



Review

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of Geneva, and was published in the *Revue de Métaphysique et de Morale* for 1912. It fulfils this purpose remarkably well. Modern logicians would hardly assent to the description of it as depicting the latest phase of development of deductive logic, since there is, for example, no notice taken of the work of Frege on such questions as definition without hypotheses and of Russell on the theory of logical types. Indeed, Frege is not mentioned, even in the Bibliography (p. 13)—in which, by the way, might be mentioned Jevons and Venn as well; and, while Russell is mentioned, it is stated (p. 61) that Russell's logic of relations is reducible to Peano's logic—a statement which cannot be admitted, but which has, fortunately, no ill effect on the rest of the book. The author, like Peano, does not distinguish clearly enough propositions and propositional functions (cf., e.g., p. 42). But otherwise the book is accurate and very readable.

PHILIP E. B. JOURDAIN.

Éléments de la Théorie des groupes de Substitutions. By J. A. DE SÉGUIER. Pp. x+238. 1912. (Paris, Gauthier-Villars.)

This book is written as a sequel to the author's former *Éléments de la Théorie des Groupes Abstraits*, published in 1904.

The theory of linear homogeneous substitutions has made much progress of late. To this, however, Dr. Séguier only devotes some ten pages, mainly occupied with the reduction of a linear substitution to canonical form, about forty-five pages giving an account of Dickson's Galois Field theory, and a note on matrices at the end.

The remainder of the book is concerned with what are now often called "permutations" (instead of "substitutions"), including a note of twenty pages devoted to the application of permutations to the Theory of Equations. The theory of permutation-groups is exhaustively treated with great ability, and Dr. Séguier's book must be for some time classical in this part of the subject. He writes as an expert for experts; and it must be confessed that his presentation will be quite unintelligible to anyone who has not already a grasp of group-theory comparable with the author's. Even the veteran in the subject will hardly find the book easy reading. There are but few examples, the style is far too condensed, the text runs on continuously without the phrasing common in mathematical books, and the proofs given are not always the simplest available.

Dr. Séguier has rather a fondness for new nomenclature; many of his terms seem convenient, but his definition of a "Monomial" substitution on p. 6 seems unusual.

Ten pages full of misprints in the *Éléments de la Théorie des Groupes Abstraits* are given. It is to be hoped that the present treatise has fewer inaccuracies; at any rate only one page of corrections is here given.

HAROLD HILTON.

(1) **Higher Algebra.** By CHARLES DAVISON, Sc.D. 6s. 1912. (Cambridge University Press.)

(2) **A New Algebra.** By S. BARNARD, M.A., and J. M. CHILD, B.A., B.Sc. Vol. II. Parts IV.-VI. 4s. 1912. (Macmillan.)

(3) **An Introduction to the Infinitesimal Calculus.** By H. S. CARSLAW, Sc.D. Second edition. 5s. net. 1912. (Longmans, Green & Co.)

(4) **The Calculus for Beginners.** By W. M. BAKER, M.A. 3s. 1912. (G. Bell & Sons.)

(5) **Elements of the Differential and Integral Calculus.** By W. A. GRANVILLE, Ph.D. Revised edition. 10s. 6d. 1911. (Ginn & Co.)

These five books differ widely in scope and still more widely in merit. At the same time they have something in common. They all profess to be "elementary," and they are all concerned, at one point or another, with some of the fundamental notions of analysis.

(1) The publication of this book by the Cambridge Press can only be attributed to reprehensible carelessness on the part of its expert advisers.

A reviewer who receives a book on "Higher Algebra" naturally turns first to the chapters on limits, convergence of series, and the exponential, logarithmic, and binomial series. Dr. Davison's treatment of all these subjects can only be described as hopelessly uneducated. He shows no kind of conception of the

logical relations between different parts of the theory, and his definitions and proofs are not only extraordinarily slovenly, but are full of the grossest blunders. It would be waste of time to justify these remarks by a large number of criticisms of detail, but I may give a few illustrations from Chapter V. Dr. Davison defines a *series* in §64, and a *convergent series* in §65, both wrongly. As he never defines a *limit* at all, and postpones to §180 any sort of explanation of what he means by a limit, this is only natural. The most important theorems in the chapter are contained in §§68, 73, 75, 78, and 79. In these sections Dr. Davison merely repeats the traditional blunders of English text-books of twenty years ago. In the first three cases, for example, he assumes what he professes to prove.

Nor can I say honestly that I think that Dr. Davison is very much happier when he gets away from the difficulties of limits and convergence. He is completely mistaken, for instance, in supposing that, in §28, he has established the possibility of expressing a given rational function as a sum of partial fractions. Chapter IV., on "Complex Quantity," is a morass of confusion. I am quite unable to disentangle what the author regards as definition and what as proof. The proof (§164) that the arithmetic mean is greater than the geometric is unsound, as has been pointed out by a previous writer in the *Gazette*.^{*} Finally, in §170, Dr. Davison proves that "every rational integral equation of the n th degree has n , and only n , roots" without a word of explanation that he is assuming the existence of at least one root.

I have selected these examples more or less at random. The book is, in my opinion, a thoroughly bad one, which ought never to have been published. The fact that it appears under the *aegis* of a University Press leads me to think that I should say so with more emphasis than I should have otherwise considered necessary.

(2) Messrs. Barnard and Child's *New Algebra* seems to me a book of an altogether higher class than any other Algebra for schools that I have seen. That such a book should be produced by two authors with a wide experience of elementary teaching is a most encouraging sign of the times, and one particularly gratifying to the professional mathematicians who have protested against the superstition that accuracy is necessarily repellent and that slipshod half-truths are all that can be interesting or intelligible to beginners.

The authors understand what is meant by a function, or a limit. They can distinguish between a quantity and a number, between a rational and an irrational number, between a rational number and a rational function, or between a limit and a value. Their standard of accuracy is a good deal higher than that of such a well-known book as Chrystal's *Algebra*; they give a satisfactory discussion, for example, of the infinite geometric series. The gulf which separates this book from the ordinary school text-book may easily be imagined.

In spite, or rather perhaps in consequence of this, the book is bright and interesting throughout, and very seldom difficult, and it has the inestimable merit, rare indeed in a text-book, of being written in clear and decent English. The examples are numerous and well selected. In short, I feel that I can hardly recommend it too strongly to teachers of mathematics.

I have noticed very little in the way of error or obscurity. I do not like the explanation (p. 593) of what is meant by saying that " x is large" or " x is small." To say that " x is large" means *nothing*. *Statements containing " x is large,"* on the other hand, may mean something. The phrase is, to use Mr. Russell's language, "incomplete": it is not a constituent of the propositions in which it occurs. I do not suggest that the authors are guilty of any real mistake, but their language seems to me confusing. My only other criticism concerns the last page of all. Here the authors are referring forward to the proof of a theorem to be given in Part VII. Exactly how much they propose to prove there, naturally I cannot say. But the proposition which they quote, regarding the rearrangement of an absolutely convergent series, does not, as ordinarily enunciated and proved, suffice for the application they propose to make of it. Here it is necessary to rearrange an absolutely convergent *double* series. Such a problem may, it is true, be reduced to a problem of a rearrangement of a simple series; but it is a rearrangement of a different and more complicated type than that contemplated in the theorem which they seem to have in mind.

^{*} Mr. Muirhead: *vide Math. Gazette*, vol. ii. p. 283.

(3) I have already reviewed Prof. Carslaw's excellent "Introduction" in the *Gazette* (vol. iii. p. 274). It is pleasant to find that it should have reached a second edition so quickly.

In the original edition the treatment of the logarithmic and exponential functions was open to criticism. That given in this edition is a great improvement. In spite of the authority of Prof. Carslaw and Prof. Love, I am still doubtful as to the advisability of making so much depend upon the limit of $\left(1 + \frac{1}{n}\right)^n$, in an elementary book in which an adequate proof of the existence of the limit is out of the question. But I am prepared to believe that this method is found as clear and satisfactory as any other. I wish, however, that in the Appendix, where Prof. Carslaw now places the "older proofs of the theorems regarding the differentiation of e^x and $\log x$," he had explained more clearly exactly where the difficulties of these "older proofs" lie.

(4) I am unable to commend this book on any grounds, or to understand why it should have any prospect of competing with much better books already on the market. It is not attractively written nor, so far as I can see, particularly "practical," and the author's knowledge of the theory may be estimated in ten minutes by any competent critic. Such a critic I would refer in particular to the first four pages, to the discussion of the differentiation of x^n (pp. 9-10), to the treatment of differentials (p. 68), or that of areas (pp. 88-92).

Mr. Baker confines himself for the most part to the reproduction of other people's mistakes, but occasionally indulges in the expression of his own opinions, as when he defines an "independent variable" as "a quantity to which we may assign any value" (p. 1), or says that the differential coefficient "always exists in functions of every kind" (p. 6).

(5) "In this revised edition of Granville's *Calculus* the latest and best methods are exhibited. . . . Those features of the first edition which contributed so much to its usefulness and popularity have been retained. . . ." The author certainly does not err on the side of bashfulness, and invites a reviewer to judge him by the severest standards. Still, the book, if sometimes a little disappointing, after what the preface has led us to expect, is on the whole quite a good one.

Dr. Granville does not seem to have any very consistent standard as to what may reasonably be regarded as elementary and what not. Thus, on p. 215 he quotes the fundamental theorem that a monotonic sequence tends to a limit or to infinity. The proof he regards as beyond his range. He is thus unable to establish the existence of the exponential limit. He does not even prove that $x^n \rightarrow 0$ if $|x| < 1$; but this is apparently because he has not seen that any proof is needed. It is only natural, in the circumstances, that his treatment of series should be sketchy and inadequate. I may add that Theorem III. on p. 215, which asserts that the condition " $\lim_{n \rightarrow \infty} (S_{n+p} - S_n) = 0$, for all values of the integer p is sufficient to ensure the existence of a limit for S_n ," is untrue (see Bromwich, *Infinite Series*, p. 46). Dr. Granville professes to be quoting Osgood's *Introduction to Infinite Series*, which I have not at hand; but I cannot believe that he is quoting correctly.

Another part of the book which is unsatisfactory, because the foundations have not been properly laid, is that which deals with integration as summation, the Fundamental Theorem of the Integral Calculus, and so on. On the other hand, formal proofs are sometimes given quite as difficult as those of more fundamental theorems that are omitted: I may instance that of the reversibility of two partial differentiations.

There is too much formal "bookwork." The reader is asked to regard the formula

$$\frac{d}{dx} (\arcsin v) = \frac{1}{v\sqrt{1-v^2}} \frac{dv}{dx}$$

as such, to "memorise" it, and to "be able to state the corresponding rule in words"! The examples are numerous, but on the whole dull and lacking in variety.

I could make many other criticisms of detail; but I do not wish to appear ungenerous to a book which, while hardly likely to excite enthusiasm, has solid

merits, and is on the whole clear, readable, and reasonably accurate. To profess that I regard its methods as the "latest and best" would, however, be an exaggeration.

G. H. HARDY.

The Method of Archimedes, recently discovered by Heiberg. A Supplement to **The Works of Archimedes.** 1897. Edited by SIR THOMAS L. HEATH, K.C.B. Pp. 51. 2s. 6d. net. 1912. (Cambridge University Press.)

The story of the discovery of this MS. by Heiberg has been told in *Hermes*, xlii., and in *Bibliotheca*, viii. The particular interest attaching to "The Method" has been most felicitously set forth by the Editor: "Nothing is more characteristic of the classical works of the great geometers of Greece, or more tantalising, than the absence of any indication of the steps by which they worked their way to the discovery of their great theorems. As they have come down to us, these theorems are finished masterpieces, which leave no traces of any rough-hewn stage, no hint of the manner by which they were evolved. We cannot but suppose that the Greeks had some method or methods of analysis hardly less powerful than those of modern analysis; yet, in general, they seem to have taken pains to clear away all traces of the machinery used and all the litter, so to speak, resulting from tentative efforts, before they permitted themselves to publish, in sequence carefully thought out, and with definitely and rigorously scientific proofs, the results obtained." A partial exception is now furnished by the Method, for here we have a sort of lifting of the veil, a glimpse of the interior of Archimedes' workshop as it were. Assuming the principle of the lever, he attacks geometrical problems through the medium of mechanics. For instance, he has already, in the Quadrature of the Parabola, proved the theorem that the area of a parabolic segment ABC is $\frac{4}{3}$ of the triangle ABC . The mechanical discussion of the Method he regards, not as a demonstration, but simply as giving reasons for suspecting that the property is true—"argument has given a sort of indication." It is interesting to find that Archimedes discovered his formula for the volume of the sphere before he found an expression for the area, and that Eudoxus was the first to discover "that the volumes of a pyramid and a cone are one-third of the volumes of a prism and a cylinder respectively which have the same base and equal height," Democritus having asserted it previously but without proof. The admirable introduction by Sir T. L. Heath is a model of editorial craftsmanship.

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