A Note on Unclosed Sets of Points defined as the Limit of a Sequence of Closed Sets of Points. By W. H. Young.

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1. The method pursued in the foregoing paper, "On Closed Sets of Points," enables us, without difficulty, to determine the most general condition that the limit of the content of such a sequence of sets G_n should be the content of the closed set Γ in which they become dense everywhere.

2. Let $G_1, G_2, \ldots$ be a countably infinite sequence of sets of points each of which is closed and nowhere dense and is contained in all the succeeding sets, and such that, given any segment, it is possible to specify a part of it which is entirely black for every set G_n, and let G be the limiting set and Γ the set obtained by adding to G those points, if any, which, without being points of G, are limiting points of G. *G and Γ will then be dense nowhere.* The following general theorem, of which the theorem of § 12 of the foregoing paper is a special case, viz., when G and Γ are identical, is the key to the whole matter.

3. *Given any two small positive quantities ϵ and σ, we can assign a definite integer m such that for all integers $n \geqslant m$ the difference between the sums of all the black intervals of Γ and of G^n which are $\geqslant \epsilon$ is less than σ.*

The proof is precisely on the lines of that of the corresponding theorem of § 12, *loc. cit.* We have, however, to notice that, if (P, Q) be a black interval of Γ which is $\geqslant \epsilon$, and we lengthen (P, Q) at each end by a length $\frac{\sigma}{2k}$ (where k is the number of black intervals of Γ which are $\geqslant \epsilon$), there must be a point P_1 of G in the little piece added on at P, though P is not necessarily a point of G, and similarly there must be a point Q_1 of G in the little piece added on at Q.

If we determine m_1 so that both P_1 and Q_1 are points of G_{m_1}, then, since (P, Q) is certainly entirely black for G_{m_1}, either (P_1, Q_1)

is a black interval of G_{m_1} or else there is a black interval of G_{m_1} whose end points lie in (P, P_1) and (Q, Q_1) respectively.

Doing this with each black interval $\geqslant \epsilon$ of Γ, we determine an integer m_2 such that the sum of those black intervals of G_{m_2} which are led up to by all the black intervals of Γ which are $\geqslant \epsilon$ differs from the sum of the latter black intervals by less than σ.

As in § 12, *loc. cit.*, however, G_{m_2} may have other black intervals $\geqslant \epsilon$; but these can be disposed of and the proof completed precisely as was done there for the closed G.

4. Thus, if I and I_n be the contents of Γ and G_n respectively, and $R(\epsilon)$ and $R_n(\epsilon)$ be the sums of those black intervals of Γ and G_n respectively which are $< \epsilon$, we have

$$I - I_n - R_n(\epsilon) + R(\epsilon) < \sigma.$$

Now we can choose ϵ so as to make $R(\epsilon)$ as small as we please, so that *it is evidently necessary and sufficient for the equality of I and $\lim I_n$ ($n = \infty$) that we may be able to choose ϵ so that, for all integers n greater than a certain integer, $R_n(\epsilon)$ may be less than any assigned small quantity.*

Summation of a certain Series. *By* A. C. Dixon.

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The object of the present note is to find the sum of the infinite series

$$1 + \frac{\alpha\beta\gamma}{\delta\epsilon} + \frac{\alpha(\alpha+1)\,\beta(\beta+1)\,\gamma(\gamma+1)}{2!\,\delta(\delta+1)\,\epsilon(\epsilon+1)} + \dots$$

in the case when $\beta + \delta = \gamma + \epsilon = \alpha + 1.$

The condition for convergency will be supposed satisfied; that is, the real part of $\delta + \epsilon - \alpha - \beta - \gamma$ will be taken to be positive.

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* The original MS. was lost in transit. The paper has been rewritten by the author, January, 1903.