

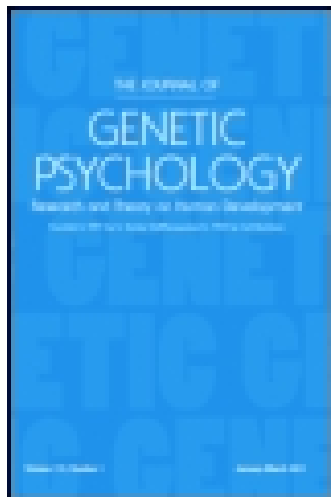
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## The Historical Development of Arithmetical Notation

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interesting study. Authority succeeds with far more children than persuasion, and should precede and prepare for it. A regime of authority can never follow one of sentiment. Wills must be broken to grow straight. None are *innately* incorrigible. Those who are made so are not by neglect, but by wrong treatment by parents, so that ignorant painstaking is worse than none. Punish rarely but severely, and avoid too much or long impression. Make no compromises. Do not buy obedience. Punish consistently and the same for the same act. Do not give too much advice, nor be too precipitate. Never give up, avoid states of chronic hostility, and whip severely sometimes if you love your child wisely; delegate no authority to domestics. A history of the whip and physical punishments is suggested, and also of parental authority before the law; so too, the eye in education, the influence of gaiety and laughter in education, children's ideas of happiness, credulity, idiosyncrasies, jealousy, *l'enfant terrible*, prodigies. Ignorance is better than bad (i. e. atheistic), science. Precocity and timidity need further study. This book is profusely anecdotal, and light, but not without suggestiveness.

*Address of Hon. J. L. M. Curry, General Agent of the Peabody Education Fund, before the General Assembly of Louisiana, May, 1890.*

We may learn profitable lessons in extinguishing illiteracy from Europe. There it is better known that ignorant voters will ruin a country, and that good soil, climate and other natural advantages are nothing without education. The bonds that tether the south to the negro are indissoluble. We must lift him up or he will drag us down. Without education representative government is a farce and life is not worth living, for every dollar saved in education costs five in prisons and reformatories. Ignorant labor is limited to a few products. There is no diversification of industries. Slaves make no inventions. There is no despotism so great as untaught public sentiment. Education is a debt due from society to children whose nature is "a vast unexplored continent of psychology." It is a levee against an overflow of ignorance, which may become constant, and leave no fertilizing sediment, but only destruction and disease. Public, are far cheaper, more responsible, and in every way better than private schools. The essence of christianity and democracy is the exaltation of the individual and then securing for society the capabilities of the individual soul.

## II—MATHEMATICAL.

### THE HISTORICAL DEVELOPEMENT OF ARITHMETICAL NOTATION.

BY LEVI L. CONANT,

Worcester Polytechnic Institute.

Arithmetic as a science is the oldest of all the sciences. Arithmetic as an art is older yet, and its origin unquestionably dates back to the time when the human race was still sunk in deepest barbarism. The modern anthropologist has upon the subject of number made extended investigation among the savage races still in existence or but recently extinct, and has as yet failed to discover a single instance in which the number concept was lacking. It may be limited to the extremest degree, however, the entire number system of a language embracing but three or four words. The Veddás of Ceylon have but two distinct numerals, "ekkamai," one, and "dekkamai," two. Beyond this they count merely by the repetition of the word "otameekai," signifying "and one more," using this expression again and again. The Wiraduroi of Australia count only to 3. For 4 they use their native word for "many," and for 5 "very many." The Puris, the Bushmen and a

few other tribes count only to 2; the New Hollanders, the low forest tribes of Brazil, and others, to 3; the Abipones, the Caribbees, the Galibi, etc., to 4; and a numerous list might be given of the savage races which have 5 as the limit of their arithmetic. Others again count to 10, 20, or 100, the latter being an exceedingly common limit among the more highly developed of the uncivilized races of the world. It should be noted, however, that the number system of a tribe is by no means an infallible index of their advancement in other directions. Certain of the most barbarious tribes of Africa, as the Yorubas, were rendered comparatively expert in numbers through their intercourse with slave traders; and it is even asserted of this particular tribe that among them the saying "You don't know 9 times 9," is equivalent to "You are a dunce." On the other hand the Peruvians, a highly civilized race, knew almost nothing of arithmetic as an art, and absolutely nothing of it as a science. Briefly stated, the method of counting among savage tribes seems to be this; they make use of a small number of numeral words, and anything beyond that is with them "many." Their scale may be "one, many," "one, two, many," "one, two, three, many," or it may extend on much farther than these limits. But their number sense is in general extremely limited, and it is not a common thing to find them able to count beyond 100. In practically all cases, the assistance of the fingers is needed. By this means counting is often done beyond the extent of their number vocabulary, the total being indicated by holding up the proper number of fingers. If the number in question exceeds 10, the toes, or the fingers of a second man are brought into requisition. The development of arithmetic itself is a subject too extended for treatment in a brief review like the present; but there are certain facts connected with the development of arithmetical notation which are of great interest, and which may be stated very briefly. From the rudest, or more properly speaking, the simplest beginnings, arithmetical notation has arrived at its present state of perfection by passing through a number of well-defined stages. This development has of course not been simultaneous all over the world. On the contrary two or more systems of notation may be found flourishing side by side, one in one country, the other in another. Nor are all these stages to be met with in any one part of the world. The different systems of notation are seven in number, and as far as the writer's investigations have gone, no people has ever used them all. Of these seven systems, four are now in common use in different parts of the world, and three of them are found in common use in all civilized countries.

1. The first principle of notation may be termed the natural principle. It consists merely in the repetition of the straight stroke, dot, or some corresponding symbol. By this method 2 would be indicated by two strokes, 11, 3 by three strokes, 111, etc. It is only among the least civilized of peoples that this system is used in its purity. The ancient Egyptians denoted all numbers under ten by the corresponding number of strokes, but with ten a new symbol was introduced. The common Roman system proceeds in the same way, introducing the new symbol at five instead of ten. The ancient Greeks and the Romans both, however, indicated numbers by simple strokes as high as ten. The Aztecs carried this system as high as twenty, but they used a small circle in place of the straight stroke. All races have unquestionably made use of this principle in the period when they were still savages, or in the early stages of their development into civilization. The use of counters or markers, such as pebbles, shells, kernels of grain, etc., and the cutting of notches in a stick, Robinson Crusoe fashion, may be looked upon as variations of this principle.

2. The repetition of strokes, the cutting of notches, the piling up of

pebbles one by one, soon produces a number of symbols so great as to become confusing. Hence, on the telling off of five, ten, twenty, or a hundred, a single stroke was made in a new place, or a pebble laid aside as the beginning of a new pile. Then the same number is again told off by repetitions of the single stroke, and a second record made. This process gives us the additive principle, so often met with in primitive systems of notation. It was employed by the Egyptians, the ancient Greeks, the Babylonians, Phoenicians, Palmyrenes, Hebrews, Romans, Aztecs, and indeed, by almost all peoples that have ever emerged from barbarism into the rudest beginnings even, of civilization. An example of this notation familiar to all is the so-called Roman notation, still used by the modern world for many purposes. This system is perhaps the best of its kind ever invented, for it uses independent symbols for 5, 50, and 500, etc., as well as for the multiples of 10. The Egyptian, Phoenician and other additive systems have only the latter symbols; and the Aztec system contains in all only four symbols—those for 1, 20, 400, and 8000.

3. A great advance on the additive method is to be found in the principle which, from the selection of its symbols, is called the alphabetical principle. In this we find the units from 1 to 9 represented by the first nine letters of the alphabet, the tens from 10 to 90 by the next nine letters, the hundreds up to nine hundred by the next nine, if the alphabet contains a number of letters sufficient for the purpose. If not, a few symbols outside the alphabet proper are used. Thousands, and higher denominations still are usually represented by variations of the letters already employed. This system was in use among the Syrians, Copts, Armenians, Ethiopians, and ancient Greeks, and was brought to its highest state of perfection by the last named people. In their system  $\alpha=1$ ,  $\beta=2$ , but  $\iota\beta=12$ ;  $\gamma=3$ ,  $\rho\kappa\gamma=123$ , etc. By this method any number may be represented by a number of *places* no greater than that required by our modern system, but the number of symbols needed is much greater. Thus,  $\eta=8$ , an entirely different symbol signifies 80, and still a different one 800, while we use but one symbol in expressing the number 888. The Greeks, however, were obliged to use three, writing the number in question,  $\omega\pi\eta$ . This system is far superior to any other used by the ancients, and is inferior only to the perfect system of the modern world.

4. In numeration we use, with all except small numbers, a system which, with a single exception, has never appeared in any mode of notation. We say "one hundred," "four thousand," etc., but when we write these numbers we do not write the one and then the hundred, or the four and then the thousand. A strictly analogous notation would require us to write five hundred thus, 5-100; eight thousand, 8-1000, etc. Uncouth as such a mode of expression would seem, it is the system actually employed by the Chinese, and it was until very recent times, universally employed by the Japanese. An idea of their system will be given by writing the number 26438 in the following manner: 2-10000 6-1000 4-100 3-10 8. Single symbols are of course used for 10, 100, 1000, as well as for higher multiples of 10, so the actual Chinese method will be seen to be rather better than would be indicated by the above illustration. But the method is unwieldy at best, and in Japan it is fast giving way to the Arabic system. A few examples of this multiplicative system are found mixed with other systems, but they are rare, and the system itself can hardly be looked upon as forming one of the successive steps in the development of notation from the piling of pebbles, up to the Arabic system.

5. An interesting variation of the alphabetical principle already mentioned is the marking principle of notation. It is never used as the basis of a system, but is not infrequently employed in an auxiliary

manner. In this system the same symbol, varied only by the addition of some distinguishing mark, may represent 1, 10, 100, etc. Thus, if  $\dot{a}$  is the symbol for 1, we might let  $\dot{a}=10$  and  $\ddot{a}=100$ ; if  $\dot{o}=8$  then  $\ddot{o}=80$ , and  $\ddot{o}=800$ . We may then write the following as illustrative numbers:  $108=\dot{a}\dot{o}$ ;  $18=\dot{a}\dot{o}$ ;  $808=\ddot{o}\dot{o}$ ; etc. The Greeks used this method with numbers from 1,000 to 10,000, and traces of the same system are found among Roman ruins. An exhumed tablet gives the number 1180600 in the following manner: I MI CLXXX D C.

6. In the early middle ages the use of the abacus, or reckoning machine, was common in Europe, just as it is to-day in China and Japan. From the abacus was developed a principle of notation which may be specifically termed the column principle. For certain kinds of work this method is very convenient, and it is still used in many ways for special purposes. It was much more employed in the early dark ages than at any other period, before or since. The following diagram will fully illustrate the method.

	M	C	X	I
2452 =	2	4	5	2
2006 =	2			6
1060 =	1		6	
450 =		4	5	

7. The perfect system of notation is that which makes use of the smallest number of symbols. The Arabic system is, mathematically considered, a perfect system; and it is safe to predict that it will never be superseded. With a number system formed on 10 as a base the smallest possible number of symbols with which it could be represented is ten. The Arabic system accomplishes this perfectly by the introduction of the symbol for zero, and by the device of place value. The introduction of this device marks a most important epoch in the history of mathematics, and without it progress in that science must have been much restricted. It was a brilliant invention, and can be matched in importance only by two other inventions in mathematical notation—logarithms and the literal notation of algebra. The name, and even the country of the inventor of the place system are unknown. Although it bears the name of the "Arabic system," it was not invented by the Arabs, but was obtained by them from the Hindus. Whether or not the Hindus were its actual inventors, or whether they in turn obtained it from some foreign source, is a disputed point and one which will probably never be fully settled. It is the universal system of the modern civilized world, and it is too perfect to admit of change until number itself shall change.

#### TEXT-BOOKS IN ARITHMETICS.

By LEVI L. CONANT,

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An inspection of the text-books which I have mentioned below brings into prominent relief certain facts respecting the methods pursued in

the teaching of arithmetic at the present time in France, Germany, England, Canada and America. The best American text books of to-day are far superior to those most used in Canada, and, judging from the meagre list of books which I have had an opportunity to examine, the same is true, and to an even greater extent with respect to American text-books as compared with English. The fetters of tradition and prejudice are, in England, very strong, and reform in educational methods gains ground but slowly. In Germany and France, on the other hand, are to be found an educational enthusiasm, a scientific investigation into methods, and above all, educational results which put to shame the best we have yet been capable of accomplishing in America. While German text books in arithmetic are in many respects not superior to those in use in this country, they are as a rule far more scientific. Instruction in the German primary schools is much better than with us, and the foundations of German scholarship are there laid broad and deep. But the best recent text-books in arithmetic to be found in the world are those published in France. In the great educational rivalry of to-day between France and Germany, there is one point in which the Germans freely confess the immense superiority of the French—the making of text-books. So marked is this superiority, that it is perhaps not too much to say that there is hardly a department of education in which French text-books are not superior to those of any other country. To take an example wholly at random, compare Clerk Maxwell's *Electricity and Magnetism* with Mascart and Joubert's work on the same subject. As a work of profound scholarship the former has made for itself a great name, but considered merely as a text-book it is both absolutely and relatively very poor. The French work, on the other hand, is a model of scholarly clearness and conciseness. In arithmetic the French do not seem to have adopted the Grube method to any extent, but their own method is so excellent that need for reform does not appear to be felt by them. The best American text-books in arithmetic are beginning to rival French works in the same field, but our elementary instruction, particularly outside our larger cities, is so hopelessly inferior to that in French primary schools, that the comparison is almost disheartening. With the better class of text-books now making their way among us, we may hope for better methods of instruction and better schools as a whole. But the system of local control which prevails in America will render progress slow. As regards methods in arithmetic, no doubt exists in my own mind that the Grube method furnishes the best foundation for subsequent work, not only in that branch, but also in other branches of mathematics. The foundation once laid, any method which is heuristic in its nature may be safely followed. But the one great fact must never be lost sight of, that the teacher is more important than the method. A good teacher can produce excellent results even with an inferior method, while in the hands of a poor teacher, even the best method will be rendered almost useless.

*Aufgaben zum Zifferrechnen für Schüler in Stadt- und Landschulen*, von A. STUBBA. Bunzlau, 1887.

A series of six Hefte or parts, containing problems only. Each Heft contains sixteen pages, and from 175 to 300 problems. The price of each is 3½ cents. The popularity of the series is attested by the sales which had at the date above given, ranged from 100,000 to 400,000 copies.

*Aufgaben für das schriftliche Rechnen*, von W. KOCH, Rektor in Berlin. 1886.

A series of Hefte, containing numerical examples. Each Heft contains 25 to 100 pages, and its price is 5 to 15 cents. The editions before me are numbered respectively 485th, 463d, 289th, 230th, 229th and 78th.

*Aufgaben zum Rechnen*, von A. BOEME. Berlin, 1886.

A series of practice Hefte containing 25 to 30 pages each. Price 5 to 7½ cents. The sale of each Hefte has been from 400,000 to 500,000 copies.

*Uebungsbuecher in Rechnen*, von A. BOEME. Berlin, 1880.

This series appears designed rather for work in closely graded schools, presumably those of some large city like Berlin. They follow the Grube method to some extent. Each Hefte contains about 40 pages and 1,300 examples. More than 100,000 copies of some of the Hefte have been sold. Price 8 to 12 cents.

*Aufgaben zum Zifferrechnen, Für Volksschulen entworfen*, von G. HENTSCHEL. Leipzig, 1887.

A series of practice Hefte containing about 30 pages and 1000 examples each. The different parts have run through from 30 to 50 editions. Price 3 to 4 cents.

*Rechenbücher für die Vorschule*, von CHR. HARME. Oldenburg, 1885.

A series containing about 2000 examples each. It follows the Grube method to some extent, but not strictly.

*Rechenbücher für Stadt und Landschulen* von FERDINAND HEUER. Hanover, 1887.

This series is prepared, as far as a series of simple exercise books can be, on the Pestalozzian method. The series is excellent and deservedly popular, the different Hefte having run through from 50 to 60 editions. Each Hefte contains 1200 to 1400 examples, and its price is 3½ cents.

*Rechenschule. Methodisch-geordnete Aufgaben zum Kopfrechnen*, von G. BERTHELT und ANDERE. Leipzig, 1877.

An excellent series of drill books; each Hefte contains about 1400 examples for mental drill. Price 4 to 6 cents.

*Aufgaben zum Zifferrechnen*. E. HENTSCHEL. 46te Auflage, Leipzig, 1887. pp. 48.

Contains 1139 miscellaneous examples, covering the four fundamental rules and factoring, together with the tables for weights and measures for reference. Price 5 cents.

*Neue Rechenfibel*. ERNST HENTSCHEL. 125te Auflage, Leipzig, 1887. pp. 32.

This book contains 3760 examples, designed to accompany the teaching of arithmetic by the Grube method. It embraces sets of examples comprising the numbers 1 to 10, 1 to 20 and 1 to 100. Price 3½ cents.

*Aufgaben zum Zifferrechnen*, von G. HENTSCHEL. 43te Auflage, Leipzig, 1887. pp. 48.

Contains 1200 well selected examples and the elementary tables of weights and measures. Price 4 cents.

*Arithmetisches Examplebuch für Schulen*, von FRIEDRICH KRANCKES. 110te Auflage, Hannover, 1887. pp. 156.

One of the best of the German exercise manuals. It contains nearly 4000 excellent examples, and full tables of weights and measures.



*Das Ganze der kaufmännischen Arithmetik*, von DR. E. E. FELLER und DR. C. G. ODERMANN. 30te Auflage, Leipzig, 1876. pp. 541.

A German commercial arithmetic, judging from the one before me, is a fearful and wonderful thing. In the early pages of the book devices are given for shortening work in the four fundamental operations; some of these devices are excellent, and some are, to say the least, peculiar. For example, to multiply by 37 we are instructed, p. 6, to multiply by 6 twice, and then to add the minuend to the result obtained. Denominate numbers are introduced almost at the outset, and an exhaustive treatment of fractions, both common and decimal. Ratio and proportion are put in their proper place, immediately after fractions, and then follow percentage, interest, discount, and all the other subjects usually introduced in a commercial arithmetic, each treated with a characteristic German minuteness of detail. Exchange alone occupies about seventy-five pages. But with all the diffuseness noted, the explanations are clear, the examples as a rule good, and the work as a whole excellent.

*Rechenfibel, oder Leitfaden und Examplebuch für den Elementar-Unterricht von Rechnen nach der Erfindungsmethode*, von FRIEDRICH KRANCKE. 13te Auflage, Hannover, 1881. pp. 111.

A primary arithmetic in five parts. Follows the Grube method. The first part deals with numbers from 1 to 10, the fifth with numbers from 1 to 10,000. Graphical devices are extensively employed. Indeed the special feature of the book is its wealth of graphical material, and, for a Grube text-book, its extreme poverty in examples. For this reason alone it can hardly be considered a successful work.

*Lehrerheft zur Ferdinand Heuer's Rechenbuch für Stadt und Landschulen*, von K. H. L. MAGNUS. Hannover, 1886.

A series of three parts, each of from 100 to 160 pages, and each published at from 12½ to 30 cents. They treat the fundamental operations and fractions, and follow a method closely analogous to that of Grube. Decimal fractions are treated first as common fractions, and then the decimal notation introduced. An enormous amount of written and oral work is given, and the development of the subject is exceedingly gradual. In the third part, numbers are used up to 10,000,000, and fractions as small as thousandths. Scarcely anything beyond decimals is attempted, the third part making bare mention of the rule of three, interest, partnership, general average and mensuration, and devoting but eight pages to these combined subjects.

*Anleitung zum Unterricht von Rechnen*, bearbeitet von A. BOEHME. 11te Auflage, Berlin, 1886. pp. 393.

Designed to accompany the series of "Rechenbuecher" mentioned elsewhere. The method is quite similar to Grube's method, but the fundamental operations are begun after the study of the number 20 instead of 100. This work is evidently intended as a basis for a mathematical education. The amount of applied work introduced is exceedingly meagre, more than four-fifths of the work being devoted to pure arithmetic. It is the most exhaustive, and one of the best grammar-school arithmetics with which I am acquainted.

*Ausführliches Lehrbuch der Arithmetik und Algebra, zum Selbstunterricht*, von H. B. LÜBSEN. 19te Auflage, Leipzig, 1887. pp. 259.

Almost as bad as a pre-Revolution arithmetic of this country. I mention the book only to call attention to the German method of combining arithmetic and algebra—a method wholly to be commended. Reform in this direction in America cannot be long delayed.

*Das Verbundene Kopf und Zifferrechnen, für ein-und zweiklassige Volksschulen bearbeitet*, von FRANZ GUTH. Stuttgart, 1888.

Quite different from most German exercise books, and in many ways better. The examples are fewer in number, but are better, and cover a wider range of subjects.

*Lehrbuch der Elementar-Mathematik*, von HALLER VON HALLERSTEIN. Erster Theil, Arithmetik, Berlin, 1885. pp. 306.

This work prepares for entrance to the Prussian Royal Military and the Imperial Marine. It is an excellent example of the more extended German works on arithmetic, and is a significant commentary on the difference between elementary instruction in Germany and in America. No such work has ever appeared in this country, nor will one like it appear for many years to come. Literal notation is introduced at the outset and is used throughout the book side by side with the ordinary notation of arithmetic. Interest is the only "practical" work introduced, and less space is given to it than to continued fractions; the book is, in fact, an extended treatise on pure arithmetic. It ranks very high among German text books, and deservedly so.

*Arithmétique Appliquée, première serie, Arithmétique Appliquée, deuxième serie*, par G. BOVIER-LAPIERRE. Paris, 1890.

Problem manuals, containing about 500 and 1000 examples, respectively, taken largely from official examination papers. Price of each 25 cents.

*La première année d'Arithmétique*, par M. P. LEYSSENNE. Cinquante-huitième édition. Paris, Armand Colin et Cie, 1887. pp. 143.

This book is designed for children of from nine to eleven years of age. In addition to the fundamental operations the tables of weights and measures are taught, being given a prominent place for so elementary a work. Simple geometric notions together with their applications to practical life are introduced. Decimal fractions are given a place among the first pages of the book, while common fractions are hardly touched upon. Multiplication and division are proved, as they should be, by the casting out of nines, a method of which American text-book compilers seem lamentably ignorant. Half-a-dozen pages are devoted to interest, and the book closes with a brief collection of moral maxims upon which the author in his preface takes occasion to plume himself.

*La deuxième année d'Arithmétique*, par M. P. LEYSSENNE. Quarante et unième édition. Paris, Armand Colin et Cie, 1887. pp. 408.

The fundamental operations are taken up in this work much more fully than in the preceding. These together with numeration and decimal fractions occupy a trifle more than one-fourth of the book. They are immediately followed by involution and by square root. Eighty pages are devoted to the tables of weights, measures and money; and common fractions, scarcely touched upon in the previous work, are here discussed fully. Commercial applications and the rule of three are given a less prominent place than is usual in American text books. The marked feature of the book is the amount of geometrical work it contains, covering more than one hundred pages, and bringing in besides the ordinary geometrical figures the ellipse, oval and spiral. Cube root, the reduction of common to decimal fractions, greatest common divisor and least common multiple are consigned to a supplement. The book contains 300 figures and more than 3,000 examples, a part of which are taken from the official examination papers of the French schools. This series has been extraordinarily successful, sales having exceeded 2,000,000 copies.

*Complément d'Arithmétique*, par une Société d'Instituteurs sous la direction de M. E. COMBETTE. Deuxième édition, Paris, 1855. pp. 71.

An admirable little book, containing 1081 examples for practice work. Composition and arrangement of the examples are exceptionally good. Price 9 cents.

*Arithmétique, Système Métrique et Géométrie Usuelle, cours élémentaire*, par une Société d'Instituteurs, sous la direction de M. E. COMBETTE. Deuxième édition, Paris, 1885. pp. 155.

Unusually good. Designed for the first year of the study of arithmetic proper. Two-thirds of its space is given to the four fundamental operations, which are set forth with characteristic French clearness, and twenty-five pages additional to the weights, measures, and money of the metric system. The elementary notions of plane and solid Geometry together with their applications are given in the latter part of the book. An examination of this and the succeeding work, the *Cours Moyen et Supérieur*, goes far toward showing how it is that pupils in the French schools gain at so early an age such an admirable knowledge of elementary geometry. The work contains 115 figures and 730 examples.

*Arithmétique, Système Métrique et Géométrie Usuelle, Cours Moyen et Supérieur*, par une Société d'Instituteurs, sous la direction de M. E. COMBETTE. Troisième édition, Paris, 1887. pp. 426.

The two books comprising this course display as their motto: "Faire aimer l'école par un enseignement attrayant et pratique," and never did two elementary text books possess a better right thus to recommend themselves. The *Cours Moyen et Supérieur* contains 3,000 examples, of which a fair proportion are mental. Several hundred examples are taken from the official government examinations. Square and cube root and the rule of three follow fractions. The subjects of Greatest Common Divisor and Least Common Multiple occupy together two pages, while Complex Fractions are omitted entirely. The method of conducting school savings banks, now so popular in France, is explained, and the elements of book-keeping are introduced. A hundred pages are devoted to weights, measures, and money, and seventy pages to geometry and its applications. If the average American text-books in arithmetic were as good as the two in this series, there would be far less complaint respecting the inefficient work done in this branch in our public schools of today.

*Traité d'Arithmétique, Théorique et Pratique*, par P. PEIN. Paris, 1884. pp. 176.

Designed to cover the work of the first four years. Although written with the characteristic French clearness, this work is far inferior to many of its competitors. It is entirely dogmatic in its method, and gives an undue importance to some of the subjects which are now recognized as of little value for a school text-book, as alligation. A feature of the book is the introduction of the elementary principles of algebra; being of course merely the application of the literal notation to certain of the principles developed.

*Eléments d'Arithmétique et de Géométrie*, par F. VINTEJONX. Cours Élémentaire. Deuxième édition, Paris, 1884. pp. 108.

*Eléments d'Arithmétique et de Géométrie*, par F. VINTEJONX. Cours Moyen. Deuxième édition. Paris, 1886. pp. 207.

These two books are thoroughly excellent. In point of selection of matter, arrangement, and method of presentation, they are models. The elementary course contains only the development of the four funda-

mental principles, together with simple notions of measurement and of the use of the geometrical concepts. The intermediate course covers the ground ordinarily found in text-books of that grade, but gives a much fuller application of arithmetic to geometrical figures than might be expected. This is, however, in harmony with the French method of giving a gradual and exceedingly concrete development to the general subject of geometry.

*Traité de Calcul Mental d'après la Méthode de Henri Mondeux*, par ÉMILE JACOBY. Paris et Bruxelles, 1860. pp. 440.

A work of great importance in French school literature. The method followed is inductive, and the work is much more extended than that of Horace Grant or Warren Colburn. Numbers of four figures are introduced and used to a limited extent, fractions are treated quite fully, and elementary notions introduced of ratio and proportion, mensuration, interest, and other topics usually considered beyond the scope of a mental arithmetic. The influence of this work on subsequent French text-books has been quite appreciable.

*L'Arithmétique des Ecoles Primaires*, par G. BOVIER-LAPIERRE, en collaboration pour le Calcul Mental avec CH. FLEURIOT. Paris, 1889.

Two books of 168 and 288 pages respectively. Distinguished by a large amount of oral work. The text is clear and full and the treatment of the various subjects, particularly of fractions, most excellent. The grouping of all the examples together in the last part of the "cours moyen," apart from the text, is not to be commended. But the course has been very successful, and ranks well among French courses in arithmetic.

*Arithmétique Élémentaire*, Cours Élémentaire, pp. 136.

*Arithmétique Élémentaire*, Cours Moyen, pp. 302.

*Arithmétique Élémentaire*, Cours Supérieur. Par S. MARIE. Paris, 1887. pp. 380.

One of the most recent of the series issued to meet the requirements of the revised courses of study in the elementary schools of France. The Cours Élémentaire is inferior to the other two works, and is not in my opinion equal to some of its rivals. But the Cours Moyen and the Cours Supérieur are fully up to the high French standard of excellence. They contain an unusually full treatment of the metric system and of elementary geometry. The series is accompanied by an exercise manual, *Problèmes d'Arithmétique*, which is one of the best problem books I have ever examined.

*Cours d'Arithmétique et d'Algebre*, par TH. CANONVILLE-DESLYS. Paris, 1886. pp. 505.

Designed for use in the elementary normal schools of France. In point of detail and in fulness of exposition it is comparable to the larger German works on arithmetic, but in system and clearness it is far superior to the best of them. A feature of the work is the space devoted to close analysis. Nearly two pages are devoted (p. 85 et seq.) to the problem: "I have spent  $\frac{2}{3}$  of what I had; I have lost  $\frac{1}{4}$  of the remainder; I have given away  $\frac{3}{8}$  of the new remainder and now have left 8 francs. How much had I to begin with?" Literal notation is introduced in the middle of the book, and the transition gradually made from arithmetic to algebra. For American schools this work would be difficult, but it is in many respects the best arithmetic I have ever examined.

*Arithmetic for Young Children*, by HORACE GRANT. London, 1861. pp. 135.

*Arithmetic for Schools and Families*. HORACE GRANT. London, 1861. pp. 350.

These books, now thirty years old, would hardly require mention were it not for the important place they have filled in the history of English elementary education. Their author, Horace Grant, seems to have done a work for English education similar to that done by Warren Colburn in this country. The plan upon which these two books are based is precisely the same as that pursued by Colburn, that is, the inductive method. The author believes and distinctly states that mental arithmetic is of far greater fundamental importance than written arithmetic. Nine-tenths of the work in these two text-books is mental, only enough written work being given to familiarize the pupil with the use of written figures, and to apply with larger numbers the principles previously developed. Little of the lumber with which so many of our more recent text-books have been encumbered finds a place here, the larger of the two books closing with common and decimal fractions and the rule of three. These two little books are constructed on the soundest pedagogical principles, and our text-book compilers of to-day would do well to study them carefully. An American edition of these books has recently been issued by Willard Small of Boston.

*A Practical Arithmetic on an Entirely New Method*, by JOHN JACKSON. London, 1886. pp. 416.

An Arithmetic on a new method is at any time an object of interest. But the "new method" here heralded is nothing more nor less than the method of the 15th century; the dogmatic method pure and simple. The only reason discoverable for calling this a new method is that the book contains a number of labor saving devices of the same general nature as the ordinary synthetic division of algebra. Definition and rule form the basis of the book, and the practice given is extended beyond all reason. The following examples will indicate sufficiently the unreason which the author displays. "Write in words 100,000,064,000,007,009,000,800,000,700,063." "Find the sq. rt. of 82,447,671,313,860,840,036." "Find the sum of the geometrical progression  $5 + \frac{1}{2} + \frac{1}{4} + \dots$  to infinity." In an appendix are nearly 100 pages of Cambridge and Oxford examination papers.

*Elementary Arithmetic on the Unitary System*, by THOMAS KIRKLAND and WILLIAM SCOTT. Two Hundredth Thousand, Toronto, 1880, pp. 182.

This impresses one as being the work of authors who recognize the shortcomings of the dogmatic method, and who would gladly break away from it, but who are bound hard and fast by the fetters of tradition. The time honored selection of material, order, and method of presentation are followed, but an unusual amount of mental work is given, and in their explanations the authors show themselves to be unconscious believers in the heuristic method. It is the best Canadian text-book I have examined.

*A Treatise on Arithmetic*, by J. HAMBLIN SMITH. Revised for Canadian Schools by THOMAS KIRKLAND and WILLIAM SCOTT. Eighteenth edition, Toronto, 1890, pp. 348.

This work, which forms with the preceding a complete course, and which is either prescribed or authorized for use in the schools of most of the provinces of Canada, may be described as a marriage between the "rule" and the "reasoning" methods. The old enemies of childhood are all here, even to alligation. About one-third of the book is devoted

to what the author calls Pure Arithmetic, and the remainder to Commercial Arithmetic. While the underlying principle is thoroughly dogmatic, reasoning has been allowed in many cases to supplant rules, and processes have been used which are thoroughly intelligible to the childish mind. As a text book this work is by no means equal to our best American arithmetics of to-day, but it is far in advance of our "written arithmetics" of a generation ago.

*High School Arithmetic*, by W. H. BALLARD, A. C. MCKAY, and R. A. THOMPSON. Toronto, 1891, pp. 391.

A unique, and in its way an excellent book. It begins with six pages of glossary, five pages of tables of weights and measures, two pages of "powers of certain numbers," and fifty-five pages devoted to common and decimal fractions, developed and explained chiefly after the dogmatic method. The remainder of the book is, practically speaking, devoted entirely to problems, familiarity with working methods evidently being presupposed. The selection of problems is in the main very good, and the book in the hands of a good teacher would form an unusually fine drill book. Some of the problems are rather difficult; and the following,  $\frac{1}{2} \times 6\frac{1}{2} \times 24\frac{1}{2} - 4\frac{1}{2} \times 3\frac{3}{4} \div 3\frac{3}{4}$ ,  $8\frac{1}{2} \times 54\frac{1}{2} \div 4\frac{1}{2} - 7\frac{1}{2} \times 5\frac{1}{2} \div 14\frac{1}{2}$ ,  $\times 4\frac{1}{2}$ , may properly be described as *mulum in parvo*. Price 60 cents.

*The Public School Arithmetic*. Toronto, 1887, pp. 183.

So good that one could heartily wish it were better. Useless rules and definitions have been minimized, and a large amount of mental work given. It is authorized for and extensively used in the public schools of Ontario.

*Primary Arithmetic*, by REV. D. H. MACVICAR. Toronto and Montreal, 1880, pp. 160.

This work aims "to train the pupil to accuracy and rapidity in the operations of the four elementary rules of arithmetic, . . . and to render him so familiar with fundamental principles and processes as to make advanced work natural and easy." This is at least a hint of the Grube method, and the author seems to understand and to appreciate the importance of that method. The mode of development is good throughout the book, but too much is made of written and not enough of mental work. No attempt is made, as is so often the case with primary arithmetics, to incorporate any except the strictly elementary subjects belonging to text books in arithmetics.

*Practical Problems in Arithmetic*, for First, Second and Third Classes, by I. WHITE. Toronto, 1889, pp. 60.

Contains between seven and eight hundred well selected examples with answers, but more errors than any text book should contain.

*Warren Colburn's First Lessons*, by WARREN COLBURN. Revised and enlarged edition, Boston, 1884, pp. 216.

This classic work, called by the author *An Intellectual Arithmetic upon the Inductive Method of Instruction*, has enjoyed a more widespread popularity than any similar work ever published in the English language, its sales having reached the enormous figure of nearly three million copies. Colburn's method rejects explanations on the part of the teacher, leaving the natural development of the child's mind to overcome difficulties as they are met. No written work is used, but mental analysis of problems is carried to the extent of solving examples of considerable complexity. The edition of 1884 is somewhat more carefully graded than the original work, and is a third or a half larger,

but, contrary to the spirit of the method adopted and distinctly enunciated by Colburn, explanations are in many cases given, and some other changes made which tend to make the work conform rather to the Grube method than to that employed by Colburn.

*Second Lessons in Arithmetic*, by H. N. WHEELER. Boston, 1890, pp. 282.

This is described by the author as "An Intellectual Written Arithmetic upon the Inductive Method of Instruction as illustrated in Warren Colburn's First Lessons." An "intellectual written arithmetic" is certainly an anomaly. The feature of this book seems to be its omission of "useless subjects and arithmetical terms known only in the school room," and the retention of an abundant amount of mental work. In fact it is a mental arithmetic with a certain amount of written work attached to each subject rather than an "intellectual written arithmetic." It is an excellent work, and fully deserves all the success with which it will meet.

*Grandpapa's Arithmetic*. A story of two little apple merchants, by JEAN MACÉ. New York, 1868, pp. 142.

An attempt at a "royal road" to the fundamental ideas of arithmetic. It is not quite the Robinson Crusoe arithmetic of the Herbartian pedagogy, but an unscientific attempt along the same lines. Of the merit of his idea and method the author entertains no more doubt than do the most ardent Herbartians of the entire superiority of their system over all others. In the preface he says, p. vi, "If the effort has not been completely successful, I hope some one will be found to make it so, for there is not the slightest doubt that this is the way in which children should be conducted." Educational research since 1868 has hardly tended to confirm this view.

*Primary Number Lessons*, pp. 149.

*Advanced Arithmetic*, pp. 288. Compiled under the direction of the California State Board of Education. Sacramento, 1887.

These two books constitute the California state series. They are hardly to be commended as school text-books. The compilers seem to have been under the shackles of the dogmatic method, and the desperate efforts they have made to render that method more "natural" have produced a result which is neither one thing nor another.

*First Year Manual and Text-Book of Arithmetic*, by J. H. HOOSE. pp. 154. Syracuse, 1882.

*Second Year Text-Book of Arithmetic*, by J. H. HOOSE. pp. 236. Syracuse, 1890.

The author has given the name "The Pestalozzian Series" to these books. They are excellent, are fully in the spirit of Pestalozzi, and the results obtained with them where they have been tried have been exceptionally good.

*The New Arithmetic*, by Three Hundred Authors. Edited by SEYMOUR EATON. Fifteenth edition, with preface by TRUMAN HENRY SAFFORD, Boston, 1889, pp. 230.

The most practical of all the practical arithmetics I have ever seen. Textual matter is abbreviated almost beyond belief, and the amount of problem work is very great. It is an excellent working text-book, and has been very successful. The preface by Professor Safford is well worth the careful perusal of every teacher of arithmetic.

*The Franklin Primary Arithmetic*, pp. 96.

*The Franklin Elementary Arithmetic*, pp. 144.

*The Franklin Written Arithmetic*, by EDWIN P. SEAVER and GEORGE A. WALTON. New York, 1878, pp. 348.

These three books constitute the now well known Franklin series of arithmetics. In their treatment of the subject they follow approximately the Pestalozzian idea, and as working text-books they can hardly be given too high praise. The Primary Arithmetic is copiously illustrated, and the method of leading the child along in the development is most judicious. The two larger books are filled with admirably selected examples, and sets of drill tables are given by which the multiplication of problems may be almost indefinitely extended. The success of the series has been phenomenal. As an illustration of the estimation in which it is held by publishers, I am informed that a rival firm, wishing to put upon the market a new series of arithmetics, placed in the hands of a compiler in their employ the Franklin series with the single instruction, "Copy these books as closely as possible." They rival in their excellence the exact, clear cut work which so distinguishes the best text-books of French writers.

*First Steps in Number. A Primary Arithmetic*, by G. A. WENTWORTH and E. M. REED. Boston, 1890, pp. 158.

Do. Teacher's Edition, pp. 474.

*Wentworth's Primary Arithmetic*, by G. A. WENTWORTH and E. M. REED. Boston, 1890, pp. 220.

*A Grammar School Arithmetic*, by G. A. WENTWORTH. Boston, 1890, pp. 330.

*A High School Arithmetic*, by G. A. WENTWORTH and REV. THOMAS HILL, D.D., LL. D., ex-President of Harvard College. Boston, 1890, pp. 362.

The most scientific and valuable series of arithmetics that has yet appeared in this country. In the elementary development of number the Grube method is followed. The whole of the first year is devoted to the study of the numbers from 1 to 10, but the authors depart from Grube in giving the second year to the numbers from 11 to 20, instead of 11 to 100, the numbers above 20 coming in the third year. The simplest of the fractions are introduced at the outset, on the theory that the fraction presents no difficulty to the mind of the child. The systematic treatment of fractions is, however, wisely postponed as long as possible. The grouping and arrangement of the matter is excellent, and where there has been departure from the order commonly followed, the departure is usually found to be a wise one. The most striking work in the series is the High School Arithmetic, in which we find one feature which calls for special mention. English and American arithmetics have from time immemorial confined their practical applications of arithmetic almost entirely to commercial work. This book, like the best French and German text-books, draws its materials from physics, chemistry, astronomy, etc., as well as from commerce and banking. Altogether the series should, and doubtless will be, epoch-making in the history of elementary mathematical instruction in America.

*A New Elementary Arithmetic*, pp. 156.

*A New Complete Arithmetic*, by E. E. WHITE. New York, Chicago, Cincinnati, 1890. pp. 360.

For twenty years White's arithmetics have been a popular series. While not the equal of either the Wentworth or the Franklin series, it



is next to them the best series published in America. In arrangement, matter, and method of presentation, these books are in harmony with modern ideas, and they deserve the high reputation they have gained. But there is just enough lack to make one wish that, good as the books are, they were better.

#### TEXT-BOOKS IN ARITHMETIC.

By TRUMAN HENRY SAFFORD,  
Williams College.

In judging of text-books it is possible to err from too great regard to method. The induction method, or, as its best form is called in mathematical subjects, the heuristic, can very well be employed with a text-book dogmatically arranged; if the teacher is sensible enough to employ the examples and ignore the text, and at the same time is able to replace the text and add to the examples. In such a case the waste is merely of type and paper.

The first set of books I shall mention are English. Rev. Barnard Smith's "Shilling Book of Arithmetic." It "has been prepared for the use of schools at the urgent request of numerous masters of schools, both at home and abroad." The arrangement is dogmatic; the subjects are introduced by Definitions; and, after some mental practice with small numbers, the operations are performed by Rules. The Tables precede the four fundamental operations one by one; Simple Division extends as far as  $2828882701578$  by  $38706$ . Fractions are taught by Rule not very plainly proved, e. g., the time honored rule for division by a fraction is illustrated in six lines. Here is a sample of the problems:

Work the following bill:— $17\frac{3}{4}$  yds. of calico at  $9\frac{1}{2}$ d. a yd.;  $35\frac{3}{4}$  yds. of flannel at  $1s. 9\frac{1}{2}$ d. a yd.;  $96\frac{3}{4}$  yds. of sheeting at  $2s. 0\frac{1}{2}$ d. a yd.;  $104\frac{3}{4}$  yds. of holland at  $1s. 0\frac{1}{2}$ d. a yd.; and  $12\frac{3}{4}$  yds. of ribbon at  $8\frac{1}{2}$ d. a yd.

On the whole the book is too difficult and too antiquated in its methods.

Rev. J. B. Lock's "Arithmetic for Beginners" is also dogmatic in form, but with rather better explanation of the processes, which are given less mechanically. His "Arithmetic for Schools" is rather more extended, but substantially according to the same method. Charles Pendlebury's "Arithmetic" is larger and the explanations are still more detailed, but the method is still dogmatic. P. Goyen's "Higher Arithmetic and Elementary Mensuration" begins with Factors, Powers and Multiples, and goes on to Fractions. It approaches much more nearly the modern idea of what a text-book should be. Although distinctly intended for the senior classes of schools and candidates preparing for public examinations, it is quite as reasonable in its examples as Barnard Smith's "Shilling Book," and is inductive in method, to its great advantage. The author is inspector of schools in New Zealand. The books so far mentioned are published by Macmillan, except Prendlebury's, which is issued by Deighton, Bell & Co. George Ricks, a London inspector of schools, is the author of two books published by Isbister.

"Elementary Arithmetic and How to Teach It," is a teacher's book for oral work. Quite possibly the pupils would need to have the examples and perhaps some of the other matter printed in a separate form. The rules are given in some of the later parts of the book; but with directions enabling the teacher to properly deduce them from easy examples done by the children. While the book is not strictly according to Grube's method, it approximates to it in its order of early problems; it contains (probably) as advanced methods as can be successfully practiced in the average English schools. Fractions are well developed, but without employing Grube's practice with individual fractions. It is likely that this subject is deferred to a later period in England than in

Germany or America. The same author's "Arithmetic for Pupil Teachers," seems admirably adapted to its purpose. In both books Mr. Ricks states the rule for division of fractions thus: "To divide by a fraction multiply by its reciprocal." He points out that if we invert the divisor the figures would be upside down.

Rev. A. D. Capel is the author of "Catch Questions in Arithmetic and Mensuration, and How to Solve Them" (London, Hughes), which seems to be an excellent book for higher classes, within about the same range as Goyen. It needs a good teacher, and is probably most in place for supplementary problems.

Dr. Thomas Muir, of the Glasgow High School, is the author of a "Text-book of Arithmetic for Use in Higher Class Schools" (London, Isbister), which is rather too difficult for common schools. The mathematical rigor at which the author aims is, in the reviewer's judgement, better attained by what the German's call "Arithmetik" in contrast to "Rechnen"; a philosophical combination of what we term Elementary Algebra and Higher Arithmetic. At the same time Dr. Muir's book is admirable as a reference book for teachers.

It will be seen from what has been said that in England the two older methods of arithmetical teaching—that by rule and that by reasoning—or the method of the 15th century and that of the early part of the 19th, flourish side by side. This corresponds in some degree to the two classes of English common schools; "Church" and "Board" schools. I think the evidence goes to show that the dogmatic method prevails in the country schools of the Established Church; while the "Board Schools" tend rather to the Pestalozzian side. Grube's method cannot but be known in England; but evidence that it is much practiced is not before me. The English currency takes up so great a space in the text-books that we can readily see how much it complicates instruction and thus renders it difficult to introduce the latest methods.

In France, advanced methods are rapidly gaining ground. Bovier-Lapierre explains Grube's method in Buisson's "Dictionnaire de Pédagogie"; and the difficulty in introducing it seems to arise from the relatively excellent system already in vogue. The weak point of the French text-books seems to lie in their scientific character; thus the three primary arithmetics by Combette, Leyssenne and Vintejoux (published by Picard-Bernheim, Armand Colin & Cie, and Hachette & Cie, respectively) begin at once with Numeration, and go on successively to Addition, Subtraction, Multiplication, Division. The result, of course, is that the numerical relations of the simpler numbers are not thoroughly apprehended, in the effort to comprehend the abstractions; and the scholar has to painfully acquire his elementary experience with numbers at a later period.

The French schools have the advantage of a decimal currency, weights, and measures; and vulgar fractions are taken up (in the main) after decimal. It seems quite probable that there should be (in any country but England) two courses of vulgar fractions; one practical, with small denomination, but quite extended and thorough, before decimals; the other more theoretical, after decimals. The weak point in much arithmetical teaching of the present day lies in the mechanical way in which decimals are taught. The French methods seem to the reviewer to err in this respect; partly by the over-use of rules, partly by laying little stress on the fractional side of decimals. If, for example, two decimal numbers are multiplied, the pupil ought to form some judgment what each one really signifies, and about what their product ought to be, to check the pointing off. If the latter is mechanically done the wildest errors may creep into the work.

Of German books, quite a number are before the writer. A few samples may suffice. The most meagre book is that of Stubba, which

had a high reputation in the time of the celebrated Prussian "Regulativ." Like most German "Rechenbücher" it contains little but examples methodically arranged. It is published in Bunzlau, and is manifestly intended for very poor children; the total price of the six numbers of the course is 78 pfennings, or about 19 cents. No. 1 reaches its 85th edition, and No. 6 its 15th by 1887. The number of editions falls off most decidedly after No. 4, which contains simple rule of three; so that the economy for the great majority of pupils is greater. No. 1 claims to have been published to the number of 435,000. The method seems to be Pestalozzian; the directions for the employment of the diminutive books are given in Bock's *Schulkunde*, a work representing the High Church Evangelicals in the Eastern part of Prussia. There is no doubt that the meagre allowance of matter taught and the conservative and loyal tone of the pedagogy of the *Regulatives* were admirably adapted to develop the great machine, the Prussian army. One can almost see in the blue covers of Stubba's *Hefte* and the painful economy of paper in the inside that they are intended for a poor outlying population in training for common soldiers. The *Regulatives* have gone by, but Bock's and Stubba's books are still republished.

In the reviewer's judgement the arithmetical training needed for practical life is best obtained from examples which are actually of a practical character; while that desirable for a basis to the higher mathematics requires an altogether different kind of examples. In other words a "higher arithmetic" should not be a book taken up before algebra; nor should it be a "commercial arithmetic," which again is desirable for the training of merchants and bankers. The order of mathematical studies in a high school or in a school preparatory for college should vary slightly as to the amount and kind of arithmetic required.

A German book which meets certain of these higher requirements is "Rechenbuch für Gymnasien, Real-Gymnasien, etc., von Christ. Horne et Dr. Alb. Kallius." Oldenburg (Stalling) 1887. Both authors have the title Professor, and are connected with an ober-Realschule and a Gymnasium respectively. The special points of interest are, first, the early treatment of powers, after the four fundamental operations; next, the employment of abbreviations for units, tens, hundreds and so on; again the early introduction of the simplest problems of mensuration; and the excellent grading of the questions for the purpose in hand (the preparation of the boys for the scientific arithmetic of the *gymnasia*). It is quite too difficult a book for common schools anywhere. There are several German arithmetics, or more properly, books of examples, with which the writer is more or less acquainted; but they are in general not very different.

Grube's "Heftfaden für das Rechnen in der Elementar-schule nach den Grundsätzen einer heuristischen Methode," 6te Aufl. Berlin (Euslie), 1881, is amply sufficient as a guide for the oral instruction which is very common in German schools. With any of the ordinary books of examples, even if their method is not precisely the same, the teacher can give successful instruction in the subject. In America it is hardly possible to employ the German oral method, save in exceptional schools. The German schoolmasters in this country, who begin with pupils more or less familiar with their language, although often "Sprachaviv," and whose habits of thought and teaching are firmly fixed in that method can of course do so; and it is quite possible that their methods influence other teachers in their neighborhood. But we have long been under the tyranny of methods of a different kind, and our body of teachers is too fluctuating to dispense in general with a rather elaborate text-book. American arithmetics are mainly such as a good teacher can employ, with judicious omissions, and a bad teacher

can rely upon as an assistant. If, however, the bad teacher is at the same time a mechanical one, the book can usually be made a torture to the pupil.

If we go back a few years, or select for our criticism some of the older books whose stereotype plates are not quite worn out, we shall find much space taken up by definitions and abstract ideas; much also occupied with rules insufficiently explained. As examples of what is here meant, the method of *long division*, and that for dividing one fraction by another, may be selected. Long division is a process of successive subtraction of multiples of the divisor; the proper multipliers must be ascertained by inspection, after long practice in multiplication, the multiplications rightly and intelligently effected, the product, omitting the correct number of zeros rightly placed, and the subtraction effected. Now a rule, or a brief explanation of the process, does no good if the child is already careful and correct in executing it, and is absolutely harmful if it is not. In other words, practice, thoroughly graded, and practice only, is the best help to teaching division; but an incompetent teacher may be assisted by the rule or the brief explanation, and there is some possibility that the same matters may help the pupil in preparing for examination or at a later stage of training in reviewing the elements.

Here is an arithmetic of a generation ago: Robinson's *Progressive Practical Arithmetick*, first copyrighted by its author (D. W. Fish) in 1858. Each portion of the book introduces quite early a series of definitions, sometimes developed by a little explanation, sometimes not. The definitions are usually quite abstract, and sometimes incorrect. The author shows an example or two, and gives a rule. An experienced teacher would naturally lay very little stress on the reading matter, and treat the book as one of examples; rather costly, it is true, and with too large numbers in many of the examples. I give a few specimens taken at random

Page 7. 1. Quantity is anything that can be increased, diminished, or measured. 2. Mathematics is the science of quantity.

Page 9. 5th. A bar or dash place over a letter increases its value *one thousand times*. Thus  $\bar{v}$  signifies five, and  $\bar{v}$  five thousand. (Roman notation is taught before Arabic).

Page 18. Rule for Notation! Rule for Numeration!

Page 55. Divide 102030405060 by 123456.

Page 163. Reduce 550355068 square inches to acres.

Page 173. Reduce 1116610'' to signs.

Page 174. If a silver dollar measure one inch in diameter, how many dollars laid side by side on the equator would reach round the earth? Answer: 1,577,511,936.

Page 198. Duodecimals.

Page 327. What is the cube root of 270671777032189896.

The writer can find nothing like these samples in any German "Rechenbuch" with which he is acquainted. Lack of time prevents the reviewer from extending this notice to other American arithmetics at present; and a historical survey of the favorite older books ought to precede a thorough notice of those now in vogue.

#### RECENT LITERATURE ON ARITHMETIC AND ARITHMETICAL TEACHING.

By LEVI L. CONANT, Worcester Polytechnic Institute.

*Industrial Primary Arithmetic*, by JAMES BALDWIN. Boston, 1891. pp. 264.

The best industrial arithmetic that has yet appeared in this country, or in the English language. While it is true to all that is implied in its name, its method is strictly heuristic, and the child is from the very outset familiarized with the relations of the numbers used, as well as

with the numbers themselves. The common, practical things of life, even to the postage stamp, are taught by actual use, but not to the exclusion of the abstract use of number. Simple geometrical concepts are freely employed, adding much to the value of the work. The course laid down is designed to cover the work of the first four years of school life.

*The Greek Method of Performing Arithmetical Operations*, by John Tetlow. School and College, January, 1892. pp. 14.

Makes no pretension to original research, but merely presents a very condensed review of the method of notation used by the Greeks, and of a few of their most common arithmetical processes. The information it contains is accurate, and the article itself very readable.

*Hindu Arithmetic*, Science, April 11, 1890.

A brief popular account of certain points in the arithmetical methods of the Hindus. Their nomenclature and their mode of instruction are such as to impart to very young pupils even an extraordinary facility in numerical computation. This is especially noticeable in the case of fractions. The Hindu multiplication table is much more extended than ours, and embraces to some extent multiplication by fractions. For example, separate tables are given for the multiplication of each number from 1 to 100 by  $1\frac{1}{2}$ ,  $1\frac{1}{3}$ ,  $1\frac{1}{4}$ ,  $2\frac{1}{2}$ ,  $3\frac{1}{2}$ , etc. In some respects the Hindu methods seem well worthy of imitation.

*Arithmetic in the Boston Schools*, by Gen. FRANCIS A. WALKER, Prest. of the Mass. Institute of Technology. The Academy, January, 1888. pp. 12.

A few years ago the School Board of Boston passed a series of orders respecting the teaching of arithmetic in the public schools of that city, which excited general interest throughout the country and provoked much discussion, pro and con. The reasons for the action are set forth in this article by Gen. Walker in his characteristic, vigorous style. The shortening of the time given to arithmetic, the prohibition of the assignment of work in that subject to be performed at home, the weeding out of subjects believed to be unnecessary and of the so-called "practical" problems, often of a nature so utterly unpractical, are handled by Gen. Walker in a spirit of candid hostility to the methods then in vogue. His arguments are sound and his reasoning unanswerable.

*Old and New Methods in Elementary Geometry*, by EUGENE L. RICHARDS. Educational Review, January, 1892.

Modern geometrical teaching aims at results; ancient teaching in this, as in other branches of study, aimed at mental training. The former is more complex, because it has at command a better arithmetic than the ancients possessed. But the ancient method had the advantage of making geometry a pure science. The treatment of ratio and proportion is now purely arithmetical—a great advantage. Important propositions are now often given as corollaries, though they often need a demonstration more elaborate even than that given to the proposition to which they are attached. The article as a whole is good, but the author in the discussion of one point falls into the absurdity of denying the right to employ a certain arithmetical process on the ground that the factors used have not been shown to be pure numbers, while in the preceding line he employs the converse process with quantities of which the same statement can be made.

*The Mathematical Preparation for College*, by TRUMAN HENRY SAFFORD, Professor of Astronomy in Williams College. The Academy, May, 1892. pp. 7.

College teaching of Mathematics is undergoing a great change. Young men with special training and with enthusiastic zeal for their work are entering the ranks of college instructors every year, but they find the number of students small who are willing to go on and pursue advanced work with them. Methods have been greatly changed during the past quarter of a century, but there exists a strong tendency on the part of teachers as well as text-books to cling to the methods of the early part of the present century. Colleges must gradually raise their standards because they cannot help themselves, and this must be met by high schools in the same way—by raising their own standards. The poorest work of the present day is to be found in grammar schools, which should give less time to that darling of their hearts, higher arithmetic, and in its place teach inductive geometry and the beginnings of algebra. The Austrian methods are very successful, and well worthy of study on the part of our educators. It is a mistake on the part of our colleges to confine themselves solely to written entrance examinations. The oral should be used to supplement it. The demand made on the grammar schools must be met by a concentration of mathematical thinking such as has long been employed in the science itself, and in the schools of nearly all other civilized countries.

*Methodisch geordnete Aufgaben zum kaufmännischen Rechnen*, von M. Löwe. Leipzig, 1892.

A series of three practice manuals in commercial arithmetic, containing 80 to 100 pages each. They are very full and complete, and contain tables adapted not only to German weights, measures, and money, but also the corresponding tables for all the other European countries. The three parts have reached the eighth, seventh and fifth editions respectively.

*Mathematische Kurzweil*. Von LOUIS MITTENZWEY. 2te Auflage, Leipzig und Wien, 1883. pp. 112.

An extreme oddity among arithmetics, and correctly described by its title, "Mathematical Recreation." It is devoted partly to arithmetic and partly to geometry. A single question from the first part is the following, p. 9: Six sparrows are sitting on a roof; how many will remain if one of them is shot off? Ans. None, for if one is shot off, the rest will fly away. Any teacher who has experienced the wearisome routine of the school-room can readily see what relief an occasional grain of spice like the above might furnish. The second part is devoted to curious and interesting arrangements of geometrical figures, to labyrinths, magic squares, Chinese puzzles, and a great variety of most interesting and, in its way, instructive work. A judicious use of this little book might well assist in the "shortening and enriching" of an ordinary school course in arithmetic.

*Die Mathematik die Fackelträgerin einer neuen Zeit*. Von E. DILLMANN. Stuttgart, 1889. pp. 214.

Mathematics is the language of languages, the best school for sharpening thought and expression, is applicable to all processes in nature; and Germany needs mathematical gymnasia. Mathematics is God's form of speech, and simplifies all things organic and inorganic. As knowledge becomes real, complete and great it approximates mathematical forms. It mediates between the worlds of mind and of matter. Such are the theses of this loosely-written, vague and incoherent book, which belies every anticipation awakened by its attractive title.