



103. Note on the Logarithmic Series

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power (which, being mechanical, would soon rust unless constantly used). No thought is given to the development of brain power; in fact such a scheme would deaden all power of thinking. The man it would produce might have a fair power of applying a formula, but would not be able to invent one, and any problem that was not of the stereotyped kind he could never attack with the slightest hope of success.

So far we have chiefly looked at the faults of Professor Perry's scheme. Let us briefly consider a few of his suggestions that could be adopted with advantage.

"The use of rough checks in arithmetical work, especially with regard to the position of the decimal point" might be more generally applied than at present. Too much stress cannot be laid on the value of the use of squared paper for mapping statistics, finding areas, solving equations, etc., etc. There is no doubt that its cost has, in the past, prevented it being used in teaching mathematics, but now that it is quite cheap it should be used freely. The practical geometry course should commence sometime before Euclid is begun, and continue side by side with it. The early introduction of trigonometrical ratios and problems in heights and distances would, with squared paper and experimental geometry, add a very real interest to our mathematical teaching.

No attempt has been made in the above to draw up a scheme for this new work (if we may call it so). There are many practical details to be considered.* It would be of great value if masters would, through the *Mathematical Gazette*, let other masters know the result of any experiments they make in improving the teaching of elementary mathematics.

Professor Perry goes on to give an advanced course. This I have left alone, though it is quite possible to modify the work in our middle and lower forms. Examinations compel us at present to teach on somewhat orthodox lines in higher forms, but if the reform is started where feasible, may it not in time become possible even in higher forms to make our methods more modern?

One noteworthy remark Professor Perry makes:—"A good teacher must understand that no examination made by anyone other than himself can be framed which will properly test the result of his teaching." This principle is already observed in the high schools of Germany.

A. W. SIDDOONS.

HARROW.

MATHEMATICAL NOTE.

103. [D. 6. b.] *Note on the Logarithmic Series.*

From the exponential series, if u is positive,

$$e^u > 1 + u,$$

hence

$$u > \log(1 + u),$$

and

$$\log \frac{1}{1-v} < \frac{v}{1-v},$$

where $u = \frac{v}{1-v}$, so that v is positive and < 1 .

Again, $e^v < \frac{1}{1-v}$, from the series, so that $\log \frac{1}{1-v} > v$.

Let $1-v = (1-x)^{\frac{1}{n}}$, so that $0 < x < 1$, and $v = 1 - (1-x)^{\frac{1}{n}}$.

* Mr. Hurst has written to *Nature* (Feb. 14th 1901) and Mr. Eggar to the *School World* (Oct. 1901) on the subject.

Thus $\frac{1}{n} \log \frac{1}{1-x} < \frac{1-(1-x)^{\frac{1}{n}}}{(1-x)^{\frac{1}{n}}}$, but $> 1-(1-x)^{\frac{1}{n}}$,

or $\log \frac{1}{1-x} < n[(1-x)^{-\frac{1}{n}} - 1]$, but $> n[1 - (1-x)^{\frac{1}{n}}]$.

Now, by the Binomial Theorem,

$$n[(1-x)^{-\frac{1}{n}} - 1] = x + \frac{1+\frac{1}{n}}{2}x^2 + \frac{\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)}{2 \cdot 3}x^3 + \dots = A, \text{ say ;}$$

$$n[1 - (1-x)^{\frac{1}{n}}] = x + \frac{1-\frac{1}{n}}{2}x^2 + \frac{\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)}{2 \cdot 3}x^3 + \dots = B, \text{ say.}$$

Let $C = x + \frac{1}{2}x^2 + \frac{1}{3}x^3 + \dots$

Then $A > C > B$,

and $A > \log \frac{1}{1-x} > B$.

But $A/B = (1-x)^{-\frac{1}{n}}$,

which can be made as near to 1 as we please by taking n great enough. Hence it must be true that

$$\log \frac{1}{1-x} = C = x + \frac{1}{2}x^2 + \frac{1}{3}x^3 + \dots$$

This holds when $0 < x < 1$. Writing x^2 for x we have

$$\log \frac{1}{1-x^2} = x^2 + \frac{1}{2}x^4 + \dots$$

Hence, by subtraction,

$$\log(1+x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \dots,$$

which therefore holds when $-1 < x < 1$.*

Again, write $\frac{1}{n}$ for x in the above. Thus

$$\log \frac{n}{n-1} < \frac{1}{n-1} \text{ and } > \frac{1}{n}.$$

Let S_n denote the sum to n terms of the series

$$1 + \left(\frac{1}{2} - \log 2\right) + \left(\frac{1}{3} - \log \frac{3}{2}\right) + \dots + \left(\frac{1}{n} - \log \frac{n}{n-1}\right) + \dots,$$

* The above proof of the logarithmic series holds only when x is real. We see however that when x is positive the sum of the series $A - C$, which consists of positive terms, tends to the limit zero as n increases. This will therefore also hold when x is complex, and thus in any case when $|x| < 1$,

$$\begin{aligned} C &= \lim_{n \rightarrow \infty} n[(1-x)^{-\frac{1}{n}} - 1] \\ &= \text{a value of } \log \frac{1}{1-x}, \end{aligned}$$

but which value is to be taken has to be decided by considerations of continuity.

which is absolutely convergent, since its n^{th} term is numerically less than $\frac{1}{n-1} - \frac{1}{n}$. Then

$$\begin{aligned} S_n &= 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n} - \log n \\ &= 2 \left(\frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \dots + \frac{1}{2n} \right) - \log n, \\ S_{2n} &= 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{2n} - \log 2n. \end{aligned}$$

By subtraction,

$$S_{2n} - S_n = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots + \frac{1}{2n-1} - \frac{1}{2n} - \log 2.$$

But $S_{2n} - S_n$ diminishes without limit as n increases, being in fact numerically $< \frac{1}{2n}$, and therefore

$$\log 2 = \lim_{n \rightarrow \infty} \left(1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots - \frac{1}{2n} \right).$$

Similarly $\log 3 = 1 + \frac{1}{2} - \frac{2}{3} + \frac{1}{4} + \frac{1}{5} - \frac{2}{6} + \frac{1}{7} + \frac{1}{8} - \frac{2}{9} + \dots$,
and so on.

A. C. DIXON.

REVIEWS AND NOTICES.

Die partiellen Differential-Gleichungen der mathematischen Physik.
Nach Riemann's Vorlesungen in vierter Auflage neu bearbeitet von HEINRICH
WEBER. Zweiter Band. Pp. xii., 528 (Braunschweig: Vieweg u. Sohn, 1901).

After an introductory section devoted to the hypergeometric series, Riemann's P-function, and the ordinary linear differential equation of the second order, Professor Weber deals in succession with the conduction of heat, elasticity, electric vibrations, and hydrodynamics. The English reader will here find himself, for the most part, on familiar ground, and may be excused if he reflects with some satisfaction that in this region at any rate his countrymen have done their full share both in discovery and in exposition. The general plan of the work (now complete) has been carried out with consistency and success, and the result cannot fail to prove very serviceable, not as a treatise on mathematical physics (which it does not profess to be), but rather as a guide to the spirit and methods of those parts of modern analysis which are most important in dealing with physical problems. By an attentive study of this treatise the student of physics will obtain a working knowledge of the Fourier analysis, spherical harmonics, Bessel functions, vector calculus, and the theory of the potential; all with direct reference to definite physical problems. Besides this, the variety of subjects dealt with tends to bring out the analogy and correspondence between different branches of applied mathematics which so often proves helpfully suggestive. Finally, it is in a certain sense advantageous to have the fundamental facts and assumptions of mathematical physics laid down in a clear-cut analytical form: this enables us to realise precisely the value of our analysis as an interpretation of Nature, and rightly used, is of service to the experimentalist, by suggesting definite lines of research, and by saving him misdirected and unprofitable labour.

The sections of this volume which will probably be found most interesting by experts are those on electric vibrations and on the motion of a gas. In the first of these Professor Weber discusses Maxwell's equations, and investigates in considerable detail the propagation of current along a wire and the electric oscillations of a spherical conductor surrounded by a dielectric. In the section on the motion of a gas we have an exposition of Riemann's theory of discontinuous motion, which, it will be remembered, has been criticised by Lord Rayleigh as being inconsistent with the principle of energy. Professor Weber's answer to