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294. Tangents to Conics

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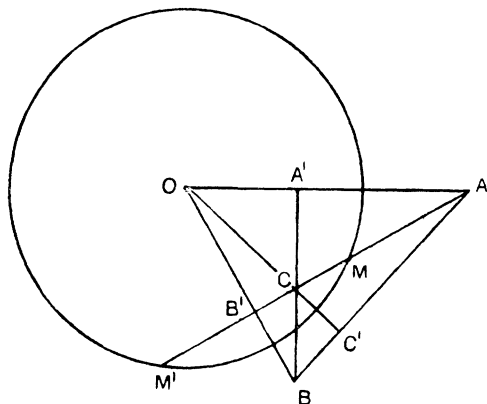
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**293. [K. 10. b.]** *Simple proof of the harmonic property of pole and polar of a circle.*

For definition of pole and polar with reference to a circle we choose the elementary one, that if  $O$  be the centre and  $O, P, P'$  a range of points such that  $OP \cdot OP' = r^2$ , then the line through  $P'$  perpendicular to  $OPP'$  is the polar of  $P$ .

Let  $O$  be the centre of a circle,  $A$  any point, and  $AMM'$  any line through  $A$  cutting the circumference in  $M$  and  $M'$  and the polar of  $A$  in  $C$ .

Complete the triangle  $OAC$  and draw the perpendiculars  $AC', OB', CA'$  from its angular points to the opposite sides, and let them meet in the orthocentre  $B$  of the triangle.



Since  $A, A', B', B$  lie on the circle whose diameter is  $AB$ , we have

$$OB \cdot OB' = OA \cdot OA' = r^2,$$

and since  $A, A', C, C'$  lie on the circle whose diameter is  $AC$ ,

$$\therefore OC \cdot OC' = OA \cdot OA' = r^2.$$

(Hence each side of the triangle  $ABC$  is the polar of the opposite angular point.)

Now

$$\begin{aligned} B'C \cdot B'A &= B'C^2 + B'C \cdot CA \\ &= B'C^2 + OC \cdot CC', \text{ since } O, B', C', A \text{ are concyclic,} \\ &= OC^2 - OB'^2 + OC \cdot CC' \\ &= OC \cdot OC' - OB'^2 \\ &= r^2 - OB'^2 = B'M^2. \end{aligned}$$

Hence  $C$  and  $A$  are harmonic conjugates with reference to  $M, M'$ .

SOLIDUS.

**294. [V. a.]** *Tangents to conics.*

In teaching students who have begun the Calculus I use the following methods:

(1) *Bifocal properties.*

Ellipse.

$$S_1P + S_2P = \text{const.},$$

$$\text{or } r_1 + r_2 = \text{const.},$$

$$\therefore \frac{dr_1}{ds} + \frac{dr_2}{ds} = 0,$$

$$\therefore \cos \phi_1 + \cos \phi_2 = 0,$$

$$\therefore \phi_1 + \phi_2 = 180^\circ,$$

Hyperbola.

$$S_1P - S_2P = \text{const.},$$

$$r_1 - r_2 = \text{const.},$$

$$\frac{dr_1}{ds} - \frac{dr_2}{ds} = 0,$$

$$\cos \phi_1 - \cos \phi_2 = 0,$$

$$\phi_1 = \phi_2,$$

Parabola.

$$SP = MP,$$

$$r = x;$$

$$\frac{dr}{ds} = \frac{dx}{ds};$$

$$\cos \phi = \cos \psi;$$

$$\phi = \psi;$$

$\therefore$  tangent bisects exterior angle between focal radii.      tangent bisects interior angle between focal radii.      tangent bisects angle between  $SP$  and  $\perp$  on directrix.

(2) *Normal property.*

$$SP = eMP; \therefore r = ex; \therefore \frac{dr}{ds} = e \frac{dx}{ds}; \therefore \cos \phi = e \cdot \cos \psi,$$

giving

$$\sin SPG = e \cdot \sin SGP \text{ or } SG = e \cdot SP.$$

The following mechanical proofs are also useful :

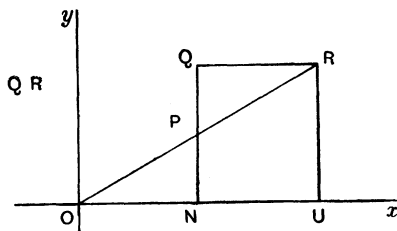
(1) *Bifocal properties.*—Let a particle constrained to move on an ellipse be pulled to the foci by equal forces  $F$ . Then since  $S_1P + SP_2 = \text{const.}$ , if the particle move along the ellipse, the increase in  $S_1P$  is equal to the decrease in  $S_2P$ ; therefore the sum of the works done by the forces is zero; therefore the particle is in equilibrium; therefore the normal is in the direction of the resultant of the pulls  $F$ , and bisects the angle  $S_1PS_2$ . For the hyperbola we must take one of the forces attractive and the other repulsive.

(2) *Normal property.*—Let the conic be placed in a vertical plane with directrix horizontal and above the curve. Let a weight  $eW$  be placed on the curve and attached to a so-called inextensible string passing through the focus and supporting a hanging weight  $eW$ . Then since when the weights move, the increase in  $SP$  is  $e$  times the increase in  $MP$ , the sum of the works done is zero, and the weights are in equilibrium. Moreover, the triangle  $SPG$  is a triangle of forces for the weight  $eW$  at  $P$ , giving at once  $SG = e \cdot SP$ .

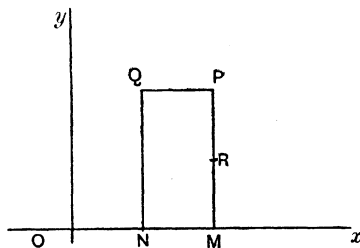
G. H. BRYAN.

295. [M<sup>1</sup>. 8. g.] *Graphic construction of the parabolic and hyperbolic curves  $y = x^n$ .*

The curves for positive or negative integral values of  $n$  may be derived in



succession from the graph of  $y=1$  by a simple construction for the graphs of  $xf(x)$ ,  $1/x \cdot f(x)$  when that of  $f(x)$  is given.



Let  $Q$  be any point on  $y=f(x)$ . Draw a parallel to  $Ox$  to cut the line  $x=1$  in  $R$ . Then the meet  $P$  of  $OR$  with the ordinate  $NQ$  of  $Q$  is a point on the graph of  $y=xf(x)$ .

For  $NP/ON = UR/OU$ ; i.e.  $NP = ONf(ON)$ .