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## On the Teaching of Elementary Mathematics in Secondary Schools

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Standard IV. Subjects of Standard III. extended ; easy proportion, plotting graphs for attendance, thermometer, etc. Algebra, use and meaning of  $a+b$ ,  $a-b$ ,  $a \times b$ ,  $\frac{a}{b}$  ; easy simple equations and problems.

Standard V. Harder exercises in I.-IV. Metric system, in which long names of multiples and submultiples are needless—they can be expressed decimally. Fractions in algebra.

Standard VI. All previous work thoroughly revised and somewhat extended. Percentages, averages, interest, discount. Graphs, charts, diagrams. Algebra, factors, simple fractional equations, simultaneous equations.

Standard VII. Simple surds ; A. and G. progressions ; easy indices and logarithms ; with a full course in Arithmetic.

Each section mentioned above opens out a well-nigh inexhaustible store of exercises and sub-exercises which only an interested and intelligent teacher can manipulate. And herein lies the secret of successful teaching of number to young children and the superlative need for careful thought and preparation on the part of the teacher to come before his class armed with all those weapons so essential to create interest in the work, and to draw the children almost unconsciously into the vortex of intensive mental effort.

Geometry might be begun much earlier, and theoretical geometry introduced.

Although this rough outline of what is being done is more or less applicable to all schools, especially in the earlier years, the work in the later years is very much curtailed and handicapped by the miserable requirements of some educational authorities for scholarships to be held in the Secondary Schools. Until this anomaly is removed, it will be utterly impossible for all schools to adopt a rational and an intelligent scheme similar to that I have just described.

Not till the fifth year do some children pass on to the County Schools with a fair knowledge of Arithmetic, Algebra, elements of Geometry and Mensuration.

Glanadda Council School, Bangor.

R. W. JONES.

## ON THE TEACHING OF ELEMENTARY MATHEMATICS IN SECONDARY SCHOOLS.\*

IN order to make this brief paper as practical as possible, I shall plunge into the subject with little introduction.

I wish to consider the question from three points of view.

- (1) What is the ideal course ?
- (2) What is the best practicable course ?
- (3) What effect the best practicable course should have on the teaching of the subject in our secondary schools, and also—since the two are so closely connected—in our elementary schools.

Of course, the chief difference between the ideal and the practicable largely arises from the fact that we are not able as teachers to frame our syllabus exactly as we wish ; external forces—chiefly our British external examination system—make us tone down our ideal into the practicable, but I hope to show that even with this limitation we can still do much to make our elementary mathematical teaching more efficient than it has been. And in what follows I wish it to be understood that I am thinking of the average child, though I may say that it is too often assumed that the elementary

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\* A paper read before the N. Wales Branch.

mathematical course suited to the average child is unsuitable for the budding mathematician. I fail to see why this should be so.

But to return to the main topic and to consider the subjects we have to deal with seriatim.

*Arithmetic.*—We must teach our pupils both to think and to calculate, and to use their powers of calculation in connection with problems more or less close to their experiences of daily life. We now allow them to shorten considerably their solutions by using algebraical methods in reason, though, while considering such algebraic solutions as correct, I myself almost invariably show my class for the benefit of the abler boys purely arithmetical solutions. “Do them by arithmetic, if you can, but do them.” In addition to the fact that  $x$  and  $y$  are not now tabooed in arithmetic, another great change (of which we have not yet seen the full development) is the use and application of approximate methods of calculation. But for such methods to be effective they must be made “part and parcel” of the whole arithmetic course. They ought to be developed so that instead of asking for answers to a certain number of significant figures or decimal places, we should be able to ask later on for answers to as many figures as are permitted by the data, considering the latter to be subject to error. An article in the *School World* for last December by the Mathematical Master at Manchester Grammar School indicates one way of tackling such problems. And may I say in an aside at the expense of our friends who teach Natural Science that such a method is more scientific than the “Always work to four figures,” which is so frequently used.

To what extent does the pressure of necessity (I mean our sacro-sanct external examination system—with most of us the Central Welsh Board) affect our arithmetic syllabus and the teaching of it? I believe it need not affect the sort of work I have referred to above. I know of no arithmetic examination in the country where the pursuing of rational methods of approximation meets with its just reward to the same extent as it does in the C.W.B.—in the Junior, though of course especially in the Senior.

I think a teacher who prefers what may be called rectangular solutions to pyramidal solutions of the old type need not fear to submit his pupils to the test of an examination like this. Of course, if we had no external examination we might be able to spend rather more time on interesting though out-of-the-way problems, but if any British examination in arithmetic known to me deserves well of modern teachers it is the Central Welsh Board.

I have dealt with the subject of approximation at full length. To what extent does it affect our elementary work and the relationship between that work and the teaching in the elementary schools from which our pupils largely come? It goes without the saying that the primal necessity is for us to teach our elementary rules, multiplication of course especially, in such a way as to help and not hinder our approximate work later on. This refers to the need of multiplying by the left hand figure of the multiplier first of all. At present the other method becomes so ingrained in the pupil's mind that there is always a chance that in the question: Multiply 16·81 by 20·575, the child uses the one method of starting, while if the question reads: Multiply 16·81 by 20·575 (correct to the nearest whole number) the other method is (reluctantly) employed. And since even the youngest children are taught multiplication before they enter our secondary schools, I should like to pass on this recommendation to our friends who teach in elementary schools, that multiplication should begin with the left-hand digit of the multiplier, and that this should be done from the first if possible, but at all events in the higher standards.

There are other matters which should claim our attention in arithmetic, but I have no time to do more than refer to a few of them by name.

Decimals and vulgar fractions should be begun almost together, decimals

if anything having the preference as regards order, and the metric system provides an excellent concrete basis. Recurring decimals have, I believe, gone from our midst altogether, with the exception perhaps of the notation, not to appear again until convergent geometrical progression is dealt with.

*Algebra.*—In connection with a modern syllabus in Algebra I cannot do better than refer you to Mr. C. Godfrey's pamphlet on *The Algebra Syllabus in the Secondary School*, which was published last year by the Board of Education. He draws up a syllabus for non-specialist pupils, *i.e.* for children who need mathematics only as a part of their general education, and he considers that the aims we should have should be—

(1) the solving of problems ;

(2) the study of functionality, including variation, graphs and the like.

This would give us time to introduce to our ordinary forms numerical Trigonometry and the ideas of the infinitesimal calculus, a view not so startling if we accept Prof. Love's dictum that (I quote from memory) "The principles of the calculus ought to be counted as a part of the intellectual heritage of every educated man and woman in the twentieth century, no less than the Copernican system or the Darwinian theory." Of course, the question of the Calculus hardly concerns us here, unless we treat it purely as a branch of Elementary Mathematics, a by no means impossible thing to do, but we can still consider the advisability of leaving out much of the algebra we at present deal with in order to make room for numerical Trigonometry and the Calculus. But we who are influenced by an external examination can hardly alter our syllabus to the extent indicated during this year or next year at any rate. What then can we do to make our algebra more concrete and valuable in view of our limitations? I still think that we may use the two aims I have taken from Mr. Godfrey's pamphlet as a sort of standard-ideal, if you will, though the word suggests its present impracticability. If we are trying to teach the way to solve problems, and if we hope later on to add the study of functionality (always remembering the examination), we can do much to make our algebra more reasonably modern and useful without rendering it any the less logical.

Consider the following syllabus in connection with our Junior stage :

Simple questions on notation and symbolical expression.

Easy simple equations and problems.

Addition, subtraction, multiplication and division of monomials.

"Think of a number" problems.

Harder simple equations and problems.

Linear equations in two unknowns and problems.

Linear statistical graphs in their many applications.

Now can we alter in this way the order of our algebra work with our present limitations? I think we can, though at present there may be some little element of risk. I think we have a right to ask that this risk be eliminated. Surely there is no need for such an order as "substitution, the four rules, simple equations, factors, H.C.F. and L.C.M., fractions" to be perpetuated in both the examination paper and in the syllabus, unless we are expected to consider this the best order in which we can teach the syllabus. Is it too much of a revolution to ask for a couple of equations to be solved in the first question, and then for one or two problems to follow? Our pupils are not good at problems, but I ask whether under our present method and order they are likely to be anything else.

With regard to the relation of our elementary algebra work to the algebra in the elementary school, I think it is fairly obvious that (where algebra is taught at all) it should be different from the five-worded syllabus "substitution, addition, subtraction, multiplication, division." The early part of the scheme sketched out above can be followed—you have not to consider the pressure of external examiners. Simple equations and problems can be

dealt with as soon as  $3x$  can be doubled, the questions, of course, being carefully chosen. As soon as the negative sign is understood and addition and subtraction of monomials have been attended to, slightly harder equations and problems can be taken. The "think of a number" problems can be solved and invented by quite small children (I tested it during a summer holiday with a girl of ten years who had never learnt a word of algebra). The value of the exercise is not reduced because of the pleasure the inventor shows when he can say in his oracular manner "Every one of you should have got the answer 23," or "Your answer, John, is 19; then the number you thought of was 7."

Finally, if multiplication is taught beyond that of monomials, surely the horizontal multiplication of simple expressions (especially binomials) should be taught some time before the ordinary multiplication in columns is introduced. Only by so doing can we be spared the irritation of seeing a boy work out two-thirds of a multiplication sum in the margin when he has to multiply  $2a+3b$  by  $5a-2b$ .

*Geometry.*—The superiority of the new geometry over the old Euclid is incalculable, though the tendency to looseness of thought in connection with proofs is a danger to be carefully guarded against. "Intuition" is an excellent aid to enabling a child to see the truth of a theorem, especially if carefully drawn figures are not used. (A few pupils will say, "These lines are equal, just notice their lengths," and so I am a firm believer in freehand figures, or at any rate figures not drawn to scale, being used in connection with propositions.) But surely intuition need not be always invoked in order to avoid proving a proposition. In the early work a simple proof of an axiomatic proposition like *Euc. I. 13* (the Theorem I. of nearly every textbook) is not without its value.

What should be the relation between our practical and our theoretical work? My own view is that after a very short course in practical geometry, not lasting longer than a term at most, the theoretical work should be begun, and the two henceforward carried on side by side. In the early stages practical work will generally enable a child to discern the truth of a new proposition, and as a rule a class can be made to teach the teacher a proof of it—of course, with a little judicious help.

We should ever be trying to give our pupils the power to solve riders. To me a course of propositions in geometry is something akin to drudgery; but the riders are quite as precious as are the pictures to a boy of fourteen as he reads his book of adventure. How can we make them so to our pupils? And perhaps in this connection one might be allowed to refer to Prof. Bryan's plea in the *School World* for a fixed order of propositions. I am wholly with him in that plea. I could almost agree with him when he says, "Euclid's order is better than no recognised order at all." Who has not been irritated by having I. 5 proved by I. 26 instead of by I. 4, a thing that happens largely because of the general vagueness on the question of order, especially if a boy happens to change his text-book when he changes his school? Surely it is not impossible to hope for a representative committee which shall decide upon a *numbering* of propositions. The order in which such propositions would be learnt could be varied, and this took place even in the days of the iron rule of Euclid. The freedom of the past few years has taught us much in the way of what to avoid.

*Trigonometry.*—The usefulness of Trigonometry in the way of practical applications is probably greater than that of either algebra or geometry, and therefore it is a pity it is not studied more than it is. And yet one first of all needs a certain groundwork of both of these other subjects. "Pythagoras" needs to be known at any rate, for Trigonometry, which only depends upon drawing to scale, will not carry us far. And if a pupil's algebra is so vague that the factors of  $a^2 - b^2$  are ever in doubt, I hardly see

why Trigonometry should be added to his repertoire of knowledge—or ignorance. But a child can learn much of useful Trigonometry in a very short time, and as a first course the solution of triangles without the use of the addition formulae may well be aimed at. Drawing to scale should be used—in moderation, and with pupils of the Junior Form much may safely be attempted—or might be, if only the time could be found for it in the syllabus. An hour and a half a week would be very effective as a means of introducing to young pupils the elements of numerical Trigonometry, including logarithms. It is in this subject, I think, that logarithms ought to be taught rather than in algebra, though if Trigonometry is not taught at all they ought to be included in the algebra course for the sake of their practical usefulness. The drawing of the graph  $y=2^x$  and its interpretation as the graph of  $x=\log_2 y$  forms an excellent introduction, 2 being a better base than 10 for illustrative purposes, because  $2^n$  increases with  $n$  less rapidly than  $10^n$ .

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W. J. WALKER.

### MATHEMATICAL NOTES.

**379.** [L<sup>1</sup>. 1. c.] *Pascal's Theorem*; *Brianchon's Theorem*; *Cross-Centre and Cross-Axis*.

(i) *Pascal's Theorem*.

Let  $A, B, C, D, E, F$  be six concyclic points. Let  $AB, BC, CD$  cut  $DE, EF, FA$  in  $L, M, N$ . Then  $L, M, N$  are collinear.

Let the circumcircle of  $MBE$  meet  $DE, AB$  in  $H, K$ .

Then

$$\angle MHE = \angle MBE = \angle CDE.$$

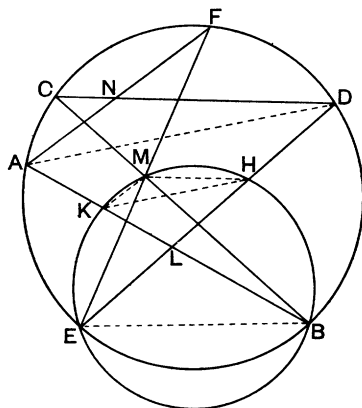
$$\therefore MH \parallel CD.$$

Similarly

$$HK \parallel DA \text{ and } KM \parallel AF,$$

i.e. sides of  $\triangle MHK \parallel$  sides of  $\triangle AND$ .

$$\therefore NM \text{ passes through } L.$$



(ii) *Brianchon's Theorem*.

Let the sides  $AB, BC, CD, DE, EF, FA$  touch the circle in  $P, Q, R, S, T, U$ .

Draw  $AH, AK$  perpendicular to  $PS, RU$ , and let  $PG$  be the diameter through  $P$ .