608. [V.] *The Pillory*, ii. (pp. 160, 204 of this volume).

Prof. Mathews' assertion that this problem is a good example of cases in which pure geometry is a more effective instrument than analytical is beyond dispute, but by his choice of axes he fails to give the analytical devil his due.

Let us use oblique axes BC, BA and identify points by their projections, not by their coordinates. Writing ω for the angle CBA, we have the projections of C as $(a, a\cos\omega)$, of A as $(c\cos\omega, c)$, and if we take those of O as (p, q), one form of the relation between p and q is

$$(p-a)(q-c)=(p-c\cos\omega)(q-a\cos\omega), \text{ ....(i)}$$

which expresses that OA and OC are parallel. Also (u, q) will serve for *any* point U on OK, and (p, v) for *any* point V on OH. The condition for B, U, V to be collinear is

$$uv=pq, \text{ ....(ii)}$$

and the condition for AU, BV to be parallel is

$$(u-c\cos\omega)(v-a\cos\omega)=(p-a)(q-c),$$

which (i) enables us to replace by

$$(u-c\cos\omega)(v-a\cos\omega)=(p-c\cos\omega)(q-a\cos\omega). \text{ ....(iii)}$$

It is obvious: (1) that the conditions (ii), (iii) cannot be identical unless the coefficients of u and v in the latter condition vanish, that is, unless $\cos\omega$ is zero; (2) that if $\cos\omega$ is zero, (iii) does reduce to (ii); (3) that for a value of $\cos\omega$ other than zero the conditions (ii), (iii) are satisfied simultaneously by $u=p$, $v=q$, and therefore also by one other pair of real values of u and v, which *inspection* of the form taken by (iii) when pq/u is written for v shows to be $u=cq/a$, $v=ap/c$. E. H. NEVILLE.

609. [R.] *Relativity Rhymes, with a Mathematical Commentary.*

A. *The Restricted Theory.*

Einstein's is a wonderful notion
That a rod will contract when in motion,
All the clocks will go slow,
And yet no one will know!
So the matter need cause no commotion.

B. *The General Theory, applied to Planetary Motion.*

If the path of a planet you'd trace,
You've Christoffel's weird symbols to face,
For an orbit, you see
Is as straight as can be
On a surface in quintuple space.

Mathematical Commentary.

Lines 2 and 3. This is as stated in the English translation of Einstein's popular book. However, Eddington distinctly states that these effects are only apparent.

Line 4. *This is the essence of the restricted theory.* Stated more fully it is the *assumption* that: It is impossible by any conceivable experiment for an observer to detect his uniform motion with respect to the "ether."

Line 5. Because all experiments (with the exception of that with Fizeau's Water Tube) yield merely negative results.

Line 7. The restricted theory leads to the conclusion that although two observers, moving relatively to each other with any uniform velocity, will disagree about the measurement of space and time, they will agree in their estimate of the velocity of light c and also of the so-called "interval between two events," $\sqrt{(c^2dt^2-dx^2-dy^2-dz^2)}$, where t is time and x, y, z cartesian coordinates. The general theory drops the assumed agreement about the velocity of light, but *assumes* that agreement still exists about an interval ds.

The four coordinates $x_1 x_2 x_3 x_4$ are *collectively* a representation of space and time, but they may be so in any of an infinite number of ways.

ds^2 is of the second degree in dx_1, dx_2, dx_3, dx_4. The complete expression contains ten terms. The coefficients of these terms are *assumed* to satisfy ten relations connecting their differential coefficients with respect to the x's. These relations are spoken of collectively as " the vanishing of the contracted Riemann-Christoffel Tensor."

This is the essence of Einstein's General Theory. It is not quite as artificial as it looks, for it is the simplest set of relations that preserve the same form when subjected to a change of coordinates.

Lines 9 and 10. That is, ds satisfies the condition involving the *four* coordinates analogous to the condition involving *two* coordinates which imply that it is an element of an arc of a geodesic on a surface in three dimensions. A geodesic on a plane is a straight line, while that on a sphere is a great circle. On any (convex) surface a string stretched between two points will lie in a geodesic. The application of the Calculus of Variations will give two differential equations for the ordinary geodesic in three dimensional space, or four for our " quintuple space." These differ slightly from the equations of motion under the inverse square law, and account for the observed anomaly in the motion of the perihelion of Mercury. H. PIAGGIO.

The following, signed " Even Chillier," have been received:

(1) *The German Jeweller's Complaint.*

Einschtein hash a shcandaloush notion
About how the univershe gosh on.
He shaysh I sha'n't know
If my glocksh all go shlow
And my money'sh shrunk up by itsh motion.

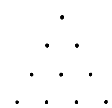
(2) *The Reveller's Charter.*

They tell me that Einstein's been thinking
A stick can't be twirled without shrinking,
And we never could know
If our clocks *were* all slow,
So I needn't say what we've been drinking.

(3) Christoffel will help you to trace
The path of the earth in a brace
Of shakes, for you see
It's only a bee-
Line followed in quintuple space.

610. [I. 25.. b] *A Mathematical Recreation.*

If points are arranged at equal distances from each other as in the adjoining figure the number of these points is $\frac{1}{2}m(m+1)$, m being the number of the points in the base line. It is well known that the ancient Greeks used the term triangular number (ἀριθμὸς τρίγωνος) for such a number, and that the Roman surveyors sometimes used the formula $\frac{1}{2}m(m+1)$ to find the area of an equilateral triangle whose side is m units long.*



If the base line of the triangle determined by the given figure remains fixed while m is increased without limit the formula $\frac{1}{2}m(m+1)$ will continue to express the number of these points in this equilateral triangle. One might therefore be led to assume that the area of this triangle could be represented to any desired degree of approximation by means of the given

* On page 345 of Bubnow's *Gerberti Opera Mathematica*, 1899, this formula is expressed in the following words: "Omnis trigonus aequilaterus unum latus in se multiplicat ipsum latus ad eam multiplicationem addit, horum dimidiam sumit et sic aream suam implet."