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ORIGINAL ARTICLES

BIOMETRICS*

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WHEN Dr. Hellman requested me to address your Society on the subject of biometrics, I was at first disposed to decline the invitation. Biometrics is the application of mathematics to biological phenomena, and while I have done some statistical work, I am neither a mathematician nor a biologist. I decided to accede to Dr. Hellman's request because there exist misconceptions of the gravest sort concerning the utility of biometrics even among some of the most distinguished biologists. The aversion from exact statistical treatment of their data has communicated itself to not a few of the younger biologists, who dogmatically reëcho their masters' prejudices. In attempting to combat these dogmas, first by a brief examination of the logic of the case, secondly by a few concrete illustrations of how mathematical treatment has actually illuminated biological questions, I thought I should be rendering something of a public service.

THE FUNCTION CONCEPT

Up to the present day the determination of causes—*rerum cognoscere causas*—has been commonly assumed as the ultimate aim of scientific investigation. But in recent times advanced thinkers among the philosophers of science have found the concept of causality lacking in definiteness and are supplanting it with the mathematical concept of function. From their point of view, the entire universe appears as an assemblage of elements more or less closely dependent on one another, and the object of science is to ascertain the *functional* relationships of these elements.†

*Lecture delivered at the Annual Meeting of the Eastern Association of Graduates of the Angle School of Orthodontia, May, 1912, New York, N. Y.

†Foremost among those holding this point of view stands Professor Ernst Mach (Vienna). Cf. his *Die Analyse der Empfindungen*, ed. 5, Jena, 1906, pp. 73-77.

What, precisely, are you to understand by the term "function?" *The quantity "y" is a function of the quantity "x," if "x" and "y" are so related that to every value which "x" may assume there correspond one or more values of "y."* Thus, the cost of a package of tea is a function of its weight because the greater the weight the greater (other things being equal) is the cost. The time required to perform a journey is a function of the distance, because (other things being equal) it varies with the distance. The algebraical expression y^2 is a function of y , for whatever values we assign to y we obtain a corresponding series of values for y^2 .

The so-called natural laws of physics are expressions of such functional relationships. Let us take one of the simplest of physical laws for an illustration. Boyle discovered that, when other conditions remained constant, the pressure and the volume of a gas are inversely proportional: the less the pressure, the greater the volume, and *vice versa*. In other words the product of the pressure and the volume is a constant quantity. Pressure is a function of volume, and volume of pressure.

Now it is of the utmost importance for you to understand how the formulation of such a rule as Boyle's Law originated. I have not at my disposal, at this moment, the record of Boyle's observations. In their place I wish to substitute a table of observations on soap bubbles leading to a similar law. The pressure of the air confined within the bubbles and the diameter of the bubbles are as follows:*

d Diameter of bubble	p Pressure of Confined Air	dp
7.55	3.00	22.65
10.37	2.17	22.50
10.55	2.13	22.47
23.35	0.98	22.88
27.58	0.83	22.89
46.60	0.48	22.37

You observe that as d increases, p decreases. The product dp is approximately the same, no matter what may be the value of d or p . For all practical purposes the slight variations are negligible. We may, accordingly, idealize our table, simplify it by abstracting from the variations, and express the somewhat cumbersome table in the form of a shorthand résumé, to use Professor Pearson's phrase, and write the equation: $dp = d_1 d_2 = \text{constant}$. The diameter is a function of the pressure, and the pressure of the diameter.

You will now be ready to grasp the fundamental proposition of biometrics, which is simply this: that the student of biology is able to apply mathematics to his problems in essentially the same way in which the physicist applies mathematics to physical phenomena,—that by abstracting from observed reality unessential conditions he is able to describe the resulting simplified group of phenomena by a mathematical expression of functional relations that represents with sufficient accuracy the phenomena observed. True, the conditions may be less readily simplified in biology than in physics: my point is that the *principle* involved in the application of mathematics is in both cases identical. Logically, there is not the slightest reason why the predictions of a physicist with regard

*Young and Linebarger: The Elements of the Differential and Integral Calculus, p. 5.

to the movement of falling bodies or those of an astronomer as to the rotation of the heavenly bodies should be less liable to error than the biological predictions of the actuary of a life insurance company. As a matter of fact, we know that the physicist's and astronomer's predictions are verified in the majority of instances. And probably you are aware that the life insurance companies are not exclusively charitable institutions, but that the calculations of the actuary, founded on a statistical study of the *biological* phenomenon of mortality results, as M. Poincaré has humorously pointed out, in the payment of dividends to shareholders. The skepticism of the biologist with regard to the application of mathematics can not consistently be limited to biometrics but must be extended to mathematical physics and celestial mechanics; and in each of these cases the pragmatic test seems to rule his skepticism out of court.

REGULARITIES IN BIOLOGICAL PHENOMENA

Mathematical treatment of biological, as of physical phenomena, however, presupposes the possibility of noting a certain *regularity* in the behavior of the facts studied. If every falling body differed very considerably in velocity from every other falling body, no law of gravitation could ever be formulated. In studying such a biological phenomenon as the stature of a definite group of human beings, it would be absolutely impossible to represent the facts by a shorthand formula, an expression of functional relationships, if people of the same kinship group and living under the same conditions belonged to quite different stature groups, if Liliputian parents, to put the case drastically, gave birth to a gigantic brood like the devourers of Mr. H. G. Wells' "Food of the Gods," and if their issue in turn fell within all conceivable orders of stature. As a matter of fact, it has been found that the distribution of statures in any one definite locality is markedly regular. Thus, Johannsen quotes from Quetelet the following figures giving the height of 1,000 American soldiers.*

<i>Stature of American Soldiers</i>		Number of individuals
5 feet	0	2
5 feet	1	2
	2	20
	3	48
	4	75
	5	117
	6	134
	7	157
	8	140
	9	121
	10	80
	11	57
6 feet	0	26
6 feet	1	13
	2	5
	3	3
		<hr/> 1000

Such figures could be multiplied almost indefinitely. Wherever a relatively

**Elemente der exakten Erblchkeitslehre*, Jena, 1909, p. 8.

pure population has been measured for height, the distribution of statures has been found to resemble that given for American soldiers. There is a congestion of frequencies about a central point corresponding to the average stature. As we take statures lower or higher than this central point, the associated frequency gradually diminishes until finally, say at the height of 4 feet or 7 feet, we find not a single member of our series. There is thus a characteristic correlation between frequency of occurrence and height within a definite group of observed individuals.

THE THEORY OF PROBABILITY

The Functional Relationship in Biology.—The fundamental question for us is this: Can the correlation between the frequency and the value of a certain measurement be definitely formulated in terms of a functional relationship? It has been shown that such is indeed the case: just as in the physicist's formula we may substitute any value we please for t (the time) in the formula for falling bodies, viz., $l = \frac{1}{2}gt^2$, and calculate l (the distance traversed), so it is possible to assign an arbitrary value to the stature and calculate therefrom the correlated frequency of occurrence. Without going into details or attempting to give the mathematical processes, a few words on the theory of probabilities will be in place here to give you some notion of the methods applied.

By the probability of an event we understand the ratio between the chances favorable to the occurrence of the event to the total number of chances for and against it. For example, if I throw a coin in the air, either the head or the tail may turn up. There are two possibilities, and one chance favorable in each case; hence the probability of a head turning up is $\frac{1}{2}$, and this is also the probability for a tail. It has been shown experimentally that when a coin has been thrown up say 16,000 times, there will be actually about 8,000 heads and 8,000 tails. If we throw up a coin, it is obvious that it *must* turn up either head or tail. The event of *either* turning up is, strictly speaking, not probable, but certain, and this certainty we denote by 1. On the other hand, if an event is absolutely impossible we denote its probability by 0.

This terminology can be readily applied to the frequencies of our table of statures. The total number of individuals measured being 1,000,* we express the probability of any measurement by one-thousandths of its frequency. Thus, there are 157 men of the stature 5 feet, 7 inches; the probability of that stature is accordingly $\frac{157}{1000}$.

It can be shown that the entire distribution of a stature series, as well as of many other variable measurements, can be briefly formulated by reference to *two* easily calculated values: the Average and the Standard Deviation (σ). The latter is simply the square root of the sum of all the squares of the deviations from the average, and serves as the measure of group variability. We can use the value of σ as a measure of variability in this sense that, provided our series conforms to the type of distribution usually found in stature series, we can calculate the probability of any deviation whatsoever from the average. It has been shown that in such a case the probability is about $\frac{68}{100}$ that a deviation

*As a matter of fact, the table given represents a reduction of the actual frequency of measurements to a total of 1,000.

will not exceed the standard deviation. In other words, if the average height of a group of 1,000 men is 5 feet, 7 inches, and the standard deviation 3 inches, about 680 men may be expected to have a height falling between 5 feet, 4 inches, and 5 feet, 10 inches. Now, that a deviation will not exceed the value of *twice* the standard deviation is much more probable, the figure being $95\frac{4}{1000}$; that is, to use our illustration, of 1,000 men only 46 will be either shorter than 5 feet 1, or taller than 6 feet 1 in. Within the limits of the Average plus 3σ and the Average minus 3σ there will be a still greater percentage,—of 10,000 men we should have 9973 not shorter than 4 feet, 10 inches or taller than 6 feet, 4 inches. Finally, when we take a deviation of 4σ , the probability of such a deviation becomes so slight that it can be practically disregarded. In the case cited, out of 100,000 men we should find only 6 taller than 6 feet 7 in. and shorter than 4 feet 7 in. Comparison with a table of probabilities (such as mathematicians have prepared) enables us at once to determine the probable frequency of an arbitrarily selected measurement.

At this point I should not like to omit mentioning that the type of distribution exemplified by our stature series is far from being the only one actually found to occur in biological phenomena. Where it does not obtain, however, the biometrician's duty is exactly the same in principle as when it does hold. He must try to find some other way to summarize the facts, and here again his task does not differ in principle from that of the mathematical physicist in a corresponding predicament. For the sake of simplicity, solely, I shall confine my examples to cases of the type indicated.

PRACTICAL APPLICATIONS

Let us now see how we can practically apply our theory of probabilities. The problem that frequently confronts us is this: Do observed differences between two series compared represent *real* differences or are they dependent on accident? You must remember that while any group we may measure is, at least theoretically, composed of an infinite number of members, we never actually measure more than a limited number of individuals. You can easily see that if I attempted to determine the average height of this audience by picking out and measuring a single individual, I might pick out one that was 6 feet 3 or 5 feet just as likely as one that approached the real mean height. If I selected two individuals, it would be much less likely that both should be extreme variants; the average of the two would probably incline somewhat more towards the general height. For a larger series of measurements, the *error* of the average as compared with that of an ideal, infinitely large series still exists, but can be measured by the ratio of the standard deviation to the square root of the number of

cases, $\frac{\sigma}{\sqrt{n}}$. If 10,000 men from the towns of a country have been measured, with an average of 170 cm. and a standard deviation of 7 cm., the error of the average is $\frac{\sigma}{\sqrt{n}} = \frac{7}{\sqrt{10000}} = \frac{7}{100} = .07$ cm.

Suppose further that in the rural districts of the same country we also measure 10,000 individuals and get the average of 169 cm. with a standard deviation

of 10 cm. Obviously, the error of our average here is $\frac{10}{100} = .10$ cm. We now have sufficient data for a comparison of the height of the urban and the rural population, for we have a rule for determining the probability of a difference of 1 cm. Without such a rule we could say nothing as to a stature difference between country and town folk, for as both series are limited, we could not know from the averages alone whether they fell within or without each other's normal accidental range. The formula in question, that is, the formula for the

$$\text{error of difference is: } \sqrt{\epsilon_1^2 + \epsilon_2^2} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$$\text{In our example } \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} = \sqrt{\frac{49}{10000} + \frac{100}{10000}} = \sqrt{\frac{149}{10000}} = 0.12 \text{ cm.}$$

Hence the error of the difference is .12. As the probabilities of errors follow a similar law to that of the probabilities of deviations from the average, it would be exceedingly improbable that our averages should be too large or too small by more than 4 times 0.12 cm. But our *observed* difference is more than 8 times as large as the error of the difference. We should then be quite safe in declaring that there is a real, not an accidental, difference between the stature of city people and of rural people.*

The method I have just outlined has innumerable applications in all the sciences dealing with variable phenomena, and as you are primarily interested in such applications rather than in the underlying mathematical considerations I will give several other illustrations.

Some years ago Dr. Channing investigated the hard palates of normal and feeble-minded individuals with the end of determining whether they revealed any real differences. Later he collaborated on this work with Dr. Wissler as biometrical adviser. From their joint publication† I take the following illustrations:

COMPARISON OF THE DISTANCE BETWEEN FIRST MOLARS

	n	Av.	σ
Normal	126	34.75	3.35
Feeble-minded	125	33.77	3.61
			<hr/>
The error of the first average is			$\frac{3.35}{\sqrt{126}} = .30$
			<hr/>
The error of the second average is			$\frac{3.61}{\sqrt{125}} = .32$

The error of the difference will be $\sqrt{.30^2 + .32^2} = .44$. The observed difference of the averages is only .98cm.,—only about twice as great as the error. In order that we could safely regard the differences as *real*, the difference would have to be four times as great as the error. Hence, we can not consider the hard palate of normal individuals and of feeble-minded individuals to differ in this particular feature.

*Westergaard; loc. cit., p. 187.

†The Hard Palate in Normal and Feeble-minded Individuals, *Anthropological Papers of the American Museum of Natural History*, i, pp. 283-369.

COMPARISON OF THE WIDTH AT THE CANINES

	n	A	σ
Normal	112	23	2.24
Feeble-Minded	124	22.36	2.61

$$\epsilon_1 = \frac{2.24}{\sqrt{112}} = .21$$

$$\epsilon_2 = \frac{2.61}{\sqrt{124}} = .23$$

$$\sqrt{.21^2 + .23^2} = .31$$

The observed difference is .64,—again only little more than twice the error of the difference. Accordingly, here again the theory of a real difference between our two series stands unproved.

Let me add an illustration from a different field. Some years ago thousands of Toronto school children were measured as to their height and other anthropometric traits, and a considerable number of facts were simultaneously noted as possibly serving to explain conditions that had to do with differences in the traits measured. Among these facts was the occupation of the children's fathers, which of course gave an approximate clue to their social position. An obvious question was whether social position and the conditions that go with it have anything to do with, for example, the children's *stature*. In order to determine this point we must separate our material into groups. Let us, then, tentatively separate the children of business and professional men from those of workingmen. Let us take the averages and standard deviations of our two groups. Suppose the ten-year-old boys of the well-to-do classes average 130 cm., while those of the poorer classes average only 128 cm. So far you do not yet know whether this difference is a really significant difference. You must accordingly resort to the rule of errors. If the number of cases is 100, and the standard deviation 6 cm. for both groups, then you have the errors

$$\epsilon_1 = \epsilon_2 = \frac{6}{\sqrt{100}} = .6$$

$$\sqrt{\epsilon_1^2 + \epsilon_2^2} = \sqrt{.36 + .36} = \sqrt{.72} = 0.848$$

The error of the difference then is 0.848.

Now, the observed difference is 2 cm. Dividing this by the error of the difference you have a ratio of 2 : 0.848. This is less than a ratio of 3 : 1, while you could not safely assume a real difference unless the ratio were at least 4 : 1. Hence it is not certain from the data at hand, whether social differences affect the stature of ten-year-old Toronto boys. But now assume that instead of having obtained the averages and standard deviations from groups of 100, you had had groups of 10,000 to deal with. Then

$$\epsilon_1 = \epsilon_2 = \frac{6}{\sqrt{10000}} = \frac{6}{100} = .06$$

$$\sqrt{\epsilon_1^2 + \epsilon_2^2} = \sqrt{.0036 + .0036} = \sqrt{.0072} = 0.085$$

The ratio of your observed difference to the error of the difference now becomes as 2 : 0.085. That is to say, the observed difference is more than twenty times the error, and accordingly you would be quite safe in asserting a real difference between the two groups, a real difference due to social position and its correlates.

This example is especially well fitted to illustrate the processes involved in biometric investigation. We started with the assumption that social position might affect the stature of children. Why did we make this assumption? It could not be suggested by biometrics or statistics any more than arithmetic could suggest to Galileo that the velocity of a falling body varied with the time. Galileo made *his* assumption because observation taught him that a body falls more and more rapidly as it descends. We made *our* assumption because observation seems to indicate that, other things being equal, the children of fairly prosperous parents are better nourished and taller than those of needy parents. In both cases the assumption must be tested by experience. If our assumption is not supported, we must frame some other assumption and see whether that works any better until we finally disengage some factor that does affect stature or whatever trait we are studying. These assumptions will depend not on our biometric, but on our biological, knowledge. We shall not waste time in considering assumptions that are quite contrary to all our biological experience, but shall select factors that bear a probable relationship to stature in the light of our present knowledge. But after we have done this, the exact formulation of our problem and the testing of our assumption becomes a matter of biometrics.

CORRELATION

In attempting an exact investigation of variable phenomena, we frequently have to ascertain not only the functional relationship between a certain measurement and its frequency or probability, but also the functional relationships between two or more measurements of a different order. We all know that tall men have in general a longer finger-reach than short men and are, on the whole, heavier than short men. We also know that this dependence is not an absolute one, for sometimes a very short man far outweighs one of superior height and, though rather more rarely, has a longer reach. Can we obtain a measure of the extent of correlation?

It would not be profitable in consideration of our limited time, to exemplify the method of computation employed, for which I should like to refer you to the popular account in Prof. Pearson's "Grammar of Science" (2nd edition, p. 392 ff.). But a single example, taken from the same work, may give you a glimpse of the logical aspect of the questions involved.

According to a popular belief, tall men and women have a tendency to marry short members of the opposite sex, and *vice versa*: in other words, there is a negative correlation between the stature of husbands and of wives.

The conceivable possibilities are as follows: Either the stature of the mate plays no part at all in the choice of a partner: in this case the correlation would be zero. Or, the statures of the mates vary with each other and are absolutely dependent on each other, so that the stature of a husband would be fully determined as soon as we knew that of his mate. This would be positive correlation in the highest possible degree, which may be indicated by the "coefficient of correlation" +1. Or, we might have the condition favored by the popular belief if put in an extreme way. That is to say, the stature of either mate would again be fully determined by that of the other mate, but measurements would vary inversely with each other. This would be complete negative correlation and might be symbolized by -1. Finally, we may cite the only possibility that is actually realized, namely, that the stature of husbands determines that of wives to some extent, but does not *fully* determine it. That is to say, there is *some* correlation, whether negative or positive, but the choice of partners is affected by influences other than stature.

In order to test our popular theory, let us take 1,000 married couples, classing either the wives or the husbands in stature-groups. Now let us ascertain what is the average of the husbands or wives correlated with each of the stature-groups. It is clear that if there were *no* correlation whatever, the "arrays" of husbands or wives with each group of wives or husbands would be the same. No matter whether we took wives of 4 feet 6 or of 6 feet 1, we should invariably get men of the general average height associated with them. If, on the other hand, the inverse relationship supposed by popular belief existed, we should have female six-footers associated with men averaging say 5 feet, while men 6 feet 4 would form the array for women of 4 feet 6. What we actually do find is this: "—if the height of the husband is above the average, then the average height of the array of wives sensibly exceeds the mean height of wives; and if the height of the husband is below the average, the average height of the array of wives is sensibly below the average height of wives. In other words, tall tends to marry tall and short to marry short."* The popular notion of a negative correlation is thus nothing but a popular fallacy. There is a real positive correlation between husbands and wives, a correlation expressed by the coefficient .2872, which indicates a closer similarity between husband and wife than between uncle and niece so far as this trait is concerned.

I am painfully conscious of the fact that my remarks have simply grazed a subject of tremendous scope and importance, but I believe I have sufficiently explained my object in presenting the subject to your notice to render further apology unnecessary. For the, at least provisional, investigation of numerous problems, I hope I have shown you that very simple mathematical processes are sufficient. In your orthodontist work you are dealing, like the anthropologist and zoologist, with variable phenomena, and the application of these simpler modes of statistical treatment would undoubtedly aid you in the formulation and solution of not a few of your problems. Perhaps my foregoing remarks will serve to stimulate at least some among you to make this application.†

*Pearson, *Loc. cit.*, p. 431.

†In addition to the works already quoted, I should like to refer to a recent textbook on the subject, viz., G. Udny Yule's *An Introduction to the Theory of Statistics*, London, Chas. Griffin and Co., 1911.