



XXVII. On the ellipticity of the earth as deduced from experiments with the pendulum

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The quantities may be arranged in the following manner :

$x = 4$	$y = 3$	$z = 3$
$x = 6$	$y = 4$	$z = 3$
$x = 8$	$y = 3$	$z = 4$
$x = 12$	$y = 5$	$z = 3$
$x = 20$	$y = 3$	$z = 5$
x infinite	$y = 3$	$z = 6$
	$y = 4$	$z = 4$
	$y = 6$	$z = 3$

x any number whatever, $y = 2$, $z =$ the same number as x .

XXVII. *On the Ellipticity of the Earth as deduced from Experiments with the Pendulum.* By J. IVORY, M.A. F.R.S.*

IN the *Conn. des Tems* 1830, lately published, there is a notice of experiments made with an invariable pendulum, by M. Duperrey, in the course of a voyage round the world. The places of observation are not numerous, and one only, namely, Toulon, is a new station. But, in the present state of this inquiry, no less instruction is to be reaped from the repetition of the experiments at the old stations, than from the extension of our knowledge by new observations. If we could compare the lengths of the pendulum determined by different observers on the same spot, the errors to which such experiments are liable would become known, and we should be able to estimate the degree of precision with which the figure of the earth can be investigated by this mode of experimenting. An attentive examination of all that has been accomplished in this research will clearly prove that the high expectations of accuracy which were at first entertained from it, have not been realized in practice. It is remarkable that the pendulum-experiments within 30° of the equator are very irregular; while in higher latitudes, if we except Drontheim, the results obtained are not chargeable with inconsistency in any great degree. Two causes only can be imagined in order to account for the irregularity near the equator: either the experiments must be erroneous, or gravity must be very unequally distributed in that quarter of the globe†.

* Communicated by the Author.

† Captain Sabine's experiments have lately appeared in this Journal, corrected from an error in estimating the divisions of the level of his repeating circle. The corrections are excessively trifling, so much so that it seemed hardly worth while to call the attention of the public to the circumstance, as the experiments must be considered as remaining *in statu quo*. That very great irregularity prevails in these experiments, and indeed in all those made near the equator, is a fact that all will allow; but the cause has not been ascertained in a satisfactory manner.

The

The pendulum which beats seconds at Paris, at the temperature of 15° Cent., being represented by unit, M. Duperrey has determined the relative lengths, at the same temperature and at the level of the sea, at the five following stations:

	Latitude.	Relative Length.
Toulon	43° 7' 20" N.	0·99950585
Isle of Ascension	7 55 48 S.	0·99729881
Isle of France. .	20 9 23	0·99789022
Port Jackson . .	33 51 40	0·99871430
Falkland Islands	51 31 44	1·00025995

Adopting 39·12929 for the length of the pendulum at Paris in English inches, at the temperature 62° Fahr., the following table shows the lengths of the pendulums at the stations of M. Duperrey, in English measure, at the standard temperature, and likewise the errors according to my formula published in this Journal for October 1826.

	Observed Pendulum.	Computed Pendulum.	Excess of Calculation.
Toulon	39·10996	39·11071	+·00075
Port Jackson . .	39·07899	39·07637	—·00261
Isle of France . .	39·04674	39·03643	—·01031
Ascension	39·02360	39·01566	—·00796
Falkland Islands	39·13945	39·13940	—·00005

The pendulums in this table at Ascension and the Isle of France coincide very nearly with the lengths previously determined by Captain Sabine and M. de Freycinet. From some cause or other, they are both anomalous, and they are treated as such by M. de Freycinet and M. Duperrey, being left out in computing the ellipticity which they adopt. But the other three pendulums fall within the limits laid down in this Journal for October 1826, and increase the number of instances that come under my formula to 31, out of 40 the total number of experiments that have been made. My formula was originally constructed from 26 experiments; and all the pendulums since determined by M. de Freycinet, Mr. Foster, and M. Duperrey, agree with it within the prescribed limits, except in the three instances of the islands of Mow, Guam, and the Isle of France, which are greatly irregular and irreconcilable with the other experiments. It is therefore very probable that the mean ellipticity of the earth will ultimately be found to approach very near $\frac{1}{300}$, as deduced from my formula.

At

At Port Jackson the pendulum determined by M. Duperrey is less than the length found by M. de Freycinet, and approaches nearer to the pendulum at Paramatta, to which it ought to be very nearly equal: and this proves the justness of the remarks I made relative to this point at pp. 351, 352, of this Journal for November 1826.

Comparing the results obtained by M. Duperrey and M. de Freycinet at the same stations and on the same spot, the pendulum of the former observer at the Falkland Islands is longer than that of the latter by $\cdot00233$; and, at Port Jackson and the Isle of France, the pendulums of the former are respectively less than those of the latter by $\cdot00145$ and $\cdot00095$. We may therefore conclude, that such experiments are liable to an error amounting from $\cdot002$ to $\cdot003$ in the length of the pendulum, or from two to three vibrations in a mean solar day. We are sure that the error may amount to the quantity mentioned; but it may be much greater. At Ascension the difference between the pendulums of Captain Sabine and M. Duperrey is very small.

In deducing the ellipticity of the earth, M. Duperrey combines his own experiments with M. de Freycinet's, exclusively of those made by other observers. The results he obtains vary between the limits $\frac{1}{286}$ and $\frac{1}{290}$. Upon the whole he concludes that the two hemispheres of the earth are similar, and he fixes definitively upon $\frac{1}{288}$, or $\frac{1}{290}$, as the ellipticity common to both. But on examining the calculations of M. Duperrey, it will be found that the ellipticity he adopts, depends entirely on the equatorial pendulum determined by M. de Freycinet at Rawak. If we suppose that the pendulum at Rawak errs in excess, which is very probable, and diminish its length by correcting the possible error, the ellipticities will come out of less quantity in all the combinations in which they were before equal to $\frac{1}{288}$ or $\frac{1}{290}$. Wherefore, although M. Duperrey finds five combinations of his experiments in which the ellipticity is $\frac{1}{288}$ or $\frac{1}{290}$; yet, whether its real value be equal to either of those numbers, or to a less fraction, will depend entirely upon the error that may exist in the pendulum at Rawak. Captain Sabine has deduced the same ellipticity, viz. $\frac{1}{288}$, by many combinations of his own experiments and those made in England and France; and I have already remarked that this uniformity of result is occasioned by the pendulums at the Islands of Ascension and St. Thomas, which enter into all the computations. If these two stations be left out in any, or in all, the combinations, the ellipticity will no longer be the same as before, but a less fraction. The arriving at the same result by a multiplicity of arithmetical operations

rations is in reality a play of calculation caused by one common datum, or several common data; and therefore it does not furnish independent proofs in favour of the number in question. There must, in all probability, be some error in the pendulum at Rawak; and we know that the two pendulums of Captain Sabine are anomalous, involving an irregularity much greater than usually occurs in such experiments; from which considerations we must conclude that the evidence for the ellipticity $\frac{1}{288}$ rests but on slender foundations.

It is reasonable to think that every new set of experiments with the pendulum should be joined to the stock we already possess, in order to add to the number of unexceptionable experiments, and to correct such as are of doubtful authority. By proceeding in this manner our *data* for obtaining an exact knowledge of the figure of the earth and of the distribution of gravity on its surface, would continually increase and improve in accuracy; and, by applying proper methods of calculation, we might hope to bring this great question to a satisfactory solution. But if every new set is to be taken by itself, we may, by making arbitrary combinations of the experiments, be led into great error, and to entertain speculations that have little foundation in nature.

The number of experiments made with the pendulum by good observers at present amounts to 40, contained in the subsequent table; but of these, six, placed last in the table, are decidedly anomalous, and must be separated from the rest. In setting aside these six experiments I do not now proceed on my own opinion; I follow the example of M. de Freycinet and M. Duperrey. There remains 34 experiments which may be compared together; and we have now to inquire, What mode of calculation must be adopted in order to obtain a result entitled to the greatest confidence the case will admit of. On reflection it will appear that we cannot expect to attain what we wish for by the method of the least squares as usually applied, nor by the modification of that method I used in this Journal for October 1826. These more direct methods are useful in bringing out a first approximation; but it seems necessary to correct the approximate elements thus obtained by the methods used with success in so many problems of astronomy. It is on these principles that the following investigation is conducted.

If we put $39 + \delta$ for the pendulum in English inches at any station; λ for the latitude; and $39 + \Delta$ for the equatorial pendulum; we shall have this equation,

$$\Delta + f \sin^2 \lambda = \delta + e, \quad (1)$$

e being

e being the error of observation, and f a quantity which is the same at all points of the earth's surface. The equation (1) being general, if we add all the like equations at the 34 stations we propose to include in our investigation, we shall get

$$34\Delta + f\Sigma(\sin^2\lambda) = \Sigma(\delta) + \Sigma(e).$$

Now I have found,

$$\begin{aligned}\Sigma(\sin^2\lambda) &= 15\cdot9316, \\ \Sigma(\delta) &= 3\cdot72872;\end{aligned}$$

and hence by dividing all the terms by 34, we get,

$$\Delta + \cdot4686f = \cdot10966, \quad (2)$$

neglecting the term containing e , which must be very small, both because the errors must in some degree compensate one another, and on account of the great divisor 34. The equation (2) must be very exact, provided we take care not to enhance its error by introducing small divisors.

Again, when the ellipticity is $\frac{1}{288}$, we have $f = \cdot202$ nearly, as appears from the calculations of Captain Sabine; and, when the ellipticity is $\frac{1}{300}$, f is nearly $\cdot208$, according to my formula in this Journal for October 1826. The mean is $\cdot205$, which cannot err from the true value of f more than $\pm \cdot003$. Now it is evident that this small error will not affect the product $f\sin^2\lambda$ near the equator, and so long as $\sin^2\lambda$ is less than $0\cdot2$. I have therefore computed the values of Δ by means of the formula, $\Delta = \delta - f\sin^2\lambda$, on the supposition that $f = \cdot205$, for all the stations in the table where $\sin^2\lambda$ is less than $0\cdot2$, as follows:

	Δ
Maranham.....	$\cdot01173$
Bahia.....	$\cdot01398$
Rio Janeiro....	$\cdot01264$
Trinidad.....	$\cdot01188$
Madras.....	$\cdot01289$
San Blas.....	$\cdot01066$
	$6)\cdot07378(\cdot01230, \text{ mean of 6.}$
Rawak.....	$\cdot01479$
Sierra Leone...	$\cdot01550$
Jamaica.....	$\cdot01560$
	$9)\cdot11967(\cdot01330, \text{ mean of 9.}$

The inequality of the several values of Δ is very remarkable; in particular the three stations placed last, greatly exceed the mean of the first six. But whatever be the cause of the irregularity, it cannot arise from the approximate value assigned

to f . The mean of the nine stations must be an approximation sufficient for our present purpose. Let $\cdot 01330$ be substituted for Δ in the equation (2), then $f = \cdot 2057$, which will serve for a first value of f . Now put

$$\begin{aligned} a &= \cdot 01330, & \Delta &= a + s, \\ b &= \cdot 2057 & f &= b + \tau, \end{aligned}$$

s and τ being the corrections of the approximate quantities: then since Δ and f must satisfy the equation (2), and a and b likewise satisfy the same equation, we get,

$$s + \cdot 4686 \tau = 0.$$

The formula (2) is one of the equations of the method of the least squares applied to all the 34 experiments; and in order to find the other equation of the same method, let all the terms of equation (1) be multiplied by the coefficient of f : then,

$$\Delta \sin^2 \lambda + f \sin^4 \lambda = \delta \sin^2 \lambda + e \sin^2 \lambda;$$

by substituting the values of Δ and f ,

$$s \sin^2 \lambda + \tau \sin^4 \lambda = \sin^2 \lambda (\delta - a - b \sin^2 \lambda) + e \sin^2 \lambda;$$

and the sum of the like equations for all the 34 stations will be the equation sought, viz.

$$s \Sigma (\sin^2 \lambda) + \tau \Sigma (\sin^4 \lambda) = \Sigma . (\sin^2 \lambda (\delta - a - b \sin^2 \lambda)),$$

the term containing e being neglected. Now,

$$\Sigma (\sin^2 \lambda) = 15 \cdot 9316,$$

$$\Sigma (\sin^4 \lambda) = 10 \cdot 4381;$$

wherefore,

$$15 \cdot 9316 s + 10 \cdot 4381 \tau = \Sigma . (\sin^2 \lambda (\delta - a - b \sin^2 \lambda));$$

and, by a former equation,

$$15 \cdot 9316 s + 7 \cdot 4654 \tau = 0;$$

consequently,

$$2 \cdot 973 \tau = \Sigma . (\sin^2 \lambda (\delta - a - b \sin^2 \lambda)).$$

The sum on the right-hand side consists of thirty-four terms, which must be separately calculated by substituting for δ and $\sin^2 \lambda$ their values at the several stations. The computation being made, and all the results combined according to their respective signs, I have found

$$2 \cdot 973 \tau = - \cdot 00061.$$

Hence, $\tau = - \cdot 00020$; $s = - \cdot 4686 \tau = + \cdot 00009$;

$$f = \cdot 2055, \quad \Delta = \cdot 01339.$$

If

If therefore $l = 39 + \delta$, be the length of the pendulum at the latitude λ , we have this general formula, viz.

$$l = 39.01339 + .2055 \sin^2 \lambda,$$

$$\text{ellipticity} = .00865 - \frac{.2055}{39.01339} = .00338 = \frac{1}{295.5}.$$

Stations.	Latitude.	Observed	Computed	Excess of	Observers.
		Pendulum.	Pendulum.		
		inches.	inches.		
Falkland Islands	51° 31' 44" S.	39.13945	39.13935	—00010	Duperrey.
Ca. Good Hope	33 55 15	39.07817	39.07739	—00078	Freycinet.
Port Jackson	33 51 40	39.07899	39.07719	—00180	Duperrey.
Paramatta	33 48 43	39.07622	39.07702	+00080	Sir T. Brisbane.
Rio Janeiro	22 55 22	39.04374	39.04457	+00083	Hall and Foster.
Bahia	12 59 21	39.02433	39.02377	—00056	Sabine.
Marauham	2 31 43	39.01213	39.01379	+00166	Sabine.
Rawak	0 1 34	39.01479	39.01339	—00140	Freycinet.
Sierra Leone	8 29 58 N.	39.01997	39.01787	—00210	Sabine.
Trinidad	10 38 56	39.01888	39.02041	+00153	Sabine.
Madras	13 4 9	39.02338	39.02390	+00052	Goldingham.
Jamaica	17 56 7	39.03503	39.03288	—00215	Sabine.
San Blas	21 32 24	39.03829	39.04109	+00280	Hall and Foster.
Formentera	38 39 56	39.09424	39.09361	—00063	Biot.
New York	40 42 43	39.10120	39.10081	—00039	Sabine.
Toulon	43 7 20	39.10996	39.10941	—00055	Duperrey.
Figeac	44 36 45	39.11322	39.11476	+00154	Biot.
Bourdeaux	44 50 26	39.11303	39.11557	+00254	Biot.
Clermont	45 46 48	39.11809	39.11893	+00084	Biot.
Paris	48 50 14	39.12929	39.12929	+00056	Biot.
Dunnose	50 37 24	39.13614	39.13618	+00004	Kater.
Dunkirk	51 2 10	39.13773	39.13762	—00011	Biot.
London	51 31 8	39.13929	39.13932	+00003	Kater.
Arbury Hill	52 12 55	39.14250	39.14174	—00076	Kater.
Clifton	53 27 43	39.14600	39.14605	+00005	Kater.
Leith	55 48 41	39.15554	39.15454	—00100	Kater.
Portsoy	57 40 59	39.16159	39.16016	—00143	Kater.
Stockholm	59 20 34	39.16541	39.16546	+00005	Svanberg.
Unst	60 45 28	39.17146	39.16985	—00161	Kater.
Drontheim	63 25 54	39.17456	39.17778	+00322	Sabine.
Hammerfest	70 40 5	39.19475	39.19636	+00161	Sabine.
Port Bowen	73 13 39	39.20347	39.20177	—00170	Foster.
Greenland	74 39 19	39.20328	39.20428	+00100	Sabine.
Spitzbergen	79 49 58	39.21464	39.21248	—00216	Sabine.
Ascension	7 55 48 S.	39.02385	39.01730	—00655	Sabine & Duperrey.
Isle of France	20 9 40	39.04721	39.03780	—00941	Freycinet & Dup ^y .
St. Thomas	0 24 41 N.	39.02074	39.01339	—00735	Sabine.
Galapagos	0 32 19	39.01717	39.01341	—00376	Hall.
Guam	13 27 51	39.03023	39.02453	—00570	Freycinet.
Mowi	20 52 7	39.04737	39.03946	—00791	Freycinet.

This table shows the errors of the formula. The greatest error is at Drontheim, and it is nearly as much with the ellipticity $\frac{1}{293.3}$. The next greatest error is at San Blas; but this would be considerably diminished by adopting the length found by Captain Foster, viz. 39.03881, instead of the mean of the lengths found by that gentleman and Captain Hall, which stands in the table. The excess at Galapagos is little greater than the defect at Drontheim; and as we know that the former pendulum was determined in unfavourable circumstances, the experiment seems to be sufficiently well represented by the formula. We may therefore affirm that, if we adopt Captain Foster's pendulum at San Blas, the formula represents 35 experiments out of 40, within the limits of the probable errors: for the difference between the observed and the calculated lengths is less than what would arise from an error of 2 vibrations in a mean solar day, except at Sierra Leone, Jamaica, San Blas, Bourdeaux, and Spitzbergen, where the error is $2\frac{1}{2}$ vibrations; and at Drontheim and Galapagos, where it amounts to $3\frac{1}{2}$ and 4 vibrations. But at the five anomalous stations, the errors are between 6 and 10 vibrations in a day.

Without laying any stress on the mode of investigation we have here followed, it cannot, I think, be denied that the ellipticity $\frac{1}{293.3}$ represents the observations much better than $\frac{1}{293.8}$; it ought therefore to be adopted until another shall be found, if it be possible, that will represent the same observations still better, or until future experimental researches shall have corrected and enlarged our present knowledge on this subject. What other reason can be alleged for preferring one ellipticity to another?

But if the experiments can be represented within the limits of the probable errors, what becomes of the splendid speculation about local attraction, which connects the apparent inequality of gravity with the variation of the soil. It is evident that we can have no measure of the excess or defect of gravity caused by the attraction of the adjacent matter, if the errors of observation be sufficient to account for the irregularity in the length of the pendulum. This theory too is quite uncertain, so long as there remains any doubt about the exact quantity of the ellipticity. At Maranham and Trinidad, Captain Sabine makes the observed pendulums gain upon the calculated ones upwards of 4 vibrations in a mean solar day; in my table the acceleration is only $1\frac{1}{2}$, which may be attributed to the inaccuracy of experiment. According to the same gentleman,

tleman, the observed pendulum at Spitzbergen falls short of the calculated one $3\frac{1}{2}$ vibrations in a day ;—in my table the defect is only $2\frac{1}{2}$. The density of the matter near the surface of the earth, varying at the different stations of the pendulum, must in some degree influence the time of vibration : but the speculation is premature ; and there is good reason to think, at least in far the greater number of cases, that the effect in question will never be separated from the unavoidable errors of observation.

What has just been said will not, however, apply to the five anomalous experiments, all of which show an excess of gravity far surpassing the ordinary measure of experimental error. Besides, at two of the stations the pendulums have been verified by two independent experiments varying little in the results. It is remarkable that in all the five cases, and at Galapagos, there is an excess of gravity ; and, according to my table, there is a like excess, though much less in quantity, at Rawak, Sierra Leone, and Jamaica. On the other hand, there appears to be a defect of gravity at Maranham, Trinidad, and San Blas. Thus there are nine instances of excess, and only three of defect ; and we may suspect that there is some cause of error, different from local attraction, tending to make these tropical pendulums longer than they should be. At the Isle of France, the excess of gravity is no less than 10 vibrations a day ; yet this is a small island, surrounded on all sides by an extensive sea ; and we might infer, *à priori*, that a pendulum placed upon it very little above the level of the sea, instead of being accelerated, would be retarded by the great defect of density in the waters of the ocean. But the purpose of this paper is to deduce from the experiments the consequences that necessarily flow from them by a strict investigation ; and I shall refrain from entering into the region of conjecture and opinion.

Feb. 11, 1828.

J. IVORY.

XXVIII. *The Climate of Penzance, Cornwall.—Meteorological Results of the Temperature, Wind and Weather deduced from diurnal Observations made at Penzance for 21 Years : (the Thermometrical Observations made at 8 a.m. and 2 p.m.) to which are added the Maxima, Minima, and Media of the Register Thermometer for 7 Years, with the Inches of Rain fallen during that Period.* By EDWARD COLLINS GIDDY, Curator of the Cabinet of the Royal Geological Society of Cornwall.

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