqrt{T}$)

60. With what velocity does an electro-magnetic wave travel in vacuum.
3×10^8 ms^{-1}

61. What is a pulse?
Short duration wave motion is called a pulse.

62. What is the condition to be obeyed for receiving an echo?
The separation between the reflecting surface and source of sound 'd' should be greater than (1/10) th of the velocity of sound, since persistence of sound in human ears is 1/10 sec.

63. Name the factors affecting the velocity of sound in a medium.
Temperature, density and ratio γ affect the velocity of sound in a medium.

64. Why stationary waves do not transport energy?
Since nodes and antinodes formed remain stationary, the energy remains confined to one region. It cannot overcome the pressure maxima at nodes. So energy is not transmitted by standing waves.

65. When will Doppler effect in sound be symmetrical?
When the velocity of the source or observer is very much less than the velocity of sound.

66. Define wavelength. Name its S.I unit.
It is defined as the distance traveled by the disturbance in a medium during the time any particle of the medium completes one vibration about its mean position. It is denoted by γ . Its S.I. units is m.

67. Define amplitude. Name its S.I unit.
It is the maximum displacement of a particle from its mean position. Its S.I. unit is m.

68. If density of oil higher than density of water is used in a resonance tube, how will the frequency change?
The frequency will not change. Because frequency depends on length of air column above the liquid surface in the tube.

69. Why should a bat be able to sense high frequencies?
Due to less inertia, the eardrum of bats can resonate faster than human ears. So they can receive high frequency.

70. Why bells are made of metal and not of wood?
Metals have less damping than wood. So bells are made of metals.

71. Air gets thinner as we go up in the atmosphere. Will the velocity of sound change?
As we move up the pressure (P) of air, both decrease.
Therefore velocity of sound will not change. So long as temperature T of air remains constant.

72. Why do we see the flash of lighting first and hear the thunder later?
Since light travels much faster than sound, the flash of lightning is seen first, and the thunder is heard later.
73. The apparent frequency of the whistle of an engine changes in the ratio 3 : 2 as the engine passes a stationary observer. If the velocity of sound is 330 m/s, calculate the velocity of the engine?
$v_1/v_2 = (c+u_0) / (c-u_0) = 3/2$
Solving we get, $u_0 = c/5 = 330/5 = 66\ ms^{-1}$

Two Mark questions:

1. What are mechanical waves? Give two examples.
The waves that requires material medium for their propogation (transmission) are called as mechanical waves. Eg. Waves on a surface of water, sound waves, seismic waves etc.

2. What are non-mechanical waves? Give two examples.
The waves that do not require material medium for their propogation are called as non-mechanical waves. Eg. Radio waves, light waves, x-rays etc.

3. What are longitudinal waves? Give two examples
The waves in which the particles of the medium oscillates parallel (along) to the direction of wave propagation are called longitudinal waves. Eg. Sound waves, waves set up in air column.

4. What are Transverse waves? Give two examples.
The waves in which the particles of the medium oscillate perpendicular to the direction of wave propagation are called Transverse waves. Eg. Light waves, waves on the surface of water, waves on a string.

5. Obtain the relation connecting v, u and l where symbols have their usual meaning.
Consider a wave traveling with a velocity 'V' let u be its frequency & l be its wavelength. In a time equal to its time period T, the wave covers a distance equal to its wavelength l.
By the definition of wave velocity we have
Wave velocity V = [distance travelled / Time taken] $= \lambda/T = \upsilon.\lambda$

6. Calculate the period of a wave of wavelength 0.005 m which travels with a speed of 50 $cm.s^{-1}$.
Given l = 0.005 m ; V = 50 cm s-1 = 50 × 10-2 ms-1
We have $V = \lambda/T$, $\therefore$ $T = \lambda/v = 0.005/50x10^{-2} = 0.01$ sec.

7. The frequency of a tuning fork is 256 Hz and sound travels a distance of 25m while the fork executes 20 vibrations. Calculate the wavelength and velocity of the sound wave.
Wavelength l = 25/20 = 1.25 m
$V = \upsilon\lambda$
V = 256 x 1.25
$V = 320\ ms^{-1}$

8. Velocity of sound in air is 340 ms-1. Two sound waves of frequency 1 KHz each interfere to produce stationary wave. What is the distance between two successive nodes?
Given $v = 340\ ms^{-1}$; υ = 1 KHz = 1000 Hz.
We have $\lambda = v/\upsilon = 340/1000 = 0.34$ m
\ Distance between two successive node $= \lambda/2 = 0.34/2 = 0.17$ m

9. When is the fundamental frequency of the sound emitted by a closed pipe is same as that emitted by an open pipe?

Let ℓ_o & ℓ_c be the length of open & closed pipe respectively

Given $(\upsilon_{\text{fundamental}})_{\text{Open pipe}} = (\upsilon_{\text{fundamental}})_{\text{closed pipe}}$

$$\frac{V}{2\ell_o} = \frac{V}{4\ell_c}$$

$$\frac{\ell_c}{\ell_o} = \frac{1}{2}$$

$$\therefore \ell_c = \frac{\ell_o}{2}$$

10. A closed pipe & open pipe have same frequency for the first overtone. What is the ratio of their lengths?

Let ℓ_o & ℓ_c be the length of open and closed pipe respectively

Given & $(\upsilon_{\text{first overtone}})_{\text{closed pipe}} = (\upsilon_{\text{first overtone}})_{\text{open pipe}}$

$$\frac{3V}{4\ell_c} = \frac{2V}{2\ell_o}$$

$$\frac{\ell_c}{\ell_o} = \frac{3}{4}$$

11. For what wavelength of waves, does a closed pipe of length 30 cm emit the first overtone?

The frequency of the first overtone in a closed pipe is given by

$$\upsilon = \frac{3V}{4\ell} = \frac{3V}{4\times 30} = \frac{V}{40}$$

But $V = \upsilon\lambda$ $\quad \therefore \upsilon = \frac{\upsilon\lambda}{40}$

$$\therefore \lambda = 40 \text{ cm}$$

12. The second overtone of closed pipe of length 1 m is in unison with the third overtone of an open pipe. What is the length of the open pipe?

Given $(\upsilon_{\text{II overtone}})_{\text{Closed pipe}} = (\upsilon_{\text{III overtone}})_{\text{open pipe}}$

$$\frac{5V}{4\ell_c} = \frac{4V}{2\ell_o}$$

$$\ell_o = \frac{8}{5}\ell_c$$

$$\ell_o = \frac{8}{5}(1) = 1.6 \text{ m}$$

13. The velocity of a sound wave decreases from 330 ms-1 to 220 ms-1 on passing from one medium to another. If the wavelength in the first medium is 3m. What is the wavelength in the second medium?

We have $v = \upsilon\lambda$

$$\therefore \frac{V_1}{V_2} = \frac{\cancel{\nu}\lambda_1}{\cancel{\nu}\lambda_2}$$

Where V_1 & V_2 are velocity of sound in first and second medium respectively and λ_1 & λ_2 are the corresponding wavelength.

$$\lambda_2 = \frac{V_2}{V_1}\lambda_1$$

$$\lambda_2 = \left(\frac{220}{330}\right)\times 3$$

$$\lambda_2 = 2\text{m}$$

14. Can sound waves of wavelength 33mm be heard in air? Justify.

Frequency $\upsilon = v/\lambda$

$$\upsilon = \frac{330}{33\times10^{-3}} = 10 \text{ KHz}$$

Since this wave belong to audible range of sound they can be heard.

15. At what temperature will the velocity of sound becomes 1.25 times that at 27oC?

We have $V \;\alpha\; \sqrt{T} = \sqrt{t+273}$

Given $V_{t^\circ C} = 1.25\, V_{27^\circ C}$

$$\therefore \frac{V_{t^\circ C}}{V_{27^\circ C}} = \frac{\sqrt{t+273}}{\sqrt{27+273}}$$

$$\frac{1.25V_{27^\circ C}}{V_{27^\circ C}} = \frac{\sqrt{t+273}}{\sqrt{300}}$$

$\therefore$ t = 195.75°C

16. A musical note produces 2 beats per second. When sounded with a tuning fork of frequency 340Hz & 6 beats per second when sounded with a tuning fork of frequency 344 Hz. Find the frequency of the musical note?

We have $\upsilon_b = \upsilon_1 \sim \upsilon_2$

Given I case υ_1 = 340 Hz & υ_b = 2 beats per second

$\therefore$ possible values of υ_2 = 342 Hz (or) 338 Hz.

II case υ_1 = 344 Hz & υ_b = 6 beats per second

$\therefore$possible values of υ_2 = 338 Hz (or) 350 Hz

$\therefore$frequency of the musical note is υ_2 = 338 Hz.

17. At which positions (or) locations of the stationary wave, the pressure changes are maximum and minimum.

Pressure changes are maximum at Node and pressure changes are minimum at antinode.

18. At which positions (or) location of the stationary wave, the displacement is maximum and minimum.

Displacement is maximum antinode and displacement is minimum at node.

19. Calculate the velocity of sound at -30oC and 30oC given the velocity of sound at 0oC is 330 ms-1.

We have $V \propto \sqrt{T}$

$$\therefore \frac{V_{-30^o C}}{V_{0^o C}} = \frac{\sqrt{-30+273}}{\sqrt{0+273}}$$

$$V_{-30^o C} = 311.3 \text{ ms}^{-1}$$

$$\frac{V_{30^o C}}{V_{0^o C}} = \frac{\sqrt{30+273}}{\sqrt{0+273}}$$

$$V_{30^o C} = 347.7 \text{ ms}^{-1}$$

20. Give any two applications of Doppler's effect?

Sonography, echocardiogram and speed of vehicles.

21. With what velocity should a sound source travel towards a stationary observer so that the apparent frequency may be double of the actual frequency.

When a source travel towards a stationary listener, apparent, frequency is given by

$$\upsilon = \upsilon_o \left(\frac{V}{V - V_S}\right)$$

$$2\upsilon_o = \upsilon_o \left(\frac{V}{V - V_S}\right)$$

$$\therefore V_S = \frac{V}{2}$$

22. A bat emits ultrasonic sound of frequency 1000KHz in air. If sound meets a water surface, what is the wavelength of a) reflected sound b) transmitted sound?
(Given speed of sound in air is 340 ms^{-1} & in water 1486 ms^{-1}?

$$\lambda_{\text{reflected sound}} = \frac{V_{air}}{\upsilon}$$

$$\lambda_\upsilon = \frac{340}{1000\times10^3}$$

$$\lambda_\upsilon = 3.4 \times 10^{-4} \text{ m}$$

$$\lambda_{\text{transmitted sound}} = \frac{V_{water}}{\upsilon}$$

$$\lambda_t = \frac{1486}{1000\times10^3}$$

$$\lambda_t = 1.486 \times 10^{-3} \text{ m}$$

23. The sitar strings A & B playing the note 'Ga' are slightly out of tune & produce beats of frequency 6 Hz. The tension in the string A is slightly reduced & the beat frequency is found to reduce to 3 Hz. If the original frequency of A is 324 Hz. What is the frequency of B?

We have $\upsilon_b = \upsilon_A - \upsilon_B$

I case $\upsilon_b = 6$ Hz, $\upsilon_A = 324$ Hz

$\therefore$ possible values of $\upsilon_B = 318$ Hz (or) 330 Hz

II case when tension in the string A is reduced, its frequency (υ_A) also decreases, the new beat frequency is given to be 3 Hz. This is possible only if $\upsilon_B = 318$ Hz.

24. A sinusoidal wave propogating through air has a frequency of 200 Hz. If the wave speed is 300 ms^{-1}, how far apart are the two points (path difference) with a phase of difference of 60^o.

Given $\upsilon = 200$ Hz ; v = 300 ms^{-1}; Phase difference = 60^o (or) $\pi/3$ radian

$$\text{We have } \lambda = \frac{V}{\upsilon} = \frac{300}{200} = 1.5 \text{ m}$$

$$\text{Path difference} = \frac{\lambda}{2\pi} \text{ phase difference}$$

$$\therefore \text{path difference} = \frac{1.5}{2\cancel{\pi}} \times \frac{\cancel{\pi}}{3}$$

Path difference = 0.25m

4 and 5 marks questions :

1. Give the differences between progressive and stationary waves.

Progressive Wave	Stationary wave
a) The wave travel continuously with certain velocity called wave velocity	The waves does not move. It remains localized.
b) The propogation of the disturbance from particle due to elastic properties of the medium give rise to a progressive wave.	The superposition of two identical waves traveling in opposite direction along the same line results in a stationary wave.
c) Amplitude of vibration is the same for every particle of the medium along the wave	The amplitude of vibration varies from zero at node & maximum at antinode.
d) Different particles over a distance λ different phases at a given instant of time.	All particles lying in a loop have same phase at a given instant of time.
e) No particles in the medium is completely at rest.	The particles at node are permanently at rest.
f) There is a net transfer of energy in the direction of propogation of wave.	There is no net transfer of energy across any section of the medium.
g) The wave equation is of the form $y(x, t) = A \sin(wt-kx)$ i.e , $y(x, t) = f(x, t)$	The wave equation is of the form. $y(x,t) = 2A \cos(Kx) \sin wt$. i.e. $y(x,t) = f(x). g(t)$

2. Give the differences between mechanical and a non mechanical (electromagnetic) waves.

Mechanical wave	Non-mechanical wave
a) Requires a material medium for their propogation	Do not require a material medium for their propogation.
b) Particles of the medium oscillate.	Electric and magnetic field oscillate.
c) Can be longitudinal (or) transverse in nature.	Are always transverse in nature.
d) Travels at relatively lower speed in a medium.	Travels at relatively higher speed in a medium
e) Doppler effect is asymmetric Eg. Sound waves	Doppler effect is symmetric Eg.Light waves.

3. Give the differences between longitudinal and transverse waves.

Longitudinal wave	Transverse wave
a) The particles of the medium oscillate along (parallel to) the direction of propogation of the wave.	The particles of the medium (electric & magnetic fields) oscillate at right angles (perpendicular) to the direction of the propogation of the waves.
b) These waves travels in alternate compression & rarefaction. (compression are the region of higher density & rarefaction are the regions of lower density).	These waves travels in alternate crests and troughs. (crests are highly raised portions of a wave & troughs are highly depressed portion of a wave).
c) Are always mechanical wave.	They are either mechanical (or) non mechanical wave.
d) The pressure and density varies as the wave propogates	The pressure and density donot vary as the wave propogates.
e) They can travel in solids, liquids and gases.	They can travel in solids and on the Surface of liquids, if the waves are mechanical.
f) These waves cannot be polarized.	These waves can be polarized.
g) Velocity of a longitudinal wave in a gas is given by $v = \sqrt{\frac{B}{\rho}}$ where B is the bulk modulus and ρ is the density. Eg. Sound waves.	Velocity of a transverse wave on a stretched string is given by $v = \sqrt{\frac{T}{\mu}}$ Where T is the tension & μ is linear Density. Eg. Waves on a string. Light wave.

4. Write Newton's formula for speed of sound in a gas. Discuss Laplace correction & arrive at the formula modified by him.
Refer Notes

5. Mention the characteristics of a progressive mechanical wave.
Characteristics of a progressive mechanical wave :
• The waves (disturbance) produced at any point in a medium is propogated by continuous periodic oscillation of the particles about their mean positions.
• The elastic and inertial properties of the medium are responsible for wave propogation.
• There is a transfer of energy and momentum from the source of vibration away from it in the form of disturbance. However, the particles of the matter themselves do not move away and they perform only simple harmonic motion about their mean position.
• Waves in a homogeneous medium travel with constant velocity at a given temperature. However, particle velocity is different.
• Every particle along the wave vibrate with same frequency and amplitude about their mean position but the phase of the different particles are different at given instant of time.
• The waves undergo reflection, refraction, interference and diffraction.
• Longitudinal waves do not show polarization whereas the transverse waves show polarization.
• Wave propogation is longitudinal inside the liquids and gases. Wave propogation can be transverse on a liquid surface and on the strings. Wave propogation can be either longitudinal (or) transverse inside a solid.

6. Mention the characteristics of a stationary wave.
Characteristics of a stationary wave :
• Stationary wave remain localized between two fixed points. i.e., waves do not move in a medium
• In a stationary wave, there exist certain points called Nodes and antinodes. At a Node, the amplitude of vibration is zero and at an antinode, the amplitude of vibration is maximum.
• Nodes and antinode are equally spaced. The distance two consecutive nodes (or) antinodes is equal to half the wavelength (λ) [i.e., $\lambda/2$]
• Always a node exist between two successive antinodes and vice-versa. The distance between a node and next antinode is (1/4)th the wavelength (λ) i.e., $\lambda/4$.
• The amplitude of vibration increases from zero to maximum between a node and an neighbouring antinode.
• Except at nodes, all the points of the medium in a segment (or) loop vibrate with the same phase, but the points in the adjacent segment vibrate in opposite phase.
• There is no net transfer of energy across any segment of the stationary wave.

7. What are beats? Give the theory of beats.
Refer Notes

8. What is Doppler effect? Derive an expression for the apparent frequency when a source moves towards a stationary listener.
Refer Notes

9. What is Doppler effect? Derive an expression for the apparent frequency when a listener moves towards a stationary source.
Refer Notes

10. What is Doppler effect? Derive an expression for the apparent frequency when the source and listener are moving in the same direction.
Refer Notes

11. Discuss the effect of pressure, temperature & humidity on the velocity of the sound through air.
Refer Notes

12. Discuss different modes of vibration on a stretched string.
Refer Notes

13. Discuss different modes of vibration (first three harmonics) produced in a open pipe.
Refer Notes

14. Discuss different modes of vibration (first three harmonics) produced in a closed pipe.
Refer Notes

5 marks problems

1. A stone dropped from the top of tower of height 300 m high splashes into the water of a pond near the base of a tower. When is the splash heard at the top given that the speed of sound in air is 340 ms^{-1}? (given g = 9.8 ms^{-2})
ANS :
Let time taken by stone to reach the surface of water dropped from top of the tower of height 300 m be t_1

$\therefore$ we have $x = \upsilon_o t_1 + \frac{1}{2} a t_1^2$

$-300 = o\,(t_1) + \frac{1}{2}(-9.8)\, t_1^2$

$\therefore\ t_1 = 7.82$ s

Let the time taken by sound to reach the person on the top of tower be t_2.

$\therefore\ t_2 = \dfrac{distance\ \ covered\ \ by\ \ sound\ \ wave}{velocity\ \ of\ \ sound\ \ wave}$

$t_2 = \frac{300}{340} = 0.88$ s

$\therefore$ The splash of sound is heard after a time (t) equal $t_1 + t_2$.
i.e., $t = t_1 + t_2$
$= 7.82 + 0.88$
$T = 8.7$ s

2. A transverse harmonic wave on a string is described by $y(x, t) = 3 \sin (36\ t + 0.018x + \frac{\pi}{4})$ where x & y are in cm and t is in s.
i) Is the wave traveling (or) stationary.
ii) What is the direction of its propogation

iii) What is its frequency?
iv) What is its initial phase?
v) What is the distance between two consecutive crests in the wave?
ANS :

Given equation is y =(x, t) = 3 sin (36t + 0.018x + $\frac{\pi}{4}$) → (1)

This equation is of form y (x, t) = a sin (ωt + kx + ϕ) → (2)

i) It is a traveling wave
ii) It travels from right to left (i.e, along the negative x direction)
iii) Comparing equation (1) & (2) we get

$\omega = 36$

$2\pi\upsilon = 36$

$\upsilon = \frac{36}{2\pi}$

$\upsilon = 5.73$ Hz

iv) Initial phase = $\frac{\pi}{4}$radian

v) Distance between two consecutive crests is its wavelength (λ)

Comparing (1) & (2) we have

$K = \frac{2\pi}{\lambda} = 0.018$

$\therefore \lambda = \frac{2\pi}{0.018}$

$\lambda = 349$ cm

$\therefore \lambda = 3.49$ m

3. The transverse displacement of a string (clamped at both ends) is given by y(x,t) = 0.06 sin $\left(\frac{2\pi}{3}\right)$ cos(120 pt) where x, y are m & t is in s. The length of the string is 1.5 m & its mass is 3 x 10^{-2} kg.
i) Does the function represent a traveling wave (or) a stationary wave?
ii) Interpret the wave as a superposition of two waves traveling in opposite directions what is the wavelength, frequency and speed of each wave?
iii) Determine the tension in the string.

Given equation is y(x, t) = 0.06 sin $\left(\frac{2\pi}{3}\right)$x. cos(120 πt) → (1)

Given equation is of form y(x, t) = 2a sin kx cos ωt → (2)

i) It is a stationary wave

ii) Comparing equation (1) and (2); we get $K = \frac{2\pi}{\lambda} = \frac{2\pi}{3}$

$\therefore \lambda = 3\text{m}$

$\omega = 2\pi\upsilon = 120\pi$

$\therefore \upsilon = 60 \text{ Hz}$

$V = \upsilon\lambda = 60 \times 3 = 180 \text{ ms}^{-1}$

$V = 180 \text{ ms}^{-1}$

iii) Given $m = 3 \times 10^{-2}$ kg

$\ell = 1.5$ m

$\therefore$ linear density $\mu = \dfrac{m}{\ell} = \dfrac{3\times10^{-2}}{1.5} = 2 \times 10^{-2} \text{ kg m}^{-1}$

We have $v = \sqrt{\dfrac{T}{\mu}}$

$\therefore T = V^2\mu$

$T = (180)^2 \times 2 \times 10^{-2}$

$T = 648$ N

4. A progressive wave is described by the equation $y = 1.2 \sin \pi \left(\frac{2t}{5} - \frac{x}{4}\right)$ where x & y are in m and t is in s. Determine the amplitude, wavelength, time period & speed of the wave?

Given equation is $y = 1.2 \sin\pi \left(\dfrac{2t}{5} - \dfrac{x}{4}\right)$

It can be rewritten as $y = 1.2 \sin \left(\dfrac{2\pi t}{5} - \dfrac{\pi x}{4}\right) \rightarrow \quad (1)$

This equation is of the form $y = a \sin(\omega t - kx) \rightarrow \quad (2)$

Comparing equation (1) & (2)

We get, a = 1.2 m

$K = \dfrac{2\pi}{\lambda} = \dfrac{\pi}{4}$

$\therefore \lambda = 8$ m

$\omega = \dfrac{2\pi}{T} = \dfrac{2\pi}{5}$

$\therefore T = 5$ s.

$V = \dfrac{\lambda}{T}$

$V = \dfrac{8}{5} = 1.6 \text{ ms}^{-1}$

$V = 1.6 \text{ ms}^{-1}$

5. A closed pipe of length 0.42 m and an open pipe both contain air at 35°C. The frequency of the first overtone of the closed pipe is equal to the fundamental frequency of the open pipe. Calculate the length of the open pipe and the velocity of sound in air at 0°C. Given that the closed pipe is in unison in the fundamental mode with a tuning fork of frequency 210 Hz.

Let ℓ_c & ℓ_o be the length of closed pipe and open pipe respectively

Given $\ell_c = 0.42$m t = 35°C

(frequency of first overtone) = (fundamental frequency)

Closed pipe open pipe

$$\text{i.e., } \frac{3V_{35^\circ C}}{4\ell_c} = \frac{V_{35^\circ C}}{2\ell_o} \rightarrow \quad (1)$$

Fundamental frequency of closed pipe = 210 Hz

$$\text{i.e., } \frac{V_{35^\circ C}}{4\ell_c} = 210 \text{ Hz} \quad \rightarrow \quad (2)$$

from (1) we get, $\ell_o = \frac{4\ell c}{6}$

$$\ell_o = \frac{4(0.42)}{6}$$

$$\ell_o = 0.28 \text{ m}$$

From (2) we get $V_{35^\circ C} = 210 \times 4\ \ell_c$

$$= 210 \times 4 \times 0.42$$

$$V_{35^\circ C} = 352.8 \text{ ms}^{-1}$$

We have $V \alpha \sqrt{T}$

$$\therefore \frac{V_{0^\circ C}}{V_{35^\circ C}} = \frac{\sqrt{0+273}}{\sqrt{35+273}}$$

$$\therefore V_{0^\circ C} = \sqrt{\frac{273}{308}} \times V_{35^\circ C}$$

$$V_{0^\circ C} = \sqrt{\frac{273}{308}} \times 352.8$$

$$V_{0^\circ C} = 332.15 \text{ ms}^{-1}$$

6. Two cars are moving with speeds of 54 kmhr^{-1} & 18 kmhr^{-1} in opposite direction along a straight road. The faster car sounds the horn with a note of frequency of 240 Hz. Calculate the number of waves received per second by a listener sitting in the other car when it (i) approaches (ii) recedes from the listener if the speed of the sound in air is 340 ms^{-1}.

The frequency (u) as heard by an observer due to motion of source and observer is given by

$$\upsilon = \upsilon_o \left(\frac{V+V_o}{V+Vs}\right) \rightarrow (1)$$

$V_o = V_{\text{second Car}} = 18$ km h^{-1}

$$= \frac{18 \times 7000}{3600}$$

$V_o = 5$ ms^{-1}

$V_S = V_{\text{first car}} = 54$ km h^{-1}

i) O → ← S

$V_o = 5$ ms^{-1} $V_S = 15$ ms^{-1}

$$\upsilon = \upsilon_o \left(\frac{V+V_o}{V+Vs}\right)$$

$$= \frac{54 \times 1000}{3600} = VS = 15 \text{ ms}^{-1}$$

Here V_o = +ve
& V_S = -ve

$\therefore$ equation (1) becomes $\upsilon = \upsilon_o \left(\frac{V + V_o}{V - Vs} \right)$

$$\upsilon = 240 \left[\frac{340 + 5}{340 - 15} \right]$$

$$\upsilon = 254.76 \text{Hz}$$

ii)

Here V_o = - ve
V_s = +ve

$\therefore$ equation (1) becomes $\upsilon = \upsilon_o \left(\frac{V - V_o}{V + Vs} \right)$

$$\upsilon = 240 \left[\frac{340 - 5}{340 + 15} \right]$$

$$\upsilon = 226.47 \text{Hz}$$

7. y = 1.4 sin π (300t – x) represents a progressive wave where x, y are in m & t is in s. Calculate the wave velocity & the phase difference between oscillatory motion of two points separated by a distance of 0.25 m.

Given equation is $y = 1.4 \sin \pi (300\, t - x)$

This can be rewritten as $y = 1.4 \sin (300\, \pi t - \pi x)$ ------- (1)

This equation is of the form $y = a \sin (\omega t - kx)$ -------- (2)

We have $V = \upsilon\lambda$ ------- (3)

Comparing equation (1) & (2)

We get, $\omega = 2\pi\upsilon = 300\, \pi$

$\therefore\ \upsilon = 150$ Hz

$$K = \frac{2\pi}{\lambda} = \pi$$

$\therefore \lambda = 2$m

$\therefore$ from (3) $V = 150 \times 2$

$V = 300 \text{ ms}^{-1}$

We have phase difference = $2\pi/\lambda$ (path difference)

$$\Delta\phi = \frac{2\pi}{\lambda} (0.25)$$

$$= \frac{2\pi}{2} (0.25)$$

$$\Delta\phi = \frac{\pi}{4} \text{ radian}$$

8. Compare the time taken by sound to travel a given distance in air and argon at NTP. Given that the density of air and argon are 1.293 & 1.789 respectively ratio of specific heat capacity for air and organ are 1.402 & 1.667 respectively.

Given distance traveled by sound in air = distance traveled by sound in argon.

i.e., $d_{air} = d_{argon}$

$$\frac{\gamma_{air}}{\gamma_{argon}} = \frac{1.402}{1.667} \quad \& \quad \frac{\rho_{air}}{\rho_{argon}} = \frac{1.293}{1.789}$$

& we know that $V_{sound} = \sqrt{\frac{\gamma P}{\rho}} \rightarrow (1)$

We have $\frac{V_{air}}{V_{argon}} = \frac{d_{air} \times t_{argon}}{t_{air} \times d_{argon}}$

$$\therefore \frac{t_{air}}{t_{argon}} = \frac{V_{argon}}{V_{air}} = \sqrt{\frac{\gamma_{argon} \times P \times \rho_{air}}{\gamma_{argon} \times \rho_{air} \times P}} \qquad \text{(from (1))}$$

$$\frac{t_{air}}{t_{argon}} = \sqrt{\frac{\gamma_{argon} \times \rho_{air}}{\rho_{argon} \times \gamma_{air}}}$$

$$\therefore \frac{t_{air}}{t_{argon}} = \sqrt{\frac{1.667}{1.402} \times \frac{1.293}{1.789}}$$

$$\frac{t_{air}}{t_{argon}} = 0.927$$

9. The speed of sound in hydrogen is 1270 ms^{-1}. What will be the speed of sound in a mixture of oxygen and hydrogen mixed in a volume ratio 1: 4?

If 'm' is the molecular mass of the given gas then, we have velocity of sound in gas is given by

$$V = \sqrt{\frac{\gamma RT}{m}}$$

$$\therefore \frac{V_{mixture}}{V_{hydrogen}} = \sqrt{\frac{\gamma RT}{m_{mixture}} \times \frac{m_{hydrogen}}{\gamma RT}}$$

$$\therefore \frac{V_{mixture}}{V_{hydrogen}} = \sqrt{\frac{m_{hydrogen}}{m_{mixture}}} \rightarrow (1)$$

$$m_{mixture} = \frac{(32\times1)+(2\times4)}{1+4}$$

$$m_{mixture} = 8$$

$$\therefore \text{ equation (1) becomes } \frac{V_{mixture}}{V_{hydrogen}} = \sqrt{\frac{2}{8}}$$

$$\therefore V_{mixture} = \frac{1}{2}\times V_{hydrogen}$$

$$= \frac{1270}{2}$$

$$V_{mixture} = 635 \text{ ms}^{-1}$$

10. A wire stretched between two rigid supports vibrates in its fundamental mode with a frequency of 45 Hz. The mass of the wire is 3.5×10^{-2} kg & its linear density is 4×10^{-2} kg m^{-1}. What is i) the speed of wave on the string. ii) the tension in the string ?

$$\text{Length of the wire } L = \frac{mass \quad of \quad the \quad wire}{Linear \quad density}$$

$$L = \frac{3.5\times10^{-2}}{4\times10^{-2}}$$

$$L = 0.875 \text{ m}$$

When the wire vibrates in its fundamental mode then

$$L = \frac{\lambda}{2} \Rightarrow \lambda = 2\times 0.875$$

$$\lambda = 1.75 \text{ m}$$

$\therefore$ speed of the transverse wave on the string is

$$\text{Given } V = \upsilon\lambda = 45\times 1.75$$

$$V = 78.75 \text{ ms}^{-1}$$

$$\text{ii) } V = \sqrt{\frac{T}{\mu}}$$

$$\therefore T = V^2\mu$$

$$T = (78.75)^2\times 4\times 10^{-2}$$

$$T = 248 \text{ N}$$

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TEXT BOOK EXERCISES

15.1 A string of mass 2.50 kg is under a tension of 200 N. The length of the stretched string is 20.0 m. If the transverse jerk is struck at one end of the string, how long does the disturbance take to reach the other end?

ANS :
Mass of the string, M = 2.50 kg
Tension in the string, T = 200 N
Length of the string, l = 20.0 m
Mass per unit length, $\mu = M/l = 2.50/20 = 0.125$ Kg m^{-1}
The velocity (v) of the transverse wave in the string is given by the relation:

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{200}{0.125}} = \sqrt{1600} = 40 \text{ m/sec}$$

∴ Time taken by the disturbance to reach the other end, t = l/v = 20/40 = 0.50 s.

15.2 A stone dropped from the top of a tower of height 300 m high splashes into the water of a pond near the base of the tower. When is the splash heard at the top given that the speed of sound in air is 340 m s^{-1} ? (g = 9.8 m s^{-2})

ANS :
Height of the tower, s = 300 m
Initial velocity of the stone, u = 0
Acceleration, a = g = 9.8 m/s^2
Speed of sound in air = 340 m/s
The time (t_1) taken by the stone to strike the water in the pond can be calculated using the second equation of motion, as: $s = ut_1 + 1/2\,(gt_1^2)$ => $300 = 0 + 1/2 \times t_1^2$

$$\therefore\ t_1 = \sqrt{\frac{300 \times 2}{9.8}} = 7.82 \text{ s}$$

Time taken by the sound to reach the top of the tower, $t_2 = 300/340 = 0.88$ s
Therefore, the time after which the splash is heard, $t = t_1 + t_2$
= 7.82 + 0.88 = 8.7 s

15.3 A steel wire has a length of 12.0 m and a mass of 2.10 kg. What should be the tension in the wire so that speed of a transverse wave on the wire equals the speed of sound in dry air at 20 °C = 343 m s^{-1}.

ANS :
Length of the steel wire, l = 12 m
Mass of the steel wire, m = 2.10 kg
Velocity of the transverse wave, v = 343 m/s
Mass per unit length, $\mu = m/l = 2.10/12 = 0.175$ kg m^{-1}
For tension T, velocity of the transverse wave can be obtained using the relation:

$$v = \sqrt{\frac{T}{\mu}}$$

$T = v^2.\mu$ = $(343)2 \times 0.175 = 20588.575 = 2.06 \times 10^4$ N

15.4 Use the formula $v = \sqrt{\frac{\gamma P}{\rho}}$ to explain why the speed of sound in air
(a) is independent of pressure,
(b) increases with temperature,
(c) increases with humidity.

ANS :

(a) Take the relation:

$$v = \sqrt{\frac{\gamma P}{\rho}} \quad \text{------- (i)}$$

where density ρ = mass/volume = M/V
M = Molecular weight of the gas and V = Volume of the gas.
Hence Eqn. (1) reduces to

$$v = \sqrt{\frac{\gamma PV}{M}} \quad \text{------ (ii)}$$

Now from the ideal gas equation for n = 1: PV = RT
For constant T, PV = Constant
Since both M and γ are constants, v = Constant
Hence, at a constant temperature, the speed of sound in a gaseous medium is independent of the change in the pressure of the gas.

(b) Take the relation:

$$v = \sqrt{\frac{\gamma P}{\rho}} \quad \text{------- (i)}$$

For one mole of an ideal gas, the gas equation can be written as:

PV = RT

P = RT/V … (ii)

Substituting equation (ii) in equation (i), we get:

$$v = \sqrt{\frac{\gamma RT}{V\rho}} = \sqrt{\frac{\gamma RT}{M}} \quad \text{---------- (iv)}$$

Where,
Mass, M = ρV is a constant
γ and R are also constants
We conclude from equation (iv) that $\upsilon \propto \sqrt{T}$.
Hence, the speed of sound in a gas is directly proportional to the square root of the temperature of the gaseous medium, i.e., the speed of the sound increases with an increase in the temperature of the gaseous medium and vice versa.

(c) Let v_m and v_d be the speeds of sound in moist air and dry air respectively.
Let ρ_m and ρ_d be the densities of moist air and dry air respectively.
Take the relation:

$v = \sqrt{\frac{\gamma P}{\rho}}$

Hence, the speed of sound in moist air is: $v_m = \sqrt{\frac{\gamma P}{\rho_m}}$ ------ (i)

And the speed of sound in dry air is: $v_d = \sqrt{\frac{\gamma P}{\rho_d}}$ ------ (ii)

On dividing equations (i) and (ii), we get:

$\frac{v_m}{v_d} = \sqrt{\frac{\gamma P}{\rho_m} \times \frac{\rho_d}{\gamma P}} = \sqrt{\frac{\rho_d}{\rho_m}}$

However, the presence of water vapour reduces the density of air, i.e.,

$\rho_d < \rho_m \qquad \therefore v_m > v_d$

Hence, the speed of sound in moist air is greater than it is in dry air. Thus, in a gaseous medium, the speed of sound increases with humidity.

15.5 You have learnt that a travelling wave in one dimension is represented by a function $y = f(x, t)$ where x and t must appear in the combination $x - v\,t$ or $x + v\,t$, i.e. $y = f(x \pm v\,t)$. Is the converse true? Examine if the following functions for y can possibly represent a travelling wave :
(a) $(x - vt)^2$
(b) $\log[(x + vt)/x_0]$
(c) $1/(x + vt)$

ANS :
No;
(a) Does not represent a wave
(b) Represents a wave
(c) Does not represent a wave

The converse of the given statement is not true. The essential requirement for a function to represent a travelling wave is that it should remain finite for all values of x and t.

Explanation:
(a) For x = 0 and t = 0, the function $(x - vt)^2$ becomes 0.
Hence, for x = 0 and t = 0, the function represents a point and not a wave.
(b) For x = 0 and t = 0, the function
$\log[(x + vt)/x_0] = \log 0 = \infty$
Since the function does not converge to a finite value for x = 0 and t = 0, it represents a travelling wave.
(c) For x = 0 and t = 0, the function
$1/(x + vt) = 1/0 = \infty$
Since the function does not converge to a finite value for x = 0 and t = 0, it does not represent a travelling wave.

15.6 A bat emits ultrasonic sound of frequency 1000 kHz in air. If the sound meets a water surface, what is the wavelength of (a) the reflected sound, (b) the transmitted sound? Speed of sound in air is 340 m s^{-1} and in water 1486 m s^{-1}.

ANS :
(a) Frequency of the ultrasonic sound, υ = 1000 kHz = 10^6 Hz

Speed of sound in air, v_a = 340 m/s
The wavelength (λ_r) of the reflected sound is given by the relation:
$\lambda_r = \upsilon/v = 340/10^6 = 3.4 \times 10^{-4}$ m

(b) Frequency of the ultrasonic sound, υ = 1000 kHz = 10^6 Hz
Speed of sound in water, v_w = 1486 m/s
The wavelength of the transmitted sound is given as:
$\lambda_t = 1486/10^6 = 1.49 \times 10^{-3}$ m

15.7 A hospital uses an ultrasonic scanner to locate tumours in a tissue. What is the wavelength of sound in the tissue in which the speed of sound is 1.7 km s^{-1} ? The operating frequency of the scanner is 4.2 MHz.
ANS :
Speed of sound in the tissue, v = 1.7 km/s = 1.7×10^3 m/s
Operating frequency of the scanner, υ = 4.2 MHz = 4.2×10^6 Hz
The wavelength of sound in the tissue is given as:
$\lambda = \upsilon/v = \frac{1.7 \times 10^3}{4.2 \times 10^6} = 4.1 \times 10^{-4}$ m

15.8 A transverse harmonic wave on a string is described by $y(x, t) = 3.0 \sin (36\,t + 0.018\,x + \pi/4)$ where x and y are in cm and t in s. The positive direction of x is from left to right.
(a) Is this a travelling wave or a stationary wave ? If it is travelling, what are the speed and direction of its propagation ?
(b) What are its amplitude and frequency ?
(c) What is the initial phase at the origin ?
(d) What is the least distance between two successive crests in the wave ?
ANS :
(a) Yes; Speed = 20 m/s, Direction = Right to left
(b) 3 cm; 5.73 Hz
(c) $\pi/4$
(d) 3.49 m
Explanation:
(a) The equation of a progressive wave travelling from right to left is given by the displacement function:
$y (x, t) = a \sin (\omega t + kx + \phi)$ … (i)
The given equation is:
$y (x, t) = 3.0 \sin (36t + 0.018x + \pi/4)$ …... (ii)
On comparing both the equations, we find that equation (ii) represents a travelling wave, propagating from right to left.
Now, using equations (i) and (ii), we can write:
ω = 36 rad/s and k = 0.018 m^{-1}
We know that: $\upsilon = \omega/2\pi$ and $\lambda = 2\pi/k$
Also $v = \upsilon\lambda$
$\therefore \upsilon = (\omega/2\pi) \times 2\pi/k = \omega/k$
= 36/ 0.018 = 2000 cm/s = 20 m/sec.
Hence, the speed of the given travelling wave is 20 m/s.

(b) Amplitude of the given wave, a = 3 cm
Frequency of the given wave:
$\upsilon = \omega/2\pi = 36/(2 \times 3.14) = = 5.73$ Hz
(c) On comparing equations (i) and (ii), we find that the initial phase angle, $\phi = \pi/4$
(d) The distance between two successive crests or troughs is equal to the wavelength of the wave. Wavelength is given by the relation: $k = 2\pi/\lambda$
$\therefore \lambda = 2\pi/k = (2 \times 3.14)/0.018 = 348.89$ cm = 3.49 m

15.9 For the wave described in Exercise 15.8, plot the displacement (*y*) versus (*t*) graphs for *x* = 0, 2 and 4 cm. What are the shapes of these graphs? In which aspects does the oscillatory motion in travelling wave differ from one point to another: amplitude, frequency or phase ?
ANS :
All the waves have different phases.

The given transverse harmonic wave is:

$$y(x,t) = 3.0\sin\left(36t + 0.018x + \frac{\pi}{4}\right) \qquad ...(i)$$

For x = 0, the equation reduces to:

$$y(0,t) = 3.0\sin\left(36t + \frac{\pi}{4}\right)$$

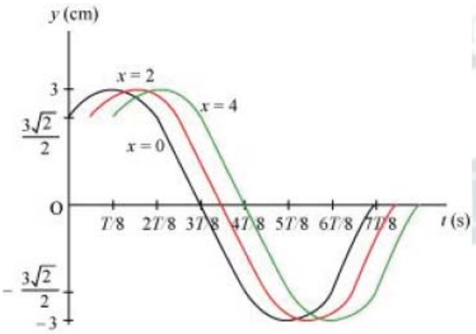
$$\text{Also, } \omega = \frac{2\pi}{T} = 36 \text{ rad/s}^{-1}$$

$$\therefore T = \frac{\pi}{8} \text{ s}$$

Now, plotting *y* vs. *t* graphs using the different values of *t*, as listed in the given table.

t (s)	0	$\frac{T}{8}$	$\frac{2T}{8}$	$\frac{3T}{8}$	$\frac{4T}{8}$	$\frac{5T}{8}$	$\frac{6T}{8}$	$\frac{7T}{8}$
y (cm)	$\frac{3\sqrt{2}}{2}$	3	$\frac{3\sqrt{2}}{2}$	0	$\frac{-3\sqrt{2}}{2}$	−3	$\frac{-3\sqrt{2}}{2}$	0

For *x* = 0, *x* = 2, and *x* = 4, the phases of the three waves will get changed. This is because amplitude and frequency are invariant for any change in *x*. The *y*-*t* plots of the three waves are shown in the given figure.



15.10 For the travelling harmonic wave $y(x, t) = 2.0 \cos 2\pi (10t - 0.0080x + 0.35)$ where x and y are in cm and t in s. Calculate the phase difference between oscillatory motion of two points separated by a distance of
(a) 4 m,
(b) 0.5 m,
(c) $\lambda/2$,
(d) $3\lambda/4$
ANS :
Equation for a travelling harmonic wave is given as:
$y(x, t) = 2.0 \cos 2\pi (10t - 0.0080x + 0.35)$
$= 2.0 \cos (20\pi t - 0.016\pi x + 0.70\pi)$
Where,
Propagation constant, $k = 0.0160\pi$
Amplitude, $a = 2$ cm
Angular frequency, $\omega = 20\pi$ rad/s
Phase difference is given by the relation: $\phi = kx = 2\pi/\lambda$
(a) For $x = 4$ m $= 400$ cm
$\phi = 0.016\pi \times 400 = 6.4\pi$ rad
(b) For 0.5 m = 50 cm
$\phi = 0.016\pi \times 50 = 0.8\pi$ rad
(c) For $x = \lambda/2$, $\phi = 2\pi/\lambda \times \lambda/2 = \pi$ rad
(d) For $x = 3\lambda/4$, $\phi = 2\pi/\lambda \times 3\lambda/4 = \pi$ rad

15.11 The transverse displacement of a string (clamped at its both ends) is given by $y(x, t) = 0.06 \sin 23x \cos(120\pi t)$
where x and y are in m and t in s. The length of the string is 1.5 m and its mass is 3.0×10^{-2} kg. Answer the following :
(a) Does the function represent a travelling wave or a stationary wave?
(b) Interpret the wave as a superposition of two waves travelling in opposite directions. What is the wavelength, frequency, and speed of each wave ?
(c) Determine the tension in the string.
ANS :
(a) The general equation representing a stationary wave is given by the displacement function:
$y(x, t) = 2a \sin kx \cos \omega t$
This equation is similar to the given equation:

$$y(x, t) = 0.06 \sin\left(\frac{2}{3}x\right) \cos(120\pi t)$$

Hence, the given function represents a stationary wave.
(b) A wave travelling along the positive x-direction is given as:

$$y_1 = a\sin(\omega t - kx)$$

The wave travelling along the negative x-direction is given as:

$$y_2 = a\sin(\omega t + kx)$$

The superposition of these two waves yields:

$$y = y_1 + y_2 = a\sin(\omega t - kx) - a\sin(\omega t + kx)$$
$$= a\sin(\omega t)\cos(kx) - a\sin(kx)\cos(\omega t) - a\sin(\omega t)\cos(kx) - a\sin(kx)\cos(\omega t)$$
$$= -2a\sin(kx)\cos(\omega t)$$
$$= -2a\sin\left(\frac{2\pi}{\lambda}x\right)\cos(2\pi \nu t) \qquad \ldots (i)$$

The transverse displacement of the string is given as:

$$y(x,t) = 0.06\sin\left(\frac{2\pi}{3}x\right)\cos(120\pi\, t) \qquad \ldots (ii)$$

Comparing equations (*i*) and (*ii*), we have:

$$\frac{2\pi}{\lambda} = \frac{2\pi}{3}$$

∴ Wavelength, $\lambda = 3$ m

It is given that:

$120\pi = 2\pi\nu$

Frequency, $\nu = 60$ Hz

Wave speed, $v = \nu\lambda \quad = 60 \times 3 = 180$ m/s

(c) The velocity of a transverse wave travelling in a string is given by the relation:

$$v = \sqrt{\frac{T}{\mu}} \qquad \ldots (i)$$

Where,

Velocity of the transverse wave, $v = 180$ m/s

Mass of the string, $m = 3.0 \times 10^{-2}$ kg

Length of the string, $l = 1.5$ m

Mass per unit length of the string, $\mu = m/l$

$$= \frac{3.0}{1.5} \times 10^{-2} \qquad = 2 \times 10^{-2}\ \text{kg m}^{-1}$$

Tension in the string = T

From equation (*i*), tension can be obtained as:

$T = v^2\mu \quad = (180)2 \times 2 \times 10^{-2} \qquad = 648$ N

15.12 (i) For the wave on a string described in Exercise 15.11, do all the points on the string oscillate with the same (a) frequency, (b) phase, (c) amplitude? Explain your answers. (ii) What is the amplitude of a point 0.375 m away from one end?

ANS :

(i)

(a) Yes, except at the nodes

(b) Yes, except at the nodes

(c) No

(ii) 0.042 m

Explanation:

(i)

(a) All the points on the string oscillate with the same frequency, except at the nodes which have zero frequency.

(b) All the points in any vibrating loop have the same phase, except at the nodes.

(c) All the points in any vibrating loop have different amplitudes of vibration.
(ii) The given equation is:

$$y(x,\ t) = 0.06\sin\left(\frac{2\pi}{3}x\right)\cos(120\pi t)$$

For $x = 0.375$ m and $t = 0$

$$\text{Amplitude} = \text{Displacement} = 0.06\sin\left(\frac{2\pi}{3}x\right)\cos 0$$

$$= 0.06\sin\left(\frac{2\pi}{3}\times 0.375\right)\times 1$$

$$= 0.06\sin(0.25\pi) = 0.06\sin\left(\frac{\pi}{4}\right)$$

$$= 0.06\times\frac{1}{\sqrt{2}} = 0.042\text{ m}$$

15.13 Given below are some functions of x and t to represent the displacement (transverse or longitudinal) of an elastic wave. State which of these represent (i) a travelling wave, (ii) a stationary wave or (iii) none at all:
(a) $y = 2\cos(3x)\sin(10t)$
(b) $y\ x\ vt = -2$
(c) $y = 3\sin(5x - 0.5t) + 4\cos(5x - 0.5t)$
(d) $y = \cos x\sin t + \cos 2x\sin 2t$
ANS:
(a) The given equation represents a stationary wave because the harmonic terms kx and ωt appear separately in the equation.
(b) The given equation does not contain any harmonic term. Therefore, it does not represent either a travelling wave or a stationary wave.
(c) The given equation represents a travelling wave as the harmonic terms kx and ωt are in the combination of $kx - \omega t$.
(d) The given equation represents a stationary wave because the harmonic terms kx and ωt appear separately in the equation. This equation actually represents the superposition of two stationary waves.

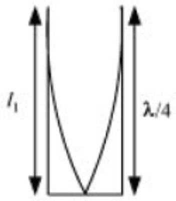
15.14 A wire stretched between two rigid supports vibrates in its fundamental mode with a frequency of 45 Hz. The mass of the wire is 3.5×10^{-2} kg and its linear mass density is 4.0×10^{-2} kg m^{-1}. What is (a) the speed of a transverse wave on the string, and (b) the tension in the string?
ANS:
(a) Mass of the wire, $m = 3.5 \times 10^{-2}$ kg
Linear mass density, $\mu = m/l = 4.0 \times 10^{-2}$ kg m^{-1}
Frequency of vibration, $\nu = 45$ Hz
∴ Length of the wire,

$$l = \frac{m}{\mu} = \frac{3.5\times 10^{-2}}{4.0\times 10^{-2}} = 0.875\text{ m}$$

The wavelength of the stationary wave (λ) is related to the length of the wire by the relation:
$\lambda = 2l/n$ where n is number of nodes in the wire.

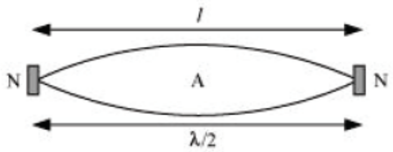
For fundamental node, $n = 1$:
$\lambda = 2l$
$\lambda = 2 \times 0.875 = 1.75$ m
The speed of the transverse wave in the string is given as:
$v = \upsilon\lambda = 45 \times 1.75 = 78.75$ m/s
(b) The tension produced in the string is given by the relation:
$T = v^2\mu$
= (78.75)2 × 4.0 × 10–2 = 248.06 N

15.15 A metre-long tube open at one end, with a movable piston at the other end, shows resonance with a fixed frequency source (a tuning fork of frequency 340 Hz) when the tube length is 25.5 cm or 79.3 cm. Estimate the speed of sound in air at the temperature of the experiment. The edge effects may be neglected.
ANS:
Frequency of the turning fork, ν = 340 Hz
Since the given pipe is attached with a piston at one end, it will behave as a pipe with one end closed and the other end open, as shown in the given figure.



Such a system produces odd harmonics. The fundamental note in a closed pipe is given by the relation: $l_1 = \lambda/4$
Where,
Length of the pipe, l_1 = 25.5 cm = 0.255 m
$\therefore \lambda = 4l_1 = 4 \times 0.255 = 1.02\,\text{m}$
The speed of sound is given by the relation:
$v = \upsilon\lambda = 340 \times 1.02 = 346.8$ m/s

15.16 A steel rod 100 cm long is clamped at its middle. The fundamental frequency of longitudinal vibrations of the rod are given to be 2.53 kHz. What is the speed of sound in steel?
ANS:
Length of the steel rod, l = 100 cm = 1 m
Fundamental frequency of vibration, ν = 2.53 kHz = 2.53 × 103 Hz
When the rod is plucked at its middle, an antinode (A) is formed at its centre, and nodes (N) are formed at its two ends, as shown in the given figure.



The distance between two successive nodes is λ/2.
$\therefore l = \lambda/2$
$\lambda = 2l = 2 \times 1 = 2$ m
The speed of sound in steel is given by the relation: $v = \nu\lambda$

= 2.53 × 103 × 2
= 5.06 × 103 m/s
= 5.06 km/s

15.17 A pipe 20 cm long is closed at one end. Which harmonic mode of the pipe is resonantly excited by a 430 Hz source ? Will the same source be in resonance with the pipe if both ends are open? (speed of sound in air is 340 m s^{-1}).

ANS:

First (Fundamental); No

Length of the pipe, l = 20 cm = 0.2 m

Source frequency = nth normal mode of frequency, νn = 430 Hz

Speed of sound, v = 340 m/s

In a closed pipe, the nth normal mode of frequency is given by the relation:

$$\nu_n = (2n-1)\frac{v}{4l} \qquad ; \; n \text{ is an integer} = 0, 1, 2, 3...$$

$$430 = (2n-1)\frac{340}{4 \times 0.2}$$

$$2n - 1 = \frac{430 \times 4 \times 0.2}{340} = 1.01$$

$$2n = 2.01$$

$$n \sim 1$$

Hence, the first mode of vibration frequency is resonantly excited by the given source.

In a pipe open at both ends, the nth mode of vibration frequency is given by the relation:

$$\nu_n = \frac{nv}{2l}$$

$$n = \frac{2l\nu_n}{v}$$

$$= \frac{2 \times 0.2 \times 430}{340} = 0.5$$

Since the number of the mode of vibration (n) has to be an integer, the given source does not produce a resonant vibration in an open pipe.

15.18 Two sitar strings A and B playing the note '*Ga*' are slightly out of tune and produce beats of frequency 6 Hz. The tension in the string A is slightly reduced and the beat frequency is found to reduce to 3 Hz. If the original frequency of A is 324 Hz, what is the frequency of B?

ANS:

Frequency of string A, fA = 324 Hz

Frequency of string B = fB

Beat's frequency, n = 6 Hz

Beat's frequency is given as

$$n = |f_A \pm f_B|$$
$$6 = 324 \pm f_B$$
$$f_B = 330\,\text{Hz or } 318\,\text{Hz}$$

Frequency decreases with a decrease in the tension in a string. This is because frequency is directly proportional to the square root of tension. It is given as:

$$\nu \propto \sqrt{T}$$

Hence beat frequency cannot be 330 Hz.
∴ f_n = 318 Hz.

15.19 Explain why (or how):
(a) in a sound wave, a displacement node is a pressure antinode and vice versa,
(b) bats can ascertain distances, directions, nature, and sizes of the obstacles without any "eyes",
(c) a violin note and sitar note may have the same frequency, yet we can distinguish between the two notes,
(d) solids can support both longitudinal and transverse waves, but only longitudinal waves can propagate in gases, and
(e) the shape of a pulse gets distorted during propagation in a dispersive medium.
ANS:
(a) A node is a point where the amplitude of vibration is the minimum and pressure is the maximum. On the other hand, an antinode is a point where the amplitude of vibration is the maximum and pressure is the minimum.
Therefore, a displacement node is nothing but a pressure antinode, and vice versa.
(b) Bats emit very high-frequency ultrasonic sound waves. These waves get reflected back toward them by obstacles. A bat receives a reflected wave (frequency) and estimates the distance, direction, nature, and size of an obstacle with the help of its brain senses.
(c) The overtones produced by a sitar and a violin, and the strengths of these overtones, are different. Hence, one can distinguish between the notes produced by a sitar and a violin even if they have the same frequency of vibration.
(d) Solids have shear modulus. They can sustain shearing stress. Since fluids do not have any definite shape, they yield to shearing stress. The propagation of a transverse wave is such that it produces shearing stress in a medium. The propagation of such a wave is possible only in solids, and not in gases.
Both solids and fluids have their respective bulk moduli. They can sustain compressive stress. Hence, longitudinal waves can propagate through solids and fluids.
(e) A pulse is actually is a combination of waves having different wavelengths. These waves travel in a dispersive medium with different velocities, depending on the nature of the medium. This results in the distortion of the shape of a wave pulse.

15.20 A train, standing at the outer signal of a railway station blows a whistle of frequency 400 Hz in still air. (i) What is the frequency of the whistle for a platform observer when the train (a) approaches the platform with a speed of 10 m s^{-1}, (b) recedes from the platform with a speed of 10 m s^{-1}? (ii) What is the speed of sound in each case ? The speed of sound in still air can be taken as 340 m s^{-1}.
ANS:

(i) (a)Frequency of the whistle, ν = 400 Hz
Speed of the train, vT= 10 m/s
Speed of sound, v = 340 m/s
The apparent frequency υ' of the whistle as the train approaches the platform is given by the relation:

$$\nu' = \left(\frac{v}{v - v_T}\right)\nu \quad = \left(\frac{340}{340 - 10}\right) \times 400 = 412.12 \text{ Hz}$$

(b) The apparent frequency υ" of the whistle as the train recedes from the platform is given by the relation:

$$\nu'' = \left(\frac{v}{v + v_T}\right)\nu \quad = \left(\frac{340}{340 + 10}\right) \times 400 = 388.57 \text{ Hz}$$

(ii) The apparent change in the frequency of sound is caused by the relative motions of the source and the observer. These relative motions produce no effect on the speed of sound. Therefore, the speed of sound in air in both the cases remains the same, i.e., 340 m/s.

15.21 A train, standing in a station-yard, blows a whistle of frequency 400 Hz in still air. The wind starts blowing in the direction from the yard to the station with at a speed of 10 m s^{-1}. What are the frequency, wavelength, and speed of sound for an observer standing on the station's platform? Is the situation exactly identical to the case when the air is still and the observer runs towards the yard at a speed of 10 m s^{-1}? The speed of sound in still air can be taken as 340 m s^{-1}.

ANS:
For the stationary observer: 400 Hz; 0.875 m; 350 m/s
For the running observer: Not exactly identical
For the stationary observer:
Frequency of the sound produced by the whistle, ν = 400 Hz
Speed of sound = 340 m/s
Velocity of the wind, v = 10 m/s
As there is no relative motion between the source and the observer, the frequency of the sound heard by the observer will be the same as that produced by the source, i.e., 400 Hz.
The wind is blowing toward the observer. Hence, the effective speed of the sound increases by 10 units, i.e., Effective speed of the sound, ve = 340 + 10 = 350 m/s
The wavelength (λ) of the sound heard by the observer is given by the relation:

$$\lambda = \frac{v_e}{\nu} = \frac{350}{400} = 0.875 \text{ m}$$

For the running observer :
Velocity of the observer, v_0 = 10 m/s
The observer is moving toward the source. As a result of the relative motions of the source and the observer, there is a change in frequency (υ').

This is given by the relation:

$$\nu' = \left(\frac{v + v_0}{v}\right)\nu \quad = \left(\frac{340 + 10}{340}\right) \times 400 = 411.76 \text{ Hz}$$

Since the air is still, the effective speed of sound = 340 + 0 = 340 m/s
The source is at rest. Hence, the wavelength of the sound will not change, i.e., λ remains 0.875 m. Hence, the given two situations are not exactly identical.

Additional Exercises

15.22 A travelling harmonic wave on a string is described by $y(x, t)$ = 7.5 sin (0.0050x +12t + π/4)
(a) what are the displacement and velocity of oscillation of a point at x = 1 cm, and t = 1 s ? Is this velocity equal to the velocity of wave propagation?
(b)Locate the points of the string which have the same transverse displacements and velocity as the x = 1 cm point at t = 2 s, 5 s and 11 s.

ANS:

(a) The given harmonic wave is:

$$y(x, t) = 7.5\sin\left(0.0050x + 12t + \frac{\pi}{4}\right)$$

For x = 1 cm and t = 1s,

$$y = (1, 1) = 7.5\sin\left(0.0050 + 12 + \frac{\pi}{4}\right)$$

$$= 7.5\sin\left(12.0050 + \frac{\pi}{4}\right) \qquad = 7.5\sin\theta$$

$$\text{Where, } \theta = 12.0050 + \frac{\pi}{4} = 12.0050 + \frac{3.14}{4} = 12.79 \text{ rad}$$

$$= \frac{180}{3.14} \times 12.79 = 732.81°$$

$$\therefore y = (1, 1) = 7.5\sin(732.81°)$$

$$= 7.5\sin(90 \times 8 + 12.81°) = 7.5\sin 12.81°$$

$$= 7.5 \times 0.2217$$

$$= 1.6629 \approx 1.663 \text{ cm}$$

The velocity of the oscillation at a given point and time is given as:

$$v = \frac{d}{dt}y(x, t) = \frac{d}{dt}\left[7.5\sin\left(0.0050x + 12t + \frac{\pi}{4}\right)\right]$$

$$= 7.5 \times 12\cos\left(0.0050x + 12t + \frac{\pi}{4}\right)$$

At x = 1 m, t= 1 s,

$$v = y(1,1) = 90\cos\left(12.005 + \frac{\pi}{4}\right)$$

$$= 90\cos(732.81°) = 90\cos(90 \times 8 + 12.81°)$$

$$= 90\cos(12.81°)$$

$$= 90 \times 0.975 = 87.75 \text{ cm/s}$$

Now, the equation of a propagating wave is given by:

$$y(x, t) = a\sin(kx + wt + \phi)$$

where k = 2π/λ $\quad\therefore$ λ = 2π/k
And ω = 2πυ $\quad\therefore$ υ = ω/2π

Speed v = υλ = ω/k where ω = 12 rsd/s and k = 0.0050 m^{-1}
∴ v = 12/(0.0050) = 2400 cm/s
Hence, the velocity of the wave oscillation at x = 1 cm and t = 1 s is not equal to the velocity of the wave propagation.

(b) Propagation constant is related to wavelength as: k = 2π/λ

$$\therefore \lambda = \frac{2\pi}{k} = \frac{2 \times 3.14}{0.0050} = 1256 \text{ cm} = 12.56 \text{ m}$$

Therefore, all the points at distances $n\lambda$ $(n = \pm 1, \pm 2, \pm 3$, ------ and so on) i.e. ± 12.56 m, ± 25.12 m, … and so on for x = 1 cm, will have the same displacement as the x = 1 cm points at t = 2 s, 5 s, and 11 s.

15.23 A narrow sound pulse (for example, a short pip by a whistle) is sent across a medium. (a) Does the pulse have a definite (i) frequency, (ii) wavelength, (iii) speed of propagation? (b) If the pulse rate is 1 after every 20 s, (that is the whistle is blown for a split of second after every 20 s), is the frequency of the note produced by the whistle equal to 1/20 or 0.05 Hz ?

ANS:
(a) (i)No
(ii)No
(iii)Yes
(b) No
Explanation:
(a) The narrow sound pulse does not have a fixed wavelength or frequency. However, the speed of the sound pulse remains the same, which is equal to the speed of sound in that medium.
(b) The short pip produced after every 20 s does not mean that the frequency of the whistle is 1/20 or 0.05 Hz. It means that 0.05 Hz is the frequency of the repetition of the pip of the whistle.

15.24 One end of a long string of linear mass density 8.0×10^{-3} kg m^{-1} is connected to an electrically driven tuning fork of frequency 256 Hz. The other end passes over a pulley and is tied to a pan containing a mass of 90 kg. The pulley end absorbs all the incoming energy so that reflected waves at this end have negligible amplitude. At t = 0, the left end (fork end) of the string x = 0 has zero transverse displacement (y = 0) and is moving along positive y-direction. The amplitude of the wave is 5.0 cm. Write down the transverse displacement y as function of x and t that describes the wave on the string.

ANS:
The equation of a travelling wave propagating along the positive y-direction is given by the displacement equation:
$y(x, t) = a \sin(wt - kx)$ … (i)
Linear mass density, μ = 8.0 × 10-3 kg m^{-1}
Frequency of the tuning fork, ν = 256 Hz
Amplitude of the wave, a = 5.0 cm = 0.05 m … (ii)
Mass of the pan, m = 90 kg
Tension in the string, $T = mg$ = 90 × 9.8 = 882 N
The velocity of the transverse wave v, is given by the relation:

$$v = \sqrt{\frac{T}{\mu}}$$

$$= \sqrt{\frac{882}{8.0 \times 10^{-3}}} = 332 \text{ m/s}$$

Angular frequency, $\omega = 2\pi \nu$

$$= 2 \times 3.14 \times 256$$

$$= 1608.5 = 1.6 \times 10^3 \text{ rad/s} \qquad ...(iii)$$

Wavelength, $\lambda = \frac{v}{\nu} = \frac{332}{256}$ m

$\therefore$ Propagation constant, $k = \frac{2\pi}{\lambda}$

$$= \frac{2 \times 3.14}{\frac{332}{256}} = 4.84 \text{ m}^{-1} \qquad ...(iv)$$

Substituting the values from equations (*ii*), (*iii*), and (*iv*) in equation (*i*), we get the displacement equation:

y (x, t) = 0.05 sin (1.6 × 103t – 4.84 x) m

15.25 A SONAR system fixed in a submarine operates at a frequency 40.0 kHz. An enemy submarine moves towards the SONAR with a speed of 360 km h^{-1}. What is the frequency of sound reflected by the submarine ? Take the speed of sound in water to be 1450 m s^{-1}.

ANS:

Operating frequency of the SONAR system, ν = 40 kHz

Speed of the enemy submarine, v_e = 360 km/h = 100 m/s

Speed of sound in water, v = 1450 m/s

The source is at rest and the observer (enemy submarine) is moving toward it.

Hence, the apparent frequency ( ) received and reflected by the submarine is given by the relation:

$$\nu' = \left(\frac{v + v_e}{v}\right)\nu$$

$$= \left(\frac{1450 + 100}{1450}\right) \times 40 = 42.76 \text{ kHz}$$

The frequency (υ') received by the enemy submarine is given by the relation:

$$\nu'' = \left(\frac{v}{v + v_s}\right)\nu'$$

Where, v_s = 100 m / s

$$\therefore \nu'' = \left(\frac{1450}{1450 - 100}\right) \times 42.76 = 45.93 \text{ kHz}$$

15.26 Earthquakes generate sound waves inside the earth. Unlike a gas, the earth can experience both transverse (S) and longitudinal (P) sound waves. Typically the speed of S wave is about 4.0 km s^{-1}, and that of P wave is 8.0 km s^{-1}. A seismograph records P and S waves from an earthquake. The first P wave arrives 4 min before the first S wave. Assuming the waves travel in straight line, at what distance does the earthquake occur ?

ANS:

Let v_S and v_P be the velocities of S and P waves respectively.

Let L be the distance between the epicentre and the seismograph.

We have:

$L = v_S t_S$ --------- (i)

$L = v_P t_P$ ------- (ii)

Where,

t_S and t_P are the respective times taken by the S and P waves to reach the seismograph from the epicentre.

It is given that: $v_P = 8$ km/s $v_S = 4$ km/s

From equations (i) and (ii), we have:

$v_S\, t_S = v_P\, t_P$

$4t_S = 8\, t_P$

$t_S = 2\, t_P$ ------- (iii)

It is also given that:

$t_S - t_P = 4$ min $= 240$ s

$2t_P - t_P = 240$

$t_P = 240$

And $t_S = 2 \times 240 = 480$ s

From equation (ii), we get:

$L = 8 \times 240 = 1920$ km

Hence, the earthquake occurs at a distance of 1920 km from the seismograph.

15.27 A bat is flitting about in a cave, navigating via ultrasonic beeps. Assume that the sound emission frequency of the bat is 40 kHz. During one fast swoop directly toward a flat wall surface, the bat is moving at 0.03 times the speed of sound in air. What frequency does the bat hear reflected off the wall ?

ANS:

Ultrasonic beep frequency emitted by the bat, ν = 40 kHz

Velocity of the bat, vb = 0.03 v

Where, v = velocity of sound in air

The apparent frequency of the sound striking the wall is given as:

$$\nu' = \left(\frac{v}{v - v_b}\right)\nu \qquad = \left(\frac{v}{v - 0.03v}\right) \times 40 \qquad = \frac{40}{0.97} \text{ kHz}$$

This frequency is reflected by the stationary wall ($\upsilon_s = 0$) toward the bat.

The frequency (υ'') of the received sound is given by the relation:

$$\nu'' = \left(\frac{v + v_b}{v}\right)\nu'$$

$$= \left(\frac{v + 0.03v}{v}\right) \times \frac{40}{0.97} \qquad = \frac{1.03 \times 40}{0.97} = 42.47 \text{ kHz}$$

Practice Problems

1) Two wires placed to each other are vibrating in their first harmonic. The sound produced by the two wires produce 5 beats per second. If the longer wire is 25 cm long and the velocity of waves in wire is 50 m / s, then find length of the smaller wire. [March, 2003]
(Ans: 23.8 cm)

2) A propagating harmonic wave expression is $y = 0.05 \sin (628t - 1.8x)$ meter. Find the values of (a) the wave-length, (b) the frequency and (c) the velocity of the wave.
[April, 2002, October, 1990]
(Ans: (a) 3.49 m, (b) 100 Hz, (c) 349 m / s)

3) Two sound waves of wavelengths 140 cm and 140.4 cm, propagating in a gas, produce 8 beats per second. Find the velocity of the sound wave in M. K. S. system. [March, 1998]
(Ans: 3931 m /s)

4) For a wave in a wire, the velocity is 10 m / s and wavelength is 4 cm. Calculate the angular frequency and the wave-vector. [October, 1997]
(Ans: 500 π rad /s, 157 rad /m)

5) A source producing a sound of the frequency 660 Hz moves with the velocity 10 m / s. The velocity of sound is 340 m / s. Compute the frequency of sound, listened by a stationary listener, for the under mentioned cases:
(1) The source is moving towards stationary listener.
(2) The source is moving away from the stationary listener. [October, 1996]
(Ans: (1) 680 Hz, (2) 642 Hz)

6) $y = 10 \sin \pi (5t - 2x)$ is the equation of a wave propagating in the positive direction of the x-axis, x and y are in m. Find velocity of a particle at a distance of 2 m from source of the wave at the end of one second. [March, 1996]
(Ans: −157 m / s)

7) Two simple harmonic waves are given by the equations $y = 0.30 \sin (314t - 1.57x)$ and $y = 0.10 \sin (314t - 1.57x + 1.57)$. Find the phase difference between them in degrees.
Also find the ratio of their intensities. [March, 1995]
(Ans: 90 0, 9)

8) The equation of a progressive wave is $y = 5 \sin (10 \pi / 3) (t - x / 40)$, where x and y are in metre and t is in second. Calculate for this wave (i) frequency, (ii) wavelength, and (iii) the phase difference between two particles separated by a distance of 6 m.
(Ans: (i) 1.67 seconds, (ii) 24 m, (iii) $\pi / 2$ radian) [October, 1994]

9) The amplitude of a transverse wave is 4 cm. If its frequency is 4 Hz and wavelength 4 cm, find the displacement of a particle 1 cm away from the origin at time t = 0.25 sec. Also give the name of this position of the particle. [March, 1993]
(Ans: 4 cm, crest)

10) If the frequency of the radio waves is 850 KHz and velocity in air is 3×10^8 m / s, calculate the wavelength and wave-vector of the radio waves. [March, 1990]
(Ans: 353 m, 1.78×10^{-2} m^{-1})

11) Amplitude of a progressive wave is 10 cm. Its frequency and wavelength are 2 Hz and 20 cm respectively. Compute the displacement of a particle involved in wave motion which is at 10 cm from the origin after 0.5 second. [March, 1989]
(Ans : 0)

12) An equation of a wave is given by $y = 5 \sin 0.5\pi (2t - x / 2)$. Find the velocity of a particle of the medium at a point, 2 cm away from the origin of the wave, after 2 sec.
(Ans: zero) [March, 1988]

13) Find the frequency of a fork which gives 4 beats per second when sounded with a fork of frequency 256 and gives 2 beats per second when one of its prongs is loaded with a small piece of wax.
(Ans: 260 Hz)

14) A set of 28 tuning forks is arranged in a series of decreasing frequencies. Each fork gives 3 beats with succeeding one. The first fork is the octave of the last. Calculate the frequency of the first and the 15th tuning fork.
(Ans: 162 Hz, 120 Hz)

15) If forks A and B produce 3 beats per second and forks B and C produce 4 beats per second, then find the number of beats per second produced by forks A and C.
(Ans: 1 or 7 beats per second)

16) Wavelengths of two forks in air are 90 / 175 m and 90 / 173 m. Each note produces 4 beats / s with the third note of a fixed frequency. Calculate the velocity of sound in air.
(Ans: 360 m / s)

17) In a string of 165 cm length, the frequencies of two consecutive harmonics are 300 Hz and 400 Hz. What will be the wave velocity ?
(Ans: 330 m / s)

18) If the displacement and velocity of a particle at a distance 1 m from one end of a stationary wave are 8 m and 6 m / s at a given instant and its time period is 2 π sec, then find its wavelength. (Ans: 4 m)

19) A train is passing a railway station with a speed of 60 km/hr and blowing continuously its whistle of frequency 320 Hz. What will be the apparent frequencies noted by a person waiting on the platform when the train is (a) approaching and (b) departing ?
(Ans: (a) 337 Hz, (b) 305 Hz)

20) A whistle is whirled in a circle of 100 cm radius and traverses the circular path twice per second. An observer is situated outside the circle but is in its plane. What will be the musical interval between the highest and lowest pitch observed if the velocity of sound is 332 m / s ? (Ans: 1.079)

21) Two sources give out identical notes of 1360 Hz. An observer halfway between them moving from one to the other hears 4 beats / second. At what speed is he moving ?
(Ans: 1.8 km / hr)

22) Two trains move towards each other at speeds of 72 km / hr. The first train whistles emitting a sound with a frequency of 800 Hz. Find the frequency of the sound which can be heard by a passenger in the second train: (a) before the trains meet, (b) after the trains meet. (velocity of sound is 340 m / s).
(Ans: (a) 887.5 Hz, (b) 722.2 Hz)

23) A source of sound and a listener are moving towards a wall with listener between the source of sound and the wall. Velocity of the source of sound is 10 m / s and that of the listener is 4 m / s. If the frequency of sound is 320 Hz and velocity of sound is 330 m / s, find the number of beats per second heard by the listener.
(Ans: 8 beats per second)

WAVE MOTION MCQ

1.	If x □ a sin $\left(\omega t+\frac{\pi}{6}\right)$ and x' □ a cos □t, then what is the phase difference between the two waves **(RAJ PMT-96)** (a) $\frac{\pi}{3}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{2}$ (d) □ Answer: (a)
2.	When a sound wave of frequency 300 Hz passes through a medium, the maximum displacement of a particle of the medium is 0.1 cm. the maximum velocity of the particle is equal to **(MNR-92)** (a) 60 □ cm/s (b) 30 □ cm/s (c) 30 cm/s (d) 60 cm/s Answer: (a)
3.	Two tuning forks A and B vibrating simultaneously produce 5 beats. Frequency of B is 512 Hz. It is seen that if one arm of a is filed, then the number of beats increases. Frequency of A will be **(PMTMP-91)** (a) 502 Hz (b) 507 Hz (c) 517 Hz (d) 522 Hz Answer: (c)
4.	A simple harmonic wave having amplitude A and time period T is represented by the equation y □ 5 sin □(t □ 4) metres. Then the value of A (in metres) and T (in seconds) are **(MNR-91)** (a) A □ 5, T □ 2 (b) A □ 10, T □ 1 (c) A □ 5, T □ 1 (d) A □ 10, T □ 2 Answer: (a)
5.	Which one of the following cannot represent a traveling wave **(NCERT-84)** (a) y □ f (x □ □t) (b) y □ y_m sin k (x □ □t) (c) y □ y_m log (x □ □t) (d) y □ f (x^2 □ □t^2) Answer: (c)
6.	Which of the following statements is wrong

	(NCERT-76) (a) Sound travels in a straight line (b) Sound travels as waves (c) Sound is a from of energy (d) Sound travels faster in vacuum that then in air Answer: (d)
7.	The equation of a progressive wave traveling on a stretched string is y □ 10 sin $\left(\frac{t}{0.02} - \frac{x}{100}\right)$ where x and y are in cm and t is in sec. what is the speed of the wave? (a) 500 cm/s (b) 50 m/s (c) 40 m/s (d) 400 cm/s Answer: (b)
8.	When a compression is incident on rigid wall it is reflected as **(MHT-CET 2006)** (a) Compression with a phase change of □ (b) Compression with no phase change (c) Rarefaction with a phase change of □ (d) Rarefaction with no phase change Answer: (a)
9.	The wavelength of sound in air is 10 cm. its frequency is, (Given velocity of sound □ 330 m/s) **(CPMT-74)** (a) 330 cycles per second (b) 3.3 kilo cycles per second (c) 30 mega-cycles per second (d) 3×10^5 cycles per second Answer: (b)
10.	Two waves are represented by y_1 □ a sin $\left(\omega t + \frac{\pi}{6}\right)$ and y_2 □ a cos □t. what will be their resultant amplitude? **(RAJ PMT-96)** (a) a (b) $\sqrt{2}a$ (c) $\sqrt{3}a$ (d) 2 a Answer: (c)
11.	Sound waves having the following frequencies are audible to human beings **(CPMT-75)** (a) 5 c/s (b) 27000 c/s (c) 5000 c/s (d) 50,000 c/s Answer: (c)
12.	A sound wave is represented by y □ a sin (1000□t □ 3x). the distance between two points having a phase difference of

	60□ is **(MHT CET 2003)** (a) $\frac{2\pi}{9}$ (b) $\frac{\pi}{18}$ (c) $\frac{\pi}{9}$ (d) $\frac{5\pi}{18}$ Answer: (c)
13.	The equation of wave traveling along string is y □ 3 cos □ (100t □ x) in C.G.S. unit then wavelength is **(CPMT-91, MPPMT-94,97, MNR-85)** (a) 1 m (b) 2 cm (c) 5 cm (d) None of above Answer: (b)
14.	A siren emitting a note of frequency n is fitted on a police van, traveling towards a stationary listener. What is the velocity of the van, if the frequency of he note heard by the listener is double the original frequency? (a) V_S □ V (b) $V_S = \frac{V}{2}$ (c) V_S □ 2V (d) $V_S = \frac{V}{3}$ Answer: (b)
15.	In the longitudinal waves the direction of vibration in medium of particle is **(MHT-CET 2005)** (a) Perpendicular to propagation of wave (b) Parallel to propagation (c) Different from each other (d) Variable for time to time. Answer: (b)
16.	The relation between frequency n, wavelength □ and velocity □ of a wave is **(CPMT 76, 85)** (a) n □ □□ (b) □ □ n□ (c) $n = \frac{\lambda}{v}$ (d) $n = \frac{1}{v}$ Answer: (b)

17.	With the propagation of a longitudinal wave through a material medium the quantities transmitted in the propagation direction are **(CBSE-92)** (a) Energy, momentum and mass (b) Energy (c) Energy and mass (d) Energy and linear momentum Answer: (d)
18.	A wave is represented by the equation y □ A sin (10□x □ 15□t □ $\frac{\pi}{3}$), where x is in meters and t is in seconds. The expression represents **(IIT-90)** (a) A wave traveling in the positive x-direction with a velocity 1.5 m/s (b) A wave traveling the negative x-direction with a velocity 1.5 m/s (c) A wave traveling in the negative x-direction have a wave-length 0.2 m (d) Both 'b' and 'c' Answer: (d)
19.	Loudness of a note of sound is **(MHT-CET 99)** (a) Directly proportional to amplitude of the wave (b) Directly proportional to square of amplitude of wave (c) Directly proportional to velocity of the wave (d) Directly proportional to square of velocity of the wave Answer: (b)
20.	The velocity of sound is maximum in **(AFMC Pune 98)** (a) Water (b) Air (c) Vacuum (d) Metal Answer: (d)
21.	A wave is represented by the equation y □ 7 sin $\left(7\pi t - 0.04\pi x + \frac{\pi}{3}\right)$, where x is in meters and t is in seconds. The speed of the wave is, (a) 175 □ m/s (b) (49 □) m/s (c) (49 / □) m/s (d) (0.28 □) m/s Answer: (a)
22.	Two wave having the intensities in the ratio of 9: 1 produce interference. The ratio of maximum to minimum intensity is equal to **(MNR-87)** (a) 10 : 8 (b) 9 : 1 (c) 4 : 1 (d) 2 : 1 Answer: (c)
23.	Two tuning forks of frequencies 256 and 258 vibrations/second are sounded together.

	Then the time interval between two consecutive maxima heard by an observer is **(PMT - MP-88)** (a) 2 sec (b) 0.5 sec (c) 250 sec (d) 252 sec Answer: (b)
24.	Two waves are y_1 □ 0.25 sin 316t, y_2 □ 0.25 sin 310t are traveling in same direction. The number of beats produced per second will be **(CPMT-93)** (a) 6 (b) 3 (c) $\frac{3}{\pi}$ (d) 3□ Answer: (c)
25.	What is phase difference between two successive troughs in the transverse wave? (a) $\frac{\pi}{2}$ (b) □ (c) $\frac{3\pi}{2}$ (d) 2□ Answer: (d)

26.	A transverse wave given by y □ 2 sin (0.01x □ 30t) moves on a stretched string from one end to another end is 0.5 sec. if x and y are in cm and t is in sec, then the length of the string is (a) 6 m (b) 9 m (c) 12 m (d) 15 m Answer: (d)
27.	Two waves are represented by x_1 □ A sin $\left(\omega t + \frac{\pi}{6}\right)$ and x_2 □ A cos □t, then the pase difference between them is (a) $\frac{\pi}{6}$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{3}$ (d) □

	Answer: (c)
28.	The displacement y (in cm) produced by a simple harmonic wave is given by $y = \frac{10}{\pi}\sin\left(2000\,\pi t - \frac{\pi x}{17}\right)$ the periodic time and maximum velocity o the particles in the medium will respectively be **(CPMT-86)** (a) 10$^{□3}$ sec and 330 m/s (b) 10$^{□4}$ sec and 20 m/s (c) 10$^{□3}$ sec and 200 m/s (d) 10$^{□2}$ sec and 2000 cm/s. Answer: (c)
29.	Two sound waves are given by y □ a sin (□t □ kx) and y’ □ b cos (□t □ kx). The phase difference between the two waves is **(EAMCET-89)** (a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$ (c) □ (d) $\frac{3\pi}{4}$ Answer: (a)
30.	The equation of a progressive wave is given by, y □ 5 sin □ $\left(\frac{t}{0.02} - \frac{x}{20}\right)$ m, then the frequency of the wave is (a) 100 Hz (b) 50 Hz (c) 25 Hz (d) 10 Hz Answer: (c)
31.	If V is the velocity of the wave and □ is the angular velocity, then the propagation constant (K) of the wave is given by (a) $K = \frac{V}{\omega}$ (b) $K = \frac{\omega}{V}$ (c) K □ 2□n (d) $K = \frac{\lambda}{2\pi}$ Answer: (b)
32.	Two sounding bodies producing progressive wave given by y_1 □ 4 sin 400 □t and y_2 □ 4 sin 404 □t are situated very near to the ears of a person who will hear **(CPMT-88)** (a) 2 beats per second with intensity ratio (4/3) between maxima and minima (b) 2 beats per second with intensity ratio (49/1) between maxima and minima

	(c) 4 beats per second with intensity ratio (7/1) between maxima and minima (d) 4 beats per second with intensity ratio (4/3) between maxima and minima Answer: (b)
33.	56 tuning forks are so arranged in series, that each fork gives 4 beats/second with the previous one. If the frequency of the last fork is 3 times that of the first, then the frequency of the first fork will be (a) 55 Hz (b) 110 Hz (c) 75 Hz (d) 220 Hz Answer: (b)
34.	The speed of sound in air at N.T.P. is 300 m/s. if air pressure becomes four times, then the speed of sound will be **(NCERT-73, CPMT-91)** (a) 150 m/s (b) 300 m/s (c) 600 m/s (d) 1200 m/s Answer: (b)
35.	Two sound waves having a phase difference of 60□ have path difference of **(CBSE 96)** (a) 2 □ (b) $\frac{\lambda}{2}$ (c) $\frac{\lambda}{6}$ (d) $\frac{\lambda}{3}$ Answer: (c)
36.	The velocity of sound is measured in hydrogen and oxygen at a certain temperature. The ratio of the velocities is, **(CPMT-76)** (a) 1:1 (b) 1:2 (c) 1:4 (d) 4:1 Answer: (d)
37.	Consider the following statements: **Assertion A :** The velocity of sound in the air increases due to presence of moisture in it **Reason (R) :** The presence of moisture in air lowers the density of air. Of these statements **(SCRA-94)** (a) Both (A) and (R) are true and (R) is the correct explanation of (A) (b) Both (A) and (R) are true but (R) is not the correct explanation of (a) (c) (A) is true but (R) is false (d) (A) is false but (R) is true Answer: (a)
38.	If the amplitude of waves at a distance r from a line source is A, then amplitude at a distance 4 r will be: (a) 2A

	(b) A (c) $\frac{A}{2}$ (d) $\frac{A}{4}$ Answer: (d)
39.	The equation of a progressive wave traveling on a string is y □ 4 sin $\frac{\pi}{2}\left(8t - \frac{\pi x}{8}\right)$ cm. the velocity of the wave is **(MPPMT-90)** (a) 64 cm/s along □ x direction. (b) 64 cm/s along □ x direction. (c) $\left(\frac{64}{\pi}\right)$ cm/s along □ x direction (d) $\left(\frac{64}{\pi}\right)$ cm/s along □ x direction Answer: (c)
40.	What is the phase difference between two successive crest in the wave? **(MHT CET 2004)** (a) □ (b) $\frac{\pi}{2}$ (c) 2□ (d) 4□ Answer: (c)
41.	If the amplitude of sound is doubled and the frequency reduced to one-fourth, the intensity of sound at the same point will be **(CBSE-92)** (a) Increasing by a factor of 2 (b) Decreasing by a factor of 2 (c) Decreasing by a factor of 4 (d) Unchanged Answer: (c)
42.	Which of the following expressions is that of a simple harmonic progressive wave **(CPMT-75)** (a) a sin □t (b) a sin (□t) cos (kx) (c) a sin (□t □ kx) (d) a cos kx Answer: (c)
43.	Frequency of tuning forks are 320 Hz. And 325 Hz. If they are sounded together, the beat period is……. **(MHT CET 2003)** (a) 5 second (b) 6 second (c) 0 second (d) 0.2 second

	Answer: (d)
44.	A wave equation which gives the displacement along the Y direction is given by y □ $10^{□4}$ sin(60t □ 2x), where x and y are in metres and t is time in seconds. This represents a wave **(MNR-83, IIT-82)** (a) Traveling with a velocity of 30 m/s in the negative x direction (b) Of wavelength x metre (c) Of frequency $\left(\frac{30}{\pi}\right)$ hertz. (d) All of the above Answer: (d)
45.	Equation of progressive wave is given by y □ 4 sin □ $\left[\left(\frac{1}{5}-\frac{x}{9}\right)+\frac{x}{6}\right]$ then which of he following is correct? **(CBSE 93)** (a) □ □ 5 cm (b) □ □ 18 m (c) a □ 0.04 m (d) f □ 50 Hz Answer: (b)
46.	The velocity of sound in any gas depends upon **(CBSE-92)** (a) Wavelength of sound only (b) Density and elasticity of gas (c) Intensity of sound waves only (d) Amplitude and frequency of sound Answer: (b)
47.	The amplitude of two waves are in the ratio 5:2. if all other conditions for the two waves are same, then what is the ratio of their energy densities? **(MHT-CET 2004)** (a) 5 : 2 (b) 10 : 4 (c) 2.5 : 4 (d) 25 :4 Answer: (d)
48.	Ultrasonic waves are those waves **(CPMT-79, NMR-83)** (a) To which man can hear (b) Man can not hear (c) Are of high velocity (d) Of high amplitude Answer: (b)
49.	The velocity of sound in air at 4 atmosphere and that at 1 atmosphere pressure would be **(CPMT-79)** (a) 1 : 1 (b) 4 : 1 (c) 1 : 4 (d) 3 : 1

	Answer: (a)
50.	To demonstrate the phenomenon of beats we need (a) Two sources which emit radiation of nearly the same frequency (b) Two sources which emit radiation of exactly the same frequency (c) Two sources which emit radiation of exactly the same frequency and have a definite phase relationship (d) Two sources which emit radiation of exactly the same wavelength Answer: (a)

51.	A tuning fork of frequency 480 Hz. Produces 10 beats per second when sounded with a vibrating sonometer string. What must have been the frequency of the string if a slight increase in tension produces fewer beats per seconds then before? **(PMT-92, NCERT-84)** (a) 460 Hz (b) 470 Hz (c) 480 Hz (d) 490 Hz Answer: (b)
52.	A whistle giving out 450 Hz approaches a stationery observer at a speed of 33 m/s. the frequency heard by the observer in Hz is (velocity of sound □ 330 m/s) **(MHT CET 2001)** (a) 409 (b) 429 (c) 517 (d) 500 Answer: (d)
53.	When a source is going away from a stationary observer, with a velocity equal to that of sound in air, then the frequency heard by the observer will be (a) Same (b) Half (c) Double (d) One third Answer: (b)
54.	A wave travels in a medium according to the equation of displacement given by y(x,t) □ 0.03 sin □(2t □ 0.01x), where y and x are in meters and 't' is seconds. The wavelength of the wave is **(EAMCET-94)** (a) 200 m (b) 100 m (c) 20 m (d) 10 m Answer: (a)
55.	Ten tuning fork are arranged in increasing order of frequency in such a way that any two nearest tuning forks produce 4 beats per second. The highest frequency is twice that of the lowest. Possible highest and lowest frequencies are

	(PMT - MP-90, MHT-CET 2002) (a) 80 & 40 (b) 100 & 50 (c) 44 & 22 (d) 72 & 36 Answer: (d)
56.	The equation of a wave is given by y □ 10 sin □ (0.01 x □ 2t) where y and x are in cm and t is in sec. its frequency is **(MNR-86)** (a) 10 sec$^{□1}$ (b) 2 sec$^{□1}$ (c) 1 sec$^{□1}$ (d) 0.01 sec$^{□1}$ Answer: (c)
57.	Velocity of sound in air is **(CPMT-79)** (a) 300 m/s (b) 3.8 □ 10^{10} m/s (c) 3 □ 10^{8} m/s (d) 9 □ 10^{19} m/s Answer: (a)
58.	If the pressure amplitude in a sound wave is tripled. Then by what factor the intensity of the sound wave increased **(CPMT-92)** (a) 3 (b) 6 (c) 9 (d) $\sqrt{3}$ Answer: (c)
59.	A tuning fork X produces 4 beats/sec with a tuning fork Y of frequency 384 Hz. When the prongs of X are slightly filed, 3 beats/sec are heard. What is the original frequency of X? (a) 388 Hz (b) 381 Hz (c) 380 Hz (d) 387 Hz Answer: (c)
60.	Which of the following phenomenon cannot take place with sound wave? **(CPMT 85)** (a) Reflection (b) Interference (c) Diffraction (d) Polarization Answer: (d)
61.	The velocity of sound in vacuum is, **(PEN 84, MP)** (a) Zero (b) 27.5 m/s (c) 156 m/s (d) 330 m/s Answer: (d)

62.	The equation of a plan progressive wave is given by y □ 0.025 sin (100t □ 0.25x). The frequency wave would be (a) $\left(\frac{50}{\pi}\right)$Hz (b) 100 Hz (c) $\left(\frac{100}{\pi}\right)$Hz (d) 50 Hz Answer: (a)
63.	The fork A of frequency 100 Hz is sounded with an other tuning fork B. the number of beats produced is 2. on putting some wax on the prong of B. the number of beats reduces to 1. the frequency of the fork B is **(NCERT-77)** (a) 101 Hz (b) 99 Hz (c) 102 Hz (d) 98 Hz Answer: (c)
64.	In a sinusoidal wave, the time required by a particular particle to move from maximum displacement to zero displacement is 0.025 sec. the frequency of the wave is (a) 2.5 Hz (b) 5 Hz (c) 7.5 Hz (d) 10 Hz Answer: (d)
65.	Transverse waves can propagate **(CPMT 84)** (a) In a gas but not in a metal (b) In a metal but not in gas (c) Neither in a gas nor in a metal (d) Either in a gas or in a metal Answer: (b)
66.	Velocity of sound waves in air is 330 m/s. for a particular sound in air, a path difference of 40 cm is equivalent to a phase difference of 1.6□. The frequency of the wave is, (a) 165 Hz (b) 150 Hz (c) 660 Hz (d) 330 Hz Answer: (c)
67.	Out of the given four waves (1), (2), (3) and (4) y □ 20 sin (100 □t)…………(1) y □ 20 sin (101 □t)…………(2) y □ 20 cos (100 □t)…………(3) y □ 20.1 sin (100 □t)…………(4) Emitted by four different sources S_1, S_2, S_3 and S_4 respectively, interference phenomena would be observed in space under appropriate conditions when

	(CPMT-88) (a) Source S_1 emits wave (1) and S_4 emits wave (4) (b) Source S_2 emits wave (2) and S_4 emits wave (4) (c) Source S_1 emits wave (1) and S_3 emits wave (3) (d) Interference phenomenon can not be observed by the combination of any of the above waves. Answer: (c)
68.	The equation of a progressive wave is y □ 8 sin $\left[\pi\left(\frac{t}{10}-\frac{x}{4}\right)+\frac{\pi}{3}\right]$. The wavelength of the wave is **(MHT CET 2002)** (a) 8 m (b) 4 m (c) 2 m (d) 10 m Answer: (a)
69.	A source is moving towards observer with a speed of 20 m/s and having frequency 240 Hz and observer is moving towards source with a velocity 20 m/s. what is the apparent frequency heard by observer if velocity of sound 340 m/s? (a) 270 Hz (b) 240 Hz (c) 268 Hz (d) 360 Hz Answer: (a)
70.	Sound wave is an example of **(AFMC PUNE-2001)** (a) Longitudinal wave (b) Transverse wave (c) Stationary wave (d) All of these Answer: (a)
71.	Two sound waves have wavelengths $\frac{85}{170}$ m and $\frac{85}{172}$ in air. Each wave produces 4 beats/seconds with a third note of fixed frequency. Find the frequency of the third note. **(MHT-CET 2005)** (a) 684 Hz (b) 342 Hz (c) 680 Hz (d) 688 Hz Answer: (a)
72.	The velocity o sound in a gas at 30□C is approximately **(CPMT-83)** (a) 332 m/s (b) 350 m/s (c) 530 m/s (d) 332 km/s Answer: (b)
73.	Transverse waves can travel through

	(a) Air, water and a copper wire (b) Air and copper wire but not through water (c) A copper wire but not through air and water (d) Water but not through air and a copper wire Answer: (c)
74.	Velocity of sound is **(CPMT-72)** (a) Directly proportional to absolute temperature (□ □ T) (b) $v \propto \sqrt{T}$ (c) $v \propto \frac{1}{\sqrt{T}}$ (d) $v \propto \frac{1}{T}$ Answer: (b)
75.	Audible waves have a frequency **(AFMC PUNE-95)** (a) 0 □ 10,000 Hz (b) 20 □ 10,000 Hz (c) 20 □ 20 kHz (d) 20 □ 40 kHz Answer: (c)

76.	A astronaut can't hear the explosion on the surface of the moon because **(CPMT-78)** (a) Frequency of explosion is out of audible range (b) Temperature at that point is very low (c) There is no medium on moon (d) None of above Answer: (c)
77.	Ultrasonic, infrasonic and Audible waves travel through a medium with speeds v_u, v_i, v_a respectively, then **(CPMT-89)** (a) v_u, v_i, v_a are nearly equal (b) $v_u \geq v_a \geq v_i$ (c) $v_u \leq v_a \leq v_i$ (d) $v_a \leq v_u$ and $v_u \approx v_i$ Answer: (a)
78.	A tuning fork when sounded together with a tuning fork of frequency 256 emits two beats. On loading the tuning fork of frequency 256 Hz. The number of beats heard are one per second. the frequency of tuning fork is **(NCERT-75)** (a) 257 Hz (b) 258 Hz (c) 256 Hz (d) 254 Hz Answer: (d)
79.	Sound travels 40 m during 20 vibrations its wavelength λ is, **(AFMC-PUNE 96)** (a) 0.5 m (b) 2 m

	(c) 3 m (d) 4 m Answer: (b)
80.	A sound wave ahs frequency 500 Hz and velocity 360 m/sec. what is the distance between two particles having phase difference of 60° **(NCERT-90, CPMT 90)** (a) 0.7 cm (b) 12.0 cm (c) 70 cm (d) 120.0 cm Answer: (b)
81.	Two adjacent piano keys are struck simultaneously. The notes emitted by them have frequencies n_1 and n_2. the number of beats heard per second is **(CPMT-74, 78)** (a) $\left(\frac{n_1 - n_2}{2}\right)$ (b) $\left(\frac{n_1 + n_2}{2}\right)$ (c) $(n_1 - n_2)$ (d) $2\,(n_1 - n_2)$ Answer: (c)
82.	Beats are produced when two sound waves given by $y_1 = A \sin 200\,\pi t$ and $y_2 = A$ sing $210\,\pi t$ are sounded together. How many beats are produced/sec? (a) 3 (b) 4 (c) 5 (d) 6 Answer: (c)
83.	Two waves of same amplitude and frequency arrive at point simultaneously. The resultant amplitude is same as amplitude of each wave. So that initial phase difference of the two waves is **(MHT CET 2001)** (a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$ (c) $\frac{2\pi}{3}$ (d) $\frac{3\pi}{4}$ Answer: (c)
84.	An astronaut can't hear his companion at the surface of the moon because **(CPMT-78)** (a) Produced frequencies are above the radio frequencies (b) There is no medium for sound propagation (c) Temperature is too low during night and too high during day moon (d) None of above Answer: (b)
85.	Elastic waves in a solid are (a) Only transverse (b) Only longitudinal

	(c) Either transverse or longitudinal (d) Neither transverse nor longitudinal Answer: (c)
86.	Which of the following statement is wrong **(NCERT-73)** (a) Changes in air temperature have no effect on the speed of sound (b) Changes in air temperature have effect on the speed of sound. (c) The speed of sound in water is lower then in air (d) Both 'a' and 'c' Answer: (d)
87.	Which one of he following represents progressive wave **(CBSE-94)** (a) y = A sin kx (b) y = A sit t (c) y = A cos (at – bx + c) (d) none of these Answer: (d)
88.	The velocity of sound in air is not affected by change in **(CPMT 72)** (a) Moisture content of air (b) Temperature of air (c) Atmospheric pressure (d) Composition of air Answer: (c)
89.	Doppler effect is applicable for **(AFMC PUNE-98)** (a) Sound wave (b) Light waves (c) None of above (d) Both 'a' and 'b' Answer: (d)
90.	When a plane wave travels in a medium in the + X-direction, the displacement of the particle is given by y (x, t) = 0.01 sin 2π (t – 0.1x) where y and x are measured in metres and t in seconds what is the wavelength of the wave? **(NCERT-84)** (a) 0.1 m (b) 10.0 m (c) 1 m (d) 100.0 m Answer: (b)
91.	If amplitude of waves at distance r from a point source is A, then amplitude at a distance 2 r will be (a) 2A (b) A (c) $\frac{A}{2}$ (d) $\frac{A}{4}$ Answer: (c)
92.	If two tuning forks A and B are sounded together, they produce 4 beats per second. A is then slightly loaded with wax, they produce two beats when sounded again. The frequency of A is 256 Hz. The frequency of B will be **(CPMT-76)** (a) 250 Hz (b) 252 Hz (c) 260 Hz

	(d) 262 Hz Answer: (b)
93.	There are three sources of sound of equal intensity with frequencies 400, and 401 and 402 vibrations/sec. the number of beats heard per second is **(MNR-80)** (a) 0 (b) 1 (c) 2 (d) 3 Answer: (b)
94.	The intensities of sound are in the ratio 1 : 16 for two waves of same frequency and traveling in same medium. Their amplitudes are in the ratio **(EAM CET 83)** (a) 1 : 4 (b) 1 : 16 (c) 4 : 1 (d) 1 : 2 Answer: (a)
95.	The waves produced in a vibrating tuning fork are (a) Longitudinal in both the stem and the prongs (b) Transverse in both stem and the prongs (c) Longitudinal in prongs and transverse in stem (d) Longitudinal in stem and transverse in prongs Answer: (d)
96.	In a wave motion, the velocity of the wave (V) and the maximum particle velocity (V_P) are related as (a) $V = \frac{2\pi A}{\lambda} V_P$ (b) $V_P = \frac{2\pi A}{\lambda} V$ (c) $V = \frac{\lambda}{2\pi A} V_P$ (d) $V_P = \frac{\lambda V}{2\pi A}$ Answer: (b)
97.	The equation of displacement of two waves are given as $y_1 = 10 \sin \left(3\pi t + \frac{\pi}{3}\right)$ and $y_2 = 5 (\sin 3\pi t + \sqrt{3} \cos 3\pi t)$ then, what is the ratio of their amplitudes **(AIIMS-97)** (a) 1 : 2 (b) 2 : 1 (c) 1 : 1 (d) None Answer: (c)
98.	A transverse wave described by the equation $y = y_0 \sin 2\pi \left(nt - \frac{x}{\lambda}\right)$. The maximum particle velocity is equal to four times the wave velocity. Then **(IIT 84)** (a) $\lambda = \frac{\pi y_0}{4}$

	(b) $\lambda = \frac{\pi y_0}{2}$ (c) $\lambda = \pi y_0$ (d) $\lambda = 2\,\pi y_0$ Answer: (b)
99.	A tunning fork C is sound with another fork D of frequency 384 Hz gives 4 beats/sec. When C is filed, the beat frequency become 3 beates/sec, then original frequency of tunning fork K is **(MHT-CET 2006)** (a) 388 Hz (b) 380 Hz (c) 387 Hz (d) 381 Hz Answer: (b)
100.	The phase difference between two points is $\frac{\pi}{3}$. If the frequency of wave is 50 Hz. Then what is the distance between two points (Given ν = 330 m/s) **(MHT CET 2004)** (a) 2.2 m (b) 1.1 m (c) 0.6 m (d) 1.7 m Answer: (b)

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PHYSICS PRACTICAL (30 Marks & 2 Hours)

(1) Principle of the experiment : 2M

(2) Formula and explaining the terms : 2M

(3) Diagram / figure / circuit with labeling : 2M

(4) Tabular column/ observation pattern : (2 Marks) :

(5) Construction of the experimental set up/ circuit : (3 Marks) :

(6) Performing experiment and entering the readings : (4 Marks) :

(7) Substitution and calculation /plotting the graph and calculation : (3 Marks) :

(8) Result with unit : (2 Marks) :

(9) Viva-Voce : 4 Simple Questions related to experiment (4 Marks) :

(10) Record Book : 13 Experiments or more out of 14 (6 Marks) :

EXPERIMENT 1 : Vernier Callipers

1. (a) Determine the diameter of a sphere & hence calculate its density. Mass of the sphere = 25.38×10^{-3} Kg.

(b) Determine internal diameter and depth of a calorimeter & Calculate its volume using vernier calipers. take three sets of readings ?

ANS :

(1) Principle of the experiment : 2M

The difference in the magnitude of one main scale division (MSD) and one vernier scale division (VSD) is the least count of the vernier callipers.

(2) Formula and explaining the terms : 2M

(i) Least Count = LC = S/N

where S = Value of one MSD, N = Number of divisions on vernier scale

(ii) Total reading = TR = MSR - (CVD × LC)

where MSR = Main scale reading, CVD = Coinciding vernier scale division

(iii) Volume of the sphere = $V = \pi D^3/6$

where D = diameter of the sphere

(iv) Density of the material of the sphere = $\rho = m/V$

where m = mass of the sphere

(v) Internal volume of the calorimeter = $V = (\pi D^2 h^2)/4$

where D = internal diameter of the calorimeter, h = Depth of the calorimeter.

(3) Diagram / figure / circuit with labeling : 2M

(4) Tabular column/ observation pattern : (2 Marks) :

(5) Construction of the experimental set up/ circuit : (3 Marks) :

(6) Performing experiment and entering the readings : (4 Marks) :

(7) Substitution and calculation /plotting the graph and calculation : (3 Marks) :

(8) Result with unit : (2 Marks) :

Density of material of sphere = ______kgm^{-3}.

Internal volume of the calorimeter = ______ m^3.

(9) Viva-Voce : 4 Simple Questions related to experiment (4 Marks) :

(1) What is the least count of a measuring instrument ?

The smallest value that can be measured by the measuring instrument is called its least count.

(2) Mention the three types of systematic error noticed in measuring a physical quantity ?

(a) **Instrumental** errors that arise from the errors due to imperfect design or calibration of the measuring instrument, zero error in the instrument, etc.

(b) **Imperfection in experimental technique or procedure** To determine the temperature of a human body, a thermometer placed under the armpit will always give a temperature lower than the actual value of the body temperature.

(c) **Personal errors** that arise due to an individual's bias, lack of proper setting of the apparatus or individual's carelessness in taking observations without observing proper precautions, etc.

(3) What is meant by systematic error ?

The systematic errors are those errors that tend to be in one direction, either positive or negative and affects each measurement by same amount.

(4) Define absolute error, fractional error and percentage of error ?

The magnitude of the difference between the true value of the quantity and the individual measurement value is called the **absolute error** of the measurement.

The relative error is the ratio of the mean absolute error $\Delta\alpha_{mean}$ to the mean value α_{mean} of the quantity measured. Relative error = $\Delta\alpha_{mean}/\alpha_{mean}$

When the relative error is expressed in per cent, it is called the percentage error ($\delta\alpha$).

$\delta a = (\Delta a_{mean}/a_{mean}) \times 100\%$

(5) Express Vernier least count in terms of 1MSD and 1 VSD

Least Count = LC = S/N

where S = Value of one MSD, N = Number of divisions on vernier scale

(6) What is error in measuring system ?

Difference between measured value and true value of a quantity is called **error**.

In general, the errors in measurement can be broadly classified as (a) systematic errors and (b) random errors.

(7) What is least count error?

It is the error associated with the resolution of the instrument.

(8) What is absolute error?

The magnitude of the difference between the individual measurement and the true value of the physical quantity is called absolute error. It is always positive.

(9) How would you determine the true value of a quantity measured several times ?

By taking arithmetic mean

(10) What is relative error?

The relative error is the ratio of the mean absolute error to the mean value of the quantity measured.

(11) What is percentage error?

The relative error expressed in percentage is called percentage error.

EXPERIMENT 2 : Screw Gauge

(a) Determine the diameter of a given wire

(b) Determine the thickness of a lamina using screw gauge. Take three sets of readings. Determine area of the lemina by graphical method. hence calculate volume of the lamina ?

ANS :

(1) Principle of the experiment : 2M

The distance moved by the screw is directly proportional to the rotation given. Using this principle, we can measure diameter of wires and thickness of sheets.

(2) Formula and explaining the terms : 2M

(i) Pitch of the screw = P = s/n

where s = Distance moved by the head on the pitch scale, n = number of rotation given to the screw.

(ii) Least Count = LC = P/N

where N = number of divisions on the head scale.

(iii) Total Reading = TR = PSR + (HSR - ZE) LC

where ZE = zero error

(iv) Volume of the Lamina = At

where A = Area of the lamina, t = thickness of the lamina

(3) Diagram / figure / circuit with labeling : 2M

(4) Tabular column/ observation pattern : (2 Marks) :

(5) Construction of the experimental set up/ circuit : (3 Marks) :

(6) Performing experiment and entering the readings : (4 Marks) :

(7) Substitution and calculation /plotting the graph and calculation : (3 Marks) :

(8) Result with unit : (2 Marks) :

(a) Diameter of the given wire = _____ mm = _______ m

(b) Thickness of the Lamina = _____ mm = _______ m

(9) Viva-Voce : 4 Simple Questions related to experiment (4 Marks) :

(1) What is the principle behind the working of a screw gauge

The distance moved by the screw is directly proportional to the rotation given. Using this principle, we can measure diameter of wires and thickness of sheets.

(2) What is the formula for measuring diameter of the wire

Diameter of wire = mean value of [PSR + (HSR - ZE) LC]

(3) Why the screw gauge is called micrometer ?

Screw gauge can be used to measure small diameters and thickness of solids in micrometers scale (10^{-6} m) .

(4) What is zero error in Screw gauge ?

The error which arises when the zero of circular scale does not coincide with the zero of the main scale upon joining the two studs.

(5) What is the full form of PSR and HSR ?

PSR = Pitch scale reading and HSR = Head scale reading

(6) What is Pitch of the screw ?

The pitch of a screw is the ratio of Distance moved by the head on the pitch scale (s) to Number of rotation given (n)

(7) What is the use of screw gauge ?

Screw gauge is used to measure the diameter and the thickness of small solid peices/wires.

(8) What is the least count of a screw gauge ?

Least count of a screw gauge is the ratio of Pitch of the screw (p) to Number of divisions of the head scale (N).

(9) What is backlash error in screw gauge ?

Ans. Within a nut there is a little space for the play of screw. Due to continuous use this space increases. Thus when the screw is turned in one direction the stud moves as usual. However, when the screw is rotated in the opposite direction, the stud does not move for a while. This error is called Back lash error. In short "Back lash error is the error introduced on reversing the direction of rotation".

(10) How back lash error is avoided?

Ans. By turning the screw in one direction only.

(11) What are "precision instrument"?

Ans. The instrument that can measure up to a fraction of a mm, e.g., vernier caliper, screw gauge and spherometer.

EXPERIMENT 3 : Spherometer

Determine the radius of curvature of a given spherical surface by a spherometer. Take three sets of readings

ANS :

(1) Principle of the experiment : 2M

The linear distance moved by the screw is directly proportional to the rotation given to it.

(2) Formula and explaining the terms : 2M

(i) Pitch of the screw = P = s/n

where, s = Distance moved by the head on the pitch scale, n = number of rotation given to the screw.

(ii) Least count = LC = P/N

(iii) Total reading = TR = PSR + (HSR × LC)

(iv) Radius of curvature of spherical surface = R = $(l^2/6h) + (h/2)$

(3) Diagram / figure / circuit with labeling : 2M

(4) Tabular column/ observation pattern : (2 Marks) :

(5) Construction of the experimental set up/ circuit : (3 Marks) :

(6) Performing experiment and entering the readings : (4 Marks) :

(7) Substitution and calculation /plotting the graph and calculation : (3 Marks) :

(8) Result with unit : (2 Marks) :

Radius of curvature of spherical surface = _____ cm.

(9) Viva-Voce : 4 Simple Questions related to experiment (4 Marks) :

(1) What is the zero error in spherometer ?

Spherometer may have a zero error.

Z.E. in spherometer = reading on the plane glass sheet

(2) When the zero error (Z.E.) is positive and when negative?

Ans. **Positive:** If the edge of the circular disc is at zero of main scale and the zero of the circular scale is ahead of the edge of main scale. If it is behind the edge of main scale, the Z.E. is **negative.**

(3) What is the principle behind working of spheroometer ?

The linear distance moved by the screw is directly proportional to the rotation given to it.

(4) What is the radius of curvature of a plane surface ?

Ans : Infinity

(5) What is the radius of curvature of a point ?

Ans : Zero

(5) Why the instrument is given the name “spherometer”?

Ans. Because it is used to determine the radius of curvature of a spherical surface.

EXPERIMENT 4 : Parallelogram Law of Vector Addition

Determine the weight of a given body using parallelogram law of vector addition. Take three sets of readings ?

ANS :

(1) Principle of the experiment : 2M

If two forces acting at a point are represented both in magnitude and direction by the adjacent sides of parallelogram drawn from that point then their resultant is represented in magnitude and direction by the diagonal of the parallelogram.

(2) Formula and explaining the terms : 2M

(i) Weight of the given body = $\vec{R} = \vec{P} + \vec{Q}$

or R = $\sqrt{(P^2 + Q^2) + 2\,PQ\,cos\theta}$

(3) Diagram / figure / circuit with labeling : 2M

(4) Tabular column/ observation pattern : (2 Marks) :

(5) Construction of the experimental set up/ circuit : (3 Marks) :

(6) Performing experiment and entering the readings : (4 Marks) :

(7) Substitution and calculation /plotting the graph and calculation : (3 Marks) :

(8) Result with unit : (2 Marks) :

Weight of the given body = ______ Kgwt.

(9) Viva-Voce : 4 Simple Questions related to experiment (4 Marks) :

(1) What is Law of forces apparatus ? (Gravesand's Apparatus) ?

Ans : It is an apparatus used to measure the mass of a wooden block using Law of forces

(2) Which vector property is used to measure the mass of a wooden block using Law of forces apparatus ?

Ans : Vector Addition

(3) State law of parallelogram of addition ?

Ans : If two forces acting at a point are represented both in magnitude and direction by the adjacent sides of parallelogram drawn from that point then their resultant is represented in magnitude and direction by the diagonal of the parallelogram.

(4) What is a vector quantity ?

A quantity which has both magnitude and direction is called vector quantity.

(5) Define resolution of vectors?

Ans. The splitting up of a single vector into two or more vectors is called resolution of vector.

EXPERIMENT 5 : Simple Pendulum

Using Simple Pendulum, plot length l versus square of Time period (T) graph. Hence find effective length of a second's pendulum. Take four sets of reading.

ANS :

(1) Principle of the experiment : 2M

The simple pendulum whose time period is equal to 2 second is called second's pendulum.

(2) Formula and explaining the terms : 2M

(i) Time Period T = $2\pi \sqrt{\frac{L}{g}}$ or $T^2 = 4\pi^2 \frac{L}{g}$

(ii) Hence $T^2 \propto L$

where L is length of pendulum, g = acceleration due to gravity.

(3) Diagram / figure / circuit with labeling : 2M

(4) Tabular column/ observation pattern : (2 Marks) :

(5) Construction of the experimental set up/ circuit : (3 Marks) :

(6) Performing experiment and entering the readings : (4 Marks) :

(7) Substitution and calculation /plotting the graph and calculation : (3 Marks) :

(8) Result with unit : (2 Marks) :

The length of the second's pendulum = ______ m.

(9) Viva-Voce : 4 Simple Questions related to experiment (4 Marks) :

(1) What is a simple Pendulum ?

An ideal simple pendulum is defined as 'single isolated particle suspended by a weightless, flexible and inextensible string with a friction-less support'.

(2) A stop watch is marked with 60 divisions to count 1 minute. What is the least count of stop watch ?

(3) How does the period of oscillations (T) depends on

(a) Length of the pendulum (b) acceleration due to gravity (c) mass of the bob

(4) Why is the oscillation of a simple pendulum a damped oscillation?

(5) What is required to overcome damped oscillations ?

Ans : A driving force is required

(6) Why is a spherical solid bob taken for a simple pendulum ? What if a hollow spherical bob like a ping-pong ball is taken for the experiment ? Can we use a cricket ball in place of the bob?
Ans. No, by definition of simple pendulum the bob must be as small as possible.

(7) What causes simple pendulum to damp in its oscillations ?
(8) Why the clock go fast in winter and slow in summer ?
(9) Define amplitude ?
(10) Give different equations for SHM of a simple pendulum ?
(11) What do you mean by oscillating motion ?
The motion from one extreme position to the other and then back to the original one.

(12) Why the word 'SIMPLE' is used before the pendulum?
Ans. Because the pendulums used in the wall clocks are 'COMPOUND PENDULUMS', in which a metallic rod is used in place of the thread.
(13) What is the difference between 'g' and 'G'?
Ans. The value of G (gravitational constant) remains constant throughout the universe, whereas the value of 'g' decreases with the increase in the height.
(14) What is the value of 'g' at the C.G. of the earth?
Ans. Zero.
(15) How the value of 'g' changes as we move from the surface towards the C.G. of the earth?
Ans. As a rule it should decrease gradually but due to variable density of the earth, it increases up to a small depth and then decreases.
(16) Where the 'g' is greater, at equator or poles?
Ans. At the poles (where the earth is slightly compressed).
(17) Can you replace the thread by a metallic wire?
Ans. No, because the wire is not flexible.
(18) Why the pendulum stops after some time?
Ans. Its energy is lost as heat.
(19) Can you replace the thread by a rubber band?
Ans. No, because it is not inextensible. By definition the string must be inextensible.

EXPERIMENT 6 : Friction

Determine the coefficient of friction between the surface of a moving block & that of a horizontal surface. Take three sets of readings.

ANS :

(1) Principle of the experiment : 2M

The limiting friction is directly proportional to the normal reaction.

(2) Formula and explaining the terms : 2M

(i) Coefficient of limiting friction = $\mu_l = \frac{F_L}{R}$

(ii) Normal Reaction (R) = W + w

(iii) Limiting Friction F_L = P + p

where W = weight of wooden box, w = added weight to wooden box

P = weight of the Pan, p = added weight to pan

(3) Diagram / figure / circuit with labeling : 2M

(4) Tabular column/ observation pattern : (2 Marks) :

(5) Construction of the experimental set up/ circuit : (3 Marks) :

(6) Performing experiment and entering the readings : (4 Marks) :

(7) Substitution and calculation /plotting the graph and calculation : (3 Marks) :

(8) Result with unit : (2 Marks) :

The coefficient of limiting friction = ______ (unit less quantity)

(9) Viva-Voce : 4 Simple Questions related to experiment (4 Marks) :

(a) What is static friction ?

(b) What is meant by limiting value of friction

(c) Define co-efficient of limiting friction

(d) How does limiting friction depends on the area of contact of the body on another surface

(e) Does limiting friction depends on nature of the surface in contact ?

(g) On what factors does the force of friction depend ?

EXPERIMENT 7 : Inclined Plane

Using inclined plane plot a graph of downward force (W) verses sine of angle of inclination (sin θ). Take four set of readings.

ANS :

(1) Principle of the experiment : 2M

The component mg sinθ of the weight acting parallel to the inclined plane downward produces motion in the body.

(2) Formula and explaining the terms : 2M

(i) Downward force, $W = mg \sin\theta - f_r$

where m = mass of roller, g = acceleration due to gravity, f_r = force of friction, θ = angle of inclination.

(3) Diagram / figure / circuit with labeling : 2M

(4) Tabular column/ observation pattern : (2 Marks) :

(5) Construction of the experimental set up/ circuit : (3 Marks) :

(6) Performing experiment and entering the readings : (4 Marks) :

(7) Substitution and calculation /plotting the graph and calculation : (3 Marks) :

(8) Result with unit : (2 Marks) :

Graph of downward force (W) verses sine of angle of inclination (sin θ) is a straight line.

(9) Viva-Voce : 4 Simple Questions related to experiment (4 Marks) :

(a) What is meant by angle of repose ?

(b) For an inclined plane, connect acceleration of a sliding body
(i) with friction (ii) without friction

(c) For a rolling body on an inclined plane, give an expression for acceleration (without friction)

(d) Give an expression for acceleration of a block lifted vertically upward as a result of load sliding on an inclined plane downwards over a pulley ?

Ans : $a = \frac{(m_1 - m_2 \sin\theta)\, g}{(m_1 + m_2)}$

(e) Say whether an inclined plane is a simple machine or not ?

EXPERIMENT 8 : Spring Constant

Determine the force constant and effective mass of the helical spring by plotting load (L) versus extension (x) graph and square of time period (T^2) verses mass (m) graph using method of oscillation.

(1) Principle of the experiment : 2M

The force is directly proportional to the displacement of the spring.

(2) Formula and explaining the terms : 2M

(i) Spring constant = k = W/x

where W = load, x = extension.

(3) Diagram / figure / circuit with labeling : 2M

(4) Tabular column/ observation pattern : (2 Marks) :

(5) Construction of the experimental set up/ circuit : (3 Marks) :

(6) Performing experiment and entering the readings : (4 Marks) :

(7) Substitution and calculation /plotting the graph and calculation : (3 Marks) :

(8) Result with unit : (2 Marks) :

The spring constant of the given spring = _________ Nm^{-1}.

Effective mass of helical spring = _______ Kg.

(9) Viva-Voce : 4 Simple Questions related to experiment (4 Marks) :

(a) What is meant by spring constant ?

(b) Give the period of oscillation (i) without the mass of the spring (ii) with low mass of the spring (iii) with a large mass of the spring

(c) If $T^2 = (4\pi^2/k)m + (4\pi^2/k)m_0$ where m_0 is the mass of the spring then a graph of T^2 along the Y-axis and m along the X-axis is a straight line with a y-intercept. Give the expression for spring constant and mass of the spring in terms of slope of the line.

(d)

EXPERIMENT 9 : Surface Tension

Determine the surface tension of water by capillary rise method. Given density of water = 1000 kgm-3, radius of the bore of the capillary tube r = m, least count of the vernier = 0.01 cm.

ANS :

(1) Principle of the experiment : 2M

The force per unit length acting in plane of interface between the liquid and the bounding surface is called surface tension.

(2) Formula and explaining the terms : 2M

(i) Surface tension = T = (ρghr)/2

where ρ = density of water, g = acceleration due to gravity, h = capillary rise, r =radius of the capillary bore.

(3) Diagram / figure / circuit with labeling : 2M

(4) Tabular column/ observation pattern : (2 Marks) :

(5) Construction of the experimental set up/ circuit : (3 Marks) :

(6) Performing experiment and entering the readings : (4 Marks) :

(7) Substitution and calculation /plotting the graph and calculation : (3 Marks) :

(8) Result with unit : (2 Marks) :

Surface tension of water = ______ Nsm^{-2}.

(9) Viva-Voce : 4 Simple Questions related to experiment (4 Marks) :

(a) What is surface tension ?

"The tangential cohesive force acting along the unit length of the surface of a liquid"

T = F / L

(b) How does surface tension depends on (i) height (ii) radius of capillary tube

(c) How are r and h related ?

(d) What are the factors affecting the surface tension?

Ans. (a) Nature of liquid (b) Nature of the surface in contact (c) Temperature

(e) Define critical temperature.

Ans. The temperature at which the surface tension is zero.

(f) Why the free surface of water is concave but that of mercury is convex?

Ans. The free surface of water is concave because:

Cohesion force between water molecules < adhesion force between water and gas molecules

Because the free surface of mercury is convex because Cohesion force $>$ adhesion force

(g) What is the effect of temperature on the surface tension?
Ans. Surface tension decreases with the rise of temperature.

EXPERIMENT 10 : Cooling Curve

Plot a graph between excess temperature of hot body ($\theta - \theta_0$) and time (t) using calorimeter. take five sets of readings.

ANS :

(1) Principle of the experiment : 2M

The rate of cooling of a body is proportional to the excess temperature of hot body ($\theta - \theta_0$) over the surroundings provided the body temperature closer to the surrounding temperature.

(2) Formula and explaining the terms : 2M

(i) Rate of cooling = $\Delta Q/\Delta t = -K(\theta - \theta_0)$

where K = constant of proportionality, θ = Temperature of the body, θ_0 = temperature of the surrounding.

(3) Diagram / figure / circuit with labeling : 2M

(4) Tabular column/ observation pattern : (2 Marks) :

(5) Construction of the experimental set up/ circuit : (3 Marks) :

(6) Performing experiment and entering the readings : (4 Marks) :

(7) Substitution and calculation /plotting the graph and calculation : (3 Marks) :

(8) Result with unit : (2 Marks) :

The time verses excess of temperature of hot body, (cooling curve) is an exponential decay curve.

(9) Viva-Voce : 4 Simple Questions related to experiment (4 Marks) :

(a) State Newton's law of cooling ?

(b) What should be the temperature of surrounding with respect to a hot object for Newton's law of cooling to hold good ?

EXPERIMENT 11 : Sonometer

Draw a graph of square of resonant length and tension for constant frequency using sonometer. Take three sets of readings.

ANS :

(1) Principle of the experiment : 2M

When applied frequency of impulse is equal to natural frequency of vibration of string, then, resonance takes place. If the frequency and the mass per unit length kept constant, then resonating length is proportional to the square root of tension.

(2) Formula and explaining the terms : 2M

(i) Resonant frequency f = $\frac{1}{2\sqrt{m}} \cdot \frac{\sqrt{T}}{l}$

where m = Mass per unit length of the string, T = tension of the string, l = resonating length.

(3) Diagram / figure / circuit with labeling : 2M

(4) Tabular column/ observation pattern : (2 Marks) :

(5) Construction of the experimental set up/ circuit : (3 Marks) :

(6) Performing experiment and entering the readings : (4 Marks) :

(7) Substitution and calculation /plotting the graph and calculation : (3 Marks) :

(8) Result with unit : (2 Marks) :

It is found that a graph of square of resonating length (l^2) verses tension (T) is a straight line. Hence resonating length is proportional to the square root of tension.

(9) Viva-Voce : 4 Simple Questions related to experiment (4 Marks) :

(a) How is frequency of vibration of a stretched string related to (i) length of the string (ii) tension of the string ? (iii) linear density of the material of the string ?

(b) What is Sonometer ?

(c) What is tension ?

EXPERIMENT 12 : Resonance Column

Determine the velocity of sound in air at room temperature using resonance column. Take readings for two different frequencies of tuning forks.

ANS :

(1) Principle of the experiment : 2M

When applied frequency of impulse is equal to natural frequency of vibration of air column then resonance takes place.

(2) Formula and explaining the terms : 2M

(i) Speed of sound = $v = 2f(l_2 - l_1)$

where f = frequency of tuning fork, l_1 = first resonating length, l_2 = second resonating length.

(3) Diagram / figure / circuit with labeling : 2M

(4) Tabular column/ observation pattern : (2 Marks) :

(5) Construction of the experimental set up/ circuit : (3 Marks) :

(6) Performing experiment and entering the readings : (4 Marks) :

(7) Substitution and calculation /plotting the graph and calculation : (3 Marks) :

(8) Result with unit : (2 Marks) :

The speed/velocity of sound at room temperature = _______ ms^{-1}.

(9) Viva-Voce : 4 Simple Questions related to experiment (4 Marks) :

(a) What is resonance air column ?

(b) What is meant by unison ?

(c) Give an expression for velocity of sound in air

(d) Why it is easier to measure resonating lengths in water using resonance pipes ?

(e) If the radius of the pipe is increased, then does it affect the resonating length of air column in it ?

(f) What changes will you observe in the intensity of sound by changing the pipes of smaller diameter to larger diameter ?

(g) What is end correction of a pipe ?

The distance between the open of the tube and the antinode at the position of tunning fork.

(h) How can you find the end correction (E)?
Ans. By using the relation E = 0.3 d
Where d is the internal diameter of the tube.

(h) Give the expression for the end correction of the pipe in terms of its radius ?
Ans : $e^{0.58\,R}$

(i) Why it is not possible to produce transverse wave in air ?

(j) How does speed of sound depends on the temperature ?

(k) What is an echo?
Ans. Echo is the effect produced when sound wave is reflected on striking a solid obstacle like wall or rock.

(l) An open end is a node or antinode?
An antinode.

(m) Is there a node or antinode at the close end of water level?
Ans. A node.

(n) Where resonance is produced?
Ans. In the air column.

EXPERIMENT 13 : Specific Heat Capacity

Determine the specific heat capacity of a given solid using calorimeter. Given specific heat capacity of material of calorimeter S_c = 385 $Jkg^{-1}K^{-1}$. Specific heat capacity of water is S_w = 4190 $Jkg^{-1}K^{-1}$.

ANS :

(1) Principle of the experiment : 2M

The specific heat capacity of the substance is the amount of heat required to raise the temperature of unit mass of that substance by unity.

(2) Formula and explaining the terms : 2M

(i) Specific heat capacity = S = $\frac{[m_1 S_c + (m_2 - m_1)S_w](\theta_3 - \theta_1)}{m_3(\theta_2 - \theta_3)}$

where m_1 = mass of empty calorimeter with the stirrer, S_c = specific heat capacity of material of calorimeter, m_2 = mass of empty calorimeter with the stirrer and water, m_3 = mass of the solid, S_w = specific heat capacity of water, θ_1 = Initial temperature of water, θ_2 = Initial temperature of solid, θ_3 = Final steady temperature of the mixture.

(3) Diagram / figure / circuit with labeling : 2M

(4) Tabular column/ observation pattern : (2 Marks) :

(5) Construction of the experimental set up/ circuit : (3 Marks) :

(6) Performing experiment and entering the readings : (4 Marks) :

(7) Substitution and calculation /plotting the graph and calculation : (3 Marks) :

(8) Result with unit : (2 Marks) :

The specific heat capacity of given solid = __________ $Jkg^{-1}K^{-1}$.

(9) Viva-Voce : 4 Simple Questions related to experiment (4 Marks) :

(a) Define thermal heat capacity of a body ?

(b) State the principle of calorimetry ?

(c) The specific heat capacity of water is S_w = 4190 $Jkg^{-1}K^{-1}$. What it signify ?

(d) Which liquid has maximum specific heat ?